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## Post-erosion mechanical response of internally unstable soil of varying size and flow regime

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2 **and flow regime**

3

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33

34 **Keywords**

35 Internal erosion; Post-erosion behavior; Flow regime; Internal stability; Suffusion

36

37 **Abstract**

38

39 One of the leading causes of dam failure is internal erosion. The impact of erosion of non-  
40 plastic fine particles, known as suffusion, on the soil structure and strength has been studied  
41 experimentally. However, influences including sample size have not been thoroughly  
42 investigated. Internally unstable gap-graded cohesionless soil samples with various sizes were  
43 investigated using an erosion-triaxial apparatus. Samples were subjected to downward inflows  
44 of different seepage velocities. The results indicated that the potential for clogging increased  
45 with an increase in specimen length, leading to less fine particle erosion. Internal erosion  
46 changed the mechanical soil behaviour even after the loss of fines equal to five percent of the  
47 overall sample volume. Eroded specimens with similar intergranular void ratios showed similar  
48 undrained post-erosion behaviour. However, the magnitude of the post-erosion initial  
49 undrained peak shear strength is a function of coarse particle interlocking, residual fine content  
50 and equivalent intergranular contact index. It was also found that the steady state line remained  
51 unchanged after erosion of fine particles and the mobilized friction angle at the steady state  
52 line is independent of the residual fine content.

53

## 54 **Introduction**

55

56 Internal erosion is one of the major causes of hydraulic structure failure (ICOLD 2015).  
57 According to ICOLD (2015), internal erosion is divided into four main mechanisms:  
58 concentrated leaks, backward erosion, contact erosion and suffusion. The focus of this research  
59 is suffusion, the migration of non-plastic fine particles from within a matrix of coarser particles  
60 due to a seepage flow within an embankment dam or its foundation. It normally occurs in gap  
61 or broadly graded internally unstable soils where fine particles are not fully involved in stress  
62 transfer.

63

64 Among the first experimental works studying of the impact of erosion of non-plastic fine  
65 particles on the post-erosion mechanical behaviour of soils, Chang and Zhang (2012) and Chen  
66 et al. (2016) investigated the drained shear strength of eroded specimens. They found a decline  
67 in the drained shear strength and alteration of soil behaviour from dilative to contractive  
68 following erosion of fines. It was believed that an increase in the void ratio due to the removal  
69 of fine particles shifted the soil to a looser state. Post-erosion undrained behaviour of granular  
70 mixtures subjected to suffusion was studied by Xiao and Shwiyhat (2012) and Ke and  
71 Takahashi (2014). It was found that the undrained shear strength increased after erosion of the  
72 fine particles. Xiao and Shwiyhat (2012) stated that this might have occurred due to a loss of  
73 saturation during the erosion stage. However, Ke and Takahashi (2014) believed that the higher  
74 undrained strength of the eroded specimen may have been attributed to formation of local  
75 reinforcement in the soil fabric due to particle rearrangement. Post-erosion drained behaviour  
76 of the same soil mixture was also studied by Ke and Takahashi (2015). Results indicated that  
77 depending on the initial fine content, the post-erosion drained shear strength may stay  
78 unchanged or decrease.

79 Despite these attempts to explain the post-erosion mechanical behaviour of internally unstable  
80 soils, no specific conclusion can be drawn on the impact of erosion on soil mechanical  
81 behaviour. Moreover, it appears that the impact of specimen size on the erosion of fine particles  
82 and post-erosion mechanical behaviour has been overlooked. From the few available studies  
83 on different types of internal erosion (e.g. Sellmeijer 1988; Li 2008; Seghir et al. 2014; Zhong  
84 et al. 2019), it is evident that the critical hydraulic gradients or hydraulic conductivity may be  
85 affected by dimensions of soil specimens although the exact impact was unclear. For instance,  
86 while Seghir et al. (2014) believed that internal erosion was independent of specimen length,  
87 Sellmeijer (1988) and Li (2008) suggested that the required hydraulic gradient for initiation of  
88 erosion had an inverse relation with the seepage length. More recently, Zhong et al. (2019)  
89 showed the critical hydraulic gradient decreases with the size of the specimen increasing.

90  
91 This paper discusses results of a series of undrained triaxial tests on eroded specimens with  
92 different dimensions subjected to downward seepage inflows while comparisons are made with  
93 undrained behaviour of non-eroded specimens.

## 94 95 **Testing Program**

96  
97 Gap-graded soil specimens with an initial fine content ( $FC_i$ ) of 25 per cent were prepared to  
98 investigate the impact of fine particle removal on the post-erosion behaviour of an internally  
99 unstable soil. An initial fine content of 25 per cent was chosen as it is believed that contribution  
100 of fine particles in the soil stress matrix is uncertain when the fine content is in the density-  
101 dependent transitional zone (i.e. between 25 and 35 per cent), with fines being active, semi-  
102 active or inactive (Shire et al. 2014). The particle size distribution and physical properties of  
103 the soil mixture are shown in **Fig. 1**. The mineralogy of particles is predominantly quartz and

104 Mehdizadeh et al. (2017a) showed that angularity of particles in coarse fraction is higher than  
105 that of fine fraction which may enhance the erosion resistance as stated by Marot et al. (2012).  
106 On the other hand, erosion of the more rounded fines will lead to an overall increase in the  
107 average angularity of particles and therefore an increase in post-erosion interlocking. The  
108 internal instability of this gradation was examined based on methods developed by Kezdi  
109 (1969), Kenney and Lau (1986), Burenkova (1993) and Indraratna et al. (2011) showing that  
110 the soil mixture is internally unstable.

111

112 <<Insert **Fig 1** about here>>

113

114 The soil specimens with diameters of 50, 75 and 100 mm were compacted layer by layer using  
115 the moist tamping technique (Mehdizadeh et al 2017a), adjusting the thickness of soil layers to  
116 ensure that the soil layers in samples with different height still receive almost the same  
117 compaction energy. To achieve a high level of saturation, carbon dioxide was injected at the  
118 bottom of the specimen using a flow controller at the low rate of 1 L/min for two hours while  
119 the cell pressure was maintained constant. The cell and back-pressure were gradually increased  
120 at a rate of 1 kPa/min to 400 and 390 kPa respectively to reach the fully saturated (B-value of  
121 0.91). All specimens were consolidated to 150 kPa after full saturation to remove the footprint  
122 of sample preparation (Frost and Park 2003) and then a downward seepage flow was applied  
123 to the top of the specimen for two hours. The eroded soil mass was collected in a collection  
124 tank, allowing the fines content to be calculated throughout the test. The collection system was  
125 designed to collect and measure the eroded particles continuously and also to record their  
126 weights, to discharge water from the triaxial chamber and to keep the bottom of the sample  
127 saturated. The collection tank was a double wall tank with a measuring container submerged  
128 under a stable water level inside a cell (inner cell) and connected to a submersible load cell

129 (with 10g resolution). The water level at the top of this cell was kept constant by discharging  
130 the water from the inner cell into the main chamber via drainage holes in the wall of the inner  
131 cell. The air above the water was pressurized to the back-pressure applied to the specimen  
132 during the test. Details of the modified apparatus, testing procedure and repeatability of tests  
133 result were discussed thoroughly by Mehdizadeh et al. (2017a). Testing was performed in four  
134 stages as shown in **Table 1**. It is currently a matter of discussion as to whether seepage velocity  
135 or hydraulic gradient should be used to predict the onset of suffusion (Vogt et al. 2015).  
136 Richards and Reddy (2008) believed that assuming Darcy's law is applicable during the  
137 seepage, an increase in hydraulic gradient leads to a decrease in hydraulic conductivity at a  
138 constant flow. Considering this effect, they suggested considering only critical hydraulic  
139 gradient for cohesionless soils may not be correct. Ke and Takahashi, (2014) stated that there  
140 is no method to accurately control and measure the head loss in tubes, valves and fittings during  
141 a laboratory erosion testing which is necessary if constant hydraulic gradient method is  
142 employed. Sibille et al. (2015) took both seepage velocity and hydraulic gradient into account  
143 to characterize the hydraulic load by computing the power expended by the seepage flow. The  
144 great influence of hydraulic loading path on the suffusion development was reported by  
145 Rochim et al (2017). Considering these limitations, it was decided to keep the seepage velocity  
146 constant instead of maintaining the hydraulic gradient during the erosion phase. Here, a flow  
147 controller was used to maintain a constant seepage velocity in preference to a constant  
148 hydraulic gradient. The inflow increased gradually to the designated velocity and then was kept  
149 constant for two hours (Fig. 2).

150

151 The initial hydraulic conductivity of the soil samples was around 0.075 cm/s based on the  
152 equation proposed by Carrier (2003) which is in the range of coarse sand as expected. Three  
153 seepage velocities of 0.086 cm/s (52 mm/min), 0.153 cm/s (92 mm/min) and 0.347 cm/s (208

154 mm/min) were applied to the top of the samples via a perforated top cap filled with glass beads  
155 to ensure that the flow was applied as uniformly as possible. Flow velocities and applied  
156 hydraulic gradients (Table 1) were comparable with previous studies (Marot et al. 2010; Chang  
157 and Zhang 2012; Ke and Takahashi 2015).

158

159 <<Insert **Table 1** about here>>

160

161 <<Insert **Fig 2** about here>>

162

## 163 **Tests result and Discussion**

164

### 165 *Impact of Erosion on the Fine Content*

166

167 According to Kenney et al. (1985) and Indraratna et al. (2007), the controlling constriction size  
168 of the tested mixture in this study is in the range 0.28 to 0.3 mm. This means that the largest  
169 fine particles (in range of 0.075 - 0.3 mm) should just be able to move through the sample  
170 under seepage forces. The normalized residual fine content ( $FC_c/FC_i$ , where  $FC_c$  is the current  
171 residual fine content and  $FC_i$  is the initial fine content) with time for test series one to three is  
172 shown in **Fig. 3**. For the first series of tests, three specimens with diameters of 50, 75 and 100  
173 mm were prepared and subjected to a seepage velocity of 52 mm/min for 120 minutes. As each  
174 erosion test progressed, it was noted that the rate of erosion for all three specimens decreased,  
175 and erosion was seen to stop by the end of the test. The rate of erosion and maximum percentage  
176 of the eroded particles were similar for the 75 and 100 mm diameter specimens (~ 45% of fines  
177 eroded). However, the 50 mm diameter specimen showed significantly larger erosion (66%)  
178 despite having similar sample preparation and test procedures, suggesting that the difference is



179 not due to soil fabric. Two possible reasons for the difference are suggested. The first scenario  
180 is attributed to the higher possibility of clogging inside the larger specimens. Following work  
181 by Kenney et al. (1985) that in a soil containing a range of constriction sizes, the chance of a  
182 fine particle encountering a smaller constriction increases with the length of a flow path. **Fig.**  
183 **4** shows the effective Constriction Size Distribution (CSD) at different heights in the sample  
184 according to the method suggested by Kenney et al (1985) with the CSD calculated according  
185 to Locke et al. (2001). An assumption of  $D_{50}$  as the layer spacing is used, as suggested by Wu  
186 et al., 2012 and Taylor et al., 2019 ( $D_{50}$  is the particle diameter in which 50 per cent by weight  
187 of coarser particles passed). It is evident from **Fig. 4** that as sample length increases the  
188 effective CSD becomes finer which increases the chance of clogging. This is in agreement with  
189 the finding in this research that a higher proportion of particles was eroded from smaller  
190 samples. **Fig. 4** also shows that there is not much difference in CSD for fine particles 115.6  
191 mm and 198.9 mm away from the base (exit point). This confirms the experimental observation  
192 that erosion of fine particles in larger samples were similar in terms of trend and magnitude.  
193 The second reason is related to inadequate seepage forces to carry the eroded particles along  
194 the larger specimens, which can lead to particle sedimentation in the downstream before  
195 washing particles out completely. However, clogging is believed to be the dominant cause.

196

197 <<Insert **Fig 3** about here>>

198

199 <<Insert **Fig 4** about here>>

200

201 In the second test series, a 75 mm diameter specimen (E-D75-V92-T120) was eroded under a  
202 higher seepage velocity (92 mm/min) for two hours (Mehdizadeh et al. 2017b). The residual  
203 fine content ( $FC_f$ ) of 10.1 per cent was very close to the residual fine content of the 50 mm

204 diameter specimen E-D50-V52-T120 in the first series of testing. The specimen with a larger  
205 diameter but higher seepage velocity (E-D75-V92-T120) initially had a lower rate of erosion,  
206 but this increased after around 15 minutes. Both specimens showed similar trends 30 minutes  
207 after the seepage initiation until the end of the erosion phase. This meant that initial clogging  
208 was more severe inside the larger specimen but after a delay, the higher seepage force allowed  
209 this to be overcome. This is interesting as theoretically it is expected to get more eroded  
210 particles under a higher seepage velocity when other influential factors such as fabric, initial  
211 condition and sample preparation are kept the same.

212

213 In the third test series, the maximum applicable seepage velocity of 208 mm/min was applied  
214 to the 50 mm diameter specimen for 120 minutes to erode the maximum possible proportion  
215 of fine particles, leading to 6.9 per cent residual fine content. Fig. 3 shows that regardless of  
216 the specimen dimensions and seepage velocity, the rate of erosion of fine particles greatly  
217 reduced despite the fact that the residual fine contents were different, and it can therefore be  
218 assumed that the majority of the inactive and semi-active particles were removed.

219

220 It is believed that inactive fine particles (sitting loose in the voids with minor participation in  
221 the force chains) are the most vulnerable to suffusion. A percentage of these free particles are  
222 washed out of the specimen, while a number of them are clogged inside the specimen. By an  
223 increase in the seepage velocity, semi-active fine particles (providing lateral support or  
224 secondary support for the coarse grains) become susceptible to suffusion if the applied  
225 hydraulic stress is high enough to overcome the current effective stress on these particles.  
226 Moreover, some of the particles clogged under a lower seepage velocity are also become prone  
227 to erosion under higher hydraulic forces. This is a plausible scenario that explains the behaviour  
228 of specimens with the same dimensions but subjected to different seepage velocities (E-D75-

229 V52-T120 and E-D75-V92-T120). **Fig. 3** also shows that even under the maximum seepage  
230 velocity (test E-D50-V208-T120), it was not possible to erode all fine particles. This could be  
231 because of full contribution of the remaining fine particles (active particles) in the soil skeleton.  
232 **Fig. 5** schematically displays erosion progress and particle rearrangement. **Fig. 5 (a)** shows the  
233 initial condition of the fine and coarse particles and the stress transferring mechanism. Free  
234 fine particles were washed first due to the seepage flow (**Fig. 5 (b)**), semi-active fines started  
235 to migrate where locally higher hydraulic gradients were raised due to clogging and released  
236 new free fine particles (**Fig. 5 (c)**). Metastable force chains were formed after the erosion of  
237 the semi-active fine particles (Mehdizadeh and Disfani 2018) which led to local coarse particle  
238 rearrangements and vertical deformations (**Fig. 5 (d)**).

239

240 <<Insert **Fig 5** about here>>

241

### 242 *Impact of Erosion on the Global Void Ratio and Particle Size Distribution*

243

244 The initial global void ratio was calculated using the soil phase relationship. The post-erosion  
245 global void ratio was estimated from the total volume of the eroded sample (calculated using  
246 deformations from the photogrammetry technique), the mass of eroded particles and the  
247 specific gravity. The erosion of fine particles increased the pre-erosion global void ratio of 0.48  
248 to post-erosion values of 0.66, 0.6 and 0.61 for specimens E-D50-V52-T120, E-D75-V52-T120  
249 and E-D100-V52-T120, respectively. The smallest soil specimen (E-D50-V52-T120) showed  
250 a higher post-erosion global void ratio although it was subjected to the same seepage  
251 experienced by the two other specimens. This was due to removal of more fine particles for  
252 specimen E-D50-V52-T120 during erosion.

253

254 Pre and post-erosion particle size distributions (PEPSD) of eroded specimens (E-D50-V52-  
255 T120, E-D75-V52-T120 and E-D100-V52-T120) are shown in **Fig. 6**. Specimens with 50 mm  
256 diameter were divided into two parts and those with 75 mm and 100 mm diameters were  
257 divided into three parts for PEPSD analysis. Top, middle and bottom PEPSDs were similar for  
258 E-D75-V52-T120 and E-D100-V52-T120, which also had similar global void ratios and  
259 residual fine contents. Regardless of sample dimension, the fine content decreased along the  
260 height of the specimens and the top region of the soil specimens lost more fine particles under  
261 downward seepage which was found to be in agreement with result of Ke and Takahashi (2012)  
262 and Zhong et al. (2018).

263

264 <<Insert **Fig 6** about here>>

265

266 ***Impact of Erosion on Vertical Deformation***

267

268 Vertical strains during the erosion phase were measured at five-minute intervals using the  
269 photogrammetry technique (Mehdizadeh et al. 2017a) and are shown in **Fig. 7**. All specimens  
270 experienced vertical strain during erosion phase; a sign of erosion of semi-active fines and local  
271 breakage of force chains. Interestingly, all specimens experienced a rapid increase in vertical  
272 strain at the beginning of seepage when the inflow velocity was very low. Almost all specimens  
273 showed step-wise changes in the vertical strain. Although the erosion rate and residual fine  
274 content were similar for the 75 and 100 mm specimens (E-D75-V52-T120 and E-D100-V52-  
275 T120), the vertical strain was larger in the 100 mm sample. The experiments here and the  
276 analyses based on the method suggested by Kenney et al. (1985) both suggest that the larger  
277 the sample is, the higher is the chance of clogging, leading to fewer eroded particles (i.e. those  
278 transported by seepage) being washed out of the sample. As fines are more likely to meet a

279 small constriction as the flow paths are longer. However, the pattern of vertical deformation  
280 mainly depends on the erosion of semi-active fine particles and the consequent buckling of the  
281 force chains. As force chain buckling is caused by local transport of semi-active fines, rather  
282 than them being washed out of the sample, there is not necessarily a relationship between fines  
283 eroded and vertical strain.

284

285 <<Insert **Fig 7** about here>>

286

### 287 ***Impact of Erosion on Post-erosion Undrained Behaviour***

288

289 The undrained stress-strain relationship up to 15 percent strain, induced excess pore pressure  
290 and stress path of all tested specimens are presented and compared in Fig. 8. To draw a better  
291 conclusion, additional erosion tests results for the same initial PSD presented by Mehdizadeh  
292 et al. (2017b) (E-D75-V52-T30 and E-D75-V92-T30) on 75 mm diameter samples are also  
293 included. Comparing tests result indicates that the post-erosion undrained behaviour of soil  
294 specimens regardless of seepage velocity and duration can be divided into three main groups.  
295 Specimens E-D75-V52-T30, E-D75-V52-T120 and E-D75-V92-T30 (Fig. 8 (a)) showed  
296 similar stress-strain relationship (similar initial peak and ultimate shear strength) and induced  
297 excess pore pressures during undrained shearing with different residual fine contents and global  
298 void ratios but with the same post-erosion intergranular void ratios ( $e_g = \frac{e + FC}{1 - FC}$ ,  $e$  is the global  
299 void ratio and  $FC$  is the fine content (Mitchell (1993))). The intergranular void ratio was found  
300 to be approximately 0.9 for these specimens after erosion. Specimens E-D75-V92-T120, E-  
301 D50-V52-T120 and E-D50-V208-T120 (Fig. 8 (b)) had similar post-erosion intergranular void  
302 ratios of 0.84-0.86 and showed similar behaviour (similar initial peak and ultimate shear  
303 strength). The residual fine content was recorded as 10.1, 10.2 and 6.9 per cent, respectively.

304 The erosion of just an additional 3.3 per cent fine content was observed for specimen E-D50-  
305 V208-T120 although it was subjected to a more powerful seepage. This suggests that erosion  
306 of the additional fine particles in specimen E-D50-V208-T120 had negligible impact on the  
307 post-erosion mechanical behaviour. The undrained behaviour of non-eroded specimens (NE-  
308 D75 and NE-D100) was shown in Fig. 8 (d). It is evident that while the hardening behaviour is  
309 more dominant in non-eroded specimens compared to all of the eroded specimens especially  
310 in higher strains, their initial undrained peak shear strength is lower than eroded specimens  
311 regardless of the erosion progress. The excess pore pressure is induced much quicker in the  
312 non-eroded specimens and also dropped much faster. The only exception was specimen E-  
313 D100-V52-T120 which showed a similar behaviour to specimens E-D75-V52-T30, E-D75-  
314 V52-T120 and E-D75-V92-T30 (Fig. 8 (a)) at small strains up to 5% then showed a hardening  
315 behaviour like specimens NE-D75 and NE-D100 at large strains. The residual fine content was  
316 similar for specimens E-D75-V52-T120 and E-D100-V52-T120, which showed similar trends  
317 in small strains (less than five per cent). However, the shear strength increased more rapidly in  
318 specimen E-D100-V52-T120 at medium and large strains. This shows that similar post-erosion  
319 particle size distribution does not necessarily lead to the same mechanical behaviour, due to  
320 sample inhomogeneity and differences in fabric. It is worth noting that the intergranular void  
321 ratio suggested by Mitchell (1993) does not consider the level of contribution of fine particles  
322 in the soil structure (their erodability potential). Therefore, it is difficult to explain how erosion  
323 of fine particles contributes to observed reductions in the intergranular void ratio. However, it  
324 seems rearrangement of coarse particles due to loss of semi-active fine particles led to vertical  
325 settlement and decrease in intergranular void ratio.

326 It can be understood from Fig. 8 that with a decrease in the fine content, the softening behaviour  
327 became more dominant and the hardening behaviour in large strains decreased. However, all  
328 non-eroded and eroded specimens (regardless of sample dimension and rate of erosion) showed

329 an “elbow” in the stress path, which signifies a transition from limited strain softening to a  
 330 quasi-steady state (initial contraction followed by dilation) (Pitman et al. 1994). In other words,  
 331 all specimens (eroded and non-eroded) were initially located between the Steady State Line  
 332 (SSL) and Isotropic Compression Line (ICL) in  $e - \log p'$  space as shown by Thevanayagam  
 333 and Mohan (2000). The mobilized friction angle at initial peak shear stress ( $\phi'_{pS}$ ), at the start  
 334 of dilation (phase transformation,  $\phi'_{pT}$ ) and at steady state ( $\phi'_{SS}$ ) have been determined for all  
 335 tested specimens using the axisymmetric principal stress ratio ( $M$ ) ( $M$  is the ratio between  $q$   
 336 and  $p'$  (**Table 2**). It was found that the mobilized friction angle at the steady state was higher  
 337 than the mobilized friction angle at initial peak shear stress and at phase transformation state  
 338 thanks to an increase of dilatancy in large strains regardless of the specimen status in terms of  
 339 erosion progress and size. It is also evident from **Fig. 8 (d, e and f)** and **Table 2** that all  
 340 specimens eventually ended up on the same Steady State Line (SSL) as suggested by Yang et  
 341 al. (2006a) for sand-silt mixtures with various non-plastic fine contents.

342

343 <<Insert **Fig 8** about here>>

344

345 <<Insert **Table 2** about here>>

346

347 To consider the contribution of active (which can also be considered as non-erodible) fine  
 348 particles in the soil structure in terms of active grain contacts, Thevanayagam et al. (2002)  
 349 proposed a density variable called equivalent intergranular contact index  $((e_c)_{eq} = \frac{e + (1-b)FC}{1 - (1-b)FC})$   
 350 when  $FC < FC_{th}$ , where the critical fine content ( $FC_{th}$ ) is a fine content above which the coarse  
 351 particles are no longer in full contact with each other and  $b$  is the fraction of active fines.  $b=0$   
 352 means all fines act exactly like voids and when  $b=1$ , they are not distinguishable from host  
 353 sand particles and they actively participate in supporting the soil skeleton. However, the

354 concept of parameter  $b$  is controversial. Some researchers (e.g. Thevanayagam et al. 2002; Ni  
355 et al. 2004; Yang et al. 2006a,b) believe  $b$  is constant for all mixtures with fine contents less  
356 than the critical fine content ( $FC_{th}$ ) and it only depends on grain size disparity ratio ( $R_d =$   
357  $D_{50}/d_{50}$ ) or particle size ratio ( $\chi = D_{10}/d_{50}$ ), where  $D_{10}$  particle size of pure sand at 10% finer,  
358  $D_{50}$  mean particle size of coarse fraction and  $d_{50}$  mean particle size of fine fraction. This means  
359 for a specific mixture and regardless of the fine content always percentage of active fine  
360 particles is constant. On the contrary, some other researchers (e.g. Rahman et al. 2008; Nguyen  
361 et al. 2017) take into account the impact of fine content. However, Chang and Deng (2019)  
362 showed that  $b$  only depends on  $D_{50}/d_{50}$  and effective stress and is independent of fines content  
363 for mixtures with small grain size disparity ratio.

364

365 The parameter  $b$  for the mixture tested in this research can be estimated from the test result on  
366 specimen E-D50-V208-T120. Here, it is assumed that all erodible particles were washed out in  
367 the sample with 50 mm diameter under the seepage velocity of 208 mm/min (the maximum  
368 applicable in the lab) under a two-hour seepage, when it can be seen that erosion rate  
369 approached zero (Fig. 3). The applied seepage was unable to erode all fine particles and 6.9  
370 percent was left unwashed at the end of the erosion. By this assumption, 6.9% residual fine  
371 content were all non-erodible fine particles and were fully active in the force chains. Therefore,  
372 the  $b$  parameter is the ratio of active fine particles (6.9%) to total fine particles (25%), i.e.  $b$   
373 = 0.28. This is in relatively good agreement with calculated  $b$  in the range of 0.25-0.4 for the  
374 mixtures with the same  $D_{50}/d_{50}$  using semi-empirical expressions proposed by Rahman et al.  
375 (2011) and Chang and Deng (2019).  $b$  can be estimated for all eroded specimens using their  
376 residual fine contents and an assumption that 6.9% of fines are initially active (**Fig. 9 (a)**).  
377 Although the amount of active fine particles can be assumed to be constant in all eroded and  
378 non-eroded specimens, the parameter  $b$ , which is a proportion of the total fine content



379 contributing to load transfer, cannot be constant. Using the calculated  $b$  and residual fine  
380 content, the equivalent intergranular contact index  $(e_c)_{eq}$  can be calculated for each specimen  
381 at the beginning of the undrained shearing. Variation of peak shear stress ratio  $(\eta_{PS} = q/p')$ ,  
382 where  $q$  and  $p'$  are deviator and mean effective stresses, respectively) with  $(e_c)_{eq}$  for all  
383 tested specimens is shown in **Fig. 9 (b)**. The  $\eta_{PS}$  increased initially with a decrease in  $(e_c)_{eq}$  (a  
384 decrease of the residual fine content down to 15.1 per cent) and then decreased with further  
385 reduction in equivalent intergranular contact index (decrease in the residual fine content down  
386 to 6.9 per cent). Mehdizadeh et al. (2017a) showed that for the soil mixture used in this research  
387 coarse particles were more angular than fine particles. Therefore, the initial improvement in  
388 the undrained shear strength (initial peak shear stress,  $\varphi'_{PS}$  in **Table 2**) could be due to a better  
389 interlock between the coarse particles but without the loss of semi-active fine particles which  
390 helps to prevent collapse at small strains. However, further erosion resulted in rearrangement  
391 of the coarse particles and loss of semi-active fine particles leading to formation of a metastable  
392 structure and a higher tendency to contractive behaviour. It is worth noting that the initial peak  
393 shear strength and in particular equivalent intergranular contact index vary over a small range.  
394 More experiments are required to validate this finding and establish a relationship between  
395  $(e_c)_{eq}$  and fine particles with different levels of contribution in the soil structure, fabric changes  
396 and re-deposition of fine particles due to clogging.

397

398 <<Insert **Fig 9** about here>>

399

## 400 **Conclusion**

401

402 The influence of internal erosion on soil structure and post-erosion mechanical behaviour of an  
403 internally unstable gap-graded soil of different specimen size and flow velocity was examined  
404 through laboratory investigation. The following points were the most important findings of this  
405 research:

406

- 407 - A step-wise trend was observed in the vertical strains during the erosion phase, which  
408 is believed to be due to erosion of semi-active fines that provided lateral support for the  
409 force chains.
- 410 - Under the same seepage velocity and duration, the erosion of fine particles decreased  
411 with an increase in length of specimen, due to higher potential of clogging for eroded  
412 particles that travel a longer distance.
- 413 - Strain softening behaviour becomes more dominant with a decrease in the residual fine  
414 content due to internal erosion.
- 415 - The experiments suggested that regardless of dimension of the soil specimens, inflow  
416 velocity and seepage duration, specimens with the same post-erosion intergranular void  
417 ratios showed similar undrained behaviour. However, more erosion-triaxial tests on  
418 samples with different fabrics need to be conducted to draw a clearer conclusion.
- 419 - It was found from the experiments in this study that erosion of fine particles up to 15%  
420 of the overall sample mass improved the initial undrained peak shear strength. This  
421 positive impact later degenerated when a greater percentage of fine particles were lost.  
422 However, more validation is required.
- 423 - It seems the initial undrained shear strength is affected by equivalent intergranular  
424 contact index. However, more experiments are required to validate this finding.
- 425 - Suffusion was found to have minimal impact on the steady state line of the mixture  
426 studied in this experiment and it seems to be independent of the residual fine content.

427 - The mixture in this study had coarse and fine particles with different angularities.  
428 Erosion of fine particles may change the global interlocking of particles and post-  
429 erosion behaviour. Impact of particle shape on erosion and post-erosion behaviour  
430 needs further investigation.

431

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- 549

550 **Figure Captions:**

551 Fig 1. Particle size distribution and Physical properties of tested soil sample

552 Fig 2. Variation of inflow velocity with time

553 Fig 3. Variation of normalized residual fine content with time

554 Fig 4. Effective Constriction Size Distribution (CSD) at different heights in the sample  
555 according to the method suggested by Kenney et al (1985)

556 Fig 5. Progress of internal erosion (a) Initial condition, (b) Erosion of the free fines, (c) Erosion  
557 of the semi-active fines and providing new free fines and (d) Particles rearrangement and  
558 vertical deformation with residual active fines

559 Fig 6. Particle size distribution plots for post-erosion specimens at different regions for (a) E-  
560 D50-V52-T120, (b) E-D75-V52-T120 and (c) E-D100-V52-T120

561 Fig 7. Vertical strains during erosion phase

562 Fig 8. Impact of internal erosion on undrained stress-strain relationship, induced excess pore  
563 pressure and stress path of eroded and non-eroded specimens during undrained shearing

564 Fig 9. (a) Variation of the parameter  $b$  with residual fine content and (b) Variation of peak  
565 shear stress ratio with equivalent intergranular contact index



566 **Table Captions:**

567 Table 1. Erosion-triaxial testing program

568 Table 2. Mobilized friction Angle

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**Table 1. Erosion-triaxial testing program**

Test Series	Sample Label	Sample Diameter (mm)	Sample Height (mm)	Seepage Velocity (mm/min)	Hydraulic Gradient <sup>a</sup>	Erosion Duration (min)	CIU <sup>b</sup> Test
1	E-D50-V52-T120 <sup>c</sup>	50	115 <sup>d</sup>	52	1.15	120	Yes
	E-D75-V52-T120 <sup>c</sup>	75	150	52	1.15	120	Yes
	E-D100-V52-T120	100	200	52	1.15	120	Yes
2	E-D75-V92-T120 <sup>c</sup>	75	150	92	2.04	120	Yes
3	E-D50-V208-T120	50	115	208	4.6	120	Yes
4	NE-75 <sup>f</sup>	75	150	-	-	-	Yes
	NE-100	100	200	-	-	-	Yes

<sup>a</sup>: assuming Darcy's Law and having initial hydraulic conductivity of 0.075 cm/s

<sup>b</sup>: Isotropically Consolidated Undrained Triaxial Test

<sup>c</sup>: E-D50-V52-T120 means E (Eroded)-D (Diameter (mm))-V (Seepage velocity (mm/min))-T (Seepage duration (min))

<sup>d</sup>: This sample had a height to diameter ratio of 2.3 which was higher than other specimens. It was attributed to the height of the mould.

<sup>e</sup>: Reported by Mehdizadeh et al. (2017b)

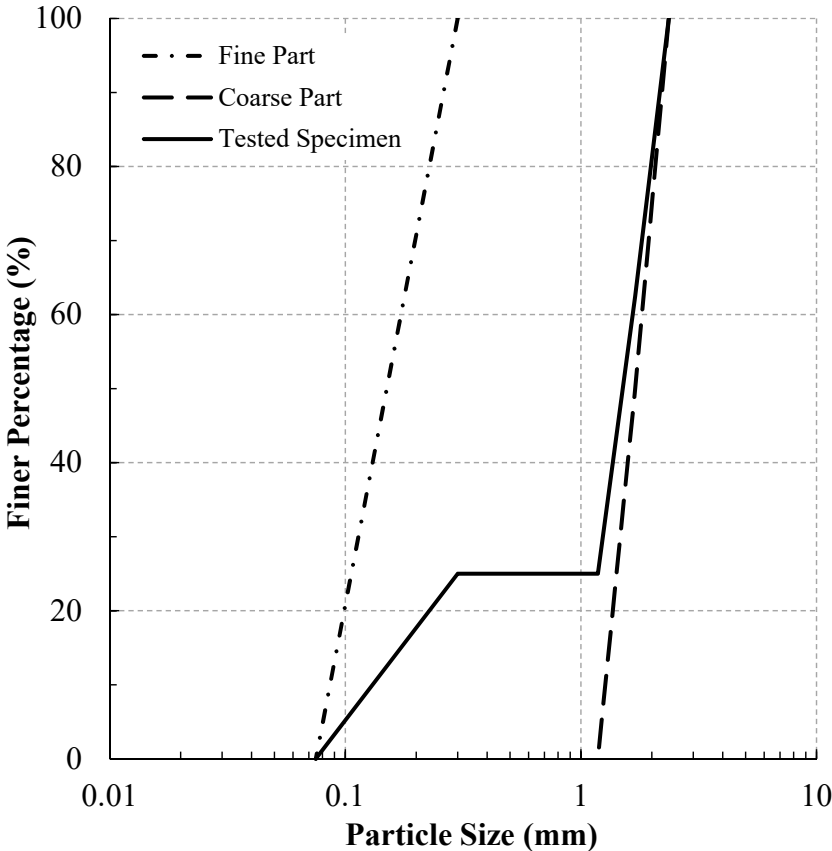
<sup>f</sup>: NE-50 means NE (Non-Eroded)-D (Diameter) and reported by Mehdizadeh et al. (2017a)

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**Table 2. Mobilized friction Angle**

Specimen	$FC_f$ (%)	$\phi'_{PS}$ (°)	$\phi'_{PT}$ (°)	$\phi'_{SS}$ (°)
E-D75-V52-T30	19.8	27	30	32
E-D75-V52-T120	15.9	27	29	32
E-D75-V92-T30	18.6	27	30	32
E-D100-V52-T120	15.1	30	30	32
E-D50-V52-T120	10.2	25	29	32
E-D50-V208-T120	6.9	24	29	32
E-D75-V92-T120	10.1	27	30	32
NE-75	25	26	30	32
NE-100	25	25	27	32

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Physical property	Value	Physical property	Value
Maximum void ratio, $e_{max}$	0.67	$D^*$ , (mm) <sup>a</sup>	0.28 - 0.3
Minimum void ratio, $e_{min}$	0.36	Initial Void Ratio, $e_i$	0.48
Initial Moisture Content, MC (%)	6	Relative Density, $D_r$ (%)	60
Uniformity coefficient, $C_u$	12.14	$(\frac{D'_{15}}{d'_{85}})_{max}$ <sup>b</sup>	5.2
Gap ratio, $G_r$	3.93	$(H/F)_{min}$ <sup>c</sup>	0.08

<sup>a</sup>: Controlling constriction size (Kenney et al. 1985 and Indraratna et al. 2007)  
<sup>b</sup>:  $D'_{15}$  is the particle diameter in which 15 per cent by weight of coarser particles passed and  $d'_{85}$  is the particle diameter in which 85 per cent by weight of fine particles passed. Soils with  $(\frac{D'_{15}}{d'_{85}})_{max} > 4$  are considered internally unstable (Keszdi 1969).  
<sup>c</sup>: F is the passed fraction by weight finer than d, and H is the weight fraction between d and 4d (Kenney and Lau 1985 and 1986).

Figure1. Particle size distribution and physical properties of tested soil sample

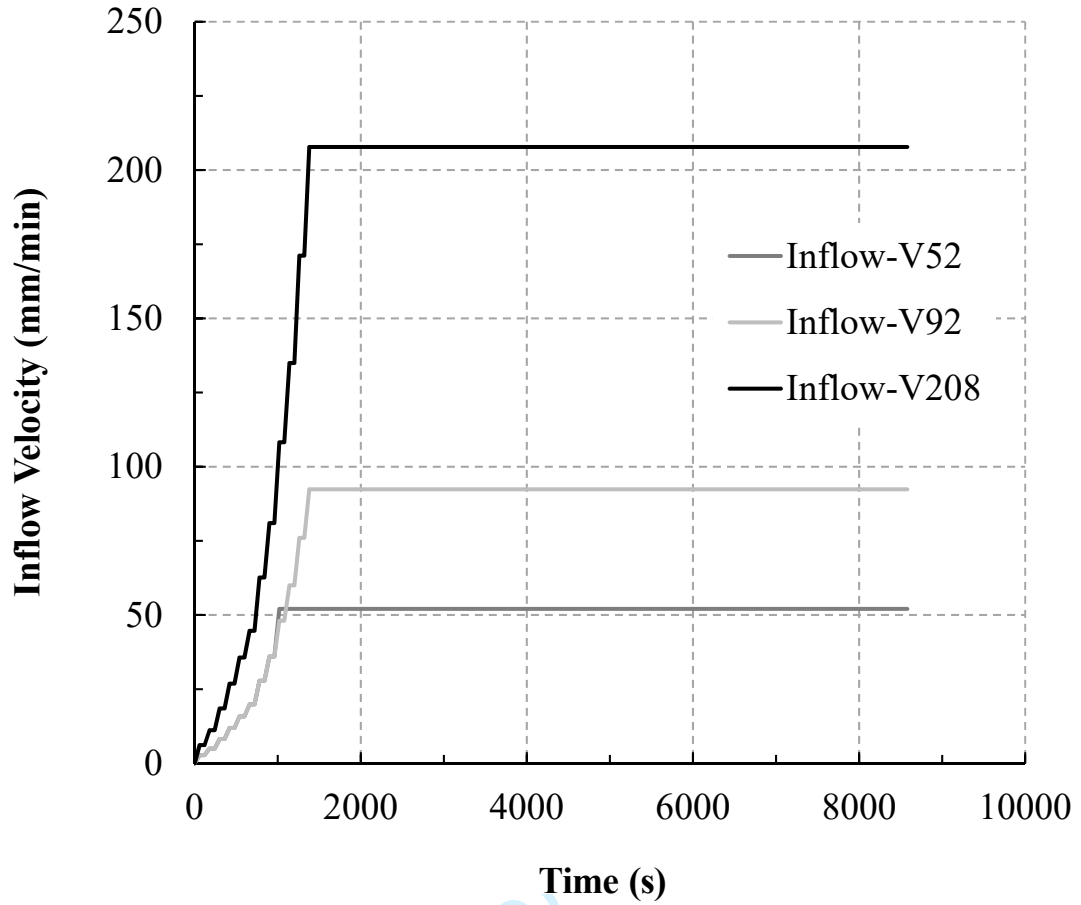


Figure2. Variation of inflow velocity with time

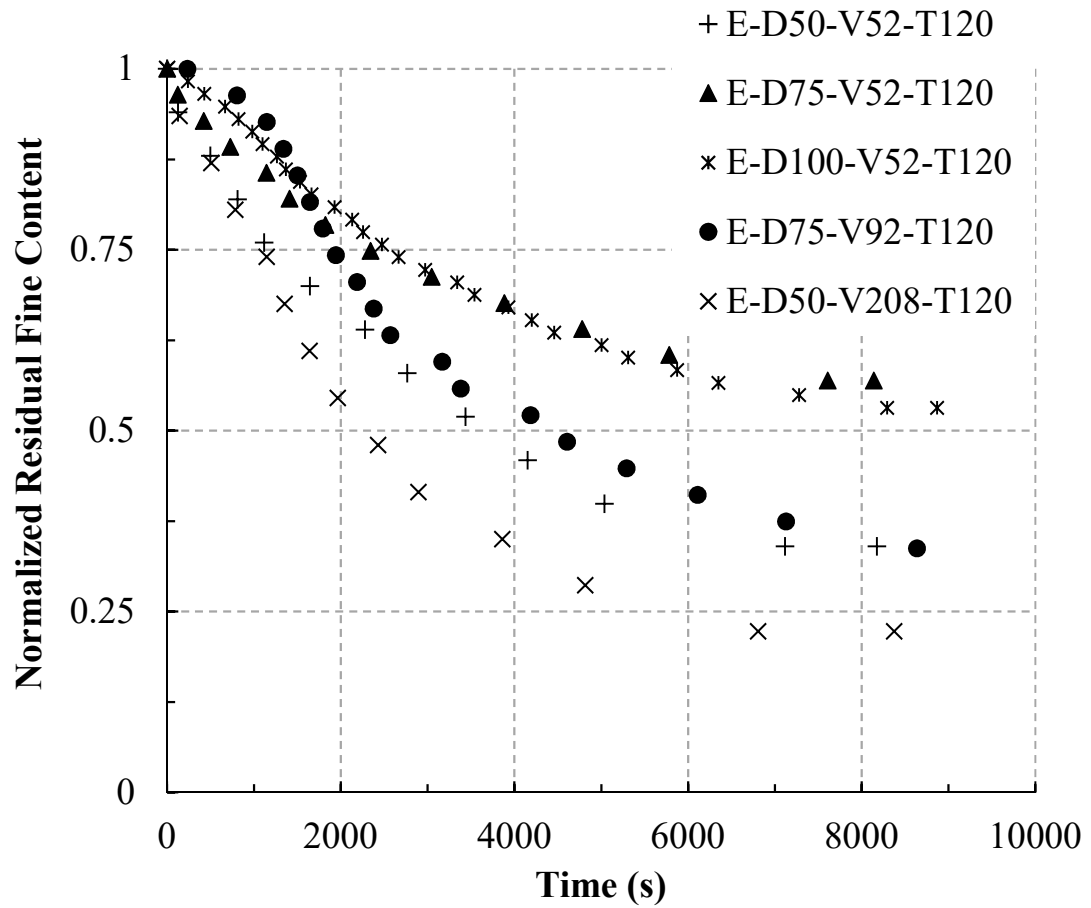


Figure 3. Variation of normalized residual fine content with time

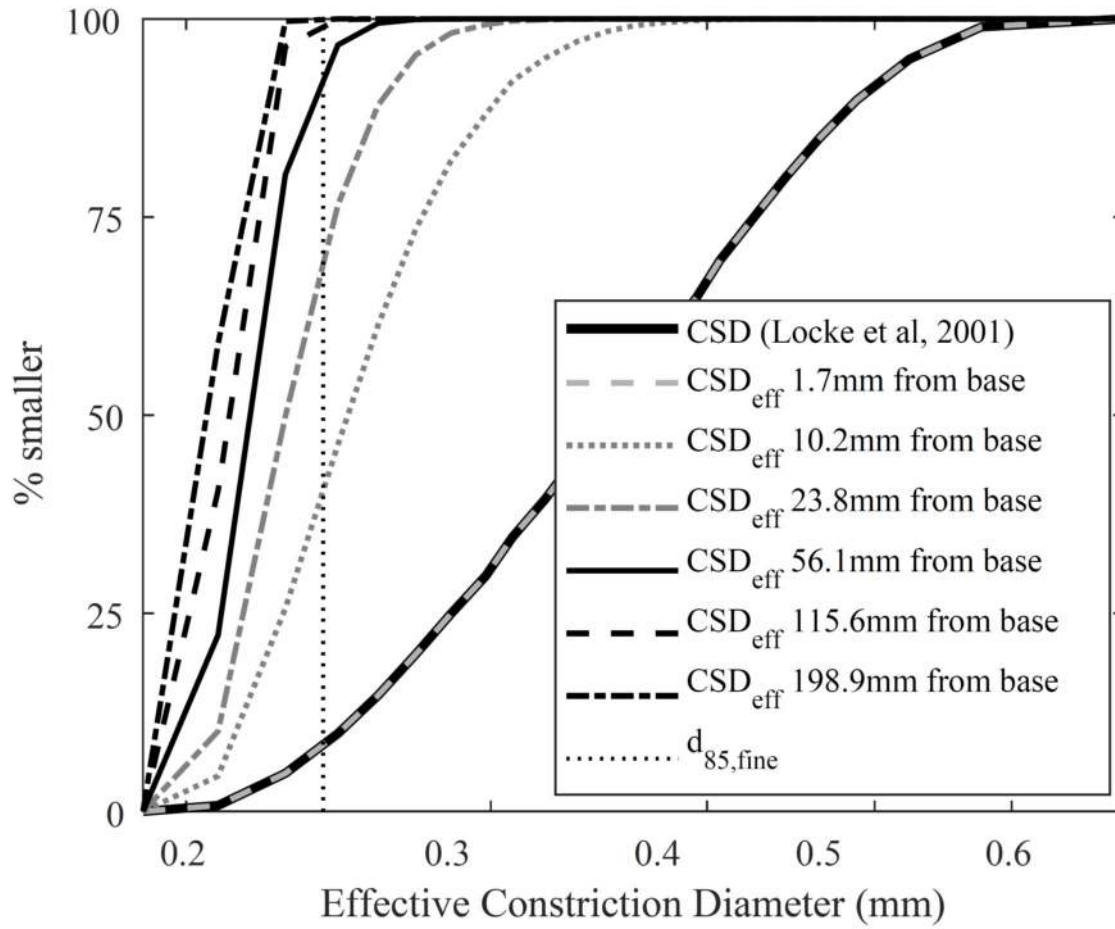


Figure 4. Effective Constriction Size Distribution (CSD) at different heights in the sample according to the method suggested by Kenney et al (1985)

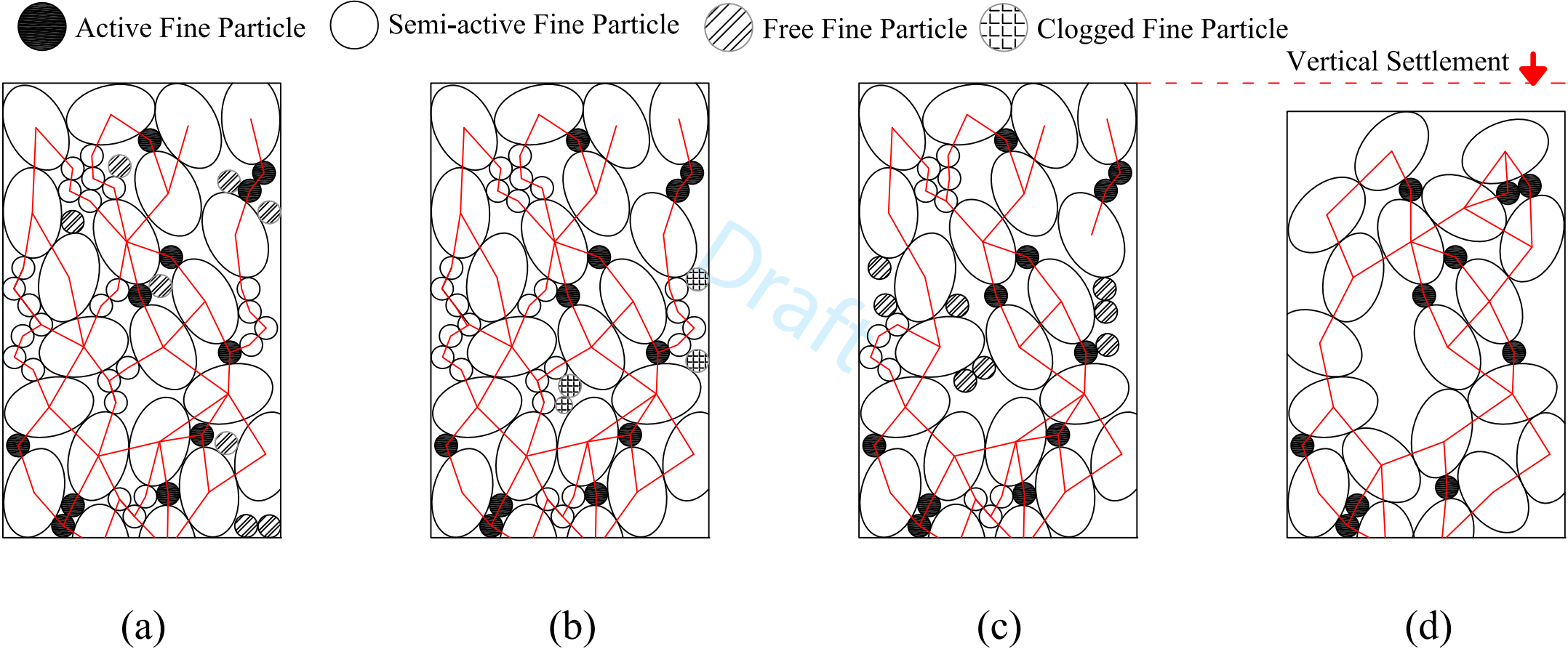


Figure 5. Progress of internal erosion (a) Initial condition, (b) Erosion of the free fines, (c) Erosion of the semi-active fines and providing new free fines and (d) Particles rearrangement and vertical deformation with residual active fines



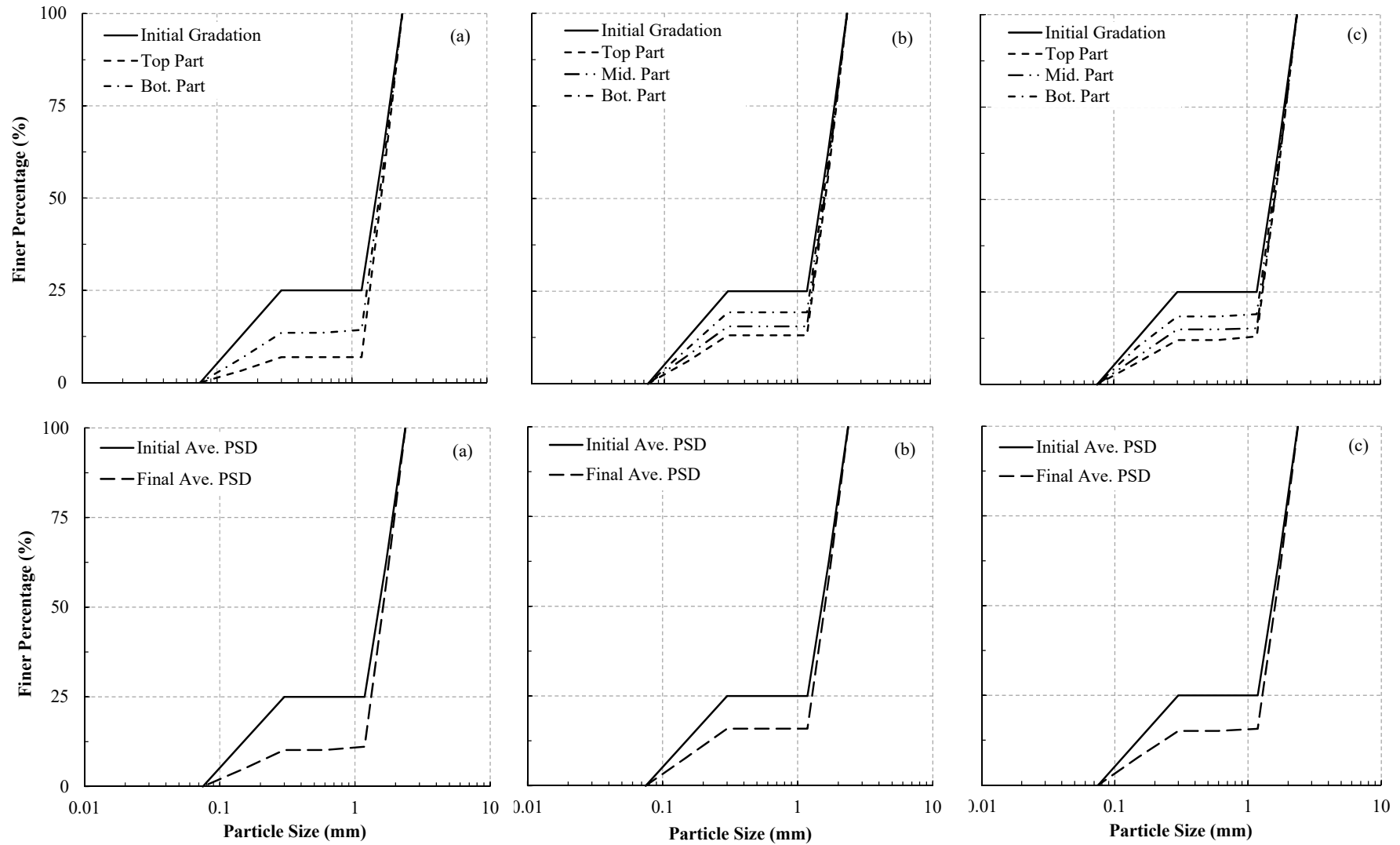


Figure 6. Particle size distribution plots for post-erosion specimens at different regions for (a) E-D50-V52-T120, (b) E-D75-V52-T120 and (c) E-D100-V52-T120

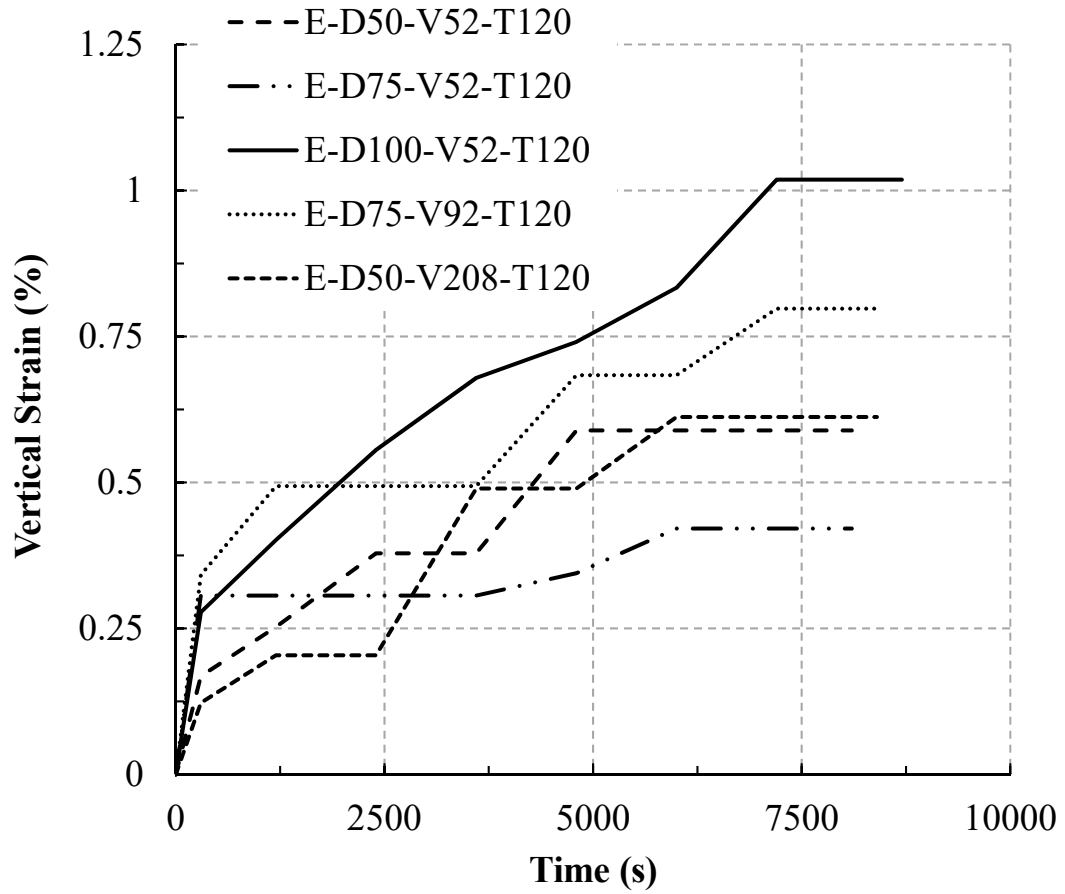


Figure7. Vertical strains during erosion phase

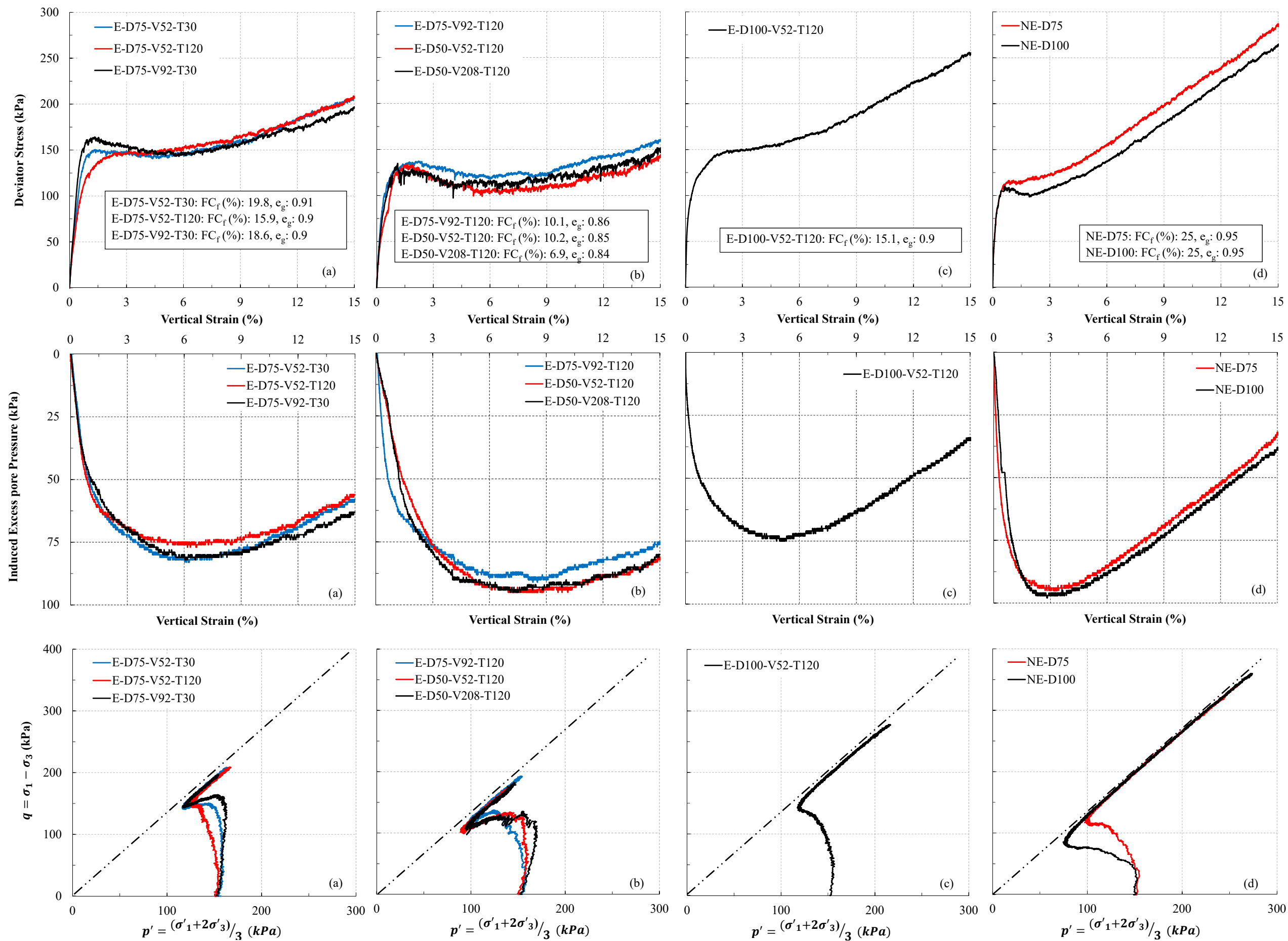


Figure 8. Impact of internal erosion on undrained stress-strain relationship, induced excess pore pressure and stress path of eroded and non-eroded specimens during undrained shearing

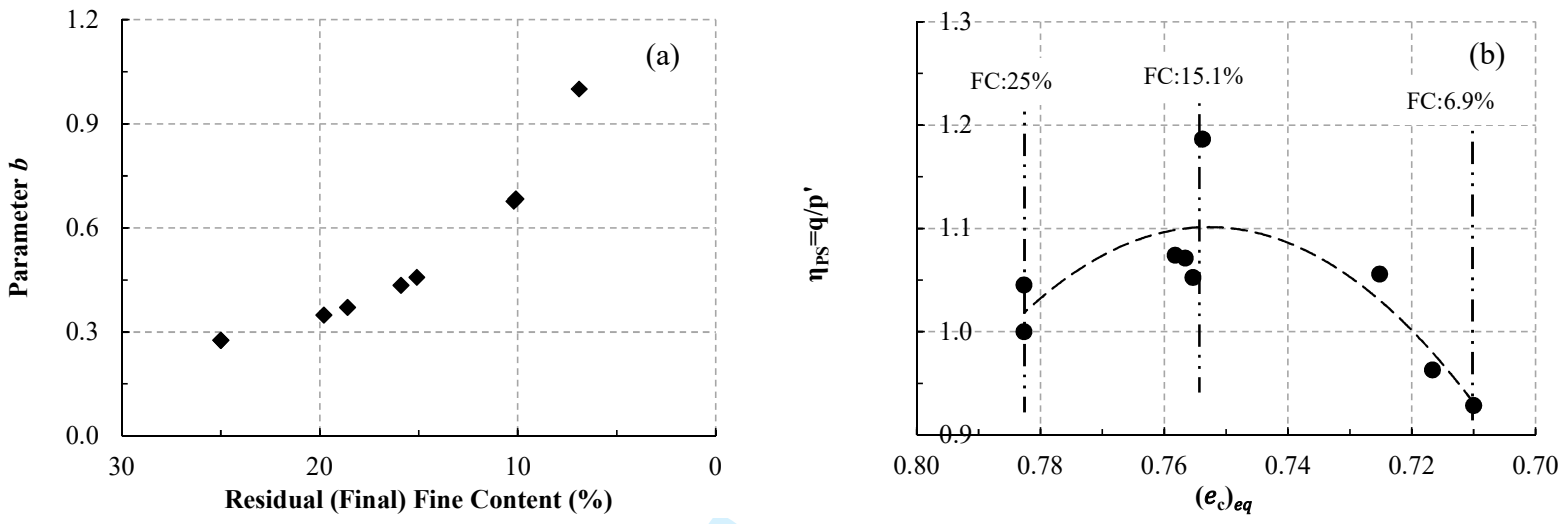


Figure 9. (a) Variation of the parameter  $b$  with residual fine content and (b) Variation of peak shear stress ratio with equivalent intergranular contact index