Post-local-buckling Behavior of Thin-walled Columns

Shien T. Wang

Yei L. Tien

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by

Shien T. Wang[1] and Yei L. Tien[2]

INTRODUCTION

Concentrically loaded columns can buckle by (1) bending about one of the principal axes; (2) twisting about the shear center; (3) simultaneous bending and twisting, i. e. torsional-flexural buckling; and (4) interaction of local and column buckling. For a compact thin-walled column, buckling will be governed by one of the first three modes, depending on the sectional geometry and the slenderness ratio of the column. For a noncompact thin-walled column, overall column buckling is coupled with local buckling.

For an unsymmetrical compact section, the buckling modes would all be torsional-flexural. For a singly symmetrical section, in which the centroid and the shear center do not coincide but are on the axis of symmetry, the buckling modes would be torsional-flexural along with buckling about the axis perpendicular to the axis of symmetry. For a doubly symmetrical section, flexural buckling about two axes of symmetry and torsional buckling are not coupled; these buckling modes can be treated separately. These problems have been treated extensively in the literature (4, 12).

For noncompact columns, the interaction of torsional-flexural and local buckling is extremely complex. Even for simple cases, the analysis could be tedious. Bijlaard (3) was one of the earliest to investigate this problem regarding the overall buckling of a locally buckled column. Several other

[1] Asst. Prof. of Civ. Engrg., Univ. of Kentucky, Lexington, Ky.
[2] Graduate Assistant, Dept. of Civ. Engrg., Univ. of Kentucky, Lexington, Ky.

investigators studied the behavior of noncompact channel sections (6, 25). Venkataramaiah studied the post-buckling behavior of columns with lipped channel sections (18). Graves-Smith (7) presented a theory to predict the ultimate strengths in compression of thin-walled rectangular columns of such proportions that they buckle locally. In his analysis, the post-buckling behavior of short columns was studied by a variational principle involving plasticity. The reduction in strength of longer columns with buckled plates, caused by overall buckling, was assessed by obtaining the apparent reduced internal bending stiffness of the locally buckled section. The bending strain was imposed on the locally buckled plates. The von Karman equations were modified to describe the new situation of an infinitesimal strain applied to the already strained column. A relaxation procedure was employed for the above solution.

Jombock and Clark (8) developed an approximate method of analysis for this type of member based on the concept of effective width of the elements. Bulson (2) suggested an interaction equation to deal with this problem. A brief review on the design aspect of this subject was presented by Sharp (15). Uribe and Winter (16) also used the effective width concept to study the noncompact column strength. Using the same concept, Vaidya and Culver (17) studied the behavior of thin-walled columns subjected to impact loading.

Rhodes and Harvey (13) studied theoretically the local buckling and post-local-buckling behavior of thin-walled lipped channel beams subjected to equal end moments. The results obtained by them showed good agreement with those obtained by using Winter's equation (1, 24) which is an experimental modification of von Karman's effective width expression (19). This effective width concept has been used successfully also in the many studies of thin-walled beams subjected to static and impact loads (16, 20, 21, 22, 23, 26).

In the present paper, a numerical method is presented to study the post-local-buckling behavior of rectangular thin-walled columns. The effective width concept is used to account for the post-local-buckling strength of the buckled plate elements. The elasto-plastic material characteristics are also considered. In addition, the effects of local buckling on the column strength and the behavior of the laterally deflected locally buckled rectangular column are studied. The results obtained in this study are then compared with the current AISI design method.

STRENGTH OF LOCALLY BUCKLED COLUMNS

Interaction of Local and Column Buckling. - The study reported herein is concerned with doubly symmetric noncompact columns under concentrical axial loading. In this case, only the interaction between local and flexural buckling need be considered. If the local buckling stress of an individual plate element is smaller than the column buckling stress about the weak axis of the section (y axis), the column will buckle locally before the overall column strength is reached. The locally buckled column can often support considerable extra load after local buckling depending on the slenderness ratio of the column (7, 8). This is due to the post-local-buckling strength of the buckled plate elements which comprise the section of the column.

The available post-buckling strength in these buckled thin compression elements can be accounted for by the concept of effective width which has been well established in light-gage steel design (1). A typical buckled rectangular thin-walled section subjected to a concentric axial load is shown in Fig. 1(a). The effective width concept of the buckled compression element is shown in Fig. 1 (b). For this study, Winter's effective width equation (1, 24) is used to account for the post-buckling strength of the buckled compression element of the column.

$$\frac{b}{t} = 1.9 \sqrt{\frac{E}{\sigma_{max}}} \left(1 - 0.415 \frac{t}{w} \sqrt{\frac{E}{\sigma_{max}}}\right) \qquad (1)$$

when $\frac{w}{t} \geq 1.288\sqrt{\frac{E}{\sigma_{max}}}$ (2)

in which b = effective width of the compression element stiffened along both unloaded edges; t = thickness; E = modulus of elasticity in compression, 29.5×10^3 ksi in this study; w = flat width of the compression element exclusive of fillets; and σ_{max} = edge stress. For values smaller than $1.288\sqrt{\frac{E}{\sigma_{max}}}$, b = w.

For columns with cross-section having small w/t ratios of the component plate elements or if the slenderness ratio of the column is large, the column buckling stress may be below the local buckling stress of the element of the section. In this case, the column strength is determined by flexural buckling about the weak axis of the cross-section, i.e. the y axis. For columns with cross-sections having large w/t ratios of the component plate elements, or if the column is fairly short, local buckling will occur before the flexural column buckling load is reached. In this case, the column strength is determined by the effective cross-sectional area of the column. This strength may be governed by the yield stress of the steel or by the flexural buckling stress of the effective area of the column about the weak axis, depending on the slenderness ratio of the column. Based on the assumptions that the column is perfectly straight, i.e. without any imperfection, and that it has a constant effective area at the instant of flexural buckling, the strength of the locally buckled column can be obtained by the procedure outlined below.

Method of Analysis. - An iterative procedure is employed to predict the post-local-buckling strength of the noncompact columns. The following steps are involved.

(1) The Euler buckling load about the weak axis, P_1, is first computed based on the full, i.e. unbuckled, cross-section of the column,

$$P_1 = \frac{\pi^2 E I_y}{\ell^2} \quad (3)$$

in which I_y = moment of inertia of the full cross-section about the y axis; and

ℓ = column length.

(2) The stress, σ_1, on the full cross-section is computed by

$$\sigma_1 = \frac{P_1}{A} \tag{4}$$

in which A = full cross-sectional area. Using this stress, the effective widths of the buckled elements can be computed by using Eq. 1. The effective area of the cross-section, A_{eff}, and the effective moment of inertia of the cross-section, $I_{y\ eff}$, can then be computed.

(3) Based on the effective moment of inertia of the section, a new load P_i can be computed by

$$P_i = \frac{\pi^2 E(I_{y\ eff})_i}{\ell^2} \tag{5}$$

in which i = number of iteration.

(4) The stress on the effective cross-section, σ_i, can be obtained by

$$\sigma_i = \frac{P_i}{(A_{eff})_i} \tag{6}$$

The effective moment of inertia $(I_{y\ eff})_{i+1}$ and $(A_{eff})_{i+1}$ can then be computed.

(5) Repeating steps 3 and 4, the load will rapidly converge to the buckling load, P_{cr}. The iteration procedure is halted when two loads, consecutively computed by Eq. 5, are within a predescribed convergence limit.

For a given section, if the buckling stress computed from Eq. 2 is larger than that computed from Eq. 4, the flexural buckling of the full section will take place before the occurrence of local buckling, i.e. $P_{cr} = P_1$. On the other hand, if the computed critical buckling stress of the effective area of the buckled column is larger than F_y, the yield stress of the steel, the column buckling load, P_{cr}, is

$$P_{cr} = F_y(A_{eff})_y \tag{7}$$

in which $(A_{eff})_y$ = effective cross-sectional area caused by the uniformly distributed yield stress.

A computer program following the above procedure has been prepared. Five sections, covering a wide range of width to thickness ratios of the α and β flanges, are considered. The dimensions of these sections are shown in Table 1 with reference to Fig. 1. It should be pointed out that for these sections, only the α and β flanges are allowed to buckle locally and the other two sides are compact. The yield stress and the modulus of elasticity of the steel considered are 50 ksi and 29.5×10^3 ksi, respectively. The computed buckling loads of the locally buckled columns are shown in Fig. 2. The slenderness ratios of the columns are based on the buckled section. It is seen from the figure that the rate of increase of load carrying capacity of the columns decreases with the increase of the w/t ratios of the α and β flanges of the cross-section.

It should be noted, however, that the effects of the deviations from the elasto-plastic stress-strain response of the sections as well as the possible cold-forming strengthening effects are not accounted for in the above analysis. These effects can be taken into account numerically by the method suggested by Peterson and Bergholm (11) and later by Karren and Winter (9).

Comparison with AISI Design Curves. - In the AISI Specification (1), the effect of local buckling on column strength is taken into account by using a form factor $Q \leq 1$ which represents the weakening influence of local buckling. The Q factor for a noncompact column composed entirely of stiffened elements may be defined as follows: For a very short column, the load carrying capacity of the column, P_{cr}, is given by Eq. 7. Dividing both sides of Eq. 7 by the original full cross-sectional area A, results in

$$\frac{P_{cr}}{A} = \frac{(A_{eff})_y}{A} F_y \quad (8)$$

$$\text{or } (F_{cr})_{avg} = Q F_y \quad (9)$$

in which $(F_{cr})_{avg}$ is the average critical column buckling stress and Q is the form factor. This Q factor is the ratio of the effective cross-sectional area due to a uniformly distributed stress F_y to the original full cross-sectional area. Eq. 9 indicates that the strength of short noncompact columns can be predicted by replacing F_y by $Q F_y$.

Using the computed P_{cr} in Fig. 2 for Section 3 of Table 1, Q = 0.548, and the slenderness ratio of the column based on the radius of gyration of the full cross-section, the computed average buckling stresses for the full cross-section are shown in Fig. 3. This is a typical column curve for a noncompact column. The slenderness ratio $(\ell/r)_y$ is the upper limit for a locally buckled column failing by yielding, while $(\ell/r)_E$ is the lower limit for Euler buckling with respect to the weak axis without the influence of local buckling. The determination of $(\ell/r)_E$ is based on the stress obtained from Eq. 2 for the largest given w/t ratio of the compression element of the section. These limiting slenderness ratios are dependent on the Q factor of the section as well as on the dimensions and arrangements of the individual component elements. The effects of local buckling on the column strength can be seen clearly. The average column buckling stress curve obtained in this manner is in a form similar to that obtained by Graves-Smith (7) using a more rigorous theoretical approach.

The following parabolic type formula is recommended in the AISI Specification for noncompact columns with moderate slenderness ratios:

$$(F_{cr})_{avg} = Q F_y - \frac{(Q F_y)^2}{4\pi^2 E} \left(\frac{\ell}{r}\right)^2 \qquad (10)$$

in which ℓ/r = slenderness ratio of the column referring to the full cross-section. Eq. 10 is applicable up to

$$\left(\frac{\ell}{r}\right)_{limit} = \pi\sqrt{\frac{2E}{Q F_y}} \qquad (11)$$

beyond which the Euler buckling without local buckling governs. This formula is not different from the one used for compact sections in both the AISC and the AISI Specifications, except that F_y has been replaced by $Q\ F_y$. In the commentary on the AISI Specification it is pointed out that this approach is conservative. This has been verified recently by tests (16) for columns with small or moderate slenderness ratios (up to 90) and with Q values slightly less than one. It is interesting to compare the results obtained in the present study with the values based on the AISI Q factor approach. Using the actual Q factor of each section and the slenderness ratio of each column based on the radius of gyration of the full cross-section, the comparison of the computed average critical stresses are shown in Fig. 4.

Examining this figure, it appears that: (1) For columns with small and moderate slenderness ratios ($\ell/r < 90$), the AISI approach using a parabolic formula modified by a Q factor is conservative. This is still true if the inelasticity and the corner strengthening effect are taken into account. This has also been verified by tests by Uribe and Winter (16). (2) For columns with intermediate to large slenderness ratios ($90 < \ell/r < (\ell/r)_E$), the Q factor approach (Eqs. 10 and 11) does not seem to be conservative. This unconservativeness increases with decreasing Q values. This is especially true for sections with a Q value smaller than 0.75. (3) For columns with very large slenderness ratios ($\ell/r > (\ell/r)_E$), the column strength is determined by the Euler buckling load and not influenced by local buckling. (4) The computed values of $(\ell/r)_E$ are larger than those obtained from Eq. 11, especially for sections with small Q factors.

It should be pointed out, however, that the limiting slenderness ratio $(\ell/r)_E$ was determined based on the critical stress computed from Eq. 2 for the largest w/t ratio of the compression elements of the cross-section. The computed critical stress from Eq. 2 is lower than the buckling stress of a perfect plate since the initial imperfections of real plates are considered in Eq. 2. This

$(\ell/r)_E$ is determined based on the local buckling considerations.

Based on these observations, it seems that the approach recommended by AISI using a parabolic formula modified by a Q factor (Eqs. 10 and 11) may not be conservative for the columns considered in this study with intermediate to large slenderness ratios and with small Q factors.

INTERACTION OF LOCAL BUCKLING AND COLUMN DEFLECTION

It should be noted that in the preceding analysis, the locally buckled column is kept straight throughout the whole loading range until the column flexural buckling load of the effective cross-section is reached. Due to column initial imperfections, the straight form equilibrium position may not be possible. This could also happen for a perfectly straight column with initial imperfections in the component plate elements or with any disturbance during loading process. It is, therefore, necessary to study the post-local-buckling behavior of thin-walled columns containing a very small imperfection since such an imperfection always exists.

Interaction of Local Buckling and Lateral Deflection. - When a bending stress distribution is imposed on a locally buckled column, the compressive stress on the concave side is increased with a further reduction in effective width. On the other hand, unloading takes place on the convex side of the column and part of the elastically buckled plate element becomes effective again. This interaction is shown in Fig. 5 at the end, quarter, and half points of a locally buckled column. After bending, the effective cross-section of the locally buckled column becomes singly symmetric. The centroidal axis of the effective area will shift towards convex side of the column. The column therefore becomes a nonprismatic member. In addition, the so called P- Δ effect must be taken into account. Therefore, the laterally deflected, locally buckled, column has to be treated as a nonprismatic beam-column. The problem can no longer be treated as a bifurcation phenomenon.

Method of Analysis. - A numerical method is presented below to study the behavior of the locally buckled column subjected to lateral deflection. In this case, the relationships between moment, curvature, and axial load for a locally buckled cross-section are required. Using a numerical approach similar to the one used earlier (20) for establishing the relationship between moment and curvature for noncompact thin-walled beams, and also similar to one which has been used for reinforced concrete columns (5, 14), a procedure has been developed which permits one to obtain the complete axial load - moment - curvature relationship of the buckled thin-walled sections directly. The following steps outline a scheme to completely analyze such a section:

(1) The section is divided into j subareas.

(2) The location of a neutral axis is assumed (parallel to the y axis).

(3) A compressive strain ε is assumed at the outmost fiber of the β flange, and the strains and stresses at the centroid of each sub-area is computed using the plane section assumption.

(4) The curvature can be computed from strain and the location of the neutral axis.

(5) The effective widths of the compression elements (α and β flanges) are computed based on the stress of the element considered using Eq. 1.

(6) The forces in each sub-area are then found.

(7) Summing forces gives P, the axial force the cross section will be resisting under the assumed condition. Summing moment of forces about the y axis, the moment the section is resisting, is obtained.

(8) A new value may be chosen for the compressive strain and the procedure repeated from Step 3.

(9) A new location of the neutral axis may be assumed and the procedure repeated from Step 2.

(10) The results are arranged in a tabular form of P - M - ϕ for the purposes of interpolation or curve fitting.

For a given P, a set of moment and curvature data can be found for a given location of neutral axis. Therefore, using many different locations of the neutral axis, a complete set of moment-curvature data can be found for a given P. The above procedure is illustrated in Figs. 6 and 7.

Since the deflected equilibrium position of the locally buckled column subjected to a certain axial load is not known, it has to be determined by iteration, taking into account the nonlinear moment-curvature relationship of the section. By dividing the column into segments, Newmark's numerical integration procedure (10) may be used over the length of the column. When the desired convergence limit is reached, the deflected shape is obtained. Using an incremental axial load, the load-deflection relationship of the buckled column can be established. When convergence is not possible, this signifies the attainment of the ultimate carrying capacity of the column. In this study, the possible torsional response of the buckled column after bending has been ignored.

Typical Load-Deflection Curve of A Locally Buckled Column. - The typical load-deflection curve for a locally buckled column is shown in Fig. 8. The local buckling load of the noncompact column, p_{cr}, is an important index which is defined as

$$p_{cr} = \sigma_{cr} A \tag{12}$$

in which σ_{cr} is equal to σ_{max} in Eq. 2 corresponding to the w/t ratio of the α and β flanges of the section. Above this load, the column will buckle locally. For w/t = 100, p_{cr} is equal to 7.125 kips for Section 3. For loads well below this, the column will converge to its original straight position when the deflection due to imperfection approaches zero. For loads above this p_{cr} with the same imperfection, the locally buckled column will converge to a deflected position which is influenced by the local buckling, i. e. by the nonlinear moment-curvature relationship shown in Fig. 7(c). The nonlinear effects become more pronounced when the axial load becomes larger. However, because of the post-buckling strength of the locally buckled column, the rate of increase of lateral deflection of the buckled column with increasing axial load is reduced until the locally buckled section is further weakened by yielding. Finally, the maximum carrying capacity P_{ult} is reached when convergence of the procedure is not attainable. This will happen when the external moment due to deflection and the axial load has reached the horizontal portion of the moment-curvature curve for this load.

From this discussion, it can be seen that the behavior of a locally buckled column is somewhat different from an ordinary compact column because of the influence of local buckling beyond p_{cr}. It would be of great interest to know whether the ultimate carrying capacity of the locally buckled column with an extremely small imperfection can go beyond the critical load derived from a bifurcation analysis without considering the lateral deflection of the column.

Load-Deflection Curves. - Load-deflection curves for Section 3 of Table 1 ($w/t = 100$ for α and ρ flanges) at slenderness ratios 88.23 and 132.35 are shown in Fig. 9. In order to obtain these curves different locations of the neutral axis were assumed to generate the P - M - ϕ data. The maximum compressive strain used was 3 ϵ_y, where ϵ_y is the strain at the yield stress of the steel. The strain increment was taken as 1/30 ϵ_y. The generated moment-curvature data for several loads are shown in Fig. 7(c). The column length was divided into 10 segments. A 20 segment model was found to yield similar results as those of the 10 segment model.

For a column with a slenderness ratio $\ell/r = 132.35$, it was found that the lateral deflection is very much influenced by the weakening effect of local buckling. For loads slightly larger than the local buckling load p_{cr}, the lateral deflections due to the load remain very small for an extremely small imperfection. For a slightly larger imperfection, the magnitude of the deflection increases. At a load about 80% of the computed P_{cr}, the deflection increases sharply. Due to the larger deflection involved, the external moment increases rapidly. Because of this large deflection, it does not seem likely that the carrying capacity of a noncompact long column with an extremely small imperfection can go beyond P_{cr} before the horizontal portion of the moment-curvature curve is reached. The maximum carrying capacity of this column with an extremely small imperfection is below the P_{cr} presented earlier in Fig. 2 (about 85%).

For a column with a slenderness ratio $\ell/r = 88.23$ with an extremely small imperfection, the lateral deflection is very small up to about 20 kips, which is well beyond the p_{cr} which is 7.125 kips. The deflection increases at a much faster rate beyond 20 kips. Due to the post-buckling strength of the buckled column, the rate of increase of deflection is decreased up to loads slightly beyond 22 kips. With additional increase of the axial load, the rate of increase of deflection is then again increased until the external moment caused by the deflection and the axial load reaches the horizontal portion of the moment-curvature curve of the particular load considered. At this point, the column fails. From the data obtained, the maximum carrying capacity of this column is in the vicinity of P_{cr}. Because the deflection and the axial load create an external moment in the vicinity of the horizontal portion of the moment-curvature curve for P_{cr}, it is likely that the maximum carrying capacity of this column will be only very slightly high than P_{cr}.

Despite the above, it is quite possible, however, that the maximum load may go beyond P_{cr} for columns with other slenderness ratios, other Q factors, other cross-sectional geometries, and other additional considerations. Research on this subject is still continuing.

CONCLUSIONS

A numerical method has been presented for predicting the post-local-buckling response of thin-walled rectangular columns. Nonlinearity due to local buckling in the post-buckling range is accounted for by the effective width concept.

For a concentrically loaded, perfectly straight noncompact rectangular column, the critical column buckling load is derived by iteration from the flexural buckling of the effective area of the buckled cross-section. The computed average buckling stresses are compared with those obtained by using the Q factor approach as recommended by the AISI. It seems that for columns with small Q factors and with intermediate and large slenderness ratios, the method recommended by the AISI may not be conservative for the cases considered.

The P-M-ϕ curves for the locally buckled section are obtained by numerical integration. The interaction between local buckling and column deflection is investigated by using the generated P-M-ϕ data. Using Newmark's numerical integration procedure, the deflected shape of the locally buckled column can be predicted. It is shown that the load-deflection curve for a locally buckled column with an extremely small imperfection is different from that of a column without local buckling. The effect of local buckling on the column behavior becomes more pronounced for loads well above the local buckling load of the column. It is also shown that the locally buckled long column ($\ell/r = 132.35$) with an extremely small imperfection can not reach the critical column buckling load P_{cr} obtained by bifurcation analyses. This does not exclude the possibility, however, that the deflected locally buckled column can not be loaded beyond P_{cr} for columns with other slenderness ratios and Q values and with geometries other than those considered. In fact, for a shorter column (ℓ/r = 88.23), the maximum carrying capacity is in the vicinity of P_{cr}. Research on this subject is still continuing.

Experimental verification will be very helpful to confirm the findings in this report. Further study on this subject and other related problems is urgently needed.

ACKNOWLEDGMENTS

The study reported herein was supported in part by the University of Kentucky through a Faculty Research Fellowship. The support given by the University of Kentucky Computing Center for the use of IBM 360/65 computer is appreciated. Thanks are due to Hans Gesund for his encouragement throughout the period of this study and for valuable discussions.

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APPENDIX II. - NOTATION

The following symbols are used in this paper:

A = original unreduced cross-sectional area;

A_{eff} = effective cross-sectional area;

$(A_{eff})_y$ = effective cross-sectional area corresponding to F_y;

B_1, B_2 = flat widths defined in Fig. 1;

E = modulus of elasticity in compression, 29.5×10^3 ksi;

F_{cr} = column buckling stress on the effective cross-sectional area;

$(F_{cr})_{avg}$ = column buckling stress on the full cross-sectional area;

I_y = moment of inertia about y axis;

$I_{y\ eff}$ = effective moment of inertia about y axis;

M = moment;

P = axial load;

P_1 = Euler buckling load of the full cross-section;

P_{cr} = buckling load of the effective cross-section;

P_{ult} = maximum carrying capacity of a column;

Q = form factor defined in Eq. 9;

R = inside radius;

b = effective width;

i = number of iteration;

j = number of sub-areas;

ℓ = column length

$(\ell/r)_E$ = lower limit of the slenderness ratio for Euler buckling of the full cross-section;

$(\ell/r)_{limit}$ = slenderness ratio limit defined in Eq. 11;

$(\ell/r)_y$ = upper limit of the slenderness ratio for column failure by yielding of the effective area;

p_{cr} = load at which causing local buckling of the column;

r = radius of gyration for the full section with respect to the y axis;

r_{eff} = radius of gyration for the effective section with respect to the y axis;

t = thickness;

w = flat width exclusive of fillets;

w_1, w_2 = flat widths exclusive of fillets defined in Fig. 1;

w/t = flat width to thickness ratio;

ε = strain;

ε_y = strain at yield stress F_y;

σ_1 = uniform stress on the full cross-sectional area;

σ_{cr} = local buckling stress defined in Eq. 2 as σ_{max};

σ_{max} = edge stress; and

ϕ = curvature.

TABLE 1. - COLUMN SECTION DIMENSIONS

Dimensions (1)	Sections				
	1 (2)	2 (3)	3 (4)	4 (5)	5 (6)
B_1 in in.	2.025	5.025	8.025	11.025	14.025
B_2 in in.	2.025	2.025	2.025	2.025	2.025
R in in.	3/16	3/16	3/16	3/16	3/16
t in in.	0.075	0.075	0.075	0.075	0.075
w_1/t	20	60	100	140	180
w_2/t	20	20	20	20	20
A* in in.^{2}	0.556	1.006	1.456	1.906	2.356
r* in in.	0.783	0.874	0.906	0.923	0.933
Q**	1.0	0.758	0.548	0.426	0.349

* For the full cross-section

**Form factor based on F_y = 50 ksi

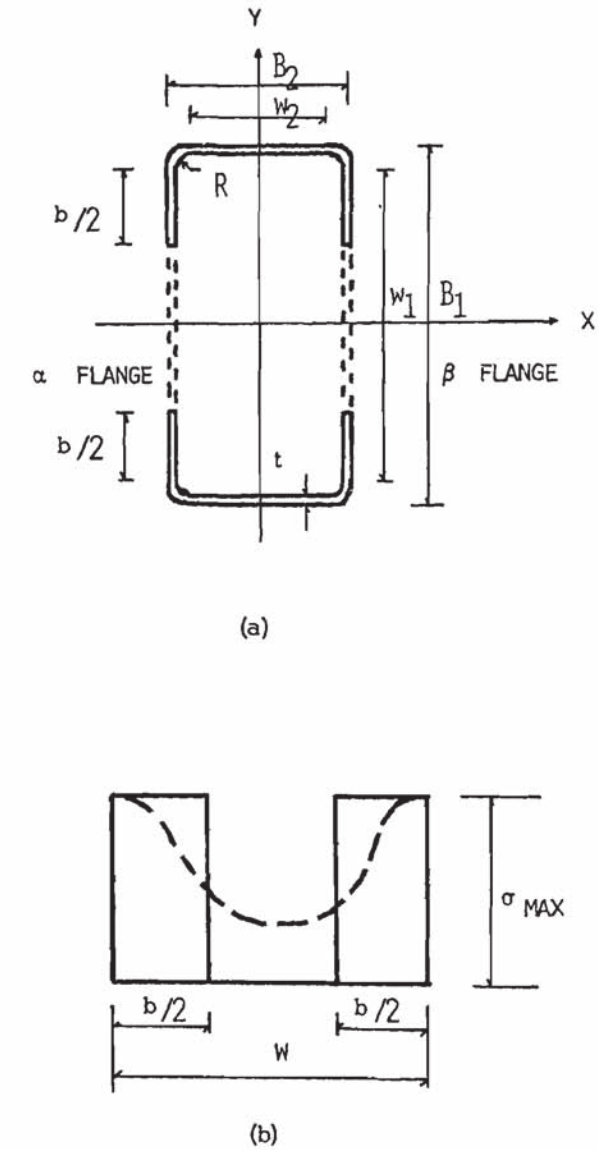

FIG. 1.- BUCKLED RECTANGULAR COLUMN SECTION AND EFFECTIVE WIDTH CONCEPT

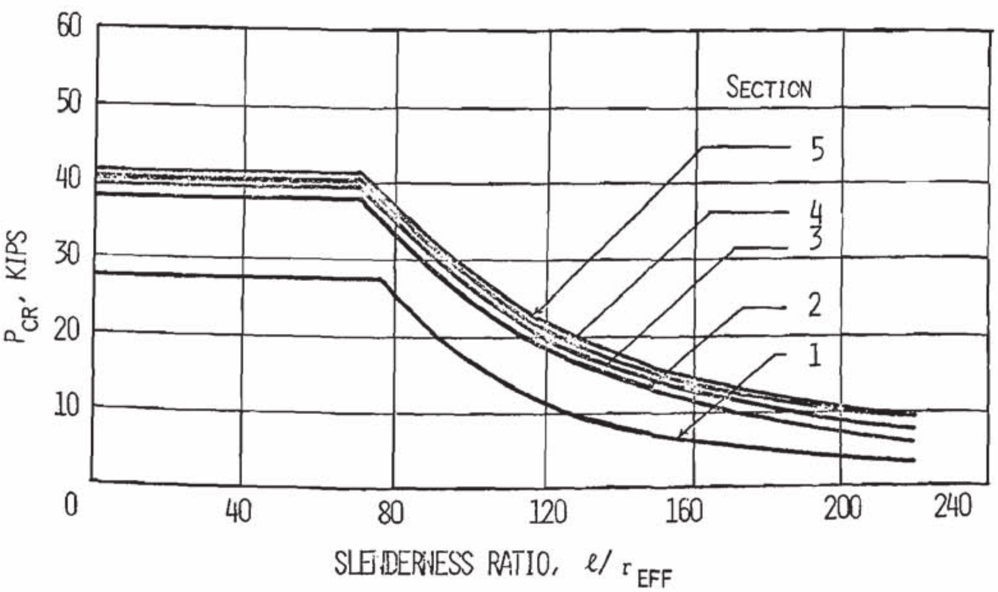

FIG. 2. - BUCKLING LOADS OF LOCALLY BUCKLED COLUMNS

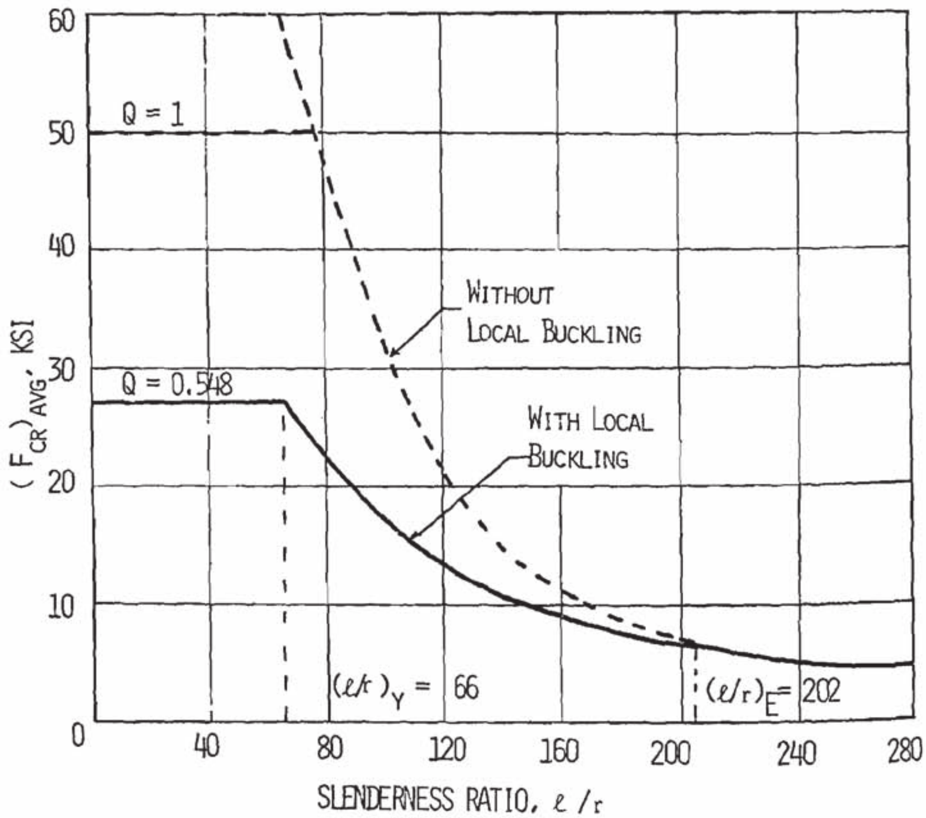

FIG. 3. - AVERAGE COLUMN BUCKLING STRESSES VS. SLENDERNESS RATIOS

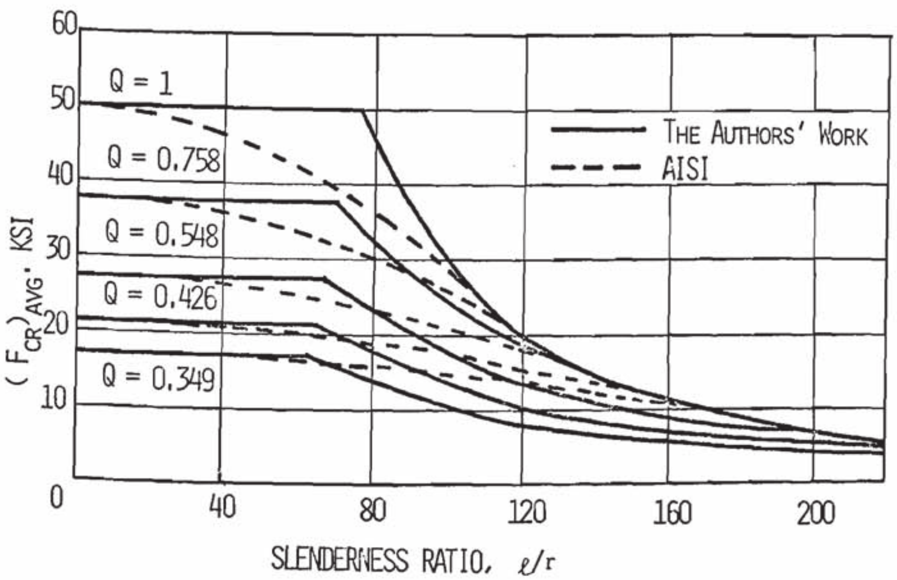

FIG. 4. - COMPARISON OF COMPUTED AVERAGE COLUMN BUCKLING STRESSES WITH AISI VALUES

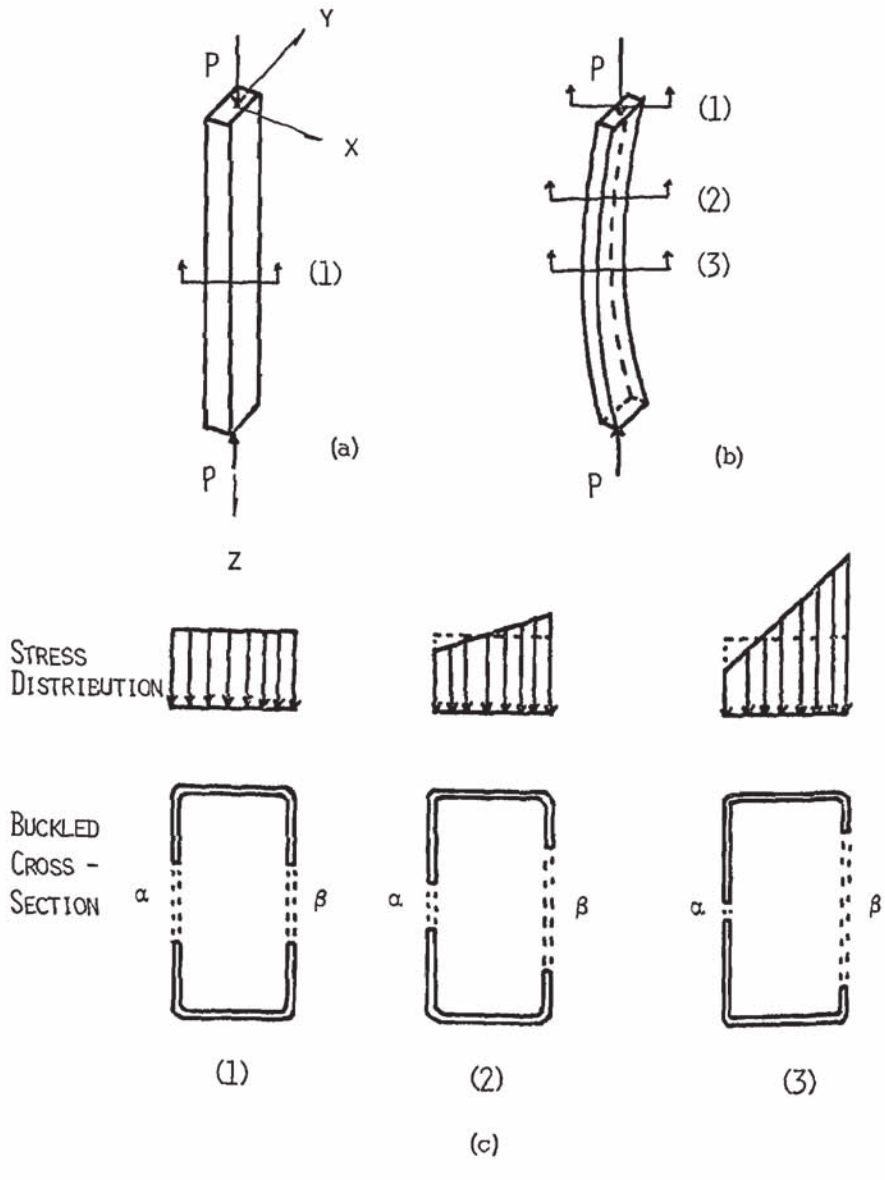

FIG. 5. - LOCALLY BUCKLED COLUMN -BEFORE AND AFTER BENDING

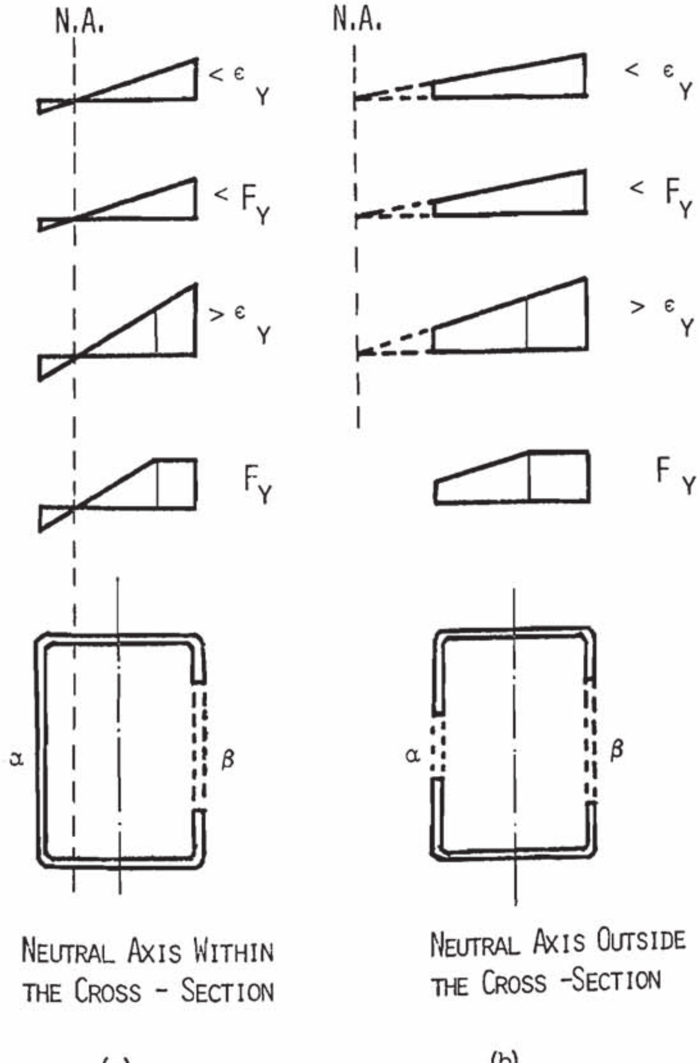

FIG. 6. - STRAIN AND STRESS DISTRIBUTIONS

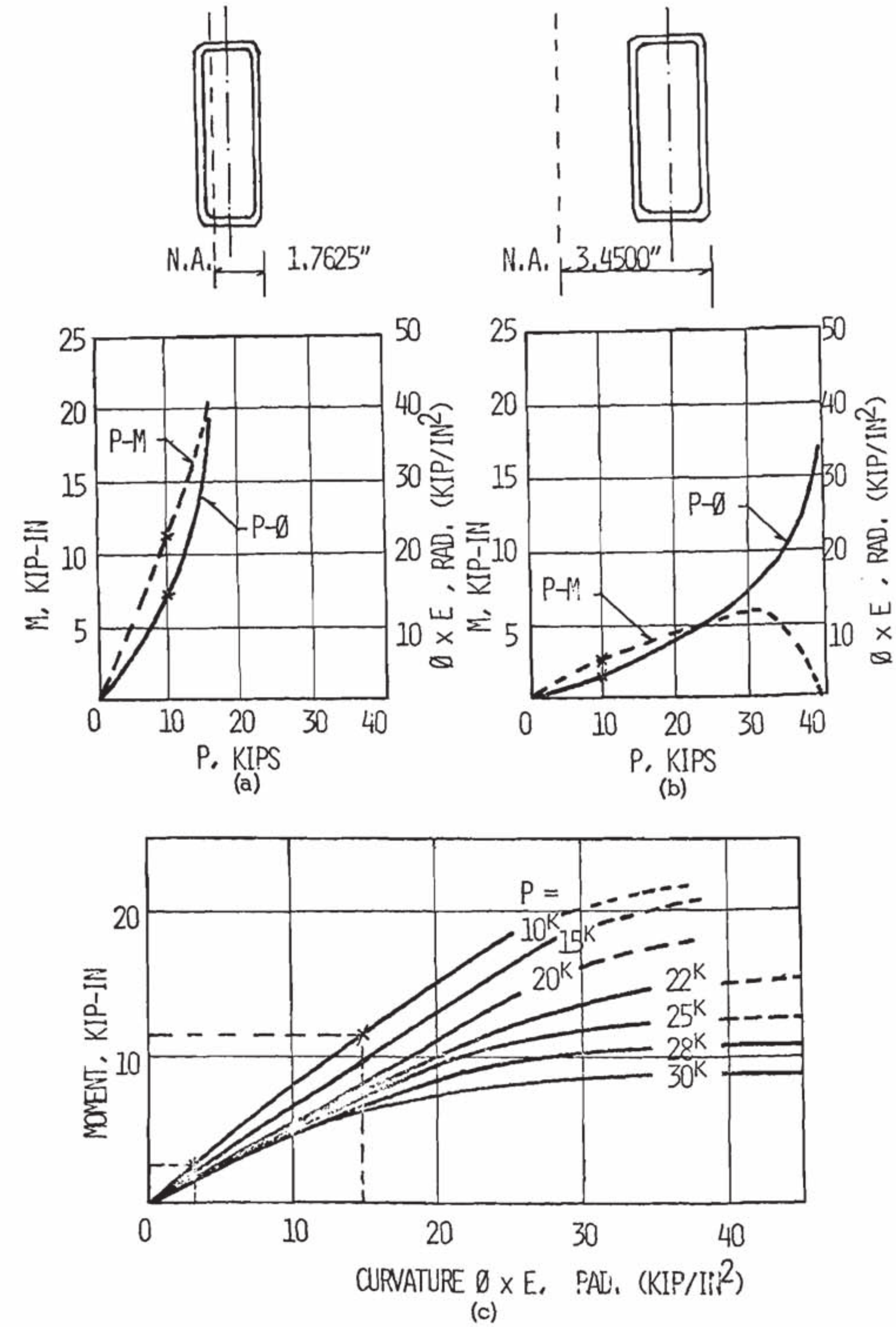

FIG. 7. - AXIAL LOAD - MOMENT - CURVATURE RELATIONSHIPS - SECTION 3

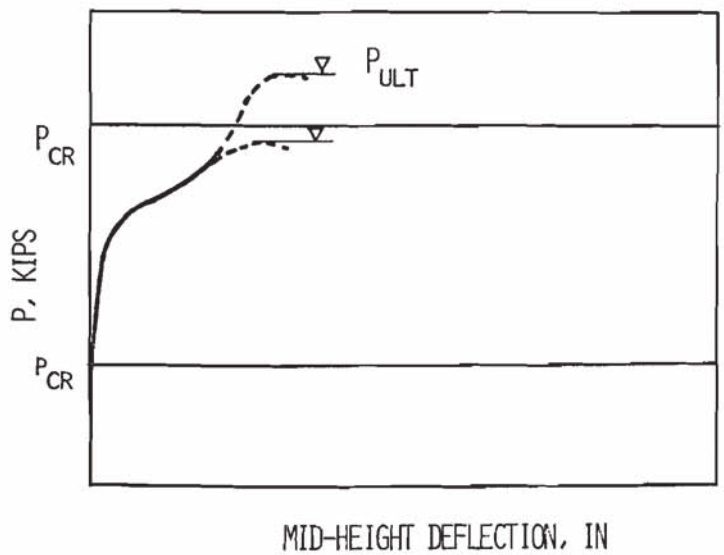

FIG. 8. - TYPICAL LOAD-DEFLECTION CURVE OF A LOCALLY BUCKLED COLUMN

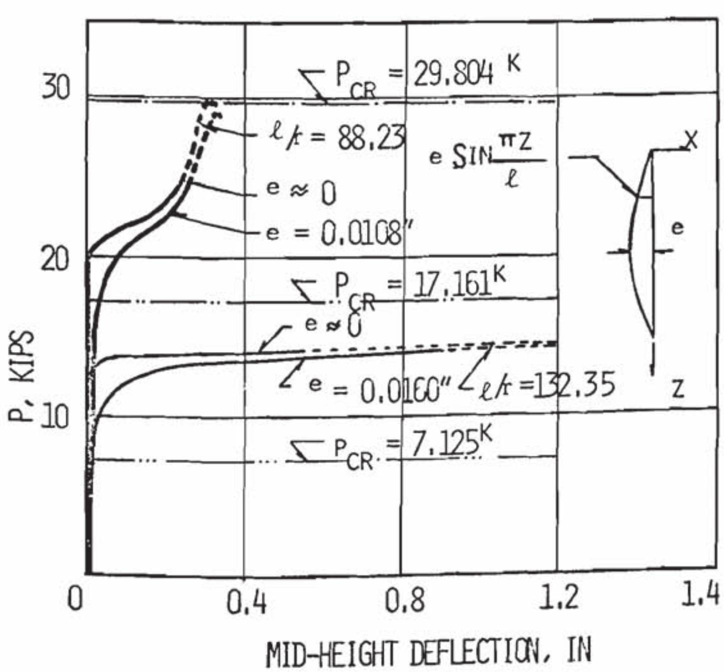

FIG. 9. - LOAD-DEFLECTION CURVES OF LOCALLY BUCKLED COLUMNS - SECTION 3