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Research Article

Power and Resource Allocation for Orthogonal Multiple Access Relay Systems

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We study the problem of joint power and channel resource allocation for orthogonal multiple access relay (MAR) systems in order to maximize the achievable rate region. Four relaying strategies are considered; namely, regenerative decode-and-forward (RDF), nonregenerative decode-and-forward (NDF), amplify-and-forward (AF), and compress-and-forward (CF). For RDF and NDF we show that the problem can be formulated as a quasiconvex problem, while for AF and CF we show that the problem can be made quasiconvex if the signal-to-noise ratios of the direct channels are at least -3 dB. Therefore, efficient algorithms can be used to obtain the jointly optimal power and channel resource allocation. Furthermore, we show that the convex subproblems in those algorithms admit a closed-form solution. Our numerical results show that the joint allocation of power and the channel resource achieves significantly larger achievable rate regions than those achieved by power allocation alone with fixed channel resource allocation. We also demonstrate that assigning different relaying strategies to different users together with the joint allocation of power and the channel resources can further enlarge the achievable rate region.

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1. INTRODUCTION

In multiple access relay (MAR) systems, several source nodes send independent messages to a destination node with the assistance of a relay node [1–4]. These systems are of interest because they offer the potential for reliable communication at rates higher than those provided by conventional [5] and cooperative multiple access schemes [6–8] (in which source nodes essentially work as relays for each other.) For example, in [4] a comparison was made between the MAR system and the system that employs user cooperation, and the MAR system was shown to outperform the user cooperation system. Full-duplex MAR systems, in which the relay is able to transmit and receive simultaneously over the same channel, were studied in [1–3, 9], where inner and outer bounds for the capacity region were provided. However, full-duplex relays can be difficult to implement due to the electrical isolation required between the transmitter and receiver circuits. As a result, half-duplex relays, which do not simultaneously transmit and receive on the same channel, are often preferred in practice. The receiver at the relay and destination nodes can be further simplified if the source nodes (and the relay) transmit their messages on orthogonal channels, as this enables "per-user" decoding rather than joint decoding.

In this paper, we will consider the design of orthogonal multiple access systems with a half-duplex relay. In particular, we will consider the joint allocation of power and the channel resource in order to maximize the achievable rate region. Four relaying strategies will be considered; namely, regenerative (RDF) and nonregenerative (NDF) decode-and-forward [6, 8], amplify-and-forward (AF) [8, 10], and compress-and-forward (CF) [11, 12]. The orthogonal half-duplex MAR system that we will consider is similar to that considered in [13]. However, the focus of that paper is on the maximization of the sum rate, and, more importantly, it is assumed therein that the source nodes will each be allocated an equal fraction of the channel resources (e.g., time or bandwidth).(This equal allocation of resources is only optimal in the sum rate sense when the source nodes experience equal effective channel gains towards the destination and equal effective channel gains towards the

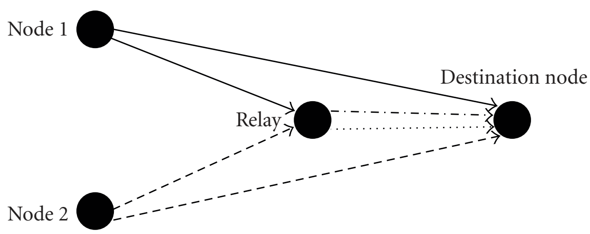


Figure 1: A simple multiple access relay channel with two source nodes.

relay.) In this paper, we will provide an efficiently solvable formulation for finding the jointly optimal allocation of power and the channel resources that enables the system to operate at each point on the boundary of the achievable rate region.

Although the problem of power allocation for an equal allocation of the channel resource was shown to be convex in [13], the joint allocation of power and the channel resource is not convex, which renders the problem harder to solve. In this paper, we show that the joint allocation problem can be formulated in a quasiconvex form, and hence, that the optimal solution can be obtained efficiently using standard quasiconvex algorithms, for example, bisection-based methods [14]. Furthermore, for a given channel resource allocation, we obtain closed-form expressions for the optimal power allocation, which further reduces the complexity of the algorithm used to obtain the jointly optimal allocation.

The practical importance of solving the problem of the joint allocation of power and channel resources is that it typically provides a substantially larger achievable rate region than that provided by allocating only the power for equal (or fixed) channel resource allocation, as will be demonstrated in the numerical results. Those results will also demonstrate the superiority of the NDF and CF relaying strategies over the RDF and AF strategies, respectively, which is an observation that is consistent with an observation in [13] for the case of power allocation with equal resource allocation. We will also demonstrate that joint allocation of the relaying strategy together with the power and channel resources, rather assigning the same relaying strategy to all users, can further enlarge the achievable rate region.

2. SYSTEM MODEL

We consider an orthogonal multiple access relay (MAR) system with N source nodes (nodes $1, 2, \ldots, N$), one destination node (node 0), and one relay (node R) that assists the source nodes in the transmission of their messages to the destination node. (The generalization of our model to different destination nodes is direct.) Figure 1 shows a simplified two-source MAR system. We will focus here on a system in which the transmitting nodes use orthogonal subchannels to transmit their signals, and the relay operates in half-duplex mode. This system model is similar to that used in [13]. The orthogonal subchannels can be synthesized in time or in frequency, but given their equivalence it is sufficient for us to focus on the case in which they are synthesized in time, that is, we will divide the total frame length into N nonoverlapping subframes of fractional length r_i, and we will allocate the ith subframe to the transmission (and relaying) of the message from source node i to the destination node. Figure 2 shows the block diagram of the cooperation scheme and the transmitted signals during one frame of such an MAR system with two source nodes. As shown in Figure 2, the first subframe is allocated to node 1 and has a fractional length r_1, while the second subframe is allocated to node 2 and has a fractional length $r_2 = 1 - r_1$. Each subframe is further partitioned into two equal-length blocks [13]. In the first block of subframe i of frame ℓ, node i sends a new block of symbols $B_i(w_{i\ell})$ to both the relay and the destination nodes, where $w_{i\ell}$ is the component of the ith user's message that is to be transmitted in the ℓth frame. In the second block of that subframe, the relay node transmits a function $f(\cdot)$ of the message it received from node i in the first block. (The actual function depends on the relaying strategy.) We will let P_i represent the power used by node i to transmit its message, and we will constrain it so that it satisfies the average power constraint $(r_i/2)P_i \leq \overline{P}_i$, where $\overline{P}_i$ is the maximum average power of node i. We will let P_{Ri} represent the relay power allocated to the transmission of the message of node i, and we will impose the average power constraint $\sum_{i=1}^{N}(r_i/2)P_{Ri} \leq \overline{P}_R$. (The function $f(\cdot)$ is normalized so that it has a unit power.) In this paper, we consider the following four relaying strategies.

(i) Regenerative decode-and-forward (RDF). The relay decodes the message $w_{i\ell}$, re-encodes it using the same code book as the source node, and transmits the codeword to the destination [6, 8].

(ii) Nonregenerative decode-and-forward (NDF). The relay decodes the message $w_{i\ell}$, re-encodes it using a different code book from that used by the source node, and transmits the codeword to the destination [15, 16].

(iii) Amplify-and-forward (AF). The relay amplifies the received signal and forwards it to the destination [8, 10]. In this case, $f(w_{i\ell})$ is the signal received by the relay, normalized by its power.

(iv) Compress-and-forward (CF). The relay transmits a compressed version of the signal it receives [11, 12].

Without loss of generality, we will focus here on a two-user system in order to simplify the exposition. However, as we will explain in Section 3.5, all the results of this paper can be applied to systems with more than two source nodes. For the two-source system, the received signals at the relay and the destination at block m can be expressed as

$$\mathbf{y}_R(m) = \begin{cases} K_{1R}\mathbf{x}_1(m) + \mathbf{z}_R(m) & m \bmod 4 = 1, \\ K_{2R}\mathbf{x}_2(m) + \mathbf{z}_R(m) & m \bmod 4 = 3, \\ \mathbf{0} & m \bmod 4 \in \{0, 2\}, \end{cases}$$
$$\mathbf{y}_0(m) = \begin{cases} K_{10}\mathbf{x}_1(m) + \mathbf{z}_0(m) & m \bmod 4 = 1, \\ K_{R0}\mathbf{x}_R(m) + \mathbf{z}_0(m) & m \bmod 4 = 2, \\ K_{20}\mathbf{x}_2(m) + \mathbf{z}_0(m) & m \bmod 4 = 3, \\ K_{R0}\mathbf{x}_R(m) + \mathbf{z}_0(m) & m \bmod 4 = 0, \end{cases} \quad (1)$$

where the vectors $\mathbf{y}_i$ and $\mathbf{x}_i$ contain the blocks of received and transmitted signals of node i, respectively; K_{ij}, $i \in \{1, 2, R\}$

$x_1(1) = \sqrt{P_1}B_1(w_{11})$ $x_2(1) = 0$ $x_R(1) = 0$	$x_1(2) = 0$ $x_2(2) = 0$ $x_R(2) = \sqrt{P_{R1}}f(w_{11})$	$x_1(3) = 0$ $x_2(3) = \sqrt{P_2}B_2(w_{21})$ $x_R(3) = 0$	$x_1(4) = 0$ $x_2(4) = 0$ $x_R(4) = \sqrt{P_{R2}}f(w_{21})$
← r_1 →		← $r_2 = 1 - r_1$ →	

FIGURE 2: One frame of the considered orthogonal cooperation scheme for the case of 2 source nodes, and its constituent subframes.

and $j \in \{R, 0\}$, represents the channel gain between nodes i and j; $\mathbf{z}_j$ represents the additive zero mean white circular complex Gaussian noise with variance σ_j^2 at node j; and $\mathbf{0}$ is used to represent blocks in which the receiver of the relay node is turned off. For simplicity, we define the effective power gain $\gamma_{ij} = |K_{ij}|^2/\sigma_j^2$.

The focus of this paper will be on a system in which full channel state information (CSI) is available at the source nodes, and the channel coherence time is long. The CSI is exploited to jointly allocate the powers P_{Ri} and the resource allocation parameters r_i, with the goal of enlarging the achievable rate region. Under the assumption of equal channel resource allocation (i.e., $r_i = r_s$, for all i, s), expressions for the maximum achievable rate for a source node under each of the four relaying considered relaying strategies were provided in [13]. The extension of those expressions to the case of not necessarily equal resource allocation results in the following expressions for the maximum achievable rate of node i as a function of P_i, the transmission power of node i, P_{Ri}, the relay power allocated to node i, and r_i, the fraction of the channel resource allocated to node i.

(i) Regenerative decode-and-forward (RDF):

$$\overline{R}_{i,\mathrm{RDF}} = \frac{r_i}{2}\min\{\log(1+\gamma_{iR}P_i),\ \log(1+\gamma_{i0}P_i+\gamma_{R0}P_{Ri})\}. \tag{2a}$$

(ii) Nonregenerative decode-and-forward (NDF):

$$\overline{R}_{i,\mathrm{NDF}} = \frac{r_i}{2}\min\{\log(1+\gamma_{iR}P_i), \log(1+\gamma_{i0}P_i) + \log(1+\gamma_{R0}P_{Ri})\}. \tag{2b}$$

(iii) Amplify-and-forward (AF):

$$\overline{R}_{i,\mathrm{AF}} = \frac{r_i}{2}\log\left(1+\gamma_{i0}P_i+\frac{\gamma_{iR}\gamma_{R0}P_iP_{Ri}}{(1+\gamma_{iR}P_i+\gamma_{R0}P_{Ri})}\right). \tag{2c}$$

(iv) Compress-and-forward (CF): assuming that the relay uses Wyner-Ziv lossy compression [17], the maximum achievable rate is

$$\overline{R}_{i,\mathrm{CF}} = \frac{r_i}{2}\log\left(1+\gamma_{i0}P_i+\frac{\gamma_{iR}\gamma_{R0}(\gamma_{i0}P_i+1)P_iP_{Ri}}{\gamma_{R0}(\gamma_{i0}P_i+1)P_{Ri}+P_i(\gamma_{i0}+\gamma_{iR})+1}\right). \tag{2d}$$

The focus of the work in this paper will be on systems in which the relay node relays the messages of all source nodes in the system using the same preassigned relaying strategy. However, as we will demonstrate in Section 4, our results naturally extend to the case of heterogeneous relaying strategies, and hence facilitate the development of algorithms for the jointly optimal allocation of the relaying strategy.

3. JOINT POWER AND CHANNEL RESOURCE ALLOCATION

It was shown in [13] that for fixed channel resource allocation, the problem of finding the power allocation that maximizes the sum rate is convex, and closed-form solutions for the optimal power allocation were obtained. However, the direct formulation of the problem of joint allocation of both the power and the channel resource so as to enable operation at an arbitrary point on the boundary of the achievable rate region is not convex, and hence is significantly harder to solve. Despite this complexity, the problem is of interest because it is expected to yield significantly larger achievable rate regions than those obtained with equal channel resource allocation. In the next four subsections, we will study the problem of finding the jointly optimal power and resource allocation for each relaying strategy. We will show that in each case the problem can be transformed into a quasiconvex problem, and hence an optimal solution can be obtained using simple and efficient algorithms, that is, standard quasiconvex search algorithms [14]. Furthermore, for a fixed resource allocation, a closed-form solution for the optimal power allocation is obtained. By exposing the quasiconvexity of the problem and by obtaining a closed-form solution to the power allocation problem, we are able to achieve significantly larger achievable rate regions without incurring substantial additional computational cost.

The jointly optimal power and channel resource allocation at each point on the boundary of the achievable rate region can be found by maximizing a weighted sum of the maximal rates $\overline{R}_1$ and $\overline{R}_2$ subject to the bound on the transmitted powers, that is,

$$\begin{aligned} \max_{P_i, P_{Ri}, r} \quad & \mu\overline{R}_1 + (1-\mu)\overline{R}_2, \\ \text{subject to} \quad & \frac{r}{2}P_{R1} + \frac{\hat{r}}{2}P_{R2} \le \overline{P}_R, \\ & \frac{r}{2}P_1 \le \overline{P}_1, \qquad \frac{\hat{r}}{2}P_2 \le \overline{P}_2, \\ & P_{Ri} \ge 0, \qquad P_i \ge 0, \end{aligned} \tag{3}$$

where $\overline{R}_i$ is the expression in (2a), (2b), (2c), or (2d) that corresponds to the given relaying strategy, $r = r_1$, $\hat{r} = r_2 = 1 - r$, and $\mu \in [0,1]$ weights the relative importance of $\overline{R}_1$ over $\overline{R}_2$. Alternatively, the jointly optimal power and channel resource allocation at each point on the boundary of the achievable rate region can also be found by maximizing $\overline{R}_i$ for a given target value of $\overline{R}_j$, subject to the bound on the transmitted powers, for example,

$$\begin{aligned} \max_{P_i, P_{Ri}, r} \quad & \overline{R}_1, \\ \text{subject to} \quad & \overline{R}_2 \geq R_{2,\text{tar}}, \\ & \frac{r}{2} P_{R1} + \frac{\hat{r}}{2} P_{R2} \leq \overline{P}_R, \\ & \frac{r}{2} P_1 \leq \overline{P}_1, \qquad \frac{\hat{r}}{2} P_2 \leq \overline{P}_2, \\ & P_{Ri} \geq 0, \qquad P_i \geq 0. \end{aligned} \tag{4}$$

Neither the formulation in (3) nor that in (4) is jointly convex in the transmitted powers and the channel resource allocation parameter r, and hence it appears that it may be difficult to develop a reliable efficient algorithm for their solution. However, in the following subsections, we will show that by adopting the framework in (4), the direct formulation can be transformed into a composition of a convex problem (with a closed-form solution) and a quasiconvex optimization problem, and hence it can be efficiently and reliably solved. The first step in that analysis is to observe that since the source nodes transmit on channels that are orthogonal to each other and to that of the relay, then at optimality they should transmit at full power, that is, the optimal values of P_1 and P_2 are $P_1^*(r) = 2\overline{P}_1/r$ and $P_2^*(r) = 2\overline{P}_2/\hat{r}$, respectively. In order to simplify our development, we will define $R_{2,\max}(r)$ to be the maximum achievable value for $\overline{R}_2$ for a given value of r and the given relaying strategy, that is, the value of the appropriate expression in (2a), (2b), (2c), or (2d) with $P_{R2} = 2\overline{P}_R/\hat{r}$ and $P_2 = 2\overline{P}_2/\hat{r}$.

3.1. Regenerative decode-and-forward

For the regenerative decode-and-forward strategy, the problem in (4) can be written as

$$\begin{aligned} \max_{P_{Ri}, r} \quad & \frac{r}{2} \min \{ \log(1 + \gamma_{1R} P_1^*), \\ & \qquad \log(1 + \gamma_{10} P_1^* + \gamma_{R0} P_{R1}) \}, \\ \text{subject to} \quad & \frac{\hat{r}}{2} \min \{ \log(1 + \gamma_{2R} P_2^*), \\ & \qquad \log(1 + \gamma_{20} P_2^* + \gamma_{R0} P_{R2}) \} \geq R_{2,\text{tar}}, \\ & \frac{r}{2} P_{R1} + \frac{\hat{r}}{2} P_{R2} \leq \overline{P}_R, \\ & P_{Ri} \geq 0. \end{aligned} \tag{5}$$

Unfortunately, the set of values for r, P_{R1}, and P_{R2} that satisfy the second constraint of (5) is bilinear, and hence the problem in (5) is not convex. However, if we define $\widetilde{P}_{R1} = rP_{R1}$ and $\widetilde{P}_{R2} = \hat{r}P_{R2}$, then the problem in (5) can be rewritten as

$$\begin{aligned} \max_{\widetilde{P}_{Ri}, r} \quad & \frac{r}{2} \min \left\{ \log\left(1 + \frac{2\gamma_{1R}\overline{P}_1}{r}\right), \right. \\ & \qquad \left. \log\left(1 + \frac{2\gamma_{10}\overline{P}_1 + \gamma_{R0}\widetilde{P}_{R1}}{r}\right) \right\}, \\ \text{subject to} \quad & \frac{\hat{r}}{2} \min \left\{ \log\left(1 + \frac{2\gamma_{2r}\overline{P}_2}{\hat{r}}\right), \right. \\ & \qquad \left. \log\left(1 + \frac{2\gamma_{20}\overline{P}_2 + \gamma_{R0}\widetilde{P}_{R2}}{\hat{r}}\right) \right\} \geq R_{2,\text{tar}}, \\ & \widetilde{P}_{R1} + \widetilde{P}_{R2} = 2\overline{P}_R, \\ & \widetilde{P}_{Ri} \geq 0. \end{aligned} \tag{6}$$

Formulating the problem as in (6) enables us to obtain the following result, the proof of which is provided in Appendix A.

Proposition 1. *For a given feasible target rate $R_{2,\text{tar}} \in (0, R_{2,\max}(0))$, the maximum achievable rate $\overline{R}_{1,\max}$ in (6) is a quasiconcave function of the channel resource sharing parameter r.*

In addition to the desirable property in Proposition 1, for any given channel resource allocation and for any feasible $R_{2,\text{tar}}$, a closed-form solution for the optimal power allocation can be found. In particular, for any given r, $\widetilde{P}_{R1}$ must be maximized in order to maximize R_1. Therefore, the optimal value of $\widetilde{P}_{R2}$ is the minimum value that satisfies the constraints in (6), and hence it can be written as

$$\widetilde{P}_{R2}^*(r) = \begin{cases} 0 & \text{if } \gamma_{2R} \leq \gamma_{20}, \\ \left(\dfrac{A - 2\gamma_{20}\overline{P}_2}{B}\right)^+ & \text{if } \gamma_{2R} > \gamma_{20}, \end{cases} \tag{7}$$

where $A = \hat{r}(2^{2R_{2,\text{tar}}/\hat{r}} - 1)$, $B = \gamma_{R0}$,and $x^+ = \max(0,x)$. The optimal value of $\widetilde{P}_{R1}$ is $\widetilde{P}_{R1}^*(r) = \min\{2\overline{P}_R - \widetilde{P}_{R2}^*(r), (2\overline{P}_1(\gamma_{1R} - \gamma_{10})/\gamma_{R0})^+\}$, where the second argument of the min function is the value of $\widetilde{P}_{R1}$ that makes the two arguments of the min function in the objective function of (6) equal. In Section 3.5, we will exploit the quasiconvexity result in Proposition 1 and the closed-form expression for $\widetilde{P}_{R2}^*(r)$ in (7) to develop an efficient algorithm for the jointly optimal allocation of power and the channel resource.

3.2. Nonregenerative decode-and-forward

Using the definition of $\widetilde{P}_{R1}$ and $\widetilde{P}_{R2}$ from the RDF case, the problem of maximizing the achievable rate region for the NDF relaying strategy can be written as

$$\begin{aligned} \max_{\widetilde{P}_{Ri}, r} \quad & \frac{r}{2} \min \left\{ \log\left(1 + \frac{2\gamma_{1R}\overline{P}_1}{r}\right), \log\left(1 + \frac{2\gamma_{10}\overline{P}_1}{r}\right) \right. \\ & \qquad \left. + \log\left(1 + \frac{\gamma_{R0}\widetilde{P}_{R1}}{r}\right) \right\}, \end{aligned}$$

$$\text{subject to} \quad \frac{\hat{r}}{2}\min\left\{\log\left(1+\frac{2\gamma_{2R}\overline{P}_2}{\hat{r}}\right),\log\left(1+\frac{2\gamma_{20}\overline{P}_2}{\hat{r}}\right)+\log\left(1+\frac{\gamma_{R0}\widetilde{P}_{R2}}{\hat{r}}\right)\right\}\geq R_{2,\text{tar}},$$
$$\widetilde{P}_{R1}+\widetilde{P}_{R2}=2\overline{P}_R,$$
$$\widetilde{P}_{Ri}\geq 0. \tag{8}$$

Using the formulation in (8), we obtain the following result in Appendix B.

Proposition 2. *For a given feasible target rate* $R_{2,\text{tar}} \in (0, R_{2,\max}(0))$, *the maximum achievable rate* $\overline{R}_{1,\max}$ *in* (8) *is a quasiconcave function of* r.

Similar to the RDF case, for a given r and a feasible $R_{2,\text{tar}}$, a closed-form expression for the optimal $\widetilde{P}_{R2}$ can be obtained. This expression has the same form as that in (7), with the same definition for A, but with B defined as $B = \gamma_{R0} + 2\gamma_{20}\gamma_{R0}\overline{P}_2/\hat{r}$. The optimal value for $\widetilde{P}_{R1}$ is $\widetilde{P}^*_{R1}(r) = \min\{2\overline{P}_R - \widetilde{P}^*_{R2}(r), (2\overline{P}_1(\gamma_{1R}-\gamma_{10})r/(\gamma_{R0}(r+2\overline{P}_1\gamma_{10})))^+\}$, where the second argument of the min function is the value of $\widetilde{P}_{R1}$ that makes the two arguments of the min function in the objective function of (8) equal.

3.3. Amplify-and-forward

In the case of amplify-and-forward relaying, problem (4) can be written as

$$\max_{\widetilde{P}_{Ri},r} \quad \frac{r}{2}\log\left(1+\frac{2\gamma_{10}\overline{P}_1}{r}+\frac{2\gamma_{1R}\gamma_{R0}\overline{P}_1\widetilde{P}_{R1}}{r(r+2\gamma_{1R}\overline{P}_1+\gamma_{R0}\widetilde{P}_{R1})}\right),$$
$$\text{subject to} \quad \frac{\hat{r}}{2}\log\left(1+\frac{2\gamma_{20}\overline{P}_2}{\hat{r}}+\frac{2\gamma_{2R}\gamma_{R0}\overline{P}_2\widetilde{P}_{R2}}{\hat{r}(\hat{r}+2\gamma_{2r}\overline{P}_2+\gamma_{R0}\widetilde{P}_{R2})}\right)\geq R_{2,\text{tar}},$$
$$\widetilde{P}_{R1}+\widetilde{P}_{R2}=2\overline{P}_R,$$
$$\widetilde{P}_{Ri}\geq 0. \tag{9}$$

Using this formulation, we obtain the following result in Appendix C. (We point out that $\gamma_{i0}\overline{P}_i$ is the maximum achievable destination SNR on the direct channel of source node i.)

Proposition 3. *If the direct channels of both source nodes satisfy* $\gamma_{i0}\overline{P}_i > 1/2$, *then for a given feasible target rate* $R_{2,\text{tar}} \in (0, R_{2,\max}(0))$, *the maximum achievable rate* $\overline{R}_{1,\max}$ *in* (9) *is a quasiconcave function of* r.

Similar to the cases of RDF and NDF relaying, for a given r and a feasible $R_{2,\text{tar}}$, in order to obtain an optimal power allocation we must find the smallest $\widetilde{P}_{R2}$ that satisfies the constraints in (9). If we define $C = A - 2\gamma_{20}\overline{P}_2$, a closed-form solution for $\widetilde{P}_{R2}$ can be written as

$$\widetilde{P}^*_{R2}(r)=\left(\frac{C(\hat{r}+2\gamma_{2R}\overline{P}_2)}{2\gamma_{2R}\gamma_{R0}\overline{P}_2-\gamma_{R0}C}\right)^+. \tag{10}$$

Hence, the optimal value of $\widetilde{P}_{R1}$ is $\widetilde{P}^*_{R1}(r) = 2\overline{P}_R - \widetilde{P}^*_{R2}(r)$.

Given $R_{2,\text{tar}} \in (0, R_{2,\max}(0))$, for $r \in (0,1)$, define $\psi(r)$ to be the optimal value of (4) for a given r if $R_{2,\text{tar}} \in (0, R_{2,\max}(r))$ and zero otherwise. Set $\psi(0)=0$ and $\psi(1)=0$. Set $t_0=0$, $t_4=1$, and $t_2=1/2$. Using the closed-form expression for the optimal power allocations, compute $\psi(t_2)$. Given a tolerance ε,
(1) set $t_1=(t_0+t_2)/2$ and $t_3=(t_2+t_4)/2$,
(2) using the closed-form expressions for the power allocations, compute $\psi(t_1)$ and $\psi(t_3)$,
(3) find $k^* = \arg\max_{k\in\{0,1,\ldots,4\}}\psi(t_k)$,
(4) replace t_0 by $t_{\max\{k^*-1,0\}}$, replace t_4 by $t_{\min\{k^*+1,4\}}$, and save $\psi(t_0)$ and $\psi(t_4)$. If $k^* \notin \{0,4\}$ set $t_2=t_{k^*}$ and save $\psi(t_2)$, else set $t_2=(t_0+t_4)/2$ and use the closed form expressions for the power allocations to calculate $\psi(t_2)$.
(5) if $t_4-t_0 \geq \varepsilon$, return to (1), else set $r^* = t_{k^*}$.

Algorithm 1: A simple method for finding r^*

3.4. Compress-and-forward

Finally, for the compress-and-forward relaying strategy, the problem in (4) can be written as

$$\max_{\widetilde{P}_{Ri},r} \quad \frac{r}{2}\min\left\{\log\left(1+\frac{2\gamma_{1R}\overline{P}_1}{r}\right),\ \log\left(1+\frac{2\gamma_{10}\overline{P}_1}{r}\right)+\log\left(1+\frac{\gamma_{R0}\widetilde{P}_{R1}}{r}\right)\right\},$$
$$\text{subject to} \quad \frac{\hat{r}}{2}\min\left\{\log\left(1+\frac{2\gamma_{2R}\overline{P}_2}{\hat{r}}\right),\ \log\left(1+\frac{2\gamma_{20}\overline{P}_2}{\hat{r}}\right)+\log\left(1+\frac{\gamma_{R0}\widetilde{P}_{R2}}{\hat{r}}\right)\right\}\geq R_{2,\text{tar}},$$
$$\widetilde{P}_{R1}+\widetilde{P}_{R2}=2\overline{P}_R,$$
$$\widetilde{P}_{Ri}\geq 0. \tag{11}$$

As we state in the following proposition (proved in Appendix D), the quasiconvex properties of the problem in (11) are similar to those of the amplify-and-forward case.

Proposition 4. *If the direct channels of both source nodes satisfy* $\gamma_{i0}\overline{P}_i > 1/2$, *then for a given feasible target rate* $R_{2,\text{tar}} \in (0, R_{2,\max}(0))$, *the maximum achievable rate* $\overline{R}_{1,\max}$ *in* (11) *is a quasiconcave function of* r.

If we define $D = \gamma_{R0}(2\gamma_{20}\overline{P}_2 + \hat{r})$, then the optimal solution for $\widetilde{P}_{R2}$ for a given r and a feasible $R_{2,\text{tar}}$ can be written as

$$\widetilde{P}^*_{R2}(r)=\left(\frac{C\hat{r}(\hat{r}+2(\gamma_{20}+\gamma_{2R})\overline{P}_2)}{D(2\gamma_{2R}\overline{P}_2-C)}\right)^+, \tag{12}$$

and the optimal $\widetilde{P}_{R1}$ is $\widetilde{P}^*_{R1}(r) = 2\overline{P}_R - \widetilde{P}^*_{R2}(r)$.

Table 1: Parameters of the two-user channel models used in the numerical results.

	$\lvert K_{10}\rvert$	$\lvert K_{1R}\rvert$	$\lvert K_{20}\rvert$	$\lvert K_{2R}\rvert$	$\lvert K_{R0}\rvert$	$\sigma_R^2 = \sigma_0^2$	$\overline{P}_1$	$\overline{P}_2$	$\overline{P}_R$
Scenario 1	0.3	1.2	0.8	0.6	0.4	1	2	2	4
Scenario 2	0.3	1.2	0.6	0.8	0.4	1	2	2	4

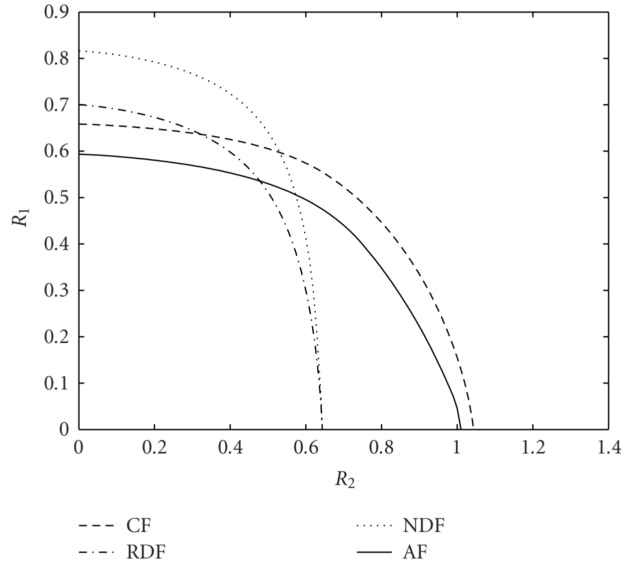


Figure 3: Achievable rate regions obtained via jointly optimal power and resource allocation in Scenario 1.

3.5. Summary and extensions

In the previous four subsections, we have shown that the problem of jointly allocating the power and the channel resource so as to enable operation at any point on the boundary of the achievable rate region is quasiconvex. In addition, we have shown that for a given resource allocation, a closed-form solution for the optimal power allocation can be obtained. These results mean that we can determine the optimal value for r using a standard efficient search method for quasiconvex problems (see, e.g., [14]). (In the AF and CF cases, these results are contingent on the maximum achievable SNR of both direct channels, being greater than -3 dB, which would typically be the case in practice. Furthermore, since the condition $\gamma_{i0}\overline{P}_i > 1/2$ depends only on the direct channel gains, the noise variance at the destination node, and the power constraints, this condition is testable before the design process commences.)

For the particular problem at hand, a simple approach that is closely related to bisection search is provided in Algorithm 1. At each step in that approach, we use the closed-form expressions for the optimal power allocation for each of the current values of r. Since the quasiconvex search can be efficiently implemented and since it converges rapidly, the jointly optimal values for r and the (scaled) powers $\widetilde{P}_{Ri}$ can be efficiently obtained.

In the above development, we have focused on the case of two source nodes. However, the core results extend directly

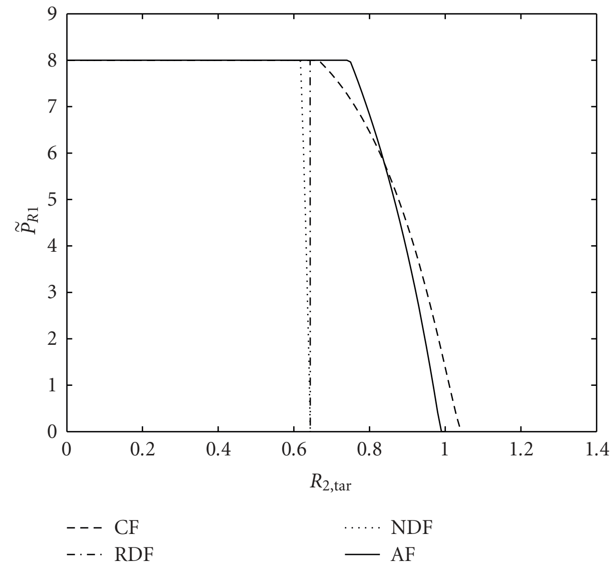


Figure 4: Powers allocated by the jointly optimal algorithm in Scenario 1.

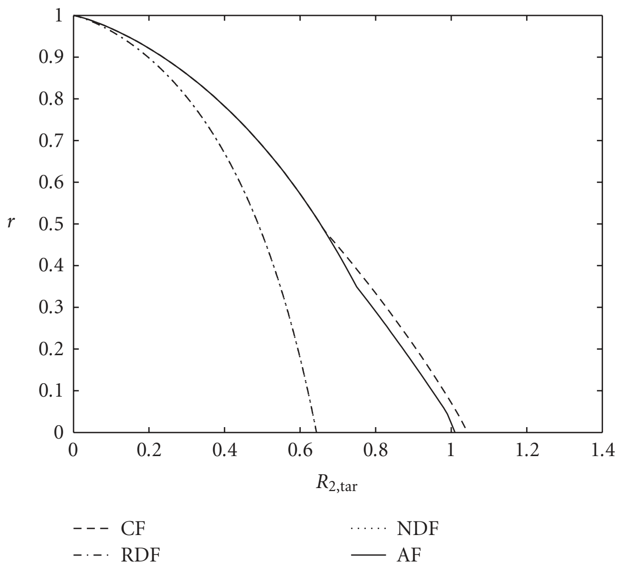


Figure 5: Resource allocation from the jointly optimal algorithm in Scenario 1.

to the case of $N > 2$ source nodes. Indeed, the joint power and resource allocation problem can be written in a form analogous to those in (6), (8), (9), and (11). To do so, we let $\overline{R}_i$ denote the appropriate maximal rate for node i from (2a), (2b), (2c), or (2d), and we define $\widetilde{P}_{Ri} = r_i P_{Ri}$, where P_{Ri} is the relay power allocated to the message of node i. If we choose to maximize the achievable rate of node j subject to target rate requirements for the other nodes, then the problem can be written as

$$\max_{\widetilde{P}_{Ri}, r_i} \quad \overline{R}_j,$$

$$\text{subject to} \quad \overline{R}_i \geq R_{i,\text{tar}} \quad i = 1, 2, \ldots, N;\ i \neq j,$$

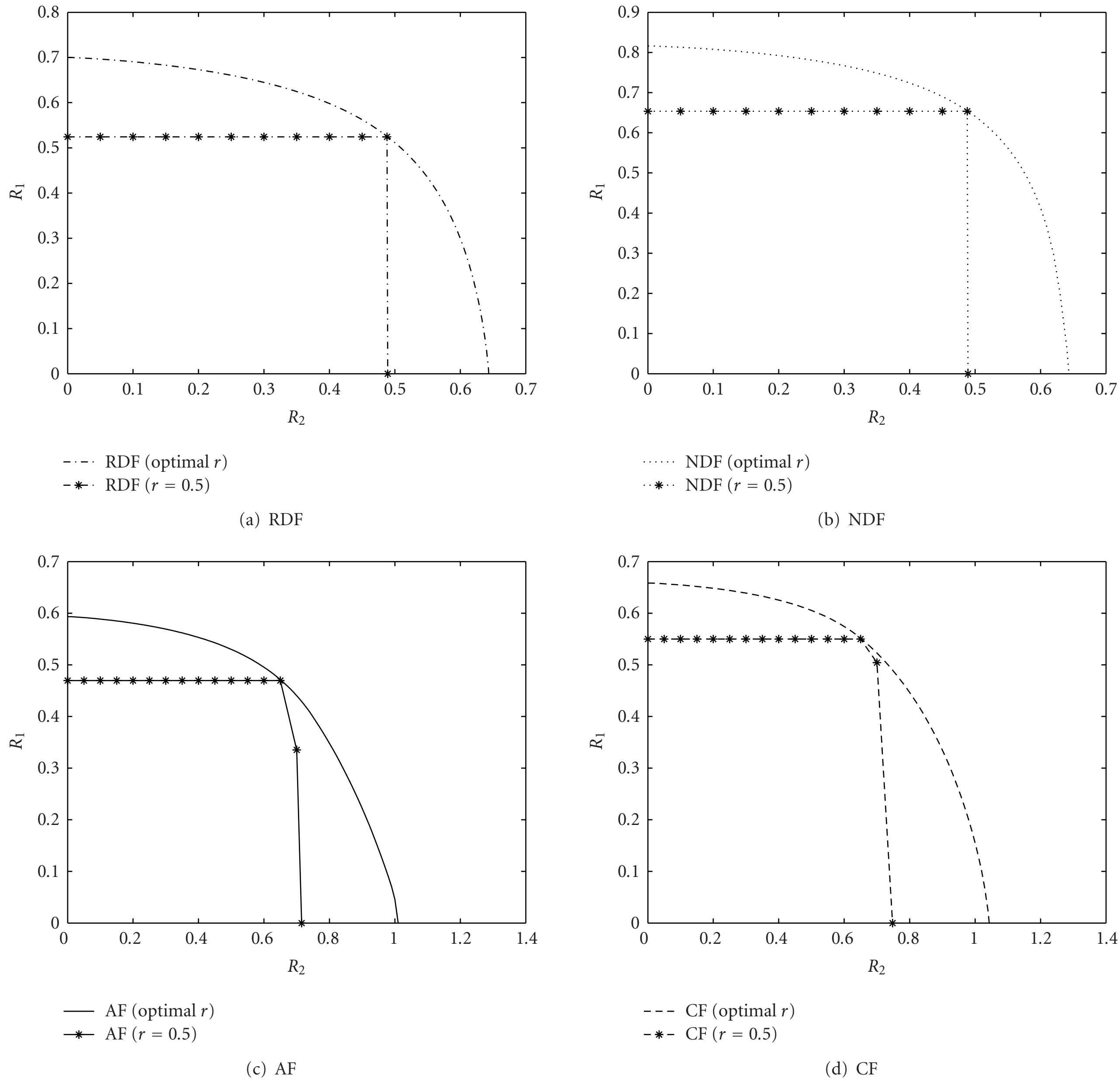


Figure 6: Comparisons between the achievable rate regions obtained by jointly optimal power and resource allocation and those obtained by power allocation only with equal resource allocation, for Scenario 1.

$$\sum_{i=1}^{N} \widetilde{P}_{Ri} \leq 2\overline{P}_R,$$
$$\widetilde{P}_{Ri} \geq 0,$$
$$\sum_{i=1}^{N} r_i = 1. \tag{13}$$

Using similar techniques to those in the previous subsections, it can be shown that this problem is quasiconvex in $(N-1)$ resource allocation parameters. The other parameter is not free as the resource allocation parameters must sum to one. (In the AF and CF cases, this result is, again, contingent on the condition $\gamma_{i0}\overline{P}_i > 1/2$ holding for all i). Furthermore, since for a given value of i, the expression $\overline{R}_i \geq R_{i,\text{tar}}$ depends only on $\widetilde{P}_{Ri}$ and r_i, for a given set of target rates for nodes $i \neq j$ and a given set of resource allocation parameters, a closed-form expression for the optimal $\widetilde{P}_{Ri}$ can be obtained (for the chosen relaying strategy). These expressions have a structure that is analogous to the corresponding expression for the case of two source nodes that was derived in the subsections above. As we will demonstrate in Section 4, problems of the form in (13) can be efficiently solved using $(N-1)$-dimensional quasiconvex search methods, in which the closed-form solution for the optimal powers given a fixed resource allocation is used at each step.

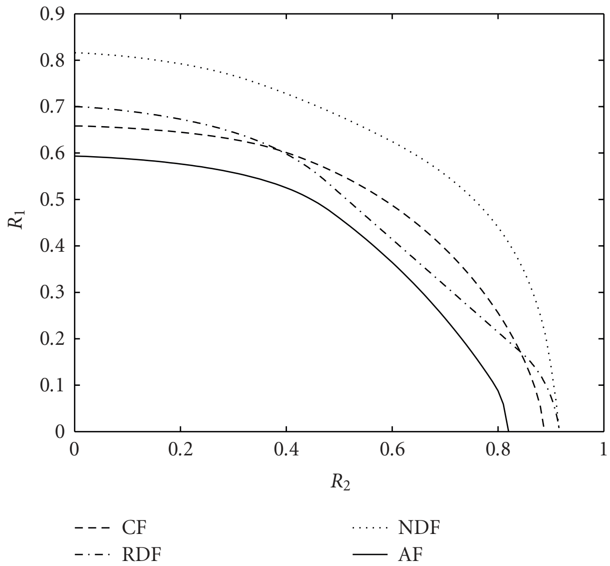


Figure 7: Achievable rate regions obtained via jointly optimal power and resource allocation in Scenario 2.

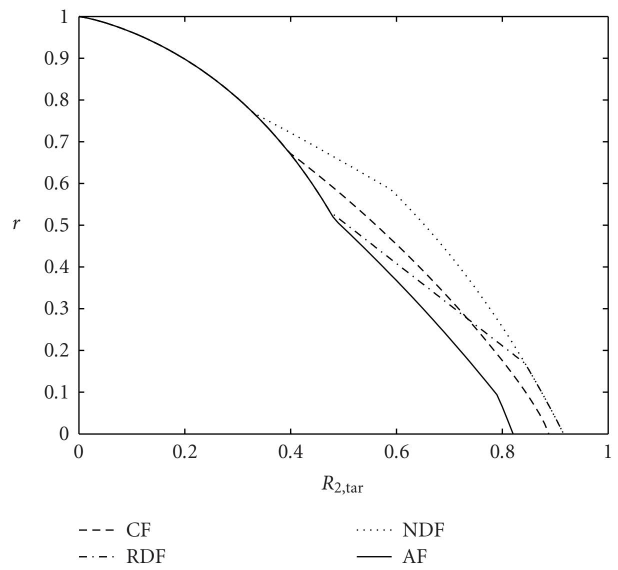


Figure 9: Resource allocation from the jointly optimal algorithm in Scenario 2.

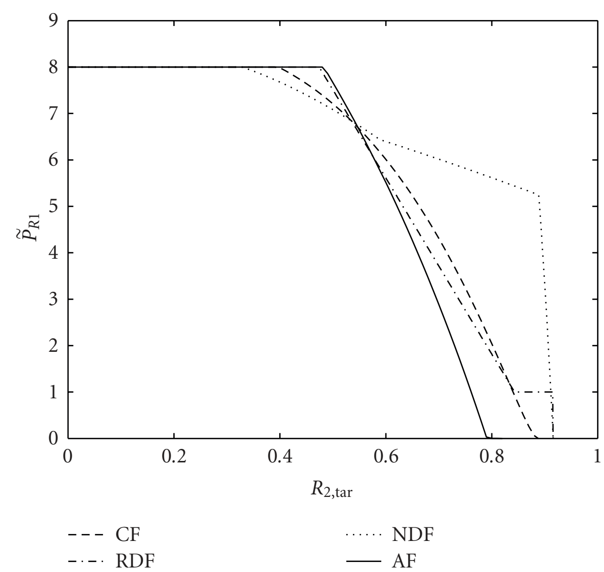


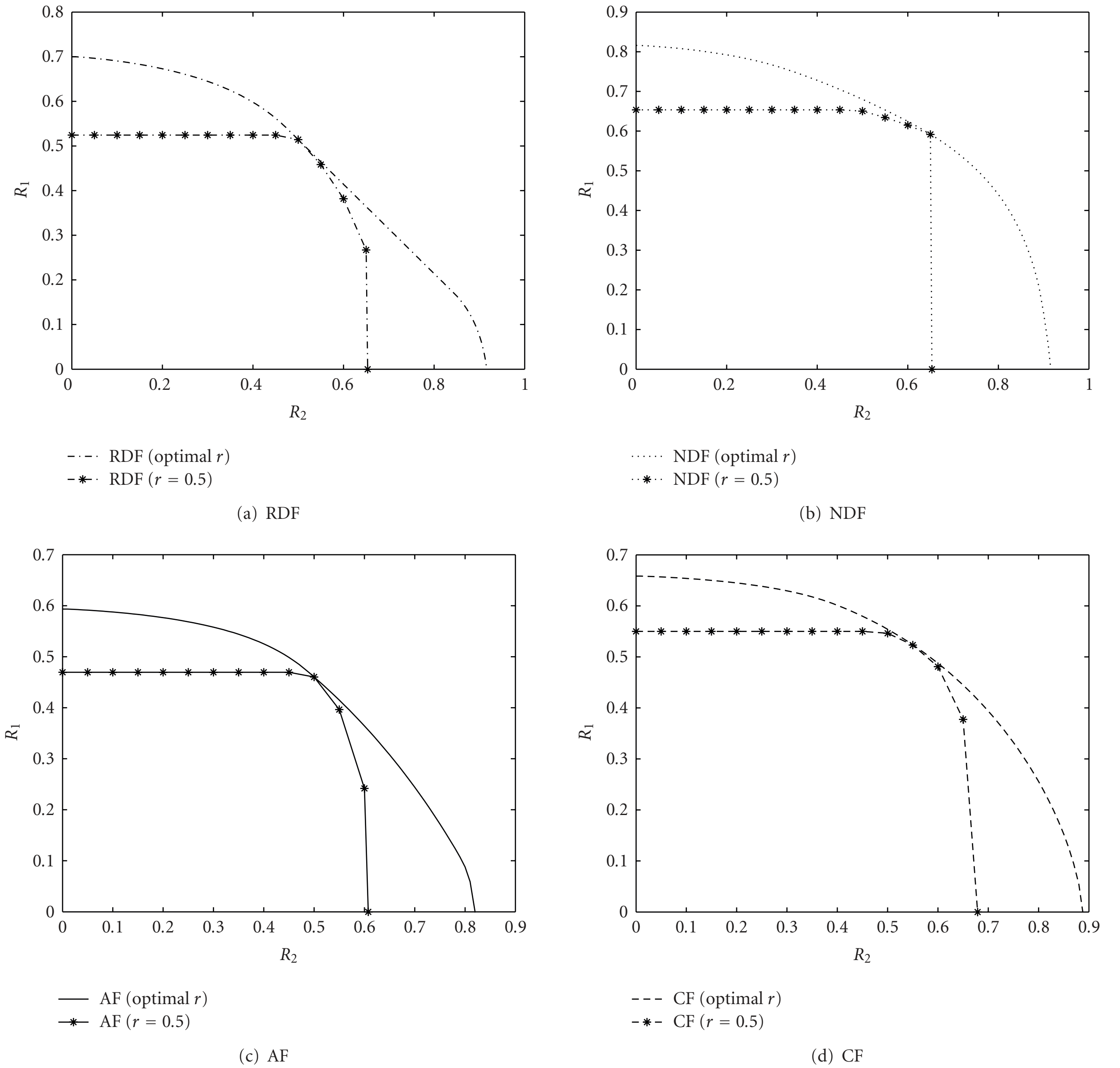
Figure 8: Powers allocated by jointly optimal algorithm in Scenario 2.

In the development above, we have considered systems in which the relay node uses the same (preassigned) relaying strategy for each node. However, since the source nodes use orthogonal channels, our results extend directly to the case of different relaying strategies, and we will provide an example of such a heterogeneous multiple access relay system in the numerical results below.

4. NUMERICAL RESULTS

In this section, we provide comparisons between the achievable rate regions obtained by different relaying strategies with the jointly optimal power and channel resource allocation derived in Section 3. We also provide comparisons between the achievable rate regions obtained with jointly optimal power and channel resource allocation and those obtained using optimal power allocation alone, with equal channel resource allocation, $r = 0.5$. We will provide comparisons for two different channel models, whose parameters are given in Table 1. Finally, we show that in some cases assigning different relaying strategies to different source nodes can result in a larger achievable rate region than assigning the same relaying strategy to all source nodes.

In Figure 3, we compare the achievable rate regions for the four relaying strategies, RDF, NDF, CF, and AF, in Scenario 1 in Table 1. In this scenario, the source-relay channel of node 1 has higher effective gain than its direct channel, whereas for node 2 the direct channel is better than the source-relay channel. Therefore, for small values of R_1 one would expect the values of R_2 that can be achieved by the CF and AF relaying strategies to be greater than those obtained by RDF and NDF, since the values of R_2 that can be achieved by RDF and NDF will be limited by the source-relay link, which is weak for node 2. Furthermore, for small values of R_2, one would expect RDF and NDF to result in higher achievable values of R_1 than CF and AF, since the source-relay link for node 1 is strong and does not represent the bottleneck in this case. Both these expected characteristics are evident in Figure 3. In Figure 4, we provide the power allocation $\widetilde{P}_{R1}$ for the four relaying strategies, and Figure 5 shows the channel resource allocation. (Note that, as expected, the optimal resource allocation is dependent on the choice of the relaying strategy.) It is interesting to observe that for the RDF strategy the relay power allocated to node 2 is zero, that is, $\widetilde{P}_{R1} = 2\overline{P}_R$ for all feasible values of $R_{2,\text{tar}}$. This solution is optimal because in Scenario 1 the achievable rate of node 2 for the RDF strategy is limited by the

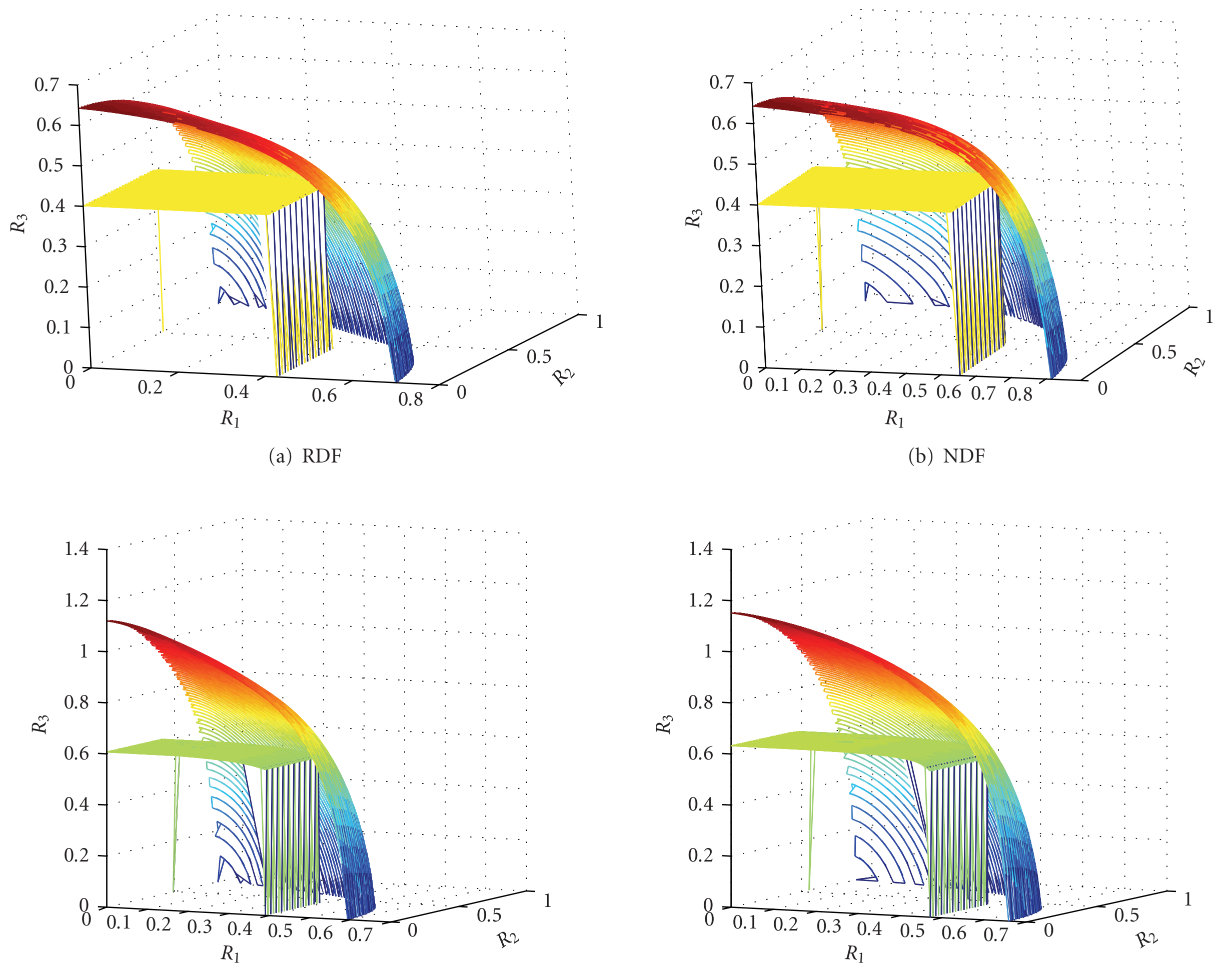


(a) RDF (b) NDF (c) AF (d) CF

Figure 10: Comparisons between the achievable rate regions obtained by jointly optimal power and resource allocation and those obtained by power allocation only with equal resource allocation, for Scenario 2.

source-relay link and there is no benefit to allocate any relay power to node 2. For the same reason, the relay power allocated to node 2 in the case of NDF relaying is also zero. However, in the case of NDF relaying, for small values of r, there is no need to use all the relay power to relay the messages of node 1, that is, $\widetilde{P}_{R1} < 2\overline{P}_R$, and it is sufficient to use only the amount of power $\widetilde{P}_{R1}$ that makes the arguments of the min function in (8) equal, that is, $\widetilde{P}_{R1} = 2\overline{P}_1(\gamma_{1R} - \gamma_{10})r/(\gamma_{R0}(r + 2\overline{P}_1\gamma_{10}))$. This can be seen in Figure 4 as the (steeply) decreasing dotted curve that represents the optimal $\widetilde{P}_{R1}$ for the case of NDF relaying. For values of R_2 in this region, the average power that the relay needs to use is strictly less than its maximum average power. We also observe from Figure 5 that the channel resource allocations for both RDF and NDF are the same. This situation arises because in both strategies the achievable rate of node 2 is limited by the achievable rate of the source-relay link. This rate has the same expression for both strategies, and hence, the same value of $\hat{r}$ will be allocated to node 2. A further observation from Figure 3 is that the achievable rate region for the CF relaying strategy is larger than that for AF and the achievable rate region for NDF is larger than that for RDF. This is consistent with the observations in [13], where the comparisons were made in terms of the expressions in (2a), (2b), (2c), and (2d) with $r = 1/2$.

To provide a quantitative comparison to the case of power allocation alone with equal resource allocation, we



(a) RDF (b) NDF (c) AF (d) CF

Figure 11: The achievable rate regions obtained by jointly optimal power and resource allocation and those obtained by power allocation alone with equal resource allocation for three-user system with $|K_{3R}| = 0.6$, $|K_{30}| = 0.9$, $\overline{P}_3 = 2$, and the remaining parameters from Scenario 2 in Table 1.

plot in Figure 6 the rate regions achieved by joint allocation and by power allocation alone for each relaying strategy. It is clear from the figure that the joint allocation results in significantly larger achievable rate regions. (The horizontal segments of the regions with $r = 0.5$ in Figure 6 arise from the allocation of all the relay power to node 1. In these cases, $R_{2,\text{tar}}$ can be achieved without the assistance of the relay, and hence all the relay power can be allocated to the message of node 1.) As expected, each of the curves for $r = 0.5$ in Figure 6 touches the corresponding curve for the jointly optimal power and channel resource allocation at one point. This point corresponds to the point at which the value $r = 0.5$ is (jointly) optimal.

In Figures 7–10, we examine the performance of the considered scheme in Scenario 2 of Table 1, in which the effective gain of the source-relay channel for node 2 is larger than that in Scenario 1, and that of the source-destination channel is smaller. As can be seen from Figure 7, increasing the gain of the source-relay channel of node 2 expands the achievable rate of the RDF and NDF strategies, even though the gain of the direct channel is reduced, whereas that change in the channel gains has resulted in the shrinkage of the achievable rate region for the CF and AF strategies. Therefore, we can see that the RDF and NDF strategies are more dependent on the quality of the source-relay channel than that of the source-destination channel (so long as the first term in the argument of the min function in (2a) and (2b) is no more than the second term), while the reverse applies to the CF and AF strategies. Figures 8 and 9 show the allocations of the relay power and the channel resource parameter, respectively. It is interesting to note that for the RDF strategy, when $R_{2,\text{tar}}$ is greater than a certain value, the relay power allocated to node 2 will be constant. The value of this constant is that which makes the two terms inside the min function on the left-hand side of the first constraint of (6) equal. This value can be calculated from the expression $\widetilde{P}_{R2} = 2(\gamma_{2R} - \gamma_{20})\overline{P}_2/\gamma_{R0}$. Figure 10 provides comparisons between the achievable rate regions obtained by the jointly optimal allocation and those obtained by optimal power allocation alone with equal resource allocation. As in

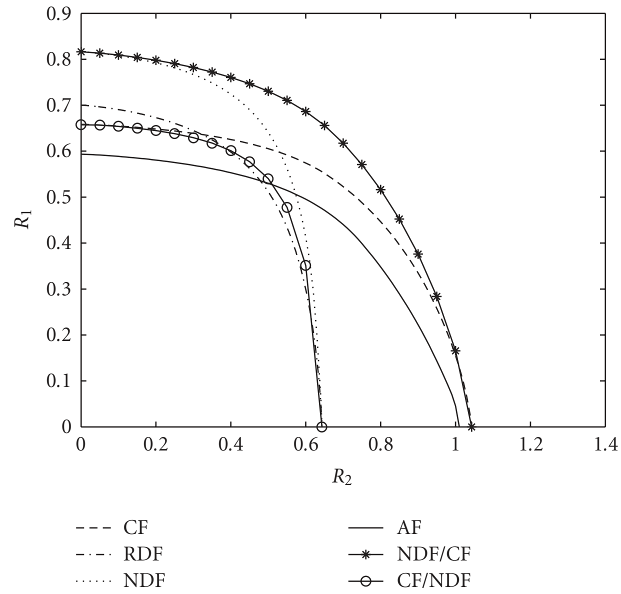


Figure 12: Comparison between the achievable rate regions when using the same relaying strategy for both users and when using different relaying strategies, for Scenario 1.

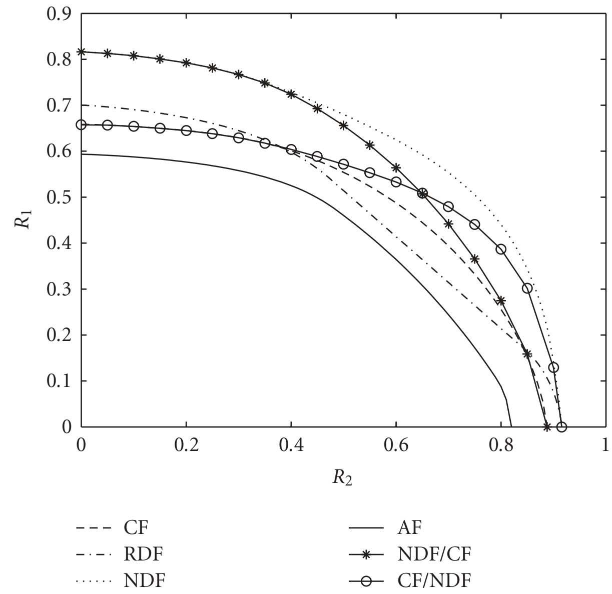


Figure 13: Comparison between the achievable rate regions when using the same relaying strategy for both users and when using different relaying strategies, for Scenario 2.

Figure 6, it is again clear that the joint allocation results in significantly larger achievable rate regions. (The horizontal segments in Figure 10 arise from all the relay power being allocated to node 1, because the corresponding values of $R_{2,\text{tar}}$ can be achieved without the assistance of the relay.)

In Figure 11, we extend this comparison to a three-user case. In order to obtain the jointly optimal power and channel resource allocations used to plot this figure, we used a two-dimensional quasiconvex search algorithm analogous to that in Algorithm 1 to solve instances of the optimization problem in (13). As in the two-user case, a substantially larger rate region can be achieved by joint allocation of the power and the channel resource. It is worth mentioning that the $R_3 = 0$ slice through the jointly optimized region is the same as that obtained in the corresponding two-user case (cf. Figure 7). This is because when $R_{3,\text{tar}} = 0$, no power and none of the channel resource will be allocated to the message of node 3. On the other hand, the $R_3 = 0$ slice through the region with fixed (and equal) resource allocation will be smaller than the corresponding region in Figure 7, because equal resource allocation in the three-user case corresponds to $r_i = 1/3$. This indicates that as the number of source nodes increases, so do the benefits of joint power and resource allocation.

In the above examples, we have considered the case in which the relay applies the same strategy to the messages of all source nodes. However, in Figures 12 and 13, we show that assigning different relaying strategies to the messages from different nodes may result in larger achievable rate regions. Figure 12 shows that in Scenario 1, if the messages of node 1 are relayed with the NDF strategy and the messages of node 2 are relayed with the CF startegy, the resulting achievable rate region will be larger than that of the homogeneous NDF and CF strategies. If the relaying strategies are reversed, it can be seen that the achievable rate region will be smaller than that of both the homogeneous NDF and CF strategies. Since the NDF achievable rate region dominates the CF achievable rate region in Scenario 2 (see Figure 7), it can be seen in Figure 13 that both combinations NDF/CF and CF/NDF provide smaller achievable rate regions than the pure NDF region. Therefore, in Scenario 2 NDF relaying for both source nodes provides the largest achievable rate region. The examples in Figures 12 and 13 suggest that one ought to jointly optimize the power allocation, the resource allocation, and the relaying strategy assigned for each node. Indeed, Figures 12 and 13 suggest that significant gains can be made by doing so. However, the direct formulation of that problem requires the joint allocation of power and the channel resource for each combination of relaying strategies, and hence the computational cost is exponential in the number of source nodes. Furthermore, as the achievable rate region of the overall system is the convex hull of the regions obtained by each combination of relaying strategies, time sharing between different combinations of relaying strategies may be required in order to maximize the achievable rate region. The approach in [18] to the design of relay networks based on orthogonal frequency division multiplexing (OFDM) offers some insight that may lead to more efficient algorithms for joint power, channel resource, and strategy allocation, but the development of such algorithms lies beyond our current scope.

5. CONCLUSION

In this paper, we have shown that the problem of jointly optimal allocation of the power and channel resource in an orthogonal multiple access relay channel is quasiconvex, and hence simple efficient algorithms can be used to obtain the optimal solution. In addition, we obtained a closed-form expression for the optimal power allocation for a given resource allocation, and we used this expression to significantly reduce the complexity of the algorithm. The numerical results obtained using the proposed algorithm show that significant rate gains can be obtained over those schemes that apply only power allocation and equal channel resource allocation. Finally, we provided an example of the joint allocation of the power, the channel resource, and the relaying strategy, and showed that this has the potential to further enlarge the achievable rate region.

APPENDICES

A. PROOF OF PROPOSITION 1

Assume that the solutions to (6) with $r = r_\alpha$ and $r = r_\beta$ are both greater than certain target rate $C_{1,\text{tar}}$. Let x_α and x_β denote the corresponding optimal values of $\widetilde{P}_{R1}$. Then, we have that

$$\frac{r}{2}\min\left\{\log\left(1+\frac{2\gamma_{1R}\overline{P}_1}{r}\right),\log\left(1+\frac{2\gamma_{10}\overline{P}_1+\gamma_{R0}P_{R1}}{r}\right)\right\} \geq C_{1,\text{tar}},$$
$$\frac{\hat{r}}{2}\min\left\{\log\left(1+\frac{\gamma_{2r}\overline{P}_2}{\hat{r}}\right),\log\left(1+\frac{2\gamma_{20}\overline{P}_2+\gamma_{R0}P_{R2}}{\hat{r}}\right)\right\} \geq C_{2,\text{tar}}, \tag{A.1}$$

for $(r = r_\alpha, \widetilde{P}_{R1} = x_\alpha)$ and $(r = r_\beta, \widetilde{P}_{R1} = x_\beta)$. The inequalities in (A.1) can be written as

$$\begin{aligned} f_1(x_\alpha) &\geq g_1(r_\alpha), & f_1(x_\beta) &\geq g_1(r_\beta), \\ f_2(x_\alpha) &\geq g_2(r_\alpha), & f_2(x_\beta) &\geq g_2(r_\beta), \end{aligned} \tag{A.2}$$

where

$$\begin{aligned} f_1(x) &= \min\{2\gamma_{1R}\overline{P}_1,\ 2\gamma_{10}\overline{P}_1+\gamma_{R0}x\}, \\ g_1(r) &= r(2^{2C_{1,\text{tar}}/r}-1), \\ f_2(x) &= \min\{2\gamma_{2r}\overline{P}_2,\ 2\gamma_{20}\overline{P}_2+\gamma_{R0}(2\overline{P}_R-x)\}, \\ g_2(r) &= \hat{r}(2^{2C_{2,\text{tar}}/\hat{r}}-1). \end{aligned} \tag{A.3}$$

Examining these functions, we observe that $f_1(x)$ and $f_2(x)$ are both concave functions. By differentiating $g_1(r)$ twice with respect to r we obtain

$$\frac{d^2g_1(r)}{dr^2} = \frac{4C_{1,\text{tar}}^2\ln(2)^2 2^{2C_{1,\text{tar}}/r}}{r^3} \geq 0, \tag{A.4}$$

and hence, $g_1(r)$ is a convex function in r. Similarly, we can show that $g_2(r)$ is convex in r.

Now, if we consider $r_\gamma = \mu r_\alpha + \hat{\mu} r_\beta$ and $x_\gamma = \mu x_\alpha + \hat{\mu} x_\beta$, where $\mu \in [0,1]$ and $\hat{\mu} = 1-\mu$, then

$$\begin{aligned} f_1(x_\gamma) &\geq_a \mu f_1(x_\alpha) + \hat{\mu} f_1(x_\beta) \\ &\geq \mu g_1(r_\alpha) + \hat{\mu} g_1(r_\beta) \\ &\geq_b g_1(r_\gamma), \end{aligned} \tag{A.5}$$

where a follows from the concavity of $f_1(x)$ and b follows from the convexity of $g_1(r)$. Similarly, it can be shown that

$$f_2(x_\gamma) \geq g_2(r_\gamma). \tag{A.6}$$

Hence, for any two values of r (namely, r_α and r_β), if there exist values of x (namely, x_α and x_β), such that the conditions in (A.1) are satisfied, then for any value of r that lies between r_α and r_β (namely, $r_\gamma = \mu x_\alpha + \hat{\mu} x_\beta$), there exists a value for x that lies between x_α and x_β (namely, $x_\gamma = \mu x_\alpha + \hat{\mu} x_\beta$) such that the conditions in (A.1) are satisfied. Therefore, the set of values of r for which the solution of the problem in (6) is greater than a certain target rate $C_{1,\text{tar}}$ is a convex set. Hence, the problem in (6) is quasiconcave in r.

B. PROOF OF PROPOSITION 2

We begin by showing that the function $(r/2)\log((1+a/r)(1+bx/r))$ is quasiconcave in the variables r and x, where a and b are nonnegative constants. To do so, we assume that the pairs (r_α, x_α) and (r_β, x_β) satisfy

$$\frac{r}{2}\log\left(\left(1+\frac{a}{r}\right)\left(1+\frac{bx}{r}\right)\right) \geq M, \tag{B.1}$$

where M is a nonnegative constant. We can write (B.1) as

$$f_0(r,x) \geq g_0(r), \tag{B.2}$$

where

$$f_0(r,x) = r + bx, \qquad g_0(r) = \frac{r^2 2^{2M/r}}{r+a}. \tag{B.3}$$

The function $f_0(r,x)$ is a linear function, while the function $g_0(r)$ can be shown to be convex function using the fact that

$$\begin{aligned} &\frac{d^2g_0(r)}{dr^2} \\ &= \left(2a^2 2^{2a/r}\left(r^2(1-\ln(2))^2 + 2ar\ln(2)(2\ln(2)-1)\right.\right. \\ &\qquad \left.\left. +\ln(2)^2(r^2+2a^2)\right)\right) \times \left((r+a)^3 r^2\right)^{-1} \\ &\geq 0. \end{aligned} \tag{B.4}$$

Now,if we consider $r_\gamma = \mu r_\alpha + \hat{\mu} r_\beta$ and $x_\gamma = \mu x_\alpha + \hat{\mu} x_\beta$, where $\mu \in [0,1]$ and $\hat{\mu} = 1-\mu$, then

$$\begin{aligned} f_0(r_\gamma, x_\gamma) &= \mu f_0(r_\alpha, x_\alpha) + \hat{\mu} f_0(r_\beta, x_\beta) \\ &\geq \mu g_0(r_\alpha) + \hat{\mu} g_0(r_\beta) \\ &\geq_a g_0(r_\gamma), \end{aligned} \tag{B.5}$$

where a follows from the convexity of $g_1(r)$. Therefore, the set of pairs (r,x) that satisfy (B.1) is a convex set, and hence the function $(r/2)\log((1+a/r)(1+bx/r))$ is quasiconcave in the variables r and x.

By obtaining its second derivative, it is straight forward to show that $(r/2)\log(1+(2\gamma_{1R}\overline{P}_1/r))$ is concave in r. Since the minimum of a concave function and a quasiconcave function is a quasiconcave function, then we can say that the function

$$\begin{aligned} \frac{r}{2}\min\Bigg\{&\log\left(1+\frac{2\gamma_{1R}\overline{P}_1}{r}\right),\ \log\left(1+\frac{2\gamma_{10}\overline{P}_1}{r}\right) \\ &+\log\left(1+\frac{\gamma_{R0}\widetilde{P}_{R1}}{r}\right)\Bigg\} \end{aligned} \tag{B.6}$$

is quasiconcave in r and P_{R1}. Similarly, the function

$$\frac{\hat{r}}{2}\min\Big\{\log\Big(1+\frac{2\gamma_{2r}\overline{P}_2}{\hat{r}}\Big),\log\Big(1+\frac{2\gamma_{20}\overline{P}_2}{\hat{r}}\Big) + \log\Big(1+\frac{\gamma_{R0}(2\overline{P}_R-\widetilde{P}_{R1})}{\hat{r}}\Big)\Big\} \tag{B.7}$$

can be shown to be quasiconcave in r and P_{R1}. Therefore, the problem in (8) is quasiconcave in r and P_{R1}. That is, the set of all pairs (r, P_{R1}) for which the solution of the problem in (8) is greater than a target rate $C_{1,\text{tar}}$, that is, the set of all pairs that satisfy

$$\begin{aligned}&\frac{r}{2}\min\Big\{\log\Big(1+\frac{2\gamma_{1R}\overline{P}_1}{r}\Big),\ \log\Big(1+\frac{2\gamma_{10}\overline{P}_1}{r}\Big) + \log\Big(1+\frac{\gamma_{R0}\widetilde{P}_{R1}}{r}\Big)\Big\}\geq C_{1,\text{tar}},\\ &\frac{\hat{r}}{2}\min\Big\{\log\Big(1+\frac{2\gamma_{2r}\overline{P}_2}{\hat{r}}\Big),\ \log\Big(1+\frac{2\gamma_{20}\overline{P}_2}{\hat{r}}\Big) + \log\Big(1+\frac{\gamma_{R0}\widetilde{P}_{R2}}{\hat{r}}\Big)\Big\}\geq C_{2,\text{tar}},\end{aligned} \tag{B.8}$$

is a convex set.

C. PROOF OF PROPOSITION 3

Consider the function

$$f(r,x)=r\log\Big(1+\frac{a}{r}+\frac{bcx}{r(r+b+cx)}\Big), \tag{C.1}$$

where a, b, and c are positive constants and $(r,x)\in(0,1)\times\mathbb{R}_{++}$. We will avoid the cases where $r = 0$ or $r = 1$ because these cases correspond to scenarios in which one of the source nodes does not transmit. In those scenarios, the problem is easy to solve because all the relay power and all the channel resource will be allocated to the transmission of the message of the other source node. We will show that $f(r,x)$ is quasiconvex using the the second-order condition for the quasiconvexity which states that [14] for any vector $\mathbf{z}$ such that $\mathbf{z}^T\nabla f = 0$, if the function f satisfies $\mathbf{z}^T\nabla^2 f\mathbf{z} < 0$, then f is quasiconcave.

For the function f, we denote the gradient by $\nabla f = [f_r,\ f_x]^T$, where $f_w = \partial f/\partial w$. Since $\nabla f \in \mathbb{R}^2$, the subspace orthogonal to ∇f will be a one-dimensional subspace. Since the vector $\mathbf{z} = [-f_x,\ f_r]^T$ is orthogonal to ∇f, then all the vectors in the subspace orthogonal to ∇f are parallel to the vector $\mathbf{z}$. Examining the quantity $\mathbf{z}^T\nabla^2 f\mathbf{z}$, we have that

$$\mathbf{z}^T\nabla^2 f\mathbf{z} = -A\frac{f(r,x)^2}{r^2}+B\frac{f(r,x)}{r}-C, \tag{C.2}$$

where A, B, and C are positive quantities that depend on the constants a, b, and c and the variables r and x. Equation (C.2) can be written as

$$\mathbf{z}^T\nabla^2 f\mathbf{z} = -\Big[\Big(\sqrt{A}\frac{f(r,x)}{r}-\sqrt{C}\Big)^2+(2\sqrt{AC}-B)\frac{f(r,x)}{r}\Big]. \tag{C.3}$$

From (C.3), it can be seen that it is sufficient that $2\sqrt{AC} > B$ for the quantity $\mathbf{z}^T\nabla^2 f\mathbf{z}$ to be negative and consequently for the function $f(r,x)$ to be quasiconcave in (r,x). Since both the quantities $2\sqrt{AC}$ and B are positive, we can examine the quantities $4AC$ and B^2. In particular, it can be shown that

$$\begin{aligned}4AC-B^2 = 4\big(&\underbrace{5abcxr^2-bcxr^3}+\underbrace{3ab^2r^2-b^2r^3}+\underbrace{abr^3-br^4}\\ &+4ab^3cx+10ab^2cxr+2b^3cxr+2a^2c^2x^2r\\ &+6a^2b^2r+6a^2br^2+2a^2r^3+2a^2b^3\\ &+4bac^2x^2r+4ab^2c^2x^2+b^2r^2cx+2a^2bc^2x^2\\ &+4a^2r^2cx+8a^2bcxr+4a^2b^2cx+2arb^3\\ &+2b^2c^2x^2r+2b^3c^2x^2\big)\times r^2b^3c^4(b+r)^2\\ &\times(rb+rcx+r^2+ab+acx+ar+bcx)^{-5}\\ &\times(b+cx+r)^{-4}.\end{aligned} \tag{C.4}$$

The underbraced terms in (C.4) contain the negative terms in (C.4), each paired with a corresponding positive term. It can be seen that if $a \geq r$, then each of these underbraced terms is nonnegative. Therefore, $a \geq 1$ is a sufficient condition for $4AC > B^2$, and hence is a sufficient condition for the function $f(r,x)$ to be quasiconcave.

By making the substitutions $a = 2\gamma_{10}\overline{P}_1$, $b = 2\gamma_{1R}\overline{P}_1$, $c = \gamma_{R0}$, and $x = P_{R1}$, the sufficient condition becomes $2\gamma_{10}\overline{P}_1 \geq 1$, that is, if the maximum achievable SNR of the direct channel of node 1 is at least -3 dB, then the objective function in (9) is quasiconcave in (r, P_{R1}). Similarly, we can obtain that $2\gamma_{20}\overline{P}_2 \geq 1$ is a sufficient condition for the function on the left hand side of the first constraint in (9) to be quasiconcave in $(\hat{r}, P_{R2})$. Therefore, the problem in (9) is quasiconcave in (r, P_{R1}) if the maximum achievable SNR of the direct channel of both nodes is at least -3 dB.

D. PROOF OF PROPOSITION 4

Following a similar proof to that in Appendix C, consider the function

$$f(r,x)=r\log\Big(1+\frac{a}{r}+\frac{bc(a+r)x}{r(r^2+(a+b)r+c(a+r)x)}\Big), \tag{D.1}$$

where a, b, and c are positive constants and $(r,x)\in(0,1)\times\mathbb{R}_{++}$. Define $\mathbf{z}$ to be the vector orthogonal to the gradient subspace of the function $f(r,x)$, that is, $\mathbf{z} = [-f_x,\ f_r]^T$. Examining the quantity $\mathbf{z}^T\nabla^2 f\mathbf{z}$, we have that

$$\begin{aligned}\mathbf{z}^T\nabla^2 f\mathbf{z} &= -\frac{A}{r^2}f(r,x)^2+\frac{B}{r}f(r,x)-C,\\ &= -\Big[\Big(\sqrt{A}\frac{f(r,x)}{r}-\sqrt{C}\Big)^2+(2\sqrt{AC}-B)\frac{f(r,x)}{r}\Big],\end{aligned} \tag{D.2}$$

where of course A, B, and C are different positive functions of a, b, c, r, and x than those in Appendix C.

From (D.2), it can be seen that it is sufficient that $2\sqrt{AC} > B$ for $\mathbf{z}^T\nabla^2 f\mathbf{z}$ to be negative, and consequently for the function $f(r,x)$ to be quasiconcave in (r,x). Since both $2\sqrt{AC}$ and B are positive, we can examine

$$\begin{aligned}
4AC - B^2 &= 4b^3c^4r^5\big(\underbrace{20ba^3r^4 - 3bar^6} + \underbrace{18b^2a^3r^3 - 2b^2ar^5} \\
&\quad + \underbrace{20bcxa^2r^4 - bcxr^6} + \underbrace{8b^2a^2r^4 - b^2r^6} \\
&\quad + \underbrace{7ba^5r^2 - br^7} + 12a^2r^2b^2c^2x^2 + 2r^5cxb^2 \\
&\quad + 2r^4c^2x^2b^2 + 4ar^4bc^2x^2 + 2a^2r^4c^2x^2 \\
&\quad + 2r^4cxb^3 + 10ar^4b^2cx + 2a^2r^6 \\
&\quad + 24a^2r^3b^2cx + 8ar^3b^2c^2x^2 + 11a^5bcxr \\
&\quad + 10a^4xcrb^2 + 8a^5r^3 + 2a^6r^2 \\
&\quad + 16a^2r^3bc^2x^2 + 3a^3xcrb^3 + 37a^4r^2cxb \\
&\quad + 16a^4x^2c^2rb + 9b^2r^2a^4 + 5a^2xcr^2b^3 \\
&\quad + 2a^6x^2c^2 + a^2r^2b^4 + 8a^3x^2c^2rb^2 \\
&\quad + 26a^3b^2cxr^2 + 4a^2br^5 + 4a^2r^5cx \\
&\quad + 24a^3br^2c^2x^2 + 44a^3br^3cx + 8x^2c^2a^3r^3 \\
&\quad + 12a^4r^2c^2x^2 + 24a^4r^3cx + 5b^3r^3a^2 \\
&\quad + ar^5bcx + 16a^3r^4cx + 12a^4r^4 \\
&\quad + 4ar^3b^3cx + 8a^3r^5 + 2a^4x^2c^2b^2 \\
&\quad + 4a^6rcx + 8a^5c^2x^2r + 4a^5x^2c^2b \\
&\quad + 16a^5r^2cx + 5b^3r^2a^3 + 21a^4r^3b\big) \\
&\quad \times \left(2ar + r^2 + a^2 + ab + rb\right)^{-2}(cx + r)^{-4} \\
&\quad \times \left(acx + rcx + ar + rb + r^2\right)^{-5}.
\end{aligned} \tag{D.3}$$

The underbraced terms of (D.3) contain the negative terms in (D.3), each paired with a corresponding positive term. It can be seen that if $a \geq r$, each of these underbraced terms is nonnegative. Therefore, $a \geq 1$ is a sufficient condition for $4AC > B^2$, and hence for the function $f(r,x)$ to be quasiconcave.

Making the substitutions $a = 2\gamma_{10}\overline{P}_1$, $b = 2\gamma_{1R}\overline{P}_1$, $c = \gamma_{R0}$, and $x = P_{R1}$, the sufficient condition becomes $2\gamma_{10}\overline{P}_1 \geq 1$. That is, if the maximum achievable SNR of the direct channel of node 1 is at least -3 dB, then the objective function in (11) is quasiconcave in (r, P_{R1}). Similarly, we can obtain that $2\gamma_{20}\overline{P}_2 \geq 1$ is a sufficient condition for the function on the left hand side of the first constraint in (11) to be quasiconcave in $(\hat{r}, P_{R2})$. Therefore, the problem in (11) is quasiconcave in (r, P_{R1}) if the maximum achievable SNR of the direct channel of both nodes is at least -3 dB.

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