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Possible Unified Models of Elementary Particles with Two Neutrinos*)

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Possible unified models of elementary particles are discussed assuming the existence of two kinds of neutrino accompanying with electron and muon respectively. The discussions are focused on the Nagoya model which is based on the Sakata model of baryons and mesons and the Gamba-Marshak-Okubo symmetry. In its connection the following assumptions are taken: i) Fundamental particles among baryons and mesons have one-to-one correspondence with leptons or their linear combinations. The correspondence is realized through a kind of "matter". ii) Basic leptons do not transmute each other by the strong interaction between the fundamental baryons.

There are two essentially different types of model. One depends on the existence of two neutrinos which are Dirac particles, and the other is related with two Majorana neutrinos. With regard to the models, the difference between electron and muon and the asymmetry of Λ-pionic decay are also discussed.

§ 1. Introduction

Various models have been proposed successively for understanding of the systematics of elementary particles. The Sakata model[1] is an attempt to understand baryons and mesons in a unifying manner. Baryons, mesons and probably their resonance states are also interpreted as the composite entities consisting of proton, neutron, Λ-particle and their anti-particles. It seems that this model has succeeded in explaining many phenomena concerning the strong interactions with the aid of the full symmetry among the fundamental baryons[2] and its slight deviation.

In the case of leptons, the mass difference between neutrino and electron is so small that the main part of electron mass may be of electromagnetic origin. This fact suggests the possibility of understanding some relations among neu-

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*) A part of the models discussed in this work is also proposed independently by Nagoya group (Z. Maki, M. Nakagawa and S. Sakata, preprint). But we discuss it without regard to repetition, for our standpoint is somewhat different from theirs. Although Taketani proposes another possible scheme (private communication), we do not make any discussions on it here, because we can not yet get definite information about it.

trino, electron and muon through an appropriate mechanism concerning the electric charge. The São Paulo model[3] has been proposed on a line of such an attempt with the assumption of two alternative ways for the electric charge loading on one kind of neutrino.

The symmetry which Gamba, Marshak and Okubo[4] pointed out between leptons (neutrino, electron and muon) and fundamental baryons (proton, neutron and Λ-particle) through the weak interactions suggests one clue to the unified scheme. This unification has been formulated through a kind of "matter" B^+ in the Nagoya model.[5] The advantages of this model are: i) There is a one-to-one correspondence between three leptons and three baryons with the same order of mass values, ii) The weak interaction current of baryons may be predicted from that of leptons, and iii) Since only B^+-matter is responsible for the strong interactions between baryons, the conservation laws in the strong interactions can be understood by the fact that the basic leptons are not affected by this interactions at all.

If one were to succeed in making a consistent theory with the above features, a unified understanding may be possible starting from one neutrino and employing two analogous mechanisms of the electric charge loading and the baryonic charge (or B^+-matter) loading. One such model has been illustrated by Taketani and one of the authors (Y.K.)[6] under the name of the neutrino unified model.

There still remains, however, several difficult problems for further advancement: How can we explain the approximate selection rule $\Delta I = \pm 1/2$ for the weak interaction between baryons and mesons? How can we explain the smallness of the (leptonic) decay coupling constants of the strangeness changing interactions[7] compared with the so-called universal coupling constant in beta decay of nucleon, pion decay, muon decay and muon capture process? How can we get the opposite asymmetry of the Λ-pionic decay compared with one from the V-A theory[8]? How can we explain the appearance of K^0-leptonic decay with $\Delta Q/\Delta S = -1$ besides the one with $\Delta Q/\Delta S = +1$ if it will be definite (S means the strangeness quantum number)[9]? It seems that a definite answer has not yet be given, though many discussions and devices have been made.

There is also an assumption of the existence of only one kind of neutrino in these models. If there really exist two kinds of neutrino, the considerable modification of these models, especially of the Nagoya model and of the São Paulo model will be needed. Two kinds of neutrino may reveal in the way that one connects with electron and the other with muon,*) thus destroying the beautiful correspondence between three leptons and three fundamental baryons and losing the footing of the three conservation laws in the strong interactions.

*) The preliminary experiment at Brookhaven seems to show this feature.

The purpose of the present work is to investigate the possible modifications of the above raised unified models in the cases with two neutrinos. From the standpoint of modification, we retain the basic assumptions similar to ones taken in the Nagoya model. They are:

i) Fundamental baryons are created by attaching a kind of "matter" to all or some of leptons.

ii) The above procedure can predict the weak current of baryons from that of leptons.

iii) Strong interactions between baryons are solely due to this new "matter" and basic leptons are not affected by this interaction, that is, they do not transmute each other through this interaction.

§ 2. The neutrino mixtures and the related models*)

We define two neutrinos as follows: Both of them are assumed to be Dirac particles in this section.

(A) Neutrino e^0

i) This neutrino plays a dominant role in beta-decay process and pion beta-decay process:

$$n \rightarrow p+e^-+e^{0c},$$
$$\pi^- \rightarrow e^-+e^{0c}.$$

Here e^{0c} is anti-particle of e^0.

ii) Then e^0 and e^- have the same lepton number, sum of which is

$$n_e=N(e^-)-N(e^+)+N(e^0)-N(e^{0c}), \tag{2·1}$$

when we assume the conservation of the lepton number.

iii) e^0 is almost completely left-handed, according to the polarization experiments.

iv) The mass of e^0 determined by the upper limit of beta spectrum is less than 200 ev.

(B) Neutrino μ^0

i) This neutrino plays a dominant role in muon capture and its reversed process and pion-muon decay:

$$\mu^-+p \rightleftarrows n+\mu^0,$$
$$\pi^- \rightarrow \mu^-+\mu^{0c}.$$

Here μ^{0c} means anti-particle of μ^0.

*) We use the standard notations in the literature, so the explanation of each notation will be omitted.

ii) Then μ^0 and μ^- have the same lepton number, sum of which is

$$n_\mu = N(\mu^-) - N(\mu^+) + N(\mu^0) - N(\mu^{0c}). \qquad (2\cdot 2)$$

iii) μ^0 is left-handed as e^0.

iv) The mass determination of μ^0 has been quite unprecise and then the upper limit of mass is as high as 5 Mev at present.

There are three possible ways of assigning the lepton number with the above definition. If we take only

$$L_+ = n_e + n_\mu, \quad L_- = n_e - n_\mu,$$

three choices are: (a) (e^-, e^0) and (μ^-, μ^0) have the same lepton number, that is, the weak interaction must satisfy

$$\Delta L_+ = 0.$$

We call this the Lee-Yang choice.[10] (b) (e^-, e^0) and (μ^-, μ^0) have the opposite lepton number, that is, the weak interaction must satisfy

$$\Delta L_- = 0.$$

We call it the Konopinski-Mahmoud choice.[11] (c) (e^-, e^0) and (μ^-, μ^0) have good quantum numbers separately, that is, the weak interaction must satisfy

$$\Delta L_+ = 0 \text{ and } \Delta L_- = 0.$$

When the case (b) is true and both e^0 and μ^0 are massless, one can reduce to three kinds of lepton e^-, μ^+ and ν, all of which are four component Dirac particles, i.e. the neutrino being constructed by

$$e^0 = \frac{1+\gamma_5}{2}\nu, \quad \mu^{0c} = \frac{1-\gamma_5}{2}\nu.$$

With this reduction, however, it can be easily seen that we can not construct any consistent unified model in the sense considered here.*) One profit of the assignment of (b) and (c) is to have the well-known selection rule which can forbid the processes of $\mu^- \to e^- + e^+ + e^-$ and $\mu^- \to e^- + \gamma$.

Then we have only two choices (a) and (c) which seem to be compatible with the consistent unified scheme. But one can see later that the latter case is too strong to get some interesting results.

If we take the Lee-Yang choice, we must make two assumptions. One is that the weak current of leptons must be of the charge type so as to exclude $\mu \to 3e$ process and $\mu \to e + \gamma$ as in

$$j_\lambda = \bar{e}^0 o_\lambda{}^+ e^- + \bar{\mu}^0 o_\lambda{}^+ \mu^-, \quad o_\lambda{}^+ = \gamma_\lambda \frac{1+\gamma_5}{2}. \qquad (2\cdot 3)$$

*) C. Iso also pointed out this possibility and tried to construct one model on it (preprint). His model, however, fails to give any relationship between leptons and baryons except a correspondence $e^- \to n$, $\nu \to p$, $\mu^+ \to \Lambda$ only.

Though the form of the weak current of leptons is not unique at the present experimental informations and its modification of coefficient of each term and the possible additional current will be discussed later, we start with it tentatively. The other is that we take four components particle for each neutrino corresponding to baryon.

When two neutrinos have the same mass, namely zero mass, we also have the following mixture states for them:

$$\nu_1=\frac{1}{\sqrt{1+s^2}}[e^0+s\,\mu^0],\ \nu_2=\frac{1}{\sqrt{1+s^2}}[-se^0+\mu^0];\ 0\leq s\leq 1. \qquad (2\cdot 4)$$

If we take the weak current of leptons, (2.3), these mixture states do not appear in any real leptonic processes, since the charged leptons can not form similar mixtures because of their large mass difference. The effect of the mixture, if it works, comes out when the correspondence between baryons and leptons is made. We therefore investigate what kinds of model arise from this base or how we can determine the rate of mixture s through the unified schemes.

2–A. Model without new baryon

To make a correspondence between four four-components leptons and three fundamental baryons in the Sakata model, one of the leptons should be sacrificed in the course of this connection. It means to assume that B^+-matter can not be attached or bound to one of neutrino or of their mixture states. We express this assumption as

$$p=\langle\nu_1\ B^+\rangle,\ n=\langle e^-\ B^+\rangle,\ \Lambda=\langle\mu^-\ B^+\rangle,\ no=\langle\nu_2\ B^+\rangle, \qquad (2\cdot 5)$$

where the additional assumption that the B^+-matter can be attached only to the one of the mixture states, not to the one of pure states, is made. For example, if B^+-matter can be described by some vector field B_ρ, the above assumption means to have a baryon-forming interaction

$$ib(\partial_\rho B_\sigma{}^* B_\sigma-B_\sigma{}^*\partial_\rho B_\sigma)(\bar{e}^-\gamma_\rho e^-+\bar{\nu}_1\gamma_\rho\nu_1+\bar{\mu}^-\gamma_\rho\mu^-), \qquad (2\cdot 5')$$

where b is its coupling constant. However, B^+-matter is not necessarily considered to be an ordinary field.

Since the above procedure gives a prescription of getting the weak current of baryons which is given by

$$\begin{aligned}\mathcal{J}_\lambda&=j_\lambda+\langle j_\lambda\rangle_B\\&=\bar{e}^0 o_\lambda{}^+ e^-+\bar{\mu}^0 o_\lambda{}^+\mu^-+\frac{1}{\sqrt{1+s^2}}\bar{p}\,o_\lambda{}^+(n+s\Lambda),\end{aligned} \qquad (2\cdot 6)$$

the weak interaction becomes

$$\frac{G}{2}\mathcal{J}_\lambda\,\mathcal{J}_\lambda{}^H, \qquad (2\cdot 7)$$

where $\mathcal{J}_\lambda^H$ means the hermitian conjugate to $\mathcal{J}_\lambda$. We have a new situation for the universality of coupling constants, that is, the squared coupling constants for various decays are generally different and are given by

$$\begin{aligned}
&G_0^2=G^2 && \text{for } \mu\text{-decay,}\\
&G_\beta^2=G_\mu^2=\frac{G^2}{1+s^2} && \text{for } \beta\text{-decay, } \mu\text{-capture and } \pi_{\mu_2}\text{-decay,} \qquad (2\cdot 8)\\
&G_\Lambda^2=\frac{s^2 G^2}{1+s^2} && \text{for } \Lambda\text{-leptonic decay and } K_{\mu_2}\text{-decay,}\\
&G_S^2=\frac{s^2 G^2}{(1+s^2)^2} && \text{for } \Lambda\text{-pionic decay}
\end{aligned}$$

with the relation

$$G_0^2=G_\beta^2+G_\Lambda^2. \qquad (2\cdot 9)$$

Then the rate of neutrino mixture is determined from $G_\Lambda^2/G_\beta^2=s^2$, after the radiative correction is fully taken into consideration. For instance, neglecting the radiative correction, we have $|s|\sim 0.4$ from $G_\Lambda^2/G_\beta^2\sim 1/6$ and the universality between G_0^2 holds approximately.

Before entering into a detail of this model, deeper studies will be needed to clarify the questions: i) Why is there no baryon corresponding to ν_2? (This question seems to be similar to that of the reason why only left-handed neutrino appears in the real nature.) ii) What is the cause of neutrino mixtures? iii) Is there any possibility of having the rules $|\Delta I|=1/2$ and $\Delta Q/\Delta S=\pm 1$, and the correct asymmetry for Λ-pionic decay?

2–B. Model with new baryon

If there are baryons corresponding to both of the neutrino mixture states, we have

$$p=\langle \nu_1 B^+\rangle,\ n=\langle e^- B^+\rangle,\ \Lambda=\langle \mu^- B^+\rangle,\ V=\langle \nu_2 B^+\rangle \qquad (2\cdot 10)$$

in place of the correspondence $(2\cdot 5)$. Here V is a newly introduced baryon, which has positive charge and is probably in iso-singlet state ($I=0$). Unless the masses of p and V are different, we have two baryons which correspond not to the mixture states ν_1 and ν_2, but to the pure states e^0 and μ^0. Then the assumption that B^+-matter can be attached only to the mixture states compels us to distinguish the mechanisms of attachment of B^+-matter to ν_1 and ν_2. For instance, when B^+-matter is described by some vector field, the similar expression to $(2\cdot 5')$ may be given by

$$ib(\partial_\rho B_\sigma^* B_\sigma - B_\sigma^* \partial_\rho B_\sigma)(\bar{e}^-\gamma_\mu e^- + \bar{\mu}^-\gamma_\rho \mu^- + \bar{\nu}_1\gamma_\rho \nu_1 + k\bar{\nu}_2\gamma_\rho \nu_2), \qquad (2\cdot 11)$$
$$k\neq 1,$$

where $k \neq 1$ is essential, otherwise the expression reduces to the attachment of B^+-matter on the pure states e^0 and μ^0.

The fundamental current for strong interaction of baryons which is given by

$$\frac{g}{2} J_\lambda J_\lambda \tag{2·12}$$

is

$$J_\lambda = \bar{p}\gamma_\lambda p + \bar{n}\gamma_\lambda n + \bar{\Lambda}\gamma_\lambda \Lambda + K(k)\,\bar{V}\gamma_\lambda V, \tag{2·12'}$$

where $K(k)$ is a constant arising from k in the expression (2·11) through the modification by B^+-matter. Since $K(k)$ may not be 1 as k, the strong interaction due to the current J_λ conserves the symmetry among baryons except V-particle.

We need a new quantum number T to introduce V-particle into Nishijima-Gell-Mann scheme.[12] Their relation must be replaced by

$$Q = I_3 + \frac{1}{2}(N + S + T). \tag{2·13}$$

The new quantum number T acts only on V-particle with the cooperation of $I_3 = S = 0$. Thus the strong interaction satisfies the four selection rules

$$\Delta I = 0,\ \Delta N = 0,\ \Delta S = 0,\ \Delta T = 0, \tag{2·14}$$

which are equivalent to the conservation of numbers of each baryon

$$\begin{aligned} &\Delta N(p) = \Delta Q - \Delta T = 0,\ \Delta N(\Lambda) = -\Delta S = 0, \\ &\Delta N(n) = \Delta N - \Delta Q + \Delta S = 0,\ \Delta N(V) = \Delta T = 0. \end{aligned} \tag{2·15}$$

They are automatically satisfied in the expression of (2·12). If we recall the correspondence (2·10), the selection rules mean that the leptons (as the bases of baryons) do not transmute each other in the strong interactions.

There still remains other possibility of retaining the original Nishijima-Gell-Mann scheme, which makes V-particle have $S = +1$. But it may need other assumption in harmony with the usual associate productions.

The mass of V-particle is determined by the value of $K(k)$. If $K(k) > 1$, it may be expected that V-particle will be considerably heavier than proton. For instance, when we adopt the conjecture by Nambu and Jona-Lasinio[13] and consider that the baryon masses are given by a non-trivial solution in a sense of the theory of superconductivity, the characteristic equations are given by

$$\begin{aligned} &1 - \frac{m_N^2}{\Lambda^2}\log\left(1 + \frac{\Lambda^2}{m_N^2}\right) = \frac{2\pi^2}{g^2\Lambda^2} && \text{for nucleon,} \\ &1 - \frac{m_V^2}{\Lambda^2}\log\left(1 + \frac{\Lambda^2}{m_V^2}\right) = \frac{1}{K^2(k)}\,\frac{2\pi^2}{g^2\Lambda^2} && \text{for } V\text{-particle,} \end{aligned} \tag{2·16}$$

where Λ is the cutoff parameter. The mass of V-particle m_V is larger than that of nucleon with the same cutoff parameter if $K^2(k)>1$.

The weak current is given similarly as in 2–A,

$$\mathcal{J}_\lambda = j_\lambda + \langle j_\lambda \rangle_B \tag{2·17}$$

$$= \bar{e}^0 o_\lambda^+ e^- + \bar{\mu}^0 o_\lambda^+ \mu^- + \frac{1}{\sqrt{1+s^2}} [\bar{p} o_\lambda^+ (n+s\Lambda) + K^{1/2}(k) \overline{V} o_\lambda^+ (-sn+\Lambda)].$$

The parameter s can be again determined from the ratio of G_Λ/G_β.

The crucial point of this model is the existence of new baryon V. The detailed study should be made as to whether it can give new bosons with four baryons and their anti-baryons and whether it will be observed through the production accompanied by these bosons. In addition to this, the same problems as indicated in 2–A must also be clarified in connection with this model.

2–C. *Modified model with opposite asymmetry for Λ-decay*

According to the recent experiment the asymmetry factor for negative pionic decay of Λ-particle has an opposite sign to that of Marshak-Okubo-Sudarshan's approximation[14] and that of Oneda-Pati-Sakita's one,[18] based on the V–A theory. It seems that there exists some deviation from V–A type in beta-decay too.[15] If it is true, a slight modification of models A and B will be needed.

A simple modification is done by introducing an additional term in the weak current of leptons

$$\Delta j_\lambda = t[\bar{\mu}^0 o_\lambda^- e^- + \bar{e}^0 o_\lambda^- \mu^-], \quad o_\lambda^- = \gamma_\lambda \frac{1-\gamma_5}{2}, \tag{2·18}$$

where t means a small constant. The form of the additional current is chosen so as not to be included in the main current and to exclude the $\mu \rightarrow e+\gamma$ transition even when the intermediate boson scheme is adopted.

The determination of this modification, i.e. of the value t can be done through the informations on muon decay resulting from the following current and their coupling:

$$j_\lambda' = j_\lambda + \Delta j_\lambda = (\bar{e}^0 o_\lambda^+ e^- + \bar{\mu}^0 o_\lambda^+ \mu^-) + t(\bar{\mu}^0 \bar{o}_\lambda e^- + \bar{e}^0 o_\lambda^- \mu^-), \tag{2·19}$$

$$\frac{G}{2} j_\lambda' j_\lambda'^H. \tag{2·20}$$

Four channels of muon decay are allowed with the following relative rates:

$$\mu^- \rightarrow \begin{cases} e^- + \mu^0 + e^{0c} & 1, \\ e^- + e^0 + e^{0c} & t^2, \\ e^- + \mu^0 + \mu^{0c} & t^2, \\ e^- + e^0 + \mu^{0c} & t^4. \end{cases}$$

The asymmetry parameter ξ and the Michel parameter ρ become to be

$$\xi \simeq 1-2t^2,\ \rho \simeq \frac{3}{4}(1-2t^2), \tag{2·21}$$

which give

$$0 \lesssim |t| \lesssim 0.4 \tag{2·22}$$

from the experimental informations of $0.85 \leq \xi \leq 1$ and $0.68 \leq \rho \leq 0.80$.[16] The total weak current after the similar procedure as before is given by

$$\begin{aligned}\mathcal{J}_\lambda' &= j_\lambda' + \langle j_\lambda' \rangle_B \\ &= (\bar{e}^0 o_\lambda^+ e^- + \bar{\mu}^0 o_\lambda^+ \mu^-) + (\bar{\mu}^0 o_\lambda^- e^- + \bar{e}^0 o_\lambda^- \mu^-) \qquad (2·23) \\ &+ \frac{1}{\sqrt{1+s^2}}\Big[(1+st)\bar{p}\gamma_\lambda \frac{1}{2}(1+\frac{1-st}{1+st}\gamma_5)n + (s+t)\bar{p}\gamma_\lambda \frac{1}{2}(1+\frac{s-t}{s+t}\gamma_5)\Lambda \\ &+ (0 \text{ for Model } A, \text{ part of } V\text{-particle for Model } B)\Big].\end{aligned}$$

The negative pionic asymmetry α_Λ^- of Λ-decay depends on the values of s and t through the factor $(s-t)/(s+t)$, for instance α_Λ^- becomes positive for $|s|>|t|$ and negative for $|s|<|t|$ not only in the M–O–S approximation,[14] but also in the O–P–S approximation,[18] the case of $\pm s=t$ (and $st=\pm 1$) being excluded from the evidence of parity violation. The evidence of $\alpha_\Lambda^- \simeq -1$[8] favours that $|s|$ is rather less than $|t|$. As the magnitude of coupling constant of Λ-leptonic decay is proportional to $(s+t)/(1+st)$ compared with one of beta decay, the values of $0 \lesssim |t| \lesssim 0.4$ and $s \simeq 0$ seems to be consistent with the experiment of this decay, and also with the negative pionic decay of Λ-particle in the *O–P–S* approximation.[18]*) The conclusion of this modification is that we need only the additional term to the weak current but not the neutrino mixtures if we accept the opposite asymmetry $\alpha_\Lambda^- \simeq -1$.

2–D. Electron-muon mass difference

It might be natural to assume some relationship between the existence of two neutrinos and the electron-muon difference. Though there may be many probable attempts to this question, a model illustrated here stands on a similar idea to the one of the São Paulo model, which relates electron and muon with neutrino through the attachment of a kind of "matter" E.

Suppose that there are two neutrinos ν_a and ν_b and they become the two charged neutrinos ν_a' and ν_b' after the attachment of E-matter. If the field theoretical description can be used for the procedure, we take the interaction

$$\frac{f^2}{m}(\bar{\nu}_a' \nu_b' + \bar{\nu}_b' \nu_a')E^*E, \tag{2·24}$$

*) We can get the good values for the ratio of $W(\Lambda \to p+\pi^-)$ to $W(\Lambda \to n+\pi^0)$ with correct order for magnitudes, and also $\alpha_\Lambda^- = \alpha_\Lambda^0 \simeq -1$ using the O–P–S approximation.[18] On the contrary, we have $\alpha_\Lambda^- \simeq -1$ and $\alpha_\Lambda^0 \gtrsim 0.3$[17] by the M–O–S one[14] sacrificing the ratio and the magnitude.

where f means its coupling constant and m is the normalized mass which is taken to be the electron mass. This procedure will give also the electromagnetic interaction for ν_a' and ν_b'

$$-i\mathcal{E}(\bar{\nu}_a'\gamma_\lambda\nu_a'+\bar{\nu}_b'\gamma_\lambda\nu_b')A_\lambda-\frac{i\mathcal{E}\delta}{2m}(\bar{\nu}_a'\gamma_\lambda\gamma_\sigma\nu_a'+\bar{\nu}_b'\gamma_\lambda\gamma_\sigma\nu_b')F_{\lambda\sigma}. \tag{2·25}$$

Here $\mathcal{E}$ is the electric charge and δ is about 10^{-5} with $m=m_e$ from the discrepancy between the experimental value and the existing theoretical value for the anomalous magnetic moments of electron and muon.

The total Hamiltonian of system is diagonalized by taking

$$\nu_a'=\frac{1}{\sqrt{2}}(e+\mu),\quad \nu_b'=\frac{1}{\sqrt{2}}(e-\mu), \tag{2·26}$$

and their masses in the lowest approximation are given by

$$m_e=\frac{3}{8\pi}\left(\frac{\mathcal{E}^2}{4\pi}\right)\frac{\delta}{m_e}\Omega^2-\frac{f^2}{m_e}\langle E^*E\rangle_{vac},$$

$$m_\mu=\frac{3}{8\pi}\left(\frac{\mathcal{E}^2}{4\pi}\right)\frac{\delta}{m_e}\Omega^2+\frac{f^2}{m_e}\langle E^*E\rangle_{vac},$$

where Ω is a cutoff parameter. The estimation in which the vacuum expectation value $\langle E^*E\rangle_{vac}$ of the unknown field is replaced with the boson of mass M gives

$$m_e=\frac{\Omega^2}{m_e}\left[\frac{3}{4\pi}\left(\frac{\mathcal{E}^2}{4\pi}\right)\delta-\frac{1}{4\pi}\left(\frac{f^2}{4\pi}\right)\left(1-\frac{M^2}{\Omega^2}\log\left(1+\frac{\Omega^2}{M^2}\right)\right)\right],$$

$$m_\mu=\frac{\Omega^2}{m_e}\left[\frac{3}{4\pi}\left(\frac{\mathcal{E}^2}{4\pi}\right)\delta+\frac{1}{4\pi}\left(\frac{f^2}{4\pi}\right)\left(1-\frac{M^2}{\Omega^2}\log\left(1+\frac{\Omega^2}{M^2}\right)\right)\right]. \tag{2·27}$$

A choice of parameters

$$\Omega^2\sim(50m_N)^2,\quad \frac{f^2}{4\pi}\left(1-\frac{M^2}{\Omega^2}\log\left(1+\frac{\Omega^2}{M^2}\right)\right)\sim1.5\cdot10^{-7} \tag{2·28}$$

will give the mass difference between electron and muon.

From this standpoint, the scheme of the weak current of leptons is as follows: The weak current may be produced from the neutrino current

$$j_\lambda{}^0=\bar{\nu}_a o_\lambda{}^+\nu_a+\bar{\nu}_b o_\lambda{}^+\nu_b. \tag{2·29}$$

After the attachment of E-matter, it becomes the charged current

$$j_\lambda=\bar{\nu}_a o_\lambda{}^+\nu_a'+\bar{\nu}_b o_\lambda{}^+\nu_b'$$

and (2·30)

$$j_\lambda{}^H=\bar{\nu}_a' o_\lambda{}^+\nu_a+\bar{\nu}_b' o_\lambda{}^+\nu_b.$$

Then the diagonalization of the charged neutrinos (2·26) compels their partners to be rearranged

$$\nu_a=\frac{1}{\sqrt{2}}(e^0+\mu^0),\ \nu_b=\frac{1}{\sqrt{2}}(e^0-\mu^0) \tag{2·31}$$

so as to be observed as e^0 and μ^0.

It seems to be very interesting that, if we are allowed to take $s=1$ for the neutrino mixtures after the argument of Gell-Mann,[19] the states of neutrino loaded with the electric charge (or E-matter) and with the B^+-matter are the same with each other, but different from the ones appearing in real processes. The real states e^0 and μ^0 are induced by the mass diagonalization of electron and muon.

§ 3. The Majorana neutrinos and the related model

Even if we start with the four-component Dirac neutrino ν, we have a different interpretation of the conservation law of the lepton number from the one discussed in the previous section. For instance, it follows from the invariance under the transformation

$$\mu\to\exp(i\alpha)\cdot\mu,\ e\to\exp(i\alpha)\cdot e \quad\text{and}\quad \nu\to\exp(i\alpha\gamma_5)\cdot\nu. \tag{3·1}$$

The weak current can be generalized as

$$j_\lambda=(a_1\bar{\nu}+b_1\bar{\nu}^c)\,o_\lambda{}^+e^-+(a_2\bar{\nu}+b_2\bar{\nu}^c)\,o_\lambda{}^+\mu^- \tag{3·2}$$

under the assumption of CP-invariance. The parameters a_i's and b_i's can be taken to

$$a_1=\xi b_1,\ b_2=\eta a_2, \tag{3·3}$$

where ξ and η are arbitrary real constants. Then we have two combinations associated with the electron and the muon respectively,

$$\frac{i}{\sqrt{1+\xi^2}}(\xi\nu+\nu^c),\ \frac{1}{\sqrt{1+\eta^2}}(\nu+\eta\nu^c). \tag{3·4}$$

We add further the condition which excludes the $\mu^-\to e^-+\gamma$ process, by giving the relation

$$\xi=-\eta. \tag{3·5}$$

This is equivalent to that anti-commutation between

$$e^0\equiv\frac{i}{\sqrt{1+\eta^2}}[-\eta\nu+\nu^c],\ \mu^0\equiv\frac{1}{\sqrt{1+\eta^2}}[\nu+\eta\nu^c] \tag{3·6}$$

is settled to be zero, i.e.

$$\{e^0(x),\ \bar{\mu}^0(y)\}=-\{\mu^0(x),\ \bar{e}^0(y)\}=0.$$

If we further require the complete independence between these states, that is,

$$\{e^0(x),\ \mu^0(y)\}=0,$$

we arrive at the combination,

$$e^0=\frac{1}{\sqrt{2}i}(\nu-\nu^c),\ \mu^0=\frac{1}{\sqrt{2}}(\nu+\nu^c) \tag{3·7}$$

without loss of generality, and the weak current of leptons,

$$j_\lambda=\alpha\bar{e}^0 o_\lambda{}^+ e^- +\beta\bar{\mu}^0 o_\lambda{}^+ \mu^-. \tag{3·8}$$

Here α and β are constants.

The states of (3·7) are so-called Majorana particles,[20] since they satisfy

$$e^0=C\bar{e}^0,\ \mu^0=C\bar{\mu}^0. \tag{3·9}$$

The fundamental assumption made in the Nagoya model is that the electron and the muon couple with B^+-matter producing neutron and Λ-particle, while their anti-particles do not produce any baryon with B^+-matter. When the similar reasoning is used, the neutrino ν may produce proton, but anti-neutrino ν^c does not play such a role. This will be assured if we take the interaction for an attachment of B^+-matter on leptons

$$i(\partial_\rho B_\sigma{}^* B_\sigma - B_\sigma{}^* \partial_\rho B_\sigma)(b_1\bar{e}^-\gamma_\rho e^- + b_2\bar{\mu}^-\gamma_\rho\mu + b_3\bar{\nu}\gamma_\rho\nu). \tag{3·10}$$

In other words, two Majorana neutrinos e^0 and μ^0 appear as the independent particles only in the real leptonic processes, and they produce the same proton state when they coupled with B^+-matter.

We express the formation of baryon through the coupling of leptons with B^+-matter (3·10) as

$$p=Z_p^{1/2}(b_3)\langle\nu B^+\rangle, \tag{3·11}$$
$$n=Z_n^{1/2}(b_1)\langle e^- B^+\rangle,\ \Lambda=Z_\Lambda^{1/2}(b_2)\langle\mu^- B^+\rangle,$$

where $Z_p^{1/2}(b_3)$, $Z_n^{1/3}(b_1)$ and $Z_\Lambda^{1/2}(b_2)$ are the normalization constants. The combination $\langle\nu^c B^+\rangle$ does not correspond to any baryon as explained above.

The lepton current (3·5) becomes through this procedure

$$\mathcal{J}_\lambda=j_\lambda+\langle j_\lambda\rangle_B$$
$$=\alpha\bar{e}^0 o_\lambda{}^+ e^- +\beta\bar{\mu}^0 o_\lambda{}^+\mu^- +\frac{\alpha i}{\sqrt{2}}(Z_p Z_n)^{1/2}\bar{p} o_\lambda{}^+ n+\frac{\beta}{\sqrt{2}}(Z_p Z_n)^{1/2}\bar{p} o_\lambda{}^+\Lambda. \tag{3·12}$$

If the weak interaction takes a form

$$\frac{G}{2}\mathcal{J}_\lambda\mathcal{J}_\lambda{}^H, \tag{3·13}$$

the squared coupling constants responsible for various processes are given by

$$G_0{}^2\equiv\alpha^2\beta^2G^2 \qquad \text{for } \mu^-\to e^-+e^0+\mu^0,$$

$$G_\beta{}^2\equiv\frac{\alpha^4}{2}Z_pZ_nG^2 \qquad \text{for } n\to p+e^-+e^0,$$

$$G_\mu{}^2 \equiv \frac{\alpha^2\beta^2}{2} Z_p Z_n G^2 \qquad \text{for } \mu^- + p \to n + \mu^0, \tag{3·14}$$

$$G_\Lambda{}^2 \equiv \frac{\alpha^2\beta^2}{2} Z_p Z_\Lambda G^2 \qquad \text{for } \Lambda \to p + e^- + e^0,$$

$$G_\mu{}^2 G_\Lambda{}^2 / G_\beta{}^2 \equiv \frac{\beta^4}{2} Z_p Z_\Lambda G^2 \qquad \text{for } \Lambda \to p + \mu^- + \mu^0,$$

$$G_\mu{}^2 G_\Lambda{}^2 / G_0{}^2 \equiv \frac{\alpha^2\beta^2}{4} Z_p Z_n Z_\Lambda{}^2 G^2 \qquad \text{for } \Lambda \to p + p^c + n.$$

These processes will determine the relations among constants α, β, Z_p, Z_n and Z_Λ,

$$\alpha^2 = \frac{G_0 G_\mu}{G G_\mu}, \quad \beta^2 = \frac{G_0 G_\mu}{G G_\beta}, \quad Z_p Z_n = 2\frac{G_\mu{}^2}{G_0{}^2}, \quad Z_p Z = 2\frac{G_\Lambda{}^2}{G_0{}^2}. \tag{3·15}$$

For instance, if we take $G_\mu \simeq G_\beta$, $\alpha^2 \simeq \beta^2 \simeq G_0/G$ with the assumption $Z_p \simeq Z_n$, we have

$$\frac{Z_\Lambda}{Z_n} \simeq \frac{G_\Lambda{}^2}{G_\mu{}^2} \simeq \frac{G_\Lambda{}^2}{G_\beta{}^2}. \tag{3·16}$$

So we can presume Z_Λ is smaller than Z_n.

The current in the strong interaction becomes

$$\begin{aligned} J_\lambda &= Z_p(b_3)\bar{p}\gamma_\lambda p + Z_n(b_1)\bar{n}\gamma_\lambda n + Z_\Lambda(b_2)\bar{\Lambda}\gamma_\lambda \Lambda \\ &\simeq Z_n(b_1)\left[\bar{p}\gamma_\lambda p + \bar{n}\gamma_\lambda n + \frac{Z_\Lambda(b_2)}{Z_n(b_1)}\bar{\Lambda}\gamma_\lambda \Lambda\right]. \end{aligned} \tag{3·17}$$

Then the symmetry between nucleon and Λ-particle will be destroyed according to the difference between G_Λ and G_β in the weak interaction, or if the full symmetry among baryons deviates in their strong interaction, then we have the difference between G_Λ and G_β automatically. The deviation of the full symmetry is usually interpreted from the mass difference between nucleon and Λ-particle. Whether a deviation occurs also in the interaction or not must be discussed in connection with this model.

§ 4. Conclusions and discussions

The models proposed in this paper have both advantages and disadvantages. In three models based on two Dirac neutrinos, the ratio of the coupling constant of Λ-leptonic decay to that of nucleon-leptonic processes (beta-decay, muon capture) is related with the rate of mixture state for neutrinos (Models A and B) or with the rate of additional current to the usual current of leptons (Model C). However, we have no grounds on which we predict these rates. Probably the neurino mixtures should be studied in connection with the difference between electron and muon as illustrated in D, Besides the evidence for the smallness of Λ-weak coupling constant, for the opposite asymmetry of Λ-pionic decay from that of V-A theory and for the deviation of beta-decay interaction from V-A type may decide the necessity of these devices as raised in Model C.

It is also impossible in these models to give a decisive answer to the question whether or not the fundamental baryons will be closed with p, n and Λ. The defect in the symmetry between leptons and baryons is considered to be an accidental lack in the course of connection from leptons to baryons in Model A, while a possibility of having a new baryon, V-particle, is discussed in Model B. The choice of the correct one should be done in future experiments.

We also propose a model with two Majorana neutrinos, in which the question about a new baryon does not appear. There arises, however, the stronger restriction on the relation between leptons and baryons through B^+-matter than in the previous models and some accidental effects are needed to give the universality of coupling constants in the weak interaction. In spite of these defects, it has the advantage of relating the coupling constant of Λ-decay directly with a deviation from the full-symmetry in the strong interaction. As the statement which claims the deviation from the full-symmetry just for the mass of Λ-particle is not so persuasive, it seems to be more reasonable to assume a deviation in the strong interaction itself.

The explanations with the above models do not cover all of the present experimental results concerning the weak interactions. One of the remaining problems is the $|\Delta I|=1/2$ rule. Though it seems necessary to introduce a special interaction between baryons caused by the neutral current, it may not mean the failure of models, since there is left some possibility to explain the rule approximately through some dynamical treatment as was done by Oneda et al.

Another ploblem concerns the rule of $\Delta Q/\Delta S=+1$. There is no possibility of having the process with $\Delta Q/\Delta S=-1$ within the above models because of the insufficient degrees of freedom. If one of the models is right and also the experiment which shows the existence of decay process with $\Delta Q/\Delta S=-1$ is definite, the model should be improved to include the six-fermion interaction in some way. There may be some connection between the existence of two neutrinos and the occurrence of six-fermion interaction. For instance, if we consider three body correlation among leptons, it may cause a new neutrino mode besides the original neutrino. This will give two independent physical neutrino states. This type of model, however, will generally cause a leptonic interaction of baryons with $\Delta Q/\Delta S=1/2$ together with $\Delta Q/\Delta S=-1$. Then it is a rather difficult problem to introduce such six-fermion interaction when there is no weak interaction with $|\Delta S|>1$.

All of our models and discussions are based on a line of the foregoing unified models which aim at the unification of baryons with leptons. There are, of course, many possible models if we give up this basic standpoint. But it seems to us that they must become more complicated than ours.

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Note added in proof: After the completion of this work, we saw the paper by Danby et al. which confirmed the existence of two kinds of neutrinos: G. Danby, J-M. Gaillard, K. Goulianos, L. M. Lederman, N. Mistry, M. Schwartz, and J. Steinberger, Phys Rev. Letters **9** (1962), 36.