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Power Allocation in OFDM-Based Cognitive Radio Systems

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1. Introduction

Spectrum scarcity is becoming a serious problem as the rapid developments of wireless communications. However, recent spectrum measurements show that the licensed spectrum is severely underutilized by the primary users (PUs) in both the time domain and the spatial domain (FCC, 2002; Mchenry, 2005). The secondary users (SUs) are introduced to exploit the spectrum opportunities left by the PUs in order to improve the spectrum utilization. The transmission of SU is required to satisfy the PU's interference limits to protect the PU's performance. Cognitive radio (Mitola, 1999; Haykin, 2005; Zhao & Sadler, 2007) is the key technology enabling flexible, efficient and reliable spectrum utilization by adapting the radio's operating characteristics to the real-time conditions of the environment. The SUs with cognitive radio technology are able to cleverly detect and utilize the spectrum opportunities and thus realize efficient reuse of the licensed spectrum.

It is extremely important to guarantee the transmission of the SU satisfy the interference limits of the PUs for the cognitive radios to be acceptable. Generally, the interference limits can be described by two metrics: the maximum allowable collision probability and the interference power limit (Zhao & Sadler, 2007). The former one is related to the spectrum sensing mechanism of the cognitive radios, which has been widely studied in both the physical (PHY) layer (Sahai et al., 2004; Oner & Jondral, 2007; Cabric et al., 2004) and the medium access control (MAC) layer (Zhao et al., 2007; Kim & Shin, 2007; Ghasemi & Sousa, 2007) in the literature. The latter one introduces power restrictions on the channel output signals in cognitive radio systems, which is different from the conventional wireless systems where the power constraints are only placed on the channel input signals. This feature motivates the studies on how to efficiently utilize the spectrum opportunities under the interference power limits of the PUs (Zhang et al., 2008; Shu et al., 2006; Sudhir & Jafar, 2007; Le & Hossain, 2007).

A cognitive radio system needs to be highly flexible in terms of the spectral shape of the transmit signal. Orthogonal frequency division multiplexing (OFDM) modulation is a promising candidate for such a flexible system because of its reconfigurable subcarrier structure (Weiss & Jondral, 2004; Berthold & Jondral, 2005). With OFDM, the SU has the ability to flexibly fill the spectral gaps left by PUs. Also, the fast Fourier transform (FFT)

components at the OFDM system's receiver may also be used for the SU to execute the channel detection. In
(Weiss & Jondral, 2004), Weiss and Jondral proposed that the band of the SU covers multiple PUs' licensed spectrum, then the SU modulates zero on the subcarriers which belong to the detected PUs's licensed spectrum while utilizing other subcarriers for transmission. In OFDM-based cognitive radio systems, the adaptive subcarrier configuration should consider not only the channel state information (CSI) as in conventional OFDM systems, but also the sensing results of the SU and the interference limits of the PUs. This chapter focuses on investigating the research challenges involved in the power allocation for OFDM-based cognitive radio systems.

The optimal power allocation algorithm for conventional OFDM systems that maximizes the channel capacity is the wellknown *water-filling,* which is derived by solving a convex optimization problem subject to the sum transmit power constraint(TSE & Viswanath, 2005). In OFDM-based cognitive radio systems, the band of the SU can be divided into several subchannels, each of which is corresponding to a licensed band of one PU system. Since the interference limit of each PU introduces the subchannel transmit power constraint for the SU, the power allocation in OFDM-based cognitive radio systems should not only satisfy the sum transmit power constraint but also the subchannel transmit power constraints (Zhao & Sadler, 2007). Therefore, the conventional water-filling algorithm is not applicable in such a scenario.

The transmit power in each subchannel is comprised of the power allocated to the subcarriers inside the subchannel and the sidelodes power of the subcarriers in other subchannels. In this chapter, we first formulate the power allocation problem in the case where the effects of subcarrier sidelobes can be ignored, i.e., there is sufficient guard band between any two neighboring subchannels. Based on the convex optimization theory, an algorithm named *iterative partitioned water-filling* (IPW) is proposed to obtain the optimal power allocation that maximizes the capacity while satisfying both the sum and subchannel transmit power constraints. Then, we extend the results and address the power allocation problem in general OFDM-based cognitive radio systems, where the effects of subcarrier sidelobes are specially considered. In this case, we propose a *recursive power allocation* (RPA) algorithm to obtain the optimal power allocation by decoupling the subchannel power constraints phase-by-phase. The organization of the rest of this chapter is as follows: Sections 2 presents an overview of the cognitive radio and OFDM-based cognitive radio systems. In Section 3, the power allocation problem ignoring the effects of subcarrier sidelobes for OFDM-based cognitive radio systems is formulated and analyzed. The IPW algorithm is proposed and its optimality is proved. Then Section 4 addresses the general OFDM-based cognitive radio systems where the effects of subcarrier sidelobes are considered. The RPA algorithm is proposed and its optimality is proved. Simulation results are presented in Section 5. Section 6 discusses the future research directions and concludes the chapter.

2. System Model

2.1 Cognitive radio system and interference power limit
One of the typical cognitive radio systems is shown in Fig. 1. A certain channel is licensed to the PU system. Since the PU system does not occupy the channel anywhere at any time, the

channel is underutilized in both the spatial domain and the time domain. A channel is said to be a spectrum opportunity if the interference to the PU receivers caused by the SU's transmission is tolerable. The SU is permitted to access the channel if the channel is detected to be a spectrum opportunity. With the participance of the SU, the spectrum
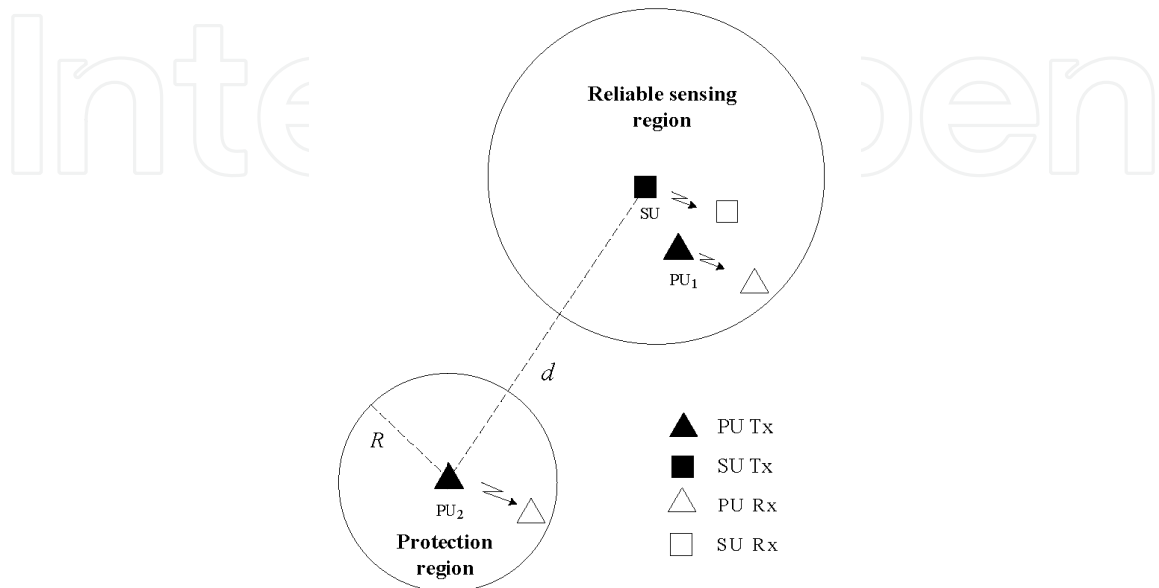

Fig. 1. Cognitive radio system model.

opportunities can be identified and exploited and thus the overall spectral efficiency can be improved.

To guarantee the interference to the PU tolerable is an extremely important problem in the implementation of cognitive radio systems. Generally, the SU has to sense the channel in order to determine whether a spectrum opportunity exists before the transmission. The SU is permitted to transmit on a licensed channel unless the corresponding PU's signal is seen as absent. A detection probability has to be achieved to reduce the collisions between the SU and the PU caused by the detection errors. As in Fig. 1, PU_1's signal can be detected when active and thus the interference to PU_1 receiver caused by the SU's transmission can be avoided. However, due to the signal to noise ratio limit, the SU can only detect the signal of the PU with the required detection probability within a certain region, as the reliable sensing region shown in Fig. 1. Therefore, for the PU_2 transmitter that is outside the SU's reliable sensing region, the SU is unable to detect PU_2's signal with the required detection probability, as depicted in Fig. 1. In this situation, as in (Zhao & Sadler, 2007), PU_2 defines a protection area whose radius is R and requires the interference power at any potential receiver in this area be lower than a certain value, say . Therefore, when one PU's signal is seen as absent, the SU's transmit power on the PU's licensed channel P_{tx} should be subject to a power constraint, which is given by

$$P_{tx} \leq \eta(d-R)^\beta,$$ (1)

where d is the distance between the SU transmitter and the nearest undetectable PU trans-mitter (the PU transmitter outside the reliable sensing region is referred to undetectable PU transmitter in the following), and β is the path attenuation factor. Note that we only consider

the distance-based path loss here for simplicity. Also, the distance between the SU and the nearest undetectable PU transmitter is assumed to be known in advance by the SU
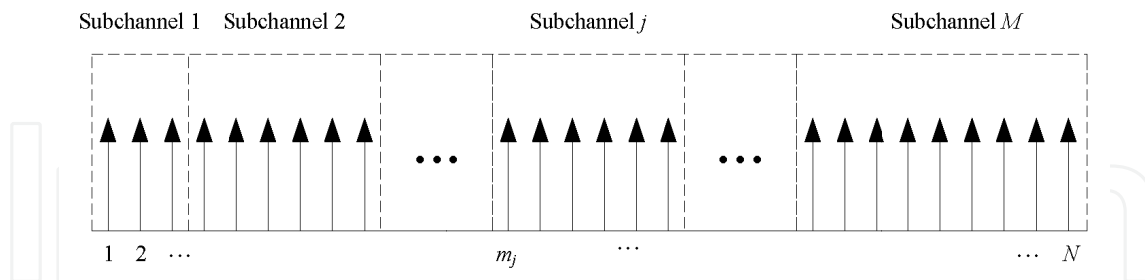

Fig. 2. Spectrum of SU in OFDM-based cognitive radio systems.

in (1). We note that if the SU is unaware of the PU's location, some conservative scheme may be used to estimate P_{tx}. For example, d is set to be the radius of the reliable sensing region, i.e., the PU's transmitter is assumed to be just at the margin of the reliable sensing region.

2.2 OFDM-based cognitive radio systems and subchannel transmit power constraints
In order to efficiently exploit the spectrum opportunities left by different PU systems, a cognitive radio system needs to be highly flexible with respect to the spectral shape of the transmit signal. OFDM modulation is a promising candidate for such a flexible system because of its reconfigurable subcarrier structure. Also, the FFT component at the OFDM receiver can be used for spectrum sensing, which reduces the overhead for the implementation of the cognitive capability.

As in Fig. 2, in an OFDM-based cognitive radio system, the spectrum that can be potentially used by the SU is divided into M subchannels. Each subchannel is corresponding to a PU's licensed band. The total number of the subcarriers is assumed to be N and m_j denotes the index of the first subcarrier in the jth subchannel.

Before the transmission, the SU senses whether each subchannel is occupied firstly, by proper spectrum sensing methods. Then according to the sensing results and the CSI of each subcarrier over SU transmission link, the SU can decide the proper power allocation scheme, modulation type and other parameters for SU's transmission. Therefore, the total transceiver diagram can be shown in Fig. 3. At the transmitter in Fig. 3, the transmission parameters should be decided before serial-to-parallel (S/P) conversion, IFFT operation, parallel-to-serial (P/S) conversion, insertion of a Cyclic Prefix (CP) and filtering. Accordingly, at the receiver, the information on transmission parameters over the link should be obtained through signaling to receive and demodulate OFDM signals. Then the received signal of the ith subcarrier within one OFDM symbol can be expressed as:

$$y_i = h_i x_i + n_i \tag{2}$$

where x_i is the signal transmitted on the ith subcarrier by the SU transmitter, h_i is the channel gain and n_i is the additive white Gaussian noise with mean 0 and variance 1. If the corresponding PU transmitter is detected in a subchannel, all the subcarriers in this subchannel will be modulated by zero during the transmission, i.e., the sum power of the subcarriers in this subchannel is set to be zero. Otherwise, the SU can use this subchannel
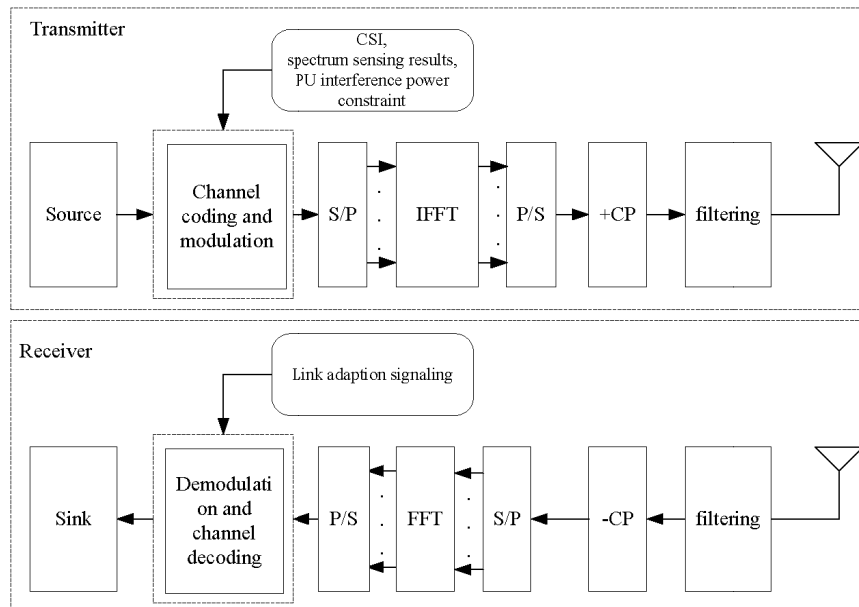
Fig. 3. Transceiver diagram of SU in OFDM-based cognitive radio systems.

but with a certain power constraint which has been described in Section 2.1. Assume G_j is the power constraint on the jth subchannel after the sensing, we have

$$G_j \triangleq \begin{cases} 0 & \text{PU}_j \text{ is detected} \\ \eta_j (d_j - R_j)^{\beta_j} & \text{PU}_j \text{ is not detected} \end{cases} \tag{3}$$

where η_j is the interference power limit of PU_j, R_j is the radius of the protection area of PU_j, d_j is the distance between the SU's transmitter and the nearest undetectable PU_j's transmitter, and β_j is the path attenuation factor.

3. Power Allocation for OFDM-based Cognitive Radio Systems without Considering Subcarrier Sidelobes

In conventional OFDM systems, given the sum transmit power constraint or the target rate, the optimal power allocation that aims at maximizing the sum rate or minimizing the required power can be achieved by the well-known water-filling algorithm. In OFDM-based cognitive radio systems, the power allocation should also satisfy the subchannel transmit power constraints which are introduced by the interference power limits of the PUs. Therefore, the water-filling algorithm needs to be modified. In this section, we first review the power allocation problem in conventional OFDM systems and then formulate that in the cognitive radio scenario.
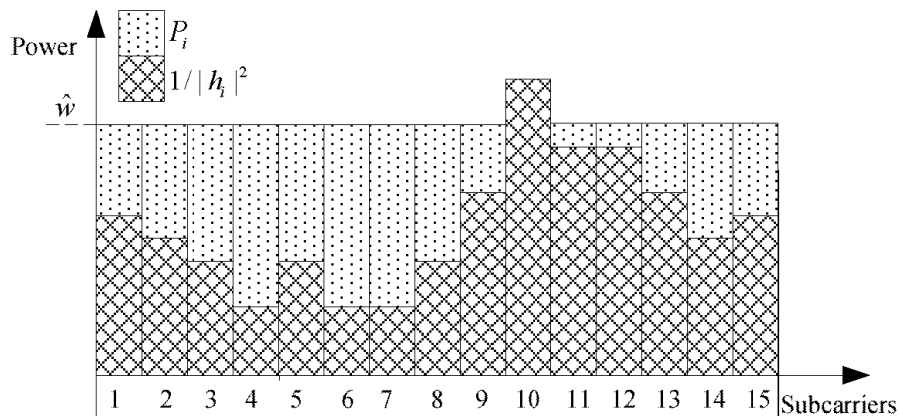
Fig. 4. Optimal power allocation in conventional OFDM systems.

3.1 Power allocation in conventional OFDM systems

The signal model for conventional OFDM systems is as (2) shows. The power allocation problem can be classified into two categories in conventional OFDM-based systems. The first one is to optimize the power allocation across the subcarriers such that the sum rate is maximized with a given sum transmit power constraint. Assume the sum transmit power constraint of an OFDM block is P_t, the allocated signal power on the ith subcarrier is P_i and the number of the subcarriers is L. The optimal power allocation can be given by

$$P_i = \left(\widehat{w} - \frac{1}{|h_i|^2} \right)^+ , i = 1, 2, ..., N \tag{4}$$

where $(\cdot)^+$ denotes $\max\{\cdot, 0\}$ and $\widehat{w}$ is determined by

$$\sum_{i=1}^{L} \left(\widehat{w} - \frac{1}{|h_i|^2} \right)^+ = P_t \tag{5}$$

The algorithm to obtain the optimal solution is the well-known water-filling, where the so-called water-level $\hat{w}$ is calculated based on (5). The other category of power allocation problem can be formulated as optimizing the power allocation across the subcarriers such that the required transmit power is minimized while a given target rate is satisfied. Assume the target rate is R_t. The optimal power allocation has the same expression of (4) while the water-level $\widehat{w}$ is determined by

$$\prod_{i=1}^{L} \left(1 + |h_i|^2 \left(\widehat{w} - \frac{1}{|h_i|^2} \right)^+ \right) = 2^{R_t} \tag{6}$$

The algorithm to obtain the optimal solution is also the water-filling, where $\widehat{w}$ is calculated based on (6). An example of the result of the conventional water-filling is shown in Fig. 4 where $L = 15$.

3.2 Power allocation in OFDM-based cognitive radio systems

The power allocation problem in OFDM-based cognitive radio should consider the subchannel transmit power constraints in addition to the sum transmit power constraint or the target rate. Assume $F_j \triangleq \sum_{i=m_j}^{m_{j+1}-1} P_i$ is the power allocated to the jth subchannel[1]. The power allocation problem with the sum transmit power constraint P_t can be formulated as the following optimization problem:

$$
\begin{aligned}
P^* = \quad & \arg\ \max \sum_{i=1}^{N} \log(1 + |h_i|^2 P_i) \\
\text{s.t.} \quad & P_i \geq 0 \qquad\qquad i = 1, 2, \ldots, N \\
& \sum_{i=1}^{N} P_i \leq P_t \\
& F_j \leq G_j \qquad\qquad j = 1, 2, \ldots, M.
\end{aligned} \tag{7}
$$

Similarly, the power allocation problem with the target rate R_t can be formulated as

$$
\begin{aligned}
P^* = \quad & \arg\ \max \sum_{i=1}^{N} P_i \\
\text{s.t.} \quad & P_i \geq 0 \qquad\qquad i = 1, 2, \ldots, N \\
& \sum_{i=1}^{N} \log(1 + |h_i|^2 P_i) = R_t \\
& F_j \leq G_j \qquad\qquad j = 1, 2, \ldots, M.
\end{aligned} \tag{8}
$$

Note that the SU also suffers interference caused by the PU. The interference can be regarded as white noise by the SU, which may lead to different noise levels in different subchannels. However, although the noise power in each subchannel is set to be equal to each other, this fact does not affect our problem formulation since we can rescale the channel gain. Obviously, the above two optimization problems are convex optimization problems with linear constraints, which can be numerically solved by the standard optimization algorithms. However, we intend to develop analytical as well as more efficient algorithms rather than numerical methods in the following two subsections.

3.3 Iterative Partitioned Water-filling: Sum Transmit Power Constraint

We first consider (7), i.e., the power allocation problem with the sum transmit power constraint. The algorithm is developed by three steps. The fist step is to analyze the structure of the optimal solution of (7) based on the convex optimization theory. Then the algorithm is proposed heuristically based on the analysis in the first step. The algorithm is proved to converge to the optimal point at last. We begin with the first step in this subsection.

3.3.1 Analysis of the structure of optimal solution

In (7), if $P_t \geq \sum_{j=1}^{M} G_j$, the sum transmit power constraint is actually meaningless. (7) is degraded to the problem of power allocation with only subchannel transmit power

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[1] Here, without considering subcarrier sidelobes, the allocated power is exactly the transmit power.

constraints and the solution is simply water-filling on the subcarriers in each subchannel individually with the corresponding subchannel transmit power constraint. Therefore, we only emphasize on the situation that $P_t < \sum_{j=1}^{M} G_j$. A theorem is first derived to describe the structure of the optimal power allocation vector.

Theorem 1 *Under the assumption that $P_t < \sum_{j=1}^{M} G_j$, a power allocation vector $\mathbf{P}$ is the solution for (7) if and only if it satisfies:*

$$P_i = \left(w_j - \frac{1}{|h_i|^2} \right)^+ \tag{9}$$

where i = 1, 2, …,N and j is the index of the subchannel which the ith subcarrier belongs to. Assume $A \triangleq \left\{ j \mid F_j < G_j \right\}, B \triangleq \left\{ j \mid F_j < G_j \right\}$. Then w_j is determined by 1. for $j \in A$:

$$w_j = \widehat{w}, \tag{10a}$$

$$\sum_{j \in A} \sum_{i=m_j}^{m_{j+1}-1} \left(\widehat{w} - \frac{1}{|h_i|^2} \right)^+ = P_t - \sum_{j \in B} G_j \tag{10b}$$

2. for $j \in B$:

$$\sum_{i=m_j}^{m_{j+1}-1} \left(w_j - \frac{1}{|h_i|^2} \right)^+ = G_j \tag{11a}$$

$$w_j \leq \widehat{w}. \tag{11b}$$

Proof: The problem (7) can be reformulated into a standard convex optimization form:

$$\begin{aligned}
\min \quad & \sum_{i=1}^{N} \log \left(1 + |h_i|^2 P_i \right) \\
\text{s.t.} \quad & -P_i \leq 0 && i = 1, 2, ..., N \\
& \sum_{i=1}^{N} P_i - P_t \leq 0 \\
& F_j - G_j \leq 0 && j = 1, 2, ..., M.
\end{aligned} \tag{12}$$

The constraint conditions obviously satisfy the Slater's conditions, so the Karush-Kuhn-Tucker(KKT) conditions are sufficient and necessary for the optimal vector $\mathbf{P}$ (Boyd & Vandenberghe, 2004). The first two KKT conditions are the constraint conditions of (12) and the others are given by

$$\lambda_i \geq 0, \tag{13a}$$

$$\alpha_j \geq 0, \tag{13b}$$

$$v \geq 0, \tag{13c}$$

where $i = 1,2,...,N$, $j = 1,2,...,M$.

$$\lambda_i P_i = 0, \tag{14a}$$

$$\alpha_j \left(F_j - G_j \right) = 0, \tag{14b}$$

$$v \left(\sum_{i=1}^{N} P_i - P_t \right) = 0, \tag{14c}$$

where $i = 1, 2, ..., N$, $j = 1, 2, ..., M$.

$$-\frac{1}{P_i + 1 / |h_i|^2} - \lambda_i + v + \alpha_j = 0 \tag{15}$$

where $i = 1, 2, ..., N$ and j is the index of the subchannel which the ith subcarrier belongs to.

I. *Proof of **only if** part*

(15) can be written as

$$\lambda_i = v - \frac{1}{P_i + 1 / |h_i|^2} + \alpha_j \tag{16}$$

Substituting (16) into (13a) and (14a) yields

$$v + \alpha_j \geq \frac{1}{P_i + 1 / |h_i|^2}, \tag{17}$$

$$\left(v - \frac{1}{P_i + 1 / |h_i|^2} + \alpha_i \right) P_i = 0. \tag{18}$$

If $v + \alpha_j < |h_i|^2$, we have $P_i > 0$ by (17). Then (18) leads to

$$P_i = \frac{1}{v + \alpha_j} - \frac{1}{|h_i|^2}. \tag{19}$$

If $v + \alpha_j \geq |h_i|^2$, from (16), we obtain $v - \frac{1}{P_i + 1/|h_i|^2} + \alpha_i > 0$. Then based on (17), it follows that $P_t = 0$. From (16), we also have

$$v + \alpha_j > 0. \quad \text{where } j = 1, 2, ..., M \tag{20}$$

Let $w_j \triangleq 1/(v + \alpha_j)$. Then

$$P_i = \left(w_j - \frac{1}{|h_i|^2} \right)^{+}, \tag{21}$$

where $i = 1, 2, ..., N$ and j is the index of the subchannel which the ith subcarrier belongs to.

The subchannels are divided into two sets A and B. Since we assume $P_t < \sum_{j=1}^{M} G_j$, there must exists at least such one subchannel that $Fj < Gj$, i.e., $A \neq \emptyset$.

1) For all $j \in A$, since $Fj < Gj$, we have $\alpha_j = 0$ based on (14b). Let $\widehat{w} = 1/v$, then Wj satisfies the condition (10a) $w_j = \widehat{w}$. From (20), we have $v > 0$. Consequently, (14c) leads to $\sum_{i=1}^{N} P_i = P_t$. Therefore, we come to the equation (10b)

$$\sum_{j \in A} \sum_{i=m_j}^{m_{j+1}-1} \left(0, \hat{w} - \frac{1}{|h_i|^2} \right)^+$$

$$= \sum_{j \in A} P_j \tag{22}$$

$$= \sum_{i=1}^{N} P_i - \sum_{j \in B} F_j$$

$$= P_t - \sum_{j \in B} G_j.$$

2) For all $j \in B$, the condition (11a) is obviously satisfied. Since $\alpha_j \geq 0$, we have

$$w_j = \frac{1}{v + \alpha_j} \leq \frac{1}{v} = \hat{w} \tag{23}$$

Therefore, the inequality (11b) also holds.

So far, we have proved the *only if* part in Theorem 1.

*II. Proof of **if** part*

We need to prove that all the KKT conditions can be derived by the power allocation vector defined in Theorem 1. It is easy to see that the first two KKT conditions, i.e., the constraint conditions in (12), hold inherently.

1) Based on (10b) and A≠∅, we have $\hat{w} > 0$. Define $v \triangleq 1/\hat{w}$, then $v > 0$. From (10b), we can also obtain

$$\sum_{i=1}^{N} P_i$$

$$= \sum_{j \in A} F_j + \sum_{j \in B} F_j$$

$$= \sum_{j \in A} \sum_{i=m_j}^{m_{j+1}-1} \left(\hat{w} - \frac{1}{|h_i|^2} \right)^+ + \sum_{j \in B} G_j \tag{24}$$

$$= P_t$$

Therefore, (13c) and (14c) both hold.

2) Define $\alpha_j \triangleq 1/w_j - 1/\hat{w}$, from (10a) and (11b), we can conclude that

$$\begin{cases} \alpha_j = 0, & j \in A \\ \alpha_j \geq 0. & j \in B \end{cases} \tag{25}$$

Therefore, (13b) holds. Since for $j \in B$, we have $Fj = Gj$, (14b) also holds.

3) Define $\lambda_i \triangleq v - \dfrac{1}{P_i + 1/|h_i|^2} + \alpha_j$, where j is the index of the subchannel which the ith

subcarrier belongs to.Then (15) holds inherently. If $w_j > 1/|h_i|^2$, based on (9), we have

$$P_i = w_j - \frac{1}{|h_i|^2}. \tag{26}$$

Since we can derive $w_j = 1/(v + a_j)$ from the definitions of α_j and v, (26) can be written as

$$v - \frac{1}{P_i + 1/|h_i|^2} + \alpha_j = 0. \tag{27}$$

Then, λ=0. Therefore, given $w_j > 1/|h_i|^2$, (13a) and (14a) hold. On the other hand, if $w_j \le 1/|h_i|^2$, it follows that $P_i = 0$ from (9). Then, we have

$$\lambda_i = v - |h_i|^2 + \alpha_j = \frac{1}{w_j} - |h_i|^2 \ge 0. \tag{28}$$

Hence, given $w_j \le 1/|h_i|^2$ (13a) and (14a) also hold.

In conclusion, we have derived all of the KKT conditions and thus the *if* parts also holds. This completes the proof of Theorem 1. □

Comparing (9) with (4), we can find that the power allocation within each subchannel is the same to the conventional water-filling result. However, the water-levels of different sub-channels may be different in (9). From Theorem 1, we know that the subchannels with the optimal power allocation can be divided into two sets: the set A, i.e., the subchannels whose allocated power is strictly smaller than the corresponding subchannel transmit power constraint and the set B, i.e., the subchannels whose allocated power is equal to the cor-responding subchannel transmit power constraint. For the subchannels in *A*, the allocated power is the result of water-filling on all the subcarriers that belong to these subchannels with the power $P_t - \sum_{j \in B} G_j$. Therefore, all the subchannels in *A* have a *common water-level* W. For each subchannel in B, e.g., subchannel j that $j \in B$, the allocated power is the result of water-filling on the subcarriers that belong to subchannel *j* with the power *Gj*. Therefore, each subchannel in *B* has a *unique water-level* w_j and satisfies $w_j \le \hat{w}$, i.e., the water-level of the subchannel which has a *unique water-level* is less or equal to the *common water-level*.
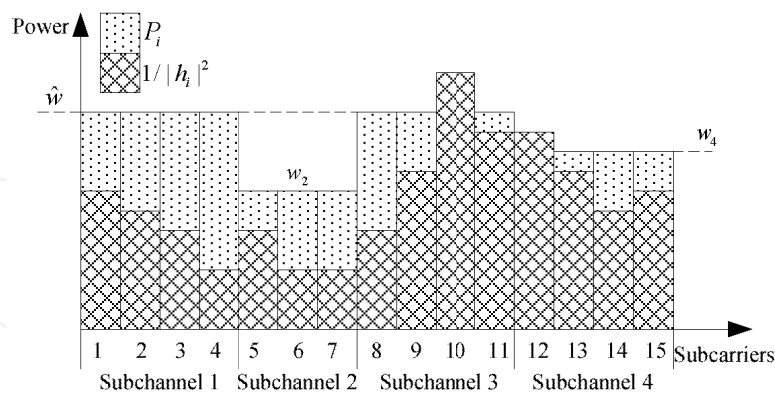
Fig. 5. Optimal power allocation with subchannel transmit power constraints.

A power allocation result satisfying Theorem 1 is shown in Fig. 5. Within each subchannel, it is the same to the conventional water-filling result. However, the water-level of each subchannel is not equal to each other compared to Fig. 4. Subchannel 1 and 3 have the same water-level W, as each allocated power is strictly lower than the corresponding subchannel transmit power constraint. Subchannel 2 and 4 have their unique water-levels w_2 and w_4 respectively, as each allocated power is equal to the corresponding subchannel transmit power constraint. Also, as in Fig. 5, we see that the *unique water-levels w_2, w_4 are lower than the common water-level* w.

3.3.2 Iterative partitioned water-filling

From the above analysis, we can infer that if the partition of the subchannels, i.e., the elements of *A* and B, is known, the optimal power allocation can be obtained by first water-filling on the subcarriers in each subchannel that belongs to the set *B* individually with the corresponding subchannel transmit power constraint, and then water-filling on the rest of the subcarriers with the power $P_t - \sum_{j \in B} G_j$. Therefore, the partition of the subchannels plays an important role in the development of the algorithm. An exhaustive search algorithm can be developed intuitively. The basic idea is to consider all the possible partitions and then verify each partition if the conditions in Theorem 1 are satisfied, which is described as follows:

1. Divide the subchannels into two sets, say *A* and *B* and there are 2^M kinds of partitions in total.

2. For each partition, perform the conventional water-filling on the subcarriers in each subchannel that belongs to the set *B* individually with the corresponding subchannel transmit power constraint. Then the water-levels *wj* where *j ∈ B* can be obtained.

3. Remove such partitions that $P_t - \sum_{j \in B} G_j < 0$. Perform the conventional water-filling on the subcarriers in all the subchannels that belong to the set *A* with the power $P_t - \sum_{j \in B} G_j$. Then the *common water-level* w can be obtained.

4. Verify each partition whether $F_j < G_j$ where $j \in A$ and $w_j \le \hat{w}$ where $j \in B$ are satisfied. According to Theorem 1, there is only one available partition and the corresponding power allocation vector is the solution.

I. *Initialization:*

$A = \{ j \mid j = 1, 2, ..., M \}, B = \varnothing, C = A, \hat{R} = R_t.$

II. *Iterations:*

1. Perform the conventional water-filling on the subcarriers in all the subchannels that belong to the set A with the target rate $\hat{R}$.

2. $F_j = \sum_{i=m_j}^{m_{j+1}-1} P_i$ where $j \in A, C = \{ j \mid F_j \ge G_j, j \in A \}$;

3. $A = A \setminus C, B = B \cup C$;

4. Perform the traditional water-filling on the subcarriers in each subchannel j that $j \in C$ individually with the corresponding subchannel transmit power constraint G_j;

5. $R_j = \sum_{i=m_j}^{m_{j+1}-1} \log \left(1 + |h_i|^2 P_i \right)$ where $j \in C$, $\hat{R} = \hat{R} - \sum_{j \in C} R_j$.

III. *End.*

Table 1. The IPW algorithm under Sum Transmit Power Constraint.

However, the complexity of the exhaustive search algorithm is far too high. In the extreme situation, we need to consider 2^M kinds of partitions throughout the algorithm. Therefore, we develop a more efficient algorithm named as *iterative partitioned water-filling* (IPW). The basic idea is to determine the elements of A and B iteratively rather than by exhaustive search. The IPW algorithm is described in Table 1.

In the beginning of each iteration, the conventional water-filling with only sum transmit power constraint P is done on the subcarriers in all the subchannels that belong to the set A. Then those subchannels whose power exceeds its subchannel transmit power constraint are taken out from the set A and then be performed by water-filling with the corresponding subchannel transmit power constraints individually. The iteration stops when each subchannel satisfies its subchannel transmit power constraint. The IPW algorithm is more efficient than the exhaustive search algorithm. In the worst case, the algorithm converges after M iterations.

3.3.3 Proof of optimality

In order to prove the optimality of the IPW, we need to explain that the IPW algorithm converges to the point that satisfies the conditions in Theorem 1. Actually, we only need to prove the inequality (11b) as other conditions are inherently satisfied.

We outline two simple lemmas about the conventional water-filling before the proof. Assume **P** is the power allocation vector of water-filling on N subcarriers with power $\mathbf{1}^T\mathbf{P}$ and the water-level is w, where **1** is the column vector of all ones.

Lemma 1 *If we take out n subcarriers from these N subcarriers and the corresponding power allocation vector of these n subcarriers is $\mathbf{P}_n$, $\mathbf{P}_n$ is also the power allocation vector of water-filling on the n subcarriers with the power $\mathbf{1}^T \mathbf{P}_n$.*

Lemma 2 *If the water-filling is done on these N subcarriers with power P' and the corresponding water-level is w' . We have $w' \leq w$ if $P' \leq 1^T P$ and vice versa. The two lemmas can be easily obtained from (4) and (5). Based on Lemma 1 and Lemma 2, the following theorem can be proved.*

Theorem 2 *The IPW algorithm converges to the point satisfying wj < w, where j e B, i.e., each unique water level is less or equal to the common water-level.*

Proof: Assume in the fcth iteration, the power $\hat{P}$ used in Step II-1 , the water-level of the subcarriers that belong to the subchannels in the set A after Step II-1 and the temporary set C are denoted as $\hat{P}_k$, w_k and C_k respectively. If we find such j that $Fj \geq Gj$, then the subchannel j needs to be taken out from the set A and put into the set C_k. At Step II-4, the conventional water-filling is done on the subcarriers in each subchannel j that $j \in C_k$ with the corresponding subchannel transmit power constraint Gj individually. The resulting water-levels are just the *unique water-level* w_j when the algorithm converges. From Lemma 1, if the conventional water-filling is done on the subcarriers in each subchannel j that $j \in C_k$ with the power Fj individually, we also get the water-level w_k. From Lemma 2, since $Fj \geq Gj$, we have

$$w_j \leq \widehat{w}_k, \forall j \in C_k \qquad (29)$$

after the kth iteration.

Based on Lemma 1, for the rest of the subchannels in the set A satisfying $F_j < G_j$, if the conventional water-filling is done on the corresponding subcarriers with the power $P_k - \sum_{j \in C_k} F_j$, we also get the water-level w_h. In the next iteration, we need to perform the conventional water-filling on these subcarriers with the power $\hat{P}_{k+1} = \hat{P}_k - \sum_{j \in C_k} G_j$ and the resulting water-level is assumed to be w_h+_1. Based on Lemma 2, since $\hat{P}_k - \sum_{j \in D} F_j < \hat{P}_{k+1}$ we have

$$\widehat{w}_k \leq \widehat{w}_{k+1}. \qquad (30)$$

When the algorithm converges after $\hat{k}$ iterations, we have $\widehat{w} = \widehat{w}_{\hat{k}}$. From (30), it follows that $\widehat{w}_{\hat{k}} \leq \widehat{w}$ where $k = 1, 2, \ldots, \hat{k}$. Also, based on (29), since $B = \bigcup_{k=1,2,\ldots,\hat{k}} C_k$, we have $\forall j \in B, \exists k$, so that $w_j \leq \widehat{w}_k$. Therefore, we come to the conclusion that $\forall j \in B, w_j \leq \widehat{w}$, i.e., each *unique water-level* is less or equal to the *common water-level*. □

So far, we have proved that the IPW algorithm converges to the point satisfying the conditions in Theorem 1 and thus the optimal power allocation can be obtained by the IPW algorithm.

3.4 Iterative Partitioned Water-filling: Target Rate Constraint

In this subsection, we consider (8), i.e., the power allocation problem with the target rate constraint. If the target rate is given, we should first verify if the target rate can be achieved

under the subchannel transmit power constraints. It is easy to see that the maximum rate that can be achieved under the subchannel transmit power constraints is the result of water-filling on the subcarriers in each subchannel with the corresponding subchannel transmit power constraint individually. Therefore, the maximum achievable rate can be expressed as

$$R_{\max} = \sum_{i=1}^{N} \log\left(1 + |h_i|^2 P_i\right), \tag{31}$$

where Pi is determined by

$$P_i = \left(w_j - \frac{1}{|h_i|^2}\right)^+, \tag{32}$$

where $i = 1, 2, \ldots, N$ and j is the index of the subchannel which the ith subcarrier belongs to. wj can be determined by

$$\sum_{i=m_j}^{m_{j+1}-1} \left(w_j - \frac{1}{|h_i|^2}\right)^+ = G_j. \; j = 1, 2, \ldots, M. \tag{33}$$

Then, if $R_t > R_{max}$, Rt can not be achieved by any power allocation vector under the subchannel transmit power constraints. Also, the optimal power allocation can be determined by (32) and (33) when $R_t = R_{max}$. In the case $R_t < R_{max}$, the algorithm for solving (8) can be developed following the similar derivation as that in Section 3.3. A similar theorem as Theorem 1 can be described as follows.

Theorem 3 *Under the assumption that $R_t < R_{max}$, a power allocation vector $\boldsymbol{P}$ is the solution for (8) if and only if it satisfies:*

$$P_i = \left(w_j - \frac{1}{|h_i|^2}\right)^+, \tag{34}$$

where $i = 1, 2, \ldots, N$ and j is the index of the subchannel which the ith subcarrier belongs to. Assume $A \triangleq \left\{j \mid F_j < G_j\right\}, B \triangleq \left\{j \mid F_j = G_j\right\}..$ Then w_j is determined by 1. for $j \in A$:

$$w_j = \widehat{w}, \tag{35a}$$

$$\sum_{j \in A} \sum_{i=m_j}^{m_{j+1}-1} \log\left(1 + |h_i|^2 \left(\widehat{w} - \frac{1}{|h_i|^2}\right)^+\right) \tag{35b}$$

$$= R_t - \sum_{j \in B} R_j,$$

where Rj is determined by

$$R_j = \sum_{i=m_j}^{m_{j+1}-1} \log\left(1 + |h_i|^2 \left(w_j - \frac{1}{|h_i|^2}\right)^+\right).$$

2. for j ∈ B :

$$\sum_{i=m_j}^{m_{j+1}-1} \left(w_j - \frac{1}{|h_i|^2}\right)^+ = G_j, \tag{36a}$$

$$w_j \leq \widehat{w}. \tag{36b}$$

The proof of Theorem 3 is also similar to that of Theorem 1 and thus is omitted in this paper. Based on Theorem 3, the IPW algorithm for solving (8), which can be developed in a similar way as that in Section 3.3, is described in Table 2.

In each iteration, the conventional water-filling with only target rate constraint $\widehat{R}$ is first done on the subcarriers in all the subchannels that belong to the set *A*. Then those sub-channels whose power exceeds its subchannel transmit power constraint are taken out from the set *A* and put into the set *C*. The conventional water-filling with the corresponding sub-channel transmit power constraints is performed on the subcarriers in each subchannel that belongs to *C* individually. Then the target rate $\widehat{R}$ is updated by subtracting the rates already achieved by the subchannels in the set *C*. The iteration stops when each subchannel satisfies its subchannel power constraint. In the worst case, the algorithm converges after *M* iterations. The proof of the optimality is omitted here as it is also similar to that in Section 3.3.3.

I. *Initialization:*

$\quad A = \{j \mid j = 1,2,...,M\}, B = \varnothing, C = A, \widehat{R} = R_t.$

II. *Iterations:*

    1. Perform the conventional water-filling on the subcarriers in all the

       subchannels that belong to the set *A* with the target rate $\widehat{R}$.

    2. $F_j = \sum_{i=m_j}^{m_{j+1}-1} P_i$ where $j \in A, C = \{j \mid F_j \geq G_j, j \in A\}$;

    3. $A = A \setminus C, B = B \cup C$;

    4. Perform the traditional water-filling on the subcarriers in each subchannel *j* that

       $j \in C$ individually with the corresponding subchannel transmit power constraint

       G_j;

    5. $R_j = \sum_{i=m_j}^{m_{j+1}-1} \log\left(1 + |h_i|^2 P_i\right)$ where $j \in C$, $\widehat{R} = \widehat{R} - \sum_{j \in C} R_j$.

III. *End.*

Table 2. The IPW algorithm under Target Rate Constraint.

4. Power Allocation for OFDM-based Cognitive Radio Systems Considering Subcarrier Sidelobes

The *iterative partitioned water-filling* (IPW) algorithm proposed in the above section can obtain the optimal power allocation for OFDM-based cognitive radio systems in the case

where only the interference to the PU caused by the subcarriers inside the corresponding subchannel was considered. However, it is clear that the PU also suffers interference introduced by the subcarrier sidelobes of the neighboring subchannels. Therefore, the assumption in the above section actually implied that there is sufficient guard band between any two subchannels so that the effects of subcarrier sidelobes could be ignored. In this section, we extend the results in the above section and address the power allocation problem in general OFDM-based cognitive radio systems. The effects of subcarrier sidelobes are specially considered in the optimization problem. We propose the power allocation algorithm that maximizes the capacity while satisfying both the sum and subchannel transmit power constraints.

The effects of subcarrier sidelobes in OFDM-based cognitive radio systems were first addressed in (Weiss et al., 2004). The subcarrier sidelobes suppression techniques were further studied in (Cosovic et al., 2005; Pagadarai et al., 2008; Mahmoud & Arslan, 2008). In (Bansal et al., 2007), the authors proposed a power loading algorithm for OFDM-based SU systems whose bands are not overlapped with any PU's. The aim was to maximize the capacity while keeping the interference to the PUs whose licensed bands are in the proximity of the band of SU below a certain threshold.

In the following, we will firstly reformulate the system model and the optimization problem with subcarrier sidelobes in consideration. Then an algorithm will be proposed for a special case with only two non-zero weighted linear inequality constraints. Finally, this algorithm mentioned above will be extended to general cases with multiple non-zero weighted linear inequality constraints, where the extended algorithm is called *recursive power allocation* (RPA) algorithm.

4.1 Power allocation problem reformulation

The basic system model used in this section is the same as that in Fig. 1 and Fig. 2. The SU's transmit power on the PU's licensed channel P_{tx} should still be subject to the power constraint as (1). However, considering the effects of subcarrier sidelobes, it is not proper to require the transmit power in the unavailable subchannel to be zero as (3), since the sidelobes of the subcarriers in other subchannels certainly lead to non-zero transmit power in the unavailable subchannel. So we set a transmit power threshold for the jth subchannel when PUj is detected, say γ_j. The transmit power constraint on the *j*th subchannel after the sensing Gj is modified from (3) to

$$G_j \begin{cases} \gamma_j & PU_j \text{ is detected} \\ \eta_j\left(d_j - R_j\right)^{\beta_j} & PU_j \text{ is not detected} \end{cases} \tag{37}$$

As mentioned in Section 3, in OFDM-based cognitive radio systems, the subchannel transmit power constraints impose further restrictions on the power allocation in addition to the sum transmit power constraint. In fact, the transmit power in one subchannel is comprised of not only the power allocated to the subcarriers inside the subchannel but also the sidelobes power of the subcarriers in other subchannels. We define Jij as the transmit power in the jth subchannel caused by the ith subcarrier with unit power. According to (Weiss et al., 2004), we have

$$J_{i,j} = \frac{1}{\Delta f} \int_{(m_j - \frac{1}{2})\Delta f}^{(m_{j+1} - \frac{1}{2})\Delta f} \left(\frac{\sin\left(\frac{\pi}{\Delta f}(f - i\Delta f)\right)}{\frac{\pi}{\Delta f}(f - i\Delta f)} \right)^2 df$$

$$= \int_{m_j - \frac{1}{2}}^{m_{j+1} - \frac{1}{2}} \left(\frac{\sin(\pi(x - i))}{\pi(x - i)} \right)^2 dx. \tag{38}$$

Assume P_i be the power allocated to the ith subcarrier. Then the transmit power on the jth subchannel can be expressed as $\sum_{i=1}^{N} J_{i,j} P_i$. Therefore, the subchannel transmit power constraints in (37) introduce M inequality constraints on the power allocation:

$$\sum_{i=1}^{N} J_{i,j} P_i \leq G_j, \quad j = 1, 2, ..., M. \tag{39}$$

Then, the original optimization problem as (7) for optimal power allocation in OFDM-based cognitive radio systems can be modified as the following one:

$$
\begin{aligned}
P^* = \ &\arg\max \sum_{i=1}^{N} \log\left(1 + |h_i|^2 P_i\right), \\
s.t. \quad &P_i \geq 0 \qquad i = 1, 2, ..., N \\
&\sum_{i=1}^{N} P_i \leq P_t \\
&\sum_{i=1}^{N} J_{i,j} P_i \leq G_j \qquad j = 1, 2, ..., M
\end{aligned} \tag{40}
$$

where **P*** is the optimal power allocation vector, h_i is the ith subcarrier's channel gain, and Pt is the maximum transmit power of the SU transmitter.

Section 3, the subchannel power constraints are approximated by ignoring the subcarrier sidelobes, i.e.,

$$
\begin{cases}
J_{i,j} = 1 & i \in \left[m_j, m_{j+1} - 1\right], \\
J_{i,j} = 0 & j \notin \left[m_j, m_{j+1} - 1\right].
\end{cases} \tag{41}
$$

When (41) is satisfied, it can be seen that the M subchannel power constraints in (40) are actually decoupled with each other in terms of the optimization variables. Based on this characteristic, we manage to find an efficient algorithm named iterative partitioned water-filling. In the case where the subcarrier sidelobes are considered, however, the problem turns to be more complicated.

It is clear that (40) is a convex optimization problem with linear inequality constraints. (40) can be expressed in a compact form:

$$P^* = \arg\max \sum_{i=1}^{N} \log\left(1 + |h_i|^2 P_i\right),$$
$$s.t. \quad P_i \geq 0 \qquad i = 1, 2, ..., N \tag{42}$$
$$\sum_{i=1}^{N} J_{i,j} P_i \leq G_j \quad j = 1, 2, ..., M$$

where $J_{i,0} = 1$ for any $i = 1, 2, \ldots, N$ and $G_o = P_t$. While the N positivity constraints are easy to handle, the remaining $M + 1$ constraints, which are named *non-zero weighted linear inequality constraints* for ease of description, introduce the main difficulties for the algorithm development.

In order to find the power allocation algorithm for (42), we start with the problem with only two *non-zero weighted linear inequality constraints*. Then the *recursive power allocation* (RPA) algorithm is proposed for the general cases, which is a recursive extension from the above simple problem.

4.2 Power allocation for the case of two non-zero weighted linear inequality constraints

The problem with two *non-zero weighted linear inequality constraints* can be expressed as

$$P^* = \arg\max \sum_{i=1}^{N} \log\left(1 + |h_i|^2 P_i\right).$$
$$s.t. \quad P_i \geq 0 \qquad i = 1, 2, ..., N,$$
$$\sum_{i=1}^{N} J_{i,k} P_i \leq G_k \tag{43}$$
$$\sum_{i=1}^{N} J_{i,l} P_i \leq G_l.$$

The two *non-zero weighted linear inequality constraints* are numbered with C_k and C_l respectively for ease of derivation.

4.2.1 Degraded cases

We begin with solving the problem with C_k excluded. Solving this degraded problem is similar as the derivation of the conventional water-filling algorithm. The solution, which satisfies the constraint C_l with equality, can be expressed as

$$P_i = \left(\frac{1}{J_{i,l}\lambda_l} - \frac{1}{|h_i|^2}\right)^+, \quad i = 1, 2, ..., N \tag{44}$$

where λ_l is determined by

$$\sum_{i=1}^{N} J_{i,l}\left(\frac{1}{J_{i,l}\lambda_l} - \frac{1}{|h_i|^2}\right)^+ = G_l.$$

Substituting (44) into the excluded constraint C_k, if C_k is also satisfied, i.e.,

$$\sum_{i=1}^{N} J_{i,k} \left(\frac{1}{J_{i,l}\lambda_l} - \frac{1}{|h_i|^2} \right)^+ \le G_k, \tag{45}$$

then (44) is also the solution of the original problem (43).

If (45) is not satisfied, we turn to consider the problem with c_i excluded. Similarly, the solution for this degraded problem can be expressed as

$$P_i = \left(\frac{1}{J_{i,k}\lambda_k} - \frac{1}{|h_i|^2} \right)^+, \quad i = 1,2,...,N, \tag{46}$$

where λ_k is determined by

$$\sum_{i=1}^{N} J_{i,k} \left(\frac{1}{J_{i,k}\lambda_k} - \frac{1}{|h_i|^2} \right)^+ = G_k.$$

If the constraint C_l is also satisfied with (46), i.e.,

$$\sum_{i=1}^{N} J_{i,l} \left(\frac{1}{J_{i,k}\lambda_k} - \frac{1}{|h_i|^2} \right)^+ \le G_l, \tag{47}$$

then (46) is also the solution of the original problem (43).

Therefore, the optimal power allocation for (43) is easy to find in the above two degraded cases, i.e., either (45) or (47) is satisfied.

4.2.2 Algorithm for general cases

In order to obtain the solution in the case that neither (45) nor (47) is satisfied, we first propose the following lemma.

Lemma 3 *Define a convex optimization problem (OP$_0$) with differentiate objective function and inequality constraints satisfying Slater's conditions:*

$$\min_x \ h_0(x) \tag{48}$$
$$s.t. \quad h_i(x) \le 0, i = 1,...,n.$$

and let x_0 be any optimal point of OP$_0$. Assume x_k is any optimal point of a sub-problem (OP$_{-k}$) of OP$_0$ with $n - 1$ inequality constraints $h_i(x) \le 0, i = 1,..., k - 1, k + 1,..., n$. Then if $h_k(x_k) \ge 0$, we have $h_k(x_0) = 0$.

Proof: If $h_k(x_0) \ne 0$, then we have $h_k(x_0) < 0$. Since x_0 is an optimal point of the convex optimization problem OP$_0$, x_0 satisfies all the KKT conditions of OP$_0$, which are given by

$$h_i(x_0) \le 0, \ \alpha_i \ge 0, \ \alpha_i h_i(x_0) = 0, \ i = 1,...,n \tag{49a}$$

$$\nabla h_0(x_0) + \sum_{i=1}^{n} \alpha_i \nabla h_i(x_0) = 0, \tag{49b}$$

where α_i is the Lagrange multiplier. Since $h_k(x_0)<0$ according to $\alpha_k h_k(x_0) = 0$ in (49a), we have $\alpha_k = 0$. Then (49b) can be expressed as

$$\nabla h_0(x_0) + \sum_{i=1, i\neq k}^{n} \alpha_i \nabla h_i(x_0) = 0. \tag{50}$$

Based on (49a) and (50), it is clear that x_0 also satisfies the KKT conditions of **OP-k**. Therefore, x_0 is also an optimal point of **OP-k**. Consequently, we know that there is an

Step-1. Calculate the optimal power allocation **P*** with C_k excluded based on (44);
Step-2. Check if **P*** also satisfies C_k. If C_k is satisfied, exit;
Step-3. Calculate the optimal power allocation **P*** with C_l excluded based on (46);
Step-4. Check if **P*** also satisfies C_l. If C_l is satisfied, exit;
Step-5. Calculate the optimal power allocation **P*** based on (53).

Table 3. Power allocation algorithm for the problem including two non-zero weighted linear inequality constraints.

optimal point of **OP-k** satisfying $h_k(x_k) < 0$, which contradicts the premise that $h_k(x_k) > 0$. Finally, we come to the conclusion that $h_k(x_0) = 0$. □

Based on Lemma 3, the optimal solution of (43) must satisfy the two constraints C_k and C_i with equalities in the case that neither (45) nor (47) holds. Therefore, solving (43) in this case is equivalent to solving the following problem:

$$P^* = \arg\max \sum_{i=1}^{N} \log\left(1 + |h_i|^2 P_i\right).$$

$$s.t. \quad P_i \geq 0 \qquad i = 1, 2, ..., N,$$

$$\sum_{i=1}^{N} J_{i,k} P_i = G_k \tag{51}$$

$$\sum_{i=1}^{N} J_{i,l} P_i = G_l.$$

To find the solution, we form the Lagrangian:

$$L_i(P_i, \lambda_k, \lambda_l) = \log\left(1 + |h_i|^2 P_i\right) - \lambda_k\left(\sum_{i=1}^{N} J_{i,k} P_i - G_k\right) - \lambda_l\left(\sum_{i=1}^{N} J_{i,l} P_i - G_l\right), \tag{52}$$

$$i = 1, 2, ..., N.$$

The optimal power allocation can be obtained by solving the first order KKT condition $\frac{\partial L_i(P_i, \lambda_k, \lambda_l)}{\partial P_i} = 0$ with the constraint $P_i > 0$, which is given by

$$P_i = \left(\frac{1}{J_{i,k}\lambda_k + J_{i,l}\lambda_l} - \frac{1}{|h_i|^2}\right)^+, \quad i = 1, 2, ..., N \tag{53}$$

where λ_k and λ_l are determined by the following equations:

$$\begin{cases} \sum_{i=1}^{N} J_{i,k}\left(\frac{1}{J_{i,k}\lambda_k + J_{i,l}\lambda_l} - \frac{1}{|h_i|^2}\right)^+ = G_k, \\ \\ \sum_{i=1}^{N} J_{i,l}\left(\frac{1}{J_{i,k}\lambda_k + J_{i,l}\lambda_l} - \frac{1}{|h_i|^2}\right)^+ = G_l. \end{cases}$$

Combining the derivations above, the power allocation algorithm for the problem with two *non-zero weighted linear inequality constraints* is shown in Table 3.

4.3 Recursive power allocation algorithm for general cases

The above algorithm can be further extended to the problem (42) including $M + 1$ *nonzero weighted linear inequality constraints.* Similarly, we first consider $M + 1$ degraded cases each of which excludes one *non-zero weighted linear inequality constraint.* Each of the $M + 1$ degraded optimization problems can be solved by the algorithm for the problem with M *nonzero weighted linear inequality constraints.* If the solution of one of these $M + 1$ degraded

---

Initialization:

    Number the *M+1 non-zero weighted linear inequality constraints* as $C_0, C_1, ... C_M$, m=M+1,A={ $C_0, C_1, ... C_M$ }.

Begin

    P*=*PAA*(m,*A*).

end

function:PAA (m,*A*)

begin

    Case m=1:

        Return $f(A)$

    Case m > 1:

        Initialization:

            Set n=1. Define set *B*.

        Begin:

            Step-1. *B*=*A*/a$_n$,**P***=*PAA*(m-1,*B*);

            Step-2. if **P*** also satisfies the constraint a$_n$, go to Step-5;

            Step-3. *N*=*n*+1. if n≤*m*,go to Step-1;

            Step-4. **P*** = $f(A)$;

            Step-5. return **P***.

        end

end

Table 4. The RPA algorithm for the problem including $M + 1$ non-zero weighted linear inequality constraints

problems also satisfies the corresponding excluded constraint, the optimal power allocation is obtained. Otherwise, the optimal solution of the original problem must satisfy the $M + 1$ constraints with equalities based on Lemma 3. Similar as that in Section 4.2.2, we form the Lagrangian:

$$
\begin{aligned}
L_i\left(P_i, \lambda_0, \lambda_1, ..., \lambda_M\right) &= \log\left(1 + |h_i|^2 P_i\right) \\
&- \sum_{j=0}^{M} \lambda_j\left(\sum_{i=1}^{N} J_{i,j} P_i - G_j\right), \quad i = 1, 2, ..., N.
\end{aligned}
\tag{54}
$$

The optimal power allocation can be obtained by solving the first order KKT condition $\dfrac{\partial L_i\left(P_i, \lambda_0, \lambda_1, ..., \lambda_M\right)}{\partial P_i} = 0$ with the constraint $P_i > 0$, which is given by

$$P_i = \left(\frac{1}{\sum_{j=0}^{M} J_{i,j}\lambda_j} - \frac{1}{|h_i|^2}\right)^+ , \quad i = 1, 2, ..., N \tag{55}$$

where $\lambda_0, \lambda_1, ..., \lambda_M$ are determined by the following $M + 1$ equations:

$$\sum_{i=1}^{N} J_{i,k}\left(\frac{1}{\sum_{j=0}^{M} J_{i,j}\lambda_j} - \frac{1}{|h_i|^2}\right)^+ = G_k, \quad k = 1, 2, ..., M.$$

Since the algorithm for the problem including two *non-zero weighted linear inequality constraints* has been obtained in Section 4.2.2, the problem for the general case including multiple *non-zero weighted linear inequality constraints* can be solved in a recursive way, i.e., recursive power allocation (RPA) algorithm.
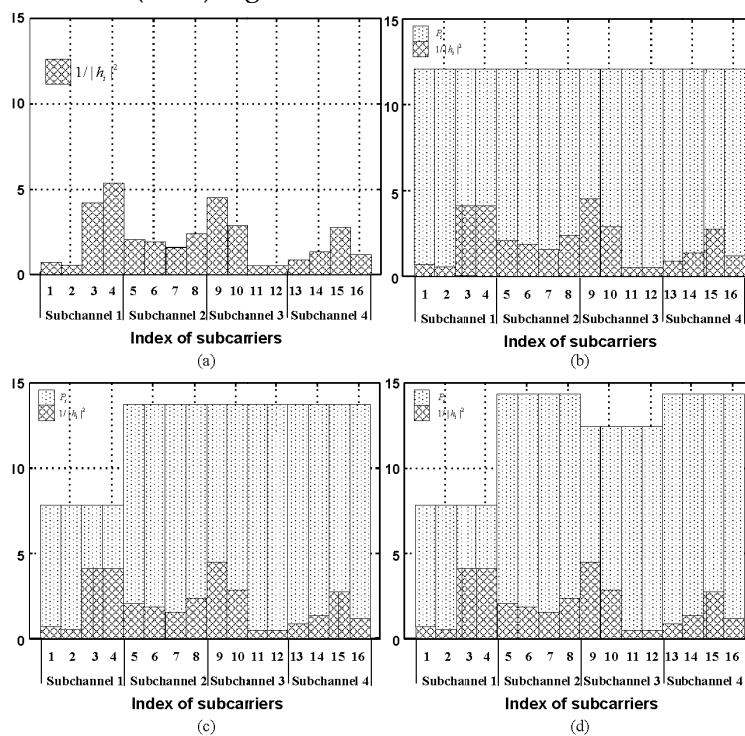
Fig. 6. Process of iterative partitioned water-filling: (a) illustrates the reciprocal of the channel gain of a realization of a frequency selective fading channel; (b), (c) and (d) are the power allocation results after Step II-l (Table 1) of the first, second and third iteration, respectively. The algorithm converges after three iterations.

For ease of derivation, we define the function f (A) where A is a set comprised of all the *non-zero weighted linear inequality constraints* of the optimization problem with the form as (42). f (A) returns the optimal solution that satisfies all the constraints contained in A with equalities. For example, $f(A)$ returns the value determined by (44) when $A = \{C_k\}$ and the value determined by (53) when $A = \{C_k, C_l\}$. We further assume that α_i denotes the ith element of A. The power allocation algorithm for general cases is shown in Table 4.

5. Simulation Results

In this section, two parts of simulation results are presented to verify our proposed algorithms of power allocation for OFDM-based cognitive radio systems. The first part focuses on the

IPW algorithm (as in Table 1)[2] for OFDM-based cognitive radio systems without considering subcarrier sidelobes. The second part focuses on the RPA algorithm (as in Table 4) for OFDM-based cognitive radio systems considering subcarrier sidelobes. ***Part A: IPW algorithm without considering subcarrier sidelobes*** In this part, we set $N = 16$, $M = 4$, i.e., each subchannel has 4 subcarriers. The IPW process is shown in Fig. 6. Fig. 6(a) illustrates the reciprocal of the channel gain given a realization of a frequency selective fading channel, i.e., the bottom of the pool for water-filling. The power allocation result after Step in Table 1 in the first iteration is shown in Fig. 6(b), where the power allocation is just the result of the conventional water-filling with only sum transmit power constraint. At Step II-2 in Table 1, we find that the power allocated to Subchannel 1 exceeds its subchannel transmit power constraint G**j.**. Therefore, Subchannel 1 should be taken out from the set A and the conventional water-filling is performed on the corresponding subcarriers with the power G_1 at the last step of the first iteration. At Step in Table 1 of the second iteration, the conventional water-filling is performed on the subcarriers in the rest of the subchannels in the set A with the power $P_t - G_1$ and the resulting power allocation is shown in Fig. 6(c). Then the power constraint on Subchannel 3 is found to be destroyed at Step II-2 in Table 1. As a result, we execute Step II-3 and Step II-4 in Table 1 again and then enter the third iteration. Fig. 6(d) illustrates the power allocation after Step II-1 in Table 1 in the third iteration, where only Subchannel 2 and 4 are still remained in the set A. At Step II-2 in Table 1, each subchannel transmit power constraint is found to be satisfied and thus we may end the iterations with $C = 0$. Finally, the power allocation in Fig. 6(d) is the optimal one that maximizing the sum rate while satisfying both the subchannel transmit power constraints and the sum transmit power constraint. ***Part B: RPA algorithm considering subcarrier sidelobes***

In this part, the frequency selective fading channel is generated according to the Typical Urban (TU) channel model, which is comprised of six paths whose delay parameters are [0.0 0.2 0.5 1.6 2.3 $5.0]\beta s$ and power profile is [0.189 0.379 0.239 0.095 0.061 0.037]. The SU's bandwidth is 5 MHz, which is equally divided into $M = 4$ subchannels. The number of the subcarriers is supposed to be $N = 64$ and each subchannel has 16 subcarriers. We use the values 640 for P_t, which implies that the signal to noise ratio is about 10 dB as the noise power on each subcarrier is set to be 1. The four subchannel power constraints are specified as 80, 480, 1.6, 480 respectively. The power constraint on Subchannel 3 is about 25 dB below its neighboring subchannels, which implies that the SU may be very close to or even just

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[2]Using the IPW algorithm in Table 2 has the similar results as mentioned in Section 3.4.

inside PU$_3$'s protection area. Thus, the constraint on the transmit power in Subchannel 3 should be very rigorous in order to satisfy PU$_3$'s interference power limit. The optimal power allocation is shown in Fig. 7 for a given realization of the TU channel. For comparison, the power allocation result using the IPW algorithm ignoring the effects of subcarrier sidelodes is also given in Fig. 8. Though it can be seen in Fig. 8 that the allocated power on each subchannel is lower than the corresponding subchannel's transmit power constraint, the actual transmit power on Subchannel 3, including the allocated power on Subchannel 3 and the sidelobe power from neighboring subchannels, is about 6 times of its transmit power constraint, which induces to serious destruction to the PU$_3$'s interference constraint. Therefore, the IPW algorithm cannot be applicable for such Subchannel 3 and its neighboring subchannels in this case. On the contrary, the RPA algorithm as shown in Fig. 7 can maximize the capacity while satisfying the subchannel transmit power constraints. Comparing Fig. 7 with Fig. 8, we can see that the power allocated to the subcarriers near Subchannel 3 is suppressed even if the channel states are fine while more power is allocated to the subcarriers that are far away from Subchannel 3. On the other hand, it can also be found that the power allocation in Subchannel 1 is almost the same as that without considering subcarrier sidelobes, which implies that it is reasonable to ignore the effects of subcarrier sidelobes when sufficient guard band between two subchannels is present. Therefore, the IPW algorithm, with higher efficiency compared to the RPA algorithm, can be applied to the subchannels such as Subchannel 1.
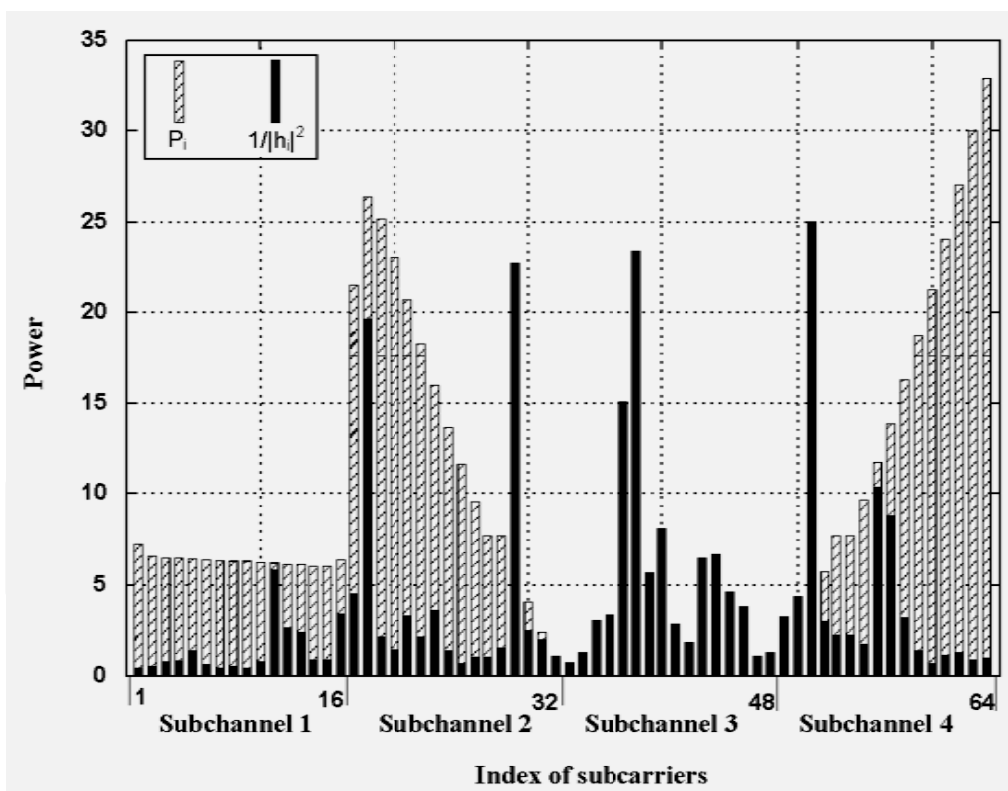

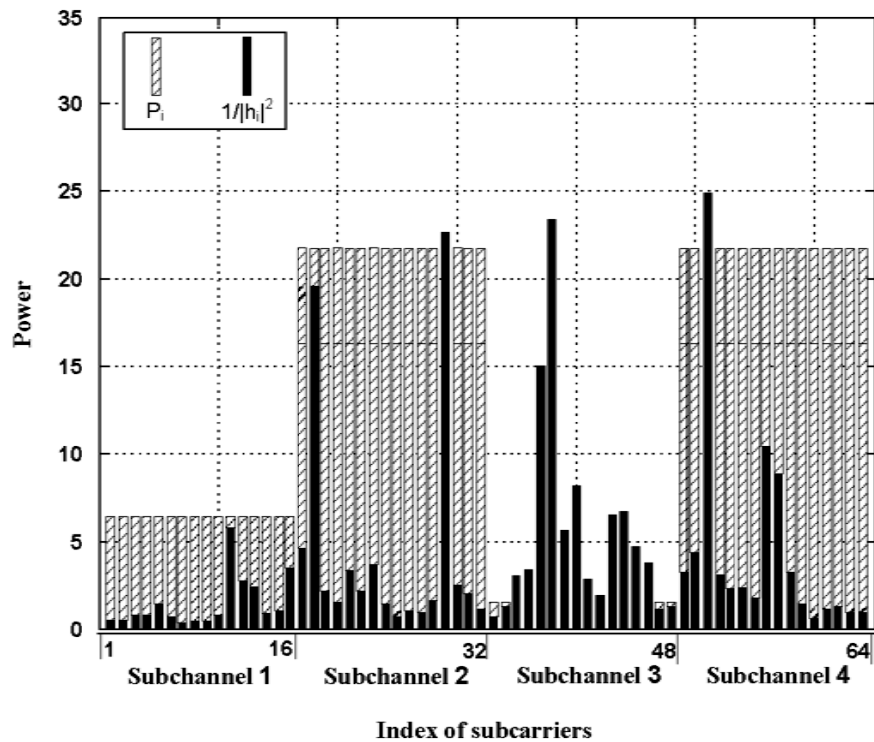Fig. 7. Optimal power allocation by the RPA.

Fig. 8. Optimal power allocation by the IPW without considering subcarrier sidelobes.

6. Conclusions

In this chapter, we study the power allocation problem in OFDM-based cognitive radio systems.

In the cognitive radio scenario, the interference limits of the primary users introduce sub-channel power constraints for the transmission of the SU. Therefore, the power allocation among the subcarriers should satisfy both the sum transmit power constraint, as that in conventional OFDM systems, as well the subchannel power constraints, which leads to the unavailability of the well-known water-filling algorithm applied in conventional OFDM systems.

In order to find the optimal power allocation, we first propose the IPW algorithms, whose basic computation units is the conventional water-filling. The algorithm is proved to converge to the optimal power allocation, which maximizes the sum rate when the sum transmit power constraint is given or minimizes the required power when the target rate is given, after a finite number of iterations.

Furthermore, we consider the effects of subcarrier sidelobes on the power allocation in prac-tical systems. We note that the actual transmit power in one subchannel is comprised of the allocated power as well as the subcarrier sidelobes power from the neighboring subchannels. The RPA algorithm is proposed for this scenario. The algorithm finds the optimal power allocation recursively by decoupling the subchannel power constraints phase-by-phase. Simulation results show that the IPW algorithm is unavailable when the guard bands between the subchannels are not wide enough, while the RPA algorithm can be

applied in such scenario. In conventional OFDM systems, bit loading is a more practical technique for the system design. Therefore, it is interesting to study bit loading algorithms in OFDM-based cognitive radio systems, where the subchannel transmit power constraints should be considered.[3]

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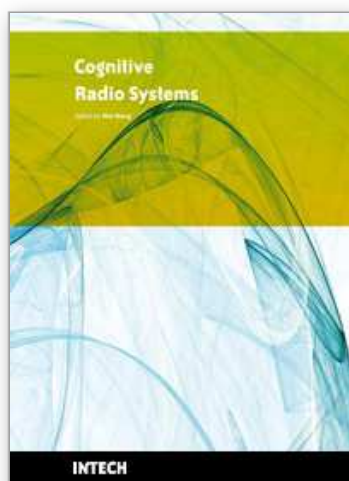

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Cognitive Radio Systems

Edited by Wei Wang

Cognitive radio is a hot research area for future wireless communications in the recent years. In order to increase the spectrum utilization, cognitive radio makes it possible for unlicensed users to access the spectrum unoccupied by licensed users. Cognitive radio let the equipments more intelligent to communicate with each other in a spectrum-aware manner and provide a new approach for the co-existence of multiple wireless systems. The goal of this book is to provide highlights of the current research topics in the field of cognitive radio systems. The book consists of 17 chapters, addressing various problems in cognitive radio systems.

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