

# Power Flow in Four-Wire Distribution Networks – General Approach

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**Abstract--** The neutral wire in most power flow software is usually merged into phase wires using Kron's reduction. Since the neutral wire and the ground are not explicitly represented, neutral wire and ground currents and voltages remain unknown. In some applications, like power quality and safety analyses, loss analysis, etc., knowing the neutral wire and ground currents and voltages could be of special interest. In this paper, a general power flow algorithm for three-phase four-wire radial distribution networks, considering neutral grounding, based on backward-forward technique, is proposed. In this novel use of the technique, both the neutral wire and ground are explicitly represented. A problem of three-phase distribution system with earth return, as a special case of a four-wire network, is also elucidated. Results obtained from several case studies using medium and low voltage test feeders with unbalanced load, are presented and discussed.

**Index Terms--** Distribution networks, power flow, three-phase four-wire, earth return, unbalance.

## I. INTRODUCTION

THREE-phase four-wire distribution networks (DNs) are widely adopted in modern power distribution systems [1], [2], [3], [4] and [5]. A multi-grounded three-phase four-wire service has higher sensitivity for fault protection than a three-phase three-wire service. The return current is due to both the unbalanced load and the non-linear characteristics of electrical equipment through the distribution feeder. The return current may be larger than the phase currents if three-phase loads are seriously unbalanced in some segments. The neutrals play an important role in power quality and safety problems [6], [7] and [8]. The design of neutrals varies widely from country to country and even from utility to utility within the same country. The different technical solutions considering the neutral in medium voltage are: systems without neutral (Brazil), systems with an isolated neutral (Italy, Finland, Switzerland), systems with multiple grounding of the neutral and connecting the utility neutral with customer grounding (USA, Greece), systems with solid neutral grounding (UK), systems with resistance neutral grounding (France, UK), systems with reactance neutral grounding (Belgium, Spain, Portugal, Netherlands), and systems compensated via a

"Petersen" coil (Germany), to name a few [3].

An efficient and robust three-phase power flow algorithm for large-scale DNs with dispersed generators is the chief tool for a Distribution Management System (DMS) today. In the last decade, different procedures have been proposed for distribution power flow [9], [10], [11], [12], [13], [14] and [15]. Experience demonstrates that very good results are obtained using the backward-forward procedure while handling large-scale real-life DNs. These methods may be classified as current summation methods, power summation methods and admittance summation methods.

The neutral wire in the existing power flow software is usually merged into phase wires using Kron's reduction. Since the neutral wire and ground are not explicitly represented, neutral wire and ground currents and voltages remain unknown. In some applications like power quality and safety analyses, loss analysis etc., it is of special interest to ascertain the neutral wire and ground currents and voltages.

In order to calculate neutral wire and ground currents and voltages, here the efficient and robust three-phase branch-oriented backward-forward procedure [10] is generalized. It is expanded the  $3 \times 3$  network representation to a  $5 \times 5$  representation, considering three-phase wires, neutral wire and assumed ground wire. The developed distribution power flow method is of general usage since it can be applied for most of the existing DNs: medium voltage (MV); low voltage (LV); three-wire, four-wire, with (solidly) grounded neutral, or isolated neutral wire; and three-wire and single-wire DNs with earth return. A problem of a three-phase distribution system with an earth return distribution [16] is also elucidated by the employment of the proposed generalized power flow algorithm, as a special case of a four-wire network, which can be found in Brazil, South Africa, Iceland and Australia, among others.

The main objectives of this work are to demonstrate that higher order line models can be included in the procedure of [10] with minor changes and then to determine the effect of this inclusion on the accuracy of the solution.

## II. POWER FLOW ALGORITHM

In the three-phase power flow algorithm, each node or line section in the network is numbered by a single index, regardless of the number of phases of this node or line section [10]. Figure 1 represents line section  $l$  between nodes  $i$  and  $j$  with shunt admittances and loads attached to the nodes.

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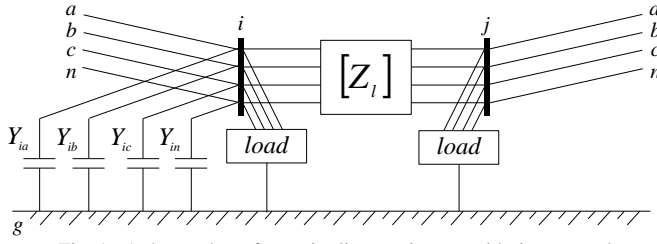


Fig. 1. A three-phase four-wire line section, considering ground.

Based on Fig. 1, a 5x5 matrix can represent the series impedance  $Z_l$  of the line section  $l$ :

$$[Z_l] = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} & Z_{ag} \\ Z_{ab} & Z_{bb} & Z_{bc} & Z_{bn} & Z_{bg} \\ Z_{ac} & Z_{bc} & Z_{cc} & Z_{cn} & Z_{cg} \\ Z_{an} & Z_{bn} & Z_{cn} & Z_{nn} & Z_{ng} \\ Z_{ag} & Z_{bg} & Z_{cg} & Z_{ng} & Z_{gg} \end{bmatrix} \quad \text{where}$$

$a, b, c$  – phase lines  
 $n$  – neutral wire  
 $g$  – ground

If any phase, neutral wire or grounding of the line section does not exist, the corresponding row and column in this matrix contain all zero entries. The shunt capacitance part of LV lines can be neglected. Branch numbering scheme for radial DN is exactly like in [10].

#### A. Model

Model of the three-phase four-wire multi-grounded distribution line is shown on Fig. 2.

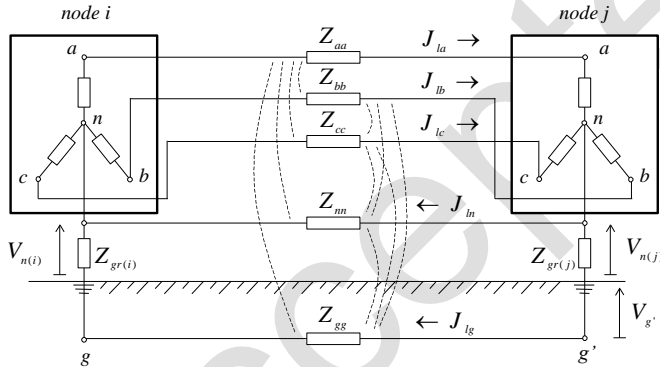


Fig. 2. Model of the three-phase four-wire multi-grounded distribution line.

#### B. Power Flow Algorithm

Assuming the root node to be the slack node with known voltage magnitude and angle, here the procedure from [10] is generalized. The proposed 5x5 iterative algorithm to solve the radial system consists of three steps, as follows.

At iteration  $k$ :

##### 1. Nodal current calculation

$$\begin{bmatrix} I_{ia} \\ I_{ib} \\ I_{ic} \\ I_{in} \\ I_{ig} \end{bmatrix}^{(k)} = \begin{bmatrix} (S_{ia}/V_{ia})^{(k-1)*} \\ (S_{ib}/V_{ib})^{(k-1)*} \\ (S_{ic}/V_{ic})^{(k-1)*} \\ -\frac{Z_{gi}}{Z_{ni} + Z_{gi}}(I_{ia}^{(k)} + I_{ib}^{(k)} + I_{ic}^{(k)}) \\ -\frac{Z_{mi}}{Z_{ni} + Z_{gi}}(I_{ia}^{(k)} + I_{ib}^{(k)} + I_{ic}^{(k)}) \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} Y_{ia} \\ Y_{ib} \\ Y_{ic} \\ Y_{in} \\ 0 \end{bmatrix} \begin{bmatrix} V_{ia} \\ V_{ib} \\ V_{ic} \\ V_{in} \\ V_{ig} \end{bmatrix}^{(k-1)}$$

where  
 $I_{ia}, I_{ib}, I_{ic}, I_{in}, I_{ig}$  are current injections at node  $i$ ;  
 $S_{ia}, S_{ib}, S_{ic}$  are scheduled (known) power injections at node  $i$ ;  
 $V_{ia}, V_{ib}, V_{ic}, V_{in}, V_{ig}$  are voltages at node  $i$ ;  
 $Y_{ia}, Y_{ib}, Y_{ic}, Y_{in}$  are admittances of all shunt elements at node  $i$ ; and  
 $Z_{gri}$  is grounding impedance at node  $i$  ( $Z_{gri} = Z_{gri} + Z_{ggi}$ ).

##### 2. Backward sweep – section current calculation

Starting from the line section in the last layer and moving towards the root node, the current in line section  $l$  is:

$$\begin{bmatrix} J_{la} \\ J_{lb} \\ J_{lc} \\ J_{ln} \\ J_{lg} \end{bmatrix}^{(k)} = \begin{bmatrix} I_{ja} \\ I_{jb} \\ I_{jc} \\ I_{jn} \\ I_{jg} \end{bmatrix}^{(k)} + \sum_{m \in M} \begin{bmatrix} J_{ma} \\ J_{mb} \\ J_{mc} \\ J_{mn} \\ J_{mg} \end{bmatrix}^{(k)} \quad (2)$$

where

$J_{la}, J_{lb}, J_{lc}, J_{ln}, J_{lg}$  are the current flows on line section  $l$ ; and  
 $M$  is the set of line sections connected downstream to node  $j$ .

##### 3. Forward sweep – nodal voltage calculation

Starting from the first layer and moving towards the last layer, the voltage at node  $j$  is:

$$\begin{bmatrix} V_{ja} \\ V_{jb} \\ V_{jc} \\ V_{jn} \\ V_{jg} \end{bmatrix}^{(k)} = \begin{bmatrix} V_{ia} \\ V_{ib} \\ V_{ic} \\ V_{in} \\ V_{ig} \end{bmatrix}^{(k)} - \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} & Z_{ag} \\ Z_{ab} & Z_{bb} & Z_{bc} & Z_{bn} & Z_{bg} \\ Z_{ac} & Z_{bc} & Z_{cc} & Z_{cn} & Z_{cg} \\ Z_{an} & Z_{bn} & Z_{cn} & Z_{nn} & Z_{ng} \\ Z_{ag} & Z_{bg} & Z_{cg} & Z_{ng} & Z_{gg} \end{bmatrix} \begin{bmatrix} J_{la} \\ J_{lb} \\ J_{lc} \\ J_{ln} \\ J_{lg} \end{bmatrix}^{(k)} \quad (3)$$

##### Voltage correction

Voltage correction in the nodes with grounded neutral and downstream nodes should be performed. Voltage in the nodes with neutral grounded at iteration  $k$  is as follows:

$$V_{in}^{(k)} = Z_{gri} J_{ig}^{(k)}, \quad i \in \{\text{Grounded nodes}\} \quad (4)$$

##### Convergence criterion

After these steps are executed in one iteration, the power

mismatches at each node for all phases, neutral wire and ground are calculated:

$$\begin{aligned}\Delta S_{ia}^{(k)} &= V_{ia}^{(k)} \left( I_{ia}^{(k)} \right)^* - Y_{ia}^* |V_{ia}^{(k)}|^2 - S_{ia} \\ \Delta S_{ib}^{(k)} &= V_{ib}^{(k)} \left( I_{ib}^{(k)} \right)^* - Y_{ib}^* |V_{ib}^{(k)}|^2 - S_{ib} \\ \Delta S_{ic}^{(k)} &= V_{ic}^{(k)} \left( I_{ic}^{(k)} \right)^* - Y_{ic}^* |V_{ic}^{(k)}|^2 - S_{ic} \\ \Delta S_{in}^{(k)} &= V_{in}^{(k)} \left( I_{in}^{(k)} \right)^* - Y_{in}^* |V_{in}^{(k)}|^2 \\ \Delta S_{ig}^{(k)} &= V_{ig}^{(k)} \left( I_{ig}^{(k)} \right)^*\end{aligned}\quad (5)$$

If the real or imaginary part of any of the power mismatches is greater than a convergence criterion, steps 1, 2 and 3 are repeated until convergence is achieved.

#### Flat start

The initial voltage for all nodes should be equal to the root node voltage:

$$\begin{bmatrix} V_{ia} \\ V_{ib} \\ V_{ic} \\ V_{in} \\ V_{ig} \end{bmatrix}^{(0)} = \begin{bmatrix} V_{ref} \\ a^2 \cdot V_{ref} \\ a \cdot V_{ref} \\ 0 \\ 0 \end{bmatrix}, \quad a = e^{j\frac{2\pi}{3}}.\quad (6)$$

Equations for calculation of the impedance matrix in a 5x5 representation are given in the Appendix. Most of the existing models of three-phase elements of the distribution network (lines, loads, shunt capacitors, distribution transformers, dispersed generators) can be applied in the proposed 5x5 power flow method according to the general form given in [15].

### III. TEST NETWORKS

The proposed power flow method using a 5x5 network representation is applied on numerous MV and LV DNs. In this paper the following networks are used:

- Four-wire three-phase MV feeder with isolated neutral (without ground conductor),
- Three-phase MV line with earth return (without neutral wire), and
- Four-wire three-phase LV distribution network.

The test network used in cases A and B is the IEEE 34-bus radial distribution network [17] and [18], Fig. 3. Base voltage of the network is  $V_b = 24.9$  kV, and the reference voltage in the root node is  $V_{ref} = 25.647$  kV. Simplifying, the autotransformer 24.9/4.16 kV/kV in the original IEEE 34-bus test feeder is replaced with the line and the network is modeled with the single voltage level. The automatic voltage regulator is also not represented. Mutual and self ground impedances  $Z_{ag}$ ,  $Z_{bg}$ ,  $Z_{cg}$ ,  $Z_{ng}$  and  $Z_{gg}$  (elements of the fifth row and fifth column of the 5x5 matrix) are computed as shown in Appendix. The values for the IEEE 34-bus DN are shown in Table I. It should be noted that mutual and self impedances of phases  $a$ ,  $b$ ,  $c$  and neutral (additional elements of the 5x5) are performed using Carson's equations, considering the ground as a perfect conductor [2]. For space reason, it is solely presented in Table II the 5x5 matrix for the 1-2 line section. The assumed ground conductor impedance is developed based

on voltage drop equations for the three-phase line with earth return [2] and full Carson's impedance calculation [16].

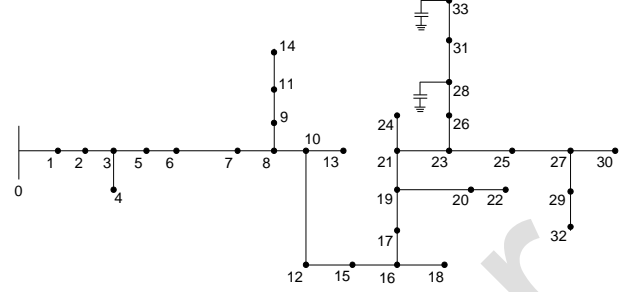


Fig. 3. IEEE 34-bus MV test feeder, Cases A and B.

TABLE I  
MUTUAL AND SELF GROUND IMPEDANCE – IEEE 34-BUS DN  
(SD: SENDING BUS AND RV: RECEIVING BUS)

SD	RV	$Z_{ag}$ ( $\Omega$ )	$Z_{bg}$ ( $\Omega$ )	$Z_{cg}$ ( $\Omega$ )	$Z_{ng}$ ( $\Omega$ )	$Z_{gg}$ ( $\Omega$ )
0	1	j 0.0560	j 0.0560	j 0.0560	j 0.0514	0.0466 + j 0.3437
1	2	j 0.0375	j 0.0375	j 0.0375	j 0.0345	0.0312 + j 0.2305
2	3	j 0.6994	j 0.6994	j 0.6994	j 0.6425	0.5816 + j 4.2939
3	4	j 0.0000	j 0.1283	j 0.0000	j 0.1157	0.1047 + j 0.7733
3	5	j 0.8138	j 0.8138	j 0.8138	j 0.7475	0.6767 + j 4.9961
5	6	j 0.6452	j 0.6452	j 0.6452	j 0.5926	0.5365 + j 3.9609
6	7	j 0.0002	j 0.0002	j 0.0002	j 0.0002	0.0002 + j 0.0013
7	8	j 0.0067	j 0.0067	j 0.0067	j 0.0062	0.0056 + j 0.0413
8	9	j 0.0378	j 0.0000	j 0.0000	j 0.0341	0.0309 + j 0.2278
8	10	j 0.2216	j 0.2216	j 0.2216	j 0.2035	0.1842 + j 1.3603
9	11	j 1.0640	j 0.0000	j 0.0000	j 0.9598	0.8688 + j 6.4149
10	12	j 0.0182	j 0.0182	j 0.0182	j 0.0167	0.0152 + j 0.1119
10	13	j 0.0000	j 0.0670	j 0.0000	j 0.0604	0.0547 + j 0.4037
11	14	j 0.3036	j 0.0000	j 0.0000	j 0.2739	0.2479 + j 1.8306
12	15	j 0.4436	j 0.4436	j 0.4436	j 0.4074	0.3688 + j 2.7232
15	16	j 0.0113	j 0.0113	j 0.0113	j 0.0104	0.0094 + j 0.0693
16	17	j 0.7993	j 0.7993	j 0.7993	j 0.7342	0.6646 + j 4.9068
16	18	j 0.0000	j 0.5155	j 0.0000	j 0.4651	0.4210 + j 3.1082
17	19	j 0.0002	j 0.0002	j 0.0002	j 0.0002	0.0002 + j 0.0013
19	20	j 0.0000	j 0.0000	j 0.0000	j 0.0000	0.0000 + j 0.0000
19	21	j 0.1063	j 0.1063	j 0.1063	j 0.0977	0.0884 + j 0.6528
20	22	j 0.2292	j 0.2292	j 0.2292	j 0.2105	0.1905 + j 1.4069
21	23	j 0.1265	j 0.1265	j 0.1265	j 0.1162	0.1052 + j 0.7767
21	24	j 0.0000	j 0.0358	j 0.0000	j 0.0323	0.0292 + j 0.2158
23	25	0.0438	0.0438	0.0438	0.0403	0.0364 + j 0.2691
23	26	j 0.0061	j 0.0061	j 0.0061	j 0.0056	0.0051 + j 0.0373
25	27	j 0.0582	j 0.0582	j 0.0582	j 0.0534	0.0484 + j 0.3571
26	28	j 0.0293	j 0.0293	j 0.0293	j 0.0269	0.0244 + j 0.1799
27	29	j 0.0061	j 0.0061	j 0.0061	j 0.0056	0.0051 + j 0.0373
27	30	j 0.0187	j 0.0187	j 0.0187	j 0.0171	0.0155 + j 0.1146
28	31	j 0.0790	j 0.0790	j 0.0790	j 0.0726	0.0657 + j 0.4850
29	32	j 0.0000	j 0.1074	j 0.0000	j 0.0969	0.0877 + j 0.6475
31	33	j 0.0115	j 0.0115	j 0.0115	j 0.0106	0.0096 + j 0.0706

TABLE II  
5X5 IMPEDANCE MATRIX FOR THE 1-2 LINE SECTION ( $\Omega$ )

0.5537 + j 0.3778	j 0.1237	j 0.1003	j 0.0994	j 0.0375
j 0.1237	0.5537 + j 0.3778	j 0.0830	j 0.0883	j 0.0375
j 0.1003	j 0.0830	0.5537 + j 0.3778	j 0.0932	j 0.0375
j 0.0994	j 0.0883	j 0.0932	0.5537 + j 0.3717	j 0.0345
j 0.0375	j 0.0375	j 0.0375	j 0.0345	0.0312 + j 0.2305

Case A considers an isolated neutral; therefore the fifth row and fifth column contain all zero elements. Since in case B the neutral wire does not exist, the fourth row and fourth column of the 5x5 impedance matrix contain all zero entries. All impedances calculations use a 100  $\Omega$ -m ground resistivity (except where ground resistivity is a parameter).

Test network used in case C is the 29-node radial real-life distribution network (LV-29 DN), Fig. 4. The base voltage of the network is  $V_b = 121.244$  V. Real and reactive load injections are shown in Table III.

Four cases of LV network are analyzed:

- LV network with isolated neutral (without ground conductor);
- Multi-grounded LV network, grounding resistance  $R_{gr} = 0.2 \Omega$  in the following nodes: 3, 4, 11, 16, 17, 18, 25,

- 28,  $R_{gr} = 0.1 \Omega$  in substation MV/LV;  
 C3. Same as in case C2, but  $R_{gr} = 0.5 \Omega$ ;  
 C4. LV network with all nodes grounded,  $R_{gr} = 0.1 \Omega$ .

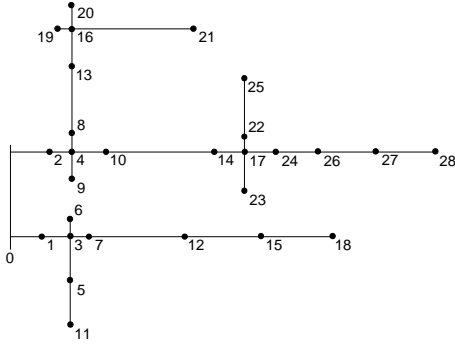


Fig. 4. LV-29 test feeder, Case C.

TABLE III  
 REAL AND REACTIVE LOAD INJECTIONS – LV-29 DN  
 (NON-ZERO ELEMENTS)

node	$P_{Lb}$ (W)	$Q_{Lb}$ (VAR)	$P_{Lc}$ (W)	$Q_{Lc}$ (VAR)	$P_{Lc}$ (W)	$Q_{Lc}$ (VAR)
2	451.4	148.4	209.7	68.9	0.0	0.0
5	0.0	0.0	18.1	5.9	329.2	108.2
7	641.7	210.9	397.9	130.8	504.9	166.0
8	188.2	61.9	268.8	88.4	0.0	0.0
9	166.0	54.6	97.2	31.9	349.3	114.8
11	262.5	86.3	142.4	46.8	120.1	39.5
12	326.4	107.3	441.0	144.9	331.3	108.9
13	147.2	48.4	310.4	102.0	127.1	41.8
14	377.1	123.9	348.6	114.6	403.5	132.6
15	1044.4	343.3	1185.4	389.6	1020.1	335.3
18	330.6	108.7	562.5	184.9	754.2	247.9
19	0.0	0.0	236.8	77.8	216.0	71.0
20	311.1	102.3	544.4	178.9	234.7	77.1
21	294.4	96.8	59.7	19.6	0.0	0.0
22	112.5	37.0	112.5	37.0	0.0	0.0
23	113.9	37.4	0.0	0.0	0.0	0.0
24	0.0	0.0	170.8	56.1	170.8	56.1
25	570.8	187.6	273.6	89.9	0.0	0.0
26	507.6	166.8	291.0	95.6	343.1	112.8
27	144.4	47.5	366.7	120.5	569.4	187.2
28	118.1	38.8	118.1	38.8	0.0	0.0

#### IV. APPLICATIONS

The proposed method for distribution power flow is tested on numerous MV and LV real life DNs. In this paper, the proposed power flow in a 5x5 network representation is applied on three DNs. Two types of load models are applied: constant admittance and constant power.

Results obtained by performing the proposed power flow method on IEEE 34-bus DN, case A, are shown from Fig. 5 to Fig. 9. Figure 5 shows voltages of the neutral wire in the four-wire three-phase IEEE 34-bus network when the loads are modeled as a constant power or constant admittance. The neutral voltage at the end node (275 V) is generated by the flow of the return current in the neutral wire, Fig.5. Figures 6 and 7 show phase voltages when the loads are modeled as a constant admittance or constant power, respectively. Figure 8 shows voltage unbalance in case A, when the loads are modeled as a constant power or constant admittance load, according to the IEEE Std. 446. Figure 9 shows neutral current in the main path in case A, for constant power and constant admittance load models. Since in the proposed model neutral wire and ground are explicitly represented, it is possible to calculate power losses in the neutral wire and in the ground. Real power losses in the neutral wire in the considered IEEE 34-bus DN are about 3% of the total real power losses.

Results from the use of the suggested power flow method on IEEE 34-bus DN with earth return, case B, are shown on Fig. 10 to Fig. 16. Differences in phase voltages obtained from the three-phase power flow method [10] and the proposed method ranged 1.5 % (Fig. 10, Fig. 13). Figures 14 and 15 show ground voltages after modeling for different ground resistivity (50 – 5000  $\Omega$ -m), for constant power and constant admittance load models, respectively [2]. Figure 16 shows ground current in the three-phase MV line with earth return.

The results obtained by performing the proposed power flow method on LV-29 DN are shown on Fig. 17 to Fig. 24. The differences in phase voltages obtained from the conventional three-phase power flow [10] and the proposed method were up to 0.2 % (Fig. 17, Fig. 18). The neutral voltage at the end node is induced by the flow of the return current in the neutral conductor (case C1) and by the ground impedance in the case of a multi-grounded LV network (Fig. 20). When it is difficult to make a proper grounding, most of the current flows through the neutral conductor. A high neutral voltage at the end node may cause not only common-mode noise problems in sensitive electronic pieces of equipment but it may also represent a safety issue [4]. The voltage unbalance, according to the IEEE Std. 446, is less than 0.3% in all cases (Fig. 21). Figures 23 and 24 show respectively neutral wire and ground currents in the evaluated LV-29 DN, when the loads are modeled as constant admittance. The current in the neutral wire is about 30% of the phase currents in the considered LV-29 DN.

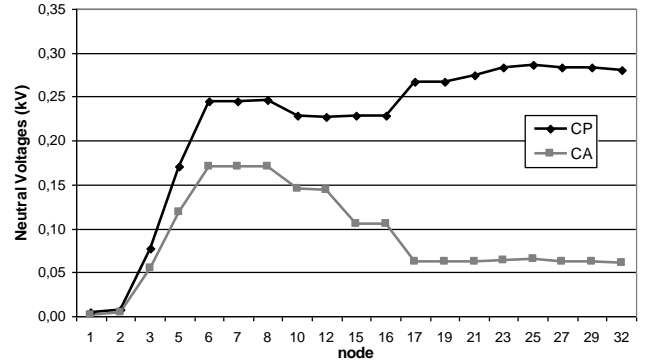


Fig. 5. Neutral Voltage: Case A with Constant Power (CP) and Constant Admittance (CA) Load Model.

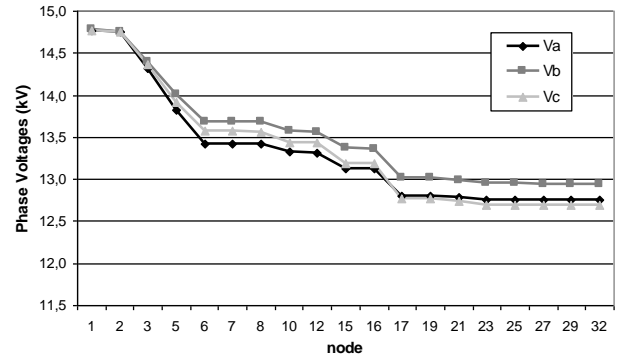


Fig. 6. Phase Voltages: Case A with Constant Admittance Load Model.

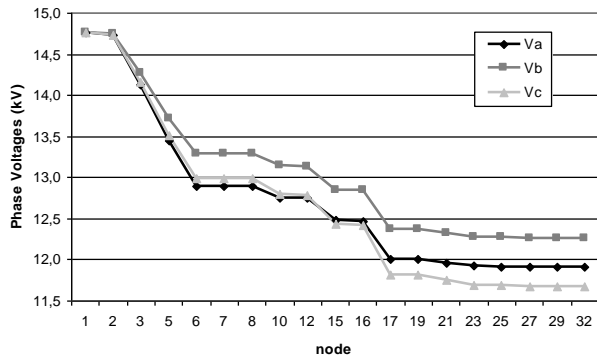


Fig. 7. Phase Voltages: Case A with Constant Power Load Model.

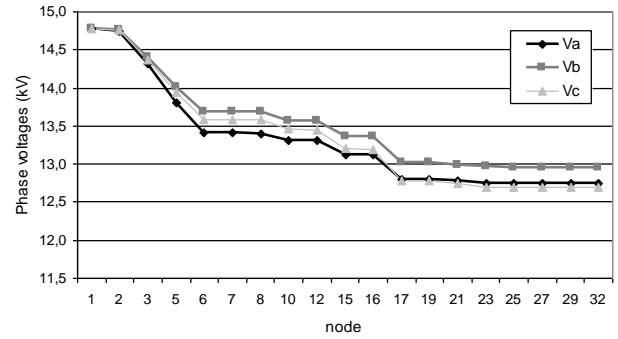


Fig. 11. Phase Voltages using proposed power flow method: Case B with Constant Admittance Load Model.

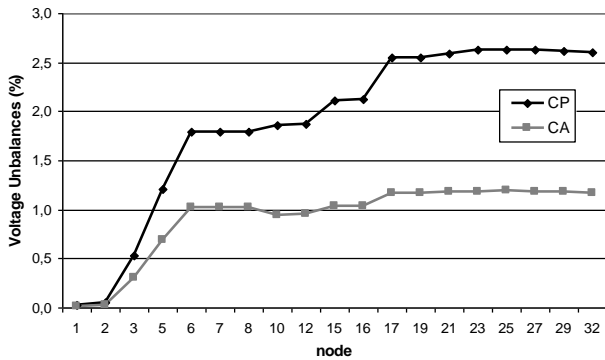


Fig. 8. Voltage Unbalance: Case A with Constant Power (CP) and Constant Admittance (CA) Load Model according to the IEEE Std. 446.

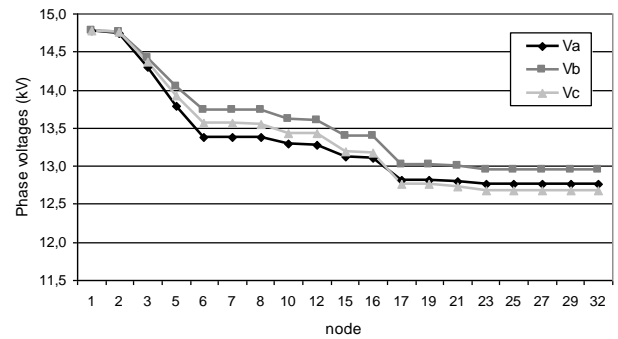


Fig. 12. Phase Voltages using ordinary three-phase power flow method: Case B with Constant Admittance Load Model.

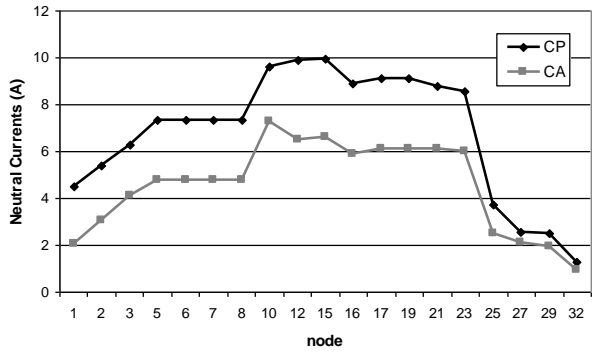


Fig. 9. Neutral current: Case A with Constant Power (CP) and Constant Admittance (CA) Load Model.

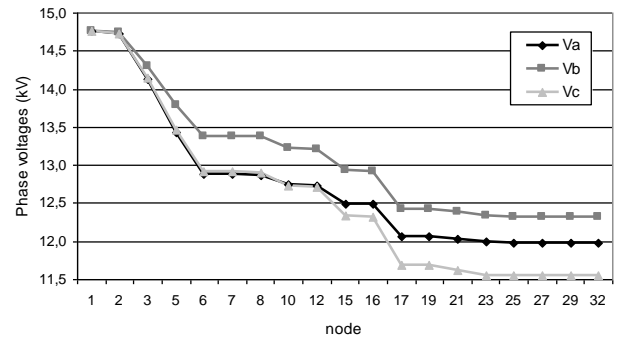


Fig. 13. Phase Voltages using ordinary three-phase power flow method: Case B with Constant Power Load Model.

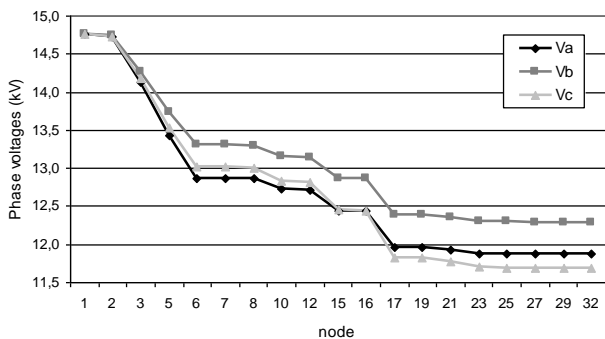


Fig. 10. Phase Voltages using proposed power flow method: Case B with Constant Power Load Model.

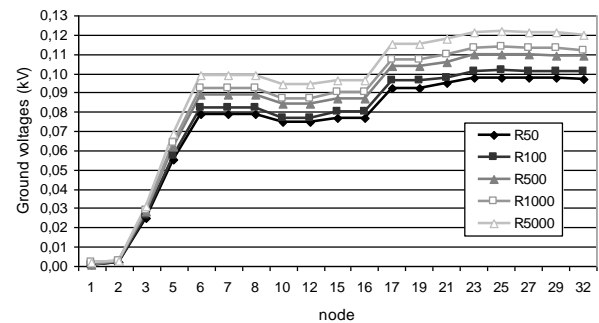


Fig. 14. Ground Voltages: Case B with different ground resistance and Constant Power Load Model.

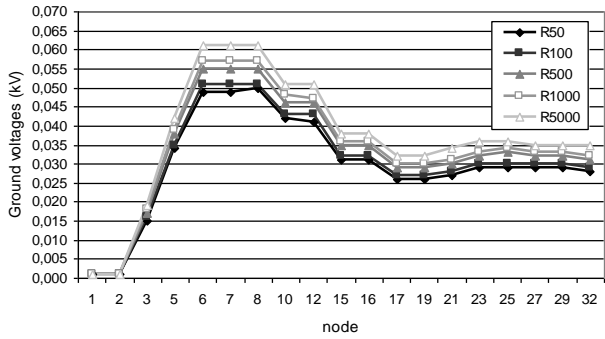


Fig. 15. Ground Voltages: Case B with different ground resistance and Constant Admittance Load Model.

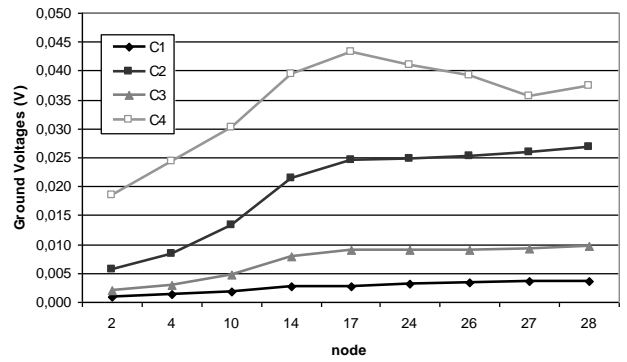


Fig. 19. Ground Voltages: Case C with Constant Admittance Load Model.

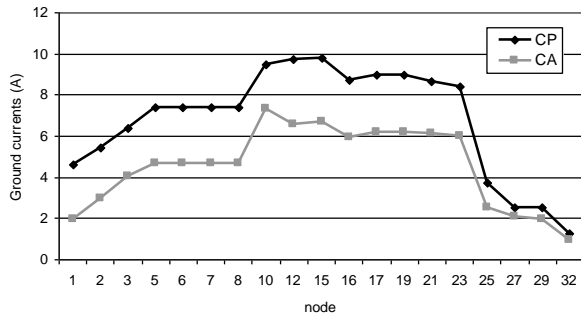


Fig. 16. Ground Current: Case B with Constant Power (CP) and Constant Admittance (CA) Load Model.

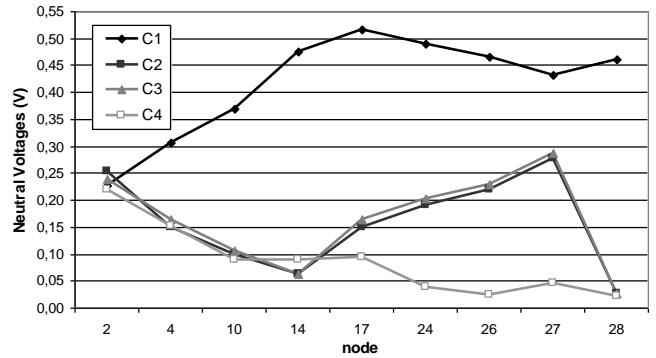


Fig. 20. Neutral Voltages: Case C with Constant Admittance Load Model.

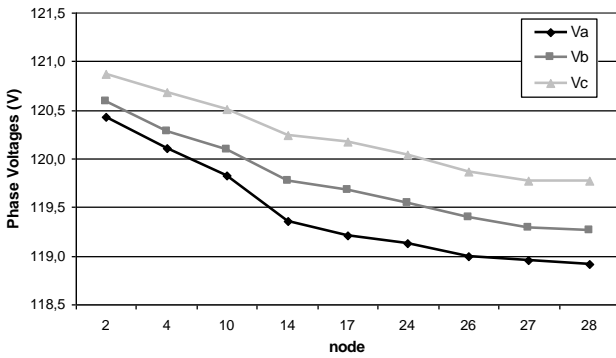


Fig. 17. Phase Voltages using conventional three-phase power flow method: Case C with Constant Admittance Load Model.

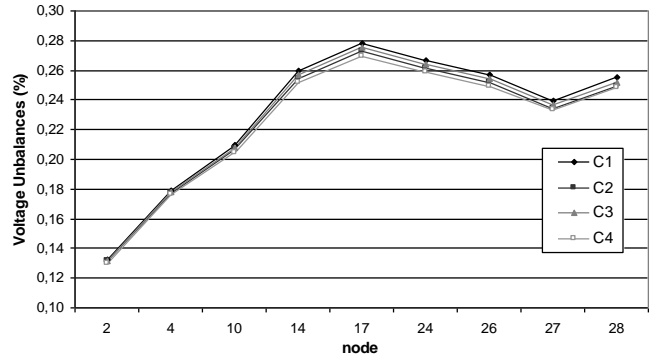


Fig. 21. Voltage Unbalances: Case C with Constant Admittance Load Model according to the IEEE Std. 446.

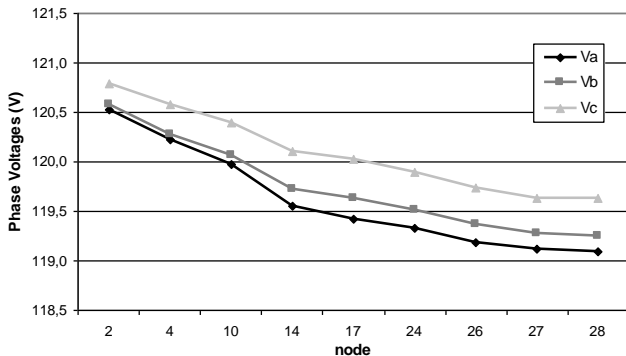


Fig. 18. Phase Voltages using proposed power flow method: Case C with Constant Admittance Load Model and Isolated Neutral (C1).

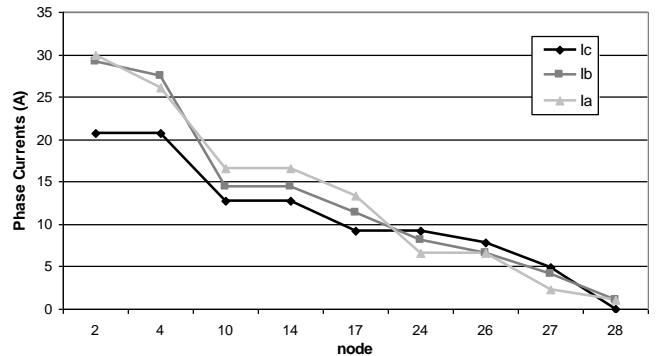


Fig. 22. Phase Currents: Case C with Constant Admittance Load Model and Isolated Neutral (C1).

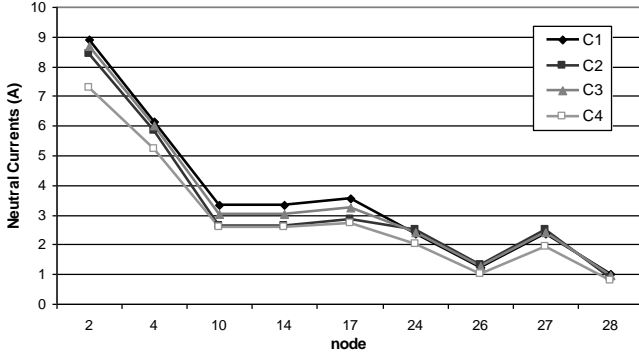


Fig. 23. Return currents: Case C with Constant Admittance Load Model.

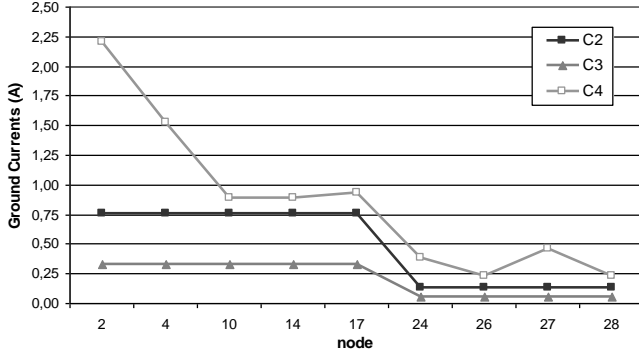


Fig. 24. Ground Currents: Case C with Constant Admittance Load Model.

Generally, load model plays a significant role in all power flow simulations. The results showed a large difference in the phase voltages due to the load model (7.0%), and a minor difference as a function of the line model (1.5%). The off-diagonal shunt admittances were neglected in all simulations, as stated in (1). However, the influence of the mutual shunt admittances on the solution was also investigated. Considering Case A, when the neutral wire and ground were explicitly represented, the difference in phase voltages with and without off-diagonal shunt admittance terms in (1), was up to 0.95 %.

Results showed that Kron's reduction is acceptable when neutral and ground current are not of specific interest. Moreover, the return currents may be calculated using 3x3 approach, after which the voltages in return paths may be obtained using the original 5x5 matrices. Kron's reduction is avoided with this proposed method when the focus is to calculate currents and voltages in an explicit manner.

The proposed method was tested on MV real-life network 475 nodes in Assis (Sao Paulo state, Brazil). When the peak load case was simulated, the 5x5 and the 3x3 backward-forward techniques converged after three iterations, with a computing time of 770 and 430 milliseconds, respectively, when a 544 MHz computer processor was used.

## V. CONCLUSIONS

In this paper, a power flow algorithm for three-phase four-wire radial DN, considering neutral wire and multi-grounding, is proposed. High-order line models (4x4, 5x5 and higher) may easily be added to a common solution method for

three-phase power flow [10]. Most of the existing models of three-phase elements of the distribution system can be well represented [15]. The introduced 5x5 network representation can easily be incorporated into a power flow solution technique for weakly meshed DN with PV nodes. The proposed power flow method enables the user to investigate the effects of neutrals and system grounding on the operation of real-life DN.

The software package based on the proposed 5x5 power flow solution technique can be useful for the evaluation of different neutral designs. Moreover, it can be an efficient tool for power quality studies, safety analysis, short circuit analysis, or other applications, where neutral and ground currents and voltages are of particular interest [19], [20].

## VI. APPENDIX

The equations for calculation of impedance matrix elements in 5x5 representation are as follows.

Using the ground return modeling of [2] for a three-phase line (Fig. A), we obtain:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} \\ z_{ab} & z_{bb} & z_{bc} \\ z_{ac} & z_{bc} & z_{cc} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (i)$$

where

$$z_{xx} = \bar{z}_{xx} + \bar{z}_{gg} - 2\bar{z}_{xg} \quad (ii)$$

$$z_{xy} = \bar{z}_{xy} - \bar{z}_{xg} - \bar{z}_{yg} + \bar{z}_{gg} \quad (iii)$$

$$x = a, b, c; y = a, b, c; \forall x \neq y$$

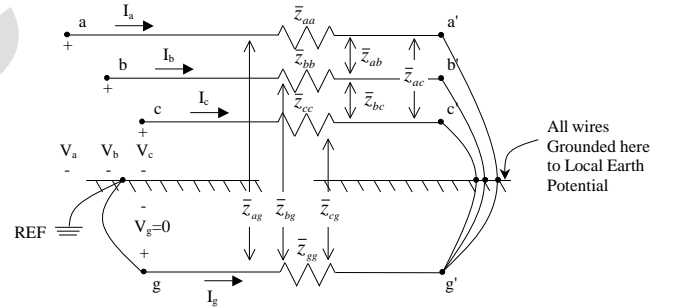


Fig. A. Three-phase line with earth return [2].

In the case of wire  $a$ ,  $\bar{z}_{aa}$  represents the self-impedance of the line (if the ground is a perfect conductor), and the term  $(\bar{z}_{gg} - 2\bar{z}_{ag})$  is the correction caused by the ground influence.

Using the effect of the finite conductivity of the ground (correction part) of the simplified Carson's equations for self-impedance, we obtain the following equation:

$$\begin{aligned} \bar{z}_{gg} - 2\bar{z}_{ag} = & \pi^2 \cdot 10^{-4} f - j0.0386 \cdot 8\pi \cdot 10^{-4} f \\ & + j4\pi \cdot 10^{-4} \cdot f \ln \frac{2}{5.6198 \cdot 10^{-3}} + j4\pi \cdot 10^{-4} \cdot f \ln \frac{\sqrt{f}}{h_a} \quad \Omega/\text{km} \quad (iv) \end{aligned}$$

where

- $f$  is the frequency (Hz);
- $h_a$  is the height of wire (m);
- $\rho$  is the ground resistivity ( $\Omega\cdot\text{m}$ ).

Looking at the right side of (iv), we can easily identify the

terms that are only frequency dependent. These three terms represent the ground self-impedance  $\bar{z}_{gg}$ . Therefore, the ground self-impedance  $\bar{z}_{gg}$  and the mutual impedance  $\bar{z}_{ag}$  from Fig. A and (iv) are, respectively

$$\bar{z}_{gg} = \pi^2 \cdot 10^{-4} \cdot f - j0.0386 \cdot 8\pi \cdot 10^{-4} \cdot f \quad \Omega/\text{km} \quad (v)$$

$$+ j4\pi \cdot 10^{-4} \cdot f \ln \frac{2}{5.6198 \cdot 10^{-3}}$$

$$\bar{z}_{ag} = -\frac{1}{2} \cdot j4\pi \cdot 10^{-4} \cdot f \ln \frac{\sqrt{p/f}}{h_a} \quad \Omega/\text{km} \quad (vi)$$

$$= j2\pi \cdot 10^{-4} \cdot f \ln \frac{h_a}{\sqrt{p/f}}$$

Hence, for a three-phase line with earth return, the 5x5 matrix representation of the line is as follows:

$$[Z_{line}] = \begin{bmatrix} \bar{z}_{aa} & \bar{z}_{ab} & \bar{z}_{ac} & 0 & \bar{z}_{ag} \\ \bar{z}_{ab} & \bar{z}_{bb} & \bar{z}_{bc} & 0 & \bar{z}_{bg} \\ \bar{z}_{ac} & \bar{z}_{bc} & \bar{z}_{cc} & 0 & \bar{z}_{cg} \\ 0 & 0 & 0 & 0 & 0 \\ \bar{z}_{ag} & \bar{z}_{bg} & \bar{z}_{cg} & 0 & \bar{z}_{gg} \end{bmatrix} \quad (vii)$$

The self and mutual impedances of phases  $a$ ,  $b$  and  $c$  in (vii), are calculated using the Carson's equations, considering the ground as a perfect conductor [2]. Self and mutual impedances of the ground use (v) and (vi). Zero elements on matrix (vii) are due to the absence of neutral wire.

For neutral wire calculations, mutual ground impedance should be computed using (vi) and mutual  $a$ ,  $b$ ,  $c$  and self impedances should be computed using the Carson's equations, considering the ground as a perfect conductor [2].

## VII. ACKNOWLEDGMENT

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