Power Allocation Schemes for Amplify-and-Forward MIMO-OFDM Relay Links

Ingmar Hammerström and Armin Wittneben

***Abstract*— We consider a two-hop MIMO-OFDM communication scheme with a source, an amplify-and-forward relay, and a destination. We examine the possibilities of power allocation (PA) over the subchannels in frequency and space domains to maximize the instantaneous rate of this link if channel state information at the transmitter (CSIT) is available. We consider two approaches: (i) separate optimization of the source or the relay PA with individual per node transmit power constraints and (ii) joint optimization of the source and the relay PA with joint transmit power constraint. We provide the optimal PA at the source (or the relay) with a node transmit power constraint that maximizes the instantaneous rate for a given relay (or source) PA. Furthermore, we show that repeating this separate optimization of the source and the relay PA alternately converges and improves the achievable rate of the considered link. Since the joint optimization of the source and the relay PA is analytically not tractable we use a high SNR approximation of the SNR at the destination. This approximation leads to rates which are quite tight to the optimum.**

***Index Terms*— Amplify and forward relaying, MIMO, OFDM, power allocation.**

I. Introduction

FIRST research results on relay channels were published in [1]–[3] during the seventies. Recently, cooperative relaying strategies have become a major topic in the wireless research community again. The encouraging results of, e.g., [4]–[6] initiated a large amount of work in this area. Relaying strategies as decode-and-forward (DF), compress-and-forward (CF), and amplify-and-forward (AF) have been proposed. In DF the relays decode the received signal prior to retransmission while in CF they retransmit a compressed version of the signal. In AF the relays only amplify and may process the signal linearly before they retransmit it again. Thus, AF leads to low-complexity relay transceivers and lower power consumption since there is no need of signal processing for decoding procedures, but suffers from the noise amplification induced by the relay. Moreover, AF relays are transparent to adaptive modulation techniques which may be employed by the source. Most of the literature available today considers frequency-flat fading. In [7]–[9], optimal PAs between single antenna source and relay (AF and DF) are discussed for the case of a joint sum transmit power constraint. In [10], the optimal gain matrix for an AF MIMO relay which optimizes the instantaneous rate for a given uniform PA at the source is presented. A gain allocation with a MIMO first hop and a second hop which considers orthogonal channels to a single antenna destination is presented in [11]. The case of cooperative relaying in frequency-selective fading channels is much less examined so far. In [12], the authors determine power allocations for multiple orthogonal AF relays (which is the same as having one relay using OFDM) maximizing the average SNR of the maximum-ratio combiner at the destination node. In [13], the capacity of OFDM and OFDMA networks consisting of one source/destination pair and multiple AF relays is examined. In the case of OFDM only one amplification gain is used for all subcarriers. Therefore, the rate is not optimized with respect to the frequency-selective channel. In the case of OFDMA only one relay is assigned to one subcarrier, which results in an optimization problem that can be solved by integer programming.

In this work we present PA schemes with respect to the subchannels in frequency and space domains to maximize the instantaneous rate of this link if CSIT is available. We give the optimal PA at the source (or relay) with a individual per node transmit power constraint that maximizes the instantaneous rate for a given PA at the relay (or the source). This result also includes the result presented in [10] for flat fading. To further enhance the achievable rate of the considered relaying scheme the subchannels of the channel matrices of both hops are paired according to their actual magnitude. Furthermore, we show that alternate, separate optimization of the source and the relay PA with node power constraints converges and provides higher achievable rates. Additionally, we present an approximated solution for the joint optimization of the source and the relay PA with a joint transmit power constraint. This PA scheme achieves rates which are quite tight compared to the rates achieved by the optimal PA.

The remainder is organized as follows. In the next section the system model is introduced. In Section III we present our PAs for separate optimization with individual per node transmit power constraints at the source and the relay, whereas in Section IV we provide the solution for the joint optimization with a joint transmit power constraint. Performance results are presented in Section V. Conclusions are given in the last section.

II. System Model

In Fig. 1 the considered relay link is depicted, which consists of two terminals and one AF relay. All nodes are equipped with multiple antennas. The nodes operate in the half-duplex mode, i.e., they are not able to receive and transmit at the same time and same frequency. The number of antennas at the source, the relay, and the destination are denoted by N_{S}, N_{R}, and N_{D}, respectively. Since we are interested in increasing the coverage by relaying, we consider the scenario where the destination is not in communication range of the

Manuscript received February 8, 2006; revised August 4, 2006 and January 16, 2007; accepted March 19, 2007. The associate editor coordinating the review of this letter and approving it for publication was P. Martin.

The authors are with the Communication Technology Laboratory, ETH Zurich, Sternwartstr. 7, CH-8092 Zurich, Switzerland (e-mail: hammerstroem@nari.ee.ethz.ch; wittneben@nari.ee.ethz.ch).

Digital Object Identifier 10.1109/TWC.2007.06071.

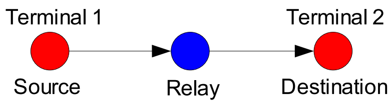

Fig. 1. Link with two terminals and one relay. Each terminal can be either source or destination.

source. Thus, the destination is not able to receive the signal from the source directly. Further, we assume that all nodes have perfect knowledge of the channels of both hops.

For broadband communication OFDM is used with a cyclic prefix that is at least as long as the channel impulse responses. Thus, the frequency-selective channel is divided into K frequency-flat subchannels. The channel matrix from the source to the relay and from the relay to the destination within the k-th OFDM subcarrier is denoted by $\mathbf{H}_{1,k} \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{S}}$ and $\mathbf{H}_{2,k} \in \mathbb{C}^{N_\mathrm{D} \times N_\mathrm{R}}$, respectively. We assume that all channel matrices have full rank. The received signal at the destination in the k-th subcarrier is given by

$$\mathbf{y}_k = \mathbf{H}_{2,k}\mathbf{F}_k\mathbf{H}_{1,k}\mathbf{x}_k + \mathbf{H}_{2,k}\mathbf{F}_k\mathbf{z}_k + \mathbf{w}_k, \qquad (1)$$

where $\mathbf{x}_k \in \mathbb{C}^{N_\mathrm{S}}$ denotes the transmit vector of the source, and $\mathbf{F}_k \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{R}}$ denotes the forwarding matrix at the relay. The noise contribution at the relay and the destination are denoted by $\mathbf{z}_k \sim \mathcal{CN}(0, \sigma_\mathrm{r}^2 \mathbf{I}_{N_\mathrm{R}})$ and $\mathbf{w}_k \sim \mathcal{CN}(0, \sigma_\mathrm{d}^2 \mathbf{I}_{N_\mathrm{D}})$. Due to the full CSI at the nodes we can use the singular value decomposition of the channel matrices to determine transmit- and receive-beamforming matrices at all nodes. Since these matrices are unitary they do not change the statistics of the channel and therefore preserve the mutual information and error performance of this link. The singular value decomposition of the channel matrices is given by $\mathbf{H}_{q,k} = \mathbf{U}_{q,k}\mathbf{S}_{q,k}\mathbf{V}_{q,k}^\dagger \forall\ k \in \{1, \ldots, K\}, q \in \{1, 2\}$. For some antenna configurations the number of possible spatial subchannels can be different for each hop. However, since AF relaying is linear the number of spatial subchannels per OFDM subcarrier over the two hops is bounded to the minimum number of spatial subchannels of each single hop, i.e., $N = \min(N_\mathrm{S}, N_\mathrm{R}, N_\mathrm{D})$. As a consequence, in AF relaying either in the first or the second hop only a subset of spatial subchannels can be used. These subspaces are defined by $\widetilde{\mathbf{H}}_{q,k} = \widetilde{\mathbf{U}}_{q,k}\widetilde{\mathbf{S}}_{q,k}\widetilde{\mathbf{V}}_{q,k}^\dagger$, whereby the matrices $\widetilde{\mathbf{U}}_{q,k}$ and $\widetilde{\mathbf{V}}_{q,k}$ consist of the N columns of $\mathbf{U}_{q,k}$ and $\mathbf{V}_{q,k}$ which belong to the N largest singular values of $\mathbf{S}_{q,k}$, respectively. The resulting transmit signal at the source is thus given by $\mathbf{x}_k = \widetilde{\mathbf{V}}_{1,k}\widetilde{\mathbf{x}}_k$ and the forwarding matrix has the structure $\mathbf{F}_k = \widetilde{\mathbf{V}}_{2,k}\mathbf{G}_k\widetilde{\mathbf{U}}_{1,k}^\dagger$. The destination multiplies the received signal (1) by $\widetilde{\mathbf{U}}_{2,k}^\dagger$ such that it can be expressed as

$$\widetilde{\mathbf{y}}_k = \widetilde{\mathbf{U}}_{2,k}^\dagger\mathbf{y}_k = \widetilde{\mathbf{S}}_{2,k}\mathbf{G}_k\widetilde{\mathbf{S}}_{1,k}\widetilde{\mathbf{x}}_k + \widetilde{\mathbf{S}}_{2,k}\mathbf{G}_k\widetilde{\mathbf{z}}_k + \widetilde{\mathbf{w}}_k, \qquad (2)$$

where $\widetilde{\mathbf{z}}_k = \widetilde{\mathbf{U}}_{1,k}^\dagger\mathbf{z}_k$ and $\widetilde{\mathbf{w}}_k = \widetilde{\mathbf{U}}_{2,k}^\dagger\mathbf{w}_k$ denote the equivalent noise contributions.

Up to now, we have no constraint on the structure of the gain matrix $\mathbf{G}_k$. To find the optimal structure of $\mathbf{G}_k$ we study the mutual information in the k-th subcarrier assuming Gaussian transmit signals with $\mathrm{E}\left\{\widetilde{\mathbf{x}}_k\widetilde{\mathbf{x}}_k^\dagger\right\} = \mathbf{\Lambda}_k$ given by (3). Due to Hadamard's inequality, a prerequisite for maximizing the determinant in (3) is that the rows (or columns) of the matrix within the determinant have to be orthogonal [14]. Using this prerequisite and the observation that all matrices in (3) are diagonal, we choose the gain matrix $\mathbf{G}_k$ also as a diagonal matrix. This leads to a diagonal overall matrix in the determinant which maximizes (3). We define $\mathbf{G}_k$ as

$$\mathbf{G}_k = \mathrm{diag}\left[g_{k,1}, \ldots, g_{k,N}\right],$$

with

$$g_{k,n} \equiv g_m = \sqrt{\frac{P_{\mathrm{r},m}}{P_{\mathrm{s},m} \cdot \lambda_{1,m} + \sigma_\mathrm{r}^2}},$$

where $P_{\mathrm{s},m}$, $P_{\mathrm{r},m}$, and $\lambda_{1,m}$ denote the transmit power of the source, the transmit power of the relay, and the eigenvalue of the source to relay channel matrix in the k-th subcarrier on the n-th spatial subchannel, respectively. We introduce the subscript $m = (k-1)N + n$ with $1 \leq n \leq N$ to keep the total number of subscripts small. It can be seen that the two-hop MIMO channel between the source and the destination decouples into $M = KN$ orthogonal SISO channels in which the transmit power values $P_{\mathrm{r},m}$ and $P_{\mathrm{s},m}$ can be chosen such that the instantaneous rate is maximized. The instantaneous rate (mutual information) per complex dimension is given by

$$C_\mathrm{I} = \frac{1}{2K}\sum_{m=1}^{M}\log_2\left(1 + \frac{P_{\mathrm{s},m}a_m \cdot P_{\mathrm{r},m}b_m}{1 + P_{\mathrm{s},m}a_m + P_{\mathrm{r},m}b_m}\right), \qquad (4)$$

where $a_m = \frac{\lambda_{1,m}}{\sigma_\mathrm{r}^2}$ and $b_m = \frac{\lambda_{2,m}}{\sigma_\mathrm{d}^2}$. $\lambda_{2,m}$ denotes the m-th eigenvalue of the relay to destination channel matrix. The factor $1/2$ is due to the two channel uses which are needed for the relay traffic pattern.

Convexity Properties: The rate expression (4) defines the objective function for our PA schemes. If the objective function is concave it exists only one global maximum. A sufficient condition for a function to be concave is that its Hessian is negative semidefinite [15]. Unfortunately this is not the case for the Hessian of (4) with respect to $P_{\mathrm{s},m}$ and $P_{\mathrm{r},m}$. Therefore, solutions for the joint optimization of $P_{\mathrm{s},m}$ and $P_{\mathrm{r},m}$ cannot be proven to be globally optimal. However, the second derivative of (4) with respect to $P_{\mathrm{s},m}$ or $P_{\mathrm{r},m}$ is strictly negative. Therefore, it is possible to find globally optimal solutions for the separate optimization of either the source or the relay PA.

Pairing of Subchannels: Up to now we have assumed that the signals of the source transmitted over the m-th subchannel are retransmitted by the relay also on the m-th subchannel. A higher performance in terms of mutual information can be achieved if the subchannels of both hops are paired according to their actual magnitude. In the following we want to give an intuition why this is favorable. For this we assume without

$$I\left(\widetilde{\mathbf{x}}_k; \widetilde{\mathbf{y}}_k\right) = \log_2\det\left(\mathbf{I}_N + \left(\sigma_\mathrm{d}^2\mathbf{I} + \sigma_\mathrm{r}^2\widetilde{\mathbf{S}}_{2,k}\mathbf{G}_k\mathbf{G}_k^\dagger\widetilde{\mathbf{S}}_{2,k}^\dagger\right)^{-1}\widetilde{\mathbf{S}}_{2,k}\mathbf{G}_k\widetilde{\mathbf{S}}_{1,k}\mathbf{\Lambda}_k\widetilde{\mathbf{S}}_{1,k}^\dagger\mathbf{G}_k^\dagger\widetilde{\mathbf{S}}_{2,k}^\dagger\right) \qquad (3)$$

loss of generality a system with two subchannels and $a_1 > a_2$ and $b_1 > b_2$. We define $A_i = P_{\mathrm{s}} a_i$ and $B_i = P_{\mathrm{r}} b_i$ for $i \in \{1, 2\}$. We want to show that the rate decreases if we pair the subchannels of first and second hop unordered, i.e.,

$$\log_2\left(1+\frac{A_1B_1}{1+A_1+B_1}\right)+\log_2\left(1+\frac{A_2B_2}{1+A_2+B_2}\right)\geq \log_2\left(1+\frac{A_1B_2}{1+A_1+B_2}\right)+\log_2\left(1+\frac{A_2B_1}{1+A_2+B_1}\right).$$

After some algebra we get (5). It can be seen that (5) can only be negative for $A_2 > A_1$ or $B_2 > B_1$, which is a contradiction to our assumption $a_1 > a_2$ and $b_1 > b_2$. In a system with $Z > 2$ subchannels we consider pairs of two subchannels and swap the ordering. It can be seen that the rate can be increased until $A_1 \geq A_2 \geq \ldots \geq A_Z$ is coupled with $B_1 \geq B_2 \geq \ldots \geq B_Z$.

Pairing can be applied to the different available dimensions. In a single antenna OFDM system it can be done over the frequency domain. The multiple antennas at all nodes open the spatial domain for pairing of subchannels. Thus, the subchannels can be ordered in the spatial domain only or in both, spatial and frequency domains. In the performance section we show that for multiple antenna systems the performance gain due to subchannel pairing in space domain is substantial, whereas additional pairing over frequency does not provide too much additional gain.

III. Separate Optimization With Individual Power Constraints

In the following we want to optimize the PA at the source and the relay separately, such that the instantaneous rate (4) is maximized. We consider individual per node transmit power constraints, i.e., $\sum_{m=1}^{M} P_{\mathrm{s},m} = P_{\mathrm{S}}$ and $\sum_{m=1}^{M} P_{\mathrm{r},m} = P_{\mathrm{R}}$. We assume that for the optimization of the PA at one node the PA at the other node is given. It can be seen that objective function of the optimization (4) has the same mathematical form with respect to either $P_{\mathrm{s},m}$ or $P_{\mathrm{r},m}$. Therefore, the optimization of the source or the relay PA can be expressed as

$$(p_1^\star, \ldots, p_M^\star) = \arg\max_{p_1,\ldots,p_M} \sum_{m=1}^{M} \log_2\left(1+\frac{\alpha_m\beta_m p_m}{1+\alpha_m+\beta_m p_m}\right)$$
$$\text{subject to} \quad \sum_{m=1}^{M} p_m = P, \quad p_m \geq 0 \;\forall\; m. \tag{6}$$

The parameter triplet (α_m, β_m, P) is chosen as $(P_{\mathrm{r},m} b_m, a_m, P_{\mathrm{S}})$ if the PA at the source is optimized or as $(P_{\mathrm{s},m} a_m, b_m, P_{\mathrm{R}})$ if the PA at the relay is optimized, respectively. Using the Karush-Kuhn-Tucker (KKT) [15] conditions we get the solution of the optimization problem (6) as

$$p_m = \frac{1}{\beta_m}\left[\frac{\alpha_m}{2}\left(\sqrt{1+\frac{4\beta_m}{\alpha_m \nu}}-1\right)-1\right]^+. \tag{7}$$

where $[z]^+ = \max\{0, z\}$. The Lagrange multiplier ν has to be chosen such that the sum power constraint $\sum_{m=1}^{M} p_m = P$ is fulfilled. The optimal PA for the source and the relay is therefore given by

$$P_{\mathrm{s},m} = \frac{1}{a_m}\left[\frac{P_{\mathrm{r},m} b_m}{2}\left(\sqrt{1+\frac{4a_m}{P_{\mathrm{r},m} b_m \nu}}-1\right)-1\right]^+ \tag{8}$$

and

$$P_{\mathrm{r},m} = \frac{1}{b_m}\left[\frac{P_{\mathrm{s},m} a_m}{2}\left(\sqrt{1+\frac{4b_m}{P_{\mathrm{s},m} a_m \nu}}-1\right)-1\right]^+, \tag{9}$$

respectively. Note that (9) is also presented in [10] for flat fading and $\sigma_{\mathrm{d}}^2 = \sigma_{\mathrm{r}}^2$.

A. Alternate Optimization of the PA at Source and Relay

For AF relaying the SNR per subchannel at the destination depends on the fading channels of both hops and also on the PA of the source and the relay. The separate optimization of either the source or the relay PA always assumes a given relay or source PA. Thus, the result is only optimal conditioned on the PA of the other node. This leads to the question, how can the joint optimal PA at the source and the relay with individual sum transmit power be determined? Since (4) is not jointly concave in $P_{\mathrm{s},m}$ and $P_{\mathrm{r},m}$ it is not possible to derive the solution of the global optimum analytically. The rate expression (4) has several maxima. To find one of these maxima, we propose a distributed optimization approach. The source and the relay alternately calculate their corresponding PA (8) and (9) given the resulting PA of the previous node as input for their optimization. Each optimization of the PA at one node (source or relay) improves the achievable rate of the relay link since (4) is concave in $P_{\mathrm{s},m}$ or $P_{\mathrm{r},m}$. However, since (4) is bounded for a limited transmit power the rate achieved by the proposed alternating optimization approach cannot grow without limit. We therefore conclude that the alternating optimization always converges to one of the maxima of (4). The maximum can be either a local or the global maximum depending on the chosen starting values for the given PA. We propose to use a starting vector which has equally large elements over all subchannels (i.e., uniform power distribution). Thus, no subchannel is preferred in the beginning.

IV. Joint Optimization With Joint Power Constraint

In the following we jointly optimize the PA at the source and the relay over the subchannels with respect to a joint power constraint at both nodes, i.e., $\sum_{m=1}^{M} P_{\mathrm{s},m} + \sum_{m=1}^{M} P_{\mathrm{r},m} = P_{\Sigma}$. By means of the joint power constraint this optimization is capable of responding more efficiently to the relative path losses between the first and the second hop. Although in a practical system the source and the relay have independent

$$\frac{(A_1-A_2)(B_1-B_2)(1+A_1+B_1+A_1B_1)(1+A_2+B_2+A_2B_2)}{(1+A_1+B_1)(1+A_2+B_2)(1+A_1+B_2)(1+A_2+B_1)} \geq 0 \tag{5}$$

power supply the optimization with a joint power constraint gives insight to the behavior of the required transmit power per communication link and not only per hop.

As already mentioned, the rate expression (4) is not jointly concave in $P_{\mathrm{s},m}$ and $P_{\mathrm{r},m}$. Therefore, we use a high SNR approximation of the SNR per subchannel given by

$$\rho_m = \frac{P_{\mathrm{s},m}a_m \cdot P_{\mathrm{r},m}b_m}{1+P_{\mathrm{s},m}a_m + P_{\mathrm{r},m}b_m} \leq \frac{P_{\mathrm{s},m}a_m \cdot P_{\mathrm{r},m}b_m}{P_{\mathrm{s},m}a_m + P_{\mathrm{r},m}b_m}. \quad (10)$$

This SNR expression leads to a jointly concave objective function and therefore to an unique maximum. The joint optimization problem is stated in (11). Note that the optimization of high SNR approximation of the SNR expression leads certainly to a lower bound in terms of rate. Using the KKT conditions we get the solution of the optimization problem (11) as

$$P_{\mathrm{s},m} = \frac{1}{1+\sqrt{\frac{a_m}{b_m}}}\left[\frac{1}{\nu} - \frac{\left(\sqrt{a_m}+\sqrt{b_m}\right)^2}{a_m b_m}\right]^+, \quad (12)$$

$$P_{\mathrm{r},m} = \frac{1}{1+\sqrt{\frac{b_m}{a_m}}}\left[\frac{1}{\nu} - \frac{\left(\sqrt{a_m}+\sqrt{b_m}\right)^2}{a_m b_m}\right]^+, \quad (13)$$

where the Lagrange multiplier ν has to be chosen such that the joint sum power constraint is fulfilled. Note that ν can be computed very efficiently. By adding $P_{\mathrm{s},m}$ and $P_{\mathrm{r},m}$ it turns out that the sum power of the source and the relay in subchannel m is

$$P_{\sigma,m} = P_{\mathrm{s},m} + P_{\mathrm{r},m} = \left[\frac{1}{\nu} - \frac{\left(\sqrt{a_m}+\sqrt{b_m}\right)^2}{a_m b_m}\right]^+. \quad (14)$$

The calculation of ν such that $\sum_{m=1}^{M} P_{\sigma,m} = P_\Sigma$ is done by the standard parallel Gaussian waterfilling [16] procedure.

V. Performance Results

In this section we present the performance of our proposed PA schemes for OFDM AF relay links by means of Monte-Carlo simulations. In our simulations we assume that the three nodes are located on a line. The distance between the source and the destination, the source and the relay, and the relay and the destination is denoted by d_0, d_1, and d_2, respectively. In the time-domain we model our frequency-selective channels as tapped delay line, equally spaced in symbol duration, with $L = 4$ paths. We assume a uniform power delay profile, where all taps are subject to Rayleigh fading and path-loss with a path loss exponent $\kappa = 3$. We define a reference SNR between the source and the relay placed at $d_0/2$ as

$$SNR_{\mathrm{ref}} = \frac{P_{\mathrm{S}}}{K\sigma_{\mathrm{d}}^2\left(\frac{d_0}{2}\right)^\kappa}.$$

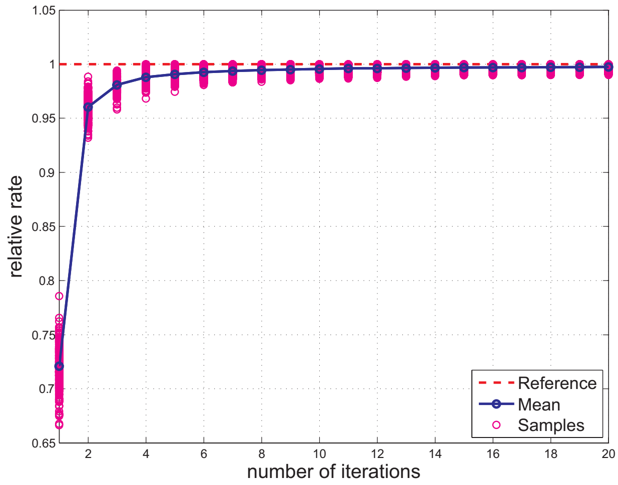

Fig. 2. Relative achieved rate of alternate optimization compared to reference vs. number of iterations; (4/4/4); $SNR_{\mathrm{ref}} = 0$ dB; $d_1 = d_2 = d_0/2$.

It is used to determine P_{S}, P_{R}, σ_{d}^2, and σ_{r}^2. If not stated otherwise in the simulations we have chosen $K = 16$, $d_0 \gg 1$, $SNR_{\mathrm{ref}} = 10$ dB, and $\sigma_{\mathrm{d}}^2 = \sigma_{\mathrm{r}}^2$. In the case of the individual transmit power constraints at the source and the relay we assume $P_{\mathrm{R}} = P_{\mathrm{S}}$, whereas for the case of the joint sum transmit power constraint we have $P_\Sigma = P_{\mathrm{S}} + P_{\mathrm{R}}$. We denote a system with N_{S}, N_{R}, and N_{D} antennas at the source, the relay, and the destination as $(N_{\mathrm{S}}/N_{\mathrm{R}}/N_{\mathrm{D}})$.

In Fig. 2 we show the convergence of the proposed alternate optimization. It shows the relative achieved rate of the alternate optimization approach compared to the numerical joint optimization of the source and the relay PA with individual transmit power constraint as reference vs. the number of iterations. We define one optimization of the relay PA or the source PA as one iteration. For the numerical reference optimization we used the Optimization Toolbox of MATLAB. Since the objective function for the numerical reference optimization is not concave we repeated the optimization with 50 randomly generated starting vectors per channel realization. The maximal achieved rate is taken for comparison. Note that only with a certain probability the numerical obtained value is the global optimum. In the figure it can be seen that the rate always converges. Furthermore, sometimes it converges to the same value as the numerical optimization. On the average the difference between the numerically obtained reference and the alternating optimization is quite small.

In Fig. 3 the average rate for the PA scheme with a joint power constraint versus the defined SNR_{ref} is shown for two system configurations. Firstly it can be seen that the achieved rate of the lower bound solution (12) and (13) is quite tight to the curve which has been obtained by the numerical optimization with the original SNR expression.

$$\left(P_{\mathrm{s},1}^\star, P_{\mathrm{r},1}^\star, \ldots, P_{\mathrm{s},M}^\star, P_{\mathrm{r},M}^\star\right) = \arg \max_{P_{\mathrm{s},1},P_{\mathrm{r},1},\ldots,P_{\mathrm{s},M},P_{\mathrm{r},M}} \sum_{m=1}^{M} \log_2\left(1+\frac{P_{\mathrm{s},m}a_m \cdot P_{\mathrm{r},m}b_m}{P_{\mathrm{s},m}a_m + P_{\mathrm{r},m}b_m}\right)$$

$$\text{subject to} \quad \sum_{m=1}^{M} P_{\mathrm{s},m} + \sum_{m=1}^{M} P_{\mathrm{r},m} = P_\Sigma, \quad P_{\mathrm{s},m} \geq 0,\ P_{\mathrm{r},m} \geq 0\ \forall\ m \quad (11)$$

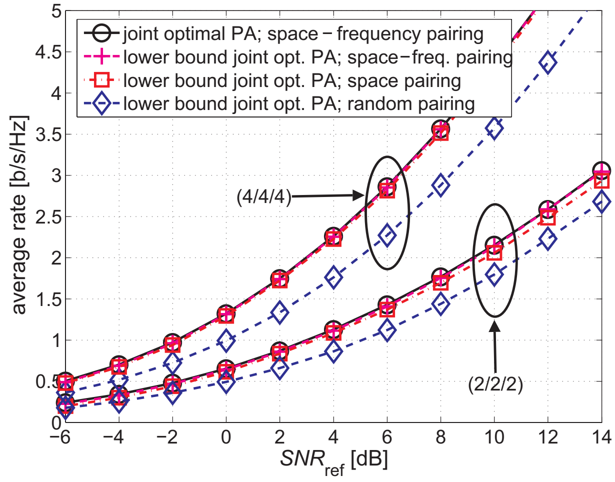

Fig. 3. Average rate vs. SNR_{ref} for (4/4/4) and (2/2/2) system; $d_1 = d_2 = d_0/2$.

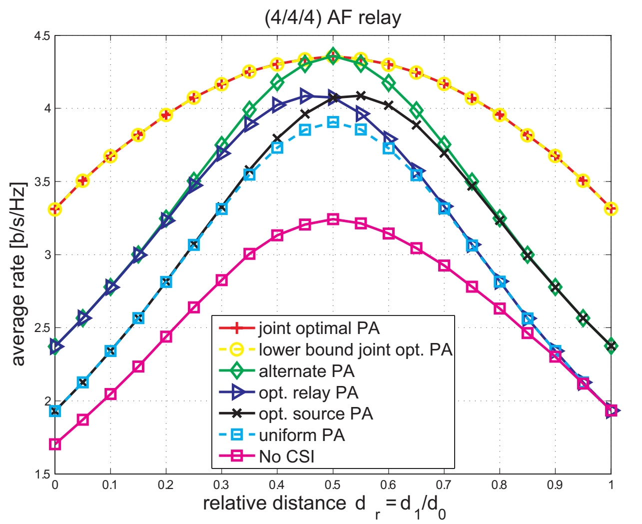

Fig. 4. Average rate vs. relative distance $d_r = d_1/d_0$; $SNR_{\text{ref}} = 10$ dB; symmetric antenna configuration $(4/4/4)$; pairing of subchannels in space and frequency domains.

Here, were also repeated the numerical optimization with 50 randomly generated starting vectors. The tightness comes from the fact that although the relative error of the lower bound compared to the reference is increased for low SNRs it only causes a small absolute error in b/s/Hz. Secondly, the impact of subchannel pairing is depicted. It can be seen that the difference for pairing in "space" and "space and frequency" domains is negligible, but both offer a considerable increase in performance compared to random subchannel pairing.

In Fig. 4 we show the average rates versus the distances between nodes for a $(4/4/4)$ system. For relative distances $d_r = d_1/d_0 < 0.5$ the optimal PA at the relay (9) with a uniformly distributed transmit power at the source ("opt. relay PA") achieves a higher average rate than an optimal PA at the source (9) with a uniform relay PA ("opt. source PA"). This is due to the fact that in this area the attenuation between the relay and the destination is larger than the attenuation between the source and the relay. Therefore, it is more important to allocate more power to the good subchannels of the second hop. For $d_r > 0.5$ this tendency changes. Now the path loss from the source to the relay is higher than from the relay to the destination. Therefore, allocating more power to good subchannels of the first hop is essential for high rates. The alternating optimization of both, the source and the relay PA, achieves the best performance of all PA schemes with individual per node power constraints. All presented PA schemes achieve a higher average rate compared to the case where the relay has no CSI ("no CSI") and only scales the received signal to a transmit power of P_{R}/K per subcarrier.

VI. Conclusions

In this work we examined the possibilities of PA in space and frequency domains within a MIMO-OFDM relaying link if CSIT is available. We considered two approaches: (i) separate optimization of the source or the relay PA with individual per node transmit power constraints and (ii) joint optimization of the source and the relay PA with a joint transmit power constraint. For (i) we presented the optimal PAs for the source and the relay. Further we showed that repeated alternate optimization of the source and the relay PA improves the achievable rate. For (ii) we presented a approximated solution since the original problem is not concave. Furthermore, we showed the impact of subchannel pairing on the achievable rate of such a communication link.

References

[1] E. C. van der Meulen, "Three-terminal communication channels," *Advances Applied Probability*, vol. 3, pp. 120-154, 1971.
[2] H. Sato, "Information transmission through a channel with relay," The Aloha System, University of Hawai, Honolulu, Tech. Rep. B76-7, Mar. 1976.
[3] T. M. Cover and A. A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. 25, no. 5, pp. 572-584, Sep. 1979.
[4] A. Sendonaris, E. Erkip, and B. Aazhang, "Increasing uplink capacity via user cooperation diversity," in *Proc. ISIT*, Aug. 1998, p. 156.
[5] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46, no. 2, pp. 388-404, Mar. 2000.
[6] J. N. Laneman, D. N. Tse, and G. W. Wornell, "An efficient protocol for realizing cooperative diversity in wireless networks," in *Proc. ISIT*, June 2001, p. 294.
[7] J. Zhang, Q. Zhang, C. Shao, Y. Wang, P. Zhang, and Z. Zhang, "Adaptive optimal transmit power allocation for two-hop non-regenerative wireless relaying system" in *Proc. 59th IEEE VTC-Spring*, May 2004, pp. 1213-1217.
[8] Q. Zhang, J. Zhang, C. Shao, Y. Wang, P. Zhang, and R. Hu, "Power allocation for regenerative relay channels with Rayleigh fading," in *Proc. 59th IEEE VTC-Spring*, May 2004, pp. 1167-1171.
[9] M. O. Hasna and M.-S. Alouini, "Optimal power allocation for relayed transmissions over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1999-2004, Nov. 2004.
[10] O. Muñoz, J. Vidal, and A. Agustín, "Non-regenerative MIMO relaying with channel state information," in *Proc. IEEE ICASSP*, Mar. 2005, vol. 3, pp. 361-364.
[11] J. Zhang, C. Shao, Y. Wang, and P. Zhang, "Optimal power allocation for multiple-input-multiple-output relaying systems," in *Proc. 60th IEEE VTC-Fall*, Sep. 2004, pp. 1405-1409.
[12] I. Maric and R. Yates, "Bandwidth and power allocation for cooperative strategies in Gaussian relay networks," in *Proc. Asilomar Conf. Signals, Syst., Comput.*, Nov. 2004, pp. 1907-1911.
[13] G. Li and H. Liu, "On the capacity of the broadband relay networks," in *Proc. Asilomar Conf. Signals, Syst., Comput.*, Nov. 2004, pp. 1318-1322.
[14] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, UK: Cambridge University Press, 1985.
[15] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge University Press, 2004.
[16] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: John Wiley & Sons, 1991.