Power Efficient Range Assignment for Symmetric Connectivity in Static Ad Hoc Wireless Networks*

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Abstract

In this paper we study the problem of assigning transmission ranges to the nodes of a static ad hoc wireless network so as to minimize the total power consumed under the constraint that enough power is provided to the nodes to ensure that the network is connected. We focus on the MIN-POWER SYMMETRIC CONNECTIVITY problem, in which the bidirectional links established by the transmission ranges are required to form a connected graph.

Implicit in previous work on transmission range assignment under asymmetric connectivity requirements is the proof that MIN-POWER SYMMETRIC CONNECTIVITY is NP-hard and that the MST algorithm has a performance ratio of 2. In this paper we make the following contributions: (1) we show that the related MIN-POWER SYMMETRIC UNICAST problem can be solved efficiently by a shortest-path computation in an appropriately constructed auxiliary graph; (2) we give an exact branch and cut algorithm based on a new integer linear program formulation solving instances with up to 35-40 nodes in 1 hour; (3) we establish the similarity between MIN-POWER SYMMETRIC CONNECTIVITY and the classic STEINER TREE problem in graphs, and use this similarity to give a polynomial-time approximation scheme with performance ratio approaching $5/3$ as well as a more practical approximation algorithm with approximation factor 11/6; and (4) we give the results of a comprehensive experimental study comparing new and previously proposed heuristics with the above exact and approximation algorithms.

*Preliminary versions of the results in this paper have appeared in [1, 11].

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1 Introduction

Ad hoc wireless networks have received significant attention in recent years due to their potential applications in battlefield, emergency disaster relief, and other application scenarios (see, e.g., [3, 8, 9, 16, 18, 22, 26, 30, 29]). Unlike wired networks or cellular networks, no wired backbone infrastructure is installed in ad hoc wireless networks. A communication session is achieved either through single-hop transmission if the recipient is within the transmission range of the source node, or by relaying through intermediate nodes otherwise. We assume that omnidirectional antennas are used by all nodes to transmit and receive signals. Thus, a transmission made by a node can be received by all nodes within its transmission range. This feature is extremely useful for energy-efficient multicast and broadcast communications.

For the purpose of energy conservation, each node can (possibly dynamically) adjust its transmitting power, based on the distance to the receiving node and the background noise. In the most common power-attenuation model [23], the signal power falls as $\frac{1}{r^\kappa}$ where r is the distance from the transmitter antenna and κ is a real *constant* dependent on the wireless environment, typically between 2 and 4. Assume that all receivers have the same power threshold for signal detection, which is typically normalized to one. With this assumption, the power required to support a link between two nodes separated by a distance r is r^κ. A crucial issue is how to find a route with minimum total energy consumption for a given communication session. This problem is referred to as *Minimum-Energy Routing* in [26, 30]. Having every link established in both directions simplifies the one-hop transmission protocols by allowing acknowledgment messages to be sent back for every packet (see, for example [27]). This motivates the study of the MIN-POWER SYMMETRIC CONNECTIVITY problem, where a link is established only if both nodes have transmission range at least as big as the distance between them, and we must ensure that established links form a connected network. Like in [3], in this paper the objective is to minimize the total power assigned to the nodes; previous research on symmetric connectivity has also addressed the objective of minimizing the maximum node power [18, 22].

Formally, given a set of points V (representing the nodes in the network) in E^2 (the two-dimensional Euclidean space) or in E^3 (the three-dimensional Euclidean space), a *transmission range assignment* (or *range assignment*, for short) is a function $r : V \to R_+$. A *unidirectional link* from node u to node v is established under the range assignment r if $r(u) \geq \|uv\|$, where $\|uv\|$ denotes the Euclidean distance between u and v. A *bidirectional link* uv is established under the range assignment r if $r(u) \geq \|uv\|$ and $r(v) \geq \|uv\|$. Let $B(r)$ denote the set of all bidirectional links established between pairs of nodes in V under the range assignment r. In this paper we study the following problem:

MIN-POWER SYMMETRIC CONNECTIVITY: Given a set of nodes V and $\kappa \geq 1$, find a transmission range assignment $r : V \to R_+$ minimizing $\sum_{v \in V} r(v)^\kappa$ subject to the constraint that the graph $(V, B(r))$ is connected.

Implicit in the work of Clementi, Penna, and Silvestri [9] is a proof that MIN-POWER SYMMETRIC CONNECTIVITY in E^2 is NP-Hard (radio "bridges" in canonical form gadgets, see Definition 3 on page 10 of [9], can be made to be bidirectional links). Also implicit in [9] is the proof that, in E^3 and in the graph model, MIN-POWER SYMMETRIC CONNECTIVITY is APX-complete, and therefore, unless $P = NP$, does not admit a polynomial-time approximation scheme. Thus, we search for polynomial-time constant approximation factor algorithms for this problem. The *approximation factor*, or *performance ratio*, of approximation algorithm A for a minimization problem is the supremum, over all possible instances I, of the ratio between the cost of the output of A when running on I and the cost of an optimal solution for I (the smaller the performance ratio, the better). We say that A is an *α-approximation algorithm* if its performance ratio is at most α. A *fully polynomial α-approximation scheme* is a family of algorithms A_ε such that, for every $\varepsilon > 0$, algorithm A_ε (1) has performance ratio at most $\alpha + \varepsilon$, and (2) runs in time polynomial in the size of the instance and $1/\varepsilon$.

Kirousis, Kranakis, Krizanc, and Pelc [16] give a minimum spanning tree (MST) based 2-approximation algorithm for MIN-POWER SYMMETRIC CONNECTIVITY (their algorithm is actually designed for a related problem, which we discuss in Section 2). In this paper we improve the performance ratio under 2 by exploiting similarities between MIN-POWER SYMMETRIC CONNECTIVITY and the classic STEINER TREE problem: given an edge-weighted graph $G = (V, E, w)$ and a set $T \subseteq V$ of *terminals*, find a minimum weight *Steiner tree* for T, i.e., a minimum weight connected subgraph of G which contains T. It is well known that computing an MST in the complete graph on T with edge-weights equal to the minimum distance in G between corresponding terminals gives a 2-approximation algorithm for STEINER TREE [6, 17]. Zelikovsky [31] gave the first algorithm with performance ratio less than 2: he used 3-restricted Steiner trees and the concept of *gain* to obtain a performance ratio of $11/6$. Promel and Steger [21] extend the results of Camerini, Galbiati, and Maffioli [5] and give a polynomial time $5/3$-approximation scheme for STEINER TREE, by finding an almost optimal 3-restricted Steiner tree.

We show that similar concepts can be used for approximating MIN-POWER SYMMETRIC CONNECTIVITY. In particular, we show that the algorithms of [21], [31], [2], and [32] can be modified to give similar performance ratios for MIN-POWER SYMMETRIC CONNECTIVITY. Our main results are a fully polynomial $5/3$ approximation scheme based on [21], and a more practical algorithm with approximation factor of 11/6 [31].

Our algorithms have the same approximation guarantees when network nodes are located in E^3. In fact, since they work on a graph model of the network, our algorithms can be applied to more general problem formulations, e.g., observing given upper-bounds on the transmission range of each node and/or taking into account obstacles that completely block the communication between certain pair of nodes.

The rest of the paper is organized as follows. In Section 2 we discuss several connectivity problems under both symmetric and asymmetric connectivity models. In particular, we show that the MIN-POWER SYMMETRIC UNICAST problem (which, for given source and destination nodes, $s, t \in V$, asks for a sequence $v_0 = s, v_1, \ldots, v_k = t$ of nodes and transmission ranges $r(v_i)$, $i = 0, \ldots, k$, under which all bidirectional links $v_i v_{i+1}$ are established) can be solved efficiently by a shortest-path computation in an appropriately constructed auxiliary graph. In Section 3 we give a new integer program formulation for the MIN-POWER SYMMETRIC CONNECTIVITY problem, and describe an exact branch and cut algorithm based on this formulation. Experimental results show that the branch and cut algorithm solves instances with 25 nodes in less than one minute and instances with up to 35-40 nodes in 1 hour. In Section 4 we show that the MST algorithm has a tight approximation factor of 2 for the MIN-POWER SYMMETRIC CONNECTIVITY problem, and discuss modifications of the MST algorithm for handling given bounds on node transmission ranges. In Section 5 we give a number of approximation algorithms for MIN-POWER SYMMETRIC CONNECTIVITY based on the concept of k-restricted decomposition and the similarity to computing k-restricted Steiner trees. In Section 6 we present the results of a comprehensive experimental study comparing new and previously proposed heuristics with the above exact and approximation algorithms. The results show that best performing algorithms give an average of 5-6% reduction in power consumption compared to the simple MST based solution. We conclude in Section 7 with open problems and directions for future research.

2 Symmetric vs. Asymmetric Connectivity Problem Formulations

Several problems have been previously studied under the related *asymmetric* connectivity model, in which unidirectional links give raise to a directed graph on V. In this section we discuss these formulations and compare them with the corresponding symmetric connectivity variants.

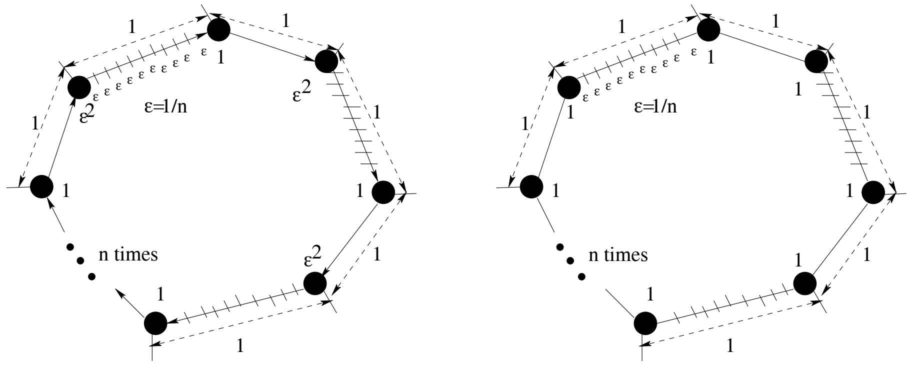
2.1 Complete Network Connectivity

In the MIN-POWER ASYMMETRIC CONNECTIVITY problem (also referred to as the complete range assignment problem) the objective is establishing a strongly connected subgraph of V. Kirousis, Kranakis, Krizanc, and Pelc [16] prove that MIN-POWER ASYMMETRIC CONNECTIVITY in E^3 is NP-Hard and, based on the minimum spanning tree, give a 2-approximation algorithm. As opposed to the MIN-POWER ASYMMETRIC BROADCAST approximation of [29], the MIN-POWER ASYMMETRIC CONNECTIVITY approximation of [16] is valid in arbitrary graphs (that is, the distance between two points could be arbitrary, not necessarily Euclidean). Clementi, Penna, and Silvestri [9] give an elaborate reduction proving that MIN-POWER ASYMMETRIC CONNECTIVITY in E^2 is also NP-Hard.

The power for the MIN-POWER ASYMMETRIC CONNECTIVITY can be half the power for MIN-POWER SYMMETRIC CONNECTIVITY as illustrated by the following example in which $\kappa = 2$. The terminal set (see Figure 1) consists of n groups of $n+1$ points each, located on the sides of a regular $2n$-gon. Each group has 2 terminals in distance 1 of each other (represented as thick circles in Figure 1) and $n-1$ equally spaced points (dashes in Figure 1) on the line segment between them. It is easy to see that the minimum range assignment ensuring asymmetric connectivity assigns a power of 1 to the one thick terminal in each group and a power of $\varepsilon^2 = (1/n)^2$ to all other points in the group. The total power then equals $n+1$. For symmetric connectivity it is necessary to assign power of 1 to all but two of the thick points, and of ε^2 to the remaining points, which results in total power of $2n - 1 - 1/n + 2/n^2$.

2.2 Unicast

The MIN-POWER ASYMMETRIC UNICAST problem requires establishing a minimum power directed path from a source s to a destination t, and is easily solved in polynomial time by shortest-path algorithms. Below we reformulate



(a) (b)

Figure 1: Total power for the MIN-POWER ASYMMETRIC CONNECTIVITY can be half the total power for MIN-POWER SYMMETRIC CONNECTIVITY ($\kappa = 2$). (a) Minimum range assignment ensuring asymmetric connectivity has total power $n + n^2\varepsilon^2 = n + n^2\frac{1}{n^2} = n + 1$. (b) Minimum range assignment ensuring symmetric connectivity has the total power $(2n-2) + (n^2 - n + 2)\varepsilon^2 = 2n - 1 - \frac{1}{n} + \frac{2}{n^2}$.

MIN-POWER SYMMETRIC UNICAST as a graph problem, and then reduce the latter problem to a single-source single-sink shortest-path computation in an appropriately constructed graph.

Let $G = (V, E, c)$ be an edge-weighted graph and uv denote the undirected edge between nodes u and v. The cost $c(uv)$ of an edge $uv \in E$ corresponds to the (symmetric) power requirement $p(u, v) = p(v, u)$. The *power cost* of an s–t path $P = (s = v_0, v_1, \ldots, v_k = t)$ is $p(P) = c(v_0v_1) + c(v_{k-1}v_k) + \sum_{i=1}^{k-1} \max\left(c(v_{i-1}v_i), c(v_i, v_{i+1})\right)$. The MIN-POWER SYMMETRIC UNICAST can thus be reformulated as follows: Given a graph $G = (V, E, c)$ with costs on edges a source $s \in V$ and a destination $t \in V$, find an s–t path in G of the minimum power-cost.

The following example in the Euclidean plane shows that a straightforward application of Dijkstra's algorithm does not work, i.e., a minimum cost s–t path does not always have minimum power-cost. Consider a network consisting of three nodes, $s = (0, 3)$, $t = (4, 0)$, and $x = (0, 0)$ (see Figure 2). If $\kappa = 2$, then the two s–t paths, namely, (s, t) and (s, v, t), have the same cost of 25 but different power-costs: the power-cost of (s, t) is 25+25=50 while the power-cost of (s, v, t) is 9+16+16=41.

Our solution of MIN-POWER SYMMETRIC UNICAST first constructs an auxiliary directed graph $G' = (V', E', c')$ from the given graph $G = (V, E, c)$ and then runs Dijkstra's algorithm on G'. The construction of G' is as follows.

For each edge (u, v) of G we add to G' two vertices $[u, v]$ and $[v, u]$ and connect them by the two arcs $([u, v], [v, u])$ and $([v, u], [v, u])$, both of cost $c(u, v)$. Every vertex v of G is also preserved in G'. For every such v, we sort the vertices adjacent to it in G, say $\{u_1, \ldots, u_k\}$, such that $c(v, u_i) \leq c(v, u_{i+1})$ for every $1 \leq i < k$. Furthermore, we connect all vertices $[v, u_i]$'s by two directed paths, $P_1 = (v, [v, u_1], \ldots, [v, u_{k-1}], [v, u_k])$ and $P_2 = ([v, u_k], [v, u_{k-1}], \ldots, [v, u_1], u)$, see Figure 3(a). The costs of the arcs on path P_1 are set to $c(v, u_1)$, $c(v, u_2) - c(v, u_1)$, …, $c(v, u_k) - c(v, u_{k-1})$, respectively, while the costs of all arcs on path P_2 are set to zero. Figure 3(b) shows the graph G' for the example in Figure 2.

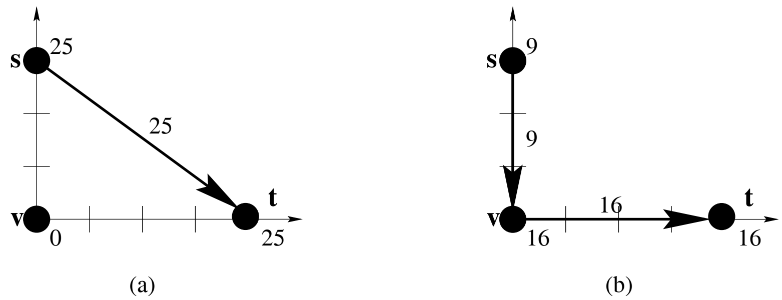


Figure 2: An example of two paths with the same cost and different power-costs. (a) The path (s, t) assigns powers 25 to s and to t. (b) The path (s, v, t) assigns powers 9 to s and 16 to v and t.

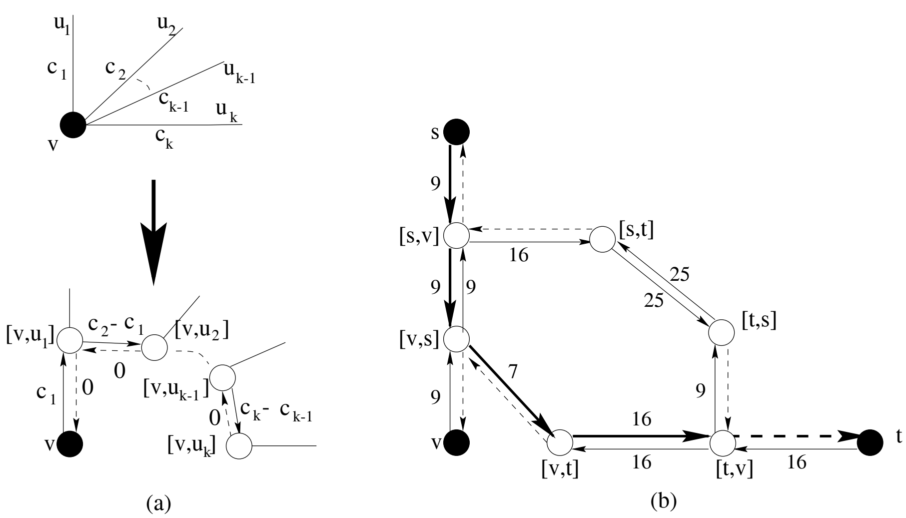


Figure 3: (a) A vertex v adjacent to k vertices $u_1, \ldots, u_k$ via edges of cost $c_1, c_2, \ldots, c_k$ and a gadget replacing v with a bidirectional path. The solid edges of the path $(v, [v, u_2])$, $([v, u_2])$, $[v, u_3]$,. . ., $([v, u_{k-1}], [v, u_k]$ have cost c_1, $c_2 - c_1, \ldots, c_k - c_{k-1}$, respectively. The dashed edges have zero cost. (b) The graph G' for the example in Figure 2. Thick edges belong to the shortest path corresponding to the path (s, v, t) in G.

We claim that every directed s–t path P in G corresponds to an s–t path P' in G' whose cost is equal to the power-cost of P. Indeed, consider a directed path $P = (s = w_1, w_2, \ldots, w_l = t)$ in G. By construction, there exists a directed path P' of G' visiting, in order, vertices w_1, $[w_1, w_2]$, $[w_2, w_1]$, …, $[w_{l-1}, w_l]$, $[w_l, w_{l-1}]$, w_l, such that

- The cost of the arc connecting w_1 to $[w_1, w_2]$ in P' is $c(w_1, w_2)$;
- The cost of the arc connecting $[w_{i-1}, w_i]$ to $[w_i, w_{i-1}]$ in P' plus the cost of the subpath connecting $[w_i, w_{i-1}]$ to $[w_i, w_{i+1}]$ in P' is equal to $\max\{c(w_{i-1}, w_i), c(w_i, w_{i+1})\}$ for every $2 \le i < l$;
- The cost of the arc connecting $[w_{l-1}, w_l]$ to $[w_l, w_{l-1}]$ is $c(w_{l-1}, w_l)$; and
- The cost of the subpath connecting $[w_l, w_{l-1}]$ to w_l is 0.

Therefore, the cost of P' equals the power-cost of P. It is not difficult to see that minimum power-cost paths in G are necessarily mapped by this correspondence to shortest paths in G' and thus MIN-POWER SYMMETRIC UNICAST reduces to computing a shortest path in G'.

Using the Fibonacci heaps implementation of Dijkstra's algorithm [10] to compute a shortest s–t path in G', and observing that $|V'| = O(|V| + |E|) = O(|E|)$ and $|E'| = O(|E|)$, we obtain the following:

Theorem 1 MIN-POWER SYMMETRIC UNICAST *is solvable in time* $O(|E| \log |V|)$.

Even in E^2, we have examples where the auxiliary graph is not planar, and we do not know faster methods to compute shortest paths in this auxiliary graph. When edge costs are integers we can use Thorup's single-source shortest path algorithm [28], reducing the runtime to $O(|V'| + |E'|) = O(|E|)$.

2.3 Broadcast and Multicast

The MIN-POWER ASYMMETRIC BROADCAST problem [26, 30] requires establishing a minimum power arborescence rooted at a given vertex s. Clementi et al. [8] prove that MIN-POWER ASYMMETRIC BROADCAST is NP-Hard when the nodes are in E^2. The best known approximation algorithm for MIN-POWER ASYMMETRIC BROADCAST [29], based on computing a minimum spanning tree, has performance ratio of at most 12 when the nodes are in E^2. We remark that, due to the need of establishing bidirectional connections, MIN-POWER SYMMETRIC BROADCAST and MIN-POWER SYMMETRIC CONNECTIVITY are the same problem. Implicit in the work of Kirousis, Kranakis, Krizanc, and Pelc [16] is the result that computing an MST gives a 2-approximation for MIN-POWER SYMMETRIC CONNECTIVITY, even in its graph formulation (see Theorem 2). In contrast, the graph version of MIN-POWER ASYMMETRIC BROADCAST cannot be approximated within a factor better than $(1 - o(1)) \ln n$ unless NP $\subseteq$ TIME($n^{O(\log \log n)}$) [14].

In MIN-POWER ASYMMETRIC MULTICAST, one is given a root s and a set of terminals T, and the goal is to establish a minimum-power branching rooted at s which reaches all vertices of T. As a generalization of MIN-POWER

ASYMMETRIC BROADCAST, MIN-POWER ASYMMETRIC MULTICAST is also NP-Hard, and the same method as in [29] implies that an approximate minimum Steiner tree gives a performance ratio of 12ρ, where ρ is the approximation for Steiner tree in graphs (the best result known at this moment, given in [24], is $\rho = 1 + \frac{1}{2}\ln 3 + \varepsilon$).

No previous results have been published for the multicast problem under the symmetric connectivity model. An immediate consequence of Theorem 2 is that a ρ-approximate minimum Steiner tree gives a performance ratio of 2ρ for MIN-POWER SYMMETRIC MULTICAST.

3 Integer Linear Program Formulation

In this section we give an integer linear program (ILP) formulation for MIN-POWER SYMMETRIC CONNECTIVITY and describe a branch and cut algorithm based on it. The results in Section 6 show that the algorithm is practical for instances with up to 35-40 nodes.

We begin by reformulating MIN-POWER SYMMETRIC CONNECTIVITY in graph theoretical terms. Let $G = (V, E, c)$ be an edge-weighted graph and uv denote the undirected edge between nodes u and v. The cost $c(uv)$ of an edge $uv \in E$ corresponds to the (symmetric) power requirement $p(u, v) = p(v, u)$. For a node $u \in V$ and a spanning tree T of G, let uu_T be the maximum cost edge incident to u in T, i.e., $uu_T \in T$ and $c(uu_T) \geq c(uv)$ for all $uv \in T$. The *power cost* of a spanning tree T is

$$p(T) = \sum_{u \in V} c(uu_T)$$

Since every connected graph contains a spanning tree, an equivalent formulation of MIN-POWER SYMMETRIC CONNECTIVITY is to ask for a spanning tree with minimum power-cost in the complete graph on V with edge costs given by $c(uv) = \|uv\|^\kappa$. Thus, MIN-POWER SYMMETRIC CONNECTIVITY can be reformulated as follows: Given a connected edge-weighted graph $G = (V, E, c)$, find a spanning tree T of G with minimum power-cost.

To formulate the problem as a linear integer program, we use two types of binary decision variables:

- x_{uv} for all $uv \in E$; x_{uv} is set to 1 if uv belongs to the selected spanning tree T and to 0 otherwise. We call these variables the *tree variables*; and
- $y_{\overline{uv}}$ for all $\overline{uv} \in \overline{E} := \{\overline{uv}, \overline{vu} \mid uv \in E\}$; $y_{\overline{uv}}$ is set to 1 if $u_T = v$ (i.e., if $uv \in T$ and $c(uv) \geq c(uw)$ for all $uw \in T$) and to 0 otherwise. We call these variables the *range variables*.

Note that there are $|E|$ tree variables and $|\overline{E}| = 2|E|$ range variables. Let ST be set of the incidence vectors of all spanning trees of G (viewed as subsets of E). Our ILP formulation is as follows.

$$\begin{aligned}
\min \quad & \sum_{\overline{uv} \in \overline{E}} c(uv) y_{\overline{uv}} & & \\
\text{s.t.} \quad & \sum_{v \in V \mid \overline{uv} \in \overline{E}} y_{\overline{uv}} = 1, & \forall u \in V & \qquad (1)
\end{aligned}$$

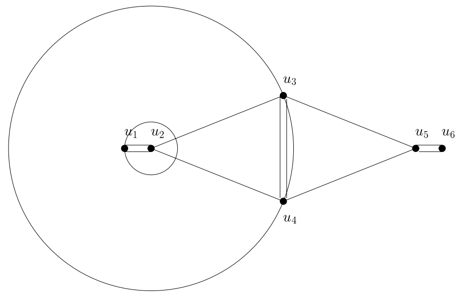


Figure 4: Let $x_e = 1/2$ for all edges in the picture ($x_e = 1$, if there are two parallel edges). Let range variables $y_{\overline{u_2 v}}$ be equal to 1/2 for $v = u_1, u_3$, and to 0 otherwise. Then constraints of type (1) and (2), are satisfied, but the constraint (4) is violated for $S = \{u_1, u_2\}$.

$$x_{uv} \leq \sum_{\overline{uw} \in \overline{E} | c(uw) \geq c(uv)} y_{\overline{uw}}, \qquad \forall \overline{uv} \in \overline{E} \tag{2}$$

$$x \in \text{conv}(ST) \tag{3}$$

$$x \in \{0,1\}^{|E|}$$

$$y \in \{0,1\}^{|\overline{E}|}$$

The constraints (1) enforce that we select exactly one range variable for every node $v \in V$, i.e., we properly define the range of each node. The constraints (2) enforce that an edge uv is included in the tree only if the range of each endpoint is at least the cost of the edge. The constraints (3) enforce that the tree variables indeed form a spanning tree. There are several well known linear descriptions for (3). We use the following, most famous formulation: $x \in \text{conv}(ST) \Leftrightarrow x \geq 0, \sum_{e \in E} x_e = |V| - 1$ and $\sum_{e \in \gamma(S)} x_e \leq |S| - 1$ for all $S \subseteq E$, where $\gamma(S)$ is the set of edges of E with both ends in S.

To solve the ILP we use branch and cut, i.e., we drop the integrality constraints and solve the corresponding LP relaxation. If the solution of the LP is integral, we have found the optimal solution, otherwise we select a variable with a fractional value and split the problem into two subproblems by setting the variable to 0 and 1 in the subproblems. We solve the subproblems recursively and disregard a subproblem if its LP bound is worse than the best known solution.

Since there are an exponential number of inequalities in this formulation of spanning trees, we can not solve the LP directly. Instead, we start with a small subset of these inequalities and algorithmically test whether the LP solution violates an inequality which is not in the current LP. If so, we add the inequality to the LP, otherwise we have found the solution of the LP with the exponential number of inequalities. The inequalities added to the LP if needed are called *cutting planes*, algorithms that find violated cutting planes are called *separation algorithms*.

In our case, the initial LP consists of the constraints (1) and (2), the constraint $\sum_{e \in E} x_e = |V| - 1$, and the bound constraints, i.e., the constraints $0 \leq x \leq 1$ and $0 \leq y \leq 1$. The only constraints added on demand are the constraints $\sum_{e \in \gamma(S)} x_e \leq |S| - 1$ for all $S \subseteq E$. A separation algorithm for these inequalities is due to Padberg and Wolsey [20].

The running time of a branch and cut algorithm can be improved by tightening the LP relaxation, i.e., by finding additional inequalities which are valid for all integer points, but may be violated by solutions to the LP relaxation (Figure 4 shows an example). We use the following class of valid inequalities. Let $S \subset V$. For every $u \in S$ let $u_S \in V \setminus S$ so that $c(uu_s) \leq c(uv)$ for all $v \in V \setminus S$. The inequality

$$\sum_{u \in S} \sum_{v \in V | c(uv) \geq c(uu_S)} y_{\overline{uv}} \geq 1 \tag{4}$$

is valid for the problem above. We can argue as follows. There is at least one edge in the spanning tree T crossing the cut S. Let uv be such an edge and $u \in S$. Then $c(uv) \geq c(uu_S)$ and the range of u is at least $c(uv)$. Thus $\sum_{v \in V | c(uv) \geq c(uu_S)} y_{\overline{uv}}$ is one and the inequality is valid.

Since we do not have a separation algorithm for these inequalities, we use the following heuristic to separate some of them. We chose an arbitrary node u. For every node $v \in V \setminus \{u\}$, we compute the minimal directed cut from u to v and from v to u, where the capacity of an edge xy is given by $\sum_{xw | c(xw) \geq c(xy)} y_{xw}$. For all computed cuts, we test whether the corresponding inequality is violated.

4 Analysis of the MST Algorithm

In this section we show that computing an MST gives a 2-approximation for MIN-POWER SYMMETRIC CONNECTIVITY; this result is implicit in the work of Kirousis, Kranakis, Krizanc, and Pelc [16]. Then we give an example showing that the approximation factor of 2 is tight, and discuss modifications of the MST algorithm for handling given bounds on node transmission ranges.

Theorem 2 *Let $G = (V, E, c)$ be an edge-weighted graph. Computing an MST with respect to c gives a 2-approximation for* MIN-POWER SYMMETRIC CONNECTIVITY.

Proof: Let $c(T) = \sum_{uv \in F} c(uv)$. Claim 2 of Theorem 3.2 in [16] is equivalent to

$$p(T) = \sum_{v \in V} \max_{u | uv \in F} c(uv) \leq \sum_{v \in V} \sum_{u | uv \in F} c(uv) = 2c(T) \tag{5}$$

Let u be a vertex incident to an edge of maximum cost. If we root the tree T at u, and use v' to denote the parent of v in T, since $\max_{u | uv \in F} c(uv) \geq c(vv')$ we conclude that $p(T) \geq c(T)$. Therefore, if MST is the minimum spanning tree with respect to c and OPT is the tree with minimum power-cost, we have

$$p(MST) \leq 2c(MST) \leq 2c(OPT) \leq 2p(OPT)$$

■

The following example shows that the ratio of 2 given in Theorem 2 is tight. Consider $2n$ points located on a single line such that the distance between consecutive points alternates between 1 and $\varepsilon < 1$ (see Figure 5) and let $\kappa = 2$. Then the minimum spanning tree MST connects consecutive neighbors and has power-cost $p(MST) = 2n$.

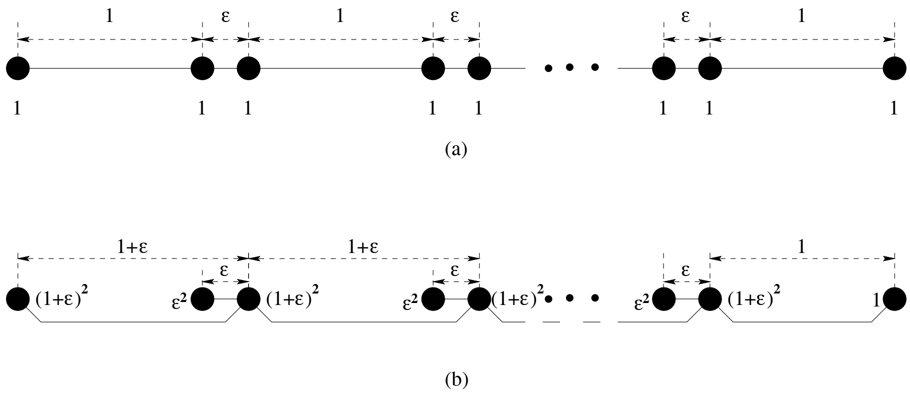


Figure 5: Tight example for the performance ratio of the MST algorithm ($\kappa = 2$). (a) The MST-based range assignment needs total power $2n$. (b) Optimum range assignment has total power $n(1+\varepsilon)^2 + (n-1)\varepsilon^2 + 1 \to n+1$.

On the other hand, the tree T with edges connecting each other node (see Figure 5(b)) has power-cost equal $p(T) = n(1+\varepsilon)^2 + (n-1)\varepsilon^2 + 1$. When $n \to \infty$ and $\varepsilon \to 0$, we obtain that $p(MST)/p(T) \to 2$.

Our MIN-POWER SYMMETRIC CONNECTIVITY formulation assumes that node transmission ranges can be arbitrary non-negative numbers. In practice node specific lower- and upper-bounds on the transmission ranges may be required. All the algorithms in this paper (including the MST algorithm) apply to the graph version of MIN-POWER SYMMETRIC CONNECTIVITY. Hence, they can easily handle upper-bounds on transmission ranges by assigning infinity cost to edges that cannot be established as bidirected links due to the imposed upper-bounds.

Handling the lower-bounds on transmission ranges is not straightforward. We propose the following modification of the MST algorithm.

1. Assign to each node the minimum allowed transmission range.

2. Compute the connected components in the graph induced by the biconnected links established by the assignment in Step 1.

3. For each two components C and C', compute a connection cost which is the minimum increase in power necessary to establish a bidirectional link between some vertex in C and some vertex in C'.

4. Construct a complete graph G' with the connected components as vertices and connection costs as edge costs.

5. Increase power ranges according to the MST in the graph G'.

Theorem 3 *The MST algorithm modified as above has an approximation factor of 2 for* MIN-POWER SYMMETRIC CONNECTIVITY *problem with lower-bounds on transmission ranges.*

5 k-Restricted Approach to Symmetric Min-Power Connectivity Approximation

We first give definitions of k-restricted decompositions and prove an upper bound on the power-cost of such decompositions. Then we will describe approximation algorithms whose approximation ratios follow from the performance ratios of Steiner tree algorithms in graphs.

5.1 k-Restricted Decompositions

A k-restricted decomposition Q of an undirected tree T is a partition of T into subtrees $T_1, T_2, \ldots, T_p$ each containing at most k vertices such that each edge of T belongs to exactly one subtree T_i. The power-cost $p(Q)$ of Q is defined to be the sum of the power-costs of all of its elements, i.e., $p(Q) = \sum_{T_i \in Q} p(T_i)$. The tight example for Theorem 5 in Figure 7 gives examples of 3-restricted decompositions.

The following theorem and its proof are similar to the results of [13, 4] on the k-restricted Steiner ratio. Our current theoretically best approximation algorithm does not make use of this theorem, but we use the theorem to establish the performance ratio of more practical algorithms derived from [2, 32].

Theorem 4 *For every weighted tree T and every $k \geq 1$, there is a 2^k-restricted decomposition Q of T such that $p(Q) \leq (1 + 1/k)p(T)$.*

Proof: Without loss of generality we can assume that all edge costs are different. Let the endpoints r and s of the heaviest edge h of T be the *roots* of T, which means that two subtrees of $T - \{h\}$ are rooted at r and s, respectively. Then each vertex v of T, except r and s, has a unique parent. We call the vertices adjacent to v, other than the parent of v (if defined), the children of v. For each vertex v of T, we sort the edges connecting v to its children in increasing order of their cost. For the most costly such edge e we define $next(e) = f$, where f is the edge connecting v to its parent (if v has a parent), or $f = h$ if v does not have a parent; for every other edge e we define $next(e) = e'$, where e' is the next edge (in the sorted order above) connecting v to one of its children.

We now construct a rooted directed binary (with arcs going toward the root) tree B as follows. The vertices of B are the edges of T and the root of B is h, the heaviest edge of T. The arcs of B consist of arcs $(e, next(e))$ for each edge e of T. It is immediate that every vertex $e = uv$ of B has at most two incoming arcs. Indeed, if $e = rs$, then only the most costly edge of $T \setminus \{e\}$ incident to r and the most costly edge of $T \setminus \{e\}$ incident to s have e as a parent. For each other edge $e = uv$ of T, where v is the parent of u, there is at most one arc coming into e from the vertex of B representing the most costly edge of $T \setminus \{e\}$ incident to u, and at most one arc coming into e from the vertex of B representing the edge of T between v and one of its children that precedes e in the sorted order above. Note that each vertex of B has an associated cost since it represents an edge of T.

Let B_i be the set of vertices of B in distance i from the root h. There is an integer $0 \leq l < k$ such that $\sum_{j \mid j \equiv l \pmod k} c(B_j) \leq \frac{1}{k} c(B) = \frac{1}{k} c(T)$, and let $\overline{B} = \cup_{j \mid j \equiv l \pmod k} B_j$. The removal of every edge outgoing from $\overline{B}$ decomposes B into subtrees Q_i corresponding to subtrees T_i of T. The number of vertices in Q_i is at most $2^k - 1$ since Q_i is a binary tree of height at most $k-1$. Therefore, each T_i has at most 2^k vertices. We denote by Q the 2^k-restricted decomposition of T into T_i's.

Let $e_i = (v_i, u_i)$ be the root of Q_i (note that $e_i \in \overline{B}$) and, if $e_i \neq (r, s)$, rename v_i and u_i such that u_i is the parent of v_i in T. By the construction of B, we have that $\max_{u \mid uu_i \in E(T_i)} c(uu_i) = c(e_i)$. Then we have:

$$p(T_i) \leq c(e_i) + \sum_{v \in V(T_i) \setminus \{u_i\}} \max_{(v,u) \in E(T)} c(v, u).$$

For $i \neq j$, the sets $V(T_i) \setminus \{u_i\}$ and $V(T_j) \setminus \{u_j\}$ are disjoint. We conclude that

$$\begin{aligned}
p(Q) &= \sum_i p(T_i) \\
&\leq \sum_{v \in V(T)} \max_{(v,u) \in E(T)} c(v, u) + \sum_i c(e_i) \\
&\leq p(T) + c(\overline{B}) \\
&\leq p(T) + \frac{1}{k} c(T) \\
&\leq (1 + \frac{1}{k}) p(T).
\end{aligned}$$

■

A subtree of T consisting of a pair of edges sharing a node is called a *fork*. So a 3-restricted decomposition Q of T consists of forks and individual edges. The following theorem is the analogue of the Steiner tree theorem in [31], but has a completely different proof.

Theorem 5 *For every tree T, there is a 3-restricted decomposition Q of T such that $p(Q) \leq \frac{5}{3} p(T)$.*

Proof: The proof proceeds in three steps. First we partition the edges of T into disjoint components using structural information derived from power requirements. Then we construct a weighted subgraph of the line graph of each component, which we refer to as the "consecutive" line graph. Finally, we show that the consecutive line graph of each component has a matching exceeding a certain weight; the edges in these matchings give the forks in the desired 3-restricted decomposition of T.

To describe how we partition the edges of T (see Figure 6(a)) we need to introduce some additional notations. Let $max(u)$ be the maximum edge of T incident to a vertex u.[1] For each vertex u, we direct the edge $max(u)$ away from u. An edge uv is called *root* if it is directed both ways (i.e., $max(u) = max(v) = uv$), and called *bridge* if it remains undirected (i.e., $max(u) \neq uv$ and $max(v) \neq uv$). In the power-cost of T, roots are counted twice (for both endpoints), bridges are not counted at all, and all other edges are counted exactly once. Thus, denoting by R the set of

[1] W.l.o.g., we assume that no two edges of T have the same cost.

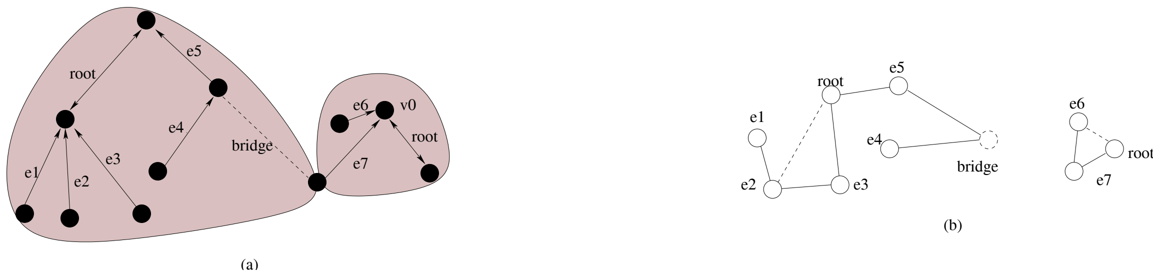


Figure 6: (a) Partitioned tree T. Each vertex has a single outgoing arc denoting its maximum incident edge, double arcs are roots and dashed edges are bridges. (b) Consecutive line graphs for the components. Vertices represent edges of T; "consecutive" forks of T are represented by the solid edges, "parity" edges are dashed.

roots and by B the set of bridges, we have:

$$p(T) = c(T) + c(R) - c(B) \tag{6}$$

The edges of T are partitioned as follows. First, we start with the connected components of $T - B$; note that each such component contains exactly one root. Then we add each bridge b of B to one of the two adjacent components of $T - B$, such that each component gets at most one bridge. A bridge assignment with this property is obtained by selecting an arbitrary vertex v_0 and assigning to each component of $T - B$ not containing v_0 the unique adjacent bridge on the path to v_0. We denote by $\mathcal{D}$ the resulting partition.

A fork $(e_1 = uv, e_2 = u'v)$ is called *consecutive* if $c(e_1) < c(e_2)$ and there is no edge e$\in D$ incident to v such that $c(e_1) < c(e) < c(e_2)$. For each component $D \in \mathcal{D}$, the *consecutive line graph* L_D is defined as follows (see Figure 6(b)):

– vertices of L_D are the edges of D

– L_D has "consecutive" edges connecting each consecutive forks of D, and at most two "parity" edges connecting the root of D and the second most expensive non-root edge incident to each end of the root

– for every edge (e_1, e_2) of L_D, $w(e_1, e_2) = \min\{c(e_1), c(e_2)\}$

By construction, each edge of L_D corresponds to a fork of D. Therefore, each matching X of L_D corresponds to a 3-restricted decomposition of D (edges of X correspond to forks and isolated vertices correspond to isolated edges) which we denote Q_X. It is easy to see that $p(Q_X) = 2c(D) - w(X)$.

The theorem follows if, for each $D \in \mathcal{D}$, we find a matching X_D in L_D such that

$$w(X_D) \geq \frac{c(D) - c(r_D) + c(b_D)}{3} \tag{7}$$

where $c(D)$ is the total cost of the edges in D, r_D is the single root in D, and b_D is the single bridge in D, if one exists. Indeed,

$$p\Big(\bigcup_{D\in\mathcal{D}} Q_{X_D}\Big) \quad = \quad \sum_{D\in\mathcal{D}} (2c(D) - w(X_D))$$

$$
\begin{aligned}
&\leq \sum_{D\in\mathcal{D}} \left(\frac{5}{3}c(D)+\frac{1}{3}c(r_D)-\frac{1}{3}c(b_D)\right)\\
&= \frac{5}{3}c(T)+\frac{1}{3}c(R)-\frac{1}{3}c(B)\\
&\leq \frac{5}{3}p(T)
\end{aligned}
$$

where the last inequality comes from (6) and the fact that $c(T) \leq p(T)$, as in the proof of Theorem 2.

By Edmonds' theorem [19] it is sufficient to construct a fractional matching X_D satisfying (7). A *fractional matching* of L_D is an assignment of nonnegative fractions $x(e_1, e_2)$ to every edge $(e_1, e_2) \in L_D$ such that

(i) the sum of fractions assigned to the edges incident to a vertex e of L_D is at most 1, and

(ii) the sum of fractions assigned to all edges with both endpoints in a set of $2k+1$ vertices of L_D is at most k.

The weight of a fractional matching X_D is given by

$$w(X_D) = \sum_{(e,e')\in E(D)} x(e,e')w(e,e')$$

We construct a fractional matching X_D by assigning $1/3$ to each consecutive edge (e_1, e_2) of L_D. This fractional matching satisfies (i) since each $e \in D$ is incident to at most 3 consecutive edges of L_D (if e is not the root r_D, then it participates to one consecutive edge of L_D as e_1, and to at most two edges as e_2; the root participates as the heaviest end in up to two edges). Condition (ii) follows from the fact that consecutive edges form a tree. Since every vertex e of L_D except the root participates in exactly one consecutive fork (e_1, e_2) as e_1, we get that the weight of X'_D is equal to $(c(D) - c(r_D))/3$.

If D has no bridge then (7) follows. Otherwise we modify X_D such that the weight increases by $c(b_D)/3$ as follows. Let $P = (b_D = e_0, f_0, e_1, f_1, ..., e_k, f_k, e_{k+1} = r_D)$ be the unique path of consecutive edges of L_D, where $f_i = (e_i, e_{i+1})$, $i = 1, \ldots, k$ are edges of L_D corresponding to consecutive forks in D. We add 1/3 to $x(f_i)$, $i = 0, 2, 4, \ldots$, and subtract 1/3 from $x(f_i)$, $i = 1, 3, \ldots$. Since both b_D and r_D participate in at most two consecutive forks, the above change leads to a feasible fractional matching (the sum of fractions assigned to the edges incident to each intermediate vertex of P remains the same). If k is even then the total weight of X_D increases by at least $c(b_D)/3$ since $w(f_{2l-1}) = c(e_{2l-1}) < c(e_{2l}) = w(f_{2l})$, $l = 1, \ldots, k/2$ and we are done.

If k is odd we add back 1/3 to $x(f_k)$ to guarantee increasing $w(X_D)$ by at least $c(b_D)/3$. If e_k has degree 2 in L_D then we are done, since the sum of all fractions assigned to the edges incident to e_k equals to 1. Otherwise, e_k has degree 3 and we need to further modify X_D in order to make it a feasible fractional matching. Let v be the vertex of T common to e_k and r_D. Since $f_k = (e_k, e_{k+1} = r_D)$ is a consecutive fork, e_k is the most expensive non-root edge of D incident to v. Let e be the second most expensive non-root edge of D incident to v. Since e and e_k form a consecutive fork, L_D contains the edge (e, e_k). Recall that L_D also contains a parity edge (e, r_D). We modify X_D as follows:

(1) If $e_{k-1} \neq e$ (i.e., e_{k-1} is not adjacent to the root), then we subtract 1/3 from $x(e, e_k)$ and set $x(e, r_D)$ to 1/3.

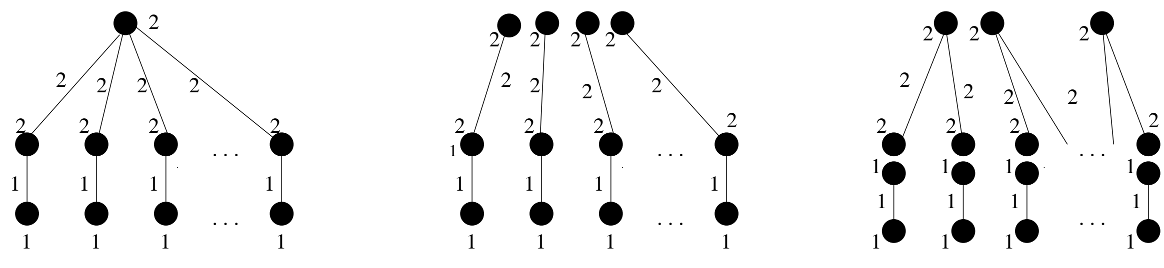


Figure 7: (a) Tight example for Theorem 5: a single node is connected via cost-2 edges to k nodes, each of which is in turn connected via a cost-1 edge to a leaf. The total power-cost of this tree is $2 + 2k + k = 3k + 2$. (b-c) Two minimum 3-restricted decompositions: the power-cost of (b) is $5k$ since each of k forks has power-cost 5; and the power-cost of (c) is $6\frac{k}{2} + 2k = 5k$ since each of $\frac{k}{2}$ upper forks has power-cost 6 and each of k single-edge components has power-cost 2.

(2) If $e_{k-1} = e$ (i.e., e_{k-1} is adjacent to the root), then we subtract 1/3 from $x(f_{k-1})$ and set $x(e = e_{k-1}, r_D)$ to 1/3.

In both cases, the resulting sums of fractions assigned to the edges incident to e_k, respectively to r_D, are equal to 1, and hence X_D satisfies (i). In case (1), the condition (ii) is valid since edges with non-zero fraction in X_D continue to form a tree. In case (2), the condition (ii) is still valid: the graph given by the edges with non-zero fraction has only one cycle, and therefore any set of $2k+1$ vertices of L_D induces a subgraph with at most $2k+1$ edges with non-zero fraction (each of them having fraction $1/3$). ∎

Remark: The bound of Theorem 5 is tight (see Figure 7).

5.2 Approximation Algorithms

All approximation algorithms described below have approximation ratios defined in terms of ρ_k, where ρ_k is the supremum, over all trees T, of the ratio of the power-cost of the minimum power-cost k-restricted decompositions to the power-cost of T. Theorem 4 implies that $\rho_k \leq 1 + \frac{1}{\lfloor \lg k \rfloor}$, in particular $\rho_4 \leq \frac{3}{2}$. Theorem 5 together with the example in Figure 7 imply that $\rho_3 = 5/3$, while Theorem 2 together with the example in Figure 5 imply that $\rho_2 = 2$.

The following *Greedy Fork-Contraction* (GFC) algorithm, originally formulated for Steiner trees, is based on the notion of *gain*, defined below. For a graph G, denote by $mst(G)$ the minimum cost of a spanning tree. For a set of vertices $V' \subseteq V(G)$, we denote by G/V' the graph obtained after contracting V', i.e., collapsing all vertices of V' into a single vertex. Let G be obtained from G_0 after contracting some subsets of vertices, H be a subtree of G_0, and $V_G(H)$ be the set of vertices of G which, seen as subsets of $V(G_0)$, intersect $V(H)$. The *gain* of H with respect to G is:

$$gain_G(H) = 2mst(G) - 2mst(G/V_G(H)) - p(H)$$

Input: Edge-weighted graph $G_0 = (V, E, c)$

Output: Spanning tree of G_0

$G \leftarrow G_0, H \leftarrow \emptyset$

Repeat forever

Find a fork K from G_0 with maximum $g = gain_G(K)$

If $g \leq 0$ **then exit repeat**

$H \leftarrow H \cup K, G \leftarrow G/V_G(K)$

Output $MST(G) \cup H$

Figure 8: The Greedy Fork-Contraction algorithm.

where $p(H)$ is the power-cost of H in the original graph G_0. It has been proved in [31] that the GFC algorithm described in Figure 8 has a performance ratio no larger than the arithmetic mean of ρ_2 and ρ_3. Thus we have:

Theorem 6 *The GFC algorithm for* MIN-POWER SYMMETRIC CONNECTIVITY *has performance ratio of* $11/6$.

A fully polynomial approximation scheme for finding optimal 3-restricted Steiner trees is given in [21], building on [5]. Theorem 5 implies our main result:

Theorem 7 *The algorithm of [21] has a performance ratio of* $\frac{5}{3} + \epsilon$ *for* MIN-POWER SYMMETRIC CONNECTIVITY.

Unfortunately, this algorithm is impractical. It is also possible to apply other Steiner tree algorithms, e.g., the algorithm in [2] gives an approximation factor of $\frac{\rho_2}{2} + \frac{\rho_3}{6} + \frac{\rho_4}{3} \leq \frac{16}{9}$, while the k-restricted Relative Greedy Algorithm in [32] gives a factor of $1 + \ln 2 + \epsilon$.

6 Experimental Study

We have implemented the exact branch and cut algorithm described in Section 3 (OPT) and the greedy fork-contraction algorithm in Figure 8 (GFC). Since there are no existing algorithms to compare against, to provide a better basis for assessing the quality of these algorithms we have included in our experimental study three simple and natural heuristics:

- A simple edge-switching (ES) heuristic that starts from the MST, and repeatedly replaces a tree edge with a non-tree edge re-establishing connectivity. At every step, the algorithm chooses the pair of edges that results in the largest reduction in power cost; the process is repeated as long as improvement is still possible. We simulated a distributed implementation of the algorithm in which only non-tree edges that connect nodes within 10 tree-hops from each other are considered for switching.

- A heuristic performing both edge and fork switching (EFS). At every step the algorithm chooses an edge or fork whose addition to the tree leads to the largest reduction in power cost. Unlike GFC, forks are not contracted, which means that an edge of an added fork can be later removed from the tree by other edge or fork switches.

- A Kruskal-like heuristic (KR) that starts with isolated nodes and iteratively adds an edge connecting two different components with *minimum increase* in power cost. A similar heuristic (called incremental search) was studied by Chu and Nikolaidis for computing low-power MIN-POWER ASYMMETRIC BROADCAST trees in a mobile environment [7].

We included in our comparison faster versions of OPT and GFC, OPT-D and GFC-D, which speed-up the computation by working on the Delaunay graph (see, e.g., [12]) defined by the nodes instead of the complete graph. We also implemented a faster version of EFS, EFS-D, in which only forks consisting of Delaunay edges (but still all non-tree edges) are considered as switching candidates.

Note that, by Theorem 2, both ES and EFS produce solutions within a factor of 2 of optimum since they improve upon an MST for the nodes. A performance of ratio of 2 can be proven for KR as well. Define a new cost function $\overline{c}(e)$ as follows: if e is not picked by the KR, then $\overline{c}(e) = c(e)$, else $\overline{c}(e)$ is the increase in power cost used by KR to pick e. It can be proven that the minimum spanning tree in $(V, E, \overline{c})$ is the same as the tree picked by KR in G, and since for every $e \in E$ we have $\overline{c}(e) \leq c(e)$, the optimum solution in $(V, E, \overline{c})$ has power at most the optimum power in G. An example showing that the performance ratio of 2 is tight for KR in the graph model is given below; the exact performance ratio in E^2 is not known. The $q + 3$ vertices are $v_0, v_1, v_2, \ldots, v_{q+2}$, and the edges have cost: for $i = 0, 1, \ldots, q$, $c(v_i v_{q+1}) = 1$ and $c(v_i v_{q+2}) = 2 - \frac{1}{2^i} - \varepsilon$, and $c(v_{q+1} v_{q+2}) = \epsilon$. KR builds a star centered at v_{q+2} with a power-cost of about $2q$, while the optimum solution is a star centered at v_{q+1} with a power-cost of about q.

All algorithms were implemented in C++, including the branch and bound algorithm whose implementation is built on SCIL [25]. The heuristics were compiled using `gpp` with `-O2` optimization, and run on an AMD Duron 600MHz PC. The experiments were run on randomly generated testcases. For each instance size n between 10 and 100, in increments of 5, 50 different instances were generated by choosing n points uniformly at random from a grid of size $10,000 \times 10,000$.

Table 1 gives the percent improvement over MST and the runtimes for the compared algorithms; solution quality is also presented in graphical form in Figure 9. We report averages over 50 instances of each size; averages marked with an asterisk do not include two instances not solved within one day. The results show that OPT has a practical running time up to 35 nodes, and produces an average improvement over MST of 5-6%. The Delaunay version of OPT has practical runtime up to 60 nodes, but gives slightly worse solutions.

The GFC algorithm, its faster Delaunay version, GFC-D, as well as the natural Kruskal-like heuristic KR are all very fast, but give less than half of the optimum improvement. KR consistently outperforms GFC, while the latter consistently outperforms GFC-D (the runtime of GFC-D is identical to that of GFC in our experiments). The EFS,

n	OPT		OPT-D		ES		EFS		EFS-D		KR		GFC		GFC-D	
	%	CPU	%	CPU	%	CPU	%	CPU	%	CPU	%	CPU	%	CPU	%	CPU
10	4.01	0.67	3.66	0.10	3.81	0.00	4.00	0.00	3.94	0.00	0.49	0.00	1.39	0.00	1.19	0.00
15	4.77	5.68	4.26	0.43	4.48	0.00	4.70	0.02	4.51	0.00	1.72	0.00	1.56	0.00	0.48	0.00
20	5.84	22.2	5.17	1.19	5.46	0.00	5.75	0.10	5.47	0.00	2.54	0.00	2.01	0.00	1.40	0.00
25	5.63	58.9	4.72	3.46	4.78	0.00	5.53	0.26	5.12	0.00	2.19	0.00	1.56	0.00	0.72	0.00
30	5.46	201	4.90	6.49	4.87	0.00	5.36	0.61	5.03	0.00	1.77	0.00	1.65	0.00	0.24	0.00
35	5.68	712	5.11	11.2	5.04	0.00	5.60	1.16	5.40	0.02	2.13	0.01	1.93	0.00	0.96	0.00
40	5.41*	4725*	4.82	52.1	5.01	0.00	5.51	2.13	5.25	0.03	1.82	0.01	1.37	0.00	0.26	0.00
45	—	—	5.37	109	5.13	0.00	5.77	3.71	5.47	0.05	2.17	0.00	2.22	0.03	0.67	0.03
50	—	—	5.36	181	5.55	0.02	5.90	5.50	5.62	0.05	2.45	0.00	2.03	0.02	0.33	0.02
55	—	—	6.09	653	5.61	0.05	6.54	9.03	6.21	0.05	2.65	0.00	2.60	0.03	1.19	0.03
60	—	—	5.46*	573*	5.25	0.05	6.06	12.48	5.73	0.06	2.31	0.00	2.15	0.05	0.50	0.05
65	—	—	—	—	5.01	0.05	5.80	17.9	5.56	0.09	2.30	0.04	1.65	0.03	0.38	0.03
70	—	—	—	—	5.12	0.03	6.01	25.5	5.60	0.10	2.41	0.04	1.94	0.01	0.24	0.01
75	—	—	—	—	5.10	0.02	5.78	33.4	5.50	0.09	2.46	0.02	1.69	0.00	0.48	0.00
80	—	—	—	—	5.14	0.05	6.03	44.9	5.77	0.12	2.88	0.00	2.00	0.00	0.64	0.00
85	—	—	—	—	4.73	0.06	5.69	55.0	5.37	0.16	2.52	0.00	1.82	0.00	0.39	0.00
90	—	—	—	—	5.42	0.09	6.30	75.5	6.01	0.21	2.84	0.00	2.18	0.00	0.38	0.00
95	—	—	—	—	5.29	0.11	6.08	101	5.81	0.26	2.35	0.00	1.73	0.05	0.19	0.05
100	—	—	—	—	5.45	0.14	6.25	123	6.09	0.32	2.56	0.00	2.30	0.05	0.99	0.05

Table 1: Average percent improvement over the MST (%) and runtime in seconds (CPU) for the compared algorithms.

EFS-D, and and even the distributed ES heuristic give significantly better solution quality, coming on the average within a fraction of a percent of the optimal improvement, still with a very well scaling runtime.

7 Conclusions

In a more realistic power-attenuation model, the power requirement for supporting a link from node u to node v separated by a distance r is given by

$$p(u,v) = \frac{r^{\kappa_{uv}}}{\chi_u \sigma_v} \tag{8}$$

where $\chi_u > 0$ is the transmission efficiency of node u, $\sigma_v > 0$ is the signal detection sensitivity threshold of node v, and κ_{uv} is the signal attenuation exponent for the link from u to v. In [1] we show that the corresponding MIN-POWER SYMMETRIC CONNECTIVITY WITH ASYMMETRIC POWER REQUIREMENTS is inapproximable within factor $(1-\epsilon)\ln|V|$ for any $\epsilon > 0$ unless $P = NP$. The proof in [1] relies on using non-uniform signal attenuation exponents κ_{uv}. An interesting open problem is to settle the approximability status of MIN-POWER SYMMETRIC CONNECTIVITY with uniform exponents.

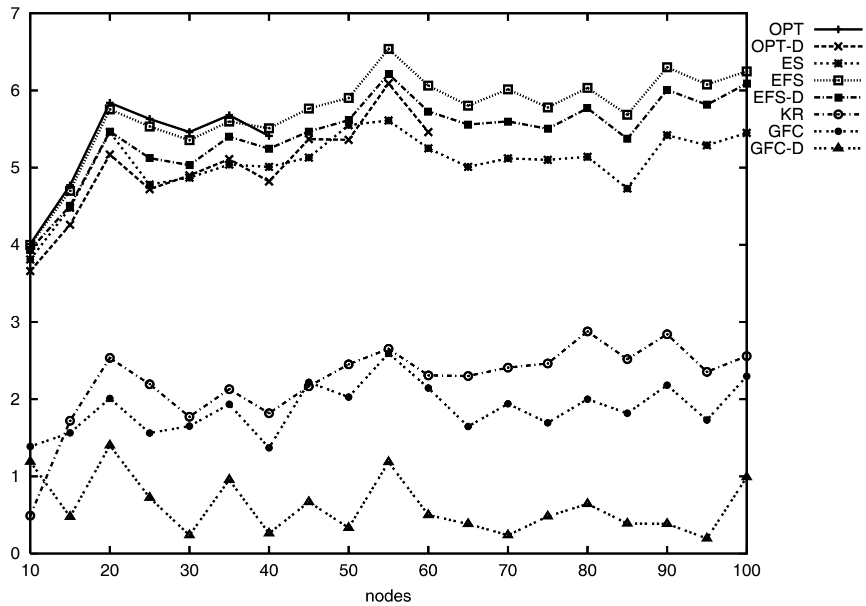


Figure 9: Average percent improvement over the MST for the compared algorithms.

It is also an open question whether MIN-POWER SYMMETRIC CONNECTIVITY can be reduced to the classical STEINER TREE problem in an approximation preserving manner. Such a reduction would allow other well-known STEINER TREE heuristics, such as the 1-Steiner algorithm [15], to be applied to MIN-POWER SYMMETRIC CONNECTIVITY.

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