The 2009 IEEE/RSJ International Conference on
Intelligent Robots and Systems
October 11-15, 2009 St. Louis, USA

Potential Based Control Strategy for Arbitrary Shape Formations of Mobile Robots

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***Abstract*— In this paper we describe a novel decentralized control strategy to realize formations of mobile robots. We first describe a methodology to obtain a formation with the shape of a regular polygon. Then, applying a bijective coordinates transformation, we show how to obtain a formation with an arbitrary shape. Our control strategy is based on the interaction of some artificial potential fields, but it is not affected by the problem of local minima.**

I. Introduction

This paper describes a novel strategy to realize formations of mobile robots with arbitrary shape.

In the literature, many different approaches to formation control can be found. The main existing approaches can be divided into two categories: centralized and distributed. Because of the intrinsic unreliability of centralized methods [8], we focus our attention to distributed ones: all the agents are equal, and if one of them stops working, the other ones can still complete their task.

Many distributed strategies have been proposed to make a group of mobile robots move in a cohesive way [5], [9], [10], imitating the behavior of large groups of animals (e.g. school of fish). However, the aim of our control strategy is quite different: we want a group of mobile robots to create a formation with an exact desired geometric shape. In fact, our target application is a group of Automated Guided Vehicles (AGVs) moving in a warehouse for goods delivery. We want a group of AGVs to cooperatively deliver a certain amount of goods, moving in a formation. The creation of a formation with the desired shape is useful to precisely limit the action zone of the AGVs, thus reducing the chance of collisions with other entities (e.g. human guided vehicles).

Potential based control strategies make robots move along the negative gradient of the composition of some artificial potential fields. Correctly shaping these potential fields allows one to impose a desired behavior to a group of robots. While most of the potential based control strategies have the aim of controlling only the overall swarm geometry (e.g. [2], [1]), recently some strategies have appeared to control the exact shape of the formation. One possible approach is to deploy a group of robots over a desired curve [11], [7], [4]. However, our control strategy allows us not only to equitably deploy the robots over a curve, but to specify their exact positions. Previous potential based strategies to obtain formation with exact geometric shape [8] have the drawback that, as the number of agents increases, many local minima appear. Local minima are asymptotically stable undesired equilibrium points. Thus, they are one of the main problem in potential based strategies [3], because they make the agents stop in undesired positions.

However, our control strategy doesn't lead to the creation of local minima. Thus, the desired formation is always created.

We will start describing a methodology to obtain a formation with the shape of a regular polygon, in Sec. II and III. In Sec. IV we will describe how to extend our control strategy to obtain formations with arbitrary shapes, by means of a coordinates transformation. In Sec. V we will show some simulations to validate the results obtained in the paper. Sec. VI contains some concluding remarks.

II. Regular polygon control law

In this paper we consider a group of n point mass holonomic agents characterized by the following dynamics:

$$\ddot{x}_i = u_i \quad i = 1, ..., n \tag{1}$$

where $x_i \in \mathbb{R}^2$ is the position of the i–th agent. The dynamic behavior we are considering is quite simple, but all the results obtained in the paper can be extended to nonholonomic vehicles. In fact, many strategies can be found (e.g. [6] and [13]) to feedback linearize several classes of nonholonomic vehicles. Furthermore, we suppose that the agents can localize themselves exactly. Referring to our target application, a group of AGVs moving in a warehouse, this can be done, for example, by means of laser triangulation.

Let W be the sensing range of each agent. Each agent knows only the positions of its neighbors, which are the agents that are closer than W.

We want the agents to create a formation with the shape of a regular polygon with n sides. More specifically, we want the length of every side (i.e. the distance between two neighboring agents) to be equal to $L \leq W$, and the circumcenter of the polygon to be in a certain position $x_c \in \mathbb{R}^2$. Let R be the radius of the circumcircle of the polygon (i.e. the distance between each agent and the circumcenter): from basic geometrical considerations, it follows that $R = L/\left[2\sin\left(\pi/n\right)\right]$.

We assume that each agent knows the position of the center of the circumcircle, x_c, the number of agents, n, and the desired distance between two neighboring agents, L. We remark that knowing the total number of agents is necessary to create a formation with an exact geometric shape.

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We implement the following control law to obtain the desired behavior:

$$u_i = f_{ci} + \sum_{j=1;j\neq i}^{n} f_{aij} - b\dot{x}_i \tag{2}$$

where b is a positive constant which implements a damping action.

The first term of Eq. (2) is defined as follows:

$$f_{ci} = -\nabla_{x_i} V_{ci}\left(x_i\right) \tag{3}$$

and

$$V_{ci}\left(x_i\right) = \frac{1}{2} K_c (d_{ci} - R)^2 \tag{4}$$

where $d_{ci}\left(t\right) = \|x_i\left(t\right) - x_c\|$, and K_c is a positive constant. The role of this term is to take each agent at distance R from the desired position for the center of the formation. In other words, if no other potential fields were present, this term would make every agent move to a circumference with center x_c and radius R.

The second term of Eq. (2) is defined by the following components:

$$f_{aij} = -\nabla_{x_i} V_{aij}\left(x_i, x_j\right) \tag{5}$$

and

$$V_{aij}\left(x_i, x_j\right) = \begin{cases} \frac{1}{2} K_a (d_{ij} - L)^2 & \text{if } d_{ij} \leq L \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where $d_{ij}\left(t\right) = \|x_i\left(t\right) - x_j\left(t\right)\|$, and K_a is a positive constant. It's easy to see that function V_{aij} is continuous (and differentiable).This term is used to regulate the distances among the agents. This interagent potential produces a repulsive force if two agents are too close, namely if $d_{ij} < L$, and produces a null force if the distance is greater than or equal to the desired one, namely if $d_{ij} \geq L$.

Thus, the composition of these potential fields produces the following behavior:

1) All the agents move toward a circumference with center x_c and radius R. No collisions among the agents can happen, because of the presence of the control action in Eq. (5).
2) When all the agents lie on the circumference, the control action in Eq. (3) is null. The control action in Eq. (5) regulates the relative distances among the agents, until they are in the desired configuration.
3) In the desired configuration, the composition of the potentials gives a null control action, because the agents are on the circumference ($f_{ci} = 0\ \forall i = 1, \ldots, n$), and the distance between each couple of agents is equal to L ($f_{aij} = 0\ \forall i, j = 1, \ldots, n$).

Proposition 1 *The regular polygon formation is an asymptotically stable configuration.*

Let $x = \left[x_1^T \ldots x_n^T\ \dot{x}_1^T \ldots \dot{x}_n^T\right]^T \in \mathcal{X}$ be the state vector of the system. Let us introduce the following Lyapunov candidate function $V : \mathcal{X} \to \mathbb{R}$, given by the total energy of the system:

$$V\left(x\right) = \sum_{i=1}^{n} \left[V_{ci}\left(x_i\right) + \sum_{j=1;j\neq i}^{n} V_{aij}\left(x_i, x_j\right) + \frac{1}{2}\|\dot{x}_i\|^2 \right] \tag{7}$$

From Eq. (4) and Eq. (6) one can trivially see that $V \geq 0$. V is the sum of three terms which are always positive or null. Thus, we need all of them to be equal to zero for V to be equal to zero. More specifically, we have $V = 0$ if and only if, simultaneously:

1) $\dot{x}_i = 0\ \forall i = 1, ..., n$; i.e. all the agents are at some steady state position (they don't move from their current position);
2) $V_{ci} = 0\ \forall i = 1, ..., n$; i.e. all the agents are on the circumference with center x_c and radius R;
3) $V_{aij} = 0\ \forall i, j = 1, ..., n$; i.e. all the agents are at a distance greater than or equal to L with respect to their neighbors ($d_{ij} \geq L\ \forall i, j = 1, ..., n$).

From basic geometrical considerations it follows that conditions 2 and 3 can hold simultaneously if and only if $d_{ij} = L$ $\forall i, j = 1, ..., n$. In other words, $V \geq 0$ always, an $V = 0$ only in the regular polygon formation (no local minima).

Consider the time derivative of this function:

$$\dot{V}\left(x\right) = \sum_{i=1}^{n} \dot{x}_i \left[\nabla_{x_i} V_{ci}\left(x_i\right) + \sum_{j=1;j\neq i}^{n} \nabla_{x_i} V_{aij}\left(x_i, x_j\right) + \ddot{x}_i \right] \tag{8}$$

From Eq. (1), Eq. (2), Eq. (3) and Eq. (5) we obtain the following equation:

$$\ddot{x}_i = -\nabla_{x_i} V_{ci}\left(x_i\right) - \sum_{j=1;j\neq i}^{n} \nabla_{x_i} V_{aij}\left(x_i, x_j\right) - b\dot{x}_i \tag{9}$$

Thus, from Eq. (8) and Eq. (9):

$$\dot{V}\left(x\right) = -\sum_{i=1}^{n} b\|\dot{x}_i\|^2 \tag{10}$$

which is always less than or equal to zero.

The asymptotic stability can be proved by invoking LaSalle's principle.

The desired configuration is not globally asymptotically stable because undesired equilibrium configurations appear when two or more vehicles are aligned with x_c. In this case the potentials never generate a force perpendicular to the alignment direction and, therefore, the aligned agents would never play their role in the creation of the desired polygonal formation. Nevertheless, these equilibrium points are not local minima but they are clearly unstable. In fact, an infinitesimal perturbation of the position of the aligned agents is sufficient for the potentials to create a force that leads the agents to the desired configuration. Thus, in order to avoid some agents to get stuck in this undesired configuration, when an agents detects that it's aligned with x_c and with another agent, it applies a random infinitesimal force that modifies its position in order destroy the alignment condition and to converge to the desired polygonal configuration. The possibility that all the aligned agents apply a force in the same direction and that, therefore, the alignment condition is preserved after the perturbation, is practically zero.

Hence, the regular polygon configuration is the only asymptotically stable configuration of the system. Thus, unlike other potential–based methods [3], in our control strategy local minima never appear.

III. Orientation of the polygon

The control strategy presented in the previous section admits a symmetry. In fact, given n agents, there are infinite regular polygons with n sides lying on the same circumcircle, and our control strategy just takes the agents in one admissible configuration. However, in many applications it is very useful to select exactly *one* of these infinite admissible configurations. To solve this problem, we have to fix the orientation of the formation. To this aim, we need to modify the control law presented in the previous section.

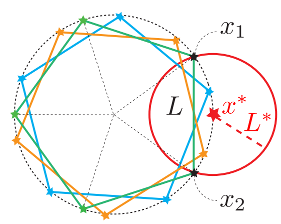


Fig. 1. The action zone of the orientation component of the control law must be such that it influences one and only one agent at the steady state

In Fig. 1 one can see three admissible configurations, obtained by rotating the polygon around its circumcenter. The system has one degree of freedom: to select one precise polygon, we need one condition to eliminate this degree of freedom. One way to do this is to select the position of one of the vertices of the polygon. Thus, define x^* as the position to be occupied by one of the vertices of the polygon. Fixing the position of one of the vertices, we can select the orientation of the polygon. Since we want all the agents to be indistinguishable, we don't want to select a priori which agent will be in the position x^*. Thus, we introduce a new potential V_{oi} which attracts to x^* every agent that is inside a proper region of attraction. It is now necessary to define this region of attraction.

Let $\mathcal{C} = \{x \text{ s.t. } \|x - x^*\| \leq L^*\}$ be a circle whose border intersects the circumcircle of the polygon in two points x_1 and x_2 such that $\|x_1 - x_2\| = L$ (Fig. 1). From simple geometrical considerations, it follows that $L^* = L/\left\{2\cos\left[\left(\arcsin\left(L/2R\right)\right)/2\right]\right\}$. Assume that $\mathcal{C}$ is the region of attraction. If one agent is inside $\mathcal{C}$, the action of V_{oi} would attract this agent to x^* taking the polygon at the right orientation. Nevertheless, if two agents are in x_1 and x_2, they are both attracted to x^* and the interaction between V_{oi} and the interagent potential creates a local minimum which deforms the final shape of the formation. On the other hand, if we exclude the border of $\mathcal{C}$ from the region of attraction, we have another pathological case. In fact, in this case, if two agents are in x_1 and x_2, none of them is attracted to x^* and the orientation of the polygon is not changed as desired. In order to avoid these undesired behaviors, we define the region of attraction as

$$\mathcal{S}^* = \{x \text{ s.t. } \|x - x^*\| < L^*\} \cup \{x_1\} \tag{11}$$

Note that x_1 can be substituted by x_2 as well.

Thus, we implement the following control law:

$$u_i = f_{ci} + \sum_{j=1;j\neq i}^{n} f_{aij} + f_{oi} - b\dot{x}_i \tag{12}$$

This control law can be obtained from Eq. (2) by adding the term f_{oi}, which is defined as follows:

$$f_{oi} = -\nabla_{x_i} V_{oi}\left(x_i\right) \tag{13}$$

and

$$V_{oi}\left(x_i\right) = \begin{cases} \frac{1}{2}K_o(d_{oi})^2 & \text{if } x_i \in \mathcal{S}^* \\ K^* & \text{otherwise} \end{cases} \tag{14}$$

where K_o and K^* are constants, with $K_o > 0$, and $d_{oi}\left(t\right) = \|x_i\left(t\right) - x^*\|$.

We have shown that, after the polygon has been created, one and only one agent would be influenced by the orientation action. But during the transient (i.e., before the polygon has been created) it can happen that two or more agents are inside $\mathcal{S}^*$. For the polygon to be correctly created, we need that the distance between two neighboring agents is equal to L, even in presence of this orientation component. Thus, if two or more agents are inside $\mathcal{S}^*$, they must get far from one other, until they reach the correct relative positions. In other words, the gain of the orientation component (K_o) must be much smaller than the gain of the interagent component (K_a). Namely, we must chose these gains such that $K_a \gg K_o$. This ensures that, in presence of both the components, the orientation one becomes negligible, and the polygonal formation is correctly created. Once the agents are in the polygonal formation, only one of them is inside $\mathcal{S}^*$, and the formation is taken to the desired orientation.

IV. Deformation of the polygon: bijective coordinates transformation

For many applications it is very useful to obtain formations with shapes different from regular polygons. Our main idea is to obtain a formation with an arbitrary shape by deforming the regular polygon, as shown in Fig. 2. In this picture, the

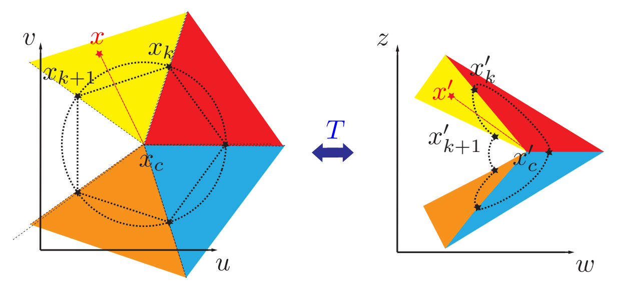


Fig. 2. To obtain an arbitrary shape, we deformed the regular polygon by means of a bijective coordinates transformation

reference frame (w,z) represents the *real* reference frame; the *real* positions of the agents are measured with respect to the coordinate set (w,z). The reference frame (u,v) is an auxiliary reference frame. We introduce a bijective coordinates transformation T that allows us to relate the desired positions for the agents in (w,z) to the positions of the vertices of a regular polygon in (u,v).

Thus, we propose the following control strategy:

1) Each agent measures its own position, and the positions of its neighbors, with respect to the real reference frame (w,z).
2) Each agent transforms these positions using the transformation T, and obtains the values of these positions with respect to the auxiliary reference frame (u,v).
3) Then, it calculates the control action as described in the previous sections, with respect to the auxiliary reference frame (u,v).
4) Finally, applying the inverse transformation, it finds the value of the control action with respect to the real reference frame (w,z), and can apply it.

Thus, we obtain a formation that has the shape of a regular polygon with respect to the auxiliary reference frame (u,v), but has the desired shape with respect to the real reference frame (w,z).

We will now define a bijective transformation of coordinates T which allows us to map n arbitrary positions into the positions of the vertices of a regular polygon. We have only to ensure that the distance between each couple of neighboring position is less than the visibility range W.

Refer to the left–hand picture in Fig. 2. We partition the (u,v) reference frame, creating n triangular zones (where n is the number of agents in the formation). The partition is created drawing n rays: each ray starts at the circumcenter of the polygon x_c and passes through a vertex. Thus, the environment is partitioned in n zones, whose borders are these n rays.

Referring to the right–hand picture in Fig. 2, the (w,z) reference frame can be partitioned in a similar way. The partition is created drawing n rays: each ray starts at x'_c and passes through the desired position of an agent in the desired formation. x'_c is the image of x_c under the transformation T. We will show in the sequel what conditions have to hold to determine its position.

Once defined the partitions in the two coordinates sets, we have to correlate them by means of a bijective relation. This relation maps each vertex of the polygon in (u,v) into the desired position of an agent in the formation in (w,z). The circumcenter of the polygon $x_c = (u_c, v_c)^T$ is mapped into the point $x'_c = (w_c, z_c)^T$. Then, each triangular zone in the (u,v) reference frame is mapped into one triangular zone in the (w,z) reference frame. For example, referring to Fig. 2, the triangular zone defined by the points (x_k, x_{k+1}) has to be mapped into the triangular zone defined by the points $\left(x'_k, x'_{k+1}\right)$, and vice versa. Thus, we define this mapping as follows: $x \in (u,v)$ is inside the k–th zone (yellow zone in the left–hand picture in Fig. 2) if the argument of the vector $(x - x_c)$ is between the arguments of the vectors $(x_k - x_c)$ and $(x_{k+1} - x_c)$:

$$x \in \ k\text{–}th\ zone \text{ iff}$$

$$\angle (x - x_c) \in [\angle (x_k - x_c), \angle (x_{k+1} - x_c)[\tag{15}$$

and $x' \in (w,z)$ is inside the k–th zone (yellow zone in the right–hand picture in Fig. 2) if the argument of the vector $(x' - x'_c)$ is between the arguments of the vectors $(x'_k - x'_c)$ and $\left(x'_{k+1} - x'_c\right)$:

$$x' \in \ k\text{–}th\ zone \text{ iff}$$

$$\angle (x' - x'_c) \in \left[\angle (x'_k - x'_c), \angle \left(x'_{k+1} - x_c\right)\right[\tag{16}$$

Let $\bar{x} = \left(x^T, 1\right)^T \in \mathbb{R}^3$ and $\bar{x}' = \left(x'^T, 1\right)^T \in \mathbb{R}^3$. For each couple of corresponding triangular zones, we exploit a projective transformation [12] that maps $\bar{x}$ into $\bar{x}'$. For the k–th couple of triangular zones:

$$\bar{x}' = M_k \cdot \bar{x} \tag{17}$$

The matrix M_k has the following structure:

$$M_k = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \tag{18}$$

where $a, b, c, d, e, f \in \mathbb{R}$. Each triangular zone is defined by three points (Fig. 2): (x_c, x_k, x_{k+1}) in the (u,v) coordinates set, and $\left(x'_c, x'_k, x'_{k+1}\right)$ in the (w,z) coordinates set. To find the matrix M_k, we impose the following conditions:

$$\begin{cases} \bar{x}'_c = M_k \cdot \bar{x}_c \\ \bar{x}'_k = M_k \cdot \bar{x}_k \\ \bar{x}'_{k+1} = M_k \cdot \bar{x}_{k+1} \end{cases} \tag{19}$$

Since $x_c, x'_c, x_k, x'_k, x_{k+1}, x'_{k+1} \in \mathbb{R}^2$, the conditions in Eq. (19) represent a linear system of six equations, to find the six components of the matrix M_k.

It's easy to show that, if x_c, x_k and x_{k+1} are different and non–collinear, the six equations are linearly independent. Since x_c, x_k and x_{k+1} are respectively the circumcenter and two adjacent vertices of a regular polygon, they are never coincident or collinear.

A projective transformation maps a straight line into a straight line [12]. Thus the line connecting x_c and x_k is transformed into the line connecting x'_c and x'_k (Fig. 2). In other words, the borders of the k–th triangular zone in the (u,v) coordinates set are mapped into the borders of the k–th triangular zone in the (w,z) coordinates set, $\forall k = 1 \dots n$.

Since any linear transformation of a convex set yelds to a convex set [14], each triangular zone is mapped into a convex set by M_k. Since the borders of each triangular zone in the (u,v) coordinates set are mapped into the borders of the corresponding triangular zone in the (w,z) coordinates set, we can conclude that the matrix M_k maps every point of the k–th triangular zone in the (u,v) coordinates set into points of the k–th triangular zone in the (w,z) coordinates set, $\forall k = 1 \dots n$.

M_k is invertible. In fact, let x_c, x_k, $x_{k+1} \in \mathbb{R}^2$ be the positions of the center of the polygon, and of two adjacent vertices, and let x'_c, x'_k, $x'_{k+1} \in \mathbb{R}^2$ be their corresponding transformed points. From geometrical considerations, it's easy to show that the matrix M_k is singular if and only if

- $x'_k = x'_{k+1}$, or
- the arguments of vectors $(x'_k - x'_c)$ and $\left(x'_{k+1} - x'_c\right)$ are equal.

The first condition means that the desired position of two different agents must be different. This appears to be a very natural condition: it doesn't have any physical meaning to obtain a formation in which two or more agents occupy the same position at the same time.

To satisfy the second condition, x'_c must be non–collinear to any couple of desired position for the agents in the formation. This is the only condition that has to be satisfied during the choice of x'_c. Since the number of agents in the formation is finite, it's always possible to find a suitable position for x'_c.

We want to remark that the coordinates transformation defined so far can be calculated by each agent without any centralized controller. Each agent must only know the desired positions that define the shape of the formation.

We have assumed so far that the triangular zones are convex sets. While this is always true in the (u, v) reference frame, because the triangular zones are defined by means of the vertices of a regular polygon, this condition can be violated in the (w, z) reference frame in many cases of interest (e.g. left–hand picture in Fig. 3).

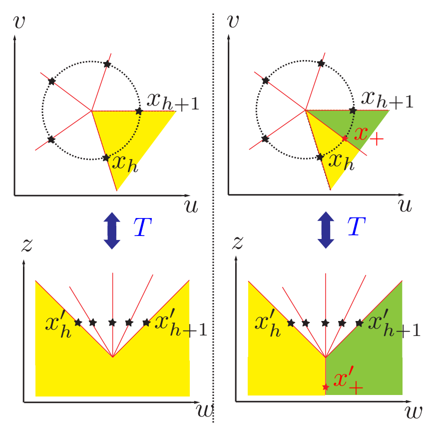


Fig. 3. Adding an auxiliary point, all the zones of the partitions are convex

The borders of the triangular zones are rays starting at x'_c and passing through the desired position of an agent in the formation. If the angle between a couple of adjacent rays is greater than π, the corresponding zone is non–convex. To apply the strategy described so far, we need to modify the partition to obtain only convex zones. More specifically, we need to divide the non–convex zone, thus obtaining two convex triangular zones. To do this, we introduce an auxiliary point, which defines and additional ray.

More specifically, let α'_h and α'_{h+1} be the arguments of vectors $(x'_h - x'_c)$ and $\left(x'_{h+1} - x'_c\right)$ respectively. Furthermore, let $\Delta\alpha'_h = \left|\alpha'_h - \alpha'_{h+1}\right|$. If $\Delta\alpha'_h > \pi$, we introduce a point x'_+, such that

$$\angle\left(x'_+ - x'_c\right) = \alpha'_+ = \alpha'_h + \Delta\alpha'_h/2 \tag{20}$$

Then, the partition of the environment (right–hand picture in Fig. 3) is done considering $n+1$ points: the desired positions of the n agents, and the auxiliary point x'_+.

To make the transformation bijective, a corresponding auxiliary point, named x_+, must be added in the (u, v) reference frame as well. Let α_h and α_{h+1} be the arguments of vectors $(x_h - x_c)$ and $(x_{h+1} - x_c)$ respectively. The argument of vector $(x_+ - x_c)$ will be the following:

$$\angle\left(x_+ - x_c\right) = \alpha_+ = \alpha_h + \left|\alpha_{h+1} - \alpha_h\right|/2 = \alpha_h + \pi/n \tag{21}$$

Thus, as stated in the introduction, our control strategy allows the creation of formations with totally arbitrary shape.

We remark that x_+ is used only for the definition of the bijective mapping: it does not directly influence the control action (it is not an attraction point for the agents).

The bijective coordinates transformation T defined so far can be described as a variable matrix:

$$x'_i = T_i\left(x_i\right) \cdot x_i \tag{22}$$

where x_i and x'_i represent the position of the i–th agents respectively in the (u, v) and in the (w, z) coordinates set. $T_i\left(x_i\right) = M_k$ if x_i is inside the k–th triangular zone. Let $\mathbf{x} = \left[\bar{x}_1^T \ldots \bar{x}_n^T\right]^T \in \mathbb{R}^{3n}$ and $\mathbf{x}' = \left[\bar{x}_1'^T \ldots \bar{x}_n'^T\right]^T \in \mathbb{R}^{3n}$. We introduce the *total* transformation matrix $\mathbf{T}$, such that

$$\mathbf{x}' = \mathbf{T}\left(\mathbf{x}\right) \cdot \mathbf{x} \tag{23}$$

The matrix $\mathbf{T}$ is a block diagonal matrix with the following structure:

$$\mathbf{T}\left(\mathbf{x}\right) = \begin{bmatrix} T_1\left(x_1\right) & 0 & \ldots & \ldots & 0 \\ 0 & T_2\left(x_2\right) & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \ldots & 0 & T_{n-1}\left(x_{n-1}\right) & 0 \\ 0 & \ldots & \ldots & 0 & T_n\left(x_n\right) \end{bmatrix} \tag{24}$$

The matrix $\mathbf{T}$ is clearly invertible, since it's the block diagonal composition of invertible matrices.

Let $\mathbf{x_D}$ be the desired configuration of the agents in the (u, v) reference frame, i.e. if $\mathbf{x} = \mathbf{x_D}$ the agents create a formation with the shape of a regular polygon in the (u, v) reference frame. Let $\mathbf{x'_D}$ be the desired configuration of the agents in the (w, z) reference frame, i.e. if $\mathbf{x}' = \mathbf{x'_D}$ the agents create a formation with the desired shape in the (w, z) reference frame. The coordinates transformation is defined such that

$$\mathbf{x'_D} = \mathbf{T}\left(\mathbf{x_D}\right) \cdot \mathbf{x_D} \tag{25}$$

We have proved in Sec. II that our control strategy is asymptotically stable, and doesn't have the problem of local minima. In other words, applying our control strategy, the regular polygon formation is always created, namely

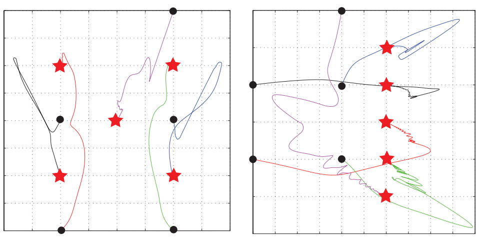
$$\lim_{t\to\infty} \mathbf{x}(t) = \mathbf{x_D} \tag{26}$$

Applying the coordinates transformation to Eq. (26), we obtain

$$\lim_{t\to\infty} \mathbf{x}'(t) = \lim_{t\to\infty} \mathbf{T}(\mathbf{x}(t)) \cdot \mathbf{x}(t) = \mathbf{T}(\mathbf{x_D}) \cdot \mathbf{x_D} = \mathbf{x}'_\mathbf{D} \tag{27}$$

In other words, with our control strategy the desired formation is always created.

V. Examples and simulations



(a) (b)

Fig. 4. Trajectories simulated with Matlab: black dots are the starting positions, red stars are the final positions

To validate our control strategy, we performed several simulations using Matlab. We considered point mass agents, with unitary mass. During our simulations, we have varied the number of the agents involved, and their desired positions. As expected, the agents always converge to the desired positions. The trajectories covered by five point mass agents realizing two different formations are represented in Fig. 4. In our simulations, we used the following parameters: $K_c = 80$, $K_a = 100$, $K_o = 30$. With these parameters, the time taken by the group to create the formation is always less then 20 seconds. Fig. 5 shows the trajectories covered by five agents

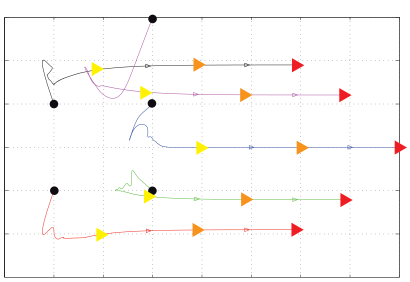


Fig. 5. Agents moving while maintaining a formation: different colors represent different instant of time

moving in the environment while keeping an arrow shaped formation. The movement of the formation is obtained by translating the point x'_c. The desired positions for the agents are represented as relative positions with respect to x'_c. Thus, as x'_c translates, even the minima of the composition of the potential fields translate. Therefore, the agents move preserving the shape of the formation, as shown in Fig. 5.

VI. Conclusions

The control strategy described in this paper is a completely decentralized algorithm: there is no need for any centralized controller. As typical in decentralized systems, this feature improves the reliability of the system. Current work aims at implementing online adaptation and scaling of the formation, in case of sudden failure or addition of an agent.

The agents only need local information about their neighbors. This kind of information can be obtained by means of proximity sensors, thus the need for explicit communication among the agents can be heavily reduced, which reduces the computational power required. Furthermore, if they don't need explicit communication, they can act in noisy environments as well, for example in presence of other radio sources which could disturb the communication.

Current work aims at including obstacle avoidance behavior in the control strategy. We are also studying how to extend our control strategy to nonholonimic systems. Afterwards, we will implement an experimental setup to test it on real robots. Once implemented our control strategy on real robots, we will investigate how to control the heading of the robots inside the formations.

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