# Power Allocation Analysis for Dynamic Power Utility in Cognitive Radio Systems

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***Abstract*—The focus of this paper is to investigate the fundamental limits of power allocation when taking into account a dynamic power pricing scheme. This paper proposes an optimal power allocation analysis for wireless systems when real time power pricing is available. We propose to minimize the total power consumption cost while ensuring minimum individual and total throughput limits. We consider different models for the power pricing function. Analytic solutions for the power allocation are derived for each model. The derived solutions are shown to be modified versions of the water-filling solution. Low-complexity algorithms are proposed for the resource allocation with each pricing model. Performance comparison and pricing effect are shown through simulations.**

***Index Terms*—Energy consumption awareness, spectrum access efficiency, power pricing.**

## I. Introduction

Optimal resource allocation is a crucial task in wireless communication systems to respond to the continuously increasing demand in terms of target data rates and network coverage which require higher and higher resources. Thus, an efficient use of the available resources, mainly the power and spectrum, became one of the principal challenges for future communications. The power consumption concerns were driven by the growing worries about the effect of this explosive demand in terms of power which were recognized as a major threat from environmental and economic perspectives. In fact, the huge demand in terms of power not only leads to higher cost but also contributes to the global warming phenomena through the increase of the carbon footprint. This tradeoff led to the emergence of the concept of 'green communication' [1], which encourages developing energy-efficient communication systems. The spectrum awareness was driven by the problem of spectrum scarcity due to increasing number of wireless devices as observed by the Federal Communications Commission (FCC) [2]. Cognitive radio systems have attracted a great interest recently as a means to enhance the spectrum efficiency and overcome the problem of spectrum scarcity [3–5]. It enables opportunistic access to the spectrum, and has then been seen as a key candidate for enabling dynamic spectrum access by taking advantage of the awareness about the surrounding environment. The challenges related to cognitive networks have been addressed in the literature, ranging from spectrum sensing and protocol design [6, 7] to spectrum access and analysis [8–10].

The resource allocation task in cognitive radio systems is of great importance. This task should allow to maximize spectrum utilization efficiency while taking into consideration the impacts of power consumption expressed in terms of cost and carbon footprint. Resource allocation has also been intensively investigated in the literature [11–13]. In [12], the authors proposed the optimal power allocation in an OFDM based underlay spectrum sharing by targeting the maximization of the total rate. The merits of the proposed scheme compared to the classical water-filling is that the latter does not account for the interference constraint. In [13], the authors formulated the problem of secondary users capacity maximization while considering a given quality of service of the primary user as well as the SU. They used a geometric program to solve the problem. Other works considered the power allocation for joint underlay and overlay in multiband cognitive systems by considering an auction based approach to deal with the competition between the secondary users [14]. In [15], the authors used a game theoretical approach to allocate the power among the secondary users while accounting for their quality of service as well as protecting the primary system.

Inspired by the emergence of dynamic pricing in future power grids (smart grids) as well as the need for providing green communication, we consider to propose an optimal power allocation for a cognitive radio system that minimizes a generic cost function of the allocated power while ensuring the performance requirements in terms of minimum individual and total throughput and at the same time protecting the licensed users by setting a threshold interference temperature. Even though the targeted power gains are not important to generate profit in the case of classic wireless systems due to small power consumption, this work can be of paramount importance for large wireless systems with high power consumption and high targeted throughput (60 Ghz communication, free space optical communication) which are expected to replace backhaul connections based on wired links. We are targeting analytical solutions to allow analysis of the system's performance and obtained gains. Obtained results can be employed later in decision algorithms for multiple service providers.

The rest of this paper is organized as follows. In Section II, we describe the system model and present the dynamic power cost models used in this work. In Section III, we formulate the resource allocation problem. In Section IV, we analyze and propose the power allocation solutions for different cost functions. Then, simulation results are presented in Section V. Finally, conclusion is drawn in Section VI.

## II. System Model and Dynamic pricing

### A. System Model

We consider a dynamic spectrum access system where a cognitive system, called secondary system, is sharing the spectrum with a licensed system, called the primary system. The cognitive system is composed of a secondary transmitter (ST) communicating with its secondary receiver (SR) while the primary system is composed of a primary transmitter (PT) and a primary receiver (PR). We assume that the communication pattern follows an underlay spectrum sharing scheme; the primary system is operating without paying attention to the presence of the secondary system while the secondary system is using power control mechanism to control the interference caused to the primary receiver (PR). Hence, the caused interference should be kept below an interference temperature $I^{\text{th}}$. We assume that $N$ channels are available for the secondary system. Note that the diversity scheme is not specified in our work. Thus, it could be applied for various access schemes (time, frequency, space, etc.). For example, this could be applied for a multi-antenna system where different channel gains are present at the secondary system. It can also be applied for a multi-carrier spectrum access where the secondary could send its data over more than one band.

We denote the instantaneous channel gains between the secondary transmitter (ST) and the secondary receiver (SR) by $\{h_1^{(c)}, ..., h_j^{(c)}, ..., h_N^{(c)}\}$ while the instantaneous channel gains of the interference channels between the secondary transmitter (ST) and the primary receiver (PR) are denoted by $\{h_1^{(p)}, ..., h_j^{(p)}, ..., h_N^{(p)}\}$. The received throughput in a channel $j$ when employing a power $P_j$ is written as

$$r_j(P_j) = \log_2\left(1 + P_j \gamma_j^{(c)}\right), \tag{1}$$

with $\gamma_j^{(c)} = \frac{|h_j^{(c)}|^2}{w_j N_0}$ where $N_0$ is the noise power density that is assumed to be constant for all the channels.

The cognitive user tries to find the optimal power allocation among the different channels with regards to the given cost function.

### B. Dynamic Power Cost

In conventional power allocation problems, the cognitive transmitter tries to minimize its used power to meet certain requirements in terms of throughput. This problem was solved in different previous works using mainly the well-known water-filling algorithm [12]. In a more sophisticated set-up, we consider a more general case where the objective is to minimize a cost function $\mathcal{C}$ that takes into account various aspects related to economic and/or environment impact of the used power. For instance, this objective function could model the cost of the power procured from a utility company with a dynamic pricing variation depending on demand level. In another context taking into consideration green communications objectives, the cost function could model the carbon emission of the used power. For generality purposes, we consider a generic cost function that could include different parts modeling both energy consumption, cost, and environment impact. Our study will not focus on modeling this cost but rather on proposing an optimal power allocation scheme for a generic cost function. Thus, Let $c_j(P_1, ..., P_N)$ be the cost of the power $P_j$ consumed by the channel $j$. Hence, the total cost in this case could be modeled as

$$\mathcal{C}(P_1, ..., P_N) \quad = \quad \sum_{j=1}^{N} c_j(P_1, ..., P_N). \tag{2}$$

where $c_j(P_1, ..., P_N)$ is continuously differentiable function, increasing and convex as of $P_j$.

## III. Problem Formulation

The focus of this work is to investigate the optimal power allocation when taking into account other aspects than the channel gains. Hence, we are seeking the optimal power level's selection that allows to minimize our cost function $\mathcal{C}$ with regards to the different system requirements. Mathematically, we formulate our optimization problem as follows

$$\min_{P_1, ..., P_N} \quad \mathcal{C}(P_1, ..., P_N) \tag{3a}$$

$$\text{Subj. to} \quad \sum_{j=1}^{N} r_j(P_j) \geq r_{\text{Tot}}^{\text{th}} \tag{3b}$$

$$r_j(P_j) \geq r_{\text{j}}^{\text{th}} \qquad \forall\, j \in \{1...N\}, \tag{3c}$$

$$P_j g_j^{(p)} \leq I^{\text{th}} \qquad \forall\, j \in \{1...N\}, \tag{3d}$$

where $r_{\text{Tot}}^{\text{th}}$ denotes the required total throughput while $r_j^{\text{th}}$ denotes the required individual throughput per channel. Equation (3b) constrains the system to achieve a total desired throughput using the $N$ channels while Eequation (3c) requires at each channel to achieve a minimum throughput that corresponds to a minimum quality of service. However, the constraint (3d) is the requirement of the underlay paradigm used to respect the interference level at the primary system with $g_j^{(p)} = |h_j^{(p)}|^2$. From practical considerations, this system is recurrent in a wide number of applications. An example of application is when the transmitter have different paths with his correspondent receiver (different time slots, bands, antennas). $r_{\text{Tot}}^{\text{th}}$ is the total throughput that the whole system should reach while $r_j^{\text{th}}$ is the minimum quality of service at each path.

In the optimization problem (3), the values for the minimum total throughput $r_{\text{Tot}}^{\text{th}}$ and the minimum throughput per channel $r_j^{\text{th}}$ along with the interference temperature $I^{\text{th}}$ may lead to some conflicting constraints. If $\sum_{j=1}^{N} r_j^{\text{th}} \geq r_{\text{Tot}}^{\text{th}}$, the constraint (3b) will automatically be guaranteed. On the other hand, the problem could be unfeasible in a number of cases; if $r_j\left(\frac{I^{\text{th}}}{g_j^{(p)}}\right) < r_j^{\text{th}}$ or if $\sum_j r_j\left(\frac{I^{\text{th}}}{g_j^{(p)}}\right) < r_{Tot}^{\text{th}}$. In these cases, the interference constraint imposed by the primary system will limit the maximum transmit power at the cognitive transmitter side and fails to reach either the minimum throughput per channel or the minimum total throughput.

## IV. Problem Analysis with Different Pricing Models

Given that the cost function of the power $c_j(P_1, ..., P_N)$ is continuously derivable and convex, the problem (3) is a convex optimization problem. Thus, we propose to alternatively solve its dual problem using the Karush-Kuhn-Tucker (K.K.T) conditions as duality gap is zero under the Slater condition [16]. For our case, the Slater condition is satisfied when the problem is feasible. Mathematically, the feasibility conditions are written as

$$r_j\left(\frac{I^{\text{th}}}{g_j^{(p)}}\right) \geq r_j^{\text{th}}, \quad \forall j \in \{1, ..., N\} \tag{4}$$

$$\sum_{j=1}^{N} r_j\left(\frac{I^{\text{th}}}{g_j^{(p)}}\right) \geq r_{Tot}^{\text{th}}. \tag{5}$$

The dual problem of the category problem (3) can be written as follows

$$\begin{aligned}
&\mathcal{L}\Big(\{P_j\}_{j=1}^N, \lambda_0, \{\lambda_j\}_{j=1}^N, \{\nu_j\}_{j=1}^N\Big) \\
&= \sum_{j=1}^{N} c_j(P_1, ..., P_N) + \lambda_0\Big(r_{Tot}^{\text{th}} - \sum_{j=1}^{N} r_j(P_j)\Big) \\
&+ \sum_{j=1}^{N} \lambda_j\Big(r_j^{\text{th}} - r_j(P_j)\Big) + \sum_{j=1}^{N} \nu_j\Big(P_j g_j^{(p)} - I^{\text{th}}\Big),
\end{aligned} \tag{6}$$

where $\lambda_0$, $\{\lambda_j\}_{j=1}^N$, and $\{\nu_j\}_{j=1}^N$ are the K.K.T. multipliers.

After simplifications, the K.K.T. constraints are written as follows

$$\sum_{i=1}^{N} \frac{\partial c_i(P_1, ..., P_N)}{\partial P_j} = \frac{\gamma_j^{(c)} \lambda_0}{(1 + P_j \gamma_j^{(c)}) \log 2} \quad \forall j \in \{1...N\} \tag{7}$$

$$\lambda_0\Big(r_{Tot}^{\text{th}} - \sum_{j=1}^{N} r_j(P_j)\Big) = 0 \tag{8}$$

$$P_j \geq \tfrac{2^{r_j^{\text{th}}} - 1}{\gamma_j^{(c)}} \quad \forall j \in \{1...N\} \tag{9}$$

$$P_j \leq \tfrac{I^{\text{th}}}{g_j^{(p)}} \quad \forall j \in \{1...N\} \tag{10}$$

$$\lambda_0 \geq 0. \tag{11}$$

Hence, the optimal power allocation for every channel $j$ is deduced such that it satisfies the following Equation:

$$\begin{gathered}
\big(1 + P_j \gamma_j^{(c)}\big) \sum_{i=1}^{N} \frac{\partial c_i(P_1, ..., P_N)}{\partial P_j} = \lambda_0' \gamma_j^{(c)} \\
\text{Subj. to} \quad P_j^- \leq P_j \leq P_j^+,
\end{gathered} \tag{12}$$

where $\lambda_0'$ is a constant proportional to $\lambda_0$ determined such that the total throughput constraint is saturated (i.e., $\sum_{j=1}^N r_j(P_j) = r_{\text{T}}^{\text{th}}$), while $P_j^-$ and $P_j^+$ are defined as

$$\begin{cases} P_j^- = \frac{2^{r_j^{\text{th}}} - 1}{\gamma_j^{(c)}} \\ P_j^+ = \frac{I^{\text{th}}}{g_j^{(p)}}. \end{cases} \tag{13}$$

Note that uniqueness of $P_j$ could be verified in the interval $[P_j^-, P_j^+]$ if it exists in this interval.

*Proof.* Let us denote

$$g_j(P_j) = \big(1 + P_j \gamma_j^{(c)}\big) \sum_{i=1}^{N} \frac{\partial c_i(P_1, ..., P_N)}{\partial P_j} - \lambda_0' \gamma_j^{(c)}. \tag{14}$$

Solving Equation (12) is equivalent to finding the zero of the function $g_j$ in the interval $[P_j^-, P_j^+]$. Thus, we compute its derivative as follows

$$\begin{aligned}
g_j'(P_j) &= \big(1 + P_j \gamma_j^{(c)}\big) \sum_{i=1}^{N} \frac{\partial^2 c_i(P_1, ..., P_N)}{\partial P_j^2} \\
&+ \gamma_j^{(c)} \sum_{i=1}^{N} \frac{\partial c_i(P_1, ..., P_N)}{\partial P_j},
\end{aligned} \tag{15}$$

which is positive (since $c_i(P_1, ..., P_N)$ are by definition increasing and convex function of $P_j$, $\forall i, j$). Thus, we prove the uniqueness of the solution. The existence of this solution could be checked by testing the interval bounds (i.e., it exists if and only if $g_j(P_j^-) \times g_j(P_j^+) \leq 0$). □

Depending on the cost function expression, the solution could be simplified further. Thus, in the following we will consider some families of cost functions and express the power allocation for each case starting from the most simple models to generic expressions.

### A. Constant Unit Price

In this section, we consider a cost function with a variable unitary power price across the channel without depending on the consumed power. In fact, as specified in the system model, the channels in our case represent generic diversity of the paths. Thus, for instance, the variable price could be applicable when channels represent different time slots or different power providers. In this case, the power cost function is expressed as follows

$$c_j(P_1, ..., P_N) = \mu_j \times P_j, \quad \forall j \tag{16}$$

where $\mu_j$ is the unitary power cost for the $j$-th channel. Given, this model, the allocated power expression is deduced from (12) as

$$P_j = \left[\lambda_0' - \frac{\mu_j}{\gamma_j^{(c)}}\right]_{P_j^-}^{P_j^+}, \tag{17}$$

where $\left[x\right]_{x^-}^{x^+} = \begin{cases} x^+ & \text{if } x > x^+, \\ x^- & \text{if } x < x^-, \\ x & \text{otherwise.} \end{cases}$

The water-level, $\lambda_0'$, is expressed as

$$\lambda_0' = \left(\frac{2^{r_{Tot}^{\text{th}}}}{\prod\limits_{j \in S_c} 2^{r_j^-} \prod\limits_{j \in S_p} 2^{r_j^+} \prod\limits_{j \notin \{S_c \cup S_p\}} (\frac{\gamma_j^{(c)}}{\mu_j})}\right)^{\frac{1}{N - (|S_c| + |S_p|)}}, \tag{18}$$

with $S_c$ and $S_p$ defind as

$$\begin{cases} S_c = \Big\{j \in \{1, ..., N\} \text{ such that } \frac{\lambda_0'}{\mu_j} - \frac{1}{\gamma_j^{(c)}} < P_j^-\Big\} \\ S_p = \Big\{j \in \{1, ..., N\} \text{ such that } \frac{\lambda_0'}{\mu_j} - \frac{1}{\gamma_j^{(c)}} > P_j^+\Big\}. \end{cases} \tag{19}$$

The obtained throughput per channel is deduced then as a function of the signal-to-noise ratio and unit price per channel as

$$r_j = \left[ \log_2 \left( \lambda_0' \frac{\gamma_j^{(c)}}{\mu_j} \right) \right]_{r_j^-}^{r_j^+}. \tag{20}$$

We obtain a water-filling expression used in resource allocation algorithms over multichannel systems [17] with the modification that the channel unit price power will affect the power allocated in each channel. The allocated power is obtained as the difference between the water-level and the ratio of the unit price by the channel gain instead of the inverse of the channel gain in ordinary water-filling.

*B. Power Consumption Dependent Unit Price*

In this section, we assume that the unitary power cost depends not only on the channel but also on the allocated power in that channel. In fact, in practical scenarios, power providers impose higher unitary power prices when the consumption increases. Similarly, to penalize high power consumers, higher factors are associated when the allocated power increases in the carbon impact computation. Thus, in this section, we study the following model for the cost function

$$c_j(P_1, ..., P_N) = \mu_j(P_j) \times P_j, \quad \forall j \tag{21}$$

. where $\mu_j(P_j)$ is the unitary cost function.

*1) Linear Unit Price function:* We consider the unitary cost as a linear function of the consumed power, i.e.,

$$\mu_j(P_j) = a_j + b_j P_j, \tag{22}$$

where $a_j$ and $b_j$ are power pricing coefficients fixed by the power provider and can be obtained in real-time through the back-haul network.

Inserting (22) and (21) in (12), we obtain the following equation to solve for the allocated power per channel

$$P_j = \left[ \frac{\lambda_0'}{a_j + 2b_j P_j} - \frac{1}{\gamma_j^{(c)}} \right]_{P_j^-}^{P_j^+}. \tag{23}$$

Although this is a quadratic equation, non-negativity of the allocated power per channel results in obtaining a unique solution which is written as

$$P_j = \left[ \frac{-(a_j\gamma_j^{(c)} + 2b_j) + \sqrt{\left(a_j\gamma_j^{(c)} - 2b_j\right)^2 + 8b_j\lambda_0'\left(\gamma_j^{(c)}\right)^2}}{4b_j\gamma_j^{(c)}} \right]_{P_j^-}^{P_j^+}. \tag{24}$$

The expression of $\lambda_0'$ can not be derived analytically in this case but it can be obtained by solving the total throughput constraint which is transformed into finding the zero of the function $f(\lambda_0')$ for $\lambda_0' \geq 0$ with

$$\begin{aligned} f(\lambda_0') = & \frac{2^{r_{Tot}^{\text{th}}}}{\prod\limits_{j\in S_c} 2^{r_j^-} \prod\limits_{j\in S_p} 2^{r_j^+}} \\ & - \prod_{j\notin\{S_c\cup S_p\}} \frac{\left(2b_j - a_j\gamma_j^{(c)} + \sqrt{(a_j\gamma_j^{(c)} - 2b_j)^2 + 8b_j\lambda_0'\left(\gamma_j^{(c)}\right)^2}\right)}{4b_j} \\ & = 0, \end{aligned} \tag{25}$$

with $S_c$ and $S_p$ defined as

$$\begin{cases} S_c = \left\{ j \in \{1, ..., N\} \text{ such that } \frac{\lambda_0'}{a_j + 2b_j P_j} - \frac{1}{\gamma_j^{(c)}} < P_j^- \right\} \\ S_p = \left\{ j \in \{1, ..., N\} \text{ such that } \frac{\lambda_0'}{a_j + 2b_j P_j} - \frac{1}{\gamma_j^{(c)}} > P_j^+ \right\}. \end{cases} \tag{26}$$

It is easy to check that $f(\lambda_0')$ is continuous and decreasing with $f(0) > 0$ and $\lim\limits_{\lambda_0' \to \infty} f(\lambda_0') = -\infty$ thus it has a unique zero which can be obtained numerically.

Although in this case we do not obtain a strictly speaking water-filling expression, a similar algorithm can be developed where $\lambda_0'$ will represent an "imaginary" water-level as it remains constant for all channels. The pseudo water-filling expression can be deduced from (23) as follows

$$P_j = \left[ \frac{\lambda_0'}{\hat{\mu_j}} - \frac{N_0}{g_j^{(c)}} \right]_{P_j^-}^{P_j^+}, \tag{27}$$

with $\hat{\mu_j} = a_j + 2b_j P_j$ is the effective power cost in the $j$-th channel. Note that this is not a water-filling equation as $\hat{\mu_j}$ depends on the allocated power $P_j$ but it only allows to analyze the allocated power function to the channel gains and the price coefficients. Thus, we obtain a system of non-linear coupled Equations (24) and (25). An iterative approach allows us to determine this water-level and thus obtain the optimal power allocation per channel by solving at each step consecutively (24) and (25) until convergence. This algorithm has the same convergence speed as the regular water-filling algorithm. The only difference is that the water-level is determined analytically in regular water-filling while it is obtained numerically by solving (25) in this case.

*2) Polynomial Unit Price function:* We consider the unitary cost as a general polynomial function of the consumed power as follows

$$\mu_j(P_j) = \sum_{i=0}^{p} a_{j,i} P_j^i, \tag{28}$$

where $p$ is the polynomial degree and $a_{j,i}$ are power pricing coefficients fixed by the power provider.

Inserting (28) and (21) in (12), we obtain the following Equation to solve for the allocated power per channel

$$P_j = \left[ \frac{\lambda_0'}{\sum_{i=0}^{p-1} a_{j,i+1}(i+1)P_j^i} - \frac{1}{\gamma_j^{(c)}} \right]_{P_j^-}^{P_j^+}. \tag{29}$$

Since the solution of this equation is unique if it exists as shown earlier, we transform the problem into root finding problem of the following polynomial

$$\sum_{i=0}^{p+1} \alpha_{j,i} P_j^i, \tag{30}$$

with

$$\alpha_{j,i} = \begin{cases} \frac{a_{j,0}}{\gamma_j^{(c)}} - \lambda_0', & \text{if } i = 0 \\ \frac{a_{j,i}}{\gamma_j^{(c)}}(i+1) + a_{j,i-1}i, & \text{if } 1 \leq i \leq p \\ a_{j,p}(p+1), & \text{if } i = p+1. \end{cases} \tag{31}$$

The obtained power will depend on the water-level $\lambda_0'$ which is obtained by solving the total throughput constraint which is transformed into finding the zero of the function $f(\lambda_0')$ for $\lambda_0' \geq 0$ with

$$\begin{aligned} f(\lambda_0') = r_{Tot}^{\text{th}} - \left[ \sum_{j \in S_c} r_j^- + \sum_{j \in S_p} r_j^+ + \sum_{j \notin \{S_c \cup S_p\}} r_j(P_j) \right] \\ = 0, \end{aligned} \tag{32}$$

with $S_c$ and $S_p$ defined as

$$\begin{cases} S_c = \left\{ j \in \{1, ..., N\} \text{ such that } \frac{\lambda_0'}{\sum_{i=0}^{p-1} a_{j,i+1}(i+1) P_j^i} - \frac{1}{\gamma_j^{(c)}} < P_j^- \right\} \\ S_p = \left\{ j \in \{1, ..., N\} \text{ such that } \frac{\lambda_0'}{\sum_{i=0}^{p-1} a_{j,i+1}(i+1) P_j^i} - \frac{1}{\gamma_j^{(c)}} > P_j^+ \right\}. \end{cases} \tag{33}$$

Although in this case we do not obtain a strictly speaking water-filling expression, a similar algorithm can be developed where $\lambda_0'$ will represent an "imaginary" water-level as it remains constant for all channels. The pseudo water-filling expression can be deduced from (29) as follows

$$P_j = \left[ \frac{\lambda_0'}{\hat{\mu}_j} - \frac{N_0}{g_j^{(c)}} \right]_{P_j^-}^{P_j^+}, \tag{34}$$

with $\hat{\mu}_j = \sum_{i=0}^{p-1} a_{j,i+1}(i+1)P_j^i$ is the effective power cost in the $j$-th channel. Note that this is not a water-filling equation as $\hat{\mu}_j$ depends on the allocated power $P_j$ but it only allows to analyze the allocated power function to the channel gains and the price coefficients. Thus, we obtain a system of non-linear coupled Equations (30) and (32). An iterative approach allows us to determine this water-level and thus obtain the optimal power allocation per channel by solving at each step consecutively (30) and (32) until convergence.

## V. Simulation Results

We consider a cognitive user randomly located in a cell with a radius $d_0 = 1$ Km. We assume that the CU is equipped with a smart meter that could provide it with (instantaneous) unit pricing in real-time. Unless notified for a different usage of the parameters, we consider $N = 20$ channels. The total required throughput $r_{Tot}^{\text{th}} = 50\ Mbps$ while the individual required throughput per channel is $r_j^{\text{th}} = 1\ Mbps,\ \forall j$. We consider a Rayleigh fading channel model. The interference threshold is fixed to be equal to the noise floor $I^{\text{th}} = N_0 = -120\ dBm$. Using the different pricing cost models presented in section IV, we compute the optimal cost of the power needed to reach the required throughput, $\mathcal{C}(\mathbf{P^{opt}})$, then compare it to the cost of the power if dynamic pricing is not available. This reference power allocation $\mathbf{P^{ref}}$ is obtained by minimizing the total power consumed instead of cost of the power (we use algorithm proposed in [17] for this reference allocation). Then we compute the relative power cost gain as follows

$$\text{Cost gain} = \frac{\mathcal{C}(\mathbf{P^{ref}}) - \mathcal{C}(\mathbf{P^{opt}})}{\mathcal{C}(\mathbf{P^{ref}})}. \tag{35}$$

In Fig. 1, we plot the cost gain with reference to the case where pricing is not considered (minimization of the total power cost) for the channel dependent unitary cost (16) as a function of the standard deviation of this unitary cost for different numbers of channels using the channel dependent pricing model (16). We observe that the gain increases as the variance of the unitary price increases. This is due to the increase of the variability between channels which allows a better exploitation of the channels. On the other hand, the gain is more important when the number of channels is lower. This can be explained by the fact that increasing the number of channels limits users' freedom to allocate the power due to additional individual constraints for the new channels.

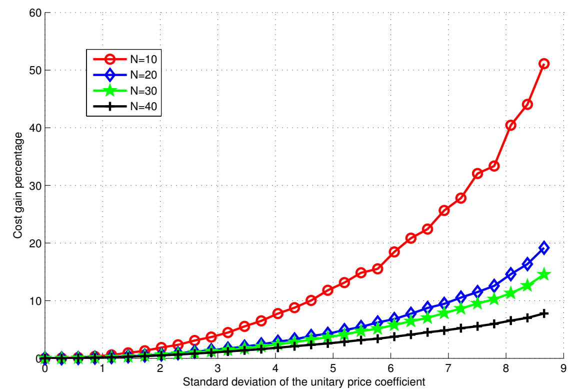


Fig. 1. Percentage of cost gain function to the unitary cost variance with $E[\gamma_j^{(c)}] = 20\ dB$.

In Fig. 2, we plot the obtained cost gain compared to absence of pricing for different values of the pricing coefficients to observe their effect on the total cost. We use the pricing model (21) with linear unit pricing as in (22) with uniform pricing coefficients for all channels. We fix $a_j = 1$ and vary $b_j$ as shown in the legend. The cost gain is increasing with the increase of the pricing coefficient since power savings became more valuable with the increase of the unitary cost.

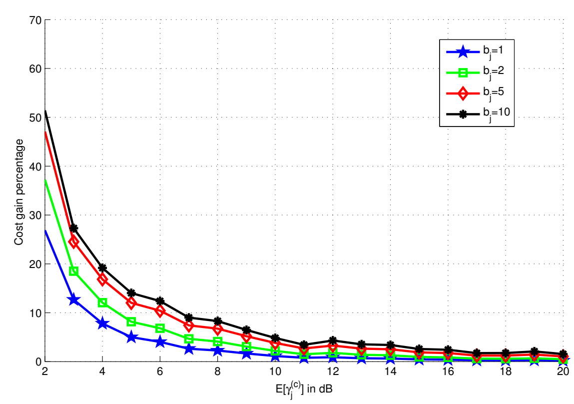


Fig. 2. Total power consumption cost with different pricing parameters function to average channels SNR.

In Fig. 3, we plot the power cost gain compared to the case where pricing is not taken in consideration as a function of the average channel gains $E[\gamma_j^{(c)}]$. We use power dependent pricing model (21) with polynomial unit pricing as in (28) with different degrees. The different polynomials are generated using Chebychev polynomial approximation [18] from the same cost function but with different degrees of approximation $p$. In this figure, we observe two behaviors of the cost gain as a function of the SNR. In the first part corresponding to low SNRs, the cost gain is a decreasing function of the SNR but it is higher with higher values of the polynomial approximation degree. In fact, for low SNRs, high power levels are needed to meet throughput constraints which results in higher cost savings when the polynomial approximation is more accurate (higher degree of the polynomial). In the second part corresponding to high SNRs, the cost gain is a slowly increasing function of the SNR and also function of the polynomial degree. In fact, for high SNRs, lower power levels are needed to reach throughput requirements. Thus, in this case, the effect of the channel SNRs on the total cost gain becomes dominant over the power effect. In addition, even though we still observe that higher degrees of polynomial approximations result in higher cost savings, the difference between the gains become negligible which justifies the use of a lower polynomial degree for this case since resource allocation is easier with low polynomial degrees.

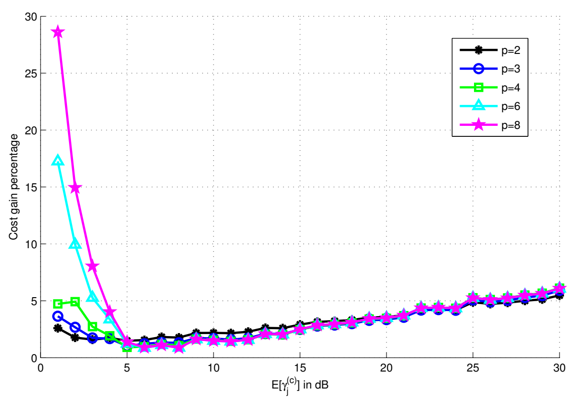


Fig. 3. Cost gain with different polynomial approximations function to average channels SNR.

## VI. Conclusion

This paper proposes a resource allocation scheme for dynamic cost of the consumed power for a cognitive radio system while ensuring total and individual throughput requirements. The proposed power allocation allows to profit from available information about the cost and the channels diversity to better employ the power to meet the throughput requirements and minimize the power cost. Analytic expressions of the allocated power are developed for different cost functions and low-cost algorithms are presented for the power allocation. Simulation results show the gain that the cognitive system achieved by profiting from the dynamic power pricing through the proposed power allocation scheme.

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