e	= coefficient of restitution	[—]
f_s	= Fanning friction factor for the solid particle	[—]
G_s	= solid mass velocity	[kg/sec]
G_a	= air mass velocity	[kg/sec]
H	= impulsive force for Y direction	[Kg·sec]
I	= moment of inertia of particle	[kg·m]
L	= pipe length	[m]
M_s	= particle mass	[kg]
m	= mass flow ratio $=G_s/G_a$	[—]
n_i	= local number of particles in unit volume	[1/cc]
P	= impulsive force for Z direction	[Kg·sec]
ΔP_a	= pressure drop due to fluid friction	[Kg/m^2]
ΔP_h	= solid static head	[Kg/m^2]
ΔP_s	= additional friction loss	[Kg/m^2]
ΔP_T	= total pressure drop	[Kg/m^2]
R	= pipe radius	[m]
r	= particle radius	[m]
u	= local air velocity	[m/sec]
u_a	= mean air velocity	[m/sec]
$u_{\max}$	= maximum air velocity	[m/sec]
u_s	= solid velocity	[m/sec]
u_t	= free falling velocity of particle in air	[m/sec]
v_x	= velocity component of X direction	[m/sec]
v_y	= velocity component of Y direction	[m/sec]
v_z	= velocity component of Z direction	[m/sec]
y	= distance from pipe axis	[m]
X	= direction at right angles to Y and Z	
Y	= opposite gravitational direction	
Z	= longitudinal direction of the pipe	
μ	= coefficient of kinetic friction	[—]
μ_s	= coefficient of static friction	[—]
ρ_a	= air density	[kg/m^3]
ρ_{ds}	= dispersed density	[kg/m^3]
ρ_s	= solid density	[kg/m^3]
ω	= angular velocity of the particle	[rad/sec]

Subscript

0 = before collision

1 = after collision

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POWER REQUIREMENTS IN THE AGITATION OF NON-NEWTONIAN FLUIDS*

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A modified Reynolds number in the agitation, based on the Ellis and Sutterby models of three rheological constants for non-Newtonian fluids, is defined in the way that the relation between friction factor and Reynolds number in the case of non-Newtonian pipe flow may be the same as that in the case of Newtonian pipe flow.

Introducing this modified Reynolds number, the same correlation of the power requirements can be applied to both experimental results of Newtonian and non-Newtonian fluids with various types of impellers.

1. Introduction

Agitation, which is one of the most important unit operations, is not easy to approach theoretically and quantitatively. The measurement of power requirements in the agitation is, however, comparatively easy, and the estimation of power consumption for Newtonian fluids is now possible except for some special cases. On the contrary, studies of power requirements in the agitation of non-Newtonian fluids are making slow progress. Such works have been carried out, by Metzner et al.[8,9], Calderbank and Moo-Young[1,2], and others[3,4,6,12]. In many of these studies, a power-law model with two constants has been used to describe the rheological properties of non-Newtonian fluids. However, the power-law model cannot express the properties of non-Newtonian fluids in a wide range

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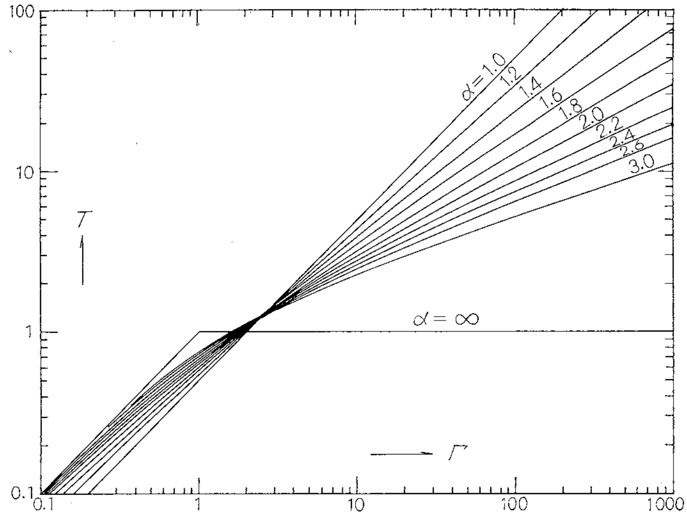

Fig. 1 Correlation of T and $\boldsymbol{\Gamma}$ of the Ellis model

of the shear stress. The authors will correlate the power requirements in the agitation of Newtonian and non-Newtonian fluids by applying three constants-models for non-Newtonian pseudoplastic fluids based on the works of Ellis[7] and Sutterby[13]. The Ellis model is considered in this paper, and the Sutterby model is treated in the Appendix.

2. Development of Correlation

2-1 Evaluation of rheological constants

The rheological equations of the Ellis model are expressed by the following equations.

$$\boldsymbol{\tau} = -\eta \boldsymbol{\Delta} \tag{1}$$

$$\eta = \eta_0 \Bigg/ \left(1 + \left|\frac{\sqrt{\frac{1}{2}(\boldsymbol{\tau}:\boldsymbol{\tau})}}{\tau_{1/2}}\right|^{\alpha-1}\right) \tag{2}$$

where, $\boldsymbol{\Delta}$ is the rate of deformation tensor with components

$$\Delta_{ij} = \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{3}$$

and

$$(\boldsymbol{\tau}:\boldsymbol{\tau}) = \sum_i \sum_j \tau_{ij}\tau_{ji} \tag{4}$$

When an Ellis model fluid which is described by Eqs. (1) and (2) flows in a circular pipe, the velocity profile is expressed by

$$v_z = \left(\frac{\tau_R \cdot R}{\eta_0}\right)\left[\frac{1}{2}\left\{1 - \left(\frac{r}{R}\right)^2\right\} + \frac{1}{\alpha+1}\left(\frac{\tau_R}{\tau_{1/2}}\right)^{\alpha-1}\left\{1 - \left(\frac{r}{R}\right)^{\alpha+1}\right\}\right] \tag{5}$$

where, τ_R is the shear stress at the wall

$$\tau_R = \frac{\Delta P \cdot R}{2L} \tag{6}$$

From Eq. (5) the volumetric flow rate Q is obtained as

$$Q = \frac{\pi R^3}{4\eta_0}\tau_R\left[1 + \frac{4}{\alpha+3}\left(\frac{\tau_R}{\tau_{1/2}}\right)^{\alpha-1}\right] \tag{7}$$

Rewriting Eq. (7),

$$\eta_0 \frac{4Q}{\pi R^3} = \tau_R\left[1 + \frac{4}{\alpha+3}\left(\frac{\tau_R}{\tau_{1/2}}\right)^{\alpha-1}\right] \tag{8}$$

Substituting the following equations into Eq. (8),

$$\boldsymbol{\Gamma} = \frac{\eta_0}{\tau_{1/2}} \cdot \frac{4Q}{\pi R^3} \tag{9}$$

$$T = \frac{\tau_R}{\tau_{1/2}} \tag{10}$$

one obtains

$$\boldsymbol{\Gamma} = T\left(1 + \frac{4}{\alpha+3}T^{\alpha-1}\right) \tag{11}$$

Eq. (11) is the equation of flow in a circular pipe for an Ellis model fluid. The relationship between T and $\boldsymbol{\Gamma}$ is plotted with α as a parameter in **Fig. 1.**

The three rheological constants can be evaluated by measuring the flow rate, Q, and the pressure drop, ΔP, with a capillary viscometer and then plotting $4Q/\pi R^3$ against τ_R on a logarithmic scale. From a curve in Fig. 1, the gradient of which is as same as that of the experimental curve, one can determine the value of α, and then $\tau_{1/2}$ and η_0 can be determined from the deviations on the ordinate and abscissa[7].

2-2 Modified Reynolds number

From Eq. (7), one obtains

$$\frac{8\eta_0\langle v_z\rangle}{\tau_{1/2}D} = \frac{\tau_R}{\tau_{1/2}}\left[1 + \frac{4}{\alpha+3}\left(\frac{\tau_R}{\tau_{1/2}}\right)^{\alpha-1}\right] \tag{12}$$

Therefore, $\tau_R/\tau_{1/2}$ is expressed as a function of α and $8\eta_0\langle v_z\rangle/\tau_{1/2}D$ as follows:

$$\frac{\tau_R}{\tau_{1/2}} = F_e\left(\alpha, \frac{8\eta_0\langle v_z\rangle}{\tau_{1/2}D}\right) \tag{13}$$

Since

$$\frac{\tau_R}{\tau_{1/2}} = T \tag{10}$$

and

$$\frac{8\eta_0\langle v_z\rangle}{\tau_{1/2}D} = \frac{\eta_0}{\tau_{1/2}} \cdot \frac{4Q}{\pi R^3} = \boldsymbol{\Gamma} \tag{14}$$

the function F_e is obtained from Fig. 1.

For Newtonian laminar flow in a circular pipe, there is the following relation between friction factor, f, and Reynolds number, Re.

$$f = \frac{2\tau_R}{\rho\langle v_z\rangle^2} = \frac{16}{R_e} \tag{15}$$

If the Reynolds number is defined so that Eq. (15) may hold also for the Ellis model, Eqs. (12) and (13) yield

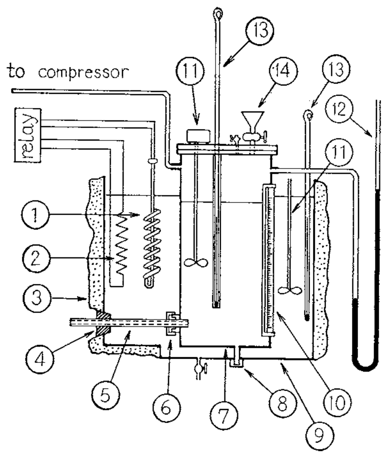

Fig. 2 Experimental apparatus for flow characteristics measurements

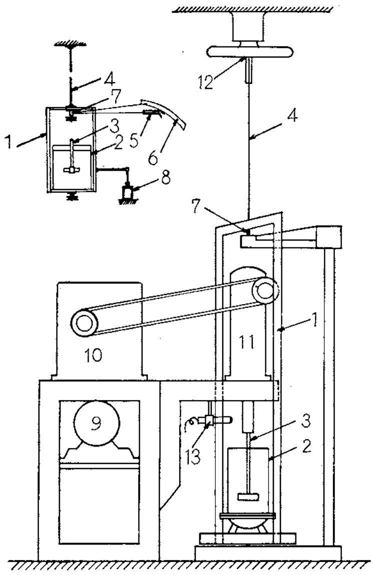

Fig. 3 Experimental apparatus for measurement of agitation power requirements

$$R_e = \frac{D\rho<v_z>}{\eta_0}\left[1 + \frac{4}{\alpha+3}\left\{F_e\left(\alpha,\ \frac{8\eta_0<v_z>}{\tau_{1/2}D}\right)\right\}^{\alpha-1}\right] \tag{16}$$

In order to apply Eq. (16) to an agitated tank, one conventionally replaces the pipe diameter D by the impeller diameter d and the mean velocity $<v_z>$ by the impeller tip velocity dn.

$$N_{R_e} = \frac{\rho n d^2}{\eta_0}\left[1 + \frac{4}{\alpha+3}\left\{F_e\left(\alpha,\ \frac{8n\eta_0}{\tau_{1/2}}\right)\right\}^{\alpha-1}\right] \tag{17}$$

F_e in Eq. (17) is then obtained from the ordinate corresponding to the value of $8n\eta_0/\tau_{1/2}$ on the abscissa in Fig. 1.

3. Experiment

3-1 Flow characteristics

The experimental apparatus, shown in **Fig. 2**, is the same one described in a previous report[10]. The internal

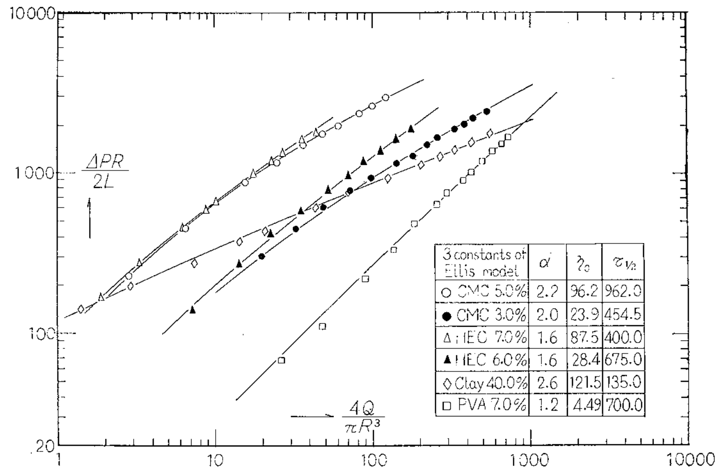

Fig. 4 Typical results of fluid flow characteristics

Table 1 Values of K for piano wires

	Diameter [mm]	Length [mm]	K[G. cm/rad]
No. 1	2.703	1560	2.895×10^{-3}
No. 2	3.710	1540	10.240 〃
No. 3	4.509	1560	22.540 〃

Table 2 Range of Variables Studied

Impeller type	d/D_T	b/D_T	H/D_T
2-Bladed-paddle	0.5	0.2	1.0
	0.8	0.2	1.0
6-Bladed-turbine	0.56	0.112	1.0
	0.67	0.134	1.0
4-Multiple-paddle	0.5	0.2	1.5
Screw with draft tube	0.6	—	1.3

Tank diameter $D_T=10,\ 30$ [cm]
Modified Reynolds number $N_{R_e}=0.13\sim520$
Impeller speed $n=0.617\sim9.70$ [rev./sec]

Fluids used
Carboxy-methyl-cellulose solutions in water
Hydroxy-ethyl-cellulose solutions in water
Polyvinyl-alcohol solutions in water
Clay suspensions in water

diameter of the pipe was determined from flow rate and pressure drop data for aqueous cane sugar solutions and "millet" jelly, which are Newtonian fluids of known viscosity. As the manometer fluid, mercury was used.

3-2 Power requirements of agitation

The experimental apparatus was similar to that of Honjō et al.[5] and is shown in **Fig. 3**. The agitated tank was hung by a torsion wire. The torque for agitation was evaluated from the angle of deflection of the turn table, which was measured by a lamp scale and a mirror. For the attenuation of vibrations, a damper was installed on the turn table. Piano wires, specifications of which are shown in **Table 1**, were used as the torsion wire.

The above-mentioned measuring device was replaced by a torque-meter only for a 30 cm diameter tank. Agitation speed was obtained by measuring the pick-up signal pulses with a photoelectric cell installed near the rotating shaft. In order to check the reliability of the

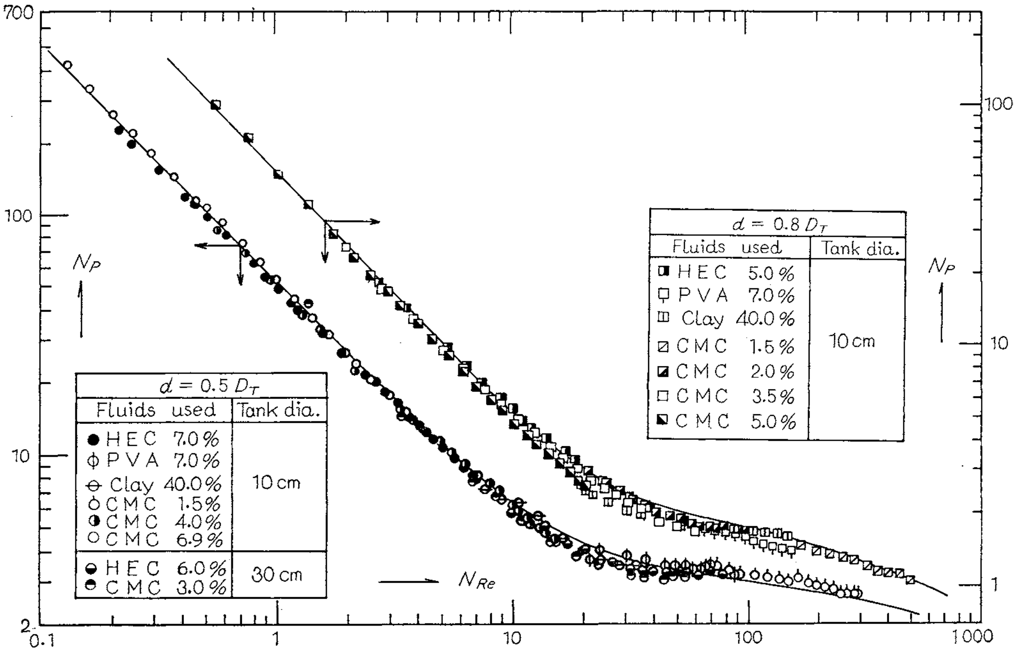

Fig. 5 Correlation of power requirements 2-Bladed-paddle (Ellis model). Each impeller was positioned 5.0cm from the bottom of a 10cm diameter unbaffled tank

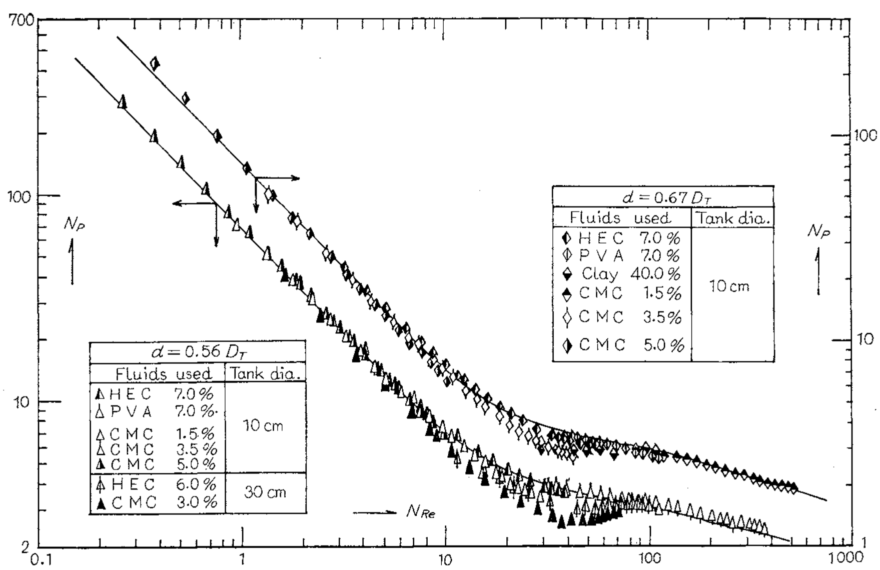

Fig. 6 Correlation of power requirements 6-Bladed-turbine (Ellis model). Each impeller was positioned 3.4cm from the bottom of a 10cm diameter unbaffled tank

performance of the apparatus, Newtonian fluids were used and experiments were carried out under the same conditions as those in a paper by Nagata et al.[11]. Dimensions of the agitators and kinds of fluids used in the experiments and the range of experimental conditions are listed in **Table 2.**

4. Discussion

Some examples of the flow characteristics are shown in **Fig. 4.** The solid lines were calculated from the Ellis model. It is difficult to describe the measured values by means of the power-law model for a wide range of the shear stress, since the measured values are not always a straight line as shown in Fig. 4. The three constants of the Ellis model are determined by the above mentioned method. The modified Reynolds number is calculated from Eq.(17) using these three constants. The measured Power Numbers as a function of the modified Reynolds Number are shown in **Figs.**

5, 6, 7 and 8. The solid lines in Fig. 5 are results for Newtonian fluids reported previously by Nagata et al.[11]. The solid lines in Figs. 6, 7 and 8 are the results for Newtonian fluids obtained by the authors. In Figs. 5, 6, 7 and 8, it is found that there is some disagreement between the Newtonian and non-Newtonian results in the transition range. However, it is evident in these figures that the correlation for non-Newtonian fluids using the modified Reynolds number of Eq. (17) are in good agreement with Newtonian fluids except in the transition range. This fact supports the applicability of the correlations of Newtonian fluids in estimating power requirements of agitation for non-Newtonian fluids when the modified Reynolds number defined by Eq. (17) is used.

The results from the application of the Sutterby model to non-Newtonian fluids are given in Appendix. Good agreement is obtained in these cases, too. In addition to the types of impeller mentioned above, the experiments on the anchor and ribbon types were executed. So far, however, the correlation has not been successful.

5. Conclusion

A modified Reynolds number based on the Ellis model and the Sutterby model of three rheological constants for non-Newtonian fluids was proposed, and found to be useful in the correlation of power requirements in the agitation of non-Newtonian fluids.

Acknowledgement

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Appendix

Correlation of Sutterby model

$$\tau = -\eta(\mathrm{II})\Delta \quad \text{(A 1)}$$

$$\eta(\mathrm{II}) = \eta_0 \left[\frac{\operatorname{arc\,sinh} B\sqrt{\frac{1}{2}\mathrm{II}}}{B\sqrt{\frac{1}{2}\mathrm{II}}} \right]^{A} \quad \text{(A 2)}$$

where,

$$\Delta_{ij} = \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad \text{(A 3)}$$

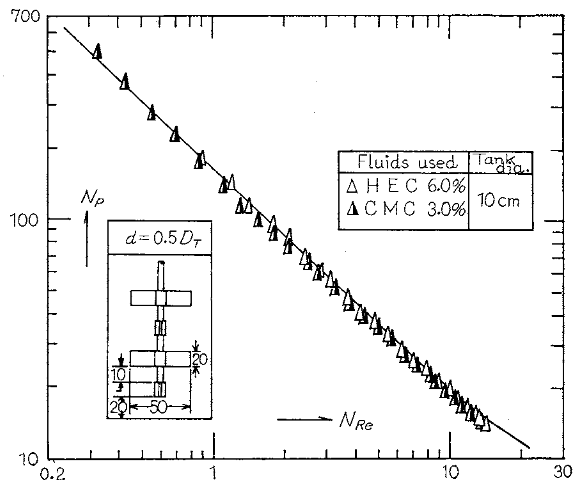

Fig. 7 Correlation of power requirements 4-Multiple-paddle (Ellis model)

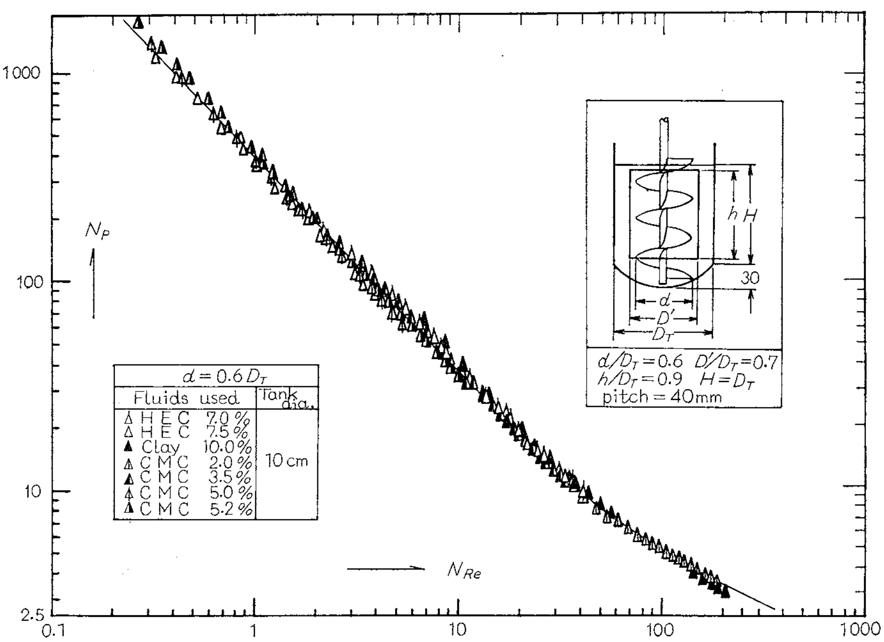

Fig. 8 Correlation of power requirements Screw with draft tube (Ellis model)

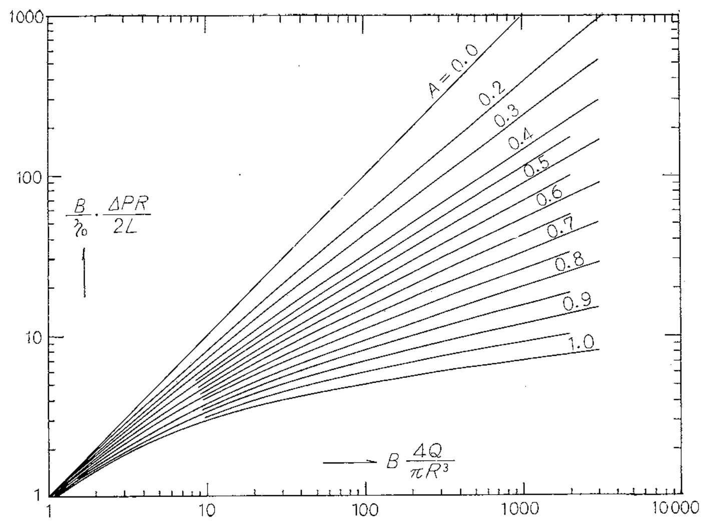

Fig. 9 Correlation of $\frac{B}{\eta_0} \cdot \frac{\Delta PR}{2L}$ and $B \cdot \frac{4Q}{\pi R^3}$ of the Sutterby model

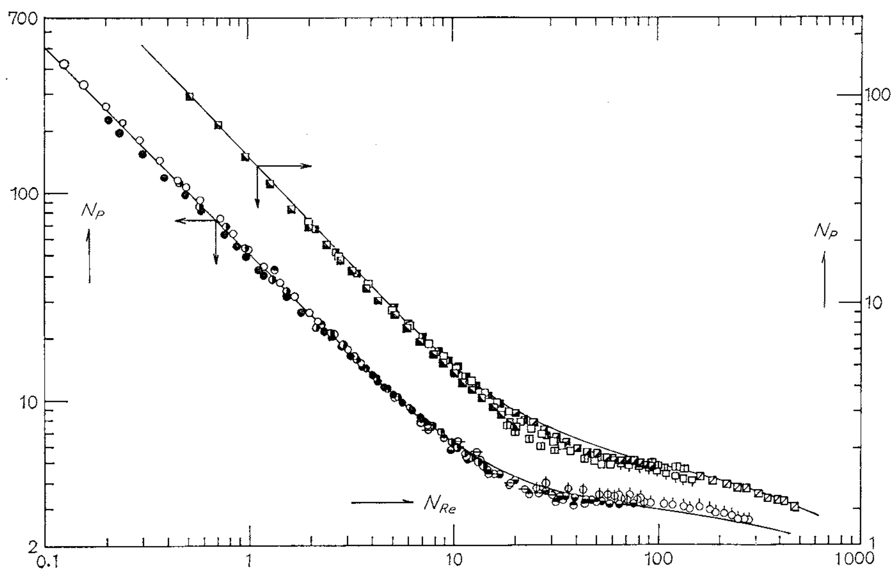

Fig. 10 Correlation of power requirements 2-Bladed-paddle (Sutterby model) see Fig. 5 for legend

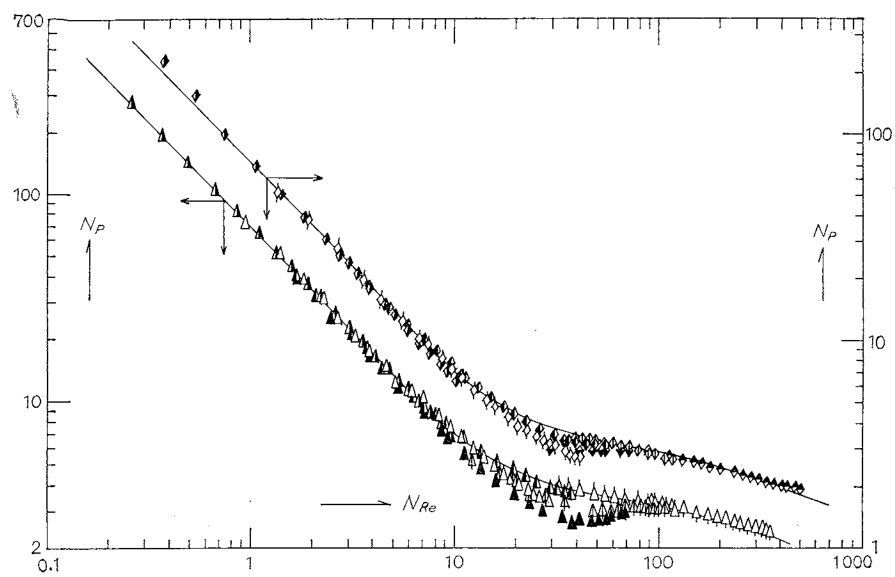

Fig. 11 Correlation of power requirements 6-Bladed-turbine (Sutterby model) see Fig. 6 for legend

and $$\text{II} = \sum_i \sum_j \Delta_{ij}\Delta_{ji} \qquad \text{(A 4)}$$

The pressure gradient and the volumetric flow rate are

$$\frac{B}{\eta_0}\cdot\frac{\Delta PR}{2L} = (B\gamma_R)\left[\frac{\text{arc sinh}(B\gamma_R)}{B\gamma_R}\right]^A \qquad \text{(A 5)}$$

$$B\cdot\frac{4Q}{\pi R^3} = \frac{4}{3}(B\gamma_R)\left\{1-\left[\frac{\text{arc sinh}(B\gamma_R)}{B\gamma_R}\right]^{-3A} \times\int_0^1\left[\frac{\text{arc sinh}(B\gamma_R\gamma^*)}{B\gamma_R\gamma^*}\right]^A\gamma^{*3}d\gamma^*\right\} \qquad \text{(A 6)}$$

where, $$\gamma^* = \gamma/\gamma_R \qquad \text{(A 7)}$$

Therefore,

$$\frac{B}{\eta_0}\cdot\frac{\Delta PR}{2L} = F_s\left(A,\ B\frac{4Q}{\pi R^3}\right) \qquad \text{(A 8)}$$

The relation between $\frac{B}{\eta_0}\cdot\frac{\Delta PR}{2L}$ and $B\cdot\frac{4Q}{\pi R^3}$ is calculated from Eqs. (A 5) and (A 6) with A as a parameter. In **Fig. 9**, is shown the correlation of $\frac{B}{\eta_0}\cdot\frac{\Delta PR}{2L}$ and $B\cdot\frac{4Q}{\pi R^3}$ of the calculated results.

Since the modified Reynolds number in a circular pipe is obtained as the following equation by considering similarly as in the case of the Ellis model,

$$R_e = \frac{D\rho<v_z>}{\eta_0}\cdot\frac{B\cdot\frac{8<v_z>}{D}}{F_s\left(A,\ B\cdot\frac{8<v_z>}{D}\right)} \qquad \text{(A 9)}$$

we can obtain Eq. (A 10) as the modified Reynolds number for an agitated tank with D replaced by d and $<v_z>$ by dn.

$$N_{Re} = \frac{\rho n d^2}{\eta_0}\cdot\frac{8nB}{F_s(A,\ 8nB)} \qquad \text{(A 10)}$$

The function F_s in Eq. (A 10) is obtained from the ordinate corresponding to the value of $8nB$ on the abscissa in Fig. 9. Results of the experiments are shown in **Figs. 10, 11, 12 and 13.**

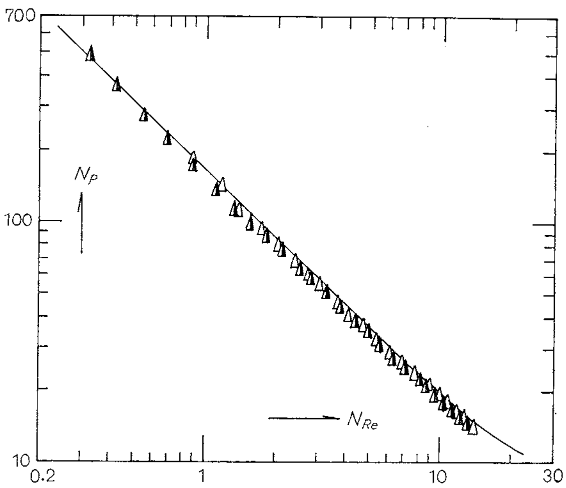

Fig. 12 Correlation of power requirements 4-Multiple-paddle (Sutterby model) see Fig. 7 for legend

Nomenclature

A	= rheological constant in the Sutterby model	[—]
B	= rheological constant in the Sutterby model	[sec]
b	= width of impeller	[cm]
D	= diameter of circular pipe	[cm]
D_T	= diameter of agitated tank	[cm]
d	= diameter of impeller	[cm]
F_e	= defined by Eq. (13)	
F_s	= defined by Eq. (A 8)	
f	= friction factor	[—]
g_c	= gravitational conversion factor	[g·cm/G·sec²]
H	= liquid depth	[cm]
K	= wire constant	[G·cm/rad]
L	= length of circular pipe	[cm]
N_p	= power number $N_p=P\cdot g_c/\rho n^3 d^5$	[—]
N_{Re}	= modified Reynolds number	[—]
n	= impeller speed	[rev/sec]
P	= power consumption	[G·cm/sec]
ΔP	= pressure drop	[dyne/cm²]
Q	= volumetric flow rate through a circular pipe	[cm³/sec]
R	= radius of circular pipe	[cm]
r	= radial distance from axis of circular pipe	[cm]
R_e	= modified Reynolds number of circular pipe	[—]
T	= defined by Eq. (10)	[—]
v_i	= i-component of the velocity vector	[cm/sec]
v_z	= fluid velocity	[cm/sec]
$<v_z>$	= mean velocity	[cm/sec]
α	= rheological constant in the Ellis model	[—]

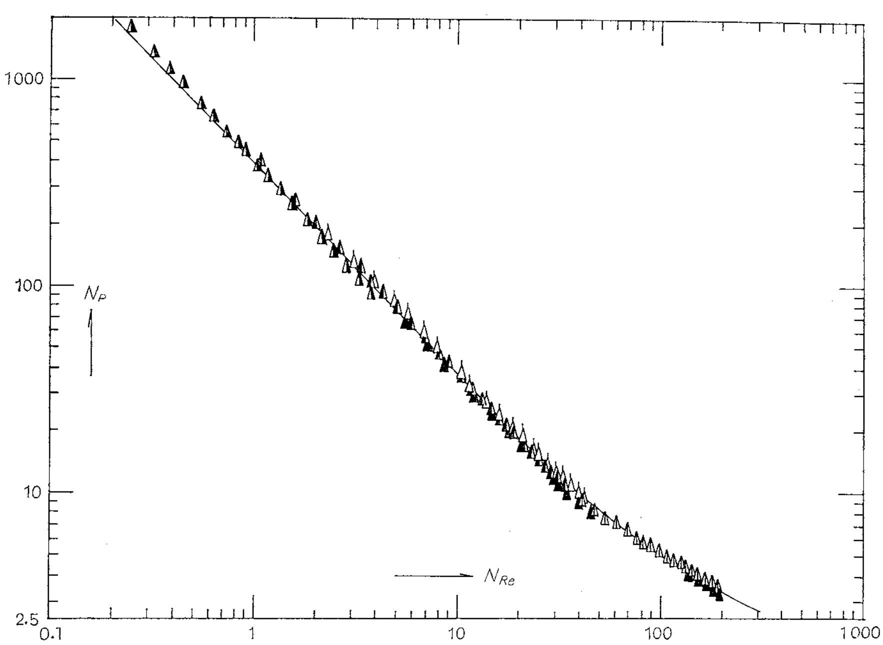

Fig. 13 Correlation of power requirements Screw with draft tube (Sutterby model) see Fig. 8 for legend

Γ	= defined by Eq. (9)	[—]
γ	= rate of deformation	[1/sec]
γ_R	= rate of deformation at the circular pipe wall	[1/sec]
γ^*	= defined by Eq. (A 7)	[—]
Δ	= rate of deformation tensor	[1/sec]
η	= non-Newtonian viscosity	[g/cm·sec]
η_0	= zero shear viscosity	[g/cm·sec]
η_R	= non-Newtonian viscosity at $\gamma=\gamma_R$	[g/cm·sec]
ρ	= fluid density	[g/cm³]
τ	= shear stress tensor	[dyne/cm²]
τ_{ij}	= i, j-component of the shear stress tensor	[dyne/cm²]
$\tau_{1/2}$	= shear stress at $\eta=\eta_0/2$	[dyne/cm²]
τ_R	= shear stress at the circular pipe wall	[dyne/cm²]
II	= second inveriant of rate of deformation tensor	[1/sec]

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POWER REQUIREMENTS FOR AGITATING AIR-FLUIDISED BEDS*

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The method of approach by "Mero-plastical Dynamics" has been applied to estimate the shearing stress and inertia force for agitating fluidised beds.**

The modified "M"number, N_M', which has been derived from the force balance for agitating fluidised solid beds, is defined as follows,

$$N_{M'} = NR/\sqrt{gL_e}\ \tan\phi$$

which is based on fluidised solid beds, radius of blade and rotational speed, and is quite similar to the Reynolds number for agitating liquids.

The general correlation of the power number, N_P, with N_M' for agitating fluidised beds, including the literature data, has been determined experimentally in the following equation similar to those for agitating fixed solid beds and for liquids in tanks.

$$N_P = 35\,N_{M'}^{-1} + 50$$

1. Introduction

The shearing stress is considered to be one of the most important factors for analysing the dynamic behaviour, such as the rising velocity of bubbles[15], the distribution of stresses[2], and the movements of solid particles[9,10], in fluidised beds. Most investigators, however, were not concerned with the characteristics of solid particles, but rather analysed the fluidised beds merely as being ideal or non-viscous fluids.

There were also many investigators who studied the apparent viscosity acting on the rotating blades[1,3~6,12,13,16] or cylinder[14] in the fluidised beds. Since fluidised beds are usually believed to have liquid-like properties, all of them explained the stresses in fluidised beds only as apparent viscosities like those of liquids, and did not notice the essential characteristics of fluidising particles. Such an analogy of fluidised beds to liquids is nothing but an expedient.

The "M" number, based on the internal friction and the inertial force of solid particles, was first introduced by the authors (1968)[7] to the analysis of the solid movements caused by rotating blades in fixed solid beds. The original "M" number, N_M, for agitating fixed solid beds, when accompanied by gas streams at a rate less than u_{mf}, has also been found to be the key factor for correlating the power number, N_P[8].

$$N_M = \text{(inertial force)}/\text{(shearing stress by the internal friction of solid bed)} = N^2R^2/gH_e \tan\phi \qquad (1)$$

The analysis of dynamic behaviour in gas-solid systems, based not only upon their hydrodynamic charac-

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** The word "mero" in Greek means "of particle".