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Power Spectrum Prediction of Amplified Dual-Band LTE-Advanced Signals

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by

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Abstract

In wireless communication, the nonlinearity of a radio frequency (RF) power amplifier is an important issue for power amplifier designers. Since the nonlinearity is generated by the properties of physical components, it is hard to avoid it in producing power amplifiers. Power amplifier designers should know about the nonlinearity in order to compensate for it.

A two-tone test is a relatively widely used method to measure the nonlinearity of a power amplifier, which means the third order intercept point (IP3) can be measured from the two-tone test. Through the two-tone test, researchers have proposed some formulae to present what the amplified Code Division Multiple Access (CDMA) signal is like. They derived formulae in terms of output power, bandwidth, IP3, and IP5 to express the amplified CDMA signal and further to Orthogonal Frequency Division Multiplexing (OFDM) signals. With the development of wireless communication, researchers put their interest increasingly in Multiple Input Multiple Output (MIMO) systems. A formula expressing amplified dual two-tone signals has been proposed. In their research, they discussed what the expressions of intermodulation and cross modulation are and what their locations are.

In this research, dual band LTE-Advanced signals, whose modulation is OFDM are utilized, which means this research proposes a formula expression about the power spectrum of dual-band LTE-Advanced signals. Intermodulation and cross modulation caused by nonlinearity of power amplifiers are then specially discussed. This study will help RF designers to continuously compensate for them.

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Chapter 1 Introduction

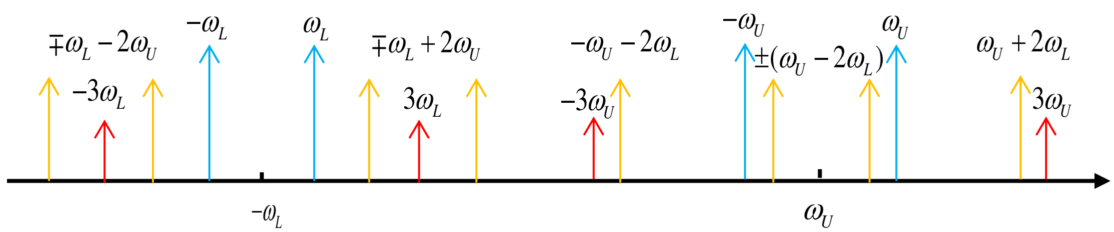
In wireless communication systems, a radio frequency (RF) power amplifier is always a significant component. To design a RF power amplifier, the nonlinear distortion effect is one of the main concerns that no designer can ignore. In communication systems, nonlinearities not only decrease the performance of our transmitted signals, but also strengthen the interference of the adjacent channels. Normally, when considering nonlinear distortion, the intercept points, especially the third-order intercept point (IP_3), are always useful nonlinear parameters to describe the nonlinearity. In addition, the coefficients of the memoryless power amplifier model can be expressed with IP_3, which can be found easily on the data sheets of power amplifiers. Therefore, the AM/AM conversion and AM/PM conversion are determined only by the input signals.

Previous researchers have made their contributions to predict the power spectrum of amplified signals. Heng [1] proposed a formula for a one-band amplified power spectrum using a CDMA signal as an input signal. Based on [2], a CDMA signal can be assumed as rectangular in the frequency domain, which is a huge convenience for computing the amplified power spectrum. The equation in [1] is expressed with IP_3 and IP_5, which are the parameters of a power amplifier. The reason to use IP_3 and IP_5 as variables is that it is easy to find IP_3 from data sheets and to calculate IP_5 from measurement [3]. Xiao [4] improved this work in OFDM by proving that the power spectrum of the OFDM signal can converge to rectangular as well.

In recent years, wireless communication systems have developed rapidly, while concurrent multi-band signals have been discussed increasingly. It is important for RF designers to predict what the power spectrum is like for concurrent dual-band signals after power amplifier operation. However, it is not reasonable to extend the conclusion from [1] directly to dual bands, because of the cross modulation between the dual-band signals which is not included in the one band equation. Shoaib Amin *et al*. [5] discussed a two-tone test with dual two tones as its input. They proposed some formulae to express the amplified dual two-tone signal and discussed intermodulation and cross modulation. Figure 1 shows the power spectrum of dual two-tone test before and after the amplifier operation.

In this research, the aim is to derive a power spectrum expression of dual-band LTE-Advanced signals after power amplifier operation. The nonlinearity behavior of a dual-band system is different from that of one band. While considering the dual-band system, the whole modulation at the output can be categorized into two parts. The first part is traditional intermodulation, which is called in-band modulation. The in-band modulation consists of the intermodulation products around each carrier center frequency that are solely due to the intermodulation between the signal within each band, same as what it is in one band. The second part is made up of components called cross modulation products. The cross-modulation products are located in a similar frequency range as in-band modulation but the reason that cross modulation is generated is the interaction between both frequency bands. For example, the upper band signal has influence on the lower band signal and the frequency regrowth locates near the lower center frequency. This frequency regrowth is called cross modulation products in lower band. Similarly, the signal in lower band has influence on upper band as well, while their frequency regrowth locates around the upper center frequency.

(a)



(b)

Figure 1. Power spectrum of dual two-tone test: (a) before amplifier and (b) after the amplifier

Chapter 2 Spectrum Model of Dual-Band LTE-Advanced Signals

2.1 Statistical Model of LTE-Advanced Signals

OFDM is the most effective method of LTE-Advanced signal modulation. According to the Law of Large Numbers and the Central Limit Theorem [6] in statistics, the larger the number of subcarriers become, the closer the OFDM signal $s(t)$ can be considered as a band-limited Gaussian stochastic process with zero mean. Based on the statement, an equivalent band-limited white Gaussian process is used to study the spectrum of the amplified LTE-Advanced signal. The Gaussian stochastic signal has been studied extensively in signal processing. Many well-known results of Gaussian stochastic signal could be used directly in the following derivation.

A bandpass Gaussian statistical equivalent of $s(t)$ could be expressed as

$$s(t) = \tilde{s}(t)\cos(2\pi f_0 t + \theta) \tag{2.1.1}$$

where θ is an arbitrary phase which does not affect the performance of $s(t)$ and $\tilde{s}(t)$ is a Gaussian wide-sense stationary process with a power spectral density (PSD) of,

$$P_{\tilde{s}}(f) = \begin{cases} \dfrac{N_0}{2} & |f| \le B \\ 0 & |f| > B \end{cases} \tag{2.1.2}$$

where B is the bandwidth of the LTE-Advanced signal and N_0 is a constant that comes from $P_0 = \dfrac{a_1^2 N_0 B}{2}$. P_0 is the power amplifier's output power and a_1 presents the linear gain of the PA.

From above, a dual-band passband LTE-Advanced signal can be considered as the input

$$s(t) = s_1(t) + s_2(t) = \tilde{s}_1(t)\cos\left(2\pi f_{c_1} t + \theta_1\right) + \tilde{s}_2(t)\cos\left(2\pi f_{c_2} t + \theta_2\right) \qquad (2.1.3)$$

In this equation (2.1.3), f_{c_1} and f_{c_2} are center frequencies, $\tilde{s}_1(t)$ and $\tilde{s}_2(t)$ are both Gaussian wide-sense stationary processes with PSD of

$$P_{\tilde{s}_1}(f) = \begin{cases} \dfrac{N_1}{2} & |f| \le B_1 \\ 0 & |f| > B_1 \end{cases} \qquad (2.1.4)$$

and

$$P_{\tilde{s}_2}(f) = \begin{cases} \dfrac{N_2}{2} & |f| \le B_2 \\ 0 & |f| > B_2 \end{cases} \qquad (2.1.5)$$

respectively.

2.2 The Nonlinear Model and Parameters of RF Power Amplifiers (PAs)

A power amplifier (PA) can hardly be a linear device, so designers usually use the Taylor series to model a PA. The Taylor series is widely used to describe power amplifiers because the high order components in the Taylor series express nonlinearity well. Normally the Taylor series model of a radio frequency (RF) PA can be represented as

$$y(t) = \sum_{i=0}^{\infty} a_{2i+1} s^{2i+1}(t) = a_1 s(t) + a_3 s^3(t) + a_5 s^5(t) + \cdots \qquad (2.2.1)$$

In equation (2.2.1), only odd order components are shown. The reason that even order terms are ignored is they are located far away from the center passband after the amplifier operation so that they can be filtered out easily. In addition, since the fifth and higher order regrowth are relatively small compared to third order regrowth, only the linear part and third order regrowth of the above equation (2.2.1) will be analyzed. The calculation would become quite complex and hard to derive an expression otherwise.

Therefore, the following amplifier expression with a linear term and a third-order term is now used to model the nonlinearity of RF PAs:

$$y(t) = a_1 s(t) + a_3 s^3(t) \tag{2.2.2}$$

The coefficient a_1 can be derived from the following equation

$$a_1 = 10^{\frac{G}{20}} \tag{2.2.3}$$

where G is the linear gain of the power amplifier in dB. The coefficient a_3 can be expressed using IP_3 and G,

$$a_3 = -\frac{2}{3} 10^{\left(-\frac{IP_3}{10} + \frac{3G}{20}\right)}. \tag{2.2.4}$$

a_3 is negative because amplifiers usually perform gain compression.

2.3 Correlation of the Dual-Band LTE-Advanced Output Signals

Substituting the input pass band signal (2.1.3) into equation (2.2.2) with the assumption that $\theta_1 = \theta_2 = 0$. The output $y(t)$ will be

$$
\begin{aligned}
y(t) = & \left[a_1\tilde{s}_1(t) + \frac{3}{4}a_3\tilde{s}_1^3(t) + \frac{3}{2}a_3\tilde{s}_1(t)\tilde{s}_2^2(t) \right]\cos\left(2\pi f_{c_1} t\right) \\
& + \left[a_1\tilde{s}_2(t) + \frac{3}{4}a_3\tilde{s}_2^3(t) + \frac{3}{2}a_3\tilde{s}_2(t)\tilde{s}_1^2(t) \right]\cos\left(2\pi f_{c_2} t\right) \\
& + \frac{1}{4}a_3\tilde{s}_1^3(t)\cos\left(6\pi f_{c_1} t\right) + \frac{1}{4}a_3\tilde{s}_2^3(t)\cos\left(6\pi f_{c_2} t\right) \\
& + \frac{3}{4}a_3\tilde{s}_1^2(t)\tilde{s}_2(t)\left[\cos\left(4\pi f_{c_1} t + 2\pi f_{c_2} t\right) + \cos\left(4\pi f_{c_1} t - 2\pi f_{c_2} t\right)\right] \\
& + \frac{3}{4}a_3\tilde{s}_1(t)\tilde{s}_2^2(t)\left[\cos\left(2\pi f_{c_1} t + 4\pi f_{c_2} t\right) + \cos\left(4\pi f_{c_2} t - 2\pi f_{c_1} t\right)\right]
\end{aligned}
\tag{2.3.1}
$$

The benefit of the phase assumption is not only that the phase won't affect the performance of the equation, but also that the equation is relatively concise. In order to simplify the equation (2.3.1), six new terms $\tilde{y}_1(t)$, $\tilde{y}_2(t)$, $\tilde{y}_3(t)$, $\tilde{y}_4(t)$, $\tilde{y}_5(t)$, and $\tilde{y}_6(t)$ are defined as follows.

$$\tilde{y}_1(t) = a_1\tilde{s}_1(t) + \frac{3}{4}a_3\tilde{s}_1^3(t) + \frac{3}{2}a_3\tilde{s}_1(t)\tilde{s}_2^2(t) \tag{2.3.2}$$

$$\tilde{y}_2(t) = a_1\tilde{s}_2(t) + \frac{3}{4}a_3\tilde{s}_2^3(t) + \frac{3}{2}a_3\tilde{s}_2(t)\tilde{s}_1^2(t) \tag{2.3.3}$$

$$\tilde{y}_3(t) = \frac{3}{4}a_3\tilde{s}_1^2(t)\tilde{s}_2(t) \tag{2.3.4}$$

$$\tilde{y}_4(t) = \frac{3}{4}a_3\tilde{s}_2^2(t)\tilde{s}_1(t) \tag{2.3.5}$$

$$\tilde{y}_5(t) = \frac{1}{4}a_3\tilde{s}_1^3(t) \tag{2.3.6}$$

$$\tilde{y}_6(t) = \frac{1}{4}a_3\tilde{s}_2^3(t) \tag{2.3.7}$$

Thus, equation (2.3.1) can be rewritten using (2.3.2) – (2.3.7)

$$\begin{aligned} y(t) = {} & \tilde{y}_1(t)\cos\left(2\pi f_{c_1} t\right) + \tilde{y}_2(t)\cos\left(2\pi f_{c_2} t\right) \\ & + \tilde{y}_3(t)\left[\cos\left(4\pi f_{c_1} t + 2\pi f_{c_2} t\right) + \cos\left(4\pi f_{c_1} t - 2\pi f_{c_2} t\right)\right] \\ & + \tilde{y}_4(t)\left[\cos\left(2\pi f_{c_1} t + 4\pi f_{c_2} t\right) + \cos\left(4\pi f_{c_2} t - 2\pi f_{c_1} t\right)\right] \\ & + \tilde{y}_5(t)\cos\left(6\pi f_{c_1} t\right) + \tilde{y}_6(t)\cos\left(6\pi f_{c_2} t\right) \end{aligned} \tag{2.3.8}$$

The PSDs of $\tilde{y}_1(t)$, $\tilde{y}_2(t)$, $\tilde{y}_3(t)$, $\tilde{y}_4(t)$, $\tilde{y}_5(t)$, and $\tilde{y}_6(t)$ are determined first and then that of $y(t)$ is derived as [7]

$$\begin{aligned} P_y(f) = {} & \frac{1}{4}\left[P_{\tilde{y}_1}\left(f - f_{c_1}\right) + P_{\tilde{y}_1}\left(f + f_{c_1}\right)\right] \\ & + \frac{1}{4}\left[P_{\tilde{y}_2}\left(f - f_{c_2}\right) + P_{\tilde{y}_2}\left(f + f_{c_2}\right)\right] \\ & + \frac{1}{4}\left[P_{\tilde{y}_3}\left(f - \left(2f_{c_1} + f_{c_2}\right)\right) + P_{\tilde{y}_3}\left(f + \left(2f_{c_1} + f_{c_2}\right)\right)\right] \\ & + \frac{1}{4}\left[P_{\tilde{y}_3}\left(f - \left(2f_{c_1} - f_{c_2}\right)\right) + P_{\tilde{y}_3}\left(f + \left(2f_{c_1} - f_{c_2}\right)\right)\right] \\ & + \frac{1}{4}\left[P_{\tilde{y}_4}\left(f - \left(2f_{c_2} + f_{c_1}\right)\right) + P_{\tilde{y}_4}\left(f + \left(2f_{c_2} + f_{c_1}\right)\right)\right] \\ & + \frac{1}{4}\left[P_{\tilde{y}_4}\left(f - \left(2f_{c_2} - f_{c_1}\right)\right) + P_{\tilde{y}_4}\left(f + \left(2f_{c_2} - f_{c_1}\right)\right)\right] \\ & + \frac{1}{4}\left[P_{\tilde{y}_5}\left(f - 3f_{c_1}\right) + P_{\tilde{y}_5}\left(f + 3f_{c_1}\right)\right] \\ & + \frac{1}{4}\left[P_{\tilde{y}_6}\left(f - 3f_{c_2}\right) + P_{\tilde{y}_6}\left(f + 3f_{c_2}\right)\right] \end{aligned} \tag{2.3.9}$$

Since physical measurements only have power spectra over the positive frequency range, with which equation (2.3.9) needs to be comparable, equation (2.3.9) should be rewritten, based on the symmetry property, as

$$
\begin{aligned}
&P_y(f) \\
&= \frac{1}{2}\Big[P_{\tilde{y}_1}\left(f - f_{c_1}\right) + P_{\tilde{y}_2}\left(f - f_{c_2}\right) + P_{\tilde{y}_3}\left(f - \left(2f_{c_1} - f_{c_2}\right)\right) + P_{\tilde{y}_4}\left(f - \left(2f_{c_2} - f_{c_1}\right)\right)\Big] \quad (2.3.10) \\
&+ \frac{1}{2}\Big[P_{\tilde{y}_3}\left(f - \left(2f_{c_1} + f_{c_2}\right)\right) + P_{\tilde{y}_4}\left(f - \left(2f_{c_2} + f_{c_1}\right)\right) + P_{\tilde{y}_5}\left(f - 3f_{c_1}\right) + P_{\tilde{y}_6}\left(f - 3f_{c_2}\right)\Big]
\end{aligned}
$$

Obviously, there are eight terms in equation (2.3.10), the last six terms of which, that are centered at $\left(2f_{c_1} \pm f_{c_2}\right)$, $\left(2f_{c_2} \pm f_{c_1}\right)$, $3f_{c_1}$, and $3f_{c_2}$, are considered far away from the passband range. Therefore, the last six terms can be filtered out easily. Thus, the simplified power spectrum will be

$$
P_y(f) = \frac{1}{2}\Big[P_{\tilde{y}_1}\left(f - f_{c_1}\right) + P_{\tilde{y}_2}\left(f - f_{c_2}\right)\Big] \quad (2.3.11)
$$

To calculate the power spectrum of $y(t)$, $P_{\tilde{y}_1}(f)$ and $P_{\tilde{y}_2}(f)$ need to be derived first. By taking $P_{\tilde{y}_1}(f)$ as an example, the other $P_{\tilde{y}_2}(f)$ can be calculated in the same logic. Full expressions can be found in the Appendices.

The Wiener-Khintchine theorem [7] is used to derive the power spectrum of $\tilde{y}_1(t)$. According to the theorem, the PSD $P_{\tilde{y}_1}(f)$ is related to the correlation function $R_{\tilde{y}_1}(\tau)$ of $\tilde{y}_1(t)$ by

$$
P_{\tilde{y}_1}(f) = \int_{-\infty}^{\infty} R_{\tilde{y}_1}(\tau) e^{-j2\pi f_{c_1}\tau} d\tau \quad (2.3.12)
$$

$R_{\tilde{y}_1}(\tau)$ is described as

$$
R_{\tilde{y}_1}(\tau) = E\left\{\tilde{y}_1(t)\tilde{y}_1(t+\tau)\right\} \quad (2.3.13)
$$

where $E\{\bullet\}$ is the mathematical expectation of $\{\bullet\}$.

In order to derive the detailed expression of equation (2.3.11), equation (2.3.2) will be substituted into (2.3.11), so that $R_{\tilde{y}_1}(\tau)$ can be expressed with $R_{\tilde{s}_1}(\tau)$, which is the autocorrelation of $\tilde{s}_1(t)$.

$$
\begin{aligned}
&R_{\tilde{y}_1}(\tau)\\
&= E\left\{\tilde{y}_1(t)\tilde{y}_1(t+\tau)\right\}\\
&= E\left\{\left[a_1\tilde{s}_1(t)+\frac{3}{4}a_3\tilde{s}_1^3(t)+\frac{3}{2}a_3\tilde{s}_1(t)\tilde{s}_2^2(t)\right]\left[a_1\tilde{s}_1(t+\tau)+\frac{3}{4}a_3\tilde{s}_1^3(t+\tau)+\frac{3}{2}a_3\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right]\right\}\\
&= a_1^2E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}+\frac{3}{4}a_1a_3E\left\{\tilde{s}_1(t)\tilde{s}_1^3(t+\tau)\right\}+\frac{3}{2}a_1a_3E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right\}\\
&+\frac{3}{4}a_1a_3E\left\{\tilde{s}_1^3(t)\tilde{s}_1(t+\tau)\right\}+\frac{9}{16}a_3^2E\left\{\tilde{s}_1^3(t)\tilde{s}_1^3(t+\tau)\right\}+\frac{9}{8}a_3^2E\left\{\tilde{s}_1^3(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right\}\\
&+\frac{3}{2}a_1a_3E\left\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1(t+\tau)\right\}+\frac{9}{8}a_3^2E\left\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1^3(t+\tau)\right\}\\
&+\frac{9}{4}a_3^2E\left\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right\}
\end{aligned}
\tag{2.3.14}
$$

Since $\tilde{s}_1(t)$ is a bandlimited White Gaussian Stationary Process, the first term of equation (2.3.14), $E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}$, is exactly the correlation of $\tilde{s}_1(t)$, which can be calculated as

$$E\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\} = R_{\tilde{s}_1}(\tau) = \mathcal{F}^{-1}\left\{P_{\tilde{s}_1}(f)\right\} \tag{2.3.15}$$

where $\mathcal{F}^{-1}\left\{\bullet\right\}$ is the inverse Fourier transform of $\left\{\bullet\right\}$.

By substituting equation (2.1.4) into equation (2.3.15), $R_{\tilde{s}_1}(\tau)$, i.e. $E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}$, can be derived as

$$R_{\tilde{s}_1}(\tau) = N_1\frac{\sin\left(2\pi B_1\tau\right)}{2\pi\tau} \tag{2.3.16}$$

The rest of the terms in (2.3.14) can be calculated through the Isserlis' theorem [8].

The Isserlis' Theorem

If $(x_1, x_2, \dots x_{2n})$ is a zero-mean multivariate normal random vector, then

$$E\{X_1X_2\cdots X_{2n}\} = \sum\prod E\{X_iX_j\}, \tag{2.3.17}$$

$$E\{X_1X_2\cdots X_{2n-1}\} = 0,$$

where the notation $\sum\prod$ means summing over all distinct ways of partitioning $X_1, X_2, \dots X_{2n}$ into pairs X_i, X_j and each summand is the product of the n pairs.

In equation (2.3.13), there are five terms that contains mathematical expectation of four variables and three other terms containing expectation of six variables. Therefore, the fourth order and sixth order expressions of equation (2.3.17) are used to derive the eight terms of equation (2.3.14), which are

$$E\{X_1X_2X_3X_4\} = E\{X_1X_2\}E\{X_3X_4\} + E\{X_1X_3\}E\{X_2X_4\} + E\{X_1X_4\}E\{X_2X_3\} \tag{2.3.18}$$

and

$$\begin{aligned}
&E\{X_1X_2X_3X_4X_5X_6\} \\
&= E\{X_1X_2\}E\{X_3X_4\}E\{X_5X_6\} + E\{X_1X_2\}E\{X_3X_5\}E\{X_4X_6\} + E\{X_1X_2\}E\{X_3X_6\}E\{X_4X_5\} \\
&+ E\{X_1X_3\}E\{X_2X_4\}E\{X_5X_6\} + E\{X_1X_3\}E\{X_2X_5\}E\{X_4X_6\} + E\{X_1X_3\}E\{X_2X_6\}E\{X_4X_5\} \\
&+ E\{X_1X_4\}E\{X_2X_3\}E\{X_5X_6\} + E\{X_1X_4\}E\{X_2X_5\}E\{X_3X_6\} + E\{X_1X_4\}E\{X_2X_6\}E\{X_3X_5\} \\
&+ E\{X_1X_5\}E\{X_2X_3\}E\{X_4X_6\} + E\{X_1X_5\}E\{X_2X_4\}E\{X_3X_6\} + E\{X_1X_5\}E\{X_2X_6\}E\{X_3X_4\} \\
&+ E\{X_1X_6\}E\{X_2X_3\}E\{X_4X_5\} + E\{X_1X_6\}E\{X_2X_4\}E\{X_3X_5\} + E\{X_1X_6\}E\{X_2X_5\}E\{X_3X_4\}
\end{aligned} \tag{2.3.19}$$

respectively.

Besides, equation (2.3.14) can be rewritten as a new form for a better analysis,

$$
\begin{aligned}
R_{\tilde{y}_1}(\tau) = {} & a_1^2 E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\} \\
& \left.\begin{aligned}
&+\frac{3}{4}a_1a_3E\left\{\tilde{s}_1(t)\tilde{s}_1^3(t+\tau)\right\} \\
&+\frac{3}{4}a_1a_3E\left\{\tilde{s}_1^3(t)\tilde{s}_1(t+\tau)\right\} \\
&+\frac{9}{16}a_3^2E\left\{\tilde{s}_1^3(t)\tilde{s}_1^3(t+\tau)\right\}
\end{aligned}\right\}\text{caused by IM} \\
& \left.\begin{aligned}
&+\frac{3}{2}a_1a_3E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right\} \\
&+\frac{3}{2}a_1a_3E\left\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1(t+\tau)\right\}
\end{aligned}\right\}\text{fourth order expectations caused by CM} \\
& \left.\begin{aligned}
&+\frac{9}{4}a_3^2E\left\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right\} \\
&+\frac{9}{8}a_3^2E\left\{\tilde{s}_1^3(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right\} \\
&+\frac{9}{8}a_3^2E\left\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1^3(t+\tau)\right\}
\end{aligned}\right\}\text{sixth order expectations caused by CM}
\end{aligned}
\tag{2.3.20}
$$

In equation (2.3.20), the first group of three terms are caused by IM because the variables of mathematical expectations consist of $\tilde{s}_1(t)$ only and the results of the three expectations are shown below [1].

$$
\begin{aligned}
& E\left\{\tilde{s}_1(t)\tilde{s}_1^3(t+\tau)\right\} \\
& = 3E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} \\
& = 3N_1\frac{\sin\left(2\pi B_1\tau\right)}{2\pi\tau}N_1B_1
\end{aligned}
\tag{2.3.21}
$$

$$
\begin{aligned}
& E\left\{\tilde{s}_1^3(t)\tilde{s}_1(t+\tau)\right\} \\
& = 3E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\} \\
& = 3N_1B_1N_1\frac{\sin\left(2\pi B_1\tau\right)}{2\pi\tau}
\end{aligned}
\tag{2.3.22}
$$

$$
\begin{aligned}
&E\left\{\tilde{s}_1^3(t)\tilde{s}_1^3(t+\tau)\right\} \\
&=9E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} \\
&+6E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\} \\
&=9\left(N_1B_1\right)^2 N_1\frac{\sin\left(2\pi B_1\tau\right)}{2\pi\tau}+6\left(N_1\frac{\sin\left(2\pi B_1\tau\right)}{2\pi\tau}\right)^3
\end{aligned}
\tag{2.3.23}
$$

The rest five terms of equation (2.3.14) caused by CM cannot be derived directly, because the variables contain $\tilde{s}_1(t)$ and $\tilde{s}_2(t)$ simultaneously. By taking one of the two fourth order expectations caused by CM in (2.3.20) as an example, $E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right\}$ can be expressed as

$$
\begin{aligned}
&E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right\} \\
&=E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\}+2E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\} \\
&=N_1\frac{\sin\left(2\pi B_1\tau\right)}{2\pi\tau}N_2B_2+2E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\}
\end{aligned}
\tag{2.3.24}
$$

To derive the mathematical expectation $E\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\}$, the probability density function of the multivariate Gaussian distribution is utilized and through a detailed calculation (see the Appendix A), the mathematical expectation $E\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\}$ is:

$$
E\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\}=\rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\frac{\left(N_1B_1\right)^2}{N_2B_2}
\tag{2.3.25}
$$

where $\rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}$ is the coefficient expression of the correlation between $\tilde{s}_1(t)$ and $\tilde{s}_2(t+\tau)$.

Using the same method, the other three mathematical expectations, i.e. the mathematical expectations between $\tilde{s}_1(t+\tau)$ and $\tilde{s}_2(t+\tau)$, $\tilde{s}_1(t)$ and $\tilde{s}_2(t)$, $\tilde{s}_1(t+\tau)$ and $\tilde{s}_2(t)$, could be calculated respectively for later use. They are

$$E\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\} = \rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)} \frac{\left(N_1 B_1\right)^2}{N_2 B_2} \tag{2.3.26}$$

$$E\{\tilde{s}_1(t)\tilde{s}_2(t)\} = \rho_{\tilde{s}_1(t)\tilde{s}_2(t)} \frac{\left(N_1 B_1\right)^2}{N_2 B_2} \tag{2.3.27}$$

$$E\{\tilde{s}_1(t+\tau)\tilde{s}_2(t)\} = \rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t)} \frac{\left(N_1 B_1\right)^2}{N_2 B_2} \tag{2.3.28}$$

Upon substituting equations (2.3.25) and (2.3.26) into equation (2.3.24), equation (2.3.24) becomes

$$\begin{aligned} &E\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\} \\ &= E\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\}E\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\} \\ &+2E\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\}E\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\} \\ &= N_2 B_2 N_1 \frac{\sin(2\pi B_1 \tau)}{2\pi\tau} + 2\rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)} \frac{\left(N_1 B_1\right)^4}{\left(N_2 B_2\right)^2} \end{aligned} \tag{2.3.29}$$

Similarly, the other fourth order expectation caused by CM can be expressed as

$$\begin{aligned} &E\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1(t+\tau)\} \\ &= E\{\tilde{s}_2(t)\tilde{s}_2(t)\}E\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\} + 2E\{\tilde{s}_1(t)\tilde{s}_2(t)\}E\{\tilde{s}_1(t+\tau)\tilde{s}_2(t)\} \\ &= N_2 B_2 N_1 \frac{\sin(2\pi B_1 \tau)}{2\pi\tau} + 2\rho_{\tilde{s}_1(t)\tilde{s}_2(t)}\rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t)} \frac{\left(N_1 B_1\right)^4}{\left(N_2 B_2\right)^2} \end{aligned} \tag{2.3.30}$$

The last three sixth order terms of expectation caused by CM in (2.3.20) will be derived respectively (see the Appendix B),

$$E\{\tilde{s}_1^3(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\}$$

$$= 3N_1N_2B_1B_2N_1\frac{\sin(2\pi B_1\tau)}{2\pi\tau} + 6\rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)}\frac{(N_1B_1)^5}{(N_2B_2)^2} \tag{2.3.31}$$

$$+6\rho^2_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}N_1\frac{\sin(2\pi B_1\tau)}{2\pi\tau}\frac{(N_1B_1)^4}{(N_2B_2)^2}$$

$$E\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1^3(t+\tau)\}$$

$$= 6\rho_{\tilde{s}_1(t)\tilde{s}_2(t)}\rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t)}\frac{(N_1B_1)^5}{(N_2B_2)^2} + 3N_1N_2B_1B_2N_1\frac{\sin(2\pi B_1\tau)}{2\pi\tau} \tag{2.3.32}$$

$$+6\rho^2_{\tilde{s}_1(t+\tau)\tilde{s}_2(t)}N_1\frac{\sin(2\pi B_1\tau)}{2\pi\tau}\frac{(N_1B_1)^4}{(N_2B_2)^2}$$

$$E\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\}$$

$$= 2\rho_{\tilde{s}_1(t)\tilde{s}_2(t)}\rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t)}\frac{(N_1B_1)^4}{N_2B_2} + (N_2B_2)^2N_1\frac{\sin(2\pi B_1\tau)}{2\pi\tau}$$

$$+2N_1\frac{\sin(2\pi B_1\tau)}{2\pi\tau}\left[N_2\frac{\sin(2\pi B_2\tau)}{2\pi\tau}\right]^2$$

$$+4\rho_{\tilde{s}_1(t)\tilde{s}_2(t)}\rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)}\frac{(N_1B_1)^4}{(N_2B_2)^2}N_2\frac{\sin(2\pi B_2\tau)}{2\pi\tau} \tag{2.3.33}$$

$$+2\rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)}\frac{(N_1B_1)^4}{N_2B_2}$$

$$+4\rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t)}\frac{(N_1B_1)^4}{(N_2B_2)^2}N_2\frac{\sin(2\pi B_2\tau)}{2\pi\tau}$$

Since the modulation of LTE-Advanced signals is OFDM, the correlation between $\tilde{s}_1(t)$ and $\tilde{s}_2(t)$ can be considered as 0, which means all ρ s are 0.

By substituting (2.3.16), (2.3.21) – (2.3.23), and (2.3.29) – (2.3.33) into (2.3.20) and applying $\rho = 0$, $R_{\tilde{y}_1}(\tau)$ is derived as

$$\begin{aligned} &R_{\tilde{y}_1}(\tau) \\ &= E\left\{ \tilde{y}_1(t)\tilde{y}_1(t+\tau) \right\} \\ &= \left[a_1^2 + \frac{9}{2}a_1a_3N_1B_1 + 3a_1a_3N_2B_2 + \frac{81}{16}a_3^2\left(N_1B_1\right)^2 + \frac{27}{4}a_3^2N_1B_1N_2B_2 + \frac{9}{4}a_3^2\left(N_2B_2\right)^2 \right] R_{\tilde{s}_1}(\tau) \\ &\quad + \frac{27}{8}a_3^2R_{\tilde{s}_1}^3(\tau) + \frac{9}{2}a_3^2R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau) \end{aligned} \tag{2.3.34}$$

whereas with (2.3.16),

$$R_{\tilde{s}_2}(\tau) = N_2\frac{\sin(2\pi B_2\tau)}{2\pi\tau} \tag{2.3.35}$$

2.4 The PSD of Amplified Dual-Band LTE-Advanced Output Signals

By substituting (2.3.34) into (2.3.12), $P_{\tilde{y}_1}(f)$ can be expressed as

$$\begin{aligned} &P_{\tilde{y}_1}(f) \\ &= \mathcal{F}\left\{ R_{\tilde{y}_1}(\tau) \right\} \\ &= \left(a_1^2 + \frac{9}{2}a_1a_3N_1B_1 + \frac{81}{16}a_3^2\left(N_1B_1\right)^2 + 3a_1a_3N_2B_2 + \frac{27}{4}a_3^2N_1B_1N_2B_2 + \frac{9}{4}a_3^2\left(N_2B_2\right)^2 \right) \mathcal{F}\left\{ R_{\tilde{s}_1}(\tau) \right\} \\ &\quad + \frac{27}{8}a_3^2\mathcal{F}\left\{ R_{\tilde{s}_1}^3(\tau) \right\} + \frac{9}{2}a_3^2\mathcal{F}\left\{ R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau) \right\} \end{aligned} \tag{2.4.1}$$

where $\mathcal{F}\{\bullet\}$ is the Fourier transform of $\{\bullet\}$.

Since $R_{\tilde{s}_1}(\tau)$ is given in equation (2.3.16), its power spectrum $\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} = P_{\tilde{s}_1}(f)$ can be found in (2.1.4). The Fourier transform of $R_{\tilde{s}_1}^3(\tau)$, and $R_{\tilde{s}_1}(t)R_{\tilde{s}_2}^2(\tau)$ can be calculated through convolution (see Appendix C), assuming the bandwidths of $\tilde{s}_1(t)$ and $\tilde{s}_2(t)$ are arranged in $B_2 < B_1 < 2B_2 < 2B_1$.

$$\mathcal{F}\left\{R_{\tilde{s}_1}^3(\tau)\right\} = P_{\tilde{s}_1}(f) \otimes P_{\tilde{s}_1}(f) \otimes P_{\tilde{s}_1}(f) = \begin{cases} \dfrac{N_1^3}{8}\left(3B_1^2 - f^2\right) & |f| \le B_1 \\ \dfrac{N_1^3}{16}\left(3B_1 - |f|\right)^2 & B_1 < |f| \le 3B_1 \\ 0 & |f| > 3B_1 \end{cases} \quad (2.4.2)$$

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\} = P_{\tilde{s}_1}(f) \otimes P_{\tilde{s}_2}(f) \otimes P_{\tilde{s}_2}(f)$$

$$= \begin{cases} \dfrac{N_1N_2^2}{8}\left(-f^2 - B_1^2 + 4B_1B_2\right) & |f| \le 2B_2 - B_1 \\ \dfrac{N_1N_2^2}{16}\left[-f^2 + \left(2B_1 - 4B_2\right)|f| - B_1^2 + 4B_2^2 + 4B_1B_2\right] & 2B_2 - B_1 < |f| \le B_1 \\ \dfrac{N_1N_2^2}{16}\left(B_1 + 2B_2 - |f|\right)^2 & B_1 < |f| \le B_1 + 2B_2 \\ 0 & |f| > B_1 + 2B_2 \end{cases} \quad (2.4.3)$$

If $B_1 > 2B_2$, the result of $\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\}$ will be slightly different in both frequency segmentations and expressions associated with each segment. The result of $\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\}$ and $P_y(f)$ in the range of $B_1 > 2B_2$ can be found in Appendix D.

Substitute (2.4.2), (2.4.3) and (2.1.4) into (2.4.1), then $P_{\tilde{y}_1}(f)$ becomes (see Appendix E)

$$
P_{\tilde{y}_1}(f) =
$$

$$
\begin{cases}
\frac{1}{B_1}\left(P_{01} - 6P_{01}^2 10^{\frac{-IP_2}{10}} + 9P_{01}^3 10^{\frac{-IP_3}{5}} - 4P_{01}P_{02}10^{\frac{-IP_3}{10}} + 12P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}} + 4P_{01}P_{02}^2 10^{\frac{-IP_3}{5}} \right) \\
+\frac{3}{2}\frac{1}{B_1^3} P_{01}^3 10^{\frac{-IP_3}{5}} \left(3B_1^2 - f^2\right) + \frac{2}{B_1 B_2^2} P_{01}P_{02}^2 10^{\frac{-IP_3}{5}} \left(-f^2 - B_1^2 + 4B_1B_2\right) & |f| \le 2B_2 - B_1 \\
\frac{1}{B_1}\left(P_{01} - 6P_{01}^2 10^{\frac{-IP_3}{10}} + 9P_{01}^3 10^{\frac{-IP_3}{5}} - 4P_{01}P_{02}10^{\frac{-IP_3}{10}} + 12P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}} + 4P_{01}P_{02}^2 10^{\frac{-IP_3}{5}} \right) \\
+\frac{3}{2}\frac{1}{B_1^3} P_{01}^3 10^{\frac{-IP_3}{5}} \left(3B_1^2 - f^2\right) & 2B_2 - B_1 < |f| \le B_1 \\
+\frac{1}{B_1 B_2^2} P_{01}P_{02}^2 10^{\frac{-IP_3}{5}} \left[-f^2 + \left(2B_1 - 4B_2\right)|f| - B_1^2 + 4B_2^2 + 4B_1B_2\right] \\
\frac{3}{4}\frac{1}{B_1^3} P_{01}^3 10^{\frac{-IP_3}{5}} \left(3B_1 - |f|\right)^2 + \frac{1}{B_1 B_2^2} P_{01}P_{02}^2 10^{\frac{-IP_3}{5}} \left(B_1 + 2B_2 - |f|\right)^2 & B_1 < |f| \le B_1 + 2B_2 \\
\frac{3}{4}\frac{1}{B_1^3} P_{01}^3 10^{\frac{-IP_3}{5}} \left(3B_1 - |f|\right)^2 & B_1 + 2B_2 < |f| \le 3B_1 \\
0 & |f| > 3B_1
\end{cases}
\tag{2.4.4}
$$

where P_{01} is the linear portion of the amplifier's first output power for $\tilde{s}_1(t)$ and P_{02} is the linear portion of the amplifier's first output power for $\tilde{s}_2(t)$, which are expressed as,

$$
P_{01} = \frac{a_1^2 N_1 B_1}{2} \tag{2.4.5}
$$

$$
P_{02} = \frac{a_1^2 N_2 B_2}{2} \tag{2.4.6}
$$

Equation (2.4.4) proposes a power spectrum expression of the first band of the dual band LTE-Advanced signal. $P_{\tilde{y}_2}(f)$ can be derived using a similar method.

$$P_{\tilde{y}_2}(f) =$$

$$\begin{cases} \frac{1}{B_2}\left(P_{02} - 6P_{02}^2 10^{\frac{-IP_3}{10}} + 9P_{02}^3 10^{\frac{-IP_3}{5}} - 4P_{01}P_{02} 10^{\frac{-IP_3}{10}} + 12P_{01}P_{02}^2 10^{\frac{-IP_3}{5}} + 4P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}}\right) \\ + \frac{3}{2}\frac{1}{B_2^3} P_{02}^3 10^{\frac{-IP_3}{5}} \left(3B_2^2 - f^2\right) + \frac{2}{B_1^2 B_2} P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}} \left(-f^2 + 4B_1 B_2 - B_2^2\right) & |f| \le B_2 \\ \frac{3}{4}\frac{1}{B_2^3} P_{02}^3 10^{\frac{-IP_3}{5}} \left(3B_2 - |f|\right)^2 + \frac{2}{B_1^2 B_2} P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}} \left(4B_1 B_2 - 2B_2 |f|\right) & B_2 < |f| \le 2B_1 - B_2 \\ \frac{3}{4}\frac{1}{B_2^3} P_{02}^3 10^{\frac{-IP_3}{5}} \left(3B_2 - |f|\right)^2 + \frac{1}{B_1^2 B_2} P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}} \left(2B_1 + B_2 - |f|\right)^2 & 2B_1 - B_2 < |f| \le 3B_2 \\ \frac{1}{B_1^2 B_2} P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}} \left(2B_1 + B_2 - |f|\right)^2 & 3B_2 < |f| \le 2B_1 + B_2 \\ 0 & |f| > 2B_1 + B_2 \end{cases}$$

(2.4.7)

Equation (2.4.7) describes the power spectral density of the second band of the dual band LTE-Advanced signal.

By substituting (2.4.4) and (2.4.7) into (2.3.11), the PSD of amplified dual-band LTE-Advanced signal $P_y(f)$ is

$$
P_y(f)=
\begin{cases}
\frac{1}{2B_1}\left(P_{01}-6P_{01}^2 10^{\frac{-IP_3}{10}}+9P_{01}^3 10^{\frac{-IP_3}{5}}-4P_{01}P_{02}10^{\frac{-IP_3}{10}}+12P_{01}^2P_{02}10^{\frac{-IP_3}{5}}+4P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\right) \\ +\frac{3}{4}\frac{1}{B_1^3}P_{01}^3 10^{\frac{-IP_3}{5}}\left[3B_1^2-\left(f-f_{c_1}\right)^2\right]+\frac{1}{B_1B_2^2}P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\left[-\left(f-f_{c_1}\right)^2-B_1^2+4B_1B_2\right], & \left|f-f_{c_1}\right|\le 2B_2-B_1 \\
\frac{1}{2B_1}\left(P_{01}-6P_{01}^2 10^{\frac{-IP_3}{10}}+9P_{01}^3 10^{\frac{-IP_3}{5}}-4P_{01}P_{02}10^{\frac{-IP_3}{10}}+12P_{01}^2P_{02}10^{\frac{-IP_3}{5}}+4P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\right) \\ +\frac{3}{4}\frac{1}{B_1^3}P_{01}^3 10^{\frac{-IP_3}{5}}\left[3B_1^2-\left(f-f_{c_1}\right)^2\right] & 2B_2-B_1<\left|f-f_{c_1}\right|\le B_1 \\ +\frac{1}{2B_1B_2^2}P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\left[-\left(f-f_{c1}\right)^2+\left(2B_1-4B_2\right)\left|f-f_{c_1}\right|-B_1^2+4B_2^2+4B_1B_2\right], & \\
\frac{3}{8}\frac{1}{B_1^3}P_{01}^3 10^{\frac{-IP_3}{5}}\left(3B_1-\left|f-f_{c_1}\right|\right)^2+\frac{1}{2B_1B_2^2}P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\left(B_1+2B_2-\left|f-f_{c_1}\right|\right)^2, & B_1<\left|f-f_{c_1}\right|\le B_1+2B_2 \\
\frac{3}{8}\frac{1}{B_1^3}P_{01}^3 10^{\frac{-IP_3}{5}}\left(3B_1-\left|f-f_{c_1}\right|\right)^2, & B_1+2B_2<\left|f-f_{c_1}\right|\le 3B_1 \\
\frac{1}{2B_2}\left(P_{02}-6P_{02}^2 10^{\frac{-IP_3}{10}}+9P_{02}^3 10^{\frac{-IP_3}{5}}-4P_{01}P_{02}10^{\frac{-IP_3}{10}}+12P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}+4P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\right) \\ +\frac{3}{4}\frac{1}{B_2^3}P_{02}^3 10^{\frac{-IP_3}{5}}\left[3B_2^2-\left(f-f_{c_2}\right)^2\right]+\frac{1}{B_1^2B_2}P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\left[-\left(f-f_{c_2}\right)^2+4B_1B_2-B_2^2\right], & \left|f-f_{c_2}\right|\le B_2 \\
\frac{3}{8}\frac{1}{B_2^3}P_{02}^3 10^{\frac{-IP_3}{5}}\left(3B_2-\left|f-f_{c_2}\right|\right)^2+\frac{1}{B_1^2B_2}P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\left(4B_1B_2-2B_2\left|f-f_{c_2}\right|\right), & B_2<\left|f-f_{c_2}\right|\le 2B_1-B_2 \\
\frac{3}{8}\frac{1}{B_2^3}P_{02}^3 10^{\frac{-IP_3}{5}}\left(3B_2-\left|f-f_{c_2}\right|\right)^2+\frac{1}{2B_1^2B_2}P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\left(2B_1+B_2-\left|f-f_{c_2}\right|\right)^2, & 2B_1-B_2<\left|f-f_{c_2}\right|\le 3B_2 \\
\frac{1}{2B_1^2B_2}P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\left(2B_1+B_2-\left|f-f_{c_2}\right|\right)^2, & 3B_2<\left|f-f_{c_2}\right|\le 2B_1+B_2 \\
0, & \text{otherwise}
\end{cases}
$$

(2.4.8)

An example is used to visualize the derived power spectrum $P_y(f)$. The plot shown in Figure 2 is constructed by choosing the output power of dual bands as $P_{01}=4dBm$ and $P_{02}=9.9dBm$, $IP_3=24$. The bandwidths B_1 and B_2 of the dual band LTE-Advanced signal are selected to be $B_1=5MHz$ and $B_2=3.5MHz$. It should be noted that the power spectrum unit in the Y-axis is dBm/MHz.

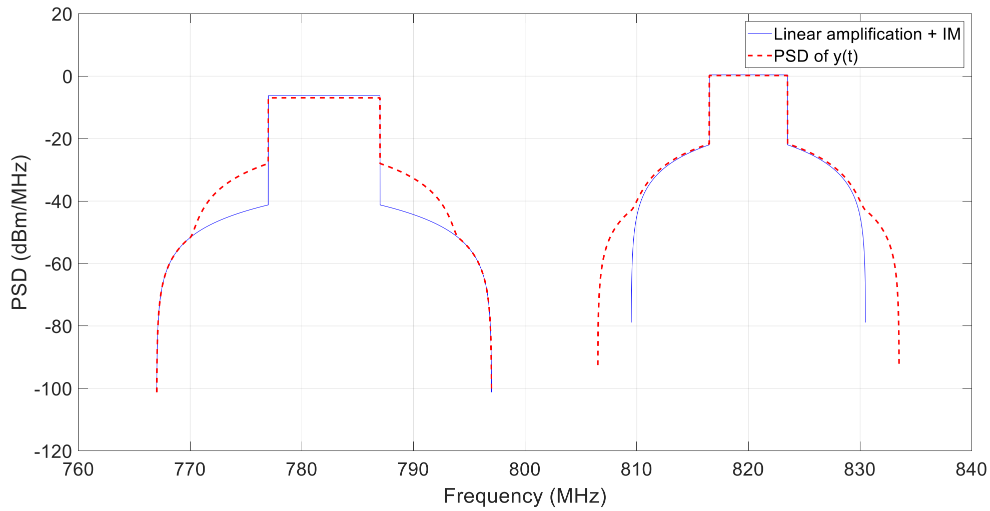


Figure 2. The theoretical PSD of amplified dual-band LTE-Advanced signal

$P_y(f)$ contains the linear amplification, the IM produced by either $\tilde{s}_1(t)$ or $\tilde{s}_2(t)$, and the CM produced by both $\tilde{s}_1(t)$ and $\tilde{s}_2(t)$, which means

$$P_y(f) = P_{y\_L+IM}(f) + P_{y\_CM}(f) \tag{2.4.9}$$

where $P_{y\_L+IM}(f)$ and $P_{y\_CM}(f)$ represent linear amplification plus IM, and CM, respectively.

The solid line represents $P_{y\_L+IM}(f)$ which is the power spectrum from the amplification of one band only, while the dashed line describes $P_y(f)$.

$P_{y\_L+IM}(f)$ can be expressed as

$$P_{y\_L+IM}(f)$$
$$= \begin{cases} \dfrac{1}{2B_1}\left(P_{01} - 6P_{01}^2 10^{\frac{-IP_3}{10}} + 9P_{01}^3 10^{\frac{-IP_3}{5}}\right) + \dfrac{3}{4B_1^3} P_{01}^3 10^{\frac{-IP_3}{5}} \left[3B_1^2 - \left(f - f_{c_1}\right)^2\right], & \left|f - f_{c_1}\right| \le B_1 \\ \dfrac{3}{8B_1^3} P_{01}^3 10^{\frac{-IP_3}{5}} \left(3B_1 - \left|f - f_{c_1}\right|\right)^2, & B_1 < \left|f - f_{c_1}\right| \le 3B_1 \\ \dfrac{1}{2B_2}\left(P_{02} - 6P_{02}^2 10^{\frac{-IP_3}{10}} + 9P_{02}^3 10^{\frac{-IP_3}{5}}\right) + \dfrac{3}{4B_1^3} P_{01}^3 10^{\frac{-IP_3}{5}} \left[3B_2^2 - \left(f - f_{c_1}\right)^2\right], & \left|f - f_{c_2}\right| \le B_2 \\ \dfrac{3}{8B_2^3} P_{02}^3 10^{\frac{-IP_3}{5}} \left(3B_2 - \left|f - f_{c_1}\right|\right)^2, & B_2 < \left|f - f_{c_2}\right| \le 3B_2 \end{cases}$$

(2.4.10)

Analysis related to (2.4.10) can be found in [1].

$P_{y\_CM}(f)$ can be expressed as

$$P_{y\_CM}(f)=$$

$$\begin{cases}
\frac{1}{2B_1}\left(-4P_{01}P_{02}10^{\frac{-IP_3}{10}}+12P_{01}^2P_{02}10^{\frac{-IP_3}{5}}+4P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\right) \\ +\frac{1}{B_1B_2^2}P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\left[-\left(f-f_{c_c}\right)^2-B_1^2+4B_1B_2\right], & \left|f-f_{c_1}\right|\le 2B_2-B_1 \\
\frac{1}{2B_1}\left(-4P_{01}P_{02}10^{\frac{-IP_3}{10}}+12P_{01}^2P_{02}10^{\frac{-IP_3}{5}}+4P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\right) \\ +\frac{1}{2B_1B_2^2}P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\left[-\left(f-f_{c_1}\right)^2+\left(2B_1-4B_2\right)\left|f-f_{c_1}\right|-B_1^2+4B_2^2+4B_1B_2\right], & 2B_2-B_1<\left|f-f_{c_1}\right|\le B_1 \\
\frac{1}{2B_1B_2^2}P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\left(B_1+2B_2-\left|f-f_{c_1}\right|\right)^2, & B_1<\left|f-f_{c_1}\right|\le B_1+2B_2 \\
\frac{1}{2B_2}\left(-4P_{01}P_{02}10^{\frac{-IP_3}{5}}+12P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}+4P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\right) \\ +\frac{1}{B_1^2B_2}P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\left[-\left(f-f_{c_2}\right)^2+4B_1B_2-B_2^2\right], & \left|f-f_{c_2}\right|\le B_2 \\
\frac{1}{B_1^2B_2}P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\left(4B_1B_2-2B_2\left|f-f_{c_2}\right|\right), & B_2<\left|f-f_{c_2}\right|\le 2B_1-B_2 \\
\frac{1}{2B_1^2B_2}P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\left(2B_1+B_2-\left|f-f_{c_2}\right|\right)^2, & 2B_1-B_2<\left|f-f_{c_2}\right|\le 2B_1+B_2 \\
0 & \text{otherwise}
\end{cases}$$

(2.4.11)

Several observations are made when inspecting (2.4.11).

1. In the passbands $\left|f-f_{c_1}\right|\le 2B_2-B_1$, $2B_2-B_1<\left|f-f_{c_1}\right|\le B_1$, and $\left|f-f_{c_2}\right|\le B_2$, these terms are added to the linear term of (2.4.10). For a linear amplifier, the CM is usually much lower than the linear output power. Thus, the CM does not affect the passband spectrum significantly.

2. In the band $B_1<\left|f-f_{c_1}\right|\le B_1+2B_2$, $B_2<\left|f-f_{c_2}\right|\le 2B_1-B_2$, and $2B_1-B_2<\left|f-f_{c_2}\right|\le 2B_1+B_2$, these non-zero PSDs are added to the third order IM of

(2.4.10). This result shows that the out-of-passband spectrum regrowth is determined by the IM and CM, which is the most harmful out-of-passband emission.

From the equation (2.4.8) and figure 2, $P_y(f)=0$ when the frequency locates in a segment that does not belong to any existing one. This result is obtained because the dual band LTE-Advanced signal is assumed to be band-limited in the derivation. In practice, no band pass signal is exactly band-limited within $\left[-B, B\right]$, which means that in the band other than any existing segment of (2.4.8), the PSD is not exactly zero. However, the emission in those band will likely be covered by noise.

Chapter 3 Experimental Verification

The proposed dual-band LTE-Advanced signal was evaluated using the measurement setup described in Figure 3, which consists of an ESG Vector Signal Generator (Agilent E4438C), a Real-time Spectrum analyzer (RSA 6120A), and a power amplifier (ZFL-1000LN+). A 16-QAM dual-band LTE-Advanced signal generated using Matlab was applied to the system. The vector signal generator was set to capture the input signal at the center carrier frequency (801 MHz). Additionally, the two fundamental frequencies f_{c_1} and f_{c_2} were set at 782 MHz and 820 MHz, respectively. When the vector signal generator captured the input signal, it transmitted the signal immediately through the power amplifier. Then, the spectrum analyzer received the amplified signal and showed its power spectrum on the screen. To compare the figure on the screen with the theoretic formula, the I/Q data of the amplified signal in the signal analyzer was saved and plotted through the pwelch function (a function used to plot PSD) in Matlab. The result is shown in Figure 4. In figure 4, Red line is the expression of the formula (2.4.8) and blue line is the plot of the experimental measurement.

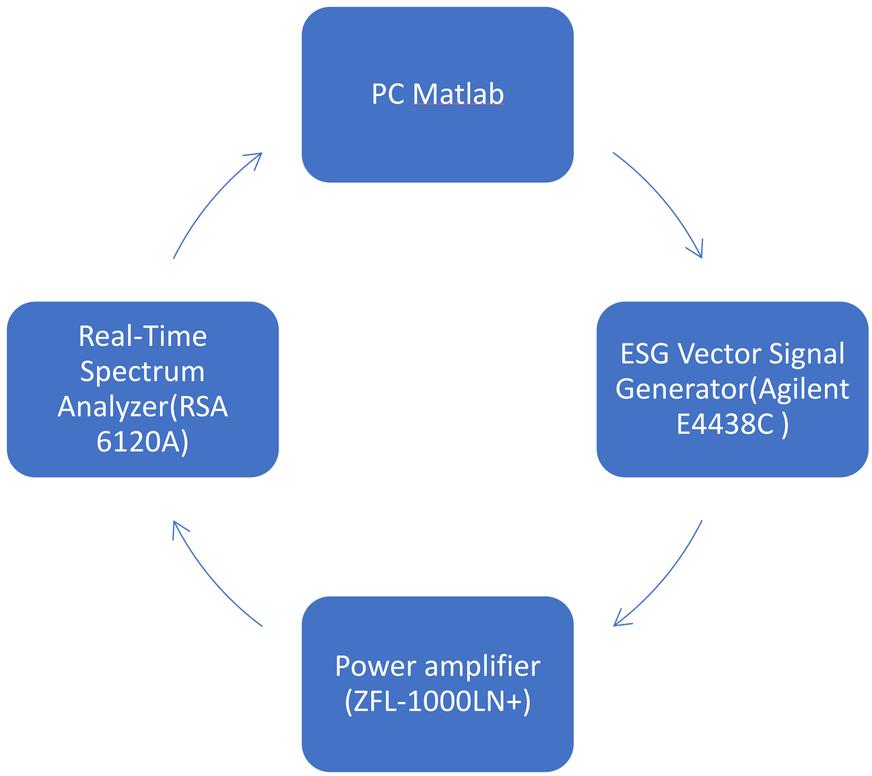


Figure 3. Measurement setup

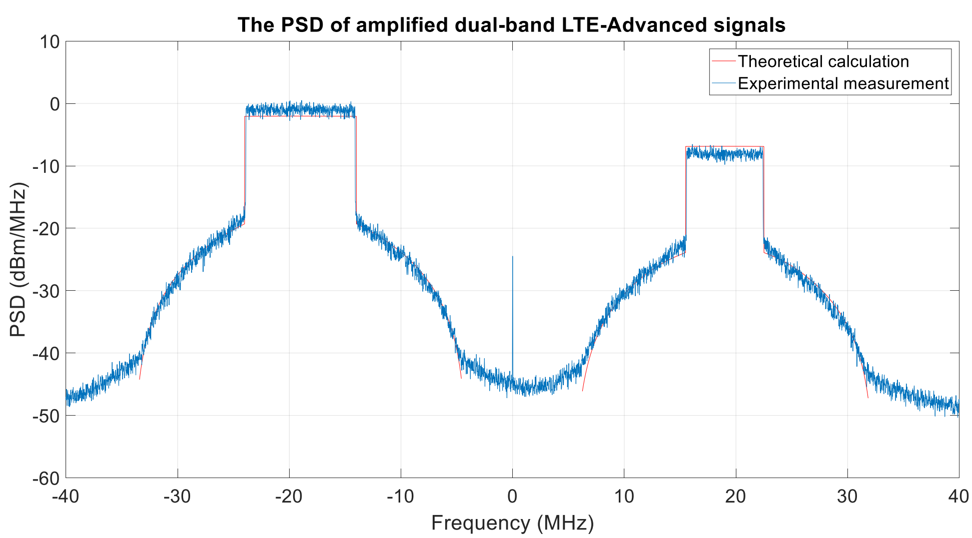


Figure 4. The PSD of amplified dual band LTE-Advanced signal

Chapter 4 Conclusion

In this study, a mathematical derivation of intermodulation and cross modulation has been presented by relating an estimation of the out-of-band power emission levels of a concurrent amplified dual band LTE-Advanced signal to the nonlinear parameter of PA, IP_3. Upon proposing a theoretical model to predict the output PSD of the LTE-Advanced signal, the method makes it possible for RF designers to specify and measure the concurrent dual band RF PAs using IP_3 description. What's more, this approach can be applied to other OFDM based communication technologies, (including FBMC-OFDM, considered as a candidate of 5G,) for the prediction of the power spectrum.

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Appendix

Appendix A The calculation of mathematical expectation $E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}$.

Using the 2-dimensional probability density function, the expression of $E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}$ is expressed as

$$\begin{aligned} &E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\} \\ &= \int_{\tilde{s}_1(t)}\int_{\tilde{s}_2(t+\tau)} \tilde{s}_1(t)\tilde{s}_2(t+\tau)\, p\left(\tilde{s}_1(t), \tilde{s}_2(t+\tau)\right) d\tilde{s}_2(t+\tau)\, d\tilde{s}_1(t) \\ &= \int_{\tilde{s}_1(t)}\int_{\tilde{s}_2(t+\tau)} \frac{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}{2\pi\left(\det\Sigma\right)^{\frac{1}{2}}} e^{-\frac{1}{2}\left[\tilde{s}_1(t)\quad \tilde{s}_2(t+\tau)\right]\Sigma^{-1}\begin{bmatrix}\tilde{s}_1(t)\\ \tilde{s}_2(t+\tau)\end{bmatrix}} d\tilde{s}_2(t+\tau)\, d\tilde{s}_1(t) \end{aligned} \tag{3.1.1}$$

where

$$\begin{aligned} &\Sigma \\ &= \begin{bmatrix} \sigma^2_{\tilde{s}_1(t)} & \rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\sigma_{\tilde{s}_1(t)}\sigma_{\tilde{s}_2(t+\tau)} \\ \rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\sigma_{\tilde{s}_1(t)}\sigma_{\tilde{s}_2(t+\tau)} & \sigma^2_{\tilde{s}_2(t+\tau)} \end{bmatrix} \\ &= \begin{bmatrix} N_1B_1 & \rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\sqrt{N_1N_2B_1B_2} \\ \rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\sqrt{N_1N_2B_1B_2} & N_2B_2 \end{bmatrix} \end{aligned} \tag{3.1.2}$$

$$\det\Sigma = N_1N_2B_1B_2 - \rho^2_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}N_1N_2B_1B_2 = \left(1-\rho^2_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\right)N_1N_2B_1B_2 \tag{3.1.3}$$

$$\Sigma^{-1} = \frac{1}{\left(1-\rho^2_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\right)N_1N_2B_1B_2}\begin{bmatrix} N_2B_2 & -\rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\sqrt{N_1N_2B_1B_2} \\ -\rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\sqrt{N_1N_2B_1B_2} & N_1B_1 \end{bmatrix} \tag{3.1.4}$$

So, substituting (3.1.4) into (3.1.1) will yield

$$E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\} =$$

$$\frac{1}{2\pi\left(\det\Sigma\right)^{\frac{1}{2}}}\times$$

$$\int_{\tilde{s}_1(t)}\int_{\tilde{s}_2(t+\tau)} \tilde{s}_1(t)\tilde{s}_2(t+\tau) e^{-\frac{1}{2(\det\Sigma)}\left(\tilde{s}_1^2(t)N_2B_2 - 2\tilde{s}_1(t)\tilde{s}_2(t+\tau)\rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\sqrt{N_1N_2B_1B_2} + \tilde{s}_2^2(t+\tau)N_1B_1\right)} d\tilde{s}_2(t+\tau)d\tilde{s}_1(t)$$

(3.1.5)

Suppose $\alpha_1^2 = N_1B_1$, $\alpha_2^2 = N_2B_2$, $\beta = \rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\sqrt{N_1N_2B_1B_2}$, then $E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}$ can be rewritten as

$$E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}$$

$$= \frac{1}{2\pi\left(\det\Sigma\right)^{\frac{1}{2}}}\times$$

$$\int_{\tilde{s}_1(t)}\int_{\tilde{s}_2(t+\tau)} \tilde{s}_1(t)\tilde{s}_2(t+\tau) e^{-\frac{1}{2(\det\Sigma)}\left[\left(\alpha_2^2 - \frac{\beta^2}{\alpha_1^2}\right)\tilde{s}_2^2(t+\tau) + \left(\alpha_2\tilde{s}_1(t) - \frac{\beta}{\alpha_2}\tilde{s}_2(t+\tau)\right)^2\right]} d\tilde{s}_2(t+\tau)d\tilde{s}_1(t)$$

$$= \frac{1}{2\pi\left(\det\Sigma\right)^{\frac{1}{2}}}\times$$

$$\int_{\tilde{s}_2(t+\tau)} \tilde{s}_2(t+\tau) e^{-\frac{1}{2(\det\Sigma)}\left(\alpha_2^2 - \frac{\beta^2}{\alpha_1^2}\right)\tilde{s}_2^2(t+\tau)} d\tilde{s}_2(t+\tau) \int_{\tilde{s}_1(t)} \tilde{s}_1(t) e^{-\frac{1}{2(\det\Sigma)}\left(\alpha_2\tilde{s}_1(t) - \frac{\beta}{\alpha_2}\tilde{s}_2(t+\tau)\right)^2} d\tilde{s}_1(t)$$

(3.1.6)

Define a new variable s as

$$s = \alpha_2\tilde{s}_1(t) - \frac{\beta}{\alpha_2}\tilde{s}_2(t+\tau) \tag{3.1.7}$$

then

$$\tilde{s}_1(t) = \frac{s}{\alpha_2} + \frac{\beta}{\alpha_2^2}\tilde{s}_2(t+\tau) \tag{3.1.8}$$

Upon using equation (3.1.8), one integration in (3.1.6) can be derived as

$$\int_{\tilde{s}_1(t)} \tilde{s}_1(t) e^{-\frac{1}{2(\det\Sigma)}\left(\alpha_2 \tilde{s}_1(t) - \frac{\beta}{\alpha_2}\tilde{s}_2(t+\tau)\right)^2} d\tilde{s}_1(t)$$

$$= \int_s \left(\frac{s}{\alpha_2} + \frac{\beta}{\alpha_2^2}\tilde{s}_2(t+\tau)\right) e^{-\frac{s^2}{2(\det\Sigma)}} d\left(\frac{s}{\alpha_2}\right)$$

$$= \frac{1}{\alpha_2}\int_s \left(\frac{s}{\alpha_2} + \frac{\beta}{\alpha_2^2}\tilde{s}_2(t+\tau)\right) e^{-\frac{s^2}{2(\det\Sigma)}} ds \tag{3.1.9}$$

$$= 0 + \frac{\beta}{\alpha_2^2}\tilde{s}_2(t+\tau)\int_s e^{-\frac{s^2}{2(\det\Sigma)}} ds$$

$$= \frac{\beta}{\alpha_2^2}\tilde{s}_2(t+\tau)\sqrt{2\pi}\left(\det\Sigma\right)^{\frac{1}{2}}$$

Substituting (3.1.9) into (3.1.6), the mathematical expectation becomes

$$E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}$$

$$= \frac{1}{2\pi\left(\det\Sigma\right)^{\frac{1}{2}}}\int_{\tilde{s}_2(t+\tau)} \tilde{s}_2(t+\tau)\frac{\beta}{\alpha_2^2}\tilde{s}_2(t+\tau)\sqrt{2\pi}\left(\det\Sigma\right)^{\frac{1}{2}} e^{-\frac{1}{2(\det\Sigma)}\left(\alpha_2^2 - \frac{\beta^2}{\alpha_1^2}\right)\tilde{s}_2^2(t+\tau)} d\tilde{s}_2(t+\tau)$$

$$= \frac{1}{\sqrt{2\pi}}\frac{\beta}{\alpha_2^3}\int_{\tilde{s}_2(t+\tau)} \tilde{s}_2^2(t+\tau) e^{-\frac{1}{2(\det\Sigma)}\left(\alpha_2^2 - \frac{\beta^2}{\alpha_1^2}\right)\tilde{s}_2(t+\tau)} d\tilde{s}_2(t+\tau) \tag{3.1.10}$$

$$= \frac{1}{\sqrt{2\pi}}\frac{\beta}{\alpha_2^3} N_1 B_1 \sqrt{2\pi N_1 B_1}$$

$$= \rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\frac{\left(N_1 B_1\right)^2}{N_2 B_2}$$

Therefore, $E\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\} = \rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\frac{\left(N_1 B_1\right)^2}{N_2 B_2}$.

Appendix B Derivation of mathematical expectation $E\left\{\tilde{s}_1^3(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right\}$,

$E\left\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1^3(t+\tau)\right\}$, and $E\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\}$.

Recall the sixth order Isserlis' Theorem,

$$
\begin{aligned}
&E\{X_1X_2X_3X_4X_5X_6\}\\
&=E\{X_1X_2\}E\{X_3X_4\}E\{X_5X_6\}+E\{X_1X_2\}E\{X_3X_5\}E\{X_4X_6\}+E\{X_1X_2\}E\{X_3X_6\}E\{X_4X_5\}\\
&+E\{X_1X_3\}E\{X_2X_4\}E\{X_5X_6\}+E\{X_1X_3\}E\{X_2X_5\}E\{X_4X_6\}+E\{X_1X_3\}E\{X_2X_6\}E\{X_4X_5\}\\
&+E\{X_1X_4\}E\{X_2X_3\}E\{X_5X_6\}+E\{X_1X_4\}E\{X_2X_5\}E\{X_3X_6\}+E\{X_1X_4\}E\{X_2X_6\}E\{X_3X_5\}\\
&+E\{X_1X_5\}E\{X_2X_3\}E\{X_4X_6\}+E\{X_1X_5\}E\{X_2X_4\}E\{X_3X_6\}+E\{X_1X_5\}E\{X_2X_6\}E\{X_3X_4\}\\
&+E\{X_1X_6\}E\{X_2X_3\}E\{X_4X_5\}+E\{X_1X_6\}E\{X_2X_4\}E\{X_3X_5\}+E\{X_1X_6\}E\{X_2X_5\}E\{X_3X_4\}
\end{aligned}
$$

Therefore, $E\left\{\tilde{s}_1^3(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right\}$ can be decomposed as

$$
\begin{aligned}
&E\left\{\tilde{s}_1^3(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right\}\\
&=E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\tilde{s}_1(t)\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\}\\
&=E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\}\\
&+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\}\\
&+E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\}\\
&+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\}E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}\\
&+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\}\\
&+E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}\\
&+E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}\\
&+E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}\\
&=3E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\}+6E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\}\\
&+6E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}
\end{aligned}
$$

(3.1.11)

Recall that

$$E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\} = N_1B_1,\ E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\} = N_2B_2,\ E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\} = \frac{N_1\sin\left(2\pi B_1\tau\right)}{2\pi\tau},$$

$$E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\} = \rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\frac{N_1^2B_1^2}{N_2B_2} = 0,$$

$$E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\} = \rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)}\frac{N_1^2B_1^2}{N_2B_2} = 0 \tag{3.1.12}$$

Substituting all the equations in (3.2.12) into (3.1.11) will yield

$$\begin{aligned}
&E\left\{\tilde{s}_1^3(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right\} \\
&= 3E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\} \\
&+6E\left\{\tilde{s}_1(t)\tilde{s}_1(t)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\} \\
&+6E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\} \\
&= 3N_1B_1N_2B_2\frac{N_1\sin\left(2\pi B_1\tau\right)}{2\pi\tau} + 6N_1B_1\rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\frac{N_1^2B_1^2}{N_2B_2}\rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)}\frac{N_1^2B_1^2}{N_2B_2} \\
&+6\frac{N_1\sin\left(2\pi B_1\tau\right)}{2\pi\tau}\left[\rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)}\frac{N_1^2B_1^2}{N_2B_2}\right]^2 \\
&= 3N_1B_1N_2B_2\frac{N_1\sin\left(2\pi B_1\tau\right)}{2\pi\tau}
\end{aligned} \tag{3.1.13}$$

$E\left\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1^3(t+\tau)\right\}$ can be expressed as

$$
\begin{aligned}
&E\left\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1^3(t+\tau)\right\} = E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\tilde{s}_2(t)\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} \\
&= E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} + E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} + E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} + E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} + E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\} + E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\} + E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} + E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\} \\
&= 6E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} + 3E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} \\
&+6E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}
\end{aligned}
\tag{3.1.14}
$$

Recall that

$$E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\} = \rho_{\tilde{s}_1(t)\tilde{s}_2(t)}\frac{N_1^2B_1^2}{N_2B_2},\ E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} = N_1B_1,$$

$$E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\} = \rho_{\tilde{s}_2(t)\tilde{s}_1(t+\tau)}\frac{N_1^2B_1^2}{N_2B_2},\ E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\} = \frac{N_1\sin\left(2\pi B_1\tau\right)}{2\pi\tau},$$

$$E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\} = N_2B_2\ . \tag{3.1.15}$$

Similarly, substitute all the equations of (3.1.15) into (3.1.14), and $E\left\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1^3(t+\tau)\right\}$ will be expressed as

$$
\begin{aligned}
&E\left\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1^3(t+\tau)\right\} \\
&= 6E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} \\
&+3E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_1(t+\tau)\right\} \\
&+6E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\} \qquad (3.1.16)\\
&= 6\rho_{\tilde{s}_1(t)\tilde{s}_2(t)}\frac{N_1^2B_1^2}{N_2B_2}N_1B_1\rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t)}\frac{N_1^2B_1^2}{N_2B_2}+3\frac{N_1\sin\left(2\pi B_1\tau\right)}{2\pi\tau}N_1B_1N_2B_2 \\
&+6\frac{N_1\sin\left(2\pi B_1\tau\right)}{2\pi\tau}\left[\rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t)}\frac{N_1^2B_1^2}{N_2B_2}\right]^2 \\
&= 3N_1B_1N_2B_2\frac{N_1\sin\left(2\pi B_1\tau\right)}{2\pi\tau}
\end{aligned}
$$

$E\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\}$ can be expressed as

$$
\begin{aligned}
&E\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\} = E\{\tilde{s}_1(t)\tilde{s}_2(t)\tilde{s}_2(t)\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\} \\
&= E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\} \\
&+E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\} \\
&= 3E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\}+E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\} \\
&+2E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}+4E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\} \\
&+2E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\}+4E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}
\end{aligned}
$$

(3.1.17)

Recall that

$$E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\} = \rho_{\tilde{s}_1(t)\tilde{s}_2(t)} \frac{N_1^2 B_1^2}{N_2 B_2},\ E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t)\right\} = \rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t)} \frac{N_1^2 B_1^2}{N_2 B_2},$$

$$E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\} = E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\} = N_2 B_2,\ E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\} = \frac{N_1 \sin\left(2\pi B_1 \tau\right)}{2\pi\tau},$$

$$E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\} = \frac{N_2 \sin\left(2\pi B_2 \tau\right)}{2\pi\tau},\ E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\} = \rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)} \frac{N_1^2 B_1^2}{N_2 B_2},$$

$$E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\} = \rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)} \frac{N_1^2 B_1^2}{N_2 B_2}. \tag{3.1.18}$$

Again, by substituting equations (3.1.18) into (3.1.17), $E\left\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\right\}$ will become

$$\begin{aligned}
&E\{\tilde{s}_1(t)\tilde{s}_2^2(t)\tilde{s}_1(t+\tau)\tilde{s}_2^2(t+\tau)\}\\
&= 3E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\} + E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t+\tau)\tilde{s}_2(t+\tau)\right\}\\
&+2E\left\{\tilde{s}_1(t)\tilde{s}_1(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\} + 4E\left\{\tilde{s}_1(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\}\\
&+2E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t)\right\}E\left\{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)\right\} + 4E\left\{\tilde{s}_1(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_2(t+\tau)\right\}E\left\{\tilde{s}_2(t)\tilde{s}_1(t+\tau)\right\}\\
&= 2\rho_{\tilde{s}_1(t)\tilde{s}_2(t)} \frac{N_1^2 B_1^2}{N_2 B_2} \rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t)} \frac{N_1^2 B_1^2}{N_2 B_2} N_2 B_2 + \frac{N_1 \sin\left(2\pi B_1\tau\right)}{2\pi\tau} N_2^2 B_2^2 + 2\frac{N_1 \sin\left(2\pi B_1\tau\right)}{2\pi\tau}\left[\frac{N_2 \sin\left(2\pi B_2\tau\right)}{2\pi\tau}\right]^2\\
&+4\rho_{\tilde{s}_1(t)\tilde{s}_2(t)} \frac{N_1^2 B_1^2}{N_2 B_2} \rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)} \frac{N_1^2 B_1^2}{N_2 B_2} \frac{N_2 \sin\left(2\pi B_2\tau\right)}{2\pi\tau} + 2\rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)} \frac{N_1^2 B_1^2}{N_2 B_2} N_2 B_2 \rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t+\tau)} \frac{N_1^2 B_1^2}{N_2 B_2}\\
&+4\rho_{\tilde{s}_1(t)\tilde{s}_2(t+\tau)} \frac{N_1^2 B_1^2}{N_2 B_2} \rho_{\tilde{s}_1(t+\tau)\tilde{s}_2(t)} \frac{N_1^2 B_1^2}{N_2 B_2} \frac{N_2 \sin\left(2\pi B_2\tau\right)}{2\pi\tau}\\
&= N_2^2 B_2^2 \frac{N_1 \sin(2\pi B_1\tau)}{2\pi\tau} + 2\frac{N_1 \sin(2\pi B_1\tau)}{2\pi\tau}\left[\frac{N_2 \sin(2\pi B_2\tau)}{2\pi\tau}\right]^2
\end{aligned} \tag{3.1.19}$$

Appendix C Derivation of $\mathcal{F}\left\{R_{\tilde{s}_1}^3(\tau)\right\}$ and $\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\}$

$$\mathcal{F}\left\{R_{\tilde{s}_1}^3(\tau)\right\} = \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}$$

Recall that $\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} = \begin{cases} \frac{N_1}{2} & |f| \le B_1 \\ 0 & |f| > B_1 \end{cases}$.

$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}$ will have four segments.

1. If $f \le -2B_1$, $\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} = 0$.

2. If $-2B_1 < f \le 0$,

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} = \int_{-\infty}^{\infty} \frac{N_1}{2}\frac{N_1}{2} d\tau = \int_{-B_1}^{f+B_1} \frac{N_1^2}{4} d\tau = \frac{N_1^2}{4}\left(f + 2B_1\right).$$

3. If $0 < f \le 2B_1$, $\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} = \int_{-\infty}^{\infty} \frac{N_1}{2}\frac{N_1}{2} d\tau = \int_{f-B_1}^{B_1} \frac{N_1^2}{4} d\tau = \frac{N_1^2}{4}\left(2B_1 - f\right)$.

4. If $f > 2B_1$, $\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} = 0$.

Therefore, $\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} = \begin{cases} \frac{N_1^2}{4}\left(2B_1 - f\right) & 0 < f \le 2B_1 \\ \frac{N_1^2}{4}\left(2B_1 + f\right) & -2B_1 < f \le 0 \end{cases}$.

$\mathcal{F}\left\{R_{\tilde{s}_1}^3(\tau)\right\} = \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}$ will have five segments.

1. If $f \le -3B_1$, $\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} = 0$

2. If $-3B_1 < f \le -B_1$,

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}\otimes\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}\otimes\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}$$

$$=\int_{-2B_1}^{f+B_1}\frac{N_1^2}{4}\left(2B_1+\tau\right)\frac{N_1}{2}d\tau=\int_{-2B_1}^{f+B_1}\frac{N_1^3}{8}\left(2B_1+\tau\right)d\tau=\frac{N_1^3}{16}\left(f+3B_1\right)^2$$

3. If $-B_1<f\le B_1$,

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}\otimes\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}\otimes\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}$$

$$=\int_{f-B_1}^{0}\frac{N_1^3}{8}\left(2B_1+f\right)d\tau+\int_{0}^{f+B_1}\frac{N_1^3}{8}\left(2B_1-f\right)d\tau=\frac{N_1^3}{8}\left(3B_1^2-f^2\right)$$

4. If $B_1<f\le 3B_1$,

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}\otimes\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}\otimes\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}=\int_{f-B_1}^{2B_1}\frac{N_1^3}{8}\left(2B_1-\tau\right)d\tau=\frac{N_1^3}{16}\left(3B_1-f\right)^2$$

5. If $f>3B_1$, $\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}\otimes\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}\otimes\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}=0$

Therefore,

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}\otimes\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}\otimes\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}$$

$$=\begin{cases}\frac{N_1^3}{16}\left(f+3B_1\right)^2 & -3B_1<f\le -B_1\\ \frac{N_1^3}{8}\left(3B_1^2-f^2\right) & -B_1<f\le B_1\\ \frac{N_1^3}{16}\left(3B_1-f\right)^2 & B_1<f\le 3B_1\\ 0 & f\le -3B_1 \text{ or } f>3B_1\end{cases}=\begin{cases}\frac{N_1^3}{8}\left(3B_1^2-f^2\right) & |f|\le B_1\\ \frac{N_1^3}{16}\left(3B_1-|f|\right)^2 & B_1<|f|\le 3B_1\\ 0 & |f|>3B_1\end{cases}$$

Derivation of $\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\}$

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\}=\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}\otimes\mathcal{F}\left\{R_{\tilde{s}_2}(\tau)\right\}\otimes\mathcal{F}\left\{R_{\tilde{s}_2}(\tau)\right\}$$

$$\mathcal{F}\left\{R_{\tilde{s}_2}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_2}(\tau)\right\} = \begin{cases} \frac{N_2^2}{4}\left(2B_2 - f\right) & 0 < f \le 2B_2 \\ \frac{N_2^2}{4}\left(2B_2 + f\right) & -2B_2 < f \le 0 \end{cases}$$ can be derived from equations above.

$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\} = \mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_2}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_2}(\tau)\right\}$ will have six segments.

1. If $|f| > B_1 + 2B_2$, $\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_2}(\tau)\right\} \otimes \mathcal{F}\left\{R_{\tilde{s}_2}(\tau)\right\} = 0$

2. If $-B_1 - 2B_2 < f \le -B_1$,

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\}$$

$$= \int_{-2B_2}^{f+B_1} \frac{N_2^2}{4}\left(2B_2 + \tau\right)\frac{N_1}{2}\,d\tau = \int_{-2B_2}^{f+B_1} \frac{N_1 N_2^2}{8}\left(2B_2 + \tau\right)d\tau = \frac{N_1 N_2^2}{16}\left(f + B_1 + 2B_2\right)^2$$

3. If $-B_1 < f \le B_1 - 2B_2$,

$$\begin{aligned}\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\} &= \int_{-2B_2}^{0} \frac{N_2^2}{4}\left(2B_2 + \tau\right)\frac{N_1}{2}\,d\tau + \int_{0}^{f+B_1} \frac{N_2^2}{4}\left(2B_2 - \tau\right)\frac{N_1}{2}\,d\tau \\ &= \int_{-2B_2}^{0} \frac{N_1 N_2^2}{8}\left(2B_2 + \tau\right)d\tau + \int_{0}^{f+B_1} \frac{N_1 N_2^2}{8}\left(2B_2 - \tau\right)d\tau \\ &= \frac{N_1 N_2^2}{16}\left(-f^2 - 2B_1 f + 4B_2 f - B_1^2 + 4B_2^2 + 4B_1 B_2\right)\end{aligned}$$

4. If $B_1 - 2B_2 < f \le 2B_2 - B_1$,

$$\begin{aligned}\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\} &= \int_{f-B_1}^{0} \frac{N_2^2}{4}\left(2B_2 + \tau\right)\frac{N_1}{2}\,d\tau + \int_{0}^{f+B_1} \frac{N_2^2}{4}\left(2B_2 - \tau\right)\frac{N_1}{2}\,d\tau \\ &= \int_{f-B_1}^{0} \frac{N_1 N_2^2}{8}\left(2B_2 + \tau\right)d\tau + \int_{0}^{f+B_1} \frac{N_1 N_2^2}{8}\left(2B_2 - \tau\right) \\ &= \frac{N_1 N_2^2}{8}\left(-f^2 - B_1^2 + 4B_1 B_2\right)\end{aligned}$$

5. If $2B_2 - B_1 < f \le B_1$,

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\} = \int_{f-B_1}^{0} \frac{N_2^2}{4}\left(2B_2+\tau\right)\frac{N_1}{2}d\tau + \int_{0}^{2B_2} \frac{N_2^2}{4}\left(2B_2-\tau\right)\frac{N_1}{2}d\tau$$

$$= \int_{f-B_1}^{0} \frac{N_1N_2^2}{8}\left(2B_2+\tau\right)d\tau + \int_{0}^{2B_2} \frac{N_1N_2^2}{8}\left(2B_2-\tau\right)d\tau$$

$$= \frac{N_1N_2^2}{16}\left(-f^2 - 4B_2 f + 2B_1 f - B_1^2 + 4B_2^2 + 4B_1B_2\right)$$

6. If $B_1 < f \le B_1 + 2B_2$,

$$\mathbb{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\} = \int_{f-B_1}^{2B_2} \frac{N_2^2}{4}\left(2B_2-\tau\right)\frac{N_1}{2}d\tau = \frac{N_1N_2^2}{16}\left(f - B_1 - 2B_2\right)^2$$

Therefore,

$$\mathbb{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\} = \begin{cases} \frac{N_1N_2^2}{8}\left(-f^2 - B_1^2 + 4B_1B_2\right) & |f| \le 2B_2 - B_1 \\ \frac{N_1N_2^2}{16}\left[-f^2 + \left(2B_1 - 4B_2\right)|f| - B_1^2 + 4B_2^2 + 4B_1B_2\right] & 2B_2 - B_1 < |f| \le B_1 \\ \frac{N_1N_2^2}{16}\left(B_1 + 2B_2 - |f|\right)^2 & B_1 < |f| \le B_1 + 2B_2 \\ 0 & |f| > B_1 + 2B_2 \end{cases}$$

Appendix D Derivation of $P_{\tilde{y}_1}(f)$ in the condition of $B_1 > 2B_2$

the result of $P_{\tilde{s}_1}(f) \otimes P_{\tilde{s}_1}^2(f)$ is

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\} = P_{\tilde{s}_1}(f) \otimes P_{\tilde{s}_2}(f) \otimes P_{\tilde{s}_2}(f)$$
$$= \begin{cases} \dfrac{N_1N_2^2}{2}B_2^2 & |f| \le B_1 - 2B_2 \\ \dfrac{N_1N_2^2}{16}\left[-f^2 + \left(2B_1 - 4B_2\right)|f| - B_1^2 + 4B_1B_2 + 4B_2^2\right] & B_1 - 2B_2 < |f| \le B_1 \\ \dfrac{N_1N_2^2}{16}\left(B_1 + 2B_2 - |f|\right)^2 & B_1 < |f| \le B_1 + 2B_2 \\ 0 & |f| > B_1 + 2B_2 \end{cases}$$

$$P_{\tilde{y}_1}(f) =$$
$$\begin{cases} \dfrac{1}{B_1}\left(P_{01} - 6P_{01}^2 10^{\frac{-IP_3}{10}} + 9P_{01}^3 10^{\frac{-IP_3}{5}} - 4P_{01}P_{02}10^{\frac{-IP_3}{10}} + 12P_{01}^2P_{02}10^{\frac{-IP_3}{5}} + 4P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\right) \\ + \dfrac{3}{2}\dfrac{1}{B_1^3}P_{01}^3 10^{\frac{-IP_3}{5}}\left(3B_1^2 - f^2\right) + \dfrac{8}{B_1}P_{01}P_{02}^2 10^{\frac{-IP_3}{5}} & |f| \le B_1 - 2B_2 \\ \dfrac{1}{B_1}\left(P_{01} - 6P_{01}^2 10^{\frac{-IP_3}{10}} + 9P_{01}^3 10^{\frac{-IP_3}{5}} - 4P_{01}P_{02}10^{\frac{-IP_3}{10}} + 12P_{01}^2P_{02}10^{\frac{-IP_3}{5}} + 4P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\right) \\ + \dfrac{3}{2}\dfrac{1}{B_1^3}P_{01}^3 10^{\frac{-IP_3}{5}}\left(3B_1^2 - f^2\right) & B_1 - 2B_2 < |f| \le B_1 \\ + \dfrac{1}{B_1B_2^2}P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\left[-f^2 + \left(2B_1 - 4B_2\right)|f| - B_1^2 + 4B_2^2 + 4B_1B_2\right] \\ \dfrac{3}{4}\dfrac{1}{B_1^3}P_{01}^3 10^{\frac{-IP_3}{5}}\left(3B_1 - |f|\right)^2 + \dfrac{1}{B_1B_2^2}P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\left(B_1 + 2B_2 - |f|\right)^2 & B_1 < |f| \le B_1 + 2B_2 \\ \dfrac{3}{4}\dfrac{1}{B_1^3}P_{01}^3 10^{\frac{-IP_3}{5}}\left(3B_1 - |f|\right)^2 & B_1 + 2B_2 < |f| < 3B_1 \\ 0 & |f| > 3B_1 \end{cases}$$

$$\mathcal{F}\left\{R_{\tilde{s}_1}^2(\tau)R_{\tilde{s}_2}(\tau)\right\}$$

$$=\begin{cases}\frac{N_1^2N_2}{8}\left(-f^2+4B_1B_2-B_2^2\right) & |f|\le B_2\\ \frac{N_1^2N_2}{8}\left(4B_1B_2-2B_2|f|\right) & B_2<|f|\le 2B_1-B_2\\ \frac{N_1^2N_2}{16}\left(2B_1+B_2-|f|\right)^2 & 2B_1-B_2<|f|\le 2B_1+B_2\\ 0 & |f|>2B_1+B_2\end{cases}$$

$$P_{\tilde{y}_2}(f)$$

$$=\begin{cases}\frac{1}{B_2}\left(P_{02}-6P_{02}^2 10^{\frac{-IP_3}{10}}+9P_{02}^3 10^{\frac{-IP_3}{5}}-4P_{01}P_{02}10^{\frac{-IP_3}{10}}+12P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}+4P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\right) \\ +\frac{3}{2}\frac{1}{B_2^3}P_{02}^3 10^{\frac{-IP_3}{5}}\left(3B_2^2-f^2\right)+\frac{2}{B_1^2B_2}P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\left(-f^2+4B_1B_2-B_2^2\right) & |f|\le B_2\\ \frac{3}{4}\frac{1}{B_2^3}P_{02}^3 10^{\frac{-IP_3}{5}}\left(3B_2-|f|\right)^2+\frac{2}{B_1^2B_2}P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\left(4B_1B_2-2B_2|f|\right) & B_2<|f|\le 3B_2\\ \frac{2}{B_1^2B_2}P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\left(4B_1B_2-2B_2|f|\right) & 3B_2<|f|\le 2B_1-B_2\\ \frac{1}{B_1^2B_2}P_{01}^2P_{02}10^{\frac{-IP_3}{5}}\left(2B_1+B_2-|f|\right)^2 & 2B_1-B_2<|f|\le 2B_1+B_2\\ 0 & |f|>2B_1+B_2\end{cases}$$

$$
P_y(f) = \frac{1}{2}\left[P_{\tilde{y}_1}(f - f_{c_1}) + P_{\tilde{y}_2}(f - f_{c_2}) \right]
$$

$$
= \begin{cases}
\frac{1}{2B_1}\left(P_{01} - 6P_{01}^2 10^{\frac{-IP_3}{10}} + 9P_{01}^3 10^{\frac{-IP_3}{5}} - 4P_{01}P_{02}10^{\frac{-IP_3}{10}} + 12P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}} + 4P_{01}P_{02}^2 10^{\frac{-IP_3}{5}} \right) \\
\quad + \frac{3}{4}\frac{1}{B_1^3} P_{01}^3 10^{\frac{-IP_3}{5}} \left(3B_1^2 - \left(f - f_{c_1} \right)^2 \right) + \frac{4}{B_1} P_{01} P_{02}^2 10^{\frac{-IP_3}{5}} & \left| f - f_{c_1} \right| \le B_1 - 2B_2 \\
\frac{1}{2B_1}\left(P_{01} - 6P_{01}^2 10^{\frac{-IP_3}{10}} + 9P_{01}^3 10^{\frac{-IP_3}{5}} - 4P_{01}P_{02}10^{\frac{-IP_3}{10}} + 12P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}} + 4P_{01}P_{02}^2 10^{\frac{-IP_3}{5}} \right) \\
\quad + \frac{3}{4}\frac{1}{B_1^3} P_{01}^3 10^{\frac{-IP_3}{5}} \left(3B_1^2 - \left(f - f_{c_1} \right)^2 \right) & B_1 - 2B_2 < \left| f - f_{c_1} \right| \le B_1 \\
\quad + \frac{1}{2B_1 B_2^2} P_{01} P_{02}^2 10^{\frac{-IP_3}{5}} \left[-\left(f - f_{c_1} \right)^2 + \left(2B_1 - 4B_2 \right)\left| f - f_{c_1} \right| - B_1^2 + 4B_2^2 + 4B_1 B_2 \right] \\
\frac{3}{8}\frac{1}{B_1^3} P_{01}^3 10^{\frac{-IP_3}{5}} \left(3B_1 - \left| f - f_{c_1} \right| \right)^2 + \frac{1}{2B_1 B_2^2} P_{01} P_{02}^2 10^{\frac{-IP_3}{5}} \left(B_1 + 2B_2 - \left| f - f_{c_1} \right| \right)^2 & B_1 < \left| f - f_{c_1} \right| \le B_1 + 2B_2 \\
\frac{3}{8}\frac{1}{B_1^3} P_{01}^3 10^{\frac{-IP_3}{5}} \left(3B_1 - \left| f - f_{c_1} \right| \right)^2 & B_1 + 2B_2 < \left| f - f_{c_1} \right| \le 3B_1 \\
\frac{1}{2B_2}\left(P_{02} - 6P_{02}^2 10^{\frac{-IP_3}{10}} + 9P_{02}^3 10^{\frac{-IP_3}{5}} - 4P_{01}P_{02}10^{\frac{-IP_3}{10}} + 12P_{01} P_{02}^2 10^{\frac{-IP_3}{5}} + 4P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}} \right) & \left| f - f_{c_2} \right| \le B_2 \\
\quad + \frac{3}{4}\frac{1}{B_2^3} P_{02}^3 10^{\frac{-IP_3}{5}} \left(3B_2^2 - \left(f - f_{c_2} \right)^2 \right) + \frac{1}{B_1^2 B_2} P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}} \left(-\left(f - f_{c_2} \right)^2 + 4B_1 B_2 - B_2^2 \right) \\
\frac{3}{8}\frac{1}{B_2^3} P_{02}^3 10^{\frac{-IP_3}{5}} \left(3B_2 - \left| f - f_{c_2} \right| \right)^2 + \frac{1}{B_1^2 B_2} P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}} \left(4B_1 B_2 - 2B_2 \left| f - f_{c_2} \right| \right) & B_2 < \left| f - f_{c_2} \right| \le 3B_2 \\
\frac{1}{B_1^2 B_2} P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}} \left(4B_1 B_2 - 2B_2 \left| f - f_{c_2} \right| \right) & 3B_2 < \left| f - f_{c_2} \right| \le 2B_1 - B_2 \\
\frac{1}{2B_1^2 B_2} P_{01}^2 P_{02} 10^{\frac{-IP_3}{5}} \left(2B_1 + B_2 - \left| f - f_{c_2} \right| \right)^2 & 2B_1 - B_2 < \left| f - f_{c_2} \right| \le 2B_1 + B_2 \\
0 & \text{otherwise}
\end{cases}
$$

Appendix E Derivation of $P_{\tilde{y}_1}(f)$

$$P_{\tilde{y}_1}(f)$$

$$=\mathcal{F}\left\{R_{\tilde{y}_1}(\tau)\right\}$$

$$=\left(a_1^2+\frac{9}{2}a_1a_3N_1B_1+\frac{81}{16}a_3^2\left(N_1B_1\right)^2+3a_1a_3N_2B_2+\frac{27}{4}a_3^2N_1B_1N_2B_2+\frac{9}{4}a_3^2\left(N_2B_2\right)^2\right)\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}$$

$$+\frac{27}{8}a_3^2\mathcal{F}\left\{R_{\tilde{s}_1}^3(\tau)\right\}+\frac{9}{2}a_3^2\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\}$$

Based on the frequency segments in (2.1.4), (2.4.2), and (2.4.3), $P_{\tilde{y}_1}(f)$ can be derived in five segments.

1. If $|f|\le 2B_2-B_1$,

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}=\frac{N_1}{2},$$

$$\mathcal{F}\left\{R_{\tilde{s}_1}^3(\tau)\right\}=\frac{N_1^3}{8}\left(3B_1^2-f^2\right),$$

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2(\tau)}^2(\tau)\right\}=\frac{N_1N_2^2}{8}\left(-f^2-B_1^2+4B_1B_2\right),$$

then

$$P_{\tilde{y}_1}(f)$$

$$=\left(a_1^2+\frac{9}{2}a_1a_3K_1+\frac{81}{16}a_3^2K_1^2+3a_1a_3K_2+\frac{27}{4}a_3^2K_1K_2+\frac{9}{4}a_3^2K_2^2\right)\frac{N_1}{2}+\frac{27}{8}a_3^2\frac{N_1^3}{8}\left(3B_1^2-f^2\right)$$

$$+\frac{9}{2}a_3^2\frac{N_1N_2^2}{8}\left(-f^2-B_1^2+4B_1B_2\right)$$

$$=\frac{1}{B_1}\left(P_{01}-6P_{01}^2 10^{\frac{-IP_2}{10}}+9P_{01}^3 10^{\frac{-IP_3}{5}}-4P_{01}P_{02}10^{\frac{-IP_3}{10}}+12P_{01}^2P_{02}10^{\frac{-IP_3}{5}}+4P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\right)$$

$$+\frac{3}{2}\frac{1}{B_1^3}P_{01}^3 10^{\frac{-IP_3}{5}}\left(3B_1^2-f^2\right)+\frac{2}{B_1B_2^2}P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\left(-f^2-B_1^2+4B_1B_2\right)$$

2. If $2B_2-B_1<\left|f\right|\le B_1$

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\}=\frac{N_1}{2},$$

$$\mathcal{F}\left\{R_{\tilde{s}_1}^3(\tau)\right\}=\frac{N_1^3}{8}\left(3B_1^2-f^2\right),$$

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\}=\frac{N_1N_2^2}{16}\left[-f^2+\left(2B_1+4B_2\right)\left|f\right|-B_1^2+4B_2^2+4B_1B_2\right],$$

then

$$P_{\tilde{y}_1}(f)$$

$$=\left(a_1^2+\frac{9}{2}a_1a_3K_1+\frac{81}{16}a_3^2K_1^2+3a_1a_3K_2+\frac{27}{4}a_3^2K_1K_2+\frac{9}{4}a_3^2K_2^2\right)\frac{N_1}{2}+\frac{27}{8}a_3^2\frac{N_1^3}{8}\left(3B_1^2-f^2\right)$$

$$+\frac{9}{2}a_3^2\frac{N_1N_2^2}{16}\left[-f^2+\left(2B_1+4B_2\right)\left|f\right|-B_1^2+4B_2^2+4B_1B_2\right]$$

$$=\frac{1}{B_1}\left(P_{01}-6P_{01}^2 10^{\frac{-IP_3}{10}}+9P_{01}^3 10^{\frac{-IP_3}{5}}-4P_{01}P_{02}10^{\frac{-IP_3}{10}}+12P_{01}^2P_{02}10^{\frac{-IP_3}{5}}+4P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\right)$$

$$+\frac{3}{2}\frac{1}{B_1^3}P_{01}^3 10^{\frac{-IP_3}{5}}\left(3B_1^2-f^2\right)+\frac{1}{B_1B_2^2}P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\left[-f^2+\left(2B_1-4B_2\right)\left|f\right|-B_1^2+4B_2^2+4B_1B_2\right]$$

3. If $B_1 < |f| \le B_1 + 2B_2$

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} = 0,$$

$$\mathcal{F}\left\{R_{\tilde{s}_1}^3(\tau)\right\} = \frac{N_1^3}{16}\left(3B_1 - |f|^2\right),$$

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\} = \frac{N_1 N_2^2}{16}\left(B_1 + 2B_2 - |f|^2\right),$$

then

$$P_{\tilde{y}_1}(f)$$

$$= \frac{27}{8}a_3^2\frac{N_1^3}{16}\left(3B_1 - |f|^2\right) + \frac{9}{2}a_3^2\frac{N_1 N_2^2}{16}\left(B_1 + 2B_2 - |f|^2\right)$$

$$= \frac{3}{4}\frac{1}{B_1^3}P_{01}^3 10^{\frac{-IP_3}{5}}\left(3B_1 - |f|\right)^2 + \frac{1}{B_1 B_2^2}P_{01}P_{02}^2 10^{\frac{-IP_3}{5}}\left(B_1 + 2B_2 - |f|\right)^2$$

4. If $B_1 + 2B_2 < |f| \le 3B_1$

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)\right\} = 0,$$

$$\mathcal{F}\left\{R_{\tilde{s}_1}^3(\tau)\right\} = \frac{N_1^3}{16}\left(3B_1 - |f|^2\right),$$

$$\mathcal{F}\left\{R_{\tilde{s}_1}(\tau)R_{\tilde{s}_2}^2(\tau)\right\} = 0,$$

then

$$P_{\tilde{y}_1}(f)$$

$$= \frac{27}{8}a_3^2\frac{N_1^3}{16}\left(3B_1 - |f|\right)^2$$

$$= \frac{3}{4}\frac{1}{B_1^3}P_{01}^3 10^{\frac{-IP_3}{5}}\left(3B_1 - |f|\right)^2$$

5. If $|f| > 3B_1$,

then

$P_{\tilde{y}_1}(f) = 0.$

Thus, $P_{\tilde{y}_1}(f)$ can be derived by combining the five segments above.