POSSIBLE VANISHING OF STRONG INTERACTION CROSS-SECTIONS AT INFINITE ENERGIES

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ABSTRACT

An algebra of currents interpretation of the Regge pole model is found to be in excellent agreement with experiment over a wide range of energy. The fit requires a Pomeranchuk intercept $\alpha_0^s \approx 0.925$, implying vanishing asymptotic total cross-sections.

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It has been proposed [1] that high energy scattering can be described in terms of two nonets of algebraic operators coupled to Regge trajectories. In this way, one obtained very satisfactory relations among total cross-sections, some of which had been previously obtained in the composite model *), in particular the asymptotic relation

$$\frac{\sigma_{\pi P}}{\sigma_{PP}} = \frac{2}{3} \tag{1}$$

A careful investigation, however, of the energy dependence of cross-sections within the model of I shows that a good fit to the entire experimental data cannot be obtained - without altering the algebraic structure - with the assumption that all total cross-sections approach non-zero asymptotic limits. Moreover, an excellent fit is obtained if one takes the value of 0.925 ± 0.008 for the intercept of the Pomeranchuk trajectory. This implies the vanishing of all total cross-sections **) as

$$\sigma \sim s^{-(0.075 \pm 0.008)} , \tag{2}$$

where s is the square of the c.m. energy.

Although this conclusion runs counter to presently held theoretical beliefs, the existing data, including cosmic ray results ***), are not sufficient to distinguish between a constant cross-section and one which vanishes as slowly as suggested by (2).

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*) See I for references.

**) Note that the value of the exponent approximates the Bond factor 0.07, a fact which may have important though obscure implications [2].

***) Cosmic ray data on proton-nucleus total cross-sections exist up to very high energy [3] indicating constant "geometric" cross-sections. G. Cocconi pointed out that this result does not imply a constancy of the P-nucleus cross-section, which can well decrease by a factor ~ 2 in 10^6 GeV, as suggested by Eq. (2). The reason is that the nucleus would still behave as an essentially opaque sphere and the cross-section would remain $\approx \pi R^2$. In fact a slowly decreasing P-nucleon cross-section could even correspond, in a large energy range, to an increasing P-nucleus cross-sections due to the shrinking of the diffraction peak.

This vanishing of the cross-sections has of course far reaching implications of a general nature, in particular, as far as dispersion relations and sum rules are concerned.

Furthermore, the fact that the Pomeranchuk trajectory has an intercept different from unity also implies that for all elastic amplitudes T,

$$\left(\frac{\mathrm{Re}\,T}{\mathrm{Im}\,T}\right)_{t=0} \xrightarrow[s\to\infty]{} -\tan\left[\frac{1-\alpha_0^s(0)}{2}\pi\right] , \qquad (3)$$

where $\alpha_0^s(0)$ is the intercept of the Pomeranchuk trajectory. For $\alpha_0^s(0) = 0.925$, this ratio becomes -0.118.

We begin by writing the results of I for the averaged total cross-sections. From the Table given in I, we have

$$S_N = 6\,t_0^s + 3\,t_8^s , \qquad (4.a)$$

$$S_\pi = 4\,t_0^s + 2\,t_8^s , \qquad (4.b)$$

$$S_K = 4\,t_0^s - t_8^s , \qquad (4.c)$$

where

$$S_N = \frac{1}{4}\left(PP + PN + \bar{P}P + \bar{P}N\right) , \qquad (5.a)$$

$$S_\pi = \frac{1}{2}\left(\pi^+P + \pi^-P\right) , \qquad (5.b)$$

$$S_K = \frac{1}{4}\left(K^+P + K^-P + K^+N + K^-N\right) , \qquad (5.c)$$

and t_0^s and t_8^s are the reduced absorptive parts for the unitary singlet and octet eighth-component even signature trajectories. AB represents the total cross-section for scattering of hadrons A and B. If we assume t_0^s to be a constant (which corresponds to $\alpha_0^s(0) = 1$), Eqs. (4) would imply that S_K approaches its asymptotic limit from below, while S_N and S_π approach their limits from above. Experimentally, all cross-sections are known to be decreasing functions of the energy, so that Eqs. (4) would not fit the data.

In order to obtain a better result, one may introduce the following modifications :

a) it was pointed out in I that an admixture of D type octet coupling in the baryon vertex strengths should be expected to occur, and is in fact necessary to achieve a fit to the data at any given energy. This in itself, however, does not supply a good fit at all energies if we assume constant t_0^s, since it multiplies t_8^s in both S_π and S_K by the same factor. We cannot therefore get both S_π and S_K decreasing properly with a D/F correction alone.

b) a mixing between t_0^s and t_8^s analogous to $\phi-\omega$ mixing. This represents SU(3) breaking and introduces differences between the asymptotic σ_{KP} and $\sigma_{\pi P}$.

Using both a) and b), formulae (4.a)-(4.c) become :

$$S_N = (\alpha\sqrt{6} + \beta\lambda\sqrt{3})^2 t_0^s + (\beta\sqrt{6} - \alpha\lambda\sqrt{3})^2 t_8^s \quad , \tag{6.a}$$

$$S_\pi = 2(\alpha\sqrt{2} + \beta)(\alpha\sqrt{2} + \beta\lambda) t_0^s + 2(\beta\sqrt{2} - \alpha)(\beta\sqrt{2} - \alpha\lambda) t_8^s \quad , \tag{6.b}$$

$$S_K = (2\alpha\sqrt{2} - \beta)(\alpha\sqrt{2} + \beta\lambda) t_0^s + (2\beta\sqrt{2} + \alpha)(\beta\sqrt{2} - \alpha\lambda) t_8^s \quad , \tag{6.c}$$

where λ describes the F/D mixing,

$$\lambda = F - \frac{1}{3}D \qquad (7)$$

and α and β represent the t_0^s, t_8^s mixture, with mixing angle ϕ :

$$\alpha = \cos\phi \quad , \quad \beta = \sin\phi \qquad (8)$$

We tried to fit Eqs. (6) to the experimental data of Galbraith et al. [4]. No acceptable fit was achieved for constant t_0^s; we observe that trying to obtain the correct decreasing behaviour for S_N, S_π and S_K destroys the fit as far as the ratio of S_π/S_K at all energies is concerned.

The only available alternative appears to consist in abandoning the idea that t_0^s represents the contribution of a single pole with intercept $\alpha_0^s(0) = 1$. One such possibility would be to adjoin to the Pomeranchuk pole an additional SU(3) singlet scalar trajectory. This has been known to give a good fit to the data [5], but it would imply some extension of our algebraic approach. In fact, at least one of the two singlets, or a linear combination thereof would be outside the algebra of $[U(6)\otimes U(6)]_\beta$. Considering that there is no natural prescription for the matrix structure of the additional operator, we would lose the asymptotic prediction of Eq. (1) which appears experimentally validated [*].

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*) If a second even signature singlet were necessary, it would seem more natural to propose a complete doubling of the whole family of trajectories, the new family being also coupled to members of a $U(6)\otimes U(6)$ algebra. In this way one could preserve the prediction of Eq. (1). It is simpler - and seems to us more natural - to assume a single family of trajectories.

We wish, however, to remain in the framework of the model of I, and we therefore consider the possibility that t_0^s corresponds to a single pole with intercept $\alpha_0^s(0) < 1$; for momentum transfer $t = 0$, we write

$$t_0^s(\nu) = t_0^s(1)\ \nu^{\alpha_0^s(0)-1} , \qquad (9)$$

$$t_8^s(\nu) = t_8^s(1)\ \nu^{\alpha_8^s(0)-1} , \qquad (10)$$

with $\nu = s - m_A^2 - m_B^2$.

We substitute Eqs. (9) and (10) into Eqs. (6.a)-(6.c) and get an excellent fit to the Galbraith et al. data, with the following parameters

$$\begin{aligned} &t_0^s(1) = 7.43 \pm 0.56 , \quad t_8^s(1) = 2.08 \pm 0.26 , \\ &\alpha_0^s(0) = 0.925 \pm 0.008 , \quad \alpha_8^s(0) = 0.76 \pm 0.04 , \\ &\tan\phi = -0.066 \pm 0.048 , \quad \lambda = 2.44 \pm 0.09 . \end{aligned} \qquad (11)$$

The χ^2 for the fit is 5.8, to be compared with an expected value of 14. The above results remain essentially unchanged when we take $\phi = 0$.

From Eqs. (6) and the above values of the parameters, we find for $s \to \infty$

$$\frac{S_N}{S_\pi} = 1.39 , \qquad \frac{S_\pi}{S_K} = 0.93 . \qquad (12)$$

If there were no mixing, one would obtain for S_N/S_π the value given by Eq. (1), and S_π/S_K would reduce to unity. Note, however, that substituting (11) into Eqs. (6), one finds $S_\pi > S_K$ up to momenta of 10^5 GeV/c, after which they cross, and the asymptotic limit of Eq. (12) is approached at extremely high energies.

The Figure displays the comparison between the Galbraith et al.[4] data for S_N, S_π, S_K and our fit.

We note that at high energy, the t_0^s contribution still dominates each cross-section, so that in a $(\log\sigma, \log\nu)$ plot, they approach a straight line asymptotically. A cross-section which approaches this asymptote from below may appear to be constant throughout a large energy region. This explains the behaviour of K^+ - nucleon and P - nucleon cross-sections above 5 GeV/c. Moreover, all elastic amplitudes in the forward direction go asymptotically to the form

$$T(\nu, 0) \propto -\frac{1 + e^{-i\pi\alpha_0^s(0)}}{\sin \pi\alpha_0^s(0)} \nu^{\alpha_0^s(0)} \qquad (13)$$

so that the ratio of real to imaginary part approaches the negative limit of Eq. (3).

We have extended our fit to include individual cross-sections at all energies. This is done by using the total cross-sections table of I, with the above-mentioned modifications for the t_0^s and t_8^s. The four contributions t_3^s, t_3^v, t_0^v, t_8^v are proportional to differences of cross-sections, so that they cannot be determined accurately. For the $I = 0$ poles, we find

$$t_0^v(\nu) = 0.65\left(\frac{\nu}{24}\right)^{-0.65},$$

$$t_8^v(\nu) = 0.65\left(\frac{\nu}{24}\right)^{-0.56}.$$

No $\phi - \omega$ mixing was required by the data, and we also assume no D/F admixture in the vector contributions. The combination $\frac{1}{2}(PP+PN)$, e.g., is expressed as :

$$\frac{1}{2}(PP + PN) = S_N - 6t_0^v - 3t_8^v .$$

This is also compared to experimental points in the Figure, showing excellent agreement. In going to individual cross-sections, one needs also t_3^S and t_3^V. These can be determined again from meson-nucleon cross-sections. Their intercepts are found to be consistent with those determined from the analysis of $\pi^- P \rightarrow \eta N$ [6] and $\pi^- P \rightarrow \pi^o N$ [7]. Using the results to compute individual nucleon-nucleon cross-sections gives again a fair agreement - albeit a very unenlightening one, given the large experimental errors in the differences PP-PN and $\bar{P}P-\bar{P}N$.

Finally, we used all of our numbers to compute Re T/Im T for π^+P, π^-P, PP and $\bar{P}P$; we list a few values in the following Table :

P_L (GeV/c)	PP	$\bar{P}P$	π^+P	π^-P
8	-0.434	-0.119	-0.244	-0.125
12	-0.385	-0.133	-0.231	-0.129
14	-0.372	-0.135	-0.227	-0.129
16	-0.362	-0.140	-0.226	-0.132
18	-0.349	-0.140	-0.219	-0.130

Ratio of real to imaginary parts of elastic amplitudes in the forward direction

The vanishing of all total cross-sections as $\nu \rightarrow \infty$ is seen to be strongly suggested by our algebraic interpretation of the Regge pole model; it is interesting that existing experimental data do not contradict the relinquishing of the constant asymptotic cross-section hypothesis. We thus feel that it would be extremely important to obtain accurate measurements of cross-sections at higher energies than now available; such as could be obtained by the use of an intersecting storage ring (ISR), or cosmic ray experiments on hydrogen. An accurate measurement of the PP cross-sections in the 30 to 70 GeV/c range – available at the new Serpukhov accelerator – would also be relevant since our fit predicts a drop of ≈ 3 mb in this range (see the Figure).

Although the conclusion that the Pomeranchuk has an intercept $\alpha_0^s < 1$ may seem at first sight shocking, we find it rather pleasing : $\alpha = 1$ is known to be an upper limit to acceptable intercepts, and therefore, in a certain sense, a point of high singularity, such that a pole with $\alpha(0) = 1$ would be entirely set apart from other poles with $\alpha(0) < 1$. Since we assume the s_0 trajectory to be a member of a nonet, we find it natural that its properties should be quantitatively – not qualitatively – different from those of other poles of even signature *).

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*) After this work had been completed, a paper by Goldberger and Jones [Phys.Rev.Letters 17, 105 (1966)] came to our attention in which it was shown that the existence of a pole with $\alpha(0)=1$ may lead to an inconsistency between the requirements of the Mandelstam representation and analyticity in the ℓ-plane.

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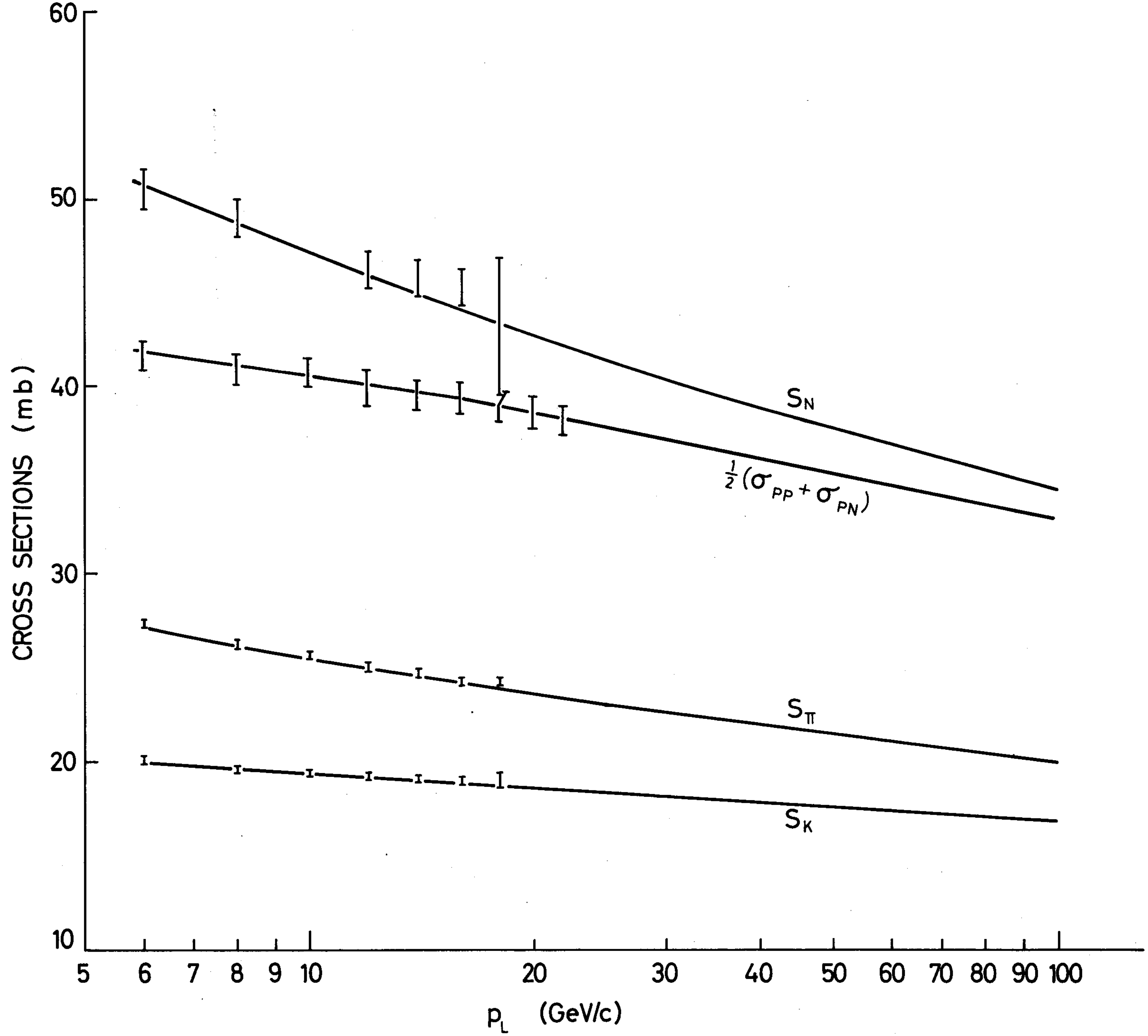
FIGURE CAPTION

Comparison between the data of Galbraith et al. [4] for S_N, S_π, S_K and $\frac{1}{2}[PP+PN]$ and our fit.


60
50
40
30
20
10
CROSS SECTIONS (mb)
5
6
7
8
9
10
20
30
40
50
60
70
80
90
100
p_L (GeV/c)
S_N
$\frac{1}{2}(\sigma_{PP} + \sigma_{PN})$
S_π
S_K

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