Additional information:

Key Points:
- We build a framework of adaptive power allocation strategy for cooperative target detection in radar networks
- We strictly prove the existence and uniqueness of Nash bargaining solution (NBS)
- We reveal the relationships between the power allocation results and the target RCS and the relative geometry

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Power allocation for target detection in radar networks based on low probability of intercept: A cooperative game theoretical strategy

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Abstract Distributed radar network systems have been shown to have many unique features. Due to their advantage of signal and spatial diversities, radar networks are attractive for target detection. In practice, the netted radars in radar networks are supposed to maximize their transmit power to achieve better detection performance, which may be in contradiction with low probability of intercept (LPI). Therefore, this paper investigates the problem of adaptive power allocation for radar networks in a cooperative game-theoretic framework such that the LPI performance can be improved. Taking into consideration both the transmit power constraints and the minimum signal to interference plus noise ratio (SINR) requirement of each radar, a cooperative Nash bargaining power allocation game based on LPI is formulated, whose objective is to minimize the total transmit power by optimizing the power allocation in radar networks. First, a novel SINR-based network utility function is defined and utilized as a metric to evaluate power allocation. Then, with the well-designed network utility function, the existence and uniqueness of the Nash bargaining solution are proved analytically. Finally, an iterative Nash bargaining algorithm is developed that converges quickly to a Pareto optimal equilibrium for the cooperative game. Numerical simulations and theoretic analysis are provided to evaluate the effectiveness of the proposed algorithm.

1. Introduction

1.1. Background and Motivation

Distributed radar network systems have received contentiously growing attention in a novel class of radar system and on a path from theory to practical use owing to their advantage of signal and spatial diversities [*Fisher et al.*, 2006; *Haimovich et al.*, 2008; *Li and Stoica*, 2009; *Pace*, 2009], where the term radar networks refer to the use of multiple-transmit as well as multiple-receive antennas. In recent years, the study of distributed radar network architectures has received sizeable impetus, which has been extensively studied from various perspectives [*Chen et al.*, 2013; *Fisher et al.*, 2006; *Godrich et al.*, 2010, 2012; *He et al.*, 2016; *Shi et al.*, 2015, 2016a, 2016c, 2016d]. In *Fisher et al.* [2006], the authors introduce the concept of distributed multiple-input multiple-output (MIMO) radar and investigate the inherent performance limitations of both conventional phased array radars and the newly proposed radars. *Niu et al.* [2012] develop the localization and tracking algorithms for noncoherent MIMO radar systems, in which it is demonstrated that the noncoherent MIMO radar can provide a significant performance improvement over traditional monostatic phased array radar with high range and azimuth resolutions. The work in *Chavali and Nehorai* [2012] addresses the problem of sensor scheduling and power allocation in a cognitive radar network for multiple-target tracking. *Yan et al.* [2015] extend the previous results in *Chavali and Nehorai* [2012] and present a performance-driven power allocation strategy for Doppler-only target tracking in unmodulated continuous wave radar network, where the Bayesian Cramer-Rao lower bound (CRLB) is derived and utilized as an optimization criterion for the optimal power allocation scheme. In *Nguyen et al.* [2015], the authors investigate the problem of target tracking in a multistatic radar system from the perspective of adaptive waveform selection, in which the transmitted waveform parameters are selected to minimize the target tracking covariance matrix. *Shi et al.* [2016c] study the problem of joint target position and velocity estimation of a Rician target in orthogonal frequency division multiplexing (OFDM)-based passive radar networks, and the modified CRLB on the Cartesian coordinates of

target position and velocity are computed. Overall, the previous studies lay a solid foundation for the problem of performance optimization in distributed radar network systems.

Game theory is a branch of mathematics traditionally investigated and applied in the areas of economics, political science, and biology, which has emerged in recent years as an effective tool for radar network systems, wireless communications, and signal processing [*Bacci et al.*, 2015]. *Rashid-Farrokhi et al.* [1998] investigate the game theory-based joint beamforming and optimal power control in wireless networks with antenna arrays, and an iterative method is proposed to jointly update the beamformer weights and the transmitting powers so that it converges to the optimum values. In *Gogineni and Nehorai* [2012], a polarimetric design algorithm is proposed for distributed MIMO radar target detection from a game-theoretic perspective, which examines the impact of all possible transmit strategies on the target detection performance with different target profiles. *Song et al.* [2014] model the interaction between a smart target and a smart MIMO radar as a two-person zero-sum game, and unilateral, hierarchical, and symmetric games are investigated based on the available information set for each player. *Moragrega et al.* [2013] present a distributed scheme for power selection in wireless sensor networks with positioning capabilities utilizing the framework provided by supermodular games. *Piezzo et al.* [2013] present a noncooperative game code design in radar networks to maximize the signal to interference plus noise ratio (SINR) of each radar. *Lan et al.* [2015] present a two-step water-filling approach for Stackelberg game between MIMO radar and target in the presence of clutter. *Bacci et al.* [2012] study the problem of power allocation in radar networks based on game theory for the first time, which presents a distributed algorithm based on game theory for efficiently allocating the transmit power in radar networks, while controlling the performance of the radar sensor networks in terms of probability of false alarm and detection at each radar node. In *Panoui et al.* [2014a], a distributed power allocation scheme is proposed for a multistatic MIMO radar network based on noncooperative game theory, whose aim is to minimize the total transmission power while maintaining a specific signal-to-disturbance ratio (SDR). Furthermore, *Panoui et al.* [2014b] also investigate the performance of the game-theoretic strategy in the presence of estimation error, while *Deligiannis et al.* [2016a] address a competitive power allocation game-theoretic problem between a MIMO radar system and multiple jammers. *Deligiannis et al.* [2016b] also investigate a game-theoretic method to tackle the problem of joint beamforming and power allocation in a distributed radar network. *Panoui et al.* [2016] employ the potential games to investigate the interaction of MIMO-based clusters of radars within a game-theoretic framework, which maximizes the SDR of the clusters of radars by selecting the most appropriate waveforms. In *Deligiannis and Lambotharan* [2017], a Bayesian game-theoretic SINR maximization and power allocation algorithm is proposed for a multistatic radar network system, where the primary goal of each radar is to maximize their SINR with the constraint of its maximum transmitting power.

However, in noncooperative game model, rational but selfish players maximize their own individual utilities in a self-interested manner, which will inevitably increase the mutual interference to other players. The objective of a noncooperative game is to find an Nash equilibrium (NE) solution, where each player has no chance to increase its utility unilaterally. Unfortunately, the sum of the individual utilities might not be maximized at the NE point. Cooperative game theory can provide an expressive and flexible framework for modeling collaboration in multiagent systems, in which players are motivated to cooperate with one another to enhance their own utility functions. In *Sun et al.* [2014] and *Chen et al.* [2015], the problem of optimal power allocation with the goal of maximizing the determinant of Bayesian Fisher information matrix in distributed MIMO radar networks is studied for target localization and tracking, wherein it is formulated as a cooperative game and the Shapley value is exploited as the solution for the proposed scheme. Simulation results show the superior performance of game-theoretic power allocation over other allocations in various scenarios. *Chen et al.* [2016] develop two power management games for cooperation localization in both asynchronous and synchronous networks.

As the notion of low probability of intercept (LPI) design is an essential part of military operations in hostile environments [*Pace*, 2009], LPI performance optimization is a primary issue that needs to be taken into account in designing radar network systems, and some of the noteworthy works include *Narykov et al.* [2013], *Narykov and Yarovoy* [2013] *Shi et al.* [2015, 2016a, 2016b], and *Zhang et al.* [2015]. *Narykov et al.* [2013] and *Narykov and Yarovoy* [2013] investigate the sensor scheduling algorithm of selecting and assigning sensors dynamically for target tracking, which can obtain a good tradeoff between the target tracking accuracy and the LPI performance. *Shi et al.* [2015, 2016a] address the LPI optimization strategies in radar networks, where it has been demonstrated that radar network architectures with multiple transmitters and receivers can

provide remarkable LPI performance advantages over traditional monostatic radar system and has triggered a resurgence of interest in radar networks. *Zhang et al.* [2015] propose a novel coordination algorithm of opportunistic array radars in the networks for target tracking, which not only has excellent target tracking accuracy in clutter but also provides better LPI performance compared with other approaches. In *Shi et al.* [2016b], the problem of LPI-based radar waveform design in signal-dependent clutter for joint radar and cellular communication systems is studied, where three different LPI-based criteria are presented to minimize the total transmitted power of the radar system by optimizing the OFDM radar waveform with a given SINR constraint and a minimum required capacity for the wireless communication systems. On the basis of the research mentioned above, power allocation problem of distributed radar network systems has been studied nicely with the framework of a cooperative game-theoretic model [*Chen et al.*, 2015, 2016; *Sun et al.*, 2014], while the problem of LPI-based power allocation game for cooperative target detection in radar networks, which has not been considered, needs to be investigated.

1.2. Major Contributions

To be specific, the main contributions of this paper are as follows:

1. *We build a framework of adaptive power allocation strategy for cooperative target detection in radar networks. A novel cooperative Nash bargaining power allocation game (NBPAG) model based on LPI is formulated subject to the transmit power constraints and the SINR constraint of each radar, which improves the LPI performance by minimizing the total transmit power in radar networks;*
2. *We strictly prove the existence and uniqueness of Nash bargaining solution (NBS) and develop an iterative Nash bargaining algorithm to solve the NBPAG model;*
3. *Numerical simulations demonstrate the superior LPI performance improvement of the proposed NBPAG strategy in radar networks compared with the NE solution of the noncooperative game;*
4. *We reveal the relationships between the power allocation results and the following two factors: target radar cross section (RCS) and the relative geometry between target and radar networks.*

1.3. Outline of the Paper

The rest of this paper is organized as follows. Section 2 describes the system model of radar networks. Section 3 presents the basic framework for the cooperative NBPAG problem based on LPI, including the basic concepts of the cooperative game, and the well-designed network utility function. An iterative Nash bargaining algorithm is developed for the NBPAG, along with analytical proofs that show the existence and uniqueness of the NBS. Numerical simulations are provided in section 4, followed by conclusion remarks in section 5.

2. System Model

Consider a radar network composed of N_t netted radars, as illustrated in Figure 1. The ith radar receives the echoes from the target due to its transmitted signals as well as the signals from the other radars, both scattered off the target and through a direct path. We assume that all the radars detect the target in the same frequency band. The transmitted signals from different radars may be correlated because of various reasons, including the absence of radar transmission synchronization [*Panoui et al.*, 2016]. Each radar can independently detect the target and send its received signals to the fusion center which takes a decision once the information coming from all the radars is collected. In the presence of a target, the received signal at the radar i can be given by as follows [*Deligiannis and Lambotharan*, 2017]:

$$\mathbf{s}_i = \alpha_i \sqrt{p_i} \mathbf{x}_i + \sum_{j=1, j \neq i}^{N_t} \beta_{i,j} \sqrt{p_j} \mathbf{x}_j + w_i, \tag{1}$$

where $\mathbf{x}_i = \phi_i \mathbf{a}_i$ denotes the transmitted signal from radar i, $\mathbf{a}_i = [1, e^{j2\pi f_{D,i}}, \dots, e^{j2\pi(N-1)f_{D,i}}]$ denotes the Doppler steering vector of radar i with respect to the target, $f_{D,i}$ is the Doppler shift associated with the radar i, N is the number of received pulses in the time-on-target, and ϕ_i is the predesigned waveform transmitted from radar i. α_i represents the channel gain at the direction of the target, p_i is the transmit power of radar i, $\beta_{i,j}$ stands for the cross gain between radar i and j, and w_i denotes a zero-mean white Gaussian noise with variance σ^2. It is assumed that $\alpha_i \sim \mathcal{CN}(0, h_{i,i}^t)$, $\beta_{i,j} \sim \mathcal{CN}(0, c_{i,j}(h_{i,j}^t + h_{i,j}^d))$, and $w_i \sim \mathcal{CN}(0, \sigma^2)$, where $h_{i,i}^t$ represents the variance of the channel gain for the radar i-target-radar i path, $c_{i,j} h_{i,j}^t$ represents the variance of the channel gain for the radar i-target-radar j path, $c_{i,j} h_{i,j}^d$ represents the variance of the channel gain for the direct radar i-radar j path, and $c_{i,j}$ denotes the cross-correlation coefficient between the ith radar and the jth radar.
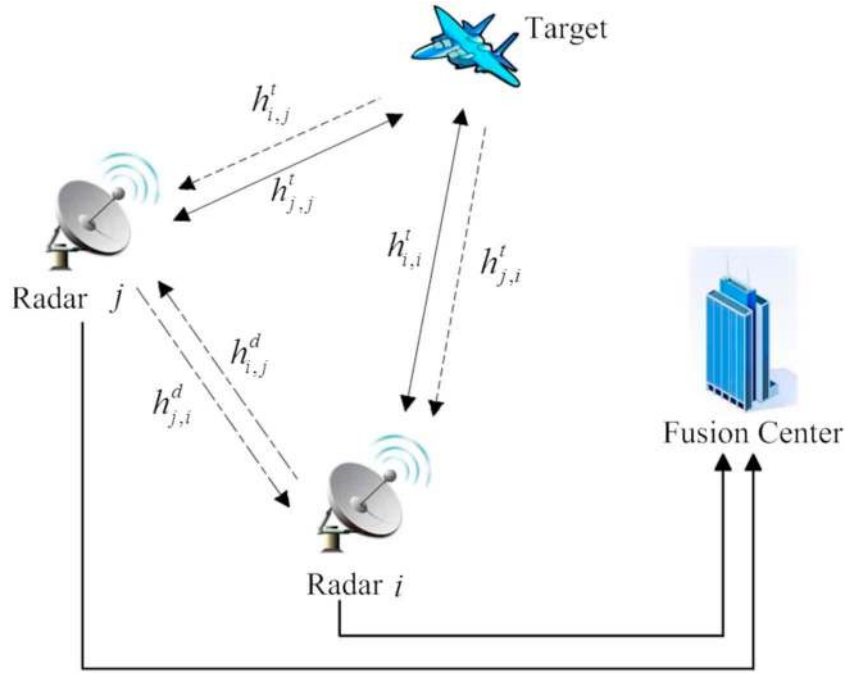
Figure 1. Radar networks system model.

Define the variances of the channel gains for the corresponding paths as follows:

$$
\begin{cases}
h_{i,i}^{t} = \dfrac{G_t G_r \sigma_{i,i}^{\mathrm{RCS}} \lambda^2}{(4\pi)^3 R_i^4}, \\[2ex]
h_{i,j}^{t} = \dfrac{G_t G_r \sigma_{i,j}^{\mathrm{RCS}} \lambda^2}{(4\pi)^3 R_i^2 R_j^2}, \\[2ex]
h_{i,j}^{d} = \dfrac{G_t' G_r' \lambda^2}{(4\pi)^2 d_{i,j}^2},
\end{cases}
\tag{2}
$$

where G_t is the radar main-lobe transmitting antenna gain, G_r is the radar main-lobe receiving antenna gain, G_t' is the radar side-lobe transmitting antenna gain, G_r' is the radar side-lobe receiving antenna gain, $\sigma_{i,i}^{\mathrm{RCS}}$ is the radar cross section (RCS) of the target with respect to the ith radar, $\sigma_{i,j}^{\mathrm{RCS}}$ is the RCS of the target between the ith radar and jth radar, λ denotes the wavelength, R_i denotes the distance from the ith radar to the target, R_j denotes the distance from the jth radar to the target, and $d_{i,j}$ denotes the distance between the ith radar and jth radar. All the variances of channel gains are assumed to be fixed during observation.

Here the generalized likelihood ratio test is used to determine the appropriate detector. The probabilities of miss detection $P_{\mathrm{MD},i}(\delta_i, \gamma_i)$ and false alarm $P_{\mathrm{FA},i}(\delta_i)$ can be derived from the following equations [*Conte et al.*, 1995; *Gini*, 1997]:

$$
\begin{cases}
P_{\mathrm{MD},i}(\delta_i, \gamma_i) = 1 - \left(1 + \dfrac{\delta_i}{1-\delta_i} \cdot \dfrac{1}{1+N\gamma_i}\right)^{1-N}, \\[2ex]
P_{\mathrm{FA},i}(\delta_i) = (1 - \delta_i)^{N-1},
\end{cases}
\tag{3}
$$

where δ_i is the detection threshold. γ_i denotes the SINR received at the ith radar, which can be given by the following:

$$
\gamma_i = \frac{h_{i,i}^{t} p_i}{\sum_{j=1, j \neq i}^{N_t} c_{i,j} \left(h_{i,j}^{d} p_j + h_{i,j}^{t} p_j\right) + \sigma^2}.
\tag{4}
$$

Equation (4) can equivalently be rewritten as follows:

$$
\gamma_i = \frac{h_{i,i}^{t} p_i}{I_{-i}},
\tag{5}
$$

where the total interference and noise received at the ith radar is defined as the following:

$$I_{-i} = \sum_{j=1, j \neq i}^{N_t} c_{i,j} \left(h_{i,j}^d p_j + h_{i,j}^t p_j \right) + \sigma^2. \tag{6}$$

Following the analysis of *Bacci et al.* [2012] and *Panoui et al.* [2014a], we can obtain the corresponding δ_i by equating $P_{\text{MD},i}(\delta_i, \gamma_i)$ and $P_{\text{FA},i}(\delta_i)$ to the predefined parameters, which can be determined by the required target detection performance. Then, using the obtained δ_i, we can compute the value of γ_i for each radar. In order to examine the interaction among radars and determine the best strategy for each radar, we propose to optimize the transmit power allocation for radar networks using a cooperative game model. Specifically, the radar network optimizes power allocation to maximize its utility function for a given SINR requirement. This competition can be modeled utilizing a cooperative power allocation game as follows.

3. Game Theoretical Formulation

This section investigates the problem of adaptive power allocation to each netted radar in a distributed fashion, where an LPI-based NBPAG model is developed for radar networks. The objective of such allocation is to minimize the total transmit power while maintaining a certain target detection requirement in radar network systems. Cooperative game-theoretic framework is a powerful tool for the resulting problem because of its distributed nature. First, we define a novel SINR-based network utility function to evaluate power allocation, which is formulated as a cooperative game. Then, the existence and uniqueness of NBS are proved analytically. Moreover, an iterative Nash bargaining algorithm is implemented that converges quickly to a Pareto optimal equilibrium for the cooperative game.

3.1. Game Theory and Utility Function

Noncooperative game theory is an excellent mathematical tool, which is very suitable for modeling interactions between selfish and rational decision makers in distributed networks [*Bacci et al.*, 2012]. A utility function of a player quantifies its degree of satisfaction as a function of the combination of all players choices [*Yang et al.*, 2015]. In distributed radar network systems, all the netted radars detect the target in the same frequency band. All the radars in a game with conflict interests will behave in a selfish and rational manner to maximize their own utility functions. The objective of noncooperative game is to find an NE point at which none of the radars desires to change its transmit strategy unilaterally. However, the sum of the individual utility functions may not be Pareto optimal at the NE point.

In a cooperative game model, the netted radars adjust their transmission strategies to maximize the network utility function [*Liu and Dong*, 2014]. The network utility function is selected as the sum of the individual utilities of the radars. With the system model described above, the cooperative power allocation game can be defined as follows:

$$\Pi = \left[\mathcal{N}, \{p_i\}_{i=1}^N, \{u_i(p_i, \mathbf{p}_{-i})\}_{i=1}^N \right]. \tag{7}$$

The mathematical structure of a cooperative game model consists of the following three primary components [*Yang et al.*, 2015]:

1. Player set: In this paper, players are netted radars, which are decision makers that choose a particular power level to transmit. A finite set of radars is denoted as $\mathcal{N} = \{1, 2, \ldots, N_t\}$, where N_t is the number of players in the game.
2. Strategy space: Herein, the strategy space is defined by the transmit power allocation strategy. The ith radar in a game selects a strategy s_i from its strategy set S. For each available power level $p_i \in s_i$, the strategy space of the game is defined as $S = \times_{i \in \mathcal{N}} p_i$.
3. Utility function: The utility function of the ith radar is denoted as $u_i(p_i, \mathbf{p}_{-i})$, where $\mathbf{p}_{-i} = [p_1, p_2, \ldots, p_{i-1}, p_{i+1}, \ldots, p_{N_t}]$ is the transmit power vector of all netted radars but radar i. It should be noticed that the ith radars utility is determined by its strategy vector $S = (p_i, \mathbf{p}_{-i})$. Each radar wants to select the appropriate transmission strategy to maximize the network utility function.

For a cooperative game model, *Yang et al.* [2010] propose *extended Nash theorem* that specify the conditions for reaching Pareto-optimal NBSs, as in *Theorem 1*.

Theorem 1 (Extended Nash Theorem): There exists a unique and fair NBS $\mathbf{p}^* = [p_1^*, \ldots, p_{N_t}^*]$, which can be obtained by maximizing a Nash product term as follows:

$$\mathbf{p}^* = \arg \max_{p_i \in S, u_i \geq u_{i,\min}, \forall i} \prod_{i=1}^{N_t} (u_i(\mathbf{p}) - u_{i,\min}), \tag{8}$$

where $S = \times_{i \in \mathcal{N}} p_i$ is the strategy space of the game, and $u_{i,\min}$ is the minimum utility requirement for player i to satisfy its basic need. Taking advantage of the strictly increasing property of logarithmic function, the optimization problem (8) can equivalently be transformed into the following problem with the objective function $\ln \left(\prod_{i=1}^{N_t} (u_i(\mathbf{p}) - u_{i,\min}) \right)$:

$$\mathbf{p}^* = \arg \max_{p_i \in S, u_i \geq u_{i,\min}, \forall i} \sum_{i=1}^{N_t} \ln(u_i(\mathbf{p}) - u_{i,\min}). \tag{9}$$

However, this game formulation is not ideal and fair, which may lead to inefficient solution as it cannot guarantee fairness and the basic requirement of each player. If the radars maximize the network utility function by increasing their own transmit power, this will inevitably result in the mutual interference between different radars and in turn degrade the overall LPI performance of radar networks. Hence, it is of high importance to select an ideal utility function when utilizing cooperative game theory. Utility function is the foundation of game theory, which will deduce the iterative algorithm [*Li et al.*, 2011]. As two sides of the cooperative game, target detection performance and transmission power should be taken into account, which should be reflected in the utility function. The primary objective of radar networks is to minimize the total transmitted power while guaranteeing a specified target detection requirement for each radar. In this paper, we utilize SINR as the target detection performance metric. A novel SINR-based utility function is constructed with the corresponding channel gain, which characterizes the radar's preference regarding LPI and fairness. Therefore, we consider the following LPI-based NBPAG model, which optimizes its transmit power allocation to maximize the network utility function for a specified SINR threshold, i.e.,

$$\max_{\{p_i, i \in \mathcal{N}\}} \sum_{i=1}^{N_t} u_i(p_i, \mathbf{p}_{-i}) = \sum_{i=1}^{N_t} h_{i,i}^t \ln(\gamma - \gamma_{th}^{\min}), \tag{10a}$$

$$s.t. : \begin{cases} \gamma_i \geq \gamma_{th}^{\min}, i \in \mathcal{N} \\ 0 \leq p_i \leq p_i^{\max}, i \in \mathcal{N} \\ \sum_{i=1}^{N_t} p_i \leq p_{tot}^{\max} \end{cases}, \tag{10b}$$

where $\gamma_{th}^{\min}$ denotes the predefined SINR threshold, $p_i^{\max}$ denotes the peak transmit power of radar i, and the total transmit power of radar networks is constrained by a maximum value $p_{tot}^{\max}$. The first constraint implies that the power allocation results should be larger than the given SINR threshold. The second constraint suggests that the transmit power of each radar is limited, while the third one stands that the total transmit power of radar networks is constrained by a maximum value. It is worth pointing out that $h_{i,i}^t$ is related to the target's RCS and the distance between radar i and target. Introducing $h_{i,i}^t$ can well guarantee the fairness among different radars locating at different places. On the basis of the interference degree, greater power would be transmitted when the radar is far away from the target with small RCS and the minor one on the contrary [*Yang et al.*, 2015]. In the simulation part, some numerical examples will be provided to reveal the relationships between the power allocation results and the following two factors: target RCS and the relative geometry between target and radar networks. Here it can be noticed from (10a) that the variances of channel gains $\{h_{i,i}^t\}_{i=1}^N$ are employed to modify the network utility function. The transmitting parameters are adjusted adaptively to guarantee the specified SINR requirement, which can improve the LPI performance by reducing the total transmitted power in radar networks.

3.2. Existence and Uniqueness of NBS
To analyze the outcome of the proposed cooperative NBPAG model, the achievement of an NBS is a well-known optimality criterion.

Theorem 1 (Existence). There is at least one NBS to the proposed NBPAG in (10) if, for all $i \in \mathcal{N}$:

1. The transmission power p_i is a non-empty, convex and compact subset of some Euclidean space.
2. The utility functions $u_i(p_i, \mathbf{p}_{-i})$ are continuous and quasi-concave in p_i.

Proof. It is apparent that our proposed NBPAG model satisfies the first condition (1), which is due to the fact that the transmission power of each radar p_i ranges from 0 to $p_i^{\max}$.

One can observe from (10a) that the utility functions $u_i(p_i, \mathbf{p}_{-i})(\forall i \in \mathcal{N})$ are continuous with respect to p_i. Taking the second derivative of $u_i(p_i, \mathbf{p}_{-i})$ with respect to p_i, we can obtain the following:

$$\frac{\partial u_i(p_i, \mathbf{p}_{-i})}{\partial p_i} = \left(\frac{\gamma_i}{p_i}\right)\frac{h_{i,i}^t}{\gamma_i - \gamma_{\text{th}}^{\min}} > 0, \tag{11}$$

and

$$\frac{\partial^2 u_i(p_i, \mathbf{p}_{-i})}{\partial p_i^2} = -\left(\frac{\gamma_i}{p_i}\right)^2\frac{h_{i,i}^t}{(\gamma_i - \gamma_{\text{th}}^{\min})^2} < 0. \tag{12}$$

Thus, $u_i(p_i, \mathbf{p}_{-i})$ is concave in p_i. As a result, the utility functions are continuous and quasi-concave. This proves the existence of NBS in the proposed NBPAG. ∎

Theorem 2 (Uniqueness). The NBS to NBPAG is unique.

Proof. For the uniqueness of the NBS in a cooperative game, it has been established that there is at most one NBS to the game if and only if the following four conditions are satisfied [*Alireza et al.*, 2009; *Kalai and Smorodinsky*, 1975; *Yang et al.*, 2010].

1. $A_i = \{p_i \in S, f(p_i) = \bar{p} - p_i \geq 0\}$ is nonempty, where $\bar{p}$ is the average transmission power.
2. There exists $p_i \in s_i$ that satisfies $f(p_i) \geq 0$.
3. The utility function $u_i(p_i, \mathbf{p}_{-i})$ of player i is continuous and quasi-concave.
4. The game model is diagonally strictly concave on its strategy set S, that is, for any $\mathbf{p}^{(0)} \neq \mathbf{p}^{(1)}$ with $\mathbf{p}^{(k)} = [p_1^k, \dots, p_{N_t}^k] \in S$ for $k = 0, 1$, and for $\mathbf{t} = [t_1, \dots, t_{N_t}] \geq 0$, the following inequality holds:

$$(\mathbf{p}^{(0)} - \mathbf{p}^{(1)})^T \mathbf{d}(\mathbf{p}^{(0)}, \mathbf{t}) + (\mathbf{p}^{(1)} - \mathbf{p}^{(0)})^T \mathbf{d}(\mathbf{p}^{(1)}, \mathbf{t}) < 0, \tag{13}$$

where the function $\mathbf{d}(\mathbf{p}, \mathbf{t})$ is defined as follows:

$$\mathbf{d}(\mathbf{p}, \mathbf{t}) = \left[t_1\frac{\partial u_1}{\partial p_1}, \dots, t_{N_t}\frac{\partial u_{N_t}}{\partial p_{N_t}}\right]^T. \tag{14}$$

Obviously, conditions 1 and 2 could be satisfied as direct results from the strategy space constraint (10b). Condition 3 has been proved in *Theorem 1*. For condition 4, we have the following:

$$\begin{aligned}
(\mathbf{p}^{(0)} &- \mathbf{p}^{(1)})^T \mathbf{d}(\mathbf{p}^{(0)}, \mathbf{t}) + (\mathbf{p}^{(1)} - \mathbf{p}^{(0)})^T \mathbf{d}(\mathbf{p}^{(1)}, \mathbf{t}) \\
&= (\mathbf{p}^{(0)} - \mathbf{p}^{(1)})^T \times \left[\mathbf{d}(\mathbf{p}^{(0)}, \mathbf{t}) - \mathbf{d}(\mathbf{p}^{(1)}, \mathbf{t})\right] \\
&= (\mathbf{p}^{(0)} - \mathbf{p}^{(1)})^T \times \left[t_1\left(\frac{\partial u_1}{\partial p_1^{(0)}} - \frac{\partial u_1}{\partial p_1^{(1)}}\right), \dots, t_{N_t}\left(\frac{\partial u_{N_t}}{\partial p_{N_t}^{(0)}} - \frac{\partial u_{N_t}}{\partial p_{N_t}^{(1)}}\right)\right]^T \\
&= \sum_{i=1}^{N_t} t_i(p_i^{(0)} - p_i^{(1)})\left(\frac{\partial u_i}{\partial p_i^{(0)}} - \frac{\partial u_i}{\partial p_i^{(1)}}\right).
\end{aligned} \tag{15}$$

Let $\alpha_i = t_i(p_i^{(0)} - p_i^{(1)})\left(\frac{\partial u_i}{\partial p_i^{(0)}} - \frac{\partial u_i}{\partial p_i^{(1)}}\right)$, where $t_i \geq 0$. From Theorem 1, $\frac{\partial u_i}{\partial p_i}$ is monotonically decreasing with respect to p_i, which is because that the utility function is concave. Thus, we can obtain $\frac{\partial u_i}{\partial p_i^{(0)}} - \frac{\partial u_i}{\partial p_i^{(1)}} < 0$ for $p_i^{(0)} > p_i^{(1)}$, and hence, $\alpha_i \leq 0$. Similarly, $\alpha_i \leq 0$ holds for $p_i^{(0)} < p_i^{(1)}$ as well. As a consequence, all the required conditions are met. It can be concluded that our proposed NBPAG model has only one unique NBS, which completes the NBS uniqueness proof. ∎

3.3. Iterative Nash Bargaining Algorithm

In this section, we present an iterative Nash bargaining algorithm that repeats the power allocation steps until convergence. Having proved the existence and uniqueness of the NBS, we now solve for this unique equilibrium by solving the constrained optimization problem in (10a) utilizing the method of Lagrange multipliers

[*Yang et al.*, 2010]. Introducing Lagrange multipliers $\{\eta_i\}_{i=1}^{N_t}$, $\{\phi_i\}_{i=1}^{N_t}$, $\{\xi_i\}_{i=1}^{N_t}$ and ψ for the multiple constraints, the Lagrangian of problem (9) can equivalently be expressed by as follows:

$$
\begin{aligned}
\mathscr{L}&\left(\{u_i(p_i,\mathbf{p}_{-i})\}_{i=1}^{N_t},\{\eta_i\}_{i=1}^{N_t},\{\phi_i\}_{i=1}^{N_t},\{\xi_i\}_{i=1}^{N_t},\psi\right)\\
&=\sum_{i=1}^{N_t}h_{i,i}^t\ln(\gamma-\gamma_{\text{th}}^{\min})-\eta_i(\gamma-\gamma_{\text{th}}^{\min})+\phi_i(p_i-p_i^{\max})-\xi_ip_i+\psi\left(\sum_{i=1}^{N_t}p_i-p_{\text{tot}}^{\max}\right).
\end{aligned}
\tag{16}
$$

In order to obtain the NBS, taking the first derivative of (16) with respect to p_i and setting it to zero, we can observe that

$$
\frac{\partial\mathscr{L}}{\partial p_i}=\frac{h_{i,i}^t}{\gamma_i-\gamma_{\text{th}}^{\min}}\frac{h_{i,i}^t}{I_{-i}}-\eta_i\frac{h_{i,i}^t}{I_{-i}}+\phi_i-\xi_i+\psi=0.
\tag{17}
$$

After basic algebraic manipulations, we can reach the optimal solution p_i^* as a function of the Lagrange multipliers:

$$
p_i^*=\frac{I_{-i}}{h_{i,i}^t}\gamma_{\text{th}}^{\min}+\frac{h_{i,i}^t}{\eta_i\frac{h_{i,i}^t}{I_{-i}}-\phi_i+\xi_i-\psi}.
\tag{18}
$$

In this paper, the fixed-point method is utilized to derive an iterative procedure that updates the power allocation results in radar network system. Obviously, according to (5) and (18), (18) can be used to obtain p_i^* through iterations as follows:

$$
p_i^{(n+1)}=\left[\frac{p_i^{(n)}}{\gamma_i^{(n)}}\gamma_{\text{th}}^{\min}+\frac{h_{i,i}^t}{\eta_i^{(n)}\frac{\gamma_i^{(n)}}{p_i^{(n)}}-\phi_i^{(n)}}+\xi_i^{(n)}-\psi^{(n)}\right]_0^{p_i^{\max}},
\tag{19}
$$

where $[x]_a^b=\max\{\min(x,b),a\}$, and n denotes the iteration index. The Lagrange multipliers $\{\eta_i^{(n)}\}_{i=1}^{N_t}$, $\{\phi_i^{(n)}\}_{i=1}^{N_t}$, $\{\xi_i^{(n)}\}_{i=1}^{N_t}$ and $\psi^{(n)}$ need to carefully be chosen to ensure fast convergence. Here, the subgradient method is employed to update these multipliers through the following steps:

$$
\begin{cases}
\eta_i^{(n+1)}=\left[\eta_i^{(n)}+s_t(\gamma_i-\gamma_{\text{th}}^{\min})\right]_0^+,\\
\phi_i^{(n+1)}=\left[\phi_i^{(n)}+s_t(p_i^{\max}-p_i)\right]_0^+,\\
\xi_i^{(n+1)}=\left[\xi_i^{(n)}+s_tp_i\right]_0^+,\\
\varphi^{(n+1)}=\left[\varphi^{(n)}+s_t\left(p_{\text{tot}}^{\max}-\sum_{i=1}^{N_t}p_i\right)\right]_0^+,
\end{cases}
\tag{20}
$$

where $[x]_a^+=x$ if $x>0$, and $[x]_a^+=a$ if $x\le 0$. s_t is a small step size, $n\in\{1,\ldots,N_{\max}\}$, and $N_{\max}$ is the maximum number of iterations. Apparently, $\eta_i^{(n)}$, $\phi_i^{(n)}$ and $\xi_i^{(n)}$ are locally updated, whereas $\varphi^{(n)}$ is updated through cooperation. Each netted radar updates its action at each iteration step such that the network utility function in (10) is maximized. The overall iterative procedure of applying the proposed cooperative NBPAG model is given in detail as follows:

Algorithm 1 :Nash Bargaining for Adaptive Power Allocation

1: **Initialization:** Initialize $\gamma_{\text{th}}^{\min}$, the multipliers $\{\eta_i^{(0)}\}_{i=1}^{N_t}$, $\{\phi_i^{(0)}\}_{i=1}^{N_t}$, $\{\xi_i^{(0)}\}_{i=1}^{N_t}$, and $\varphi^{(0)}$ to some values; Initialize iterative index $n=1$, the tolerance $\epsilon>0$.
2: **Repeat Until:** $\left|p_i^{(n+1)}-p_i^{(n)}\right|<\epsilon$ or $n=N_{\max}$
    for $i=1,\ldots,N_t$, **do**
        Calculate $p_i^{(n)}$ by solving (19);
        Update $\eta_i^{(n)}$, $\phi_i^{(n)}$, $\xi_i^{(n)}$ and $\varphi^{(n)}$ by (20);
    end for
    Set $n\leftarrow n+1$;
3: **Update:** Update $p_i^*\leftarrow p_i^{(n)}$ for $\forall i$.

Table 1. Radars Positions in Radar Networks

Radar	Position (km)	Radar	Position (km)
1	[0, 0]	3	[50, 50]
2	[50, 0]	4	[0, 50]

Remark 1. In the foregoing procedure, the transmit power iteration function $p_i^{(n+1)}$ can be updated according to (19), wherein the optimal power allocation results can be decided locally. For the ith radar, compute the difference of utility function for $n + 1$ and n iteration. If the difference in allowable error scope ϵ, the iteration stops; otherwise, it returns to step 2.

Remark 2. In order to implement *Algorithm 1* in a distributed manner, each radar has to collect the variances of its adjacent channel gains $\{c_{i,j}h_{i,j}^t\}_{j=1,j\neq i}^{N_t}$ and $\{c_{i,j}h_{i,j}^d\}_{j=1,j\neq i}^{N_t}$. The variances of channel gains $\{h_{i,i}^t\}_{i=1}^{N_t}$ also need to be obtained. This can be done by having each radar measures the channel and feed back to the transmitter. Here the best response of the ith radar p_i^* depends on the strategies of all the other radars, that is, $\mathbf{p}_{-i}^*$. In order to obtain this knowledge, each radar has to broadcast its transmission strategy to the other radars. In the following section, the convergence behavior of the iterative Nash bargaining algorithm will be verified via numerical simulations.

4. Numerical Simulations and Analysis

In the following, several numerical simulations are dedicated to demonstrate the improvement of the LPI performance brought by the power allocation strategy and reveal the effects of several factors on the power allocation results.

4.1. Description

In this paper, we consider a radar network with $N_t = 4$ spatially diverse radars. The positions of the netted radars are given in Table 1. To evaluate the effect of the relative geometry between the target and the radar networks on the power allocation, two different target positions with respect to the radar networks are
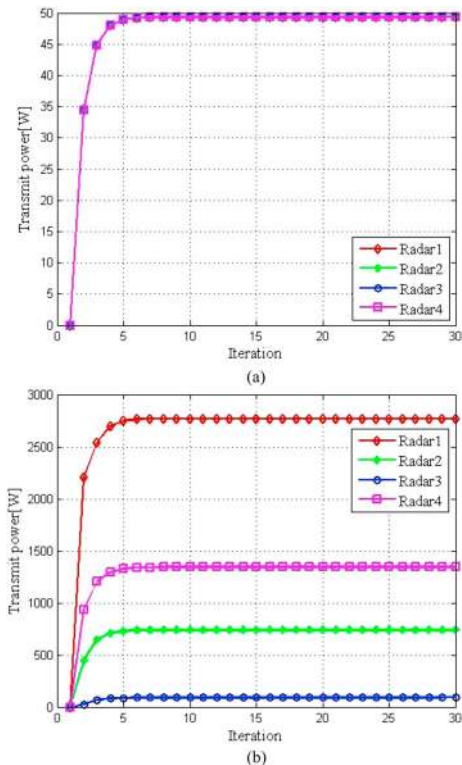

Figure 2. Power convergence of the proposed NBPAG algorithm in Case 1: (a) Case 1 with $\overrightarrow{\sigma_1^{RCS}}$ and (b) Case 1 with $\overrightarrow{\sigma_2^{RCS}}$.
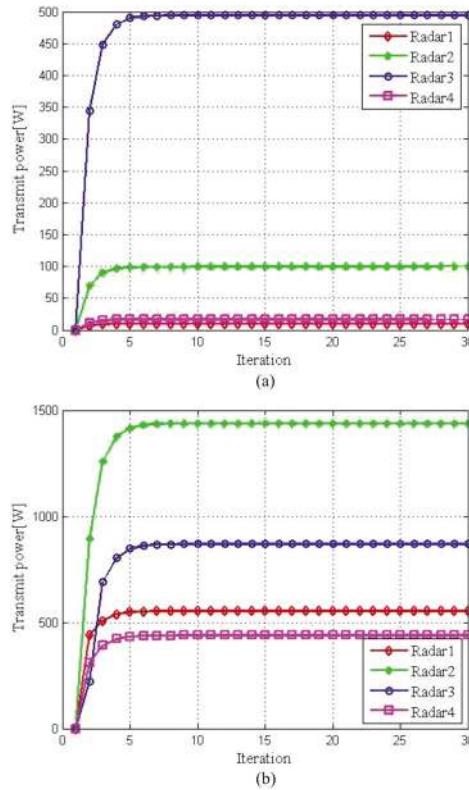
Figure 3. Power convergence of the proposed NBPAG algorithm in Case 2: (a) Case 2 with $\overrightarrow{\sigma_1^{RCS}}$ and (b) Case 2 with $\overrightarrow{\sigma_2^{RCS}}$.

chosen. In the first case, we assume that the target is located at [25, 25] km. In the second case, we simulate a scenario in which the target is located at [80, 60] km. The cross-correlation coefficient between different radars is $c_{i,j} = 0.01 (i \neq j)$. The system parameters are set as follows: the radar antenna gains are $G_t = G_r = 30$dB, $G_t' = G_r' = -30$dB, the wavelength is $\lambda = 0.03$ m, the maximum transmit power of each radar is limited to $p_i^{max} = 0.5 p_{tot}^{max} = 5$ KW, the number of received pulses is $N = 512$, $P_{MD,i} = 2.7 \times 10^{-3}$, $P_{FA,i} = 10^{-6}$. The SINR can be computed using (2), which is $\gamma_i = 10$dB for all radars, and the corresponding δ_i is equal to 0.0267 for $\forall i$. The noise power $\sigma^2 = 10^{-18}$W, $\epsilon = 10^{-15}$. We initialize $\eta_i^{(0)} = 10$, $\phi_i^{(0)} = 10$, $\xi_i^{(0)} = 10(\forall i)$, and $\varphi^{(0)} = 10$. The step size s_t is 0.001.

Two target RCS models are adopted in this paper. The first model is uniform reflectivity, where $\overrightarrow{\sigma_1^{RCS}} = [1, 1, 1, 1]$ m². On the other hand, in order to evaluate the effect of the target RCS on the power allocation strategy, we also adopt the second RCS model $\overrightarrow{\sigma_2^{RCS}} = [5, 0.5, 0.1, 3]$ m².

4.2. Numerical Results

Figures 2 and 3 testify the transmit power convergence of the proposed algorithm. There are four curves in all subfigures, which means the power conditions of the four radars. The steady state transmit power values are relatively ideal, which are much smaller than the maximum value. The difference between them resides in the target RCS with respect to different radars and their different distances to the target. The convergence to the equilibrium of the NBPAG model is visible in Figures 2 and 3, and the transmit power of each radar converges fast to the equilibrium value after 4–8 iterations. Since the equilibrium of the proposed NBPAG model is unique, the proposed algorithm will converge to it independently of the initial set of transmit power values that were used. Besides, it is worth pointing out that the choice of the Lagrange multipliers is crucial to the convergence behavior [*Liu and Dong*, 2014].

In order to disclose the effects of several factors on the power allocation results, Figures 2 and 3 plot the power allocation results of the proposed algorithm. In Figure 2b, less transmit power is assigned to Radar 2 and Radar 3, as they are closer to the target. In other words, more power tends to be allocated to the radar farther from the target. Analyzing the power allocation results given in Figure 2 reveals that the different
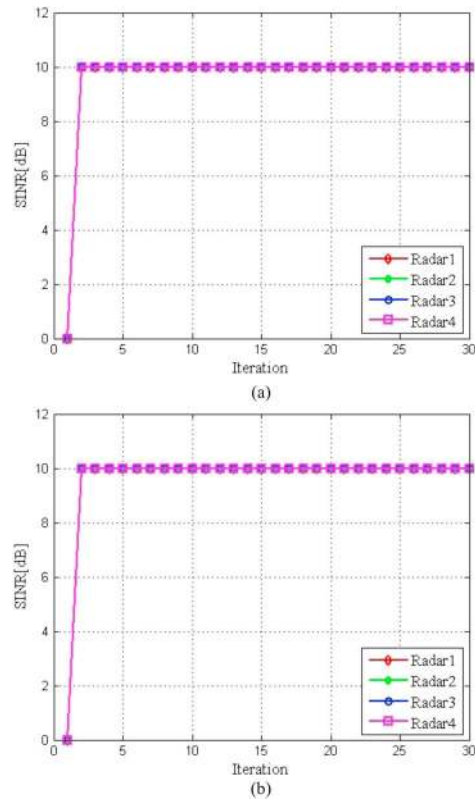
Figure 4. SINR convergence of the proposed NBPAG algorithm in Case 1: (a) Case 1 with $\overrightarrow{\sigma_1^{RCS}}$ and (b) Case 1 with $\overrightarrow{\sigma_2^{RCS}}$.
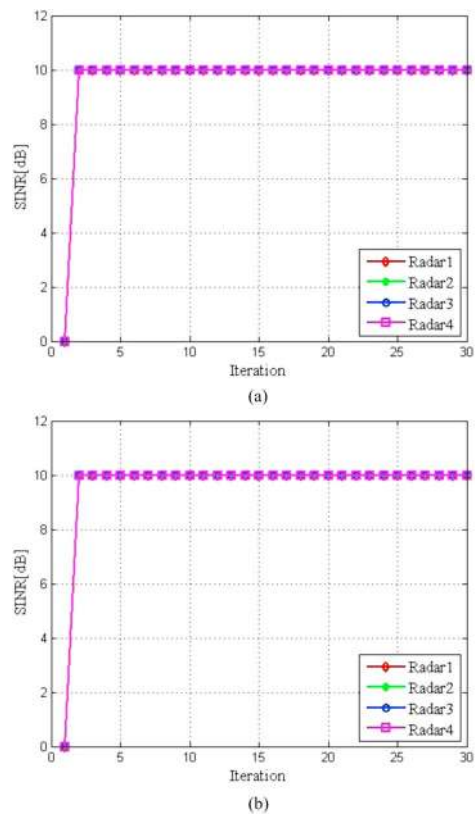

Figure 5. SINR convergence of the proposed NBPAG algorithm in Case 2: (a) Case 2 with $\overrightarrow{\sigma_1^{RCS}}$ and (b) Case 2 with $\overrightarrow{\sigma_2^{RCS}}$.
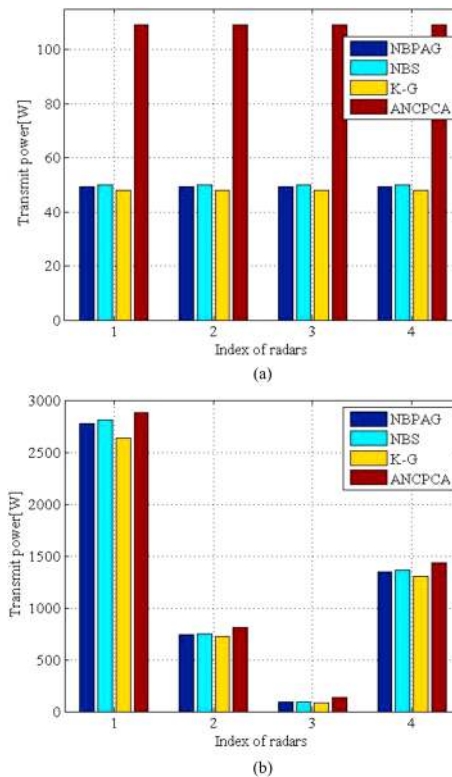
Figure 6. Comparisons of equilibrium transmit power levels in Case 1 with various power allocation algorithms: (a) Case 1 with $\overrightarrow{\sigma_1^{RCS}}$ and (b) Case 1 with $\overrightarrow{\sigma_2^{RCS}}$.

deployment of radar networks may lead to different gain in LPI performance. Moreover, the results also show that in an overdetermined scenario, most of the available total transmit power is allocated to a smaller subset of radars, while others are kept to a minimal power [*Yan et al.*, 2015]. We then expand the simulation with the consideration of the losses due to target RCS and provide the power allocation results in Figure 3. The results illustrate that the radars with smaller RCS are favorable over others, when it comes to power allocation. In the optimization process, higher transmit power is assigned to the radars with relative weaker propagation channels. The above results imply that the allocation of transmit power is determined by the following two factors: radar network deployment and target RCS.

Figures 4 and 5 illustrate the SINR achieved at each radar receiver for each player, utilizing our proposed cooperative game-theoretic algorithm for the considered radar network topology. We can clearly notice that the achieved SINR converges fast to the predetermined threshold after three to five iterations, which can meet the target detection requirement of each radar, confirming that the NBPAG model can overcome the near-far effect. As aforementioned, the proposed NBPAG model will converge to its equilibrium independently of the initial set of transmit power values. Thus, the players will reach the same Nash equilibrium regardless of the choice of the initial values and employing the publicly known information. Due to the fact that the convergence rate of the proposed algorithm is not slow under current computation conditions, there is no need to study the acceleration approach of our algorithm, which may be an interesting topic for further research.

To demonstrate the superior advantages of the proposed algorithm further, we compare the LPI-based NBPAG algorithm with a couple of benchmark algorithms for power allocation: the standard NBS for cooperative game, *Koskie and Gajic*'s [2005, hereinafter K-G] algorithm, and the adaptive noncooperative power control algorithm (ANCPCA) in *Yang et al.* [2015], as depicted in Figures 6–9. In Figures 6 and 7, we compare the transmit power levels of four power allocation algorithms with different radars. It turns out that the ANCPCA transmits the most power due to the radars' self-interested noncooperative behavior in the game process, which is consistent with the results in *Yang et al.* [2015]. To be specific, when one of the netted radars cannot reach or maintain its minimum SINR, it resorts to the only means of increasing the transmit power to
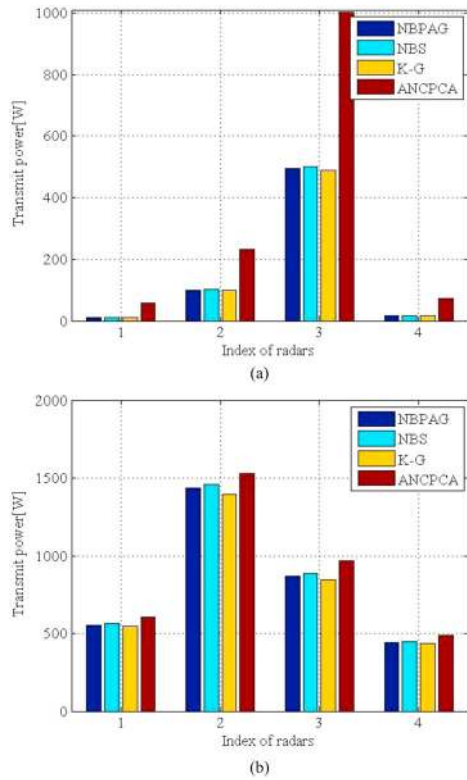
Figure 7. Comparisons of equilibrium transmit power levels in Case 2 with various power allocation algorithms: (a) Case 2 with $\overrightarrow{\sigma_1^{\text{RCS}}}$ and (b) Case 2 with $\overrightarrow{\sigma_2^{\text{RCS}}}$.
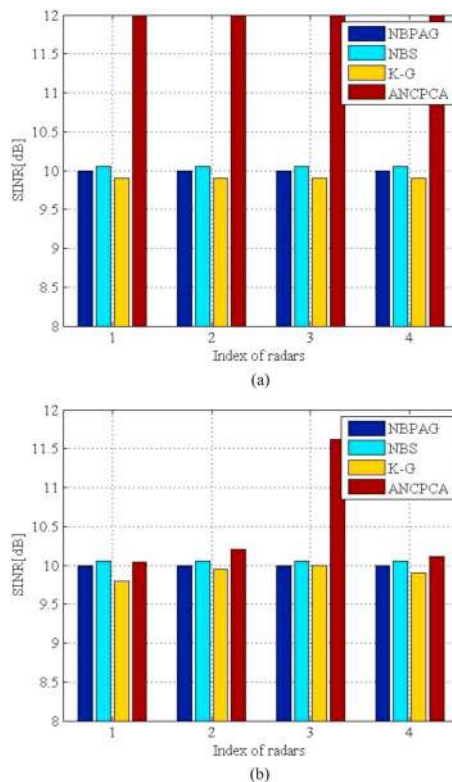

Figure 8. Comparisons of equilibrium SINR values in Case 1 with various power allocation algorithms: (a) Case 1 with $\overrightarrow{\sigma_1^{\text{RCS}}}$ and (b) Case 1 with $\overrightarrow{\sigma_2^{\text{RCS}}}$.
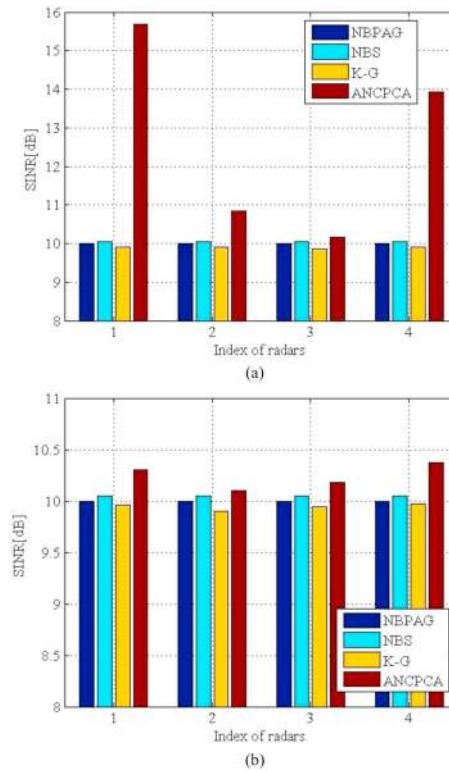
Figure 9. Comparisons of equilibrium SINR values in Case 2 with various power allocation algorithms: (a) Case 2 with $\overrightarrow{\sigma_1^{RCS}}$ and (b) Case 2 with $\overrightarrow{\sigma_2^{RCS}}$.

guarantee the SINR requirement, as do other radars in a similar situation [*Yang et al.*, 2010, 2015]. As a result, the LPI performance of radar networks degrades. While for the NBPAG algorithm, the netted radars can perceive the interference environment well and accordingly make the most appropriate transmit power adjustment decision.

From Figures 8 and 9, it can be seen that the SINR values of the proposed algorithm and ANCPCA can reach the target SINR threshold. However, the standard NBS method and K-G algorithm are not ideal because they sacrifice radars' SINRs, where part of SINR values achieved at radar receivers are below the specified SINR threshold. Thus, the standard NBS method and K-G algorithm cannot guarantee the fairness among different radars. As we can observe, the SINR of each radar of the proposed algorithm converges to the SINR threshold, which shows that the NBPAG algorithm can overcome the near-far effect. Overall, the K-G algorithm consumes the least power, which cannot meet the radars' target detection need. However, the NBPAG approach can accommodate each radar's transmit power to satisfy its SINR requirement, although it might consume high transmit power. Generally speaking, those results demonstrate that the NBPAG approach not only guarantees the SINR requirements of all the netted radars but also improves the LPI performance of radar networks.

5. Conclusion

We have considered an LPI-based distributed power allocation strategy for radar networks in a cooperative game-theoretic framework. In our proposed algorithm, the LPI performance of radar networks can be improved by optimizing the transmit power allocation for a predefined target detection threshold. A novel SINR-based network utility function is developed as a metric to evaluate power allocation, which guarantees the existence and uniqueness of the NBS. In addition, an iterative Nash bargaining algorithm with low complexity and fast convergence is utilized to play the game among the netted radars. Simulation results demonstrate that compared with the NE solution of the noncooperative game, the NBS of the cooperative NBPAG can remarkably improve the LPI performance for radar networks. In future work, we will concentrate on other practical distributed approaches to enhance the LPI performance for distributed radar network systems.

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