# Postselected versus nonpostselected quantum teleportation using parametric down-conversion

Pieter Kok[1,*] and Samuel L. Braunstein[1,2]
[1]*SEECS, University of Wales, Bangor LL57 1UT, United Kingdom*
[2]*Hewlett-Packard Labs, Mail Box M48, Bristol BS34 8QZ, United Kingdom*
(Received 23 March 1999; published 7 March 2000)

We study the experimental realization of quantum teleportation as performed by Bouwmeester *et al.* [Nature (London) **390**, 575 (1997)] and the adjustments to it suggested by Braunstein and Kimble [Nature (London) **394**, 841 (1998)]. These suggestions include the employment of a detector cascade and a relative slow-down of one of the two down-converters. We show that coincidences between photon pairs from parametric down-conversion automatically probe the non-Poissonian structure of these sources. Furthermore, we find that detector cascading is of limited use, and that modifying the relative strengths of the down-conversion efficiencies will increase the time of the experiment to the order of weeks. Our analysis therefore points to the benefits of single-photon detectors in non post selected-type experiments, a technology currently requiring roughly 6 °K operating conditions.

PACS number(s): 03.67.Hk, 42.50.Dv

Quantum entanglement, an aspect of quantum theory already recognized in the early days, clearly sets quantum mechanics apart from classical mechanics. More recently, fundamentally new phenomena involving entanglement such as cryptography, error correction, and dense coding have been discovered [1–3]. In particular, the field has witnessed major steps forward with the experimental realization of *quantum teleportation* [4–9].

We speak of quantum teleportation when a (possibly unknown) quantum state $|\phi\rangle$ held by Alice is sent to Bob without actually traversing the intermediate space. The protocol uses an entangled state of two systems which is shared between Alice and Bob. To bring teleportation about, Alice and Bob proceed as follows: a Bell measurement of $|\phi\rangle$ and Alice's half of the entangled pair will correlate Bob's half to the original state $|\phi\rangle$. Bob then uses the outcome of Alice's measurement to determine which unitary transformation brings his state into the original one $|\phi\rangle$.

In this paper, we study the experimental realization of quantum teleportation of a single polarized photon as performed in Innsbruck, henceforth called the ''Innsbruck experiment'' (Bouwmeester *et al.* [5]). Our aim is to evaluate the suggestions to ''improve'' the experiment in order to yield nonpostselected operation, as given by Braunstein and Kimble [10], (also see Ref. [11]). These suggestions include the employment of a so-called detector cascade in the state preparation mode, and enhancement of the photon-pair source responsible for the entanglement channel relative to the one responsible for the initial-state preparation.

Subsequently, we hope to clarify some of the differences in the interpretation of the Innsbruck experiment. As pointed out by Braunstein and Kimble [10], to lowest order the teleported state in the Innsbruck experiment is a mixture of the vacuum and a single-photon state. However, we cannot interpret this *state* as a low-efficiency teleported state, where sometimes a photon emerges from the apparatus and sometimes not. This reasoning is based on what we call the ''partition ensemble fallacy,'' or PEF for short. It relies on a particular partition of the outgoing density matrix, and this is not consistent with quantum mechanics [12]. Circumventing the PEF leads to the notion of *postselected* teleportation, in which the teleported state is detected. The postselected teleportation indeed has a high fidelity and a low efficiency. Although generally the PEF is harmless (it might even be considered a useful tool in understanding aspects of quantum theory), to our knowledge, this is the first instance where it leads to a *quantitatively* different evaluation of an experiment.

The main result of this paper is that the suggested improvements require near-perfect efficiency photodetectors or a considerable increase in the time needed to run the experiment. The remaining practical alternative in order to obtain *nonpostselected* quantum teleportation (i.e., teleportation *without* the need for detecting the teleported photon) is to employ a single-photon detector in the state-preparation mode (a technology currently requiring approximately 6 °K operating conditions).

We start in Sec. I by reviewing photon-pair creation using parametric down-conversion. In Sec. II we present the fidelity of teleportation and discuss some of its interpretations. Finally, Sec. III is devoted to an analysis of a generalized version of the Innsbruck experiment, and requirements are given to sufficiently enhance the fidelity.

## I. INNSBRUCK EXPERIMENT

In this section we review the Innsbruck experiment. In Sec. I A we calculate the probability distribution of finding $n$ photon pairs, and subsequently we compare this with the Poisson distribution for $n$ photon pairs. The difference between the two distributions, in terms of distinguishability, is evaluated by means of the so-called *statistical distance* in Sec. I B.

In the Innsbruck experiment, parametric down-conversion is used to create two entangled photon-pairs. One pair constitutes the entangled state shared between Alice and Bob,

*Electronic address: pieter@sees.bangor.ac.uk

 

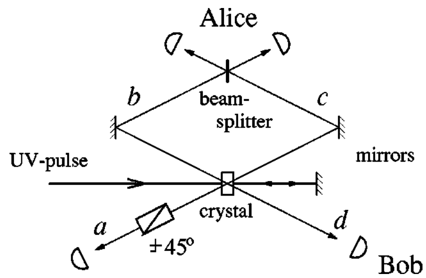


FIG. 1. Schematic representation of the experiment conducted in Innsbruck. An UV pulse is sent into a nonlinear crystal, thus creating an entangled photon pair. The UV pulse is reflected by a mirror and returned into the crystal again. This reflected pulse creates the second photon pair. Photons *b* and *c* are sent into a beam splitter and are detected. This is the Bell measurement. Photon *a* is detected to prepare the input state, and photon *d* is the teleported output state Bob receives. In order to rule out the possibility that there are no photons in mode *d*, Bob detects this mode.

while the other is used by Victor to create an ''unknown'' single-photon polarization state $|\phi\rangle$: Victor detects mode *a*, shown in Fig. 1 to prepare the single-photon input state in mode *b*. This mode is sent to Alice. A coincidence in the detection of the two outgoing modes of the beam splitter (Alice's—incomplete—Bell measurement) tells us that Alice's two photons are in a $|\Psi^-\rangle$ Bell state [13,14]. The remaining photon (held by Bob) is now in the same unknown state as the photon prepared by Victor because in this case the unitary transformation Bob has to apply coincides with the identity, i.e., doing nothing. Bob verifies this by detecting his state along the same polarization axis which was used by Victor. A fourfold coincidence in the detectors of Victor's state preparation, Alice's Bell measurement and Bob's outgoing state indicate that quantum teleportation of a single-photon state is complete.

There is, however, a complication which gave rise to a different interpretation of the experiment [10,11]. Analysis shows that the state detected by Bob is a mixture of the vacuum and the original state [5,10] (to lowest order). This vacuum contribution occurs when the down-converter responsible for creating the input state $|\phi\rangle$ yields two photon pairs, while the other gives nothing. The detectors used in the experiment cannot distinguish between one or several photons coming in, so Victor's detection of mode *a* in Fig. 1 will not reveal the presence of more than one photon. A threefold coincidence in the detectors of Victor and Alice is still possible, but Bob has not received a photon and quantum teleportation has not been achieved. Bob therefore needs to detect his state in order to identify successful quantum teleportation. When Victor uses a detector which can distinguish between one or several photons this problem vanishes. However, currently such detectors require an operating environment of roughly 6 °K [15–17].

In Sec. III we give a detailed analysis of the Innsbruck experiment, and the suggestions for improvement given in Ref. [11]. Here we investigate the creation of entangled photon-pairs using weak parametric down-conversion [18]. In this process, there is a small probability of creating more than one photon pair simultaneously. One might expect that

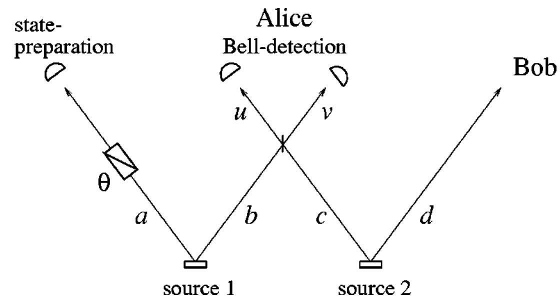


FIG. 2. Schematic ''unfolded'' representation of the teleportation experiment with two independent down-converters and a polarization rotation in mode *a*. The state-preparation detector is actually a detector cascade, and Bob does not detect the mode he receives.

for sufficiently weak down-conversion the two pairs created by one source (which give rise to the vacuum contribution in the teleported output state) can be considered independent from each other. However, we show that this is not the case. In what follows we find it convenient to ''unfold'' the experimental setup according to Fig. 2.

### A. Probability for *n* pairs

In this section we study the statistics of parametric down-conversion. We show that the probability $P_{\rm PDC}(n)$ for finding $n$ photon pairs deviates from the Poisson distribution, even in the weak limit.

Let *a* and *b* be two field modes with a particular polarization along the *x* and *y* axes of a given coordinate system. We are working in the interaction picture of the Hamiltonian which governs the dynamics of creating two entangled field modes *a* and *b* using weak parametric down-conversion. In the rotating-wave approximation this Hamiltonian reads ($\hbar=1$)

$$H=i\kappa(a_x^\dagger b_y^\dagger - a_y^\dagger b_x^\dagger)+\text{H.c.} \tag{1.1}$$

In this equation H.c. means Hermitian conjugate, and $\kappa$ is the product of the pump amplitude and the coupling constant between the electromagnetic field and the crystal. The operators $a_i^\dagger$, $b_i^\dagger$ and $a_i$, $b_i$ are creation and annihilation operators for polarizations $i\in\{x,y\}$ respectively. They satisfy the commutation relations

$$\begin{aligned} [a_i,a_j^\dagger]&=\delta_{ij}, \quad [a_i,a_j]=[a_i^\dagger,a_j^\dagger]=0,\\ [b_i,b_j^\dagger]&=\delta_{ij}, \quad [b_i,b_j]=[b_i^\dagger,b_j^\dagger]=0, \end{aligned} \tag{1.2}$$

where $i,j\in\{x,y\}$. The time evolution due to this Hamiltonian is given by

$$U(t)\equiv\exp(-iHt), \tag{1.3}$$

where $t$ is the time it takes for the pulse to travel through the crystal. By applying this unitary transformation to the vacuum $|0\rangle$, the state $|\Psi_{\rm src}\rangle$ is obtained:

$$|\Psi_{\rm src}\rangle=U(t)|0\rangle=\exp(-iHt)|0\rangle. \tag{1.4}$$

We are interested in the properties of $|\Psi_{\rm src}\rangle$. Define the $L_+$ and $L_-$ operators to be

$$L_+ = a_x^\dagger b_y^\dagger - a_y^\dagger b_x^\dagger = L_-^\dagger . \tag{1.5}$$

This will render Eqs. (1.1) and (1.3) into

$$\begin{aligned} H &= i\kappa L_+ - i\kappa^* L_- ,\\ U(t) &= \exp[\kappa t L_+ - \kappa^* t L_-]. \end{aligned} \tag{1.6}$$

Applying $L_+$ to the vacuum will yield a singlet state (up to a normalization factor) in modes $a$ and $b$:

$$\begin{aligned} L_+|0\rangle &= |\leftrightarrow,\updownarrow\rangle_{ab} - |\updownarrow,\leftrightarrow\rangle_{ab}\\ &= |1,0;0,1\rangle_{a_xa_yb_xb_y} - |0,1;1,0\rangle_{a_xa_yb_xb_y}. \end{aligned} \tag{1.7}$$

We henceforth use the latter notation, where $|i,j;k,l\rangle_{a_xa_yb_xb_y}$ is shorthand for $|i\rangle_{a_x}\otimes|j\rangle_{a_y}\otimes|k\rangle_{b_x}\otimes|l\rangle_{b_y}$, a tensor product of photon number states. Applying this operator $n$ times gives a state $|\Phi^n\rangle$ (where we have included a normalization factor $N_n$, so that $\langle\Phi^n|\Phi^n\rangle = 1$),

$$\begin{aligned} |\Phi^n\rangle &\equiv N_n L_+^n |0\rangle\\ &= N_n \sum_{m=0}^{n} n!(-1)^m |m_x,(n-m)_y;(n-m)_x,m_y\rangle_{ab}, \end{aligned} \tag{1.8}$$

with

$$N_n^2 = \frac{1}{n!(n+1)!}. \tag{1.9}$$

We interpret $|\Phi^n\rangle$ as the state of $n$ entangled photon pairs.

We want the unitary operator $U(t)$ in Eq. (1.6) to be in a normal ordered form, because then the annihilation operators will ''act'' on the vacuum first, in which case Eq. (1.4) simplifies. In order to obtain the normal ordered form of $U(t)$ we examine the properties of $L_+$ and $L_-$. Given the commutation relations (1.2), it is straightforward to show that

$$\begin{aligned} [L_-,L_+] &= a_x^\dagger a_x + a_y^\dagger a_y + b_x^\dagger b_x + b_y^\dagger b_y + 2\\ &\equiv 2L_0,\\ [L_0,L_\pm] &= \pm L_\pm . \end{aligned} \tag{1.10}$$

An algebra which satisfies these commutation relations (together with the properties $L_- = L_+^\dagger$ and $L_0 = L_0^\dagger$) is an su(1,1) algebra. The normal ordering for this algebra is known [19] (with $\hat\tau = \tau/|\tau|$):

$$\begin{aligned} \exp(\tau L_+ - \tau^* L_-) &= \exp(\hat\tau \tanh|\tau| L_+)\\ &\quad\times \exp[-2\ln(\cosh|\tau|)L_0]\\ &\quad\times \exp(-\hat\tau^* \tanh|\tau| L_-). \end{aligned} \tag{1.11}$$

The scaled time $\tau$ is defined as $\tau\equiv\kappa t$. Without loss of generality we can take $\tau$ to be real. Since the ''lowering'' operator $L_-$ is placed on the right, it will yield zero when applied to the vacuum, and the exponential reduces to the identity. Similarly, the exponential containing $L_0$ will yield a $c$ number, contributing only to the normalization.

We can now ask the question whether the pairs thus formed are independent of each other, i.e., whether they yield the Poisson distribution. Suppose $P_{\rm PDC}(n)$ is the probability of creating $n$ photon pairs with parametric down-conversion, and let

$$r \equiv \tanh\tau \quad \text{and} \quad q \equiv 2\ln(\cosh\tau), \tag{1.12}$$

then the probability of finding $n$ entangled photon pairs is

$$\begin{aligned} P_{\rm PDC}(n) &\equiv |\langle\Phi^n|\Psi_{\rm src}\rangle|^2\\ &= |\langle 0|(L_-^n N_n)(e^{rL_+}e^{-qL_0}e^{-rL_-})|0\rangle|^2\\ &= e^{-2q}\left|\langle 0|L_-^n N_n\left[\sum_{l=0}^{\infty}\frac{r^l}{l!}L_+^l\right]|0\rangle\right|^2\\ &= (n+1)r^{2n}e^{-2q}. \end{aligned} \tag{1.13}$$

It should be noted that this is only a normalized probability distribution in the limit of $r,q\to 0$.

Given Eqs. (1.12), $P_{\rm PDC}(n)$ deviates from the Poisson distribution, and the pairs are therefore not independent. For weak sources, however, one might expect that $P_{\rm PDC}(n)$ approaches the Poisson distribution sufficiently closely. This hypothesis can be tested by studying the distinguishability of the two distributions.

### B. Distinguishability

Here we study the distinguishability of the pair distribution calculated in Sec. I A and the Poisson distribution. The Poisson distribution for independently created objects is given by

$$P_{\rm Poisson}(n) = \frac{p^n e^{-p}}{n!}. \tag{1.14}$$

Furthermore, rewrite the pair distribution in Eq. (1.13) as

$$P_{\rm PDC}(n) = (n+1)\left(\frac{p}{2}\right)^n e^{-p} \quad \text{for } p \ll 1, \tag{1.15}$$

using $q\approx r^2$ and $p\equiv 2r^2 = 2\tanh^2\tau$ for small scaled times. Here $p$ is the probability of creating one entangled photon pair. Are these probability distributions distinguishable? Naively one would say that for sufficiently weak down-conversion (i.e., when $p\ll 1$) these distributions largely coincide, so that instead of the complicated pair distribution (1.15) we can use the Poisson distribution, which is much easier from a mathematical point of view. The distributions are distinguishable when the ''difference'' between them is larger than the size of an average statistical fluctuation of the difference. This fluctuation depends on the number of samplings.

Consider two nearby discrete probability distributions $\{p_j\}$ and $\{p_j + \mathrm{d}p_j\}$. A natural difference between these distributions is given by the so-called (infinitesimal) *statistical distance* $ds$ [20–22]:

$$ds^2 = \sum_j \frac{dp_j^2}{p_j}. \tag{1.16}$$

When the typical statistical fluctuation after $N$ samplings is $1/\sqrt{N}$, the two probability distributions are distinguishable if

$$ds \gtrsim \frac{1}{\sqrt{N}} \Leftrightarrow N ds^2 \gtrsim 1. \tag{1.17}$$

The statistical distance between Eqs. (1.14) and (1.15), and therefore the distinguishability criterion, is

$$ds^2 \propto \frac{p^2}{8} \to N \gtrsim \frac{8}{p^2}. \tag{1.18}$$

On the other hand, the average number of trials in the teleportation experiment required to obtain one photon pair from both down-converters is

$$N = \frac{1}{p^2}. \tag{1.19}$$

The minimum number of trials in the experiment thus almost immediately renders the two probability distributions distinguishable, and we therefore cannot approximate the actual probability distribution with the Poisson distribution.

Since the Poisson distribution in Eq. (1.14) is derived by requiring the statistical independence of $n$ pairs, and the pair distribution is distinguishable from the Poisson distribution, the photon pairs cannot be considered to be independently produced, *even in the weak limit*. In the analysis of the Innsbruck experiment we need to take extra care due to this property of parametric down-converters.

## II. TELEPORTATION FIDELITY

In this section we introduce the so-called *fidelity* for quantum teleportation. This is already recognized as an important tool in quantum information theory, and it is therefore natural to consider teleportation criteria based upon it. Subsequently, we discuss different points of view of the Innsbruck experiment emerging from this concept. We restrict our discussion to the subset of events where successful Bell-state and state-preparation detections have occurred (all subsequent statements are *conditioned* on such events). Since the interpretation of the experiment has become a slightly controversial issue, we treat this in some detail.

In order to define the fidelity, denote the input state by $|\phi\rangle$ (which is here assumed to be pure) and the outgoing (teleported) state by a density matrix $\rho_{\mathrm{out}}$. The fidelity $F$ is the overlap between incoming and outgoing states:

$$F = \mathrm{Tr}[\rho_{\mathrm{out}} |\phi\rangle\langle\phi|]. \tag{2.1}$$

This corresponds to the lower bound for the probability of mistaking $\rho_{\mathrm{out}}$ for $|\phi\rangle$ in any possible (single) measurement [23]. When $\rho_{\mathrm{out}}$ is an exact replica of $|\phi\rangle$, then $F=1$, and when $\rho_{\mathrm{out}}$ is an imprecise copy of $|\phi\rangle$ then $F<1$. Finally, when $\rho_{\mathrm{out}}$ is completely othogonal to $|\phi\rangle$ the fidelity is zero.

In the context of this paper, the fidelity is used to distinguish between quantum teleportation and teleportation which could have been achieved ''classically.'' Classical teleportation is the disembodied transport of some quantum state from Alice to Bob by means of a classical communication channel. There is *no* shared entanglement between Alice and Bob. Since classical communication can be duplicated, such a scheme can lead to many copies of the transported output state (so-called *clones*). Classical teleportation with perfect fidelity (i.e., $F=1$) would then lead to the possibility of *perfect* cloning, thus violating the no-cloning theorem [24]. This means that the maximum fidelity for classical teleportation has an upper bound which is less than 1.

Quantum teleportation, on the other hand, can achieve perfect fidelity (and circumvents the no-cloning theorem by disrupting the original). To demonstrate *quantum* teleportation therefore means [25] that the teleported state should have a higher fidelity than possible for a state obtained by any scheme involving classical communication *alone*.

For classical teleportation of randomly sampled polarizations, the maximum attainable fidelity is $F=2/3$. When only linear polarizations are to be teleported, the maximum attainable fidelity is $F=3/4$ [26–28,23]. These are the values which the quantum teleportation fidelity should exceed.

In the case of the Innsbruck experiment, $|\phi\rangle$ denotes the ''unknown'' linear polarization state of the photon issued by Victor. We can write the undetected outgoing state to lowest order as

$$\rho_{\mathrm{out}} \propto |\alpha|^2 |0\rangle\langle 0| + |\beta|^2 |\phi\rangle\langle\phi|, \tag{2.2}$$

where $|0\rangle$ is the vacuum state. The overlap between $|\phi\rangle$ and $\rho_{\mathrm{out}}$ is given by Eq. (2.1). In the Innsbruck experiment the fidelity $F$ is then given by

$$F \equiv \mathrm{Tr}[\rho_{\mathrm{out}} |\phi\rangle\langle\phi|] = \frac{|\beta|^2}{|\alpha|^2 + |\beta|^2}. \tag{2.3}$$

This should be larger than 3/4 in order to demonstrate quantum teleportation. The vacuum contribution in Eq. (2.2) arises from the fact that Victor cannot distinguish between one photon or several photons entering his detector, i.e., Victor's inability to properly prepare a single-photon state.

As pointed out by Braunstein and Kimble [10], the fidelity of the Innsbruck experiment remains well below the lower bound of 3/4 due to the vacuum contribution (the exact value of $F$ will be calculated in Sec. III). Replying to this, Bouwmeester and co-workers [11,29] argued that ''when a photon appears, it has all the properties required by the teleportation protocol.'' The vacuum contribution in Eq. (2.2) should therefore only affect the efficiency of the experiment, with a consequently high fidelity. However, this is a potentially ambiguous statement. If by ''appear'' we mean ''appearing in a photodetector,'' we agree that a high fidelity (and low efficiency) can be inferred. However, this yields a so-called *postselected* fidelity, where the detection destroys the teleported state. The fidelity prior to (or without) Bob's detection is called the *non*postselected fidelity. The question is now whether we can say that a photon appears when *no* detection is made, thus yielding a high *non*postselected fidelity.

This turns out not to be the case. Making an *ontological* distinction between a photon and *no* photon in a mixed state (without a detection) is based on what we call the ''partition ensemble fallacy.'' We now study this in more detail.

Consider the state $\rho_{\mathrm{out}}$ of the form of Eq. (2.2). To lowest order, it is the sum of two pure states. However, this is not a unique ''partition.'' Whereas in a chemical mixture of, say, nitrogen and oxygen there is a unique partition (into $N_2$ and $O_2$), a quantum mixture can be decomposed many ways. For instance, $\rho_{\mathrm{out}}$ can equally be written in terms of

$$|\psi_1\rangle = \alpha|0\rangle + \beta|\phi\rangle \quad \text{and} \quad |\psi_2\rangle = \alpha|0\rangle - \beta|\phi\rangle \tag{2.4}$$

as

$$\rho_{\mathrm{out}} = \frac{1}{2}|\psi_1\rangle\langle\psi_1| + \frac{1}{2}|\psi_2\rangle\langle\psi_2|. \tag{2.5}$$

In fact, this is just one of an infinite number of possible decompositions. Quantum mechanics dictates that all partitions are equivalent to each other [12]. They are indistinguishable. To elevate one partition over another is to commit the partition ensemble fallacy.

Returning to the Innsbruck experiment, we observe that in the absence of Bob's detection, the density matrix of the teleported state (i.e., the *non*postselected state) may be decomposed into an infinite number of partitions. These partitions do not necessarily include the vacuum state at all, as exemplified in Eq. (2.5). It would therefore be incorrect to say that teleportation did or did not occur except through some operational means (e.g., a detection performed by Bob).

Bob's detection thus leads to a high postselected fidelity. However, the vacuum term in Eq. (2.2) contributes to the *non*postselected fidelity, decreasing it well below the lower bound of 3/4 (see Sec. III). Due to this vacuum contribution, the Innsbruck experiment did *not* demonstrate *non*postselected quantum teleportation. Nonetheless, teleportation was demonstrated using postselected data obtained by detecting the teleported state. By selecting events where a photon was observed in the teleported state, a postselected fidelity higher than 3/4 could be inferred (estimated at roughly 80% [29]). (We recall that this entire discussion is restricted to the subset of events where successful Bell state and state preparation have occurred.)

## III. GENERALIZED EXPERIMENT

In this section we present a generalized scheme for the Innsbruck experiment which enables us to establish the requirements to obtain nonpostselected quantum teleportation (based on a threefold coincidence of Victor and Alice's detectors). The generalization consists of a detector cascade [30] for Victor's state-preparation detection and parametric down-converters with different specifications, rather than two identical down-converters. We consider a detector cascade since single-photon detectors currently require roughly 6 °K operating conditions [17]. Furthermore, an arbitrary polarization rotation in the state-preparation mode allows us to consider any superposition of $x$ and $y$ polarizations.

First, we give an expression for detectors with a finite efficiency. Then we calculate the output state and give an expression for the teleportation fidelity in terms of the detector efficiencies and down-converter probabilities.

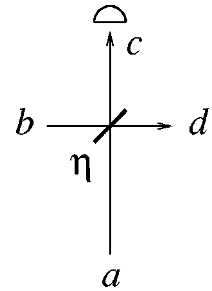


FIG. 3. A model of an inefficient detector. The beam splitter will reflect part of the incoming mode $a$ to mode $d$, which is thrown away. The transmitted part $c$ will be sent into a ideal detector. Mode $b$ is vacuum.

### A. Detectors

There are two sources of errors for a detector: it might fail to detect a photon, or it might give a signal although there was not actually a photon present. The former is called a ''detector loss,'' and the latter a ''dark count.'' Dark counts are negligible in the teleportation experiment because the UV pump is fired during very short-time intervals, and the probability of finding a dark count in such a small interval is negligible. Consequently, the model for real, finite-efficiency detectors we adopt here only takes into account detector losses. Furthermore, the detectors cannot distinguish between one or several photons.

To simulate a realistic detector we make use of projection operator valued measures, or POVM's for short [31]. Consider a beam splitter in the mode which is to be detected so that part of the signal is reflected (see Fig. 3). The second incoming mode of the beam splitter is the vacuum (we neglect higher photon number states because they hardly contribute at room temperature). The transmitted signal $c$ is sent into an ideal detector. We identify mode $d$ with the detector loss.

Suppose in mode $a$ there are $n$ $x$-polarized and $m$ $y$-polarized photons. Furthermore, let these photons all be reflected by the beam splitter (since the detectors cannot distinguish between one or more photons, we do not consider the case where only some of the photons are reflected; we are interested in a ''click'' in the detector and partially reflected modes still give a click). The projector for finding these photons in the $d$ mode is given by

$$E_d = |n,m\rangle_{d_x d_y}\langle n,m| = \frac{1}{n!m!}(d_x^\dagger)^n (d_y^\dagger)^m |0,0\rangle_{d_x d_y}\langle 0,0| d_x^n d_y^m . \tag{3.1}$$

The beam-splitter equations are taken to be ($\tilde{\eta} \equiv \sqrt{1-\eta^2}$)

$$c = \eta a + \tilde{\eta} b \quad \text{and} \quad d = \tilde{\eta} a - \eta b. \tag{3.2}$$

Substituting these equations in Eq. (3.1), summing over all $n$ and $m$, and using the binomial expansion yields

$$E_{ab} = \sum_{n,m} \binom{n}{k}^2 \binom{m}{l}^2 \frac{(-1)^{2(k+l)}}{n!m!} (\tilde{\eta} a_x^\dagger)^{n-k} (\eta b_x^\dagger)^k (\tilde{\eta} a_y^\dagger)^{m-l} \times (\eta b_y^\dagger)^l |0\rangle_{ab}\langle 0| (\tilde{\eta} a_x)^{n-k} (\eta b_x)^k (\tilde{\eta} a_y)^{m-l} (\eta b_y)^l . \tag{3.3}$$

Since the $b$ mode is the vacuum, the only contributing term is $k=l=0$. So the POVM $E_a^{(0)}$ of finding *no* detector counts in mode $a$ is

$$E_a^{(0)} = \sum_{n,m} \frac{\tilde{\eta}^n (a_x^\dagger)^n \tilde{\eta}^m (a_y^\dagger)^m}{n!m!} |0\rangle_{a_x a_y}\langle 0| \tilde{\eta}^n a_x^n \tilde{\eta}^m a_y^m = \sum_{n,m} \tilde{\eta}^{2(n+m)} |n,m\rangle_{a_x a_y}\langle n,m| . \tag{3.4}$$

The required POVM for finding a detector count is

$$E_a^{(1)} = I - E_a^{(0)} = \sum_{n,m} [1 - \tilde{\eta}^{2(n+m)}] |n,m\rangle_{a_x a_y}\langle n,m| , \tag{3.5}$$

where $I$ is the unity operator, $\eta^2$ is the detector efficiency, and $\tilde{\eta}^2$ is the detector loss. When we let $E_a^{(1)}$ act on the total state and trace out mode $a$, we have inefficiently detected this mode. However, it is worth noting that this model only applies for short periods of detection. In the case of continuous detection we need a more elaborate model (see, e.g., Ref. [32]).

In order for Victor to distinguish between one or more photons in the state-preparation mode $a$, we consider a detector cascade (Victor does not have a detector which can distinguish between one photon or several photons coming in). When there is a detector coincidence in the cascade, more than one photon was present in mode $a$, and the event should be dismissed. In the case of ideal detectors, this will improve the fidelity of the teleportation up to an arbitrary level (we assume there are no beam-splitter losses). Since we employ the cascade in the $a$ mode (which was used by Victor to project mode $b$ onto a superposition in the polarization basis) we need to perform a *polarization-sensitive* detection.

In order to model this we separate the incoming state $|n,m\rangle_{a_x a_y}$ of mode $a$ into two spatially separated modes $|n\rangle_{a_x}$ and $|m\rangle_{a_y}$ by means of a polarization beam splitter (see Fig. 4). The modes $a_x$ and $a_y$ will now be detected. The POVM's corresponding to inefficient detectors are derived along the same lines as Sec. II, and read

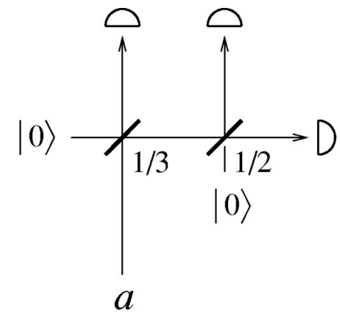


FIG. 4. A simple detector cascade. The fractions 1/2 and 1/3 are the beam splitter's intensity transmission coefficients. Several photons in mode $a$ are likely to enter different detectors, thus revealing that more than one photon was present in this mode.

$$E_{a_j}^{(0)} = \sum_n \tilde{\eta}^{2n} |n\rangle_{a_j}\langle n| , \qquad E_{a_j}^{(1)} = \sum_n [1 - \tilde{\eta}^{2n}] |n\rangle_{a_j}\langle n| . \tag{3.6}$$

with $j \in \{x,y\}$. We choose to detect the $x$-polarized mode. This means that we only have to make sure that there are no photons in the $y$ mode. The output state will include a product of the two POVM's, one for finding a photon in mode $a_x$, and one for finding *no* photons in mode $a_y$: $E_{a_x}^{(1)} E_{a_y}^{(0)}$.

To make a cascade with two detectors in $a_x$ and one in $a_y$ employ another 50:50 beam splitter in mode $a_x$, and repeat the above procedure of detecting the outgoing modes $c$ and $d$ [Eq. (3.6)]. Since we can detect a photon in either one of the modes, we have to include the sum of the corresponding POVM's, yielding a transformation $E_{c_x}^{(1)} E_{d_x}^{(0)} + E_{c_x}^{(0)} E_{d_x}^{(1)}$. This is easily expandable to larger cascades by using more beam splitters and summing over all possible detector hits.

### B. Output state

In this section we incorporate the finite-efficiency detectors and the detector cascade in our calculation of the undetected teleported output state. This calculation includes the creation of two photon pairs (lowest order) and three photon pairs (higher-order corrections due to four or more photon pairs in the experiment are highly negligible). A formula for the vacuum contribution to the teleportation fidelity is given for double-pair production (lowest order).

Let the two down-converters in the generalized experimental setup yield evolutions $U_{\text{src1}}$ and $U_{\text{src2}}$ on modes $a$, $b$ and $c$, $d$ respectively (see Figs. 1 and 2) according to Eq. (1.4). The beam splitter which transforms modes $b$ and $c$ into $u$ and $v$ (see Fig. 2) is incorporated by a suitable unitary transformation $U_{\text{BS}}$, as is the polarization rotation $U_\theta$ over an angle $\theta$ in mode $a$. The $n$ cascade will be modeled by $n-1$ beam splitters in the $x$-polarization branch of the cascade, and can therefore be expressed in terms of a unitary transformation $U_{a_1 \ldots a_n}$ on the Hilbert space corresponding to modes $a_1$ to $a_n$ (i.e., replace mode $a$ with modes $a_1$ to $a_n$):

$$|\Psi_\theta\rangle\langle\Psi_\theta| = U_{a_1\ldots a_n} U_\theta U_{\mathrm{BS}} U_{\mathrm{scr1}} U_{\mathrm{scr2}} |0\rangle \times \langle 0| U^\dagger_{\mathrm{scr1}} U^\dagger_{\mathrm{scr2}} U^\dagger_{\mathrm{BS}} U^\dagger_\theta U^\dagger_{a_1\ldots a_n}. \quad (3.7)$$

Detecting modes $a_1 \ldots a_n$, $u$ and $v$ with real (inefficient) detectors means taking the partial trace over the detected modes, including the POVM's derived in Sec. III A,

$$\rho_{\mathrm{out}} = \mathrm{Tr}_{a_1\ldots a_n uv}[E_{n\text{-cas}} E_u^{(1)} E_v^{(1)} |\tilde{\Psi}_\theta\rangle_{a_1\ldots a_n uvd} \langle \tilde{\Psi}_\theta|], \quad (3.8)$$

with $E_{n\text{-cas}}$ the superposition of POVM's for a polarization-sensitive detector cascade having $n$ detectors with finite efficiency. In the case $n=2$ this expression reduces to the two-cascade POVM superposition derived in Sec. II. Equation (3.8) is an analytic expression of the undetected outgoing state in the generalization of the Innsbruck experiment.

The evolutions $U_{\mathrm{src1}}$ and $U_{\mathrm{src2}}$ are exponentials of creation operators. In the computer simulation (using MATHEMATICA) we truncated these exponentials at first and second orders. The terms that remain correspond to double- and triple-pair production in the experimental setup. To preserve the order of the creation operators we put them as arguments in a function *f*. We defined the following algebraic rules for *f*:

$$\begin{aligned} f[x,y+w,z] &:= f[x,y,z] + f[x,w,z], \\ f[x,na,y] &:= n f[x,a,y], \\ f[x,na^\dagger,y] &:= n f[x,a^\dagger,y], \end{aligned} \quad (3.9)$$

where $x,y,z$, and $w$ are arbitrary expressions including creation and annihilation operators ($a^\dagger$ and $a$), and $n$ some expression *not* depending on creation or annihilation operators. The last entry of $f$ is always a photon number state (including the initial vacuum state).

Since we now have functions of creation and annihilation operators, it is quite straightforward to define (lists of) substitution rules for a beam splitter [see also Eq. (3.2)], polarization rotation, POVM's, and the trace operation. We then use these substitution rules to ''build'' a model of the generalized experimental setup.

### C. Results

The probability of creating one entangled photon pair using the weak parametric down-conversion source 1 or 2 is $p_1$ or $p_2$, respectively (see Fig. 2). We calculated the output state both for an $n$ cascade up to order $p^2$ (i.e., $p_1^2$ or $p_1p_2$) and for a 1 cascade up to the order $p^3$ ($p_1^3$, $p_1^2p_2$ or $p_1p_2^2$). The results are given below. For brevity, we take

$$\begin{aligned} |\Psi_\theta\rangle &= \cos\theta|0,1\rangle + \sin\theta|1,0\rangle, \\ |\Psi_\theta^\perp\rangle &= \sin\theta|0,1\rangle - \cos\theta|1,0\rangle \end{aligned} \quad (3.10)$$

as the ideally prepared state and the state orthogonal to it. Suppose $\eta_u^2$ and $\eta_v^2$ are the efficiencies of the detectors in mode $u$ and $v$, respectively, and $\eta_c^2$ the efficiency of the detectors in the cascade (for simplicity we assume that the detectors in the cascade have the same efficiency). Define $g_{uvc} = \eta_u^2\eta_v^2\eta_c^2$. The detectors in modes $u$ and $v$ are polarization insensitive, whereas the cascade consists of polarization-sensitive detectors. Bearing this in mind, we have up to order $p^2$ for an $n$ cascade in mode $a_x$, and find no detector click in the $a_y$ mode,

$$\rho_{\mathrm{out}} \propto \frac{p_1}{8} g_{uvc} \left\{ \frac{p_1}{n}[1+(5n-3)(1-\eta_c^2)]|0\rangle\langle 0| + p_2|\Psi_\theta\rangle\langle\Psi_\theta| \right\} + O(p^3), \quad (3.11)$$

where the vacuum contribution formula was calculated and found to be correct for $n \leqslant 4$ (and $n \neq 0$).

In order to have nonpostselected quantum teleportation, the fidelity $F$ must be larger than 3/4 [28,23]. Since we only estimated the two lowest-order contributions (to $p^2$ and $p^3$), the fidelity is also correct up to $p^2$ and $p^3$, and we write $F^{(2)}$ and $F^{(3)}$, respectively. Using Eqs. (2.3) and (3.11), we have

$$F^{(2)} = \frac{np_2}{p_1[1+(5n-3)(1-\eta_c^2)]+np_2} \geqslant \frac{3}{4} \quad (3.12)$$

$$\Rightarrow \eta_c^2 \geqslant \frac{(15n-6)p_1 - np_2}{(15n-9)p_1}. \quad (3.13)$$

This means that in the limit of infinite detector cascading ($n\to\infty$) and $p_1 = p_2$ the efficiency of the detectors must be better than 93.3% to achieve nonpostselected quantum teleportation. When we have detectors with efficiencies of 98%, we need at least four detectors in the cascade to obtain unequivocal quantum teleportation. *The necessity of a lower bound on the efficiency of the detectors used in the cascade might seem surprising*, but this can be explained as follows. Suppose the detector efficiencies become smaller than a certain value *x*. Then upon a two-photon state entering the detector, finding only one click becomes more likely than finding a coincidence, and ''wrong'' events end up contributing to the output state. Equation (3.13) places a severe limitation on the practical use of detector cascades in this situation.

In the experiment in Innsbruck, no detector cascade was employed and also the $a_y$ mode was left undetected. The state entering Bob's detector therefore was (up to order $p^2$)

$$\rho_{\mathrm{out}} \propto \frac{p^2}{8} g_{uvc}[(3-\eta_c^2)|0\rangle\langle 0| + |\Psi_\theta\rangle\langle\Psi_\theta|] + O(p^3). \quad (3.14)$$

Remember that $p_1 = p_2$, since the experiment involves one source which is pumped twice. The detector efficiency $\eta_c^2$ in the Innsbruck experiment was 10% [33], and the fidelity without detecting the outgoing mode therefore would have been $F^{(2)} \simeq 26\%$ (conditioned only on successful Bell detection and state preparation). This clearly exemplifies the need for Bob's detection. Braunstein and Kimble [10] predicted a theoretical maximum of 50% for the teleportation fidelity, which was conditioned upon (perfect) detection of both the $a_x$ and the $a_y$ modes.

Rather than improving the detector efficiencies and using a detector cascade, Eq. (3.12) can be satisfied by adjusting the probabilities $p_1$ and $p_2$ of creating entangled photon pairs [10]. From Eq. (3.12) we have

$$p_1 \leqslant \frac{n}{3[1+(5n-3)(1-\eta_c^2)]} p_2 . \tag{3.15}$$

Experimentally, $p_1$ can be diminished by employing a beam splitter with a suitable reflection coefficient rather than a mirror to reverse the pump beam (see Fig. 1). Bearing in mind that $\kappa$ is proportional to the pump amplitude, the equation $p_i = 2\tanh^2(\kappa_i t)$ [see the discussion following Eq. (1.15), with $i=1$ and 2] gives a relation between the pump amplitude and the probability of creating a photon pair. In particular, when $p_2 = x p_1$,

$$\frac{\tanh(\kappa_2 t)}{\tanh(\kappa_1 t)} = \sqrt{x}. \tag{3.16}$$

Decreasing the production rate of one photon-pair source will increase the time needed to run the experiment. In particular, we have from Eq. (3.14) that

$$p_2 \geqslant 3(3-\eta_c^2) p_1 . \tag{3.17}$$

With $\eta_c^2 = 10\%$, we obtain $p_2 \geqslant 8.7 p_1$. Using Eq. (1.19) we estimate that diminishing the probability $p_1$ by a factor 8.7 will increase the running time by that same factor (i.e., running the experiment about nine days, rather than 24 h).

The third-order contribution to the outgoing density matrix without cascading and without detecting the $a_y$ mode is

$$\begin{aligned}\rho_{\mathrm{out}} \propto & \frac{p_1}{8} g_{uvc}(4-\eta_u^2-\eta_v^2)\frac{1}{16}[6p_1^2(6-4\eta_c^2+\eta_c^4)|0\rangle \\ & \times\langle 0| + 2p_1p_2(2-\eta_c^2)(|\Psi_\theta\rangle\langle\Psi_\theta| + |\Psi_\theta^\perp\rangle\langle\Psi_\theta^\perp|) \\ & + 8p_1p_2(3-\eta_c^2)\rho_1 + 12p_2^2\rho_2],\end{aligned} \tag{3.18}$$

with

$$\begin{aligned}\rho_1 &= \frac{1}{2}(|1,0\rangle\langle 1,0| + |0,1\rangle\langle 0,1|), \\ \rho_2 &= \frac{1}{6}\bigg[(2+\cos 2\theta)|0,2\rangle\langle 0,2| + (2-\cos 2\theta)|2,0\rangle \\ & \quad \times\langle 2,0| + 2|1,1\rangle\langle 1,1| + \frac{1}{2}\sqrt{2}\sin 2\theta(|2,0\rangle\langle 1,1| + |1,1\rangle \\ & \quad \times\langle 2,0| + |0,2\rangle\langle 1,1| + |1,1\rangle\langle 0,2|)\bigg].\end{aligned} \tag{3.19}$$

We have explicitly extracted the state which is to be teleported ($|\Psi_\theta\rangle\langle\Psi_\theta|$) from the density-matrix contribution $\rho_1$ (this is not *necessarily* the decomposition with the largest $|\Psi_\theta\rangle\langle\Psi_\theta|$ contribution). As expected, this term is less important in the third order than it is in the second. In the Appendix it is shown that the $n$-photon contribution to the outgoing density matrix is always proportional to $p_2^n$.

The teleportation fidelity including the third-order contribution (3.18) can be derived along the same lines as Eq. (3.12). Assuming that all detectors have the same efficiency $\eta^2$ and $p_1 = p_2 = p$, the teleportation fidelity up to third order is

$$F^{(3)} = \frac{4+p(2-\eta^2)^2}{4(4-\eta^2)+p(80-76\eta^2+34\eta^4-3\eta^6)}. \tag{3.20}$$

With $p=10^{-4}$ and a detector efficiency of $\eta^2 = 0.1$, this fidelity differs from Eq. (3.12), with only a few parts in ten thousand:

$$\frac{F^{(2)}-F^{(3)}}{F^{(2)}} \propto p \sim 10^{-4}. \tag{3.21}$$

On the other hand, let us compare two experiments in which the cascades have different detector efficiencies (but all the detectors in one cascade still have the same efficiency). The ratio between the teleportation fidelity with detector efficiencies $\eta_-^2$ and $\eta_+^2$ (with $\eta_-^2$ and $\eta_+^2$ the lower and higher detector efficiencies respectively) up to lowest order is

$$\frac{F_{95\%}^{(2)}-F_{10\%}^{(2)}}{F_{95\%}^{(2)}} \propto \frac{\Delta\eta^2}{2-\eta_-^2} \sim 10^{-1}, \tag{3.22}$$

where $\Delta\eta^2$ is the difference between these efficiencies. This shows that detector efficiencies have a considerably larger influence on the teleportation fidelity than the higher-order pair production.

To summarize our results, we have found that detector cascading is only useful when the detectors in the cascade have near-unit efficiency. In particular, there is a lower bound to the efficiency below which an increase in the number of detectors in the cascade actually decreases the ability to distinguish between one or several photons entering the cascade. Finally, enhancement of the photon-pair source responsible for the entanglement channel relative to the one responsible for the state preparation increases the time needed to run the experiment by an order of magnitude.

## IV. CONCLUSIONS

We studied the experimental realization of quantum teleportation as performed in the Innsbruck experiment [5] including possible improvements suggested by Braunstein and Kimble to achieve a high nonpostselected fidelity [10]. The creation of entangled photon pairs using parametric down-conversion was analyzed, and we presented a discussion about the teleportation fidelity. Finally, we determined the usefulness of detector cascading and the slowdown of one down-converter relative to the other for the generalized experiment.

The difficulties of the Innsbruck experiment can be traced to the state preparation (i.e., to the sources of the entangled photon pairs, see Fig. 2). In particular, there is a probability that the source responsible for creating entangled photon pairs produces two pairs simultaneously. We studied these sources in some detail, and have found that *photon pairs created in a parametric down-converter are not independent of each other*. Employing two parametric down-converters therefore automatically probes the non-Poissonian structure of these sources.

The teleported *state* in the Innsbruck experiment is a mixture of the vacuum and a single-photon state. However, we cannot interpret this state as a low-efficiency teleported state, where sometimes a photon emerges from the apparatus and sometimes not. This reasoning is based on a particular partition of the outgoing density matrix, and this is not consistent with quantum mechanics (to our knowledge, this is the first instance where the PEF leads to a different evaluation of an experiment). In Sec. II we showed how a high fidelity in the Innsbruck experiment could only be interpreted in a postselected manner.

The interpretation of what quantum teleportation is, gives rise to different evaluations of the Innsbruck experiment. When one holds that the freely propagating output state of quantum teleportation should resemble the input state sufficiently closely (i.e., nonpostselected quantum teleportation), the nonpostselected teleportation fidelity in the Innsbruck experiment should be at least 3/4. This requirement was not met. Nonetheless the Innsbruck experiment demonstrated *postselected* quantum teleportation (i.e., teleportation conditioned on the detection of the outgoing state).

In the generalized version of the Innsbruck experiment (*à la* Braunstein and Kimble) we have modeled a detector cascade in the state-preparation mode. However, for the cascade to work, the detectors need to have near unit efficiency. In particular, for infinite cascading the efficiency of the detectors should be at least 93%. Finite cascading requires even higher detector efficiencies. This places a severe limitation on the practical use of detector cascades in this situation. Detector losses in the cascade have an immediate influence on the teleportation fidelity, yielding an effect which is much stronger than the higher-order corrections due to multiple-pair creation (three pairs or more) of the down-converters.

If the stability of the experimental setup can be maintained for a longer time (the order of weeks), it is possible to slow down the down-converter responsible for creating the unknown input state. This can improve the fidelity up to arbitrary level. Nevertheless, we feel that our analysis demonstrates the definite benefits of single-photon detectors for such experiments or applications in the future. This technology currently requires roughly 6 °K operating conditions.

## ACKNOWLEDGMENT

This research was funded in part by EPSRC Grant No. GR/L91344.

## APPENDIX: CROSS-TERMS

In this appendix we show that all the cross-terms of the density matrix in Eq. (3.8) must vanish. The density matrix consists of several distinct parts: a vacuum contribution, a contribution due to one photon in mode $d$, two photons, and so on. Suppose there are $n$ photon pairs created in the whole system, and $m$ photon pairs out of $n$ are produced by the second source (modes $c$ and $d$). The outgoing mode must then contain $m$ photons. Reversing this argument, when we find $m$ photons in the outgoing mode the probability of creating this particular contribution must be proportional to $p_1^{n-m}p_2^m$. Expanding the $n$th-order output state into parts of definite photon number, we can write

$$\rho_{\mathrm{out}}^{(n)}=\sum_{m=0}^{n-1} p_1^{n-m}p_2^m\rho_m^{(n)}, \tag{A1}$$

where $\rho_m^{(n)}$ is the (unnormalized) $n$th-order contribution containing all terms with $m$ photons.

An immediate corollary of this argument is that all the cross-terms between different photon number states in the density matrix must vanish. The cross-terms *are* present in Eq. (3.7), and we must therefore show that the partial trace in Eq. (3.8) makes them vanish. Suppose there are $n$ photons in the total system. A cross-term in the density matrix will have the form

$$|j,k,l,m\rangle_{auvd}\langle j',k',l',m'|,$$

with $m\neq m'$. We also know that $j+k+l+m=j'+k'+l'+m'=n$, so that at least one of the other modes must have the cross-term property as well. Suppose $k$ is not equal to $k'$. Since we have $\mathrm{Tr}[|k\rangle\langle k'|]=\delta_{k,k'}$, the cross-terms must vanish.

[1] A.K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
[2] P.W. Shor, Phys. Rev. A **52**, R2493 (1995).
[3] C.H. Bennett and S.J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
[4] C.H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W.K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[5] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).
[6] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).
[7] M.A. Nielsen, E. Knill, and R. Laflamme, Nature (London) **396**, 52 (1998).
[8] A. Furusawa, J.L. Sørensen, S.L. Braunstein, C.A. Fuchs, H.J. Kimble, and E.S. Polzik, Science **282**, 706 (1998).
[9] See also item 6 of the top ten scientific achievements in 1998, Science **282**, 2159 (1998).
[10] S.L. Braunstein and H.J. Kimble, Nature (London) **394**, 840 (1998).
[11] D. Bouwmeester, J.-W. Pan, M. Daniell, H. Weinfurter, M. Zukowski, and A. Zeilinger, Nature (London) **394**, 841 (1998).

[12] A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer, Dordrecht, 1995), p. 75; E. Merzbacher, *Quantum Mechanics*, 3rd ed. (Wiley New York, 1998), p. 366.
[13] H. Weinfurter, Europhys. Lett. **25**, 559 (1994).
[14] S.L. Braunstein and A. Mann, Phys. Rev. A **51**, R1727 (1995).
[15] P.G. Kwiat, A.M. Steinberg, R.Y. Chiao, P.H. Eberhard, and M.D. Petroff, Appl. Opt. **33**, 1844 (1994).
[16] P. G. Kwiat and R. Hughes (private communication).
[17] S. Takeuchi, J. Kim, Y. Yamamoto, and H.H. Hogue, Appl. Phys. Lett. **74**, 1063 (1999); J. Kim, S. Takeuchi, Y. Yamamoto, and H.H. Hogue, *ibid.* **74**, 902 (1999).
[18] P.G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A.V. Sergienko, and Y. Shih, Phys. Rev. Lett. **75**, 4337 (1995).
[19] D.R. Truax, Phys. Rev. D **31**, 1988 (1985).
[20] W.K. Wootters, Phys. Rev. D **23**, 357 (1981).
[21] S.L. Braunstein and C.M. Caves, Phys. Rev. Lett. **72**, 3439 (1994).
[22] J. Hilgevoord and J. Uffink, Found. Phys. **21**, 323 (1991).
[23] C.A. Fuchs, N. Gisin, R.B. Griffiths, C.-S. Niu, and A. Peres, Phys. Rev. A **56**, 1163 (1997).
[24] W.K. Wootters and W.H. Zurek, Nature (London) **299**, 802 (1982).
[25] The fidelity captures one particular feature of quantum teleportation very well, and has been extensively studied so far.
[26] C. A. Fuchs, Ph.D. thesis, University of New Mexico, 1996 (unpublished).
[27] S. Massar and S. Popescu, e-print quant-ph/9907066.
[28] S. Massar and S. Popescu, Phys. Rev. Lett. **74**, 1259 (1995).
[29] D. Bouwmeester, J.-W. Pan, H. Weinfurter, and A. Zeilinger, e-print quant-ph/9910043 [J. Mod. Opt. (to be published)].
[30] S. Song, C.M. Caves, and B. Yurke, Phys. Rev. A **41**, R5261 (1990).
[31] K. Kraus, *States, Effects and Operations: Fundamental Notions of Quantum Theory* (Springer, Berlin, 1983).
[32] H.M. Wiseman and G.J. Milburn, Phys. Rev. A **47**, 642 (1993).
[33] H. Weinfurter (private communication).