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# Power characteristics of single-mode semiconductor lasers

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The basic aspects of power calculations for high-gain semiconductor lasers are briefly reviewed, and a straightforward one-dimensional model is described. The relative importance of spontaneous emission, distributed losses, band-to-band absorption, and high single-pass gain are investigated in detail.

## I. INTRODUCTION

One of the basic and easily measured properties of a laser oscillator is the total output power. Accordingly, it would be especially useful if analytic models were available for calculating laser power, at least for those lasers of greatest practical interest. In fact, many such models have been developed, and for most lasers, power characteristics can be predicted with confidence. Semiconductor lasers have proven to be among the most difficult to model, and many aspects of diode laser performance are still only poorly understood. On the other hand, these same lasers are among the most important in terms of practical applications, and successful models might have a substantial economic value. The purpose of this study is to examine in some detail the power characteristics predicted by relatively simple, diode laser models.

Laser action in semiconductors was first reported by Hall *et al.*,[1] Nathan *et al.*,[2] Holonyak and Bevacqua,[3] and Quist *et al.*,[4] in late 1962. It was not long afterward that the unique difficulties of modeling semiconductor laser characteristics began to be fully appreciated. One particular difficulty concerns the very high gain of the laser medium and the low reflectivities of typical cleaved laser reflectors. The power characteristics of diode lasers including the effects of large single-pass gain were first investigated by Scott.[5–7] Those studies also included the possibility of longitudinal spatial hole burning due to standing wave effects in the regions of the laser where the right and left traveling waves are of approximately equal amplitude. Similar calculations were carried out by Rigrod, excluding any standing wave effects.[8] In these studies, neither Scott nor Rigrod included the possibility of distributed losses or spontaneous emission input. The strong $z$ dependence of the fields in a high-gain semiconductor laser was also demonstrated experimentally by Ulbrich and Pilkuhn.[9]

The specific topic of longitudinal spatial hole burning in low-gain semiconductor lasers was studied by Statz *et al.* in connection with multimode oscillation.[10] However, Danielmeyer argued that under most circumstances carrier diffusion would effectively eliminate longitudinal hole burning in semiconductors.[11] More recently, Streifer *et al.* have suggested again that hole burning is a signficant factor in determining a laser's mode structure,[12] but other authors consider this effect to be actually or potentially unimportant.[13–15] For simplicity and consistency with most previous work, longitudinal spatial hole burning is not included in the following analysis.

The possibility of analytically including distributed losses as well as high gain in calculating a laser output was first discussed in detail in a later paper by Rigrod,[16] and for extremely high-power levels by Whiteaway and Thompson.[17] In principle at least, such results could be important for diode lasers, where the level of distributed losses is always very high. The exact solutions obtained by Rigrod are in the form of implicit transcendental equations, and a more extensive discussion of those solutions has been given by Schindler.[18]

Spontaneous emission is an effect that influences very strongly the power curves of diode lasers. Compared to most other laser systems, the amount of spontaneous emission noise which enters a diode laser mode is always very high. Analytic expressions for the power characteristics of noisy low-gain single-mode lasers were derived by Haug[19] and by Siegman.[20] The corresponding multimode results were obtained by Casperson and compared with diode laser data.[21] Approximate solutions of the same equations have been discussed by Suematsu *et al.*, and detailed experimental confirmation has been obtained.[22] The implication of those studies is that spontaneous emission is extremely important in diode lasers operating below threshold or in the threshold transition region.

It would be useful to include all of the effects mentioned above in a single model of semiconductor laser oscillation, and obviously there are many additional effects that might be included as well. However, if one is not a bit selective in developing a laser model, the analysis quickly becomes intractable. One generalization of the previously mentioned results has been given by Hunter and Hunter.[23] Those authors have analyzed a one-dimensional single-mode laser including high gain, distributed losses, and a spontaneous emission input, and the results are again in the form of implicit transcendental equations. However, since band-to-band absorption is not included, diode laser behavior can not be directly interpreted. A recent analysis by Streifer *et al.* has been oriented specifically to semiconductor lasers and has included high-gain, band-to-band absorption, spontaneous emission, and multimode effects but excluding saturation.[24] Very recently, Cassidy has carried out a semianalytical comparison of diode laser models assuming high and low single-pass gain but excluding saturable band-to-band absorption.[25] Other authors have chosen to place their laser models directly on a computer without any analytical simplifications.[26] This procedure has the obvious advantage that in principle any model that can be formulated can be solved. On the other hand, the solutions may be very slow and costly while providing relatively little insight into the underlying physical phenomena. The study presented here has been carried out in the belief that there is still value in formulations which, at least in part, can be solved analytically.

At the outset, it is appropriate to comment on the general idea of developing a single-mode model for use with semiconductor diode lasers. All laser models inevitably involve some approximations (often well obscured), and, in the present case, the principal limitation may be found to be our restriction to a single-cavity mode. Because of this restriction, some quantitative discrepancies can occur when the models are applied to multimode devices below or around threshold. However, most useful diode lasers are effectively single mode when operated above threshold. Also, some newer devices use very short cavities or frequency-selective filters which make the single-mode model appropriate for all operating conditions. Furthermore, as shown below, the diode equations are simply related to the equations for other laser types, so the results that we obtain are also applicable to nondiode single-mode laser systems.

In Sec. II, a diode laser formalism is developed including the effects of spontaneous emission, band-to-band absorption, and a large value of the single-pass gain. Some analytic solutions are possible, and exact numerical results are given in Sec. III. For several limiting cases, the equations can be solved exactly, and a discussion of such limits is included in Sec. IV. An approximate explicit formula for power output above threshold in high-gain lasers is described in Appendix A, and the results can be expressed in terms of measurable quantities using the formulas found in Appendix B. For many purposes, the simpler approximations provide an adequate description of diode laser performance.

## II. THEORY

The basic rate equations governing the populations and the intensity in many types of single-mode laser can be written:

$$\frac{\partial n_2}{\partial t} = S_2 - A_2 n_2 - B(n_2 - n_1)(I^+ + I^-), \tag{1}$$

$$\frac{\partial n_1}{\partial t} = S_1 + A_{21} n_2 - A_1 n_1 + B(n_2 - n_1)(I^+ + I^-), \tag{2}$$

and

$$\frac{n}{c}\frac{\partial I^\pm}{\partial t} \pm \frac{\partial I^\pm}{\partial z} = h\nu B(n_2 - n_1)I^\pm - \alpha I^\pm + h\nu A_{21} C n_2, \tag{3}$$

where $n_2$ and $n_1$ are the population densities of the upper and lower states of the laser transition, $S_2$ and $S_1$ are the corresponding pump rates, $A_2$ and $A_1$ are the corresponding total

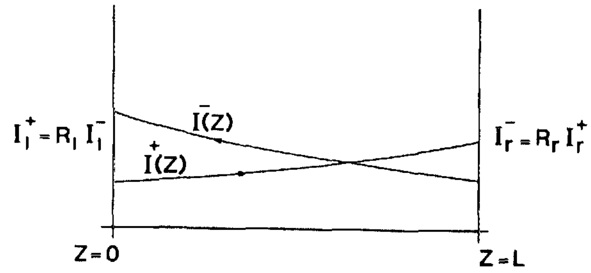


FIG. 1. Schematic distribution of light intensity in the longitudinal direction for a laser with mirror reflectivities $R_l$ and $R_r$.

Einstein $A$ coefficients for spontaneous decay, and $A_{21}$ represents that part of the spontaneous decay that goes directly from level 2 to level 1. The intensities of the right and left traveling waves are represented by $I^+$ and $I^-$, respectively, $B$ is an effective Einstein $B$ coefficient for the transition, $C$ indicates the fraction of the total spontaneous emission that is added to the lasing mode, $n$ is the index of refraction, and $\alpha$ is the internal absorption loss coefficient. The distribution of light intensities in a laser with the mirror reflectivities $R_l$ and $R_r$ is shown schematically in Fig. 1.

GaAs diode lasers are, for many purposes, two level systems, and with slight modification Eqs. (1)–(3) can be used to represent most of their important saturation characteristics. Thus, in the spontaneous emission terms, $n_2$ might be replaced by $n_c$, the excess above the thermal equilibrium concentration of electrons in the conduction band, and $n_1$ can become $n_v$, the excess of electrons in the valence band. In the stimulated emission terms, $n_2$ and $n_1$ are replaced by total concentrations of electrons in the corresponding bands. Then, with some obvious simplifications for a two-level system, Eqs. (1)–(3) become

$$\frac{\partial n_c}{\partial t} = S_c - A n_c - B(n_c - n_v + \Delta n_0)(I^+ + I^-), \tag{4}$$

$$\frac{\partial n_v}{\partial t} = S_v + A n_c + B(n_c - n_v + \Delta n_0)(I^+ + I^-), \tag{5}$$

and

$$\frac{n}{c}\frac{\partial I^\pm}{\partial t} \pm \frac{\partial I^\pm}{\partial z} = h\nu B(n_c - n_v + \Delta n_0)I^\pm - \alpha I^\pm + h\nu A C n_c, \tag{6}$$

where $\Delta n_0$ is the thermal equilibrium concentration difference ($\Delta n_0 < 0$). If one assumes in addition that charge neutrality is maintained ($n_c + n_v = 0$), only two equations are necessary, and these can be written

$$\frac{\partial n_c}{\partial t} = S - A n_c - B(2n_c + \Delta n_0)(I^+ + I^-) \tag{7}$$

and

$$\frac{n}{c}\frac{\partial I^\pm}{\partial t} \pm \frac{\partial I^\pm}{\partial z} = h\nu B(2n_c + \Delta n_0)I^\pm - \alpha I^\pm + h\nu A C n_c. \tag{8}$$

In one form or another, these equations have been the starting point for most analytical treatments of diode laser power characteristics.

It is convenient to introduce the net intensity gain coefficient $g = h\nu B(2n_c + \Delta n_0)$. With this definition, the steady-state solution of Eq. (7) is

$$g = \frac{2h\nu BS/A + h\nu B\Delta n_0}{1 + (2B/A)(I^+ + I^-)} = \frac{g_0 - g'}{1 + x^+ + x^-}, \tag{9}$$

where we have introduced the unsaturated gain $g_0 = 2h\nu BS/A$, the unsaturated band-to-band absorption $g' = -h\nu B\Delta n_0$, and the normalized intensities $x^\pm = (2B/A)I^\pm$. With these definitions, Eq. (8) can be written

$$\pm\frac{dx^\pm}{dz} = (g - \alpha)x^\pm + (g + g')C. \tag{10}$$

Equations (9) and (10) are a complete set which, in principle, can be integrated subject to the boundary conditions imposed by the resonator mirrors. First, it is helpful to introduce the new dependent variables $u^\pm = x^\pm + C$ so that Eqs. (9) and (10) reduce to

$$g = \frac{g_0 - g'}{1 - 2C + u^+ + u^-} \tag{11}$$

and

$$\pm\frac{du^\pm}{dz} = (g - \alpha)u^\pm + (\alpha + g')C. \tag{12}$$

Next, it is useful to introduce the normalized intensity sum and difference $v^\pm = u^+ \pm u^-$, and now Eqs. (11) and (12) can be combined and written

$$g = \frac{g_0 - g'}{1 - 2C + v^+}, \tag{13}$$

$$\frac{dv^+}{dz} = (g - \alpha)v^-, \tag{14}$$

and

$$\frac{dv^-}{dz} = (g - \alpha)v^+ + 2(\alpha + g')C. \tag{15}$$

If Eq. (15) is divided by Eq. (14) and combined with Eq. (13), one obtains the single equation

$$v^- dv^- = \left(v^+ + \frac{2(\alpha + g')C(1 - 2C + v^+)}{g_0 - g' - \alpha(1 - 2C + v^+)}\right) dv^+. \tag{16}$$

This equation may be readily integrated, and the result is

$$\frac{v^{-2}}{2} = \frac{v^{+2}}{2} - \frac{2(\alpha + g')C}{\alpha} \times \left[v^+ + \frac{g_0 - g'}{\alpha}\ln\left(\frac{g_0 - g'}{\alpha} - (1 - 2C + v^+)\right)\right] + a, \tag{17}$$

where $a$ is an integration constant. Equation (17) is a relationship between the intensities of the right and left traveling waves, which is valid for all values of position $z$ in the laser. The constant can be expressed in terms of the intensity at one end of the laser by means of the boundary conditions at the laser mirrors. These conditions can be written

$$x_r^- = R_r x_r^+ \tag{18}$$

and

$$x_l^+ = R_l x_l^- \tag{19}$$

where the subscripts $l$ and $r$ refer to the left and right ends of the laser, respectively, and the $R$'s refer to the corresponding mirror reflectivities. For example, in terms of $x_l^+$, the integration constant is

$$a = -2(x_l^+ + C)\left(\frac{x_l^+}{R_l + C}\right) + \frac{2C(\alpha + g')}{\alpha}\left(x_l^+\left(1 + \frac{1}{R_l}\right) + 2C + \frac{g_0 - g'}{\alpha}\ln\left\{\frac{g_0 - g'}{\alpha} - \left[1 + x_l^+\left(1 + \frac{1}{R_l}\right)\right]\right\}\right). \tag{20}$$

Alternatively, with the replacements $x_l^+ \to x_r^+$ and $R_l \to 1/R_r$, the constant can be written in terms of $x_r^+$. Furthermore, since $a$ can be expressed in terms of either $x_l^+$ or $x_r^+$, equating the two expressions yields an algebraic relationship between $x_l^+$ and $x_r^+$. Finally, the actual intensity at any point in the laser can be obtained by using Eq. (17) to eliminate $v^-$ from Eq. (14). This yields the first-order differential equation:

$$\frac{dv^+}{dz} = \left(\frac{g_0 - g'}{1 - 2C + v^+} - \alpha\right)\left\{v^{+2} - \frac{4(\alpha + g')C}{\alpha}\left[v^+ + \frac{g_0 - g'}{\alpha}\ln\left(\frac{g_0 - g'}{\alpha} - 1 + 2C - v^+\right)\right] + 2a\right\}^{1/2}, \tag{21}$$

where $a$ is given by Eq. (20). This equation can be integrated numerically over the resonator length $L$ subject again to the boundary conditions. Equation (21) can also be integrated analytically in several limiting cases: (i) at high power levels above the lasing threshold where the spontaneous emission can be neglected ($C \simeq 0$); (ii) at low power levels below the lasing threshold ($v^+ \simeq C \ll 1$); and (iii) for $\alpha = 0$ as is the case in many gas lasers. In any case, integration of Eq. (21) yields yet another relationship (either numerical or analytical) between intensities at the ends of the laser, and a combination of the two relationships can be solved for, say, $x_r^+$.

## III. GENERAL RESULTS

As indicated above, general numerical solutions of the laser equations can be readily obtained. To simplify the presentation, it is helpful to introduce a normalized threshold parameter in place of the unsaturated gain, and the conventional definition is

$$r = g_0/g_{th} \tag{22}$$

where the threshold gain $g_{th}$ can be related to the unsaturating internal absorption, the band-to-band absorption, and the mirror losses by

$$g_{th} = g' + \alpha - \ln(R_l R_r)/2L. \tag{23}$$

Next, it is necessary to specify some typical numerical values of the various parameters of GaAs(Al) diode lasers. As an example, we assume a laser length of 300 $\mu$m and mirror reflectivities of $R_l = R_r = 0.3$. For a length of 300 $\mu$m, Petermann calculates the spontaneous emission factor to be in the range $10^{-5}$ to $10^{-4}$, depending on the laser astigmatism.[27] Estimates for $\alpha$ vary from 15 $cm^{-1}$ up to 100 $cm^{-1}$, depending on the laser structure and the material quality,[28,29] with typical values in the range of 35–45 $cm^{-1}$.[12,30] We have adopted the value $\alpha = 40$ $cm^{-1}$ for our initial calculations. Stern has calculated the spectral dependence of the band-to-band absorption coefficient obtaining the result $g' = 190$ $cm^{-1}$ for GaAs.[31] Other authors have used 150,[12] 190,[32] 217 $cm^{-1}$,[33] and other values. We consider here only the fundamental transverse mode, for which the confinement factor $\Gamma$ of optical power in the active region ranges between 0.1 and 0.6. The precise value of $\Gamma$ depends on the aluminum content in the adjacent confining layers and on geometrical parameters such as, e.g., the active layer thickness. With a confinement factor of $\Gamma = 0.2$ the effective value of the band-to-band absorption coefficient would be reduced from about 190 $cm^{-1}$ to $g' = 38$ $cm^{-1}$, and this is a reasonable value for CDH LOC lasers with an active layer thickness of about 0.1 $\mu$m.[28,32].

Numerical solutions of the equations given above can be readily obtained using a Runge–Kutta method for the $z$ integrations with an iterative technique to match the boundary conditions at the resonator mirrors. Typical results are shown in Fig. 2 using the parameter values mentioned above

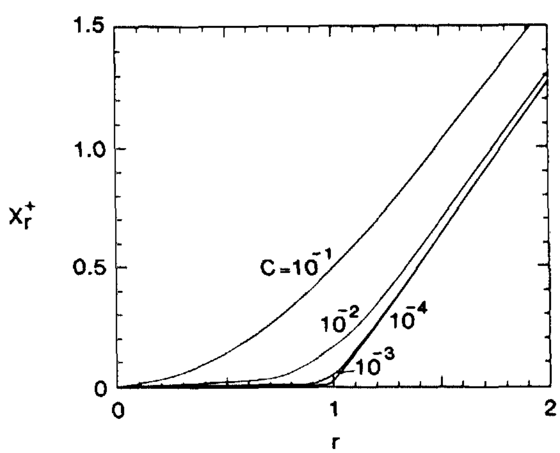


FIG. 2. General curves of normalized laser intensity incident on the right-hand laser facet $x_r$ as a function of the threshold parameter $r$ for various values of the noise parameter $C$. The other values used in this figure include the mirror reflectivities $R_l = 0.3$ and $R_r = 0.3$, nonsaturating distributed intensity loss rate $\alpha = 40$ $cm^{-1}$, band-to-band absorption coefficient $g' = 38$ $cm^{-1}$, and length $L = 300\,\mu$m.

with a series of values for the spontaneous emission parameter $C$. These results have the general form that is commonly observed in threshold data for all types of lasers. The lowest curve corresponds to a noise value $C = 10^{-4}$ and is almost indistinguishable on this scale from the results that would be obtained in the limit $C = 0$. A comparison with experimental data for semiconductor lasers suggests that, if all other parameter values are about right, the noise parameter is perhaps somewhat larger than the values mentioned above. This is due, in part, to the enhancement factor for narrow-stripe gain-guided lasers, and this factor can be as large as a factor of 10.[34] Multimode effects can also influence the threshold transition.[21,22]

## IV. SPECIAL CASES

Even though the laser equations that have been developed in the previous sections can be integrated numerically without approximations, it is still worthwhile to consider certain limiting cases and simplifications. These limits are generally much easier to evaluate and yield additional insight into the relative importance of the various laser parameters. In some cases, quantitatively useful results can be obtained without the need for complex numerical calculations.

### A. Below threshold

In one well-known limit, the laser is considered to always remain below threshold, so that the intensity is too weak to saturate the amplifying medium. While this limit can be obtained directly from Eq. (21), it is actually simpler to start from the unsaturated version of Eq. (10).

$$\pm \frac{dx^\pm}{dz} = (g_0 - g' - \alpha)x^\pm + g_0 C$$
$$= \bar{g} x^\pm + g_0 C, \tag{24}$$

where $\bar{g}$ is the net effective gain. This equation can be integrated for the right and left propagating intensities, and the results are

$$x^+ = \frac{g_0 C}{\bar{g}}[\exp(\bar{g}z) - 1] + x_l^+ \exp(\bar{g}z) \tag{25}$$

and

$$x^- = \frac{g_0 C}{\bar{g}}[\exp(\bar{g}z) - 1] + x_r^- \exp(\bar{g}z). \tag{26}$$

With the boundary conditions given in Eqs. (18) and (19), these equations may be solved to obtain, for example,

$$x_r^+ = \frac{C g_0 L}{\bar{g}} \frac{[1 - \exp(\bar{g}L)][1 + R_l \exp(\bar{g}L)]}{R_l R_r \exp(2\bar{g}L) - 1}. \tag{27}$$

Evidently, in this limit the light output is directly proportional to the spontaneous emission coefficient $C$.

Equation (27) is plotted in Fig. 3 together with the corresponding exact solution of the laser equations. As one should anticipate, the approximation is excellent for operation well below threshold but it diverges to infinity as the threshold is approached. The numerical coefficients used in these plots are the same as those used in Fig. 2.

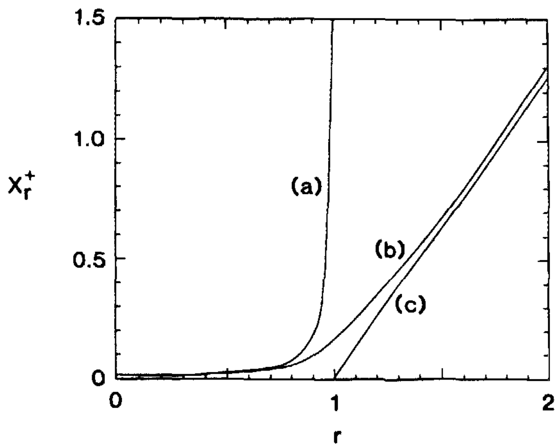


FIG. 3. Comparison of the simple (a) below-threshold and (c) above-threshold limits with the (b) general solution of the laser equations. All parameter values are the same as in Fig. 2, except that only a noise value of $C = 10^{-2}$ has been used.

### B. Above threshold

When a laser operates above threshold, the spontaneous emission has relatively little effect on the output power. Mathematically, this fact is represented by the condition $x^{\pm} \gg C$. Thus, for operation well above threshold, Eq. (17) reduces to the relationship

$$v^{-2} = v^{+2} + 2a, \tag{28}$$

and in terms of the normalized intensities this is

$$x^+ x^- = -a/2. \tag{29}$$

From Eq. (20), the integration constant can be written

$$a = -2x_l^{+2}/R_l. \tag{30}$$

Also, in the limit of small $C$, Eq. (21) reduces to

$$\frac{dv^+}{dz} = \left(\frac{g_0 - g'}{1 + v^+} - \alpha\right)(v^{+2} + 2a)^{1/2}. \tag{31}$$

Using $v^+ = x^+ + x^-$ with Eq. (29), Eq. (31) becomes

$$\frac{dx^+}{dz} = x^+\left(\frac{x^+(g_0 - g' - \alpha) - \alpha(x^{+2} - a/2)}{x^{+2} + x^+ - a/2}\right), \tag{32}$$

and if $g'$ were equal to zero, this would be the same as Eq. (2) of Rigrod.[8] Integrating Eq. (32) and using Eq. (30) in the resulting equation to eliminate $a$ yields the transcendental equation for $x_r^+$

$$\alpha L - \ln(R_l R_r)^{1/2} = \frac{g_0 - g'}{\Delta^{1/2}} \ln\left(\frac{F(x_r^+)}{F(x_l^+)}\right), \tag{33}$$

where

$$\Delta = (g_0 - g' - \alpha)^2 - 4\alpha^2 R_r x_r^{+2} \tag{34}$$

and

$$F(x_i^+) = \frac{g_0 - g' - \alpha - 2\alpha x_i^+ - \Delta^{1/2}}{g_0 - g' - \alpha - 2\alpha x_i^+ + \Delta^{1/2}}. \tag{35}$$

Equations (33)–(35) can be evaluated by any of several iterative techniques. A typical plot of these results is given in Fig. 3 using the numerical coefficients obtained previously. This plot is in reasonable agreement with the exact solution, and the agreement would be still better with more realistic (smaller) values of the noise parameter $C$.

Although straightforward in their derivation, the preceding equations only determine the intensity implicitly. It would be very useful if explicit formulas could be obtained. In high-gain lasers above threshold, the distributed losses represented by the parameter $\alpha$ are less than the gain from stimulated emission. In this event, approximate explicit formulas for the intensity can be obtained by appropriately expanding Eq. (33). This possibility is discussed in Appendix A and the resulting formula [Eq. (A2) with the parameter $G$ given by Eq. (A7)] is plotted in Fig. 6. Notice that (i) to derive Eq. (A2), no assumption about the homogeneity[16] of the saturation was made; (ii) the final formula (A2) reduces to its correct limit set by the absorption losses as the laser is lengthened; and (iii) the plot is in very good agreement with Fig. 2 which was prepared by solving the transcendental Eq. (33).

### C. Negligible distributed losses

The general equations governing laser oscillation also simplify substantially for lasers in which distributed losses can be neglected entirely ($\alpha \ll |g|$). In this limit, Eq. (17) reduces to

$$\frac{v^{-2}}{2} = \frac{v^{+2}}{2} + \frac{2g'C}{g_0 - g'}\left[(1 - 2C)v^+ + \frac{v^{+2}}{2}\right] + a', \tag{36}$$

where $a' = a + b$ and where $b$ is independent of $v^{\pm}$. Using Eq. (36) and boundary conditions at the laser mirrors, the integration constant can be written in terms of the intensity at one end of the laser (e.g., $v_l^{\pm}$) as

$$a' = \frac{v_l^{-2}}{2} - \frac{pv_l^{+2}}{2} - \frac{qv_l^+}{2}. \tag{37}$$

Also, the end intensities can be related by the algebraic expression

$$v_l^{-2} - pv_l^{+2} - qv_l^+ = v_r^{-2} - pv_r^{+2} - qv_r^+, \tag{38}$$

where $p$ and $q$ are given by

$$p = 1 + \frac{2g'C}{g_0 - g'} \tag{39}$$

and

$$q = \frac{4g'C(1 - 2C)}{g_0 - g'}. \tag{40}$$

Finally, if Eq. (36) is used to eliminate $v^-$, Eq. (14) for the $z$ dependence of the intensity is

$$\frac{dv^+}{dz} = \frac{g_0 - g'}{1 - 2C + v^+}(pv^{+2} + qv^+ + 2a')^{1/2}. \tag{41}$$

Equation (41) can be integrated analytically. Using Eq. (37) and a similar equation for $v_r^{\pm}$ in the resulting equation yields

$$\frac{p(g_0 - g')L + v_l^- - v_r^-}{1 - 2C}p^{1/2} = \ln\frac{2p^{1/2}v_r^- + 2pv_r^+ + q}{2p^{1/2}v_l^- + 2pv_l^+ + q}. \tag{42}$$

Equation (42) is a transcendental equation relating the intensities at the ends of the laser. When combined with Eq. (38)

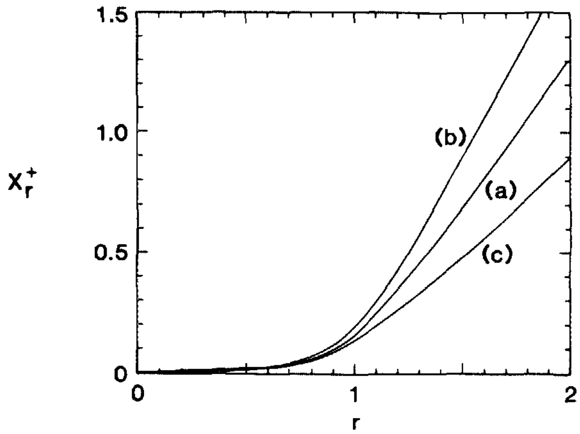


FIG. 4. Comparison of the (a) general light-pump curve (using $\alpha = 40$ cm$^{-1}$, $C = 10^{-2}$, $g' = 38$ cm$^{-1}$, and other parameters as in Fig. 2) with the low-loss special cases (b) ($\alpha = 0$ cm$^{-1}$, $C = 10^{-2}$, and $g' = 38$ cm$^{-1}$) and (c) ($\alpha = 0$ cm$^{-1}$, $C = 10^{-2}$, and $g' = 0$ cm$^{-1}$).

and the boundary conditions, these equations can be solved for the intensities at the ends of the laser. The results obtained here, neglecting distributed losses, are plotted in Fig. 4 together with the more general solution described previously, which assumes a distributed loss of $\alpha = 40\,\text{cm}^{-1}$. It can be observed that in these units the main effect of neglecting $\alpha$ is to scale up the output intensity for a given value of the threshold parameter.

A further substantial simplification occurs if $g'$ is also negligible. This limit would correspond to the many laser systems in which absorption by the lower state is unimportant. If $g'$ is set to zero, Eqs. (36) and (38) simplify to, respectively,

$$(x^+ + C)(x^- + C) = -a'/2 \tag{43}$$

and

$$(x_l^+ + C)(x_l^- + C) = (x_r^+ + C)(x_r^- + C), \tag{44}$$

and this result has also been discussed in connection with mirrorless lasers.[35] The final formula given in Eq. (42) reduces to

$$\frac{g_0 L + (x_l^+ - x_l^-) - (x_r^+ - x_r^-)}{1 - 2C} = \ln\left(\frac{x_r^+ + C}{x_l^+ + C}\right). \tag{45}$$

Equations (44) and (45) can now be combined with the boundary conditions to obtain a relationship for the intensity incident on the laser mirrors, and these results are also plotted in Fig. 4. It is clear that the main effect of neglecting $g'$ is to reduce the laser output.

### D. Low gain approximation

The power characteristics of a diode laser can be calculated in a relatively straightforward way if the net gain per pass is assumed to be small. This assumption forms the basis for many laser studies, but its validity for diode lasers is not self evident. Even when the low-gain approximation is not strictly valid, however, the enormous simplifications that result may still justify its use.

If the gain per pass is small, both $x^+$ and $x^-$ may be approximated by a single parameter $x$,[36] and Eqs. (9) and (10) become

$$\frac{dx}{dz} = (g - \alpha)x + (g + g')C \tag{46}$$

and

$$g = \frac{g_0 - g'}{1 + 2x}. \tag{47}$$

Equating the increase in intensity in one round trip with the mirror losses leads to the result

$$\Delta x = \frac{2(g_0 - g')L(x + C)}{1 + 2x} - 2L(\alpha x - g'C)$$
$$= (1 - R_l)x + (1 - R_r)x. \tag{48}$$

This is a quadratic equation in $x$, and in standard form it looks like

$$x^2 - \frac{x}{2\alpha'}(g_0 - g' + 2Cg' - \alpha') - \frac{g_0 C}{2\alpha'} = 0, \tag{49}$$

where distributed losses and mirror losses have been combined in the parameter

$$\alpha' = \alpha + \frac{(1 - R_l) + (1 - R_r)}{2L}. \tag{50}$$

Also, in this approximation Eq. (23) is replaced by

$$g_{\text{th}} = g' + \alpha'. \tag{51}$$

The solutions of Eq. (48) are plotted in Fig. 5 and compared with the exact solutions of the high-gain laser equations. Evidently, the main effect of neglecting the large single-pass gain is to underestimate the laser output.

## V. DISCUSSION

In this study, we have reviewed techniques for calculating the power characteristics of single-mode diode lasers, and several new analytical and semianalytical procedures have been developed. By comparing these various procedures, it becomes possible to decide in a rational way what might be the most appropriate model to use in characterizing a particular laser configuration. Many of these models involve simple solutions of transcendental equations rather than numerical integration of differential equations, and in some cases explicit intensity formulas can be obtained.

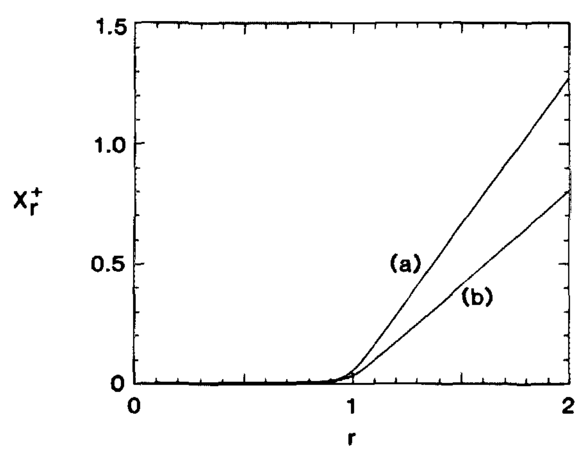


FIG. 5. Comparison of (a) the general solution of the laser equations (using $C = 10^{-3}$ and otherwise the same parameters as in Fig. 2), and (b) the corresponding low-gain approximation.

Some more specific consequences of these models can also be noted. First, for most laser applications, the effective noise parameter $C$ can be neglected entirely if its value is below about $10^{-4}$. For diode lasers the spontaneous emission noise has a significant effect near the laser threshold, but for other laser types, noise could be neglected even near threshold with a consequent simplification of the analysis. In particular, a transcendental equation is obtained; and if the distributed losses are less than the gain from stimulated emission, the intensity above threshold is given by an explicit formula.

Several additional effects have also been considered here including distributed losses, band-to-band absorption, and large single-pass gain. Distributed losses tend to reduce the slope of the power curve above threshold, while the effect of neglecting band-to-band absorption is to underestimate the laser output for a given value of the pumping parameter $r$. Using the low-gain approximation may also reduce the theoretical laser output. Thus, in all cases, neglect of these effects leads to quantitative errors in the predicted laser power characteristics, though for some qualitative applications the results may still be adequate. The techniques and graphs presented here should provide an improved basis for deciding how detailed a laser model is needed for a particular application.

Finally, it may be useful to relate the threshold condition to device physical and geometrical parameters. This possibility is discussed in Appendix B for "well-behaved" lasers with a constant thickness of the active layer. An explicit formula for the light intensity above threshold is described in Appendix A. Using this formula and integrating over the laser cross section, Appendix B also gives an explicit expression for the output power in injection lasers above threshold in terms of the injection current and other device parameters.

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## APPENDIX A: EXPLICIT EXPRESSION FOR $x_r^+$ ABOVE THRESHOLD

Since Eqs. (33)–(35) only determine $x_r^+$ implicitly, they are somewhat cumbersome to use. To more clearly exhibit the main features of the power output characteristics, an explicit formula would be desirable. An approximate formula can be obtained by expanding Eq. (33) in the limit that the internal absorption loss is less than the gain from stimulated emission, or more specifically when

$$\alpha < \frac{g_0 - g'}{1 + 2x_r^+ (R_r)^{1/2}}. \tag{A1}$$

Keeping only the lowest order corrections in $(g_0 - g_{\text{th}})$, this yields, after some algebra

$$x_r^+ \simeq \frac{L(g_0 - g_{\text{th}})}{R} \frac{1 - \dfrac{\alpha}{g_0 - g'}}{\left(1 + \dfrac{2\alpha L(g_0 - g_{\text{th}})}{(g_0 - g')} G\right)}, \tag{A2}$$

where $g_{\text{th}}$ is defined by Eq. (23) of the main text and $R$ and $G$ are given by

$$R = [1 + (R_r/R_l)^{1/2}][1 - (R_r R_l)^{1/2}] \tag{A3}$$

and

$$G = -(R_r/2R^2)\ln(R_r R_l). \tag{A4}$$

Equation (A2) provides a qualitatively correct description of the light output above threshold. Notice that for $\alpha = 0$, Eq. (A2) reduces to the familiar expression that is linear with the excess gain $(g_0 - g_{\text{th}})$ and is used for describing the light output for many types of lasers. For diode lasers, $\alpha$ is not negligible, resulting in a reduced slope. Notice, furthermore, that for very large $\alpha$, Eq. (A2) shows that the light output can become nonlinear; with increasing pumping levels, the output is first superlinear and then approaches a linear asymptote with a reduced slope

$$\lim_{g_0 \gg g_{\text{th}}} x_r^+ \frac{R}{L(g_0 - g_{\text{th}})} = \frac{1}{1 + 2\alpha LG}. \tag{A5}$$

However, since only the lowest order corrections in $(g_0 - g_{\text{th}})$ were kept in Eq. (A2), the numerical value of $G$ is not expected to be exactly correct. A better value of $G$ can be obtained by requiring that the output approach an upper limit set by the absorption losses as the laser is lengthened. Using Eqs. (33) and (34) one obtains

$$\lim_{L \to \infty} x_r^+ = \frac{g_0 - g' - \alpha}{2\alpha (R_r)^{1/2}}. \tag{A6}$$

Taking the corresponding limit of $L \to \infty$ in Eq. (A2) and comparing the result with Eq. (A6) suggests that

$$G = (R_r)^{1/2}/R. \tag{A7}$$

A typical plot of Eq. (A2) with $G$ defined by Eq. (A7) is given in Fig. 6. The plot is in very good agreement with Fig. 2 of the main text which was prepared by solving the transcendental Eq. (33). Notice that rather than to introduce nonlinearities, the effect of $\alpha$ is to (nearly) uniformly reduce the slope everywhere above theshold in Fig. 6.

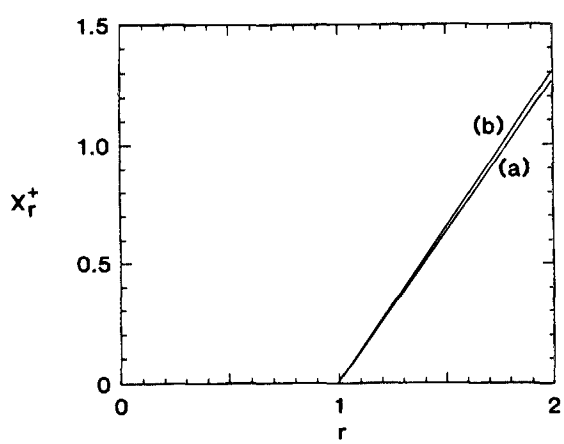


FIG. 6. Comparison of (a) the general solution of the laser equations (using $C = 0$ and otherwise the same parameters as in Fig. 2), and (b) the corresponding explicit approximation.

It may be important to know how good an approximation Eq. (A2) really is. Notice, first, that unlike Eq. (16) of Rigrod's[16] paper, though somewhat similar in form, derivation of Eq. (A2) required no assumption about homogeneous saturation. Furthermore, Eq. (A2) is applicable also to lasers in which the absorption by the lower level may be large ($g' \neq 0$). On the other hand, constraint (A1), to which Eq. (A2) is subjected, may be of concern. A careful inspection reveals however that Eq. (A1) is always satisfied above threshold, unless the total distributed losses are very large, i.e., unless $\alpha L > 1$. Thus, Eq. (A2), with $G$ defined by Eq. (A7), is a good approximation to the exact solution in high-gain lasers everywhere above threshold.

## APPENDIX B

It may be useful to relate the threshold condition to device geometrical and physical parameters. This is accomplished by integrating[24] the starting Eqs. (7) and (8) over the laser cross section. In the so-called effective index approximation for a constant thickness of the active layer,[32] this yields equations which are particularly simple; the equations are of the same form as Eqs. (7) and (8) except that the carrier density and flux intensities are replaced by their total and, where appropriate, overlapping populations. Thus $I^{\pm} \rightarrow P^{\pm}$, $n_c \rightarrow N_c$, $n_c I^{\pm} \rightarrow N_c P^{\pm}/S_{\text{eff}}$, and $\Delta n_0 I^{\pm} \rightarrow \Gamma \Delta n_0 P^{\pm}$, where $P^{\pm}$ is the total energy flux, $N_c$ is the electron population per unit length, $\Gamma$ is the modal confinement factor in the transverse direction, and the laser effective cross-section area is

$$S_{\text{eff}} = \frac{d}{\Gamma} W_{\text{eff}}, \tag{B1}$$

where $d$ is the active layer thickness and $W_{\text{eff}}$ is the effective width in the lateral direction first introduced by Petermann[27]

$$W_{\text{eff}} = \frac{\int n_c dy \int |I^{\pm}| dy}{\int n_c |I^{\pm}| dy}. \tag{B2}$$

The pumping rate $S$ is replaced by the integrated rate $\overline{S} = \int S dx dy$, and for injection lasers is related to the injection current $J$ as $\overline{S} = \eta_i J/eL$ where $\eta_i$ is the internal quantum efficiency and $e$ is the electronic charge.

Introducing normalized intensities $x^{\pm} = (2BP^{\pm}/AS_{\text{eff}})$ and $g_0 = (2h\nu B\overline{S}/AS_{\text{eff}})$, the resulting equations are of the same form and are integrated in the same way as are Eqs. (9) and (10) of the main text. The threshold current $J_{\text{th}}$ is then calculated by setting $g_0 = g_{\text{th}}$ from which

$$J_{\text{th}} = (eL/\eta_i)\left(\frac{AS_{\text{eff}}}{2h\nu B}\right) g_{\text{th}}, \tag{B3}$$

where $g_{\text{th}}$ is given by Eq. (23) and in which $g'$ is reduced by the factor $\Gamma$.

In injection lasers, $A$ has the meaning of the spontaneous recombination lifetime, $A = \tau_s^{-1}$. Using that $-h\nu B \Delta n_0 = g'$, the unsaturated band-to-band absorption coefficient, Eqs. (B3) and (A5) for power output above threshold can finally be rewritten in terms of measurable quantities as

$$J_{\text{th}} = -\frac{eL}{\eta_i} \frac{\Delta n_0 S_{\text{eff}}}{2g'} \frac{1}{\tau_s} g_{\text{th}} \tag{B4}$$

and

$$P^{+}_{r,\text{out}} = \frac{h\nu\eta_i}{e} \frac{T_r}{R(1 + 2\alpha LG)} (J - J_{\text{th}}), \tag{B5}$$

where $J$ is the injection current, $R$ and $G$ are defined by Eqs. (A3) and (A7), respectively, and $T_r$ is the intensity transmission coefficient of the right-hand mirror. We note that other results of this paper, the general ones or for other limiting cases, can be related to measurable quantities using similar transformations.


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