

Power-Law Time Distribution of Large Earthquakes

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We study the statistical properties of time distribution of seismicity in California by means of a new method of analysis, the diffusion entropy. We find that the distribution of time intervals between a large earthquake (the main shock of a given seismic sequence) and the next one does not obey Poisson statistics, as assumed by the current models. We prove that this distribution is an inverse power law with an exponent $\mu = 2.06 \pm 0.01$. We propose the long-range model, reproducing the main properties of the diffusion entropy and describing the seismic triggering mechanisms induced by large earthquakes.

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The search for correlation in the space-time distribution of earthquakes is a major goal in geophysics. At the short-time and the short-space scales the existence of correlation is well established. Recent geophysical observations indicate that main fracture episodes can trigger long-range as well as short-range seismic effects [1–4]. However, clear evidence in support of these geophysical indications has not yet been provided. This is probably the reason why one of the models adopted to describe the time distribution of earthquakes is still the generalized Poisson (GP) model [5–9]. Basically, the GP model assumes that the earthquakes are grouped into temporal clusters of events and these *clusters are uncorrelated*: in fact, the clusters are distributed at random in time and therefore the time intervals between one cluster and the next one follow a Poisson distribution. On the other hand, the *intracluster earthquakes are correlated* in time as it is expressed by Omori's law [10,11], an empirical law stating that the main shock, i.e., the highest magnitude earthquake of the cluster, occurring at time t_0 is followed by a swarm of correlated earthquakes (aftershocks) whose number (or frequency) $n(t)$ decays in time as a power law, $n(t) \propto (t - t_0)^{-p}$, with the exponent p being very close to 1. Omori's law implies [12] that the distribution of the time intervals between one earthquake and the next, denoted by τ , is a power law $\psi(\tau) \propto \tau^{-p}$. This property has been recently studied by the authors of Ref. [12] by means of a unified scaling law for $\psi_{L,M}(\tau)$, the probability of having a time interval τ between two seismic events with a magnitude larger than M and occurring within a spatial distance L . This has the effect of taking into account also space and extending the correlation within a finite time range τ^* , beyond which the authors of Ref. [12] recover Poisson statistics.

In this Letter, we provide evidence of an intercluster correlation mechanism by studying a catalog of seismic events in California with a new technique of analysis called diffusion entropy (DE) [13,14]. This technique, scarcely sensitive to predictable events such as the Omori cascade of aftershocks, is instead very sensitive to the deviation from Poisson statistics that generates Lévy diffusion [14,15]. This deviation, on the other hand, implies that the geophysical process generating clusters has some memory. In Fig. 1 we report the sketch of the typical earthquake frequency vs time in the catalog

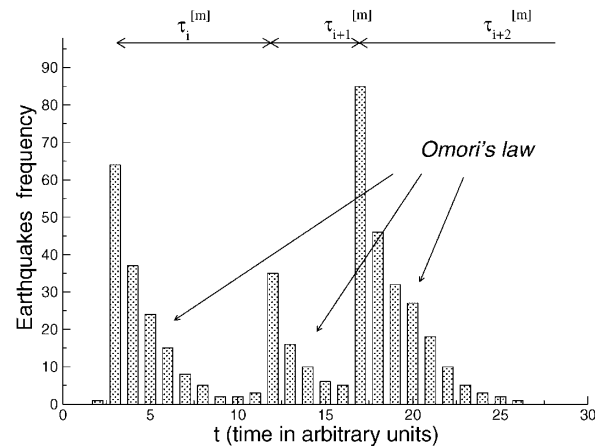


FIG. 1. A sketch of the typical earthquake frequency vs time. In the correspondence to each main shock we observe a frequency peak determined by the aftershock swarm. The peaks decay according to Omori's law. The horizontal arrows indicate the time intervals $\tau_i^{[m]}$ between two consecutive main shocks. The DE method gives information on the distribution of these time intervals.

considered. By $\tau_i = t_{i+1} - t_i$ we indicate the time interval between an earthquake and the next. Each peak of frequency (cluster) in the figure includes the time location of a main shock. The time interval between one peak and the next is reported in the figure and is denoted by the symbol $\tau_i^{[m]}$, where the superscript m stands for main shock. We assume that two distinct time intervals $\tau^{[m]}$ are not correlated, i.e., $\langle \tau_i^{[m]} \tau_j^{[m]} \rangle = \langle (\tau^{[m]})^2 \rangle \delta_{i,j}$. This assumption does not conflict with the departure of the distribution of the time intervals $\tau^{[m]}$ from Poisson. On the other hand, the interoccurrence times τ_i 's are correlated. In fact, after a main shock the earthquake frequency is high. Consequently, the τ 's are short and a short value of τ is followed with a large probability by another short value. For the same reason we expect that, far from a main shock, and prior to the next one, a long value of τ is followed by another long value of τ . This implies that the correlation function $\langle \tau_i \tau_j \rangle$ does not decay to zero after one step and that it survives for the whole time interval between two consecutive main shocks. This means that the long-time relaxation of the correlation function $\langle \tau_i \tau_j \rangle$ is determined by the $\tau^{[m]}$ statistics [15], being faster or slower, according to whether $\psi(\tau^{[m]})$ is Poisson or not. As we shall see, the non-Poissonian condition is straightforwardly assessed by the DE, if $\psi(\tau^{[m]})$ has an infinite second moment. It is important to stress that the model in Fig. 1 will be used to support the results of this Letter with the study of artificial sequences, but in no way does it imply an *a priori* identification of the main shocks for the DE method to work.

The DE method, as almost all the techniques used to detect correlation in a time series $\xi(t)$, is based on the diffusion process of the auxiliary x space through the equation

$$\frac{dx}{dt} = \xi(t). \quad (1)$$

In the case under study here, the stochastic variable $\xi(t)$ is constructed by setting $\xi(t) = 1$ [or $\xi(t) = M$, with M being the earthquake magnitude] if an earthquake occurs at time t and setting $\xi(t) = 0$ otherwise. In practice, $\xi(t)$ is a string of long patches of 0's occasionally interrupted by 1's. With this prescription we build up a diffusion process in the x space [16]. We construct many distinct trajectories, labeled by the integer index $n = 1, 2, \dots$, according to the prescription

$$x_n(t) = \int_{n\Delta t}^{n\Delta t + t} \xi(t') dt', \quad (2)$$

where the generic walker $x_n(t)$ takes a step ahead, of either length 1 or M , every time an earthquake occurs. Note that Δt is the resolution time, set in our case to be equal to 1 min. All the trajectories $x_n(t)$ occupy at time $t = 0$ the position $x = 0$ and then spread up over the x

axis as a result of their partial or total random nature. We study the probability distribution $p(x, t)dx$ of finding the walker position at time t in the interval $[x, x + dx]$. In the reference frame moving with velocity $v = W$, where W is the average number of earthquakes in the time interval Δt , the diffusion process is expected to fulfill the scaling condition

$$p(x, t) = \frac{1}{t^\delta} F\left(\frac{x - Wt}{t^\delta}\right), \quad (3)$$

where $F(y)$ is a function with a form dictated by the statistics of the process, and δ is the so-called scaling parameter. According to the results of Refs. [13,17], the evaluation of the scaling parameter δ requires the use of $S(t) = - \int_{-\infty}^{+\infty} dx p(x, t) \ln[p(x, t)]$. In fact, using Eq. (3), we get, after some simple algebra,

$$S(t) = A + \delta \ln(t). \quad (4)$$

This means that the entropy of the diffusion process is a linear function of $\ln(t)$ and a measure of the slope is equivalent to the determination of the scaling parameter δ . It is now important to observe that the DE method has the interesting property of detecting the statistics of really random events, as recently discovered by the authors of Refs. [15,18]. We refer the reader to these papers for mathematical proofs. For the purposes of this Letter, the following remarks should suffice. The time intervals τ 's are correlated, as shown with the help of Fig. 1, while the DE method rests on the Shannon entropy, and the Shannon entropy increases only as a consequence of the occurrence of really random events. For the time being, we rule out the possibility that the Shannon entropy increase is determined by a deterministic bias [19] and so by a nonstationary condition. In the stationary condition, the only source of entropy increase is given by the occurrence of clusters of seismic events because the $\tau^{[m]}$'s are not correlated. The deviation from the Poisson statistics is easily detected by the DE method if $\psi(\tau^{[m]})$ produces anomalous diffusion. Let us consider the non-Poissonian waiting time distribution:

$$\psi(\tau^{[m]}) \approx \frac{1}{(\tau^{[m]})^\mu}. \quad (5)$$

The condition $\mu > 3$ implies a finite second moment and, consequently, ordinary diffusion with $\delta = 0.5$. The condition $2 \leq \mu < 3$, on the contrary, produces an infinite second moment and, consequently, through the generalized central limit theorem [14], the anomalous scaling

$$\delta = \frac{1}{\mu - 1}. \quad (6)$$

The condition $1 \leq \mu \leq 2$ produces an anomalous scaling with

$$\delta = \mu - 1 \quad (7)$$

and would imply nonstationarity (as in the presence of a deterministic bias). As we shall see, the non-Poissonian statistics of the distance between two clusters (i.e., between large events) is detected by the DE method, yielding the anomalous scaling parameter $\delta = 0.94 \pm 0.01$.

The catalog we have studied covers the period 1976–2002 in the region of southern California spanning 20° N– 45° N latitude and 100° W– 125° W longitude [20]. This region is crossed by the most seismogenic part of the San Andreas fault, which accommodates by displacement the primarily strike-slip motion between the North America and the Pacific plates, producing velocities up to 47 mm/yr [21]. The total number of recorded earthquakes in the catalog is 383 687 and includes the 28 June 1992 Landers earthquakes [$M = 7.3$, where M is for magnitude on the Richter scale], the 17 January 1994 Northridge earthquake ($M = 6.7$), and the 16 October 1999 Hector Mine earthquake ($M = 7.1$). Geophysical observations point out that these large earthquakes have triggered a widespread increase of seismic events at remote distances in space and in time [2,4]. The coupling of the sources of stress change (i.e., large earthquake occurrence) and seismicity triggering mechanisms is a primary target of geophysical investigations and, as shown below, is revealed by the DE analysis. In Fig. 2 we report the results of the DE method. The analysis was performed by setting $\xi(t) = 1$ when an earthquake occurs at time t (independently whether it is a main shock or an aftershock), and $\xi(t) = 0$ if no earthquake happens. In solid circles we plot the entropy $S(t)$ as a function of time when all the seismic events of the catalog are considered (independently of their magnitude M). After a short transient, the function $S(t)$ is characterized by a linear dependence on $\ln t$. A fit in the linear region gives a value of the scaling parameter $\delta = 0.94 \pm 0.01$ at 95%

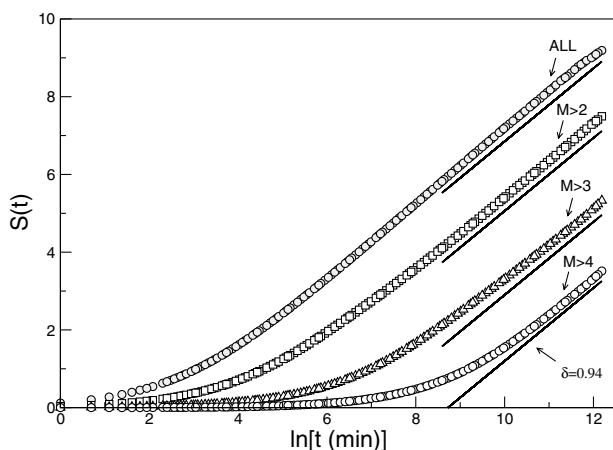


FIG. 2. The Shannon entropy $S(t)$ of the diffusion process as a function of time, in a logarithmic time scale. From top to bottom, the curve refers to all events (solid circles) and to events with threshold $\bar{M} = 2, 3, 4$ (open symbols). The straight lines have the slope $\delta = 0.94$.

of confidence level. We next consider (open symbols in Fig. 2) only the earthquakes with magnitude larger than a fixed value $\bar{M} = 2, 3, 4$. We see that, regardless of the value of the threshold \bar{M} adopted, the function $S(t)$ is characterized by the same long-time behavior with the same slope. This indicates that we are observing a property of the time location of large earthquakes. This leads us to conclude that the time intervals between two large events fit the distribution of Eq. (5), with the value of μ related to δ through Eq. (6), $\mu = 2.06 \pm 0.01$. In fact, numerical checks on the time series under study have supported the stationary assumption and ruled out the alternative condition of Eq. (7) [22]. Our conclusion is also supported by the use of two different walking prescriptions. The former rests on assuming $\xi(t)$ equal to the magnitude M of the earthquake, at each time when an earthquake occurs. The latter sets with equal probability either $\xi(t) = 1$ or $\xi(t) = -1$ when an earthquake occurs [14]. Both methods yield the same exponent $\mu = 2.06 \pm 0.01$. The results of our statistical analysis support the geophysical arguments that earthquakes of large magnitude produce strain diffusion. Unlike coseismic deformation, which is practically instantaneous, the strain diffusion ensuing an earthquake of large magnitude produces postseismic stress changes, generating a remarkable increase of rate of seismicity at locations hundred of kilometers away and over a time span up to several years. Consistently the distribution of the time intervals between two large earthquakes is not a Poisson function. Our conclusion is reached under the important assumption that the sequence $\{\tau_i^{[m]}\}$ is not affected by any deterministic bias, this being a possible source of ballistic scaling [19]. If this condition applies, cluster occurrence would show deterministic trends on the scale of the whole sequence. This is an attractive possibility that does not seem to be ruled out by the current literature on this subject [23]. However, here we adopt the explanation that the waiting time distribution is given by Eq. (5) with $\mu > 2$, this being the unique consequence of the stationary assumption.

We now illustrate how the DE method works on two artificial earthquake time series: the first generated by means of the GP model and the second generated by a new model, the long-range (LR) model, that we propose as a better model to reproduce the properties of the catalog considered. In the LR model the earthquakes are grouped into temporal clusters, and as in the GP model the number of earthquakes in a cluster follows Pareto's law, i.e., a power law distribution with exponent equal to 2.5 [5,7,8]. The events within the same cluster are distributed according to Omori's law: the interval τ follows a power law with exponent $p = 1$. However, in the LR model the time distance $\tau^{[m]}$ between one cluster and the next follows a power law with exponent $\mu = 2.06$, rather than a Poisson prescription as in the GP. Notice that this value of μ is close to the border between stationary

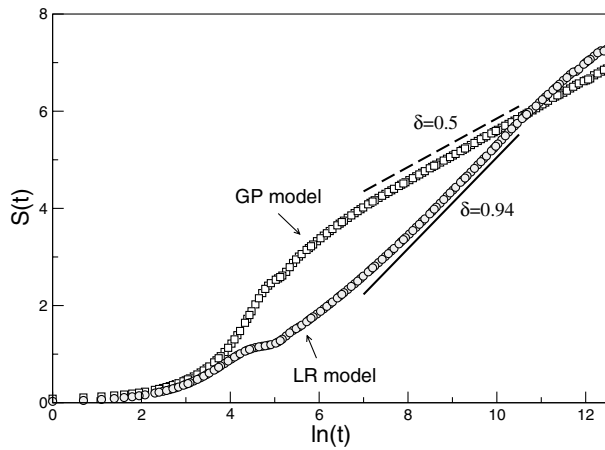


FIG. 3. The Shannon entropy $S(t)$ of the diffusion process as a function of time, in a logarithmic time scale. Open squares and solid circles are, respectively, the results of the GP and of the LR model. The two straight lines have the slopes $\delta = 0.5$ and $\delta = 0.94$.

and nonstationary condition [14]. The two sequences have the same time length. We choose the number of clusters in order to have the same total number of earthquakes as in the real data [24]. The result of the DE on the artificial sequences is reported in Fig. 3. The GP model is characterized by a long-time behavior that, as expected, fits very well the prescription of ordinary statistical mechanics, with $\delta = 0.5$. The LR model yields the quite different scaling $\delta = 0.94$. It is also clear that the LR model yields a behavior qualitatively similar to that produced by the real data of Fig. 2 as well as the same scaling parameter $\delta = 0.94$, while the GP fails in reproducing both properties.

In conclusion, this Letter is the first application of the DE method to study the statistical properties of earthquake time distribution. We found that there exists a correlation mechanism beyond Omori's law. Both intra-cluster swarms and intercluster distances obey an inverse power law prescription, the former being $\psi(\tau) \propto \tau^{-1}$ and the latter $\psi(\tau^{[m]}) \propto (\tau^{[m]})^{-\mu}$ with $\mu = 2.06 \pm 0.01$. We have proposed a new model, the LR model, better than the GP model in reproducing real data. The method proposed is based on the fact that the asymptotic properties of the diffusion process generated by the seismic events are scarcely sensitive to the memory stemming from Omori's law. They are, on the contrary, sensitive to the anomalous statistics generated by the non-Poissonian nature of the time distance between two consecutive large earthquakes.

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- [1] Y. Y. Kagan and D. D. Jackson, *Geophys. J. Int.* **104**, 117 (1991).
- [2] D. P. Hill *et al.*, *Science* **260**, 1617 (1993).
- [3] L. Crescentini, A. Amoroso, and R. Scarpa, *Science* **286**, 2132 (1999).
- [4] T. Parsons, *J. Geophys. Res.* **107**, 2199 (2001).
- [5] S. Shlien and M. N. Toksoz, *Bull. Seismol. Soc. Am.* **60**, 1765 (1970).
- [6] J. K. Gardner and L. Knopoff, *Bull. Seismol. Soc. Am.* **64**, 1363 (1974).
- [7] P. Gasperini and F. Mulargia, *Bull. Seismol. Soc. Am.* **79**, 973 (1989).
- [8] R. Console and M. Murru, *J. Geophys. Res.* **106**, 8699 (2001).
- [9] C. Godano and V. Caruso, *Geophys. J. Int.* **121**, 385 (1995).
- [10] F. Omori, *J. Coll. Sci. Imp. Univ. Tokyo* **7**, 111 (1895).
- [11] T. Utsu, *Geophys. Mag.* **30**, 521 (1961).
- [12] P. Bak, K. Christensen, L. Danon, and T. Scanlon, *Phys. Rev. Lett.* **88**, 178501 (2002).
- [13] N. Scafetta, P. Hamilton, and P. Grigolini, *Fractals* **9**, 193–208 (2001).
- [14] P. Grigolini, L. Palatella, and G. Raffaelli, *Fractals* **9**, 439–449 (2001).
- [15] P. Allegrini, P. Grigolini, P. Hamilton, L. Palatella, and G. Raffaelli, *Phys. Rev. E* **65**, 041926 (2002).
- [16] S. V. Buldyrev, A. L. Goldberger, S. Havlin, C.-K. Peng, M. Simons, and H. E. Stanley, *Phys. Rev. E* **47**, 4514 (1993).
- [17] The work of Ref. [13] proves that the adoption of the Tsallis entropy would be beneficial to explore the transition to the scaling regime, but it would not determine the scaling parameter δ .
- [18] P. Allegrini, R. Balocchi, S. Chillemi, P. Grigolini, L. Palatella, and G. Raffaelli, *Medical Data Analysis*, edited by A. Colosimo *et al.*, Lecture Notes Computer Sciences Vol. 2526 (Springer-Verlag, Berlin, 2002), pp. 115–126.
- [19] M. Ignaccolo, P. Allegrini, P. Grigolini, P. Hamilton, and B. J. West (to be published).
- [20] The catalog has been downloaded from the Southern California Earthquake Data Center <http://www.scecdc.scec.org/ftp/catalogs/SCSN/>
- [21] D. L. Turcotte and G. Schubert, *Geodynamics* (Cambridge University Press, Cambridge, 2002).
- [22] In any case, by using Eq. (7) one would obtain an exponent $\mu = 1.94 \pm 0.01$, a value well different from the Omori's law exponent.
- [23] D. Sornette and A. Helmstetter, *Phys. Rev. Lett.* **89**, 158501 (2002).
- [24] We made other tests by adopting different exponents of Pareto's law (3.0 ± 0.7) and different numbers of clusters, obtaining different transient behavior but the same value of δ .