Power indices of influence games and new centrality measures for agent societies and social networks*

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Abstract. We propose as centrality measures for social networks two classical power indices, Banzhaf and Shapley-Shubik, and two new measures, *effort* and *satisfaction*, related to the spread of influence process that emerge from the subjacent influence game. We perform a comparison of these measures with three well known centrality measures, *degree*, *closeness* and *betweenness*, applied to three simple social networks.

Keywords: Social Network, Centrality, Power index, Influence game, Simple game

1 Introduction and preliminaries

We propose to study networked societies, social networks or agent societies, from the social networking point of view. Social network analysis is a multidisciplinary field related to sociology, computer science and mathematics, among other topics. One of the most studied concepts is *centrality*, that measures how structurally important is an actor within a social network [4, 15, 8, 12]. Here we consider seven centrality measures: Banzhaf and Shapley-Shubik power indices through the use of *influence games* [10]; two new measures, the *effort* and the *satisfaction*; and the classic ones [8], *degree*, *closeness* and *betweenness*. We perform an experimental comparison on three simple real social networks, *monkeys' interaction* [3, 8], *dining-table partners* [11, 2], and *student Government discussion* [6, 2].

A *social network* is a directed edge-labeled graph (G, w), where $G = (V, E)$ is a graph without loops, V is the set of nodes representing individuals, actors, players, etc., E is the set of edges representing interpersonal ties between actors, and $w : E \to \mathbb{R}$ is a *weight function* which assigns a weight to every edge, representing the strength of each interpersonal tie. An actor $i \in V$ has *influence* over another $j \in V$ if and only if $(i, j) \in E$.

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Now we consider three of the most well-known (normalized) centrality measures [8], which study the relevance of a node inside a network [15]. We use the notation $deg^-(i) = |\{j \in V \mid (j,i) \in E\}|$ and $deg^+(i) = |\{j \in V \mid (i,j) \in E\}|$.

Degree centrality (C_D): measures the average indegree or outdegree of each actor, $C_D^-(i) = deg^-(i)/(n-1)$, or $C_D^+(i) = deg^+(i)/(n-1)$. For undirected networks, $deg(i) = deg^-(i) = deg^+(i)$, so we set $C_D = C_D^- = C_D^+$.

Closeness centrality (C_C): It is based on the inverse of the sum of the shortest distances from i to the other actors. Let D be the usual distance matrix of the network in which, if there is no path from i to j, we set $(D)_{ij} = n$. We define $C_C(i) = (n-1)/\sum_{i \neq j}(D)_{ij}$.

Betweenness centrality (C_B): Let b_{jk} the number of shortest paths from the node j until k, and b_{jik} the number of these shortest paths that pass through i. If there is no path from j to k, we assume that $b_{jik}/b_{jk} = 0$. We define $C_B(i) = \sum_{j \neq k} \frac{b_{jik}}{b_{jk}}/((n-1)(n-2))$.

Notation related to simple and influence games comes from [13, 10]. An *influence graph* is a tuple (G, w, f) where (G, w) is asocial network and $f : V \to \mathbb{N}$ a labeling function that quantifies how influenciable each actor is.

Given an influence graph (G, w, f) and an initial activation set $X \subseteq V$, the *spread of influence* [7], in the linear threshold model, is denoted by $F(X)$, where $F(X) \subseteq V$ is formed by the actors activated through an iterative process in which initially only the nodes in X are activated. Let $F^t(X)$ be the set of nodes activated at some iteration t, then at the next $t+1$ iteration a node $i \in V$ will be activated iff $\sum_{j \in F^t(X)} w((j,i)) \geq f(i)$. The process stops when no additional activation occurs.

A *simple game* is a tuple $(N, \mathcal{W})$ where N is a finite set of players and $\mathcal{W}$ is a monotonic family of subsets of N formed by the *winning coalitions*, such that if $X \in \mathcal{W}$ and $X \subseteq Z$, then $Z \in \mathcal{W}$. An *influence game* is a simple game defined by a tuple (G, w, f, q) where (G, w, f) is an influence graph and q is a *quota* $0 \leq q \leq |V| + 1$. $X \subseteq V$ is a *winning coalition* iff $|F(X)| \geq q$, otherwise X is a *losing coalition*. Note that every simple game is an influence game [10]. From now on, we assume $N = V$ and $n = |N| = |V|$.

2 Power indices and new centrality measures

A *power index* is a measure of the relevance of the players in a game [1, 5]. We consider the two main power indices of a given simple game $(N, \mathcal{W})$. The *Banzhaf index* $\texttt{Bz}(i) = |C_i|/\sum_{i \in N}|C_i|$ and the *Shapley-Shubik index* $\texttt{SS}(i) = (\sum_{S \in C_i}(|S|-1)!\,(n-|S|)!)/n!$, where $C_i = \{S \in \mathcal{W} \mid S \backslash \{i\} \notin \mathcal{W}\}$.

Power indices, in influence games, can be considered centrality measures because an actor is more central in a network while more necessary is for generating of winning coalitions. Moreover, influence games also provide other *new* criteria to determine measures of centrality. Let $f(S) = \sum_{i \in S} f(i)$, for a coalition $S \subseteq N$. For an influence game (G, w, f, q), $\texttt{Effort}(i) = \min\{f(S) \mid |F(S \cup \{i\})| \geq q\}$, the (minimum) effort required by the network to choose a winning coalition that

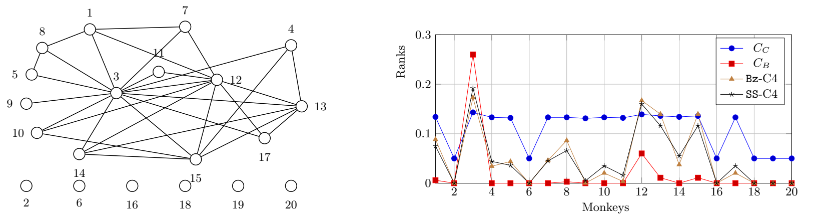

Fig. 1. Monkeys' interaction network and comparisons among Bz-C4, SS-C4, C_C, C_B.

contains a required actor. While greater is the required effort for a node, this node should be less central. Therefore, the *effort centrality measure* is the effort required to make the social network follows the opinion of an actor, i.e., $C_E(i) = (f(N) - \texttt{Effort}(i))/f(N)$.

The second new measure is the *satisfaction centrality measure*, based on the *satisfaction score* [14], representing the level of satisfaction of each actor applied to an influence game (G, w, f, q), i.e., $C_S(i) = (|\mathcal{W}_i| + |\mathcal{L}_{-i}|)/2^n$, where $\mathcal{W}_i = \{X \subseteq V(G) \mid i \in X, |F(X)| \geq q\}$ and $\mathcal{L}_{-i} = \{X \subseteq V(G) \mid i \notin X, |F(X)| < q\}$.

3 Cases of study

We consider three simple real social networks to compare the new centrality measures Bz, SS, C_E and C_S, with some traditional ones, C_D, C_C and C_B. In each comparative table the three more central values will be highlighted in bold. We used enough significant digits to distinguish all the different values.

Monkeys' interaction. This is a network representing the real interactions amongst a group of 20 monkeys observed during three months next to a river provided in [3]. It is represented by an undirected graph with an edge $\{i, j\}$ whenever monkeys i and j were witnessed together in the river. See Figure 1, on the left.

In order to analyze this network $((V, E), w)$ we assume, as usual, that every undirected edge $\{i, j\}$ with $i, j \in V$ represents in fact two arcs (i, j) and (j, i) of E, and the weight function is defined by $w(e) = 1$, for all $e \in E$. In our context, this means that a monkey can influence and be influenced by other monkey if and only if they have interacted. To define an influence game we have to set the quota and define the labeling function. We select $q = 14$, which corresponds to the maximum spread of influence which can be obtained from a monkey. We consider four labeling functions representing different influence requirements. For every node $i \in V$, (C1) minimum, $f(i) = 1$; (C2) average, $f(i) = \lceil deg(i)/2 \rceil$; (C3) majority, $f(i) = \lfloor deg(i)/2 \rfloor + 1$; and (C4) maximum, $f(i) = deg(i)$.

The Bz, SS, C_E and C_S measures have been computed for all these cases (Table 1). Note that only isolated nodes for Bz, SS and C_E, as well as the last column of C_E assume a score exactly equal to zero. For (C1), the new measures are not good representatives. As the spread of influence is fluid, i.e., actors

				Bz				SS				C_E				C_S			
Node	C_D	C_C	C_B	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
1	0.21	0.134	0.006	0.07	0.038	0.0708	0.0885	0.07	0.025	0.068	0.075	0.9	0.43	0.14	0	0.501	0.521	0.575	0.598
2	0.00	0.050	0.000	0.00	0.000	0.0000	0.0000	0.00	0.000	0.000	0.000	0.0	0.00	0.00	0	0.500	0.500	0.500	0.500
3	**0.68**	**0.143**	**0.260**	0.07	**0.156**	**0.1214**	**0.1730**	0.07	**0.219**	**0.150**	**0.192**	0.9	0.36	0.07	0	0.501	**0.589**	**0.644**	**0.736**
4	0.16	0.133	0.000	0.07	0.059	0.0673	0.0343	0.07	0.047	0.062	0.044	0.9	**0.50**	0.14	0	0.501	0.537	0.547	0.580
5	0.11	0.132	0.000	0.07	0.019	0.0373	0.0438	0.07	0.013	0.032	0.036	0.9	0.43	0.14	0	0.501	0.510	0.543	0.578
6	0.00	0.050	0.000	0.00	0.000	0.0000	0.0000	0.00	0.000	0.000	0.000	0.0	0.00	0.00	0	0.500	0.500	0.500	0.500
7	0.16	0.133	0.000	0.07	0.049	0.0497	0.0460	0.07	0.032	0.043	0.045	0.9	0.43	0.14	0	0.501	0.528	0.551	0.583
8	0.16	0.133	0.003	0.07	0.048	0.0282	0.0863	0.07	0.040	0.024	0.066	0.9	0.43	0.07	0	0.501	0.527	0.532	**0.601**
9	0.05	0.131	0.000	0.07	0.028	0.0281	0.0003	0.07	0.017	0.022	0.005	0.9	0.43	0.14	0	0.501	0.516	0.531	0.548
10	0.16	0.133	0.000	0.07	0.074	0.0538	0.0205	0.07	0.069	0.050	0.035	0.9	**0.50**	0.14	0	0.501	0.536	0.555	0.582
11	0.11	0.132	0.000	0.07	0.037	0.0470	0.0035	0.07	0.023	0.040	0.016	0.9	**0.50**	0.14	0	0.501	0.520	0.553	0.574
12	**0.47**	**0.139**	**0.060**	0.07	**0.154**	0.1004	**0.1671**	0.07	**0.180**	**0.107**	**0.160**	0.9	0.43	0.14	0	0.501	**0.580**	**0.604**	**0.625**
13	**0.32**	**0.136**	**0.011**	0.07	**0.091**	**0.1197**	**0.1395**	0.07	**0.096**	**0.125**	**0.116**	0.9	0.43	0.07	0	0.501	**0.546**	**0.586**	0.596
14	0.21	0.134	0.000	0.07	0.081	**0.1028**	0.0375	0.07	0.075	0.100	0.055	0.9	**0.50**	0.14	0	0.501	0.541	0.569	0.584
15	**0.32**	**0.136**	**0.011**	0.07	**0.091**	**0.1197**	**0.1395**	0.07	**0.096**	**0.125**	**0.116**	0.9	0.43	0.07	0	0.501	**0.546**	**0.586**	0.596
16	0.00	0.050	0.000	0.00	0.000	0.0000	0.0000	0.00	0.000	0.000	0.000	0.0	0.00	0.00	0	0.500	0.500	0.500	0.500
17	0.16	0.133	0.000	0.07	0.074	0.0538	0.0205	0.07	0.069	0.050	0.035	0.9	**0.50**	0.14	0	0.501	0.536	0.555	0.582
18	0.00	0.050	0.000	0.00	0.000	0.0000	0.0000	0.00	0.000	0.000	0.000	0.0	0.00	0.00	0	0.500	0.500	0.500	0.500
19	0.00	0.050	0.000	0.00	0.000	0.0000	0.0000	0.00	0.000	0.000	0.000	0.0	0.00	0.00	0	0.500	0.500	0.500	0.500
20	0.00	0.050	0.000	0.00	0.000	0.0000	0.0000	0.00	0.000	0.000	0.000	0.0	0.00	0.00	0	0.500	0.500	0.500	0.500

Table 1. Comparison for the Monkeys' interaction network for $q = 14$.

do not require too many restrictions to form winning coalitions, then all the non-isolated nodes have the same value. However, for the other cases in which differences between influence are relevant, only the pair of monkeys $(10, 17)$ and $(13, 15)$ assume the same value for Bz, SS and C_S, allowing a more relevant classification.

Dining-table partners. This network represents the companion preferences of 26 girls living in one cottage at a New York state training school [11, 2]. See Figure 2, on left. Each girl was asked about who prefers as dining-table partner in first and second place. Thus, each girl is represented by a node, and there is a directed edge (i, j) per each girl i preferring girl j as dining-table partner. Every node has an outdegree equal to 2: edges with weight 1 denote the first option of the girl, and edges with weight 2 denote her second option.

We could assume that a girl has some ability to influence another one which has chosen her as a partner. Figure 2 (on right) shows the corresponding network of this influence game, reversing each arc (i, j) by (j, i), so that a node points to another when the first one has some influence over the second one. Further, the weights of the edges must be exchanged, so that an original edge (i, j) with weight 1 now becomes in an edge (j, i) with weight 2, and viceversa. Because a girl has more influence over another one if that other has chosen her in the first place rather than in the second place. Of course, now every node has an indegree equal to 2: one edge with weight 1 and the other with weight 2. We consider a quota $q = 14$, so that a coalition is winning if and only if it achieves to convince (through its spread of influence) most of the girls *absolute majority.* For every node $i \in V$, we consider the following labeling functions:
(C1) minimum, $f(i) = 1$; (C2) average, $f(i) = 2$; and (C3) maximum, $f(i) = 3$. Unlike in the previous network, here there are no isolated nodes, but we can

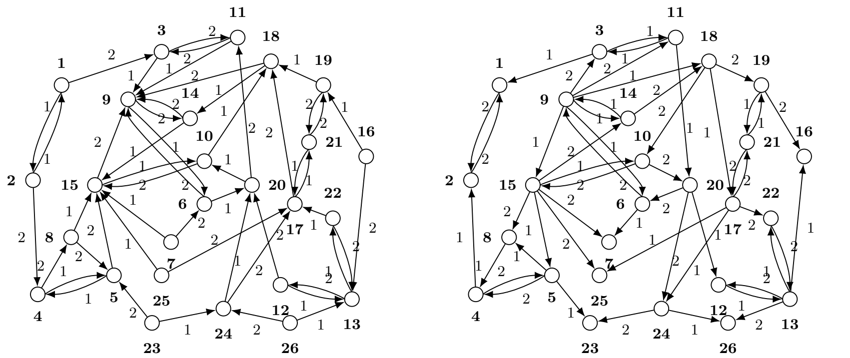

Fig. 2. Network for dining-table partners and the associated influence graph.

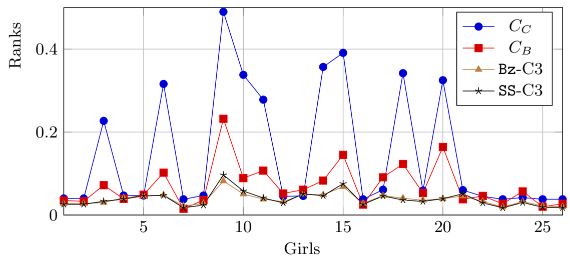

Fig. 3. Comparative between Bz-C3, SS-C3, C_C and C_B for Dining-table partners.

still obtaining scores for Bz and SS equal to zero. See the columns of Bz-C1 and SS-C1 on Table 2.

Indegree centrality C_D^- does not provide any relevant information, because the indegree for each node is always 2 (see Table 2 and Figure 3). Similarly as it succeded in the previous network, Bz-C1, SS-C1, C_E-C1 and C_S-C1 have several nodes with the same rank, but while the required influence to convincement increases, the values of the measures are more diverse for the power indices and satisfaction centrality. Measures Bz-C2, SS-C2 and C_S-C2, as well as C_D^+ and C_C, have only some values that are repeated, but measures Bz-C3, SS-C3 and C_S-C3 have the same values only for girls 1 and 2. These girls are equivalent in this sense for all the other measures except by C_B, in which, however, together with C_E, girls 23 and 26 have the same centrality.

Girl 15 has a high centrality in all measures, as well as girl 9, except in C_E-C2, as well as in Bz-C2 and SS-C2, where is far less central. Girl 13 is fairly central exclusively in C_D^+, because despite of its high outdegree, only exist paths from this node to another four, which is a severe restriction for all other measures.

					Bz			SS			C_E			C_S		
Node	C_D^-	C_D^+	C_C	C_B	C1	C2	C3	C1	C2	C3	C1	C2	C3	C1	C2	C3
1	0.08	0.04	0.0400	0.035	0.00	0.028	0.0274	0.0000	0.0103	0.0259	0.92	0.85	0.42	0.500000	0.5024	0.5300
2	0.08	0.04	0.0400	0.033	0.00	0.028	0.0274	0.0000	0.0103	0.0259	0.92	0.85	0.42	0.500000	0.5024	0.5300
3	0.08	0.08	0.2273	0.072	**0.08**	0.008	0.0302	**0.0832**	0.0014	0.0331	**0.96**	0.85	**0.54**	**0.500217**	0.5006	0.5329
4	0.08	0.08	0.0473	0.039	0.00	0.028	0.0413	0.0000	0.0103	0.0383	0.92	0.85	0.42	0.500000	0.5024	0.5451
5	0.08	0.12	0.0473	0.049	0.00	0.028	0.0452	0.0000	0.0103	0.0463	0.92	0.85	0.42	0.500000	0.5024	0.5494
6	0.08	0.08	0.3165	0.102	**0.08**	0.043	0.0481	**0.0832**	0.0142	0.0473	**0.96**	0.85	0.42	**0.500217**	0.5036	0.5526
7	0.08	0.00	0.0385	0.015	0.01	0.024	0.0216	0.0003	0.0075	0.0176	0.92	0.85	0.42	0.500027	0.5020	0.5236
8	0.08	0.04	0.0471	0.036	0.00	0.024	0.0278	0.0000	0.0075	0.0239	0.92	0.85	0.42	0.500000	0.5020	0.5303
9	0.08	**0.24**	**0.4902**	**0.232**	**0.08**	0.027	**0.0820**	**0.0832**	0.0072	**0.0965**	**0.96**	0.85	**0.54**	**0.500217**	0.5023	**0.5896**
10	0.08	0.08	0.3378	0.089	**0.08**	**0.104**	**0.0506**	**0.0832**	**0.1953**	**0.0578**	**0.96**	**0.92**	**0.54**	**0.500217**	**0.5089**	**0.5552**
11	0.08	0.08	0.2778	0.107	**0.08**	0.008	0.0383	**0.0832**	0.0014	0.0410	**0.96**	0.85	**0.54**	**0.500217**	0.5006	0.5418
12	0.08	0.04	0.0452	0.052	0.00	0.004	0.0321	0.0001	0.0007	0.0292	0.92	0.85	0.42	0.500004	0.5003	0.5350
13	0.08	**0.16**	0.0454	0.061	0.00	0.014	0.0500	0.0001	0.0041	0.0511	0.92	0.85	0.42	0.500004	0.5012	0.5546
14	0.08	0.08	**0.3571**	0.083	**0.08**	**0.104**	0.0486	**0.0832**	**0.1953**	0.0465	**0.96**	**0.92**	0.42	**0.500217**	**0.5089**	0.5531
15	0.08	**0.24**	**0.3906**	**0.145**	**0.08**	**0.104**	**0.0683**	**0.0832**	**0.1953**	**0.0755**	**0.96**	**0.92**	**0.54**	**0.500217**	**0.5089**	**0.5746**
16	0.08	0.00	0.0385	0.025	0.00	0.015	0.0279	0.0000	0.0055	0.0256	0.92	0.85	0.42	0.500000	0.5013	0.5305
17	0.08	**0.33**	0.0614	0.091	**0.08**	**0.051**	0.0469	**0.0832**	**0.0232**	0.0459	**0.96**	0.85	0.42	**0.500217**	**0.5043**	0.5512
18	0.08	0.12	0.3425	0.123	**0.08**	**0.104**	0.0404	**0.0832**	**0.1953**	0.0361	**0.96**	**0.92**	0.42	**0.500217**	**0.5089**	0.5442
19	0.08	0.08	0.0595	0.053	**0.08**	0.036	0.0356	**0.0832**	0.0126	0.0327	**0.96**	0.85	0.42	**0.500217**	0.5031	0.5389
20	0.08	0.12	0.3247	**0.164**	**0.08**	**0.075**	0.0394	**0.0832**	**0.0413**	0.0395	**0.96**	0.85	0.42	**0.500217**	**0.5064**	0.5430
21	0.08	0.08	0.0605	0.038	**0.08**	**0.051**	0.0457	**0.0832**	**0.0232**	0.0512	**0.96**	0.85	**0.54**	**0.500217**	**0.5043**	0.5499
22	0.08	0.04	0.0452	0.046	0.00	0.025	0.0325	0.0001	0.0082	0.0293	0.92	0.85	0.42	0.500004	0.5021	0.5355
23	0.08	0.00	0.0385	0.027	0.00	0.011	0.0191	0.0000	0.0029	0.0173	0.92	0.85	0.42	0.500000	0.5010	0.5208
24	0.08	0.08	0.0417	0.057	0.01	0.029	0.0324	0.0003	0.0083	0.0301	0.92	0.85	0.42	0.500027	0.5025	0.5354
25	0.08	0.00	0.0385	0.020	0.01	0.024	0.0218	0.0003	0.0075	0.0187	0.92	0.85	0.42	0.500027	0.5020	0.5239
26	0.08	0.00	0.0385	0.027	0.00	0.004	0.0197	0.0000	0.0007	0.0177	0.92	0.85	0.42	0.500000	0.5003	0.5215

Table 2. Comparison values for the Dining-table partners network for $q = 14$.

Student Government discussion. The last case of study starts with the social network illustrated in Figure 4. This network represents the communication interactions among different members of the Student Government at the University of Ljubljana in Slovenia. Data were collected through personal interviews in 1992 and published by [6], being used later by [2].

Every directed edge is a communication interaction and all of them have the same weight equal to 1. Each node is a member of the Student Government, and unlike the previous cases, here nodes are labeled beforehand: There are three *advisors* labeled 1, seven *ministers* labeled 2, and one *prime minister* labeled 3.

We modified slightly this network to obtain the influence graph of Figure 4. We assume that every communication interaction is an attempt to influence another student. Thus, the capacity to influence depends on the student's position. For instance, the advise of a prime minister does not have the same effectiveness —marked with weight 3— than the advise of an advisor —marked with weight 1. Furthermore, as the labels of the nodes should represent the difficulty of each student $i \in N$ to be influenced, according to their position in the Student Government, then they have been changed by the following values: $f(i) = 1$, if i is an advisor; $f(i) = \lceil deg^-(i)/2 \rceil$, if i is a minister; and $f(i) = deg^-(i)$, if i is the primer minister. We consider a majority influence required to win, setting $q = 6$ (see Table 3 and Figure 4).

Traditional measures provide different rankings. In fact, none of the most central nodes measured with C_C and C_B coincide, and while the most central node for C_C is the advisor 10, this is the less central according to C_B. Moreover, the ministers 3 and 1 are very central for C_C but with C_B are at the bottom of the ranking. This is because nodes 1, 3 and 10 have a high accessibility to all other nodes, but however, they are not good intermediaries for connecting

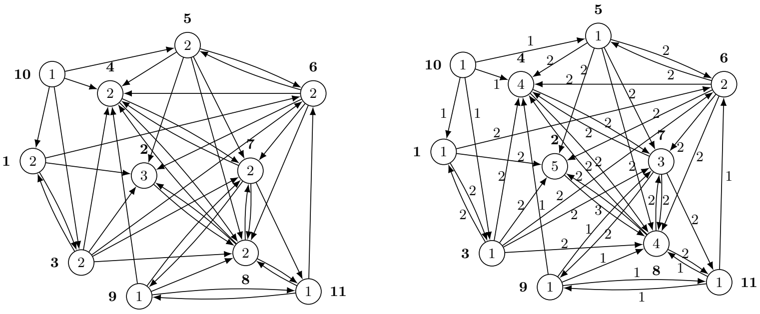

Fig. 4. Student Government discussion network and the asssociated influence graph.

Node	C_D^-	C_D^+	C_C	C_B	Bz	SS	C_E	C_S
1	0.2	0.3	**0.357**	0.130	**0.164**	**0.176**	**0.91**	**0.516**
2	0.5	0.1	0.200	0.195	0.154	0.076	0.45	0.515
3	0.2	**0.6**	**0.435**	0.169	**0.164**	**0.176**	**0.91**	**0.516**
4	**0.7**	0.2	0.208	0.204	0.005	0.009	0.55	0.500
5	0.2	**0.5**	0.238	0.211	**0.164**	**0.176**	**0.91**	**0.516**
6	0.4	**0.5**	0.238	**0.304**	**0.164**	**0.176**	0.82	**0.516**
7	**0.6**	**0.4**	0.227	**0.316**	0.005	0.009	0.64	0.500
8	**0.8**	**0.4**	0.227	0.262	0.005	0.009	0.55	0.500
9	0.2	**0.4**	0.227	0.193	0.005	0.009	0.82	0.500
10	0.0	**0.4**	**0.556**	0.111	**0.164**	**0.176**	**0.91**	**0.516**
11	0.3	0.3	0.227	**0.306**	0.005	0.009	0.82	0.500

Table 3. Comparison for the Student Government discussion networ and $q = 6$.

distant nodes through paths. Nevertheless, nodes 1, 3 and 10, as well as ministers 5 and 6, have a high score for measures Bz, SS and C_S. Thi is so since the spread of the influence over the other students, starting from the coalitions where they participate, is often necessary to overcome the required quota q. The same occurs for C_E, except for the minister 6, which is a bit less central.

4 Conclusions and future work

Our main motivation in this work was to use influence games as a way to propose additional centrality measures coming from the field of cooperative game theory. The framework of influence games derives a connection between social network analysis and spread of influence in decision processes. We exploit this link with simple game theory to propose new centrality measures: Banzhaf, Shapley-Shubik, Effort and Satisfaction. This is the first approach to apply power indices as centrality measures for social networks (for *specific* game-theoretic networks [9] has been used the Shapley-Shubik index as centrality measure). Our results do not contradict the relevance criteria provided by traditional centrality measures like degree centrality, closeness or betweenness. In some cases such measurements are similar to our measurements, but there are also cases where the results have been quite different. Indicating that an additional study on more realistic social networks is of interest.

Our proposal can be extended to other power indices [5] and measures, it will be of interest to determine which of them provide relevant rankings for social network analysis. Finally, we want to mention that there are other well known concepts related with players in simple games, such as *dummy*, *vetoer* or *dictators* [13], that could provide interesting properties of actors in a social network.

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