Power Controlled Adaptive Sum-Capacity in the Presence of Distributed CSI

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***Abstract*—We consider a distributed MAC setting with block-wise flat fading links and full receiver CSI (channel state information). Each user has individual CSI (knowledge of its own fading coefficient) and is unaware of the link quality of other links. Outage is not allowed in any communication block. In this distributed set up, throughput optimal power-rate allocation strategies were proposed recently, under identical average powers and channel statistics across users. The average sum-throughput is also known as power controlled adaptive sum-capacity in literature.**

We extend this result in two directions, solving some open problems in the process. 1) We find the power controlled adaptive sum-capacity when the users have different average power constraints and identical channel statistics, and propose bounds when the channel statistics are different. 2) we analyze the impact of finite-rate additional CSI on the fading coefficients of the other links, and compute the optimal throughput in some interesting cases.

I. Introduction

The multiple access channel (MAC) is a widely studied model in information theory, where many users communicate to a single entity using a shared medium. With its natural applications in wireless communications, the so called fading MAC with additive white Gaussian noise is one of the popular MAC models. In here, the channel from each user to the receiver is modeled by a multiplicative fading channel.

In order to find the rate-tuples at which reliable communication is possible over the fading MAC model, it is important to make assumptions about the amount of channel knowledge available at the transmitters and the receiver. It is natural to assume that the receiver has access to the fading coefficients, by means of pilot symbols. In other words, the receiver has full CSI. On the other hand, the same is not true about the transmitter. We consider a model where each transmitter is fully aware of its own fading coefficient (individual CSI), but that of no other. Towards the latter sections of this paper, we relax this assumption and equip the transmitter with partial CSI of other links.

While the literature on fading MAC is extensive, perhaps the most relevant ones here are [1], [2], [3], [4], [5], [6]. In [2], the sum-capacity under average power constraints was computed when full CSI is available at the transmitters. This was generalized in [3], in particular to weighted sum-rate maximization among other extensions. [1] considered a very general transmitter CSI set-up and showed that the optimal nature of the communication strategies is to adaptively control power, along with ergodic averaging to account for the lack of full CSI.

We consider slow-fading models, effectively modeled as a block fading MAC, where the channel fading coefficient stays constant within a block, and varies in an iid fashion across blocks. In this set-up, one may further demand that each block be outage-free, while allowing for adaptively controlling the power and transmission-rates based on the available channel knowledge. This is considered in [5], [6], where the notion of power controlled adaptive sum-capacity is introduced (see Section 23.5.2 of [5]), which is the maximal empirical average of the sum-rates achieved in each block. The evaluation of adaptive sum-capacity for Rayleigh slow-fading MAC channels is mentioned as an open problem in [6]. Our previous work solved part of this problem when the users have identical channel characteristics and average powers [4].

Throughout the paper, the usage *identical users* is synonymous with the following two constraints.

- the fading statistics are iid across users
- each user has the same average power constraint.

Whereas the first restriction is justified in many circumstances, there are instances where the second one does not occur naturally. Part of this work sheds the latter restriction, and extends the results of [4] to arbitrary power constraints at the transmitters. We term the optimal throughput maximizing strategy as **alpha midpoint strategy**. This new strategy not only extends the *midpoint* strategy of [4], but also seems to perform well over non-identical fading statistics. To bench-mark our performance, we propose an upper-bound for the non-identical channel statistics case.

Once the throughput in both the individual CSI case and the full CSI case are known, it is interesting to see the impact of additional partial CSI on the other users' channels. We turn to this problem and consider limited-rate CSI from the other links in addition to individual CSI. We compute the optimal throughput for identical channel statistics.

The organization is as follows. Section II will introduce the required definitions and some notations. The optimal strategy for the case of unequal average powers is presented in Section III, and bounds for non-identical channels are detailed in Section IV. Section V extends our results to the case where additional partial finite-rate side information on the other links is available at the transmitters. Section VI concludes the paper.

II. System Model and Definitions

Our system model and objective is similar to the adaptive sum-capacity formulation for a MAC with distributed side-information given in [5]. Consider a $L-$user real Gaussian fading MAC given by,

$$Y = \sum_{i=1}^{L} H_i X_i + Z\,,$$

where Z is a Gaussian noise process, independent of the transmissions X_i and multiplicative fading H_i. The fading space $\mathcal{H}_i$ of the i-th user is the set of values taken by H_i, and the joint fading space $\mathcal{H}$ is the set of values taken by the joint fading state $\bar{H} = (H_1, H_2, \cdots, H_L)$. We assume a block-fading model where fading remains constant within block and varies across blocks in an i.i.d fashion. We further assume that the fading gains are independent across links, and their distributions are known to all the transmitters and the receiver. The receiver knows all the fading coefficients. In addition, we have *individual* CSIT, i.e. each transmitter knows its own channel fading coefficient H_i but that of no other. In the latter parts we will consider the case where the transmitters have additional partial CSI of the other fading coefficients. The transmitters have individual average power constraints P_i^{avg}, $1 \le i \le L$, and have the freedom to adapt their rate (and power) according to their own channel conditions.

This leads naturally to the following notion of a power-rate strategy.

Definition 1. *A power-rate strategy is a collection of mappings* $(P_i, R_i) : \mathcal{H}_i \longmapsto \mathbb{R}^+ \times \mathbb{R}^+$*;* $i = 1, 2, \cdots, L$*. Thus, in the fading state* H_i*, the* i^{th} *user expends power* $P_i(H_i)$ *and employs a codebook of rate* $R_i(H_i)$*.*

Let $C_{MAC}(\bar{h}, \bar{P}(\bar{h}))$ denote the capacity region of a Gaussian multiple-access channel with fixed channel gains of $\bar{h} = h_1, \cdots, h_L$ and respective power allocations $\bar{P}(\bar{h}) = (P_1(h_1), \cdots, P_L(h_L))$. We know that,

$$C_{MAC}(\bar{h}, \bar{P}(\bar{h})) = \Bigg\{ \bar{R} \in \{\mathbb{R}^+\}^L : \forall S \subseteq \{1, 2, \cdots, L\} \\ \sum_{i \in S} R_i \le \frac{1}{2} \log \left(1 + \sum_{i \in S} |h_i|^2 P_i(h_i) \right) \Bigg\} \quad (1)$$

Definition 2. *We call a power-rate strategy as* feasible *if it satisfies the average power constraints for each user i.e.* $\forall i \in \{1, 2, \cdots, L\}, \quad \mathbb{E}_{H_i} P_i(H_i) \le P_i^{avg}$.

Definition 3. *A power-rate strategy is termed as* outage-free *if it never results in outage i.e.*

$$\forall \bar{h} \in \mathcal{H}, (R_1(h_1), \cdots, R_L(h_L)) \in C_{MAC}(\bar{h}, \bar{P}(\bar{h}))$$

Let Θ_{MAC} be the collection of all feasible power-rate strategies which are outage-free. Let us now specialize the definitions to the case of identical channel statistics, i.e. the cdf of each user is $\Psi(h)$. For any strategy $\theta \in \Theta_{MAC}$, the throughput is

$$T_\theta = \sum_{i=1}^{L} \mathbb{E} R_i^\theta(H_i) = \sum_{i=1}^{L} \int_h R_i^\theta(h)\, d\Psi(h) \\ = \int_h d\Psi(h) \left(\sum_{i=1}^{L} R_i^\theta(h) \right), \quad (2)$$

where the superscript θ is used to identify the feasible power-rate strategy employed. i.e. $R_i^\theta(h)$ is the rate allocated to user i while observing fading coefficient h. The corresponding transmit power is denoted as $P_i^\theta(h)$.

Definition 4. *The* ***power controlled adaptive sum-capacity*** *is the maximum (average) throughput achievable, i.e.* $C_{sum}(\Psi) = \max_{\theta \in \Theta_{MAC}} T_\theta$*.*

III. Power Controlled Adaptive Sum Capacity

Let us recall the *midpoint* strategy of [4]. Suppose there are L users with *fixed* transmit powers $P_1, P_2, \cdots, P_L$ and respective fading values h_i, $1 \le i \le L$ for a given block of communication. In a $L-$user *midpoint* strategy, each user assumes that all others are identical to itself and constructs a symmetrical $L-$user MAC region, and then chooses the maximal equal-rates point for operation. Thus we have, for $1 \le i \le L$

$$R_i^{mid}(h_i) = \frac{1}{2L} \log\left(1 + L|h_i|^2 P_i\right). \quad (3)$$

This communication strategy will never result in an outage, see [4]. Furthermore, it is throughput optimal in a block fading setup for identical users, see Definition 4. The best midpoint strategy requires adaptive power control according to a water-filling formula on the individual fading coefficients.

In the block fading model of [4], the underlying assumption of identical channel statistics is very reasonable. However, assuming all users to have the same average power, though justified in many circumstances, can be relaxed. We will now show that a simple rate-split adaptation of the identical-users case is throughput optimal for unequal average powers.

Theorem 5. *Given independent and identical channels according to the c.d.f* $\Psi(h)$*,*

$$C_{sum}(\Psi) = \frac{1}{2} \int d\Psi(h) \log(1 + |h|^2 P^*(h)), \quad (4)$$

where

$$P^*(h) = \left(\frac{1}{\lambda} - \frac{1}{|h|^2} \right)^+ \text{ and } \int d\Psi(h) P^*(h) = \sum_{i=1}^{L} P_i^{avg}.$$

Proof: Notice that the RHS only depends on $P_{sum} = \sum_i P_i^{avg}$. For future use, let us denote the RHS of (4) under the given constraints by $C_{sum}(\Psi, P_{sum})$. The proof of the theorem proceeds through lemmas 6 – 8. We first propose an upperbound and later show its achievability. ∎

A. An Upperbound

For a link of cdf $\Psi(h)$, let $\Theta_s(\tilde{P})$ be the collection of all single-user power allocation schemes such that

$$\int P(h) d\Psi(h) = \tilde{P}. \tag{5}$$

Lemma 6. *The throughput T_θ for $\theta \in \Theta_{MAC}$ obeys,*

$$T_\theta \leq C_{sum}(\Psi, P_{sum}), \forall \theta \in \Theta_{MAC}.$$

Proof:

$$T_\theta \overset{(a)}{\leq} \frac{1}{2} \int_h d\Psi(h) \log\left(1 + |h|^2 \sum_{i=1}^{L} P_i^\theta(h)\right) \tag{6}$$

$$\leq \max_{\Theta_s(\sum P_i^{avg})} \frac{1}{2} \int d\Psi(h) \log\left(1 + |h|^2 P(h)\right). \tag{7}$$

Here (a) follows from (2), by applying the sum-rate upper bound on a MAC with received signal power $\sum_i |h|^2 P_i^\theta(h)$. The second inequality results from relaxing the individual power constraints to a single average sum-power constraint.

It is clear that water-filling of the inverse fading gains is the optimal strategy in a point to point fading channel under an average power constraint. Thus the last expression above is indeed $C_{sum}(\Psi, P_{sum})$. ■

B. Alpha-midpoint Strategy

Let us now construct an achievable strategy. Our achievable strategy is motivated by the so called *midpoint* scheme described in (3), we call it the **alpha-midpoint strategy**. Let $\bar{\alpha}$ be a vector of non-negative values with $\sum_i \alpha_i = 1$. In alpha-midpoint strategy, the rate chosen by user i while encountering a fading coefficient of h_i is,

$$R_i^{\bar{\alpha}}(h_i) = \alpha_i \frac{1}{2} \log\left(1 + |h_i|^2 \frac{P_i(h_i)}{\alpha_i}\right), \tag{8}$$

where $P_i(h_i)$ is the transmitted power, chosen such that

$$\int P_i(h) d\Psi(h) = P_i^{avg}.$$

Lemma 7. *The alpha-midpoint strategy is outage-free.*

Proof: For any $S \subseteq \{1, 2, \cdots, L\}$,

$$\sum_{i \in S} R_i^{\bar{\alpha}}(h_i) = \sum_{i \in S} \alpha_i \frac{1}{2} \log\left(1 + |h_i|^2 \frac{P_i(h_i)}{\alpha_i}\right) \tag{9}$$

$$\leq \frac{1}{2} \log\left(1 + \sum_{i \in S} |h_i|^2 P_i(h_i)\right), \tag{10}$$

by concavity of the logarithm. Clearly the chosen rate-tuple across users is within $C_{MAC}(\bar{h}, \bar{P}(\bar{h}))$ for every block, ensuring that there is no outage. ■

We now show the optimality of alpha-midpoint schemes.

Lemma 8.

$$\max_{\theta \in \Theta_{MAC}} T_\theta = C_{sum}(\Psi, P_{sum})$$

Proof: We will specialize our alpha-midpoint strategy to achieve $C_{sum}(\Psi, P_{sum})$. To this end, choose for $1 \leq i \leq L$,

$$\alpha_i = \frac{P_i^{avg}}{\sum_{i=1}^{L} P_i^{avg}} \text{ and } P_i(h) = \alpha_i P^*(h),$$

where $P^*(h)$ is given in (4). Not only users adhere to their respective power constraints, but also the throughput is indeed $C_{sum}(\Psi, P_{sum})$, completing the proof. ■

This will suggest a very simple strategy of achieving $C_{sum}(\Psi, P_{sum})$, which has the added advantage of being achievable using single user decoding and successive cancellation. In particular, we split user k in to N_k virtual users such that each virtual user has an identical average power constraint of P_v. Thus,

$$\sum_{k=1}^{L} \sum_{i=1}^{N_k} P_v = \sum_{k=1}^{L} P_k^{avg}.$$

Evaluating the maximal average rate for the $L' = \sum_{k=1}^{L} N_k$ virtual users under the midpoint strategy of (3) will also yield $C_{sum}(\Psi, P_{sum})$. Since the midpoint rates are achievable by single user decoding techniques [4], alpha midpoint rates can also be achieved by low complexity schemes.

IV. Nonidentical Channel Statistics

Our second result is a generalization to non-identical channel statistics. In this case, we do not know the optimal schemes, but we provide an upperbound, which seems to be close for several channels of practical interest. W.l.o.g consider non-negative valued fading coefficients (by taking modulus), and let the respective cdf of the individual channels be $\{F_1(\cdot), F_2(\cdots), \cdots, F_L(\cdot)\}$. We will assume each of them to be right continuous and define the corresponding inverse functions as

$$\forall \gamma \in [0, 1],\ F_k^{-1}(\gamma) = \min h : F_k(h) \geq \gamma. \tag{11}$$

For convenience, we will denote $F_k^{-1}(\cdot)$ by $h_k^2(\cdot)$.

Lemma 9. *For non-identical channels defined by the c.d.fs $F_k(\cdot), 1 \leq k \leq L$, the maximal sum-rate C_{sum} is bounded by*

$$C_{sum} \leq \max_{\theta_{MAC}} \int_0^1 \frac{1}{2} \log\left(1 + \sum_{k=1}^{L} h_k^2(x) P_k[h_k(x)]\right) dx. \tag{12}$$

Proof: Imagine that the range of each cdf $F_k, 1 \leq k \leq L$ in $[0, 1]$ is divided into n equal segments. Let the inverse map of the j^{th} segment of cdf F_k be $h_k^2(j/n)$. The lemma states that for each segment j, the MAC formed by the corresponding inverse maps of this segment should obey the sum-rate constraint. ■

Notice that different channel values are coupled in the above bound (through their cdf structure), and we can maximize the power allocation on these coupled fading vectors, thus obtaining a bound to the RHS of (12). By using Lagrange optimization as in [2], we get the following lemma.

Lemma 10.

$$C_{sum} \le \sum_{k=1}^{L} \int_0^1 \frac{1}{2} \log(1 + h_k^2(x) P_k(x)) \alpha_k(x) dx \quad (13)$$

where

$$P_k(x) = \left(\frac{1}{\lambda_k} - \frac{1}{h_k^2(x)}\right)^+ \text{ and } \int_0^1 P_k(x)\alpha_k(x)dx = P_k^{avg}.$$

In here, $\alpha_k(x)$ are non-negative functions such that $\sum_{k=1}^{L} \alpha_k(x) = 1, \forall x \in (0,1)$.

Proof: Maximizing (12) over all coupled channel vectors will yield the bound in (13). The detailed proof is available in appendix A. ∎

While the existence of λ_k and the functions $\alpha_k(x)$ is enough for the proof, numerical algorithms are required to find these, except for special cases. One such case where the algorithm is straightforward is when the channel coefficients are generated by the same law, but scaled by different average gains. In this case, $\alpha_k(x) = P_k^{avg} / \sum P_k^{avg}$ for all $x \in (0,1)$ and the waterfilling formula can be evaluated using single-user water-filling. For example, consider a $2-$user Rayleigh faded MAC, with $E|\mathbf{h}_2|^2 = 2E|\mathbf{h}_1|^2 = 2$, and $P_1^{avg} = P_2^{avg}$. Figure 1 compares our upperbound against the rates obtained by an adaptation of the alpha-midpoint strategy.

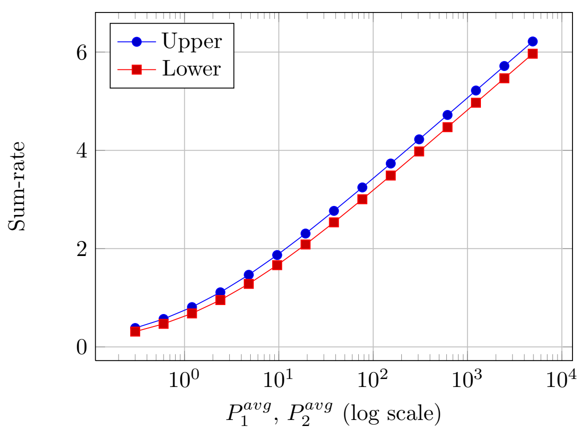

Fig. 1. Upper and Lower bounds to adaptive sum-capacity

This not only demonstrates the utility of our upper-bound, but also that the alpha mid-point strategy is a good scheme.

V. Finite-rate CSI on Other Links

Up to this, we have assumed that there is only individual CSI. We wish to study the effect of incremental information about the other links. To keep things simple, we consider *identical users* with the specified cdf $\Psi(h)$ and an average power of P^{avg}. Extensions to unequal average power constraints are possible, but not covered here. We consider 1 bit of additional partial CSI, i.e., each transmitter gets one bit of information from every other link, in addition to its own individual CSI. This information is assumed to be obtained through transmitter cooperation or cribbing. It is crucial that the receiver has no say on the partial CSI. If the receiver decides the conveyed bit, then the throughput is same as that of the full CSI [2].

The partial CSI contains link quality information, assume it to be chosen from the set $\{G, B\}$, where we used G for good and B for bad. A natural separation between G and B is a link gain threshold. In particular, the partial CSI bit $\hat{h}_k$ of transmitter k is

$$\hat{h}_k = \begin{cases} G \text{ if } |h_k| \ge h_T \\ B \text{ otherwise,} \end{cases} \quad (14)$$

for some fixed positive threshold h_T. By slight abuse of notation, we will say that link j is in state G (and call it good user), and denote the probability of that event by $\mu(G)$. Using the same token, $1 - \mu(G) = \mu(B)$. Let C_{PSI} be the maximum attainable throughput with 1 bit additional CSI on each of the other links, along with individual CSI.

Theorem 11. *For L identical users,*

$$C_{PSI} = C_{sum}\left(\Psi', LP^{avg}\right), \quad (15)$$

where the cdf $\Psi'(\cdot)$ is such that,

$$d\Psi'(h) = d\Psi(h)\left([\mu(B)]^{L-1}\mathbb{1}_{\{h\in B\}} + (1+\zeta)\mathbb{1}_{\{h\in G\}}\right)$$

and

$$\zeta = \sum_{m=1}^{L-1} \binom{L-1}{m} [\mu(B)]^m [\mu(G)]^{L-1-m} \frac{m}{L-m}. \quad (16)$$

Proof: Recall the definition of $C_{sum}(\cdot,\cdot)$ given in Section III. We explain the proof for $L = 2$, which contains all the essential features. The proof is relegated to appendix B. ∎

It is instructive to compare the advantages of 1 bit of extra CSI, which we demonstrate for a two user *identical* Rayleigh fading links of unit second moment, see Figure 2. The threshold value h_T for $1-$bit CSI was taken as one.

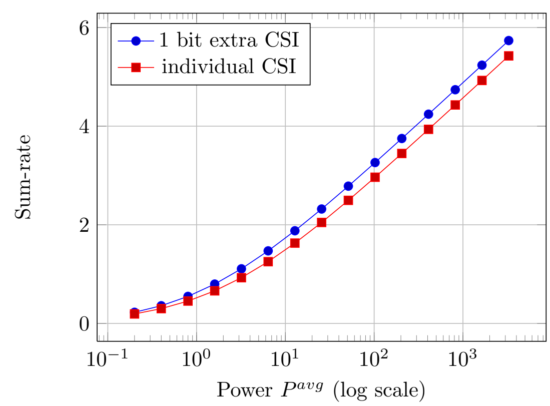

Fig. 2. Sum-rate improvement by additional CSI

VI. Conclusion

We have presented throughput optimal outage-free communication schemes for a block-fading MAC with identical channel statistics, and individual side information as well as additional partial finite-rate CSI.

While extending to multiple bits of symmetric partial CSI on the other links is straightforward, we are currently investigating the impact of asymmetric partial CSI.

APPENDIX A

Proof of Lemma 10: Notice that we assume arbitrary channel statistics for the links. The following proposition on MAC captures the essential idea behind the result.

Proposition 12. *For a given* fixed $L-$*user MAC with link gains* $h_1, \cdots, h_L$ *and respective average transmit powers* $P_1, \cdots, P_L$*, the maximal sum-rate can be achieved by time-sharing.*

Proof: The proof is reasonably easy. Let user i transmit for a fraction of time β_i with power $\frac{P_i}{\beta_i}$, at its single user capacity. By choosing $\beta_i = \frac{h_i^2 P_i}{\sum_k h_k^2 P_k}$ we get,

$$\sum R_i = \sum_{i=1}^{L} \frac{\beta_i}{2} \log(1 + \sum_{k=1}^{L} h_k^2 P_k), \tag{17}$$

which is indeed the MAC sum-rate bound. ■

Let us now relax the maximization in (12). In particular, we replace $P_k[h_k(x)]$ by $P_k(\bar{h}(x))$, where $\bar{h}(x)$ is the global fading vector corresponding to the same c.d.f. value x at each transmitter. Thus our relaxed optimization problem is,

$$\int_0^1 \log(1 + \sum_k h_k^2(x) P_k(\bar{h}(x)))dx, \tag{18}$$

such that

$$\int_0^1 P_k(\bar{h}(x))dx = P_k^{avg}, \forall k. \tag{19}$$

By defining Lagrange multipliers, $\lambda_i, 1 \leq i \leq L$, one for each constraint, we can equivalently maximize the cost J, where

$$J = \int_0^1 \log(1 + \sum_k h_k^2(x) P_k(\bar{h}(x)))dx - \sum_{k=1}^{L} \lambda_k \int_0^1 P_k(\bar{h}(x))dx. \tag{20}$$

Taking derivative w.r.t to $P_k(\cdot)$ and equating to zero,

$$\frac{h_k^2(x)}{1 + \sum_{k=1}^{L} h_k^2(x) P_k(\bar{h}(x))} - \lambda_k = 0, \, 1 \leq k \leq L \tag{21}$$

Since this has to be true for the active user-set (ones which are allocated non-zero power at a given value of x), we can conclude that power is allocated to user j only if

$$\frac{h_j^2(x)}{\lambda_j} \geq \frac{h_i^2(x)}{\lambda_i}, \forall i \neq j.$$

Let $\zeta(x)$ be the maximum value of $\frac{h_j^2(x)}{\lambda_j}$ over $1 \leq j \leq L$. Each user with $D_i(x) = 1$ will achieve $\zeta(x)$. However, Proposition 12 will suggest that the active users can time share and achieve the sum-rate. The power chosen by an active user is

$$P_i(h_i(x)) = \max\{0, \frac{1}{\lambda_i} - \frac{1}{h_i^2(x)}\}.$$

The instantaneous received power from active user i while in its transmitting time-fraction is $\frac{h_i^2(x)}{\lambda_i} - 1$. However, the fraction of time given to each active user is dependent on the channel-laws and average powers. Thus $\alpha_i(x)$ in (13) is the time-fraction for the active user i, for a given set of channels determined by the cdf index x. This concludes the proof of Lemma 10.

APPENDIX B

Proof of Theorem 11:
We will show the proof for a 2 user system for simplicity. Let $\hat{h}_i$ denote the CSI communicated from user i to all others. User 1 employs a power of $P_1(h_1, \hat{h}_2)$ and user 2 spends $P_2(\hat{h}_1, h_2)$. Let $R_1(h_1, \hat{h}_2)$ and $R_2(\hat{h}_1, h_2)$ be the respective rates chosen. We can bound the average sum-rate as,

$$\begin{aligned} R_1 + R_2 = &\int_{G\times G} \Big(R_1(h_1, \hat{h}_2) + R_2(\hat{h}_1, h_2)\Big) d\Psi(h_1, h_2) \\ &+ \int_{B\times B} \Big(R_1(h_1, \hat{h}_2) + R_2(\hat{h}_1, h_2)\Big) d\Psi(h_1, h_2) \\ &+ \int_{B\times G} \Big(R_1(h_1, \hat{h}_2) + R_2(\hat{h}_1, h_2)\Big) d\Psi(h_1, h_2) \\ &+ \int_{G\times B} \Big(R_1(h_1, \hat{h}_2) + R_2(\hat{h}_1, h_2)\Big) d\Psi(h_1, h_2). \end{aligned} \tag{22}$$

Consider the first term in the summation of the right side. By suitably integrating, it can be written as a single integral,

$$\begin{aligned} &\mu(G) \int_G (R_1(h, G) + R_2(G, h)) d\Psi(h) \leq \\ &\frac{\mu(G)}{2} \int_G \log\left(1 + h^2(P_1(h, G) + P_2(G, h))\right) d\Psi(h), \end{aligned} \tag{23}$$

which is the sum-rate bound of the corresponding MAC. Similarly, for the second term,

$$\begin{aligned} &\mu(B) \int_B (R_1(h, B) + R_2(B, h)) d\Psi(h) \leq \\ &\frac{\mu(B)}{2} \int_B \log\left(1 + h^2(P_1(h, B) + P_2(B, h))\right) d\Psi(h). \end{aligned} \tag{24}$$

As for the third and fourth terms, the information on who has the better channel is readily available to both parties here. Let us now consider only those channel states $(h_1, h_2) \in \{(G \times B) \bigcup (B \times G)\}$. Let the average power expenditure on these channel states be P_{GB}. Suppose we relax our assumption, and give full CSI to each transmitter whenever one of the links is in state G and the other in B. Furthermore, let us enforce only a average sum-power constraint of P_{GB} in these states. In such a system, only the better user transmits with an appropriate power [2]. This fact can be utilized along with (23) and (24) to obtain an upperbound to C_{sum}. We call this J^*, defined as

$$\begin{aligned} J^* = \max &\frac{\mu(B)}{2} \int_B \log\left(1 + h^2(P_1(h, B) + P_2(B, h))\right) d\Psi(h) \\ &+ \frac{\mu(G)}{2} \int_G \log\left(1 + h^2(P_1(h, G) + P_2(G, h))\right) d\Psi(h) \\ &+ \frac{\mu(B)}{2} \int_G (\log(1 + h^2 P_1(h, B)) + \log(1 + h^2 P_2(B, h))) d\Psi(h), \end{aligned}$$

where we also relax the original individual power constraint by an average sum-power constraint of the form,

$$\mu(B)\int_B (P_1(h,B)+P_2(B,h))d\Psi(h)+ \\ \mu(G)\int_G (P_1(h,G)+P_2(G,h))d\Psi(h)+ \\ \mu(B)\int_G (P_1(h,B)+P_2(B,h))d\Psi(h)\leq 2P^{avg}.$$

We can further relax the above optimization to get,

$$J^{**}=\max \mu(B)\int_B \frac{1}{2}\log\left(1+h^2(P(h)\right)d\Psi(h) \\ +(\mu(G)+2\mu(B))\int_G \frac{1}{2}\log\left(1+h^2P(h))\right)d\Psi(h) \quad (25)$$

subjected to

$$\mu(B)\int_B P(h)+(\mu(G)+2\mu(B))\int_G P(h)\leq 2P^{avg}.$$

Clearly $J^{**}\geq J^{*}$, as any allocation in the latter can be emulated by the former optimization. Notice that J^{**} can be maximized by single user waterfilling. On the other hand, it also turns out to be achievable, which we state as a lemma.

Lemma 13. *For 2 identical-users with individual cdf $\Psi(\cdot)$, the maximal throughput with partial CSI is $C(\Psi', 2P^{avg})$, where*

$$d\Psi'(h)=\begin{cases} d\Psi(h)\mu(B) \text{ if } h\in B \\ d\Psi(h)(1+\mu(B)) \text{ if } h\in G \end{cases} \quad (26)$$

Proof: From (25) it is evident that $J^{**}=C(\Psi',2P^{avg})$. To show the achievability of J^{**}, assign $P_1(h,G)=P_2(G,h)=0$, $\forall h\in B$ and

$$R_1(h,B)=R_2(B,h)=\frac{1}{2}\log(1+h^2P_1(h,B)),\forall h\in G.$$

For all other cases, employ the mid-point rates using the waterfilling power allocation of J^{**}. It is straightforward to show that the throughput is indeed J^{**}. ∎

For $L>2$ users, if there are $K\geq 1$ links in G, only those links with $h_k\in G$ will transmit at their respective $K-$ user mid-point rates. On the other hand, if no links are in G, all L users transmit at their respective $L-$user mid-point rates. The power allocation can be effectively determined by single user waterfilling of the cdf $\Psi'(h)$ given in Theorem 11.

References

[1] A. Das and P. Narayan, "Capacities of time-varying multiple-access channels with side information," *Information Theory, IEEE Transactions on*, vol. 48, no. 1, pp. 4 –25, Jan. 2002.

[2] R. Knopp and P. Humblet, "Information capacity and power control in single-cell multiuser communications," in *ICC '95 Seattle*, vol. 1, Jun. 1995, pp. 331 –335.

[3] D. Tse and S. Hanly, "Multiaccess fading channels. i. polymatroid structure, optimal resource allocation and throughput capacities," *Information Theory, IEEE Transactions on*, vol. 44, no. 7, pp. 2796 –2815, Nov. 1998.

[4] Y. Deshpande, S. R. B. Pillai, and B. K. Dey, "On the sum capacity of multiaccess block-fading channels with individual side information," in *IEEE Information Theory Workshop, Paraty*, 2011.

[5] A. El Gamal and Y.-H. Kim, *Network Information Theory*. Cambridge University Press, 2011.

[6] C.-S. Hwang, M. Malkin, A. El Gamal, and J. M. Cioffi, "Multiple-access channels with distributed CSI," in *ISIT 2007.*,pp. 1561 –1565.

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