Power System Investment Planning using Stochastic Dual Dynamic Programming

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ABSTRACT

Generation and transmission investment planning in deregulated markets faces new challenges particularly as deregulation has introduced more uncertainty to the planning problem. Traditional planning techniques and processes cannot be applied to the deregulated planning problem as generation investments are profit driven and competitive. Transmission investments must facilitate generation access rather than servicing generation choices. The new investment planning environment requires the development of new planning techniques and processes that can remain flexible as uncertainty within the system is revealed.

The optimisation technique of Stochastic Dual Dynamic Programming (SDDP) has been successfully used to optimise continuous stochastic dynamic planning problems such as hydrothermal scheduling. SDDP is extended in this thesis to optimise the stochastic, dynamic, mixed integer power system investment planning problem. The extensions to SDDP allow for optimisation of large integer variables that represent generation and transmission investment options while still utilising the computational benefits of SDDP. The thesis also details the development of a mathematical representation of a general power system investment planning problem and applies it to a case study involving investment in New Zealand's HVDC link. The HVDC link optimisation problem is successfully solved using the extended SDDP algorithm and the output data of the optimisation can be used to better understand risk associated with capital investment in power systems.

The extended SDDP algorithm offers a new planning and optimisation technique for deregulated power systems that provides a flexible optimal solution and informs the planner about investment risk associated with uncertainty in the power system.

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CONTENTS

Chapter 1

INTRODUCTION

The introduction of deregulated power markets has increased the level of uncertainty in power systems. Investors are no longer guaranteed a profitable rate of return on their investment and must assess market uncertainties and predict future pricing trends. The decentralised and competitive investment environment limits the sharing of information and cooperation between market participants. The new investment planning environment is far more complex and uncertain than in regulated power systems necessitating development of new planning techniques and tools.

The difficulties encountered by many countries with regard to investment planning in restructured power systems illustrate the underdeveloped nature of investment planning tools and processes for the deregulated environment. During the initial phase of restructuring the research and development focus was on real time operation of the new markets. Over time, as demand has increased, the issue of investment planning has become more important as the overcapacity of many systems begins to erode. Many traditional planning tools and processes are no longer relevant in a deregulated market structure. Generation investments are chosen to maximise profit and are undertaken on a competitive basis. Transmission investment no longer services generation investment decisions and is now required to facilitate fair and competitive access to all generators across the network.

Previous research into the uncertainties of investment planning has so far focussed on either generation or transmission issues and not a total system approach. It is acknowledged that tools for studying the uncertainty of combined generation and transmission planning in deregulated markets are underdeveloped [1] [2] [3]. This motivates the research presented in this thesis that develops and applies a new planning tool, using the optimisation technique of Stochastic Dual Dynamic Programming, to the investment planning problem.

Stochastic Dual Dynamic Programming (SDDP) [4] has been successfully applied to the stochastic dynamic hydrothermal scheduling problem. SDDP offers benefits over other dynamic optimisation techniques as it requires no discretisation of the state space allowing complex problems to be solved in a timely fashion. The power system investment planning problem is a stochastic dynamic optimisation problem but differs from hydrothermal scheduling due to investments being represented by integer variables. The research presented in this thesis extends the current formulation of SDDP to allow mixed integer optimisation problems to be solved. The extended SDDP model is applied to a case study and is shown to successfully optimise the problem. The development and application of the extended SDDP algorithm presents a new planning technique and tool for planners of deregulated power systems. It provides a flexible and adaptable optimal solution path and illustrates the level of risk faced by investments due to uncertainty within the planning environment.

1.1 CHAPTER SUMMARY

The layout of chapters in this thesis are as follows:

Chapter 2 - Power System Planning: This chapter discusses investment planning processes and techniques used in regulated and deregulated markets and how they differ. The current state of research regarding generation and transmission investment planning in deregulated markets is detailed and shown to be lacking with respect to dynamic planning problems where uncertainty is a major influence on investment decisions.

Chapter 3 - Planning over Time: In this chapter the power system investment planning problem is identified as a stochastic dynamic planning problem. Solution of such problems by dynamic and stochastic dynamic programming is detailed but such techniques are discounted for solving real world problems due to the 'curse of dimensionality' associated with dynamic programming.

Chapter 4 - Stochastic Dual Dynamic Programming: To overcome the 'curse of dimensionality' of dynamic programming, approximations to dynamic programming have been developed. One such approximation is Stochastic Dual Dynamic Programming (SDDP) that uses the dual variables of the problem to build a piecewise linear representation of the future cost function without discretising the state space. SDDP has been successfully applied to the stochastic dynamic hydrothermal scheduling problem, a problem similar to power system investment planning. This chapter shows the development of SDDP, details the calculations involved to obtain a piecewise linear representation of future costs functions and outlines the optimisation algorithm.

Chapter 5 - Defining and Solving Optimisation Problems: This chapter explores the process involved in developing a real world problem into an optimisation problem. The process is described in four steps; construction of the objective function, defining problem constraints, identification of solution technique and solution and analysis of the problem. The power system investment problem objective function and generalised constraints are developed followed by selection of SDDP as a suitable solution technique. The investment planning problem does not satisfy all requirements of SDDP as it is a non convex mixed integer problem. This motivates the extension to SDDP presented in Chapter 6.

Chapter 6 - SDDP and Power System Planning: This chapter details the extensions developed to SDDP to allow the investment planning problem to be optimised. The mixed integer planning problem requires three extensions to SDDP; relaxation of the mixed integer problems on the backward pass, changes to convergence criteria and development of a dynamic constraint to prevent re-use of previously selected investment options.

Chapter 7 - Generation Investment Optimisation: The mathematical representation of a generation investment problem is developed in this chapter. Modelling decisions regarding capacity and demand representation, timing of investments, reserves, demand growth and value of lost load are presented. The objective function of the problem is outlined followed by the constraints of the problem. A simple example optimisation problem is developed and the mathematical representation presented.

Chapter 8 - Transmission Investment Optimisation: This chapter presents the extension of the mathematical representation of Chapter 7 to include transmission investments and two load regions. Additional modelling decisions regarding transmission representation, changes to reserves modelling and the introduction of a second stochastic variable are presented. A generalised objective function and problem constraints are detailed.

Chapter 9 - Case Study: HVDC Upgrade: The mathematical representation developed in Chapter 8 is applied to a case study involving the interisland HVDC link in New Zealand. The case study problem data and generation scenarios are outlined.

Chapter 10 - Case Study Results and Modelling Discussion: The case study of Chapter 9 is successfully solved using the extended SDDP algorithm and the results are presented. The successful optimisation of the HVDC case study shows that the extended SDDP algorithm is a useful planning tool to investigate uncertainty in power system planning in a deregulated environment.

Chapter 2

POWER SYSTEM PLANNING

2.1 INTRODUCTION

Modern society is dependent on affordable and reliable electricity supply, making investment planning an integral part of power system management. Good investment planning processes and coordination of investment decisions ensures the secure and reliable electricity supply expected by consumers can be achieved at the least cost, or most value, to consumers. Power systems in regulated environments are centrally planned making optimisation and coordination of investment decisions simpler as information is shared freely. In deregulated markets, investments are undertaken by private companies where commercial sensitivity of investment information restricts information sharing between competing market participants. Lack of information increases the uncertainty for planning in deregulated markets potentially resulting in suboptimal system investment. The commercial environment that investments are made within results in future investment decisions having feedback effects, particularly in terms of price, on current investment decisions. This increases uncertainty within the system making investment decisions riskier and therefore less attractive.

This chapter investigates how power system planning changes in deregulated markets and focuses on the challenges faced by investment planners. Firstly investment planning in regulated systems is discussed. This is followed by discussion of the challenges faced by planners in deregulated power systems. Lastly an analysis of how the proposed research fits within the existing body of knowledge and the benefits it offers is presented.

2.2 POWER SYSTEM EXPANSION PLANNING: REGULATION

Power system planning in a centrally planned, regulated environment is not considered inherently risky even though it is a complex process [5]. As described by [6] the planning and investment process consists of three main stages. The first is the development of a load forecast, the second stage is the analysis of generation requirements (based on the load forecast) including reliability, sizes and timing of new investments and the third stage is the use of power flow studies and reliability analysis to decide where and when transmission should be built or upgraded. Each of the stages is sequential with overall goal of the process to ensure reliability of supply at least cost.

Dyner and Larsen [5] describe the main characteristics of regulated planning as stable prices, full information, demand forecasting and co-operation. Stable prices refers to the pricing principles used in regulation where prices are set based on the needs and costs of generation companies. Investment costs are recovered immediately or in the near future by using rate of return regulation or similar. In this environment the financial risk for generation companies and investors is very low or non existent. Full information is a unique characteristic of monopoly environments where there is no incentive to withhold information regarding investment, unit retirement or capacity upgrade. The resulting low level of market uncertainty facilitates the use of optimisation tools to identify optimal investment paths. Demand forecasting in regulated markets is done on a regional or national aggregated level. While there is still uncertainty surrounding forecasting on this level, good long term knowledge of the system and demand growth characteristics allows for some level of certainty in forecasting. The final characteristic described is co-operation where the regulator and generation companies are tasked with the same objective, to make decisions in the best interest of the country or region. This allows cooperative decisions to made without conflicting objectives driving investment decisions. The four characteristics described by Dyner and Larsen [5] illustrate that investment planning in a regulated market environment can be managed in order to reduce risk and uncertainty. The stable investment environment that is created itself reduces risk and helps provide certainty regarding the future state of the system.

The certainty surrounding planning in regulated markets has allowed for development of optimisation tools and techniques to aid the planning process. Linear programming was one of the first optimisation techniques to be applied to the planning problem [7]. Linear programming works well in simplified systems but fails to adequately model complex planning problems and detailed systems. With increases in complexity, the generation planning problem becomes a non linear, mixed integer optimisation and development and application of other optimisation techniques became necessary.

Dynamic programming has been one of the most widely used optimisation techniques for the planning problem [8] but suffers from the 'curse of dimensionality' [9]. The overwhelming state space resulting from real world problems has required development of heuristics such as tunnel based constraints that allow the user to specify particular allowable states e.g. as available in the WASP [10] and EGEAS [11] software packages. Other heuristic techniques aim to reduce the state space by reducing the number of scenarios studied or restricting the investments available [12] [13] [14]. Many other optimisation techniques such as genetic algorithms, stochastic optimisation, decision analysis, tradeoff analysis and artificial intelligence have to applied to various power system investment planning problems. The range of applied optimisation techniques is well covered in the literature with detailed reviews provided by Kagiannas [8], Zhang et.al. [15], Jebaraj [16] and Zhu [17]. Regardless of the type of optimisation technique used within the investment planning process, all rely on the characteristics of a regulated system such as transparency of information, financial certainty and industry co-operation. The introduction of deregulated or restructured markets has removed much of the certainty of power system investment planning and the tools used for planning and optimisation are no longer as relevant in their current forms.

2.3 POWER SYSTEM EXPANSION PLANNING: DEREGULATION

Deregulated markets introduce competition into power systems where generation companies compete to sell energy to retailers and retailers compete to sell energy to consumers. Investments are undertaken in a competitive nature and on a 'most profit' basis [8]. The commercially sensitive nature of investments results in reduced information transparency and cooperation between the market participants regarding future investment decisions. The change in market structure has introduced a high level of uncertainty into power system planning environment that includes; current and future prices, demand, levels of regulation and actions from competitors within the market [5]. The reduction in industry cooperation and information transfer and the increase in uncertainty has made investment planning a far more complex process.

The profit generators can make from an investment is directly related to the price received for the energy sold. The price of energy and hence income for generators is no longer centrally controlled and cannot be adjusted to ensure investment profitability. The electricity price is now subject to market forces such as the spot market, hedge contracts or call options, reserves markets or a combination of these. Forecasting how the price many change in the long term is uncertain and therefore identifying a profitable investment is difficult. A generation company may use hedge contracts to improve the certainty of income over the long term but the pricing structure of the hedge contract will usually be related to the long term trends of the electricity price, that is itself uncertain.

One of the goals of restructuring power markets was to increase choice in price, reliability and supply provider for consumers [5]. The competition amongst retailers allows consumers to chose how and at what price they are supplied electricity. Consumers also have a choice of electricity retailers and may switch between them. Development of small scale demand reduction technologies such as micro hydro, micro wind and solar power along with increasing awareness of human impacts on the environment is increasing demand reduction investments by consumers. The active participation by consumers in the power market makes long term demand forecasting more uncertain than in a regulated system. Consumer choice and participation has an effect on price as the spot market price is the price balance point between supply and demand. Where future demand is uncertain due to consumer participation in the market, the future price is also uncertain.

While restructured markets are often termed 'deregulated' markets, this phrase is misleading as deregulated markets usually operate under some form of regulation or market rules. The nature of the regulation varies from market to market and is often dependent on a range of factors including market size, interconnections and political climate. The regulation experienced by restructured markets is continually evolving leading to uncertainty about future regulatory structures or intervention. The unpredictability of regulatory changes and the potential effects on electricity price increases price uncertainty and therefore future income uncertainty for investors.

Investors also face uncertainty from competitors. New investments made by competitors have the ability to alter the price received for all generated electricity thus potentially reducing the income of an existing investment or the profitability of a planned investment. Competitive uncertainty means that investors must try to predict their competitors moves and or try to restrict competitors from entering the market. The gaming aspect of predicting competitive investments adds an additional layer of difficulty and uncertainty to investment planning in deregulated markets.

Removal of vertical integration between generation, transmission and retail alongside a competitive market environment has reduced the information transparency and cooperation between market participants. Investments are no longer centrally planned and coordinated. Any investor can build new generating plant subject to their own choice of capacity, location and timing. One outcome of this circumstance is an increase in uncertainty for transmission planning. Without coordination between generation and transmission investments, transmission investment may be delayed or left stranded. The construction time, including consenting, for transmission investment is often very different to that of generation [2] leading to generation being available before transmission can be built. The lack of coordination between generation and transmission requires that transmission planning in deregulated power systems must be flexible to accommodate unforseen changes in generation investments and demand growth.

2.3.1 Generation Planning

Deregulated electricity markets rely on market price signals to indicate the need for new investment. An investment opportunity becomes attractive when the market price is high enough to ensure the investment is profitable. Investors must consider both the uncertainty of the market environment and the competition from other investors in making an investment decisions. These two factors are both new in power systems planning. Two areas of active research regarding these issues are game theory as it pertains to electricity markets and real options theory. Both are described in further detail in the following sections.

2.3.1.1 Real Options Theory

The uncertainty and timing of individual generation investments in power systems has been studied using Real Options Theory [18] [19] [20] [21] [22] [23] [24]. Real Options theory originated in finance and is used to assess the value of deferring investment that is subject to uncertainty. It assesses the additional value in waiting for uncertainty to reveal itself so that the investor can delay making an irreversible investment. Botterud et al. [19] use real options theory to study optimal investment decisions when demand growth and hence future prices are uncertain. They use an explicit power market model that utilises linear supply and demand curves to calculate the electricity price. Their model is used to study investments in both restructured and regulated environments and identifies the option value of delaying investment. Their work also shows how price caps and price feedback can affect the timing of investment decisions. The time frame of their study is 10 years with restrictions placed on investment options in order to limit the state space. Keppo and Hao [20] extend the use of real options to include the effect that large investments can have on future prices. Where real options theory often assumes that all participants are price takers [19] [18], Keppo and Hao assume the investment is large enough to have an effect on prices. This is applied in the context of power systems as market prices can easily be distorted through large generation investments. Smit and Ankum [21] study the value of investment deferral using real options under the different competitive scenarios of perfect competition, oligopoly and monopoly. Their research finds that under perfect competition there is a tendency to invest early to avoid the erosion of investment value by competitive investment. In a monopoly there is no loss of value due to postponement so there is tendency to delay projects. Oligopoly lies between the competition and monopoly where cooperation between investors tends to delay investment but as soon as one makes an investment, the others will follow immediately. Marreco and Carpio [22] use real options theory to study the optimum value for capacity payments to incentivise generation investment and availability. Botterud and Korpas [23] investigate, using real options, how fixed or variable capacity payments affect investment decisions in the Norwegian power market. Their research shows that capacity payments can induce earlier investments but that the kind of payment i.e. fixed or variable, affects the timing of investment decisions.

Real options theory is a useful and promising technique for an individual investor to assess the value of an investment and the value of deferring an investment. The application of real options to power system planning has shown how uncertainty has a tendency to delay investment decisions under any kind of market structure. Real options analysis is best suited for use by individual investors to investigate investment choices and timing under uncertainty. It doesn't help with coordination of investment planning throughout the power system as each real options analysis considers a single investment option and fixed location.

2.3.1.2 Game Theory

One aspect that contributes to price volatility is competition between market participants. Competitive behaviour affects both short term prices, where generators compete in the real time spot market, and long term prices, where generation investors can influence the long term electricity price by the size, type and timing of new investments. Competition causes investors to face higher risks and therefore seek higher returns. It also results in investment decisions affecting other investors profits and decisions [25]. The interaction, competition and resulting outcomes for deregulated power markets have been studied using game theory.

Chunag et al. [25] use Cournot theory to model generation expansion in a competitive electricity market. The model incorporates operational considerations including plant capacity limitations and energy balance constraints. The solution algorithm described uses an iterative procedure to find the Cournot equilibrium. A Cournot equilibrium is reached when an investor cannot better their financial position by changing their investment decision, with respect to their competitors' investment decisions. The solution algorithm assumes that all investors make decisions simultaneously and provide this information to a central independent authority. The central authority then broadcasts this information to all competitors who adjust their own investment decisions based on the updated information. The assumed simultaneous timing of investment decisions and broadcasting of information is somewhat unrealistic and the solution algorithm does not appear to consider demand forecast variability and uncertainty. Kleindorfer et al. [26] have developed a model termed EPSIM (Electric Power Strategy Simulation Model) that can be used to study the strategic gaming interactions of market participants during real time operation and dispatch and ancillary services contracting. The focus of the work undertaken is the contractual and real time modelling of the England and Wales power pool. The model is proposed to be used by management to test strategic plans before implementation. The authors note that detailed analysis of investment strategies would require further detailed modelling of the power network. Moitre [27] studies the use of non cooperative game theory to analyse the economic behaviour of generating companies with respect to the real time power pool market. The model developed is used to investigate the market rules and their impacts on incentives and the following actions of generators. This work is based in real time and doesn't specifically investigate investment decisions. Murphy and Smeers [28] use game theory to study three different generation investment models. The first model assumes an environment similar to traditional expansion planning, the second model assumes an oligopolistic market environment where capacity is simultaneously built and sold in long-term contracts where there is no spot market (similar to power purchase agreements) and the third model assumes a spot market where investment and sales are separated. The results of the study show that prices and quantities produced in the third model (spot market) fall between the traditional planning model and the oligopolistic model. This is a result of the spot market mitigating market power with the resultant price being lower than that of the oligopolistic model. This work is an interesting approach to the study on generation investments but does not assess the effect of uncertainty on generation investment decisions and due to computational complexity uses an extremely simplified model. Finding a Cournot equilibrium, if one exists, becomes progressively more difficult for more detailed models. Smit and Ankum [21] use a combination of real options and game theory to study the interactions between competitive investors when uncertainty is considered. Previous work has shown that competition may force an investor to invest early and therefore the option value in deferring the investment is lost. Smit and Ankum consider the reduction in investment value for differing scenarios of market power of investors. They show that in an oligopoly it may be beneficial for all investors to defer investment if the project value is low and demand is uncertain. In contrast in a very competitive environment, firms are likely to invest immediately in order to preempt an investment by another party. This can result in a suboptimal solution.

Game theory has been used to study a range of competition issues in power systems. A number of the studies undertaken and discussed above offer promise in terms of identifying potential behaviour of rival investors but only Smit and Ankum consider uncertainty within the model. The use of game theory in power system planning would seem to be in identifying specific actions by investors at potential point in time but is less suited to a system wide planning perspective. One aspect of game theory that has not been discussed is the assumption that all players, or investors, act rationally. Rational actions in this context relates to an investment being made on a financial basis where an investment decision is based purely on profit. While profit and income is a driving factor for investment other factors are also likely to be considered such as work force requirements, company vision and the political environment. Where these externalities influence investment decisions an investor cannot be considered as acting rationally and the outcome predicted by game theory may not eventuate.

2.3.2 Transmission Planning

Transmission planning under regulation has been researched extensively and is well understood. In a very simplified form the traditional planning process can be considered to be three main steps; firstly a demand forecast is developed, secondly a least cost generation investment plan is found, the third stage is to optimise the transmission network based on the known future generation investments as found in the second stage of the planning process [3]. There are often a number of iterations between each stage of the process resulting in an overall optimal transmission and generation expansion plan. Each stage of planning requires in-depth technical and financial study and works with an assumed future as given by a previous planning stage. The objective of transmission investment in regulated systems is to maintain and improve system reliability and to expand the network to service new and existing generation.

The objectives of transmission planning have undergone large changes with the introduction of deregulation. Buygi et al. [29] describe the main objective of transmission planning in deregulated systems as providing a nondiscriminatory and competitive environment for all market participants, while maintaining system reliability. This is a complete change in planning objective from regulated systems where planning was undertaken to minimise cost and maintain system reliability [3]. The new planning objectives give rise to the question of whether it is the responsibility of the transmission owner to ensure sufficient transmission capacity, now and in the future to ensure a competitive market. If this is so, the transmission owner must try to foresee future generation investments in order to be able to provide equal access and encourage competition. Planning transmission before knowing how generation will develop is a reversal of the planning process under regulation and requires new planning and investment processes.

New transmission investments have the ability to affect other market participants in an unequal fashion [29] therefore requiring market impacts of investment to be acknowledged in the expansion planning process [30]. Assessing the market impacts that may arise from proposed investments is a complex problem. Much of the complexity arises from the system uncertainty, in particular, the uncertainty of generation investments [2] [1] [30] [31]. The removal of vertical integration between generation and transmission results in less coordination between generation and transmission investments [2]. The uncertainty of generation size, timing and location due to factors discussed previously in Section 2.3 combined with lack of investment coordination transfers the uncertainties of generation investment through to transmission investment planning.

The techniques and methods used for transmission planning must change to reflect the new objectives and greater levels of uncertainty within the system. Both Latorre et al. [1] and Buygi et al. [29] have undertaken comprehensive reviews of transmission planning techniques under deregulation. Latorre et al. characterise the solution methods of transmission planning into two categories. The first is mathematical optimisation models where the planning problem is posed as an optimisation problem. Most models presented in the literature are based on classical optimisation techniques such as linear programming, dynamic programming, mixed integer programming and stochastic programming. To obtain tractable solutions using optimisation techniques the problems modelled are usually extensively simplified. This requires the obtained solution be scrutinised carefully with regard to technical, financial, market and environment feasibility before implementation. The second category of planning techniques are heuristic models. Heuristic planning techniques use a selection of rules or sensitivities to locally search for a good feasible solution. The process is continued until no better solution can be found with respect to the rules or sensitivities. Heuristic models have economic solution times and find good feasible solutions but the solutions are not guaranteed to be optimal. A large number of heuristic models have been developed including game theory, simulated annealing, expert systems, fuzzy set theory and adaptive search procedures (see [1]). Buygi et al. describes stochastic approaches used in transmission planning and characterises them into five groups; probabilistic load flow, probabilistic reliability criteria, scenario analysis, decision analysis and fuzzy decision making. The common thread through each group is the uncertainty in one or more aspect of the modelled system e.g. system demand, outages, probability of scenario occurrence, investment cost uncertainties. Using a stochastic model allows the problem solver to investigate the risk associated with a particular outcome. Risk analysis of planning outcomes is an ongoing area of research particularly with regard to risk influencing investment decisions.

Despite the wealth of available literature and models regarding transmission planning for deregulated power systems, many of the models proposed are considered only in the context of static planning problems. That is, the planning problem is not interested in the timing of investments, only that they are available for a future definite situation [1]. The length of time required to plan, consent and build a new transmission investment may often be longer than that of planning, consenting and building new generation, particularly small scale wind or gas turbines [2]. The differences in timing between generation and transmission mean that it is vitally important that transmission investment plans be flexible to cope with uncertainty and consider unanticipated generation investments. These requirements necessitate the development of dynamic planning techniques for joint transmission and generation investments. Latorre et al. discuss the small number of dynamic planning models that have been developed including pseudo-dynamic optimisations that provide an optimal dynamic solution by solving a sequence of static subproblems [32] [33]. Many of the pseudo-dynamic models discussed in the literature were developed and applied before the deregulation of power systems. The model in [33] assumes a demand forecast and generation expansion plan are developed before the transmission planning is optimised. Repeated simulation results in a decision tree that the problem solver must use to decide the best investment plan. The model presented in [32] is very similar to that of [34] where a combined transmission and generation expansion plan is iteratively optimised using Bender's Decomposition. This model does not consider uncertainty of demand or uncertainty of investments. The solution obtained, while achieved in a dynamic and iterative manner, is static in that an optimal investment plan based on a deterministic demand forecast is produced. The solution does not present any flexibility to cope with unexpected demand growth over time.

2.3.3 Mitigating Investment Uncertainty

The vast majority of literature that investigates methods of mitigating investment uncertainty focuses on market and financial instruments. Market instruments include capacity payments and capacity markets. Financial instruments include hedge contracts and call options.

The spot pricing theory underpinning the operation of deregulated electricity markets has been shown by Caramanis et al. [35] [36] to provide efficient investment signals under the assumption of ideal market conditions. Vries and Hakvoort [37] assert that spot pricing theory is valid but question whether it applies in practice or if real market conditions deviate too much from the ideal. Much debate exists regarding this question and is illustrated in the wide range of market structures implemented around the world. Many European countries, New Zealand and Australia have energy only markets, that is, the only investment signals provided are from the spot market. These markets have no provision for ensuring adequacy of capacity and operate assuming that spot pricing theory is sufficient to facilitate investment. Some South American markets (Columbia, Argentina), Spain and the UK use capacity payments to stimulate capacity investment and provide additional revenue for capacity provision [38]. This approach brings a host of new difficulties to be overcome such as determining a suitable value of remuneration for capacity provision [38]. Another market implementation seen particularly in the north east of the USA are capacity markets where load serving entities are required to hold sufficient capacity credits to supply their load plus a reserve margin. Both the capacity payment and capacity market approaches presume that spot pricing is insufficient to provide adequate investment signals.

Ford [39] [40] has used the simulation technique known as system dynamics to show that business or investment cycles can exist in deregulated power systems. Investment cycles occur where after a period of no investment, prices rise as the capacity demand balance becomes strained. The increase in prices stimulates significant investment leading to an over supply of capacity within the system. The oversupply of capacity reduces the strain between supply and demand causing prices to fall, potentially below the initial high price before the investment occurred. The low prices deter investment for a period of time until demand has grown to a point where prices begin to rise and the investment cycle repeats. The price volatility experienced in investment cycles increases risk and uncertainty for investors leading to delayed investment decisions and reduced system reliability. Ford studied investment cycles for the Western United States using the Californian market rules and found that the introduction of a capacity payment can dampen the investment cycles seen otherwise. Bunn et al. [41] have also used system dynamics to study investment decisions for the UK and Wales market. Similarly to Ford they found the potential for investment cycles to exist but that the introduction of capacity payments and greater information sharing reduced the cyclic nature of investments.

Both Ford and Bunn et al. show that capacity payments theoretically have the ability to reduce price volatility and dampen investment cycles but a differing viewpoint is held by some, such as Oren [42], who dispute the use of capacity payments as they undermine the economic efficiency objectives of power system restructuring. Oren also argues that capacity payments can distort pricing signals and result in over investment. The alternative suggested is the use of financial instruments such as hedges that permit greater choices and promote demand side participation. Vazquez et al. [38] promotes the use of financial call options to produce reliability contracts that commit generators to being available when the system has scarcity of supply. Simply put, for a consumer a call option limits their financial exposure to the call option strike price. For generators, the call option limits their income to the strike price of the call option but can reduce their income risk by reducing price volatility. The reliability contract promoted by Vazquez et al. includes an additional penalty payment for the generator if they are required to generate but are unable to do so hence providing an incentive to build sufficient capacity.

Both capacity market instruments and financial tools have potential to improve investment incentives. The focus of these instruments and market changes is on improving income streams for generators by financially rewarding those who invest in additional capacity. These tools help mitigate investment uncertainty by reducing the price volatility and hence income uncertainty for investors. Capacity payments and financial instruments may make investors less hesitant about making investment decisions but they are not a complete solution to the investment planning problem as these instruments are not designed to provide information regarding size, type and timing of investments.

2.4 PLANNING UNDER DEREGULATION: ISSUES FOR FURTHER INVESTIGATION

The difficulties encountered by many countries with regard to investment planning in restructured power systems [43] illustrate the underdeveloped nature of investment planning tools and processes for the deregulated environment. Significant research has been undertaken regarding generation and transmission investment planning in restructured power systems but much of it has focused on single issues such as competitive effects, gaming, uncertainty in generation investment profit and development of new optimisation and simulation techniques.

One very important issue that is not well covered or understood is how the investment planning process can become more flexible in order to cope with the increased uncertainty of deregulated power systems. Real options theory investigates uncertainty and flexibility of the timing of investments but is focused on a specific investment opportunity at a particular point in time. Stochastic methods such as scenario analysis and monte carlo simulation have been applied to both the generation and transmission planning problems individually but few have investigated the interrelationships between uncertainty and generation and transmission investments. Latorre et al. [1] describe a number of areas that require further development including the flexibility and dynamics of the transmission planning process and the interrelationship between transmission planning and generation investment. A number of other researchers have noted the challenges of transmission planning in restructured power systems. David and Wen [2] conclude that transmission planning is a key issue for deregulated markets and that plans should be flexible and robust to cope with different future generation expansion and load growth scenarios. Xu et al. [30] state that any proposed technique for transmission planning should be able to handle the uncertainties and risks associated with deregulation. Wu et al. [3] discuss the intricate issues of transmission planning in deregulated systems but conclude that further research is required to improve methodology for transmission planning.

Numerous researchers discuss the need for additional study of transmission expansion and investment planning with particular focus on flexibility of planning processes regarding uncertainties in generation investment and load growth. Previous research has covered some aspects of these issues but further development of planning processes and tools are required with particular emphasis on flexibility, uncertainty and interactions between generation and transmission.

2.5 RESEARCH MOTIVATIONS AND AIMS

The importance of the continued development of investment planning tools and processes can be illustrated by looking at the reverse situation. Where planning is inadequate, tools underdeveloped and planning processes single faceted (considering only generation or only transmission) the uncertainty regarding future capacity and system reliability increases. This leads to long term price volatility and supply uncertainty. These uncertainties can lead to investment delays and unwillingness by investors to commit to timely investment decisions that in turn increases the uncertainty in price and supply reliability for consumers.

The self perpetuating cycle of uncertainty within power system planning delays investment and results in systems being more heavily utilised as reserve margins shrink and capacity is stretched. Utilising the system at maximum levels may achieve greater economic efficiency but at the expense of system reliability. Without appropriate available spare capacity within a network, the effect of equipment failure may be far more serious than would otherwise be expected. Improvement and development of flexible investment planning processes will allow planners and investors to better understand the impacts of uncertainty on their investment decisions. Better understanding of investment uncertainty, competitive investment interactions and the associated risks will reduce uncertainty in the investment planning process leading to a more stable investment environment.

The aim of the research undertaken and presented in this thesis is to develop a planning tool that aids in the complex investment planning process. The work does not question the validity of the market models implemented around the world nor the social outcomes of profit objectives in a deregulated environment but aims to provide a new optimisation tool that focuses on uncertainty within the investment planning process. The model presented does not quantify uncertainty but provides a flexible investment planning tool and process as the uncertainty within the system is revealed over time.

Chapter 3

PLANNING OVER TIME

3.1 STATIC VS. DYNAMIC PLANNING

Planning problems can separated into two broad categories, static or dynamic. Static planning seeks an optimal system state for a single time period in the future. The problem isn't to decide when investments are made but to decide which investments are necessary to deliver an optimal system at the future time period. Dynamic planning is used when multiple time periods are considered and an optimal investment sequence is needed for the whole planning duration. These problems must consider not only size and type of investment but also the timing of the investment.

Investment planning in power systems can be either static or dynamic but the increasing complexity of decision making means that dynamic planning will provide a more flexible and therefore useful investment strategy. Deregulation of the electricity industry has dramatically increased the number of factors that must be considered when planning and making investment decisions. These factors include competitor investments, environmental restrictions, profit and regulatory controls alongside the more traditional considerations of cost, location, timing and size. The landscape in which planning decisions are made can change rapidly and investments (or non-investments) can have direct effects on price, competitor investment decisions and consumer demand response. To successfully optimise an investment plan that incorporates these decision factors and studies their interactions, the planning model must be dynamic.

3.2 DETERMINISTIC DYNAMIC PROGRAMMING

Dynamic planning problems can be represented as a series of sequential decisions over time. As each decision is made, the state of the system changes. A static optimisation method aims to find the optimal set of investments for a fixed future system state whereas a dynamic optimisation method aims to find the optimal sequence of investments for any system state. The latter is the type of optimisation technique that provides the greatest insight into power system planning.

Dynamic programming is a dynamic optimisation method that finds an optimal policy for decision making over the time period of interest. The term *policy* refers to a rule for making a decision that leads to an allowable sequence of decisions. An *optimal policy* is a a rule for making a decision that leads to an allowable sequence of decisions where those decisions optimise a predefined function of the state variables [44]. Dynamic programming produces an optimal policy for each time period of the planning problem by utilising the *Principle of Optimality* [44].

> *"An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision".*

The Principle of Optimality is a recursive definition that gives the idea that an optimal path is composed of optimal sub-paths. Once a first decision is made and the system state changes, the following decisions must also be optimal, with respect to the new system state, if the whole path is to be considered optimal. An optimal policy gives a set of decision making rules that describe how to make an optimal decision given any particular system state. This means that decisions prior to the current one are not relevant, only the resulting system state after the previous decision.

The work of Richard Bellman [44] and Bellman and Dreyfus [45] pioneered the use of the Principle of Optimality for a large number of applications of optimisation including transportation, inventory, control theory, optimal trajectories and multistage production industries. Through this work the Principle of Optimality became formalised into the Bellman Optimality Equation as shown in Equation 3.1.

$$f_t(\mathbf{x}_t) = min\left[d(\mathbf{x}_t, \mathbf{x}_{t+1}) + f_{t+1}(\mathbf{x}_{t+1})\right] \tag{3.1}$$

Where:

- t = Represents the time period within the optimisation.
- $\mathbf{x}_t$ = Vector of state variable(s) at the end of the stage $t-1$. This is the same vector of state variable(s) at the ***beginning*** of stage t.
- $\mathbf{x}_{t+1}$ = Vector of state variable(s) ***after*** optimal decision is made in stage t.
- $f_t(\mathbf{x}_t)$ = Optimal solution cost for this time period plus future time periods.
- $d(\mathbf{x}_t, \mathbf{x}_{t+1})$ = Cost of decision made in this time period.
- $f_{t+1}(\mathbf{x}_{t+1})$ = Optimal solution cost for all future time periods. This is often termed the 'Future Cost Function'.

In terms of the Principle of Optimality, the initial state variable vector is $\mathbf{x}_t$, the initial decision is made by solving the Bellman Optimality Equation and results in the future system state being $\mathbf{x}_{t+1}$. The remaining decisions that make up an optimal policy (optimal sequence of decisions) have a cost represented by $f_{t+1}(\mathbf{x}_{t+1})$ which is dependent on the initial decision and hence the new system state $\mathbf{x}_{t+1}$. It can be seen from this mathematical representation that the crux of dynamic programming is the tradeoff between the costs in the current time period and the future costs that are a consequence of the current decision.

The Bellman Optimality Equation illustrates how dynamic programming is solved using recursive calculations starting at the final time period of the planning problem. Expanding out the equation shows how the recursive calculations combine to produce an overall optimal policy of decisions for the entire time period of the planning problem.

For a three year planning problem the Bellman Optimality Equation can be expanded to look like Equation 3.2.

$$f_1(\mathbf{x}_1) = min\big[d(\mathbf{x}_1, \mathbf{x}_2) + f_2(\mathbf{x}_2)\big] \tag{3.2}$$

Where:

$$f_2(\mathbf{x}_2) = min\big[d(\mathbf{x}_2, \mathbf{x}_3) + f_3(\mathbf{x}_3)\big]$$

Where:

$$f_3(\mathbf{x}_3) = min\big[d(\mathbf{x}_3, \mathbf{x}_4) + endVal\big]$$

Where:

- $endVal$ = The end of time period salvage value. This may be the current value of the asset dependent on its age, its resale value at this time or some other measure. It may also be zero.

Equation (3.2) is equivalent to Equation (3.3).

$$f_1(\mathbf{x}_1) = min\big[d(\mathbf{x}_1, \mathbf{x}_2) + d(\mathbf{x}_2, \mathbf{x}_3) + d(\mathbf{x}_3, \mathbf{x}_4) + endVal\big] \tag{3.3}$$

If the planning problem was modelled mathematically as Equation 3.3 it would be extremely difficult to solve due to the large number of required computations. It would require complete enumeration of every possible combination of decisions for all stages of the problem, something that may be impossible to undertake for real world sized problems. Dynamic programming offers a solution to this difficulty by solving the optimisation in stages, where each stage is a (more) simple problem to solve. By utilising the previously made recursive calculations the optimal policy of decisions can be found with fewer computations than by complete enumeration.

3.2.1 Subproblem Solution Techniques

The description of dynamic programming given in Section 3.2 is very general and gives no indication of the method used to solve each subproblem. Dynamic programming describes how the Principle of Optimality can be used to solve dynamic optimisation problems but it does not stipulate how to optimise each subproblem. This decision rests with the problem solver who must assess the optimisation problem and identify a suitable optimisation technique to solve it.

Dynamic programming can solve both linear and non linear problems as long as the optimisation technique used is suitable for the problem. Linear programs could use the simplex method, branch and bound or an interior point solution. Non linear problems can use Steepest Descent Algorithms, Simulated Annealing, Tabu Search, and Genetic Algorithms to name only a few [46]. The types of optimisation techniques applicable to dynamic programming are vast requiring the problem solver to have a good understanding of their optimisation problem before choosing a suitable technique.

3.2.2 Terminology

Dynamic programming has a range of terminology that is used to describe the optimisation technique and recursion calculation process. A number of these terms are explained below.

3.2.2.1 Stages

Dynamic programming optimises a problem over time where each time segment is described as a *stage* of the optimisation. At each stage a subproblem is solved that is used in the optimal solution to the overall problem.

3.2.2.2 State Variables

State variables completely describe the state of the system they represent. The only requirements that must be met in choosing the state variables is that they must fully describe the state of the system at the current stage without requiring any knowledge of historical system behaviour. An example of this in the power system planning problem is using total system capacity as a state variable. It is not important to the optimisation how the system arrived in the current state, e.g. what the previous investments were, it is only necessary to know the overall system capacity in order to decide on future investments.

3.2.2.3 State Equation

The state equation(s) (sometimes called the transition equation) describe, for each state variable, how the state variable changes over time based on the outcome of the subproblem optimisation.

The equation relates the current value of the state variable to the future value of the state variable by means of an optimal decision that involves the state variable. Equation 3.4 illustrates a general state equation.

$$\mathbf{x}_{t+1} = \mathbf{x}_t + \mathbf{x}_{dec} \tag{3.4}$$

Where:

- $\mathbf{x}_{t+1}$ = Vector of state variables at the end of stage t.
- $\mathbf{x}_t$ = Vector of state variables at the beginning of stage t.
- $\mathbf{x}_{dec}$ = Vector of decisions made by the optimisation.

3.2.2.4 Optimal Policy

The optimal policy for a dynamic programming problem is a set of functions that represent the future costs of the optimisation problem for a range of state variable values at each stage of the problem. This concept is described more fully in Section 3.2.3.

3.2.2.5 Subproblem

A subproblem of the planning problem is an optimal policy of investment decisions from stage t until the end of the planning period.

3.2.3 Subproblem Optimisation

As outlined in section 3.2 above, dynamic programming is an algorithm solved using recursive calculations. The Optimality Equation for the final time period is Equation 3.5.

$$f_T(\mathbf{x}_T) = min\big[d(\mathbf{x}_T, \mathbf{x}_{T+1}) + endVal\big] \tag{3.5}$$

- T = Final stage of the optimisation problem.
- $\mathbf{x}_{T+1}$ = Vector of state variable(s) ***after*** optimal decision(s) are made in stage T.
- $\mathbf{x}_T$ = Vector of state variable(s) at the end of the stage $T-1$. This is the same vector of state variables at the ***beginning*** of stage T.
- $endVal$ = The end of time period salvage value.

In order to solve this final stage equation it is necessary to know the value of $\mathbf{x}_T$. In a backwards recursion the final time period equation is the first to be solved and hence the value of $\mathbf{x}_T$ is *not* known so must be 'guessed' in some way. The method used is to discretise the state space of $\mathbf{x}_T$. Equation 3.5 is then solved for each discrete value of $\mathbf{x}_T$. The optimal solution from each discrete state variable subproblem is represented by a point on a function. This function represents the optimal subproblem solution cost across a range of state variable values and is stored for future use. Figure 3.1 illustrates the points that represent an example function $f_T(\mathbf{x}_T)$.

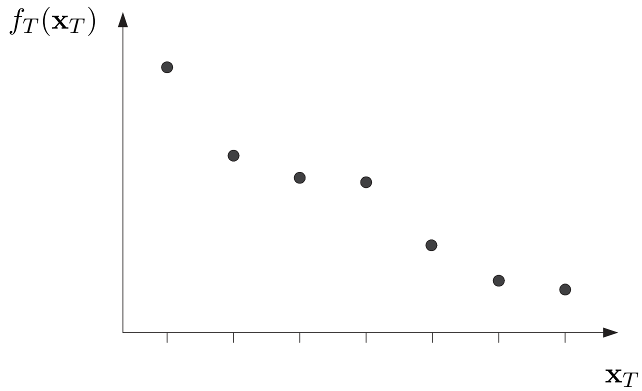


Figure 3.1 Function $f_T(\mathbf{x}_T)$

Now that $f_T(\mathbf{x}_T)$ has been calculated, the algorithm moves to solving the $f_{T-1}(\mathbf{x}_{T-1})$ subproblem as in Equation 3.6.

$$f_{T-1}(\mathbf{x}_{T-1}) = min\big[d(\mathbf{x}_{T-1}, \mathbf{x}_T) + f_T(\mathbf{x}_T)\big] \tag{3.6}$$

Where:

- $f_T(\mathbf{x}_T)$ = The function calculated in subproblem optimisation for stage T.
- $\mathbf{x}_T$ = Vector of state variable(s) ***after*** optimal decision(s) are made in stage $T-1$.
- $\mathbf{x}_{T-1}$ = Vector of state variable(s) at the end of the stage $T-2$. This is the same vector of state variable at the ***beginning*** of stage $T-1$.

To solve Equation 3.6 the state variable $\mathbf{x}_{T-1}$ must be discretised in a similar fashion to $\mathbf{x}_T$ in Equation 3.5. When the $T-1$ subproblem is solved, there is no guarantee that the value of $\mathbf{x}_T$ that is found through the state equation will match with the discretisations of $\mathbf{x}_T$ used in Equation 3.5. This means the value of $f_T(\mathbf{x}_T)$ needed for solving Equation 3.6 may not have been precalculated from the last recursion. In this instance the value for $f_T(\mathbf{x}_T)$ must be estimated in some way from the data that is available. When every subproblem at stage $T-1$ has been solved, one for each discretisation of the state variable $\mathbf{x}_{T-1}$, the function $f_{T-1}(\mathbf{x}_{T-1})$ is found.

Dynamic programming continues in this recursive fashion until first stage of the optimisation is reached. At the beginning of the optimisation problem the value of the state variable should be known by the problem solver as it represents the state of the system at a known point in time. Equation 3.7 represents the optimal subproblem at stage 1.

$$f_1(\mathbf{x}_1) = min\big[d(\mathbf{x}_1, \mathbf{x}_2) + f_2(\mathbf{x}_2)\big] \tag{3.7}$$

Where:

- $f_2(\mathbf{x}_2)$ = The function calculated in the in subproblem optimisation for stage 2.
- $\mathbf{x}_1$ = Value of state variable(s)at beginning of the optimisation problem. Their value represents the state of the system at the beginning of the dynamic planning problem.
- $\mathbf{x}_2$ = Value of state variable(s) ***after*** optimal decision(s) are made in stage 1.

The value of $\mathbf{x}_1$ is known and so does not need to be discretised for this subproblem. The subproblem optimal solution then represents the optimal decision for stage 1.

Number of discretisations of State Variables: The number of discrete values of the state variables is controlled by the problem solver who must decide the range and number of discrete state variable values. This decision is governed by the need to choose state variable values that are most likely to occur at the relevant stage in the problem. The most likely values are not always obvious and it is easy to see that for real world, large scale problems the number of discretisations required can be extremely large.

Multiple State Variables: If the dynamic programming problem was defined with multiple state variables, every time the state variables are required to be discretised, all possible combinations of discrete state variables must be solved as subproblems. For example, two state variables results in $var_1 \times var_2$ subproblem calculations at each stage. var_1 and var_2 represent the number of discretisations for each state variable respectively.

For real world problems that may have large numbers of state variables the exponential increase in the number of subproblem calculations can make the problem intractable. In situations where the problem is not easily computed, assumptions and simplifications are often used to make the problem solvable but this is done at the expense of model detail.

3.2.4 Dynamic Programming Numerical Example

The following is a simplistic power system planning problem that illustrates the dynamic programming technique.

The problem: A power system currently has generating capacity equal to 800MW and peak demand of 750MW. Demand has been forecast to grow at 60MW per year for at least the next three years. All new generation investments are costed at \$150/MW. Fixed costs for all installed capacity is \$3/MW per year and variable costs for generation are \$2/MWh. The

Stage	Demand Value (MW)
1	750
2	810
3	870
4	930

Table 3.1 System Demand over Planning Period

planning problem is to minimise the cost of investment and operation of the power system for the next three years while ensuring capacity is always greater than demand.

Dynamic Programming Problem: The state variable is chosen to be system capacity as this value completely describes the state of the system that is of interest to the optimisation problem. The subproblem to be optimised at each stage is shown in Equation 3.8

$$
\begin{aligned}
f_t(\mathbf{x}_t) =& min\Big[(150\mathbf{x}_{inv} + 3\mathbf{x}_{t+1} + 17520 d_{t+1}) + f_{t+1}(\mathbf{x}_{t+1})\Big] \\
\text{Subject to:}& \\
&\mathbf{x}_t + \mathbf{x}_{inv} = \mathbf{x}_{t+1} \\
&cap_{t+1} \geq d_{t+1}
\end{aligned}
\tag{3.8}
$$

Where:

- $\mathbf{x}_t$ = Vector of state variable(s) at beginning of stage t
- $\mathbf{x}_{t+1}$ = Vector of state variable(s) ***after*** optimal decision(s) are made in stage t.
- $f_{t+1}(\mathbf{x}_{t+1})$ = The future cost function calculated in the subproblem optimisation for stage $t+1$.
- $\mathbf{x}_{inv}$ = Represents the decision variable(in MW) that is optimised in this stage.
- 150 = Cost per MW for new generation investments.
- d_{t+1} = Forecast demand at beginning of stage $t+1$.
- 3 = Fixed cost of capacity = \$3/MW per year
- 17520 = Variable cost of generation (hours in year x \$2/MWh)
- cap_{t+1} = System capacity at the beginning of stage $t+1$

Demand: As the demand forecast is known, the value of demand at each stage can be calculated in advance. There are only 3 stages in the problem but the demand value is also calculated at a fictitious stage 4 as it is required for the final stage calculation.

$\mathbf{x}_3$	**Demand at $t = 4$**	**Optimal Investment Size (MW)**	**Optimal Cost(minimum)($)**
850	930	80	16,308,390
860	930	70	16,306,890
870	930	60	16,305,390
880	930	50	16,303,890
890	930	40	16,302,390
900	930	30	16,300,890
910	930	20	16,299,390
920	930	10	16,297,890
930	930	0	16,296,390
940	930	0	16,296,420
950	930	0	16,296,450

Table 3.2 Stage 3 Subproblem Calculations

Stage 3 - Final Time Period: The subproblem to be optimised at the final time period is shown in Equation 3.9

$$
\begin{aligned}
f_3(\mathbf{x}_3) =& min\Big[(150\mathbf{x}_{inv} + 3\mathbf{x}_4 + 17520 d_4) + endVal\Big] \\
\text{Subject to:}& \\
& \mathbf{x}_3 + \mathbf{x}_{inv} = \mathbf{x}_4 \\
& cap_4 \geq d_4
\end{aligned}
\tag{3.9}
$$

It is assumed that no investments are made past the end of the planning period. This results in $endVal = 0$ as there is no decision costs past Stage 3. This is of course untrue but is a useful assumption in this example for the sake of simplicity. In order to solve the subproblem the state variable, $\mathbf{x}_3$, has to be discretised. The choice made here is to discretise it in 10MW blocks between 850MW and 950MW. The subproblem is solved using a linear programming optimisation technique with the results shown in Table 3.2.

The function $f_3(\mathbf{x}_3)$ is shown in Figure 3.2

Stage 2: Equation 3.10 shows the subproblem to be optimised in stage 2.

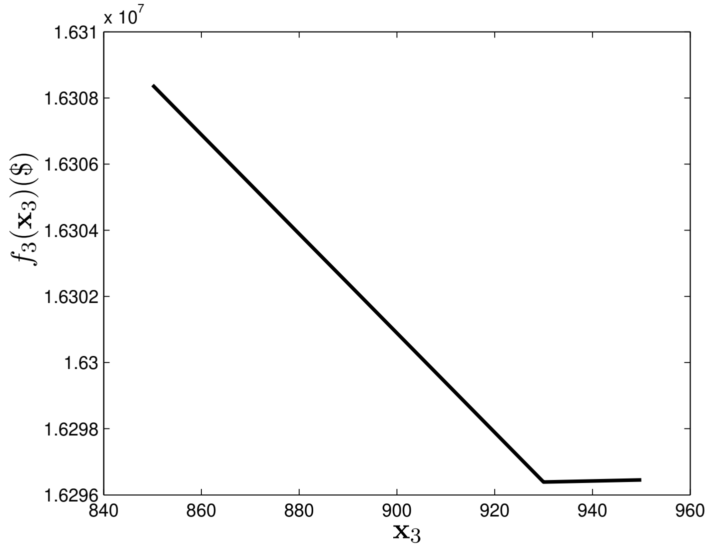


Figure 3.2 Function $f_3(\mathbf{x}_3)$

$$f_2(\mathbf{x}_2) = min\Big[(150\mathbf{x}_{inv} + 3\mathbf{x}_3 + 17520d_3) + f_3(\mathbf{x}_3)\Big]$$
Subject to: (3.10)
$$\mathbf{x}_2 + \mathbf{x}_{inv} = \mathbf{x}_3$$
$$cap_3 \geq d_3$$

The difference in subproblem between stage 3 and stage 2 is the inclusion in stage 2 of the previously calculated function $f_3(\mathbf{x}_3)$. The value this function takes in the subproblem is dependent on the value of $\mathbf{x}_3$. $\mathbf{x}_3$ is related to the value of $\mathbf{x}_2$ through the state equation and once again the value of $\mathbf{x}_2$ is unknown and needs to be discretised. The choice of discretisation is in 10MW blocks between 800MW and 900MW. The calculations of the subproblem are shown in Table 3.3.

The function $f_2(\mathbf{x}_2)$ is shown in Figure 3.3

Stage 1: Equation 3.11 shows the subproblem to be optimised in stage 2.

$$f_1(\mathbf{x}_1) = min\Big[((150\mathbf{x}_{inv} + 3\mathbf{x}_2 + 17520d_2) + f_2(\mathbf{x}_2)\Big]$$
Subject to: (3.11)
$$\mathbf{x}_1 + \mathbf{x}_{inv} = \mathbf{x}_2$$
$$cap_2 \geq d_2$$

$\mathbf{x}_2$	**Demand at $t = 3$**	**Optimal Investment Size (MW)**	**Optimal Cost(minimum) ($)**
800	870	70	31,560,900
810	870	60	31,559,400
820	870	50	31,557,900
830	870	40	31,556,400
840	870	30	31,554,900
850	870	20	31,553,400
860	870	10	31,551,900
870	870	0	31,550,400
880	870	0	31,548,930
890	870	0	31,547,460
900	870	0	31,545,990

Table 3.3 Stage 2 Subproblem Calculations

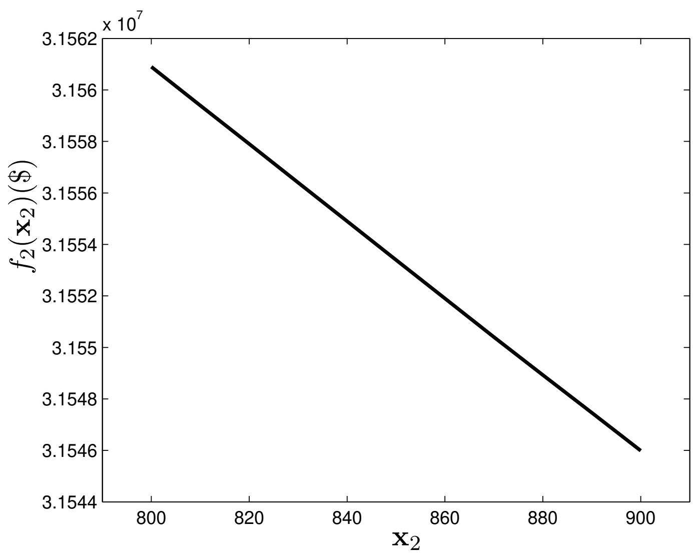


Figure 3.3 Function $f_2(\mathbf{x}_2)$

In this stage the state variable $\mathbf{x}_1$ doesn't need to be discretised as the value is already known as 800MW. The stage 1 subproblem need only be solved once. The optimal solution to stage 1 is shown in Table 3.4

The optimal solution to subproblem 1 is a single value that gives the minimum cost of investment

$\mathbf{x}_1$	**Demand at $t = 2$**	**Optimal Investment Size (MW)**	**Optimal Cost(minimum) ($)**
800	810	10	45,754,530

Table 3.4 Stage 1 Subproblem Calculations

Stage (t)	$\mathbf{x}_t$	**Investment (MW)**	$\mathbf{x}_{t+1}$	**Optimal Cost**(minimum) ($)
1	800	10	810	45,754,530
2	810	60	870	31,559,400
3	870	60	930	16,305,390

Table 3.5 Optimal Investment Path

over the three year period while satisfying the constraint that capacity must always be greater than or equal to demand.

Optimal Policy: The optimal policy for this problem is given by the set of functions calculated during the recursive calculations. The function $f_1(\mathbf{x}_1)$ is only a single point as the initial state $\mathbf{x}_1$ is known as doesn't need to be discretised. The remainder of the policy is given by the functions $f_2(\mathbf{x}_2)$ and$f_3(\mathbf{x}_3)$ as seen in Figure 3.3 and Figure 3.2. These function give the optimal cost of future decisions at a range of state variable values.

Optimal Investment Sequence: Once the recursive calculations have been completed the functions from each stage $(f_1(\mathbf{x}_1), f_2(\mathbf{x}_2), f_3(\mathbf{x}_3))$ collectively define the optimal policy of decisions for the dynamic programming problem. The optimal policy is used to identify the optimal sequence of investment sizes during the time period of the planning problem for any given value of the state variables.

The optimal investment decision from the stage 1 subproblem gives the first optimal investment. From this decision the value of $\mathbf{x}_2$ is found. Solving the stage 2 subproblem for the value of $\mathbf{x}_2$ found in stage 1 gives the second optimal investment. The value of $\mathbf{x}_3$ is now known and the stage 3 subproblem can be solved for this $\mathbf{x}_3$ value. The solution to the stage 3 subproblem is the final optimal investment for the planning problem. The optimal investment path for the example above is given in Table 3.5.

It should be noted that due to the simplicity of this example that the optimal decision in each stage resulted in a state variable value $\mathbf{x}_{t+1}$ that matched a previously discretised value of $\mathbf{x}_{t+1}$. This is not the norm in optimisation problems. If in stage 3, $\mathbf{x}_3$ had been discretised differently the known points on the function $f_3(\mathbf{x}_3)$ would be at different $\mathbf{x}_3$ values. For example, if $\mathbf{x}_3$ was instead discretised at $855, 865, 875$ etc. then solving the stage 2 problem is more difficult. Stage 2 would still have an optimal decision of 60MW and $\mathbf{x}_3 = 870$ but the optimal solution cost of future decisions $f_3(\mathbf{x}_3)$ is not known from previous recursive calculations. In this situation the value of $f_3(\mathbf{x}_3 = 870)$ is read off the known function $f_3(\mathbf{x}_3)$ using an interpolation technique.

3.3 STOCHASTIC DYNAMIC PROGRAMMING

In the example problem in Section 3.2.4 the demand forecast was considered to be 100% correct with no room for error. Having a 100% accurate forecast of any variable is an unrealistic expectation when modelling real world situations, especially if the planning period is long. The further into the future a forecast is, the more likely it is to be incorrect. When considering variable uncertainty it is desirable to be able to incorporate the uncertainty into the planning problem and hence the optimisation model. Stochastic dynamic programming is an extension to dynamic programming that models uncertainty in model variables so long as the uncertainty in the variable is known or able to be estimated in some way.

Stochastic dynamic programming is a very similar process to dynamic programming with the subproblem being solved at each stage for every discretisation of the state variables. The difference in stochastic dynamic programming is the way in which the subproblem is solved at each discrete state value. The solution of each subproblem is an *expected value.*. The optimal subproblem solutions collectively describe the future cost function for the stage. The set of future cost functions across the planning period describe the optimal policy of decisions for the planning problem.

The introduction of a stochastic variable results in an altered Bellman Optimality Equation for each subproblem as shown in Equation 3.12. The subproblem solution at stage t is now the *expected value* of the immediate decision costs plus the future costs that are a consequence of the immediate decision.

$$f_t(\mathbf{x}_t) = min \ \mathrm{E}_{y_t}\left[d(\mathbf{x}_t, \mathbf{x}_{t+1}) + f_{t+1}(\mathbf{x}_{t+1})\right] \tag{3.12}$$

Where:

- E_{y_t} = Expectation over stochastic variable y at stage t.
- y_t = Stochastic variable.
- $\mathbf{x}_t$ = Vector of state variable(s) at the end of the stage $t-1$. This is the same vector of state variable at the ***beginning*** of stage t.
- $\mathbf{x}_{t+1}$ = Vector of state variable(s) ***after*** optimal decision(s) are made in stage t.
- $f_t(\mathbf{x}_t)$ = Expected optimal solution for this subproblem.
- $f_{t+1}(\mathbf{x}_{t+1})$ = Expected value of the future cost function for the state variable vector $\mathbf{x}_{t+1}$.
- $d(\mathbf{x}_t, \mathbf{x}_{t+1})$ = Cost of optimal decision at stage t.

The expected value of the subproblem is calculated by solving the subproblem at a number of realisations of the stochastic variable and calculating the expected value of all the solutions. Equation 3.13 shows how the subproblem is calculated.

$$\begin{aligned} f_t(\mathbf{x}_t) = p_{y_{t_1}}\big(min[d(\mathbf{x}_t, \mathbf{x}_{t+1}) + f_{t+1}(\mathbf{x}_{t+1})]\big) + \\ p_{y_{t_2}}\big(min[d(\mathbf{x}_t, \mathbf{x}_{t+1}) + f_{t+1}(\mathbf{x}_{t+1})] + \ldots + \\ p_{y_{t_m}}\big(min[d(\mathbf{x}_t, \mathbf{x}_{t+1}) + f_{t+1}(\mathbf{x}_{t+1})]\big) \end{aligned} \tag{3.13}$$

Where:

- m = Number of realisations of the stochastic variable.
- $p_{y_{t_1}}$ = Probability of stochastic variable at stage t taking the value y_{t_1}.
- $p_{y_{t_m}}$ = Probability of stochastic variable at stage t taking the value y_{t_m}.

Once the expected value of the subproblem is calculated at each discretisation of the stochastic variable($\mathbf{x}_t$) it becomes a known point on the expected future cost function. The recursive calculation process continues backwards in time until the first stage problem is solved.

For a deterministic dynamic programming problem, the optimal subproblem solutions can be used to trace a path across the planning period that defines the optimal sequence of decisions. The same cannot be done in stochastic dynamic programming as the optimal decision is dependant on the realisation of the stochastic variable. The information gained from a stochastic dynamic programming problem is the *expected* cost of decisions across the planning period based on the *expected* value of the stochastic variable and a policy of optimal decisions. To understand more about the sequence of optimal decisions a Monte Carlo simulation must be run using the optimal policy (set of *expected* future cost functions) for the planning problem and the stochastic variable probability distribution.

3.3.1 Monte Carlo Simulation

Monte Carlo simulation uses the optimal policy, or set of future cost functions, for the problem to simulate the sequential planning decisions moving forwards in time. The simulation samples a value of the stochastic variable at each stage and uses the *expected* future cost function for the current stage to find the optimal decision. Each subproblem is solved for *one* realisation of the stochastic variable, turning the subproblem into a deterministic optimisation. This is repeated for all stages $t = 1, \ldots, T$. The total cost of investment decisions is the summation of the immediate decision costs from each stage. This optimal cost will be unique to the particular simulation as the simulation has a unique sampling of the stochastic variable.

The Monte Carlo simulation is repeated a number of times with the total investment cost from each simulation being stored. Upon completion of the simulation the *expected* total cost and uncertainty of this value can be calculated using the total cost of investment decisions from each simulation. Both the *expected* total cost and the uncertainty, in the form of the standard deviation, give the problem solver additional information about the range and probability of the expected costs over the planning period. This information helps illustrate the uncertainty and variability and hence risk in decision making that is introduced by the stochastic variable.

3.3.2 Stochastic Variable Description

The stochastic variable may be described as either a discrete or continuous probability distribution. With either description the problem solver must chose how many realisations of the stochastic variable will be used in calculating the expected value of the subproblem. This is a tradeoff between accuracy of the expected value and the number of calculations performed.

Regardless of the type of probability distribution the problem solver must decide how many realisations of the stochastic variable are required and how the realisations are obtained e.g. is the distribution sampled in some way so a random selection of stochastic variable values are used or are the variable values (and probabilities) chosen prior to calculation in order to represent a good range across the spectrum of possible values. If the stochasticity is described by a continuous distribution the sampled values can take any value within the distribution whereas a discrete distribution is restricted to those discrete values within the distribution.

For the Stochastic Optimality Equation (Equation 3.12) and the stochastic variable sampling procedure as described previously to be valid, the stochastic variable must be time independent. That is, the value of the stochastic variable in the current time period is not dependent on any previous value of the stochastic variable and it doesn't influence any future values of the stochastic variable. If this were not the case the stochastic variable is considered to be serially correlated and has to be included as a state variable. The serial correlation between time periods can be described as a Markov Chain. The specification of the Markov chain is supplied by the problem solver, similar to the problem solver specifying the distribution of an independent stochastic variable.

A Markov chain represents the probability p_{ij} of a random variable in state i changing to state j. An example from the power system planning problem would be that if demand growth in stage t is 50MW then the probability of demand growth being 30MW in stage $t + 1$ is 0.25. The random variable has changed from state i = 50MW to state j = 30MW so $p_{ij} = 0.25$. To represent this dependence on time the stochastic variable, in this example demand growth, must be included as a state variable. The subproblem calculation no longer uses sampled values of the stochastic variable to calculate the expected value of the subproblem but uses the states and probabilities described by the Markov chain. Equation 3.14 gives an example subproblem calculation for a serially correlated stochastic variable with the Markov chain shown in Table 3.6. The table shows the probability, $p_{t,t+1_x}$, of the stochastic variable starting in state y_t, moving to state y_{t+1_x}.

$y_t \downarrow y_{t+1} \rightarrow$	y_{t+1_1}	y_{t+1_2}	y_{t+1_3}
y_t	$p_{t,t+1_1}$	$p_{t,t+1_2}$	$p_{t,t+1_3}$

Table 3.6 Markov Chain for Stochastic Variable

$$\begin{aligned} f_t(\mathbf{x}_t, y_t) = p_{t,t+1_1}&\big[min\big(d(\mathbf{x}_t, \mathbf{x}_{t+1}, y_t, y_{t+1_1}) + (f_{t+1}(\mathbf{x}_{t+1}, y_{t+1_1}))\big)\big]+ \\ &p_{t,t+1_2}\big[min\big(d(\mathbf{x}_t, \mathbf{x}_{t+1}, y_t, y_{t+1_2}) + (f_{t+1}(\mathbf{x}_{t+1}, y_{t+1_2}))\big)\big]+ \\ &p_{t,t+1_3}\big[min\big(d(\mathbf{x}_t, \mathbf{x}_{t+1}, y_{t+1_3}) + (f_{t+1}(\mathbf{x}_{t+1}, y_{t+1_3}))\big)\big] \end{aligned} \tag{3.14}$$

Where:

- $p_{t,t+1_1}$ = Probability of state variable y_t moving to state y_{t+1_1}.
- y_t = Current value of the stochastic state variable.
- y_{t+1_x} = Stochastic state variable at stage $t+1$ with the value x. This value is obtained from the Markov chain definition, not from the state equation of the subproblem.
- $f_{t+1}(\mathbf{x}_{t+1}, y_{t+1_1})$ = Value of the future cost function for the state variables $\mathbf{x}_{t+1}$ and y_{t+1_1}.
- $\mathbf{x}_{t+1}$ = Vector of state variables found through solving the state equation of the subproblem optimisation.

The noticeable differences between the time independent stochastic dynamic programming calculation in Equation 3.13 and the serially correlated stochastic dynamic programming calculation in Equation 3.14 are the inclusion of the stochastic variable as a state variable and the way in which the probability values for the stochastic variable are obtained. The serially correlated calculation uses the Markov chain probabilities to calculate the expected value of the subproblem when moving from the state $[\mathbf{x}_t, y_t]$ to the range of states that are 'preconditioned' by the Markov chain state transitions, e.g. $[\mathbf{x}_{t+1}, y_{t+1_x}]$.

When the recursive calculations are undertaken, the added stochastic state variable introduces an additional set of state variable discretisations. The discretisations are not selected by the problem solver as the Markov chain specifies the values the stochastic variable can take.

3.4 DISCOUNTING

Optimisation problems are a balancing act between the outcomes of decisions made in a system. To compare the outcomes of potential decisions against each other in a fair manner, the comparison must be done between like outcomes. That is, all decision outcomes must be represented in same measurement unit system. A common measurement unit used is in monetary terms e.g. costs or profits. In planning problems the time frame of the problem is often large and measured in years. This results in the necessity to account for the time value of money within the optimisation. Optimisations account for this by introducing a discount factor. The discount rate is a number by which a future cost or profit is multiplied, to obtain its value in present dollar terms [47]. This discounting of the future costs or profits allows all the decision outcomes to be compared in present time monetary terms while still accounting for the time value of money. The present value of money is referred to as the 'Net Present Value' or NPV and is calculated by Equation 3.15

$$NPV = \frac{FC}{(1+r)^n} \tag{3.15}$$

Where:

- NPV = Net present value of cost or profit.
- FC = Future cost or profit.
- r = Discount rate. This is an annual percentage value. e.g. for a rate of 12%, $r = 0.12$.
- n = Number of years away from the present time the cost or profit occurs.

Incorporating the NPV calculation in dynamic programming is simple due to the structure of the Bellman Optimality Equation which optimises a subproblem in terms of the cost of the immediate decision plus the cost of future decisions that are a consequence of the immediate decision. The cost of the immediate decision is already in present value monetary terms as it is considered an immediate payment. The present value of decisions made in the future is found using Equation 3.15. The Bellman Optimality Equation can now be represented as Equation 3.16

$$f_t(\mathbf{x}_t) = min\big[d(\mathbf{x}_t, \mathbf{x}_{t+1}) + \beta(f_{t+1}(\mathbf{x}_{t+1}))\big] \tag{3.16}$$

Where all terms are the same as defined in Section 3.2 and where:

- $\beta = \frac{1}{(1+r)^1}$
- r = Discount rate.

The future cost or profit is the value of $f_{t+1}(\mathbf{x}_{t+1})$ which occurs 1 year away from the present time. While all future costs don't actually simultaneously occur only a single year away from the present time, (they are spread out across the planning period) the recursive nature of dynamic programming calculations results in each future cost function representing the present value of costs with respect to the particular subproblem. For example, in the final time period, T, the subproblem calculation is Equation 3.17.

$$f_T(\mathbf{x}_T) = min\big[d(\mathbf{x}_T, \mathbf{x}_{T+1}) + \beta(endVal)\big] \tag{3.17}$$

The resulting function $f_T(\mathbf{x}_T)$ is represented in present value terms with respect to the time period T. When $f_T(\mathbf{x}_T)$ is used in the next recursive calculation at time period $T-1$, as shown in Equation 3.18, to represent all future costs, the value of $f_T(\mathbf{x}_T)$ is now only 1 year away from the present time period. The future costs of the decision made in stage $T-1$ need only be discounted by 1 year.

$$f_{T-1}(\mathbf{x}_{T-1}) = min\big[d(\mathbf{x}_{T-1}, \mathbf{x}_T) + \beta(f_T(\mathbf{x}_T))\big] \tag{3.18}$$

The inclusion of a discount rate in dynamic programming is easily extended to stochastic dynamic programming. The expected future cost function is multiplied by the discount rate equation in exactly the same way as for dynamic programming. The Stochastic Bellman Optimality Equation is shown in Equation 3.19

$$\mathrm{E}_{y_t}\big(f_t(\mathbf{x}_t)\big) = min\ \mathrm{E}_{y_t}\big[d(\mathbf{x}_t, \mathbf{x}_{t+1}) + \beta\big(\ \mathrm{E}_{y_{t+1}}\big(f_{t+1}(\mathbf{x}_{t+1})\big)\big)\big] \tag{3.19}$$

Where all terms are the same as defined in Section 3.3 and where:

- $\beta = \frac{1}{(1+r)^1}$
- $r =$ Discount rate.

The value of discount rate used is decided upon by the business or organisation undertaking the planning optimisation. The discount rate may reflect the opportunity cost of capital i.e. the rate of return expected from an investment. The discount rate may also be used to include a risk premium that is quantified on a per investment basis [48].

3.5 CURSE OF DIMENSIONALITY

Dynamic programming is often slated to suffer from the 'Curse of Dimensionality' and as such is often dismissed as a useful optimisation technique for real world problems. The dimensionality refers to the vast increase in the number of calculations required when the number of state variables increases. As mentioned in Section 3.2.3, each additional state variable exponentially increases the number of calculations required at each stage [4]. An example of how quickly the potential state combinations grow is illustrated by considering a problem with m state variables, each discretised into n intervals. The number of state variable combinations is given by n^m. Table 3.5 illustrates the number of discrete state variable combinations and hence the number of subproblems required to be calculated at each stage, given that there are 25 discretisations for each state variable.

The number of subproblem calculations increases further for a stochastic problem where the expected value of the subproblem is found by solving the subproblem a number of times for

Number of State Variables	**Number of Calculations Required**
2	$25^2 = 625$
4	$25^4 = 390625$
6	$25^6 = 244140625$
8	$25^8 = 1.526 \times 10^{11}$
10	$25^{10} = 9.537 \times 10^{13}$

Table 3.7 Increasing Dimensionality of Dynamic Programming

different realisations of the stochastic variable.

3.6 SUMMARY

Planning problems can be considered either static or dynamic. Investment planning in power systems is usually considered dynamic due to the many complex factors involved in the decision making process. With dynamic planning problems, dynamic programming is a good solution technique as it finds a set of rules, in the form of functions, that give optimal decisions across a range of system states. Dynamic programming is characterised by a sequence of recursive calculations that calculate a future cost function at each stage. As a set, the future cost functions make up the optimal policy for the planning problem.

Dynamic programming can be extended to stochastic planning problems where one or more of the variables are defined by a probability distribution. The resulting future cost functions are *expected* future cost functions. Unlike deterministic dynamic programming, the stochastic variant doesn't provide an optimal sequence of investments across the planning horizon. The optimal policy gives a set of optimal decisions based on the realisation of the stochastic variable. To understand more about the sequential decisions in a stochastic problem a Monte Carlo simulation is undertaken. This can give information about variability and risk of decisions due to the stochastic variable.

One downfall of dynamic programming is that it is said to suffer from the 'Curse of Dimensionality'. The dimensionality refers to the vast increase in the number of calculations required when the number of state variables increases. Real world problems are often hard to solve using dynamic programming, even with modern computing power, due to this problem. Approximation techniques have been developed to try and overcome this dimensionality issue. The approximations attempt to construct the future cost function from fewer subproblem calculations by interpolation, state space reduction and analytical functions. These techniques are discussed further in Chapter 4.

Chapter 4

STOCHASTIC DUAL DYNAMIC PROGRAMMING

4.1 INTRODUCTION

Dynamic programming has the potential to solve large dynamic planning problems but suffers from the 'Curse of Dimensionality' due to the necessity of discretising the state space of the problem. This drawback results in real world problems being computationally intractable, even with the extensive computing power available today. In order to successfully use the principles of dynamic programming to solve problems, approximation techniques have been developed that aim to reduce or eliminate the state space discretisations. One approximation technique that has been successfully applied to large scale problems is Stochastic Dual Dynamic Programming (SDDP) [4]. This technique eliminates the need to completely discretise the state space and is therefore computationally tractable for many large problems. SDDP uses the technique of Benders Decomposition to find a set of linear constraints that construct a piecewise linear approximation of the future cost function.

This chapter describes the previous development and applications of Stochastic Dual Dynamic Programming and details the development and operation of the deterministic version of Dual Dynamic Programming. The development process outlined in this chapter includes discussion of the different types of dynamic programming approximations, how Dual Dynamic Programming (DDP) uses dual variables to approximate the future cost function and the Benders Decomposition Algorithm that calculates Benders Cuts. The Dual Dynamic Programming technique is demonstrated with an example that illustrates the computational savings of the technique. Finally DDP is extended to stochastic problems where sampling and Monte Carlo simulation are used to find expected values of optimal solutions.

4.1.1 Stochastic Dual Dynamic Programming: Historical Developments

Stochastic Dual Dynamic Programming is the name given to two different (though similar) approximations to dynamic programming. Both Pereira and Pinto [4] and Read and George [49] have developed dynamic programming approximation techniques titled Stochastic Dual Dynamic

Programming.

Read and George [49] [50] developed a dual dynamic programming technique that deals directly with the critical values that determine the shape of the future cost function [51]. This results in a accurate approximation across the entire state space without the need to discretise or undertake successive refinement of the state space. A similar technique was developed by Bannister and Kaye [52] and further developed by Kaye and Travers [53].

Pereira and Pinto [4] [32] developed a dual dynamic programming technique that differs from that of Read and George in that the future cost function approximation produced is a locally approximate solution. The dual dynamic programming algorithm of Pereira and Pinto successively refines the approximation of the future cost function but only in the areas of the state space that are most likely to occur in the optimisation problem. This saves computation as refinement doesn't occur in unused portions of the state space.

The dual dynamic programming developed by Read and George is considered to be best suited to finding analytical solutions to low dimensional problems whereas the technique developed by Pereira and Pinto is suited to high dimensional problems that the analytical technique of Read and George would find difficult to solve [54].

This thesis uses the dual dynamic programming technique of Pereira and Pinto as the power system investment planning problem is a high dimensional problem. Each existing asset and all potential investments individually contribute additional dimensions. The ability of this dual dynamic programming technique to refine the solution only in areas of the state space that are likely to occur allows a high dimensional problem, such as the investment problem, to be solved easily.

4.1.2 Stochastic Dual Dynamic Programming: Previous Applications

Stochastic Dual Dynamic Programming as developed by Pereira and Pinto has primarily been applied to the hydro scheduling problem [4] [32] [55] [56] [57] [58] [59]. In this application the objective of the problem is to optimise the release of water to minimise the operational costs of the system where thermal power must offset any generation shortfall from hydro power. The stochastic variable is the water inflow to reservoirs. The technique has been applied to multi-reservoir systems as well as run of river hydro systems. The application of SDDP to the hydro scheduling problem has been highly successful with the commercialisation of software by the company Power Systems Research (PSR) [60](founded by Pereira). A number of countries have trialed or implemented software that uses SDDP for hydro scheduling including; South and Central America, Austria, Spain, Norway, The Balkans, New Zealand, Shanghai and South China [60].

SDDP has been used in assessing the costs and benefits of transmission upgrades. By including transmission constraints and an optimal power flow within the optimisation Granville et al. [58]

use SDDP to find the optimal hydro thermal dispatch for the Brazilian power system. The results of their simulation include spot prices that are used as indicators of transmission constraints and areas that may need transmission reinforcement.

Extension of SDDP to the problem of profit maximisation by generators has been undertaken by Iliadis et al. [61], Mo et al. [62] and used by PSR in development of their MaxRev software. For generators in hydro systems to maximise profits they must consider both operational dispatch and price. In this situation, price becomes an additional state variable. The additional of price as a state variable prevents the problem from being solved directly by SDDP as the future cost function becomes non convex (refer Section 5.4.3.2). The future cost function is convex with respect to spot prices and concave with respect to reservoir volume. This combination generates a saddle function that SDDP cannot solve. A hybrid of SDDP and Stochastic Dynamic Programming (SDP) is used where SDDP solves the optimal dispatch problem at a selection of price points that are found through SDP. The computation time of this application will be large and is dependent on the number of discrete state variable values used for the SDP algorithm.

Campodónico et al. [34] use SDDP in an integrated generation and transmission expansion study. The study undertaken uses Benders Decomposition to optimise a static expansion plan where SDDP is used in determining the operational costs of the system. The first part of their solution process finds a feasible set of investment decisions. This is then used in an operational analysis using SDDP. The results are then checked for optimality and feedback is used to improve the feasible investment plan. This study, while performing an integrated generation and transmission optimisation, is different to the application of SDDP presented in this thesis. SDDP is used in this thesis to optimise the dynamic investment problem without iteration between feasible investment plans and operational costs. It is not used for hydro scheduling but to find approximations to the future cost functions, where these functions represent the future costs of system investment *and* operation.

The hydro scheduling application of SDDP is widely known and is promoted through the commercial software development of PSR. All applications of SDDP have so far concentrated entirely on the hydrothermal scheduling problem. SDDP has been incorporated into other problems such as transmission planning and profit maximisation but only to optimise hydrothermal dispatch. The success of the SDDP technique at solving high dimensional problems suggests it as suitable for extension, development and application to the power system investment problem.

4.2 DYNAMIC PROGRAMMING APPROXIMATION

The computational intractability of dynamic programming arises from the necessity of discretising the state space in order to develop the future cost function. In order to utilise the benefits of the Principle of Optimality that dynamic programming offers but retain problem tractability, approximation techniques have been developed. These approximation techniques fall into two general categories. The first is where techniques are used to reduce size of the state space and hence reduce the number of required discretisations, the second approximates the future cost function at a more limited range of values but uses some form of interpolation or function approximation if an 'in between' value is required.

4.2.1 Reduction of State Space Discretisations

Real world problems have a huge potential range of state variable values. This range of values must be discretised in a way that allows for a reasonable representation of the future cost function. The number of discretisations is a tradeoff between computational tractability and accuracy of the solution. Where a problem may have more than one state variable, the number of calculations required increases exponentially with the number of state variables and this must be a consideration when discretising the state space. If a less accurate solution is acceptable then simply reducing the number of discretisations may be an easy solution to improving computational tractability but this is not usually the case. To reduce the number of discretisations while retaining reasonable solution accuracy, the size of the state space being discretised must be reduced in some way. There are two methods to do this. The first is to remove sections of the state space from the solution that lead to infeasible solutions. Some problems may have unallowable solutions due to financial requirements, physical plant requirements, labour limitations etc. This results in a smaller state space that must be discretised and hence fewer calculations. The second method is to repeatedly solve the dynamic programming problem first with a coarse grid of discretisations followed by a progressively finer grid of discretisations around the values of state variables most likely to occur [63]. This method may end up with just as many calculations as if a fine grid had been used originally and therefore may be just as computationally intensive.

Approximation techniques that reduce the number state space discretisations may result in computational tractability for some problems but it still may not be suitable for some problem types. For those problems where the reduction of the state space is not sufficient to gain computational tractability, the alternative technique of interpolation between known function points may offer a better solution.

4.2.2 Approximating the Future Cost Function

The second type of technique that is used for approximating the future cost function in dynamic programming uses interpolation. This method still requires discretisation of the state space but the number of discretisations may be be reduced. Whenever a value of the future cost function is required that has not been calculated at a discretised state value, the function is interpolated between the two closest known points. It is important to ensure that the interpolated value is a 'reasonable' representation of the actual future cost function otherwise the solution will be inaccurate. An issue that can affect the accuracy of interpolation is the spacing of neighbouring function points. There is no way of knowing what the actual future cost function looks like between the points so an assumption of its form, e.g. linearity, may be inaccurate. Even with well spaced known function points the interpolated values may not accurately represent the actual function if it varies in a non linear or convex fashion.

Another technique for approximating the future cost function is to use analytical functions rather than sets of discrete values to approximate the future cost function [4]. In this approximation technique the future cost function is calculated for a small number of discretisations and an analytical function is fitted to those known points. Dual Dynamic Programming is an example of using functions to approximate the future cost function. A set of linear constraints is used to approximate the future cost function in a linear piecewise fashion. For problems with certain characteristics, such as convexity in the future cost function, this technique of dynamic programming approximation offers a simple and robust alternative to discretisation of the state space.

4.2.3 Dynamic Programming Approximation for Capacity Investment in Power Systems

With reference to the research presented in this thesis, capacity investments in power systems, both transmission and generation, are dynamic optimisations with a convex future cost function. Dual Dynamic Programming is an excellent dynamic programming approximation technique for this type of problem and offers a computationally tractable dynamic solution to the capacity investment problem. Extending Dual Dynamic Programming to incorporate stochastic variables is also possible as detailed in Section 4.4. This allows for study of the effects of uncertainty on investment planning. The Dual Dynamic Programming technique must be extended for the capacity investment problem and these extensions are discussed in Chapter 6. The basic theory of Dual Dynamic Programming is presented in the following sections with discussion of duality theory, Benders Cuts, Benders Decomposition, extensions to multistage and stochastic problems.

4.3 DUAL DYNAMIC PROGRAMMING

To avoid the need to discretise the state space the future cost function can be approximated by analytical functions. For certain types of optimisation problems, namely those with concave or convex future cost functions, a good alternative to discretisation of the state space is to use Dual Dynamic Programming. The necessity for the future cost function to be concave or convex is so that the linear constraints do not remove valid solution points from the solution space. Figure 4.1 illustrates how linear constraints can represent the future cost function.

Figure 4.2 shows how for a non convex future cost function, a representation by linear constraints is not suitable as the constraints remove part of the solution space.

The strength of the DDP technique lies in that the problem doesn't require an exact repre-

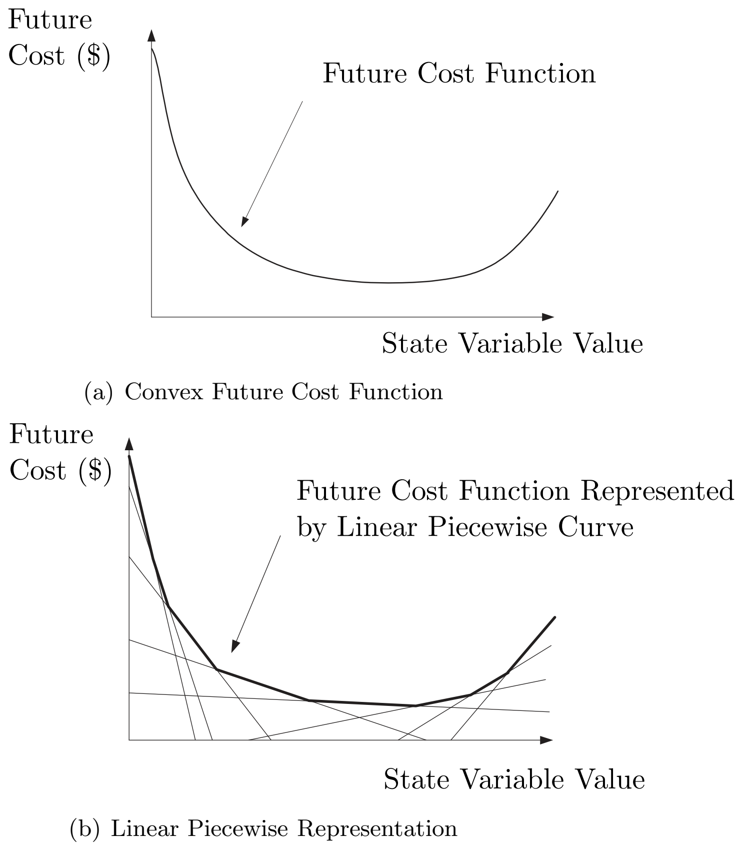


Figure 4.1 Future Cost Function Represented by Linear Cuts

sentation of the future cost function, only one that is considered ‘good enough’. This means that only a subset of all the linear constraints which provide an approximation to the exact future cost function need to be found. Calculation of the linear constraints is done by Benders Decomposition, a technique that relies on finding the dual variables of an optimisation problem.

4.3.1 Duality in Linear Optimisation

When an optimisation problem is solved, the optimal solution represents a snapshot of system variables at the particular time the problem is solved. What is not known is how the solution may change if the system variables were to change value. Study of how the optimal solution cost changes when system variables change is called sensitivity analysis. Sensitivity analysis uses Duality Theory [64] to find the dual variables of problem. Dual variables describe how the optimal solution cost will change for a change in value of the system variables. The idea of describing how the optimal solution cost will alter for differing variable values is central to the idea of approximating the future cost function with linear constraints. Each constraint represents the optimal solution cost across a range of system variable values. To construct each constraint the dual variables of the optimisation problem must be found.

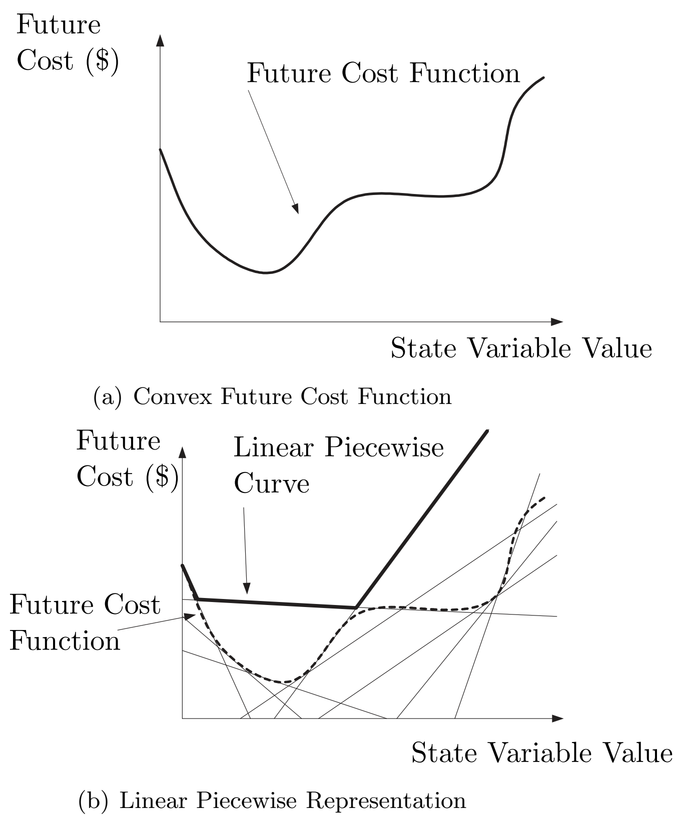


Figure 4.2 Future Cost Function Badly Represented by Linear Cuts

For all optimisation problems the two main components of the problem are an objective function and a set of constraints. The objective function sets out the function to maximised or minimised. The constraints must be satisfied by the optimal solution and define the solution space of the problem. When an optimisation problem is mathematically described for a specific situation this is referred to as the *primal* problem. The primal problem is usually a resource allocation problem [65]. The objective is to allocate the limited resources so to maximise revenue or minimise costs.The primal problem for a cost minimisation can be written as shown in Equation 4.1.

$$
\begin{aligned}
f(\mathbf{x}) &= min\left[\mathbf{c^T x}\right] \\
\text{Subject to:} & \\
\mathbf{Ax} &\geq \mathbf{b} \\
\mathbf{x} &\geq 0
\end{aligned}
\tag{4.1}
$$

Where:

- $f(\mathbf{x})$ = Optimal solution value

- $\mathbf{c}$ = Resource profit coefficients
- $\mathbf{x}$= Decision variables for the optimisation problem. These variables represent the amounts of a resources that can be chosen to be utilised.
- $\mathbf{A}$ = Resource limitation coefficients
- $\mathbf{b}$ = Limit on resource availability

The primal optimisation problem has a corresponding *dual* optimisation problem that is defined as a resource valuation problem [65]. The dual problem associated with this primal problem is shown in Equation 4.2.

$$
\begin{aligned}
f(\boldsymbol{\lambda}) = max\left[\mathbf{b}^{\mathbf{T}}\boldsymbol{\lambda}\right]& \\
\text{Subject to:}& \\
&\mathbf{A}^{\mathbf{T}}\boldsymbol{\lambda} \leq \mathbf{c} \\
&\boldsymbol{\lambda} \geq 0
\end{aligned}
\tag{4.2}
$$

Where:

- $f(\boldsymbol{\lambda})$ = Optimal solution value
- $\mathbf{b}$ = Limit on resource availability
- $\boldsymbol{\lambda}$= Dual variables, represent the marginal value of a resource.

By the Strong Duality theorem [66] [67] (refer Appendix B), if the primal problem can be solved and an optimal solution obtained, the dual problem also has an optimal solution and the objective functions of both problems are equal. Ensuring that the Strong Duality theorem holds is important for optimisation problems when the solution method chosen is Dual Dynamic Programming. Strong Duality ensures that the optimal solution to the dual is equal to the optimal solution of the primal. A linear constraint that approximates the *primal* subproblem is calculated by using the dual variables of the *dual* problem. Ensuring that the dual variables accurately represent the primal subproblem is essential to approximating the subproblem via linear constraints.

Economic Interpretation of the Dual Problem Economic theory indicates that any limited resource has a value. Primal resource allocation problems allocate limited resources therefore each resource that is used should be valued. The dual problem, as a resource valuation problem finds a marginal value of each resource. That is, if in the primal problem, the resource limitation $\mathbf{b}$ is changed by a discrete unit value, by what amount would optimal solution profit (or cost) also change? This change represents the value of that particular resource to the problem. This marginal resource value can be found by repeatedly changing $\mathbf{b}$ by a discrete unit value and studying how the optimal solution value changes but a simpler method is available - solve the dual problem. The dual variables represent the marginal value of a resource.

Example A furniture maker can make a combination of tables and chairs. Each table requires 4m^2 of wood, takes 4 hours to make and makes \$60 profit. Each chair requires 1.5m^2 of wood, takes 1 hour to make and makes \$20 profit. The furniture maker can only get 50m^2 of wood per week and is only prepared to work for 40 hours. How many tables and chairs should the furniture maker produce each week to maximise their profit?

Solution The solution to the problem is found by first constructing the primal resource allocation problem. The primal problem is a maximisation problem as the goal of the furniture maker is to maximise profit. Where the primal problem is a maximisation problem, the dual problem will be a minimisation problem. Equation 4.3 states the primal maximisation optimisation problem.

$$
\begin{aligned}
& f(\mathbf{x}) = max\left[20\mathbf{x}_{chair} + 60\mathbf{x}_{table}\right] \\
& \text{Subject to:} \\
& \qquad 1.5\mathbf{x}_{chair} + 4\mathbf{x}_{table} \leq 50 && \text{limitation on amount of wood} \\
& \qquad 1\mathbf{x}_{chair} + 4\mathbf{x}_{table} \leq 40 && \text{limitation on number of hours worked} \\
& \qquad \mathbf{x}_{table} \geq 0 \\
& \qquad \mathbf{x}_{chair} \geq 0
\end{aligned}
\tag{4.3}
$$

Where:

- $f(x)$ = Optimal solution profit
- $\mathbf{x}_{chair}$ = Decision variable for number of chairs.
- $\mathbf{x}_{table}$= Decision variable for number of tables

Solving Equation 4.3 results in $\mathbf{x}_{chair} = 20$ and $\mathbf{x}_{table} = 5$ with an optimal profit $f(x) = \$700$ per week. This solution tells the problem solver the optimal solution based on the resource limitations but doesn't give any insight into the value of each unit of resource, namely wood and labour. This resource value is given by the dual variables of the dual problem as shown in Equation 4.4.

$$f(\boldsymbol{\lambda}) = min\big[50\boldsymbol{\lambda}_{wood} + 40\boldsymbol{\lambda}_{labour}\big]$$

Subject to:

$$\begin{aligned} &1.5\boldsymbol{\lambda}_{wood} + 1\boldsymbol{\lambda}_{labour} \geq 20 && \text{Dual Constraint 1} \\ &4\boldsymbol{\lambda}_{wood} + 4\boldsymbol{\lambda}_{labour} \geq 60 && \text{Dual Constraint 2} \\ &\boldsymbol{\lambda}_{wood} \geq 0 \\ &\boldsymbol{\lambda}_{labour} \geq 0 \end{aligned} \tag{4.4}$$

Where:

- $f(\boldsymbol{\lambda})$ = Optimal solution value
- $\boldsymbol{\lambda}_{wood}$ = Dual variable representing the marginal value of the wood resource.
- $\boldsymbol{\lambda}_{labour}$=Dual variable representing the marginal value of the labour resource.
- Dual Constraint 1 = This constraint restricts the value of the resources used in constructing a chair to be greater than or equal to the value gained in producing a chair, namely $20 profit.
- Dual Constraint 2 =This constraint restricts the value of the resources used in constructing a table to be greater than or equal to the value gained in producing a table, namely $60 profit.

The dual problem aims to minimise the total marginal value of the resources used (objective function) while ensuring that the value of the resources used are at least equal to the profit gained from producing each item (dual constraints). The solution of the dual problem results in $\boldsymbol{\lambda}_{wood} = 10$, $\boldsymbol{\lambda}_{labour} = 5$ and the total optimal solution $f(\boldsymbol{\lambda}) = \$700$. The dual variables $\boldsymbol{\lambda}_{wood}$ and $\boldsymbol{\lambda}_{labour}$ are representative of the marginal value of each resource. $\boldsymbol{\lambda}_{wood}$ represents the value/profit per square metre of wood, $\$/m^2$ and $\boldsymbol{\lambda}_{labour}$ represents the value/profit per hour of labour, $/hour.

Duality and Dual Dynamic Programming The economic interpretation of the dual variables state that they represent the marginal value of the resource to the optimal solution value. To approximate the future cost function using linear constraints, the dual variables represent the slope of each linear constraint. For example, assume that the optimal solution value of a dynamic programming problem is $110 when the value of the state variable $\mathbf{x}_i = 50$. Solving the dual problem results in a dual variable value of $5/unit of $\mathbf{x}_i$. This dual variable represents the value of each unit of $\mathbf{x}_i$ to the optimisation problem, hence if the value of $\mathbf{x}_i = 51$ then the optimal solution profit/cost would become $115 and if $\mathbf{x}_i = 49$ then the optimal solution profit/cost would become $105. Figure 4.3 illustrates how the dual variable can be translated into a linear constraint.

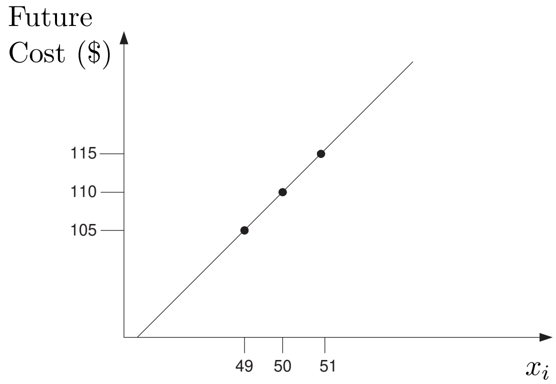


Figure 4.3 Single Linear Cut Derived from Dual Variable

Using the dual variable to construct a linear constraint is intuitively simple for problems with a single state variable by noting that the dual variable gives the slope of the constraint. Using the optimal solution to the problem and the value of trial variable the linear constraint can be calculated in the form of $y = m\mathbf{x} + c$. In problems with numerous state variables, the linear constraint becomes a multidimensional hyperplane representation that is much harder to visualise graphically. Dual Dynamic Programming is equivalent to Benders Decomposition [4], an optimisation technique that provides a framework for formulaic computation of multidimensional linear constraints.

4.3.2 Benders Decomposition

Benders Decomposition is a decomposition technique initially developed by J.F.Benders to solve large scale mixed integer optimisation problems [68]. To successfully use Benders Decomposition to solve an optimisation problem the problem must exhibit a special structure known as having *complicating variables*.

If an optimisation problem is written as shown in Equation 4.5, the problem can be decomposed into three separate subproblems Equations 4.6, 4.7 and 4.8.

$$
\begin{aligned}
&\text{minimise}\left[\mathbf{g}_1\mathbf{x}_1 + \mathbf{g}_2\mathbf{x}_2 + \mathbf{h}_1\mathbf{y}_1 + \mathbf{m}_1\mathbf{v}_1 + \mathbf{m}_2\mathbf{v}_2 + \mathbf{m}_3\mathbf{v}_3\right] \\
&\text{Subject to:} \\
&\quad \mathbf{A}_{11}\mathbf{x}_1 + \mathbf{A}_{12}\mathbf{x}_2 \geq \mathbf{d}_1 \\
&\quad \mathbf{A}_{21}\mathbf{x}_1 + \mathbf{A}_{22}\mathbf{x}_2 \geq \mathbf{d}_2 \\
&\quad \mathbf{B}_{11}\mathbf{y}_1 \geq \mathbf{e}_1 \\
&\quad \mathbf{C}_{11}\mathbf{v}_1 + \mathbf{C}_{12}\mathbf{v}_2 + \mathbf{C}_{13}\mathbf{v}_3 \geq \mathbf{f}_1 \\
&\quad \mathbf{C}_{21}\mathbf{v}_1 + \mathbf{C}_{22}\mathbf{v}_2 + \mathbf{C}_{23}\mathbf{v}_3 \geq \mathbf{f}_2
\end{aligned}
\tag{4.5}
$$

$$\begin{aligned} &\text{minimise}\left[\mathbf{g}_1\mathbf{x}_1 + \mathbf{g}_2\mathbf{x}_2\right] \\ &\text{Subject to:} \\ &\qquad \mathbf{A}_{11}\mathbf{x}_1 + \mathbf{A}_{12}\mathbf{x}_2 \geq \mathbf{d}_1 \\ &\qquad \mathbf{A}_{21}\mathbf{x}_1 + \mathbf{A}_{22}\mathbf{x}_2 \geq \mathbf{d}_1 \end{aligned} \tag{4.6}$$

$$\begin{aligned} &\text{minimise}\left[\mathbf{h}_1\mathbf{y}_1\right] \\ &\text{Subject to:} \\ &\qquad \mathbf{B}_{11}\mathbf{y}_1 \geq \mathbf{e}_1 \end{aligned} \tag{4.7}$$

$$\begin{aligned} &\text{minimise}\left[\mathbf{m}_1\mathbf{v}_1 + \mathbf{m}_2\mathbf{v}_2 + \mathbf{m}_3\mathbf{v}_3\right] \\ &\text{Subject to:} \\ &\qquad \mathbf{C}_{11}\mathbf{v}_1 + \mathbf{C}_{12}\mathbf{v}_2 + \mathbf{C}_{13}\mathbf{v}_3 \geq \mathbf{f}_1 \\ &\qquad \mathbf{C}_{21}\mathbf{v}_1 + \mathbf{C}_{22}\mathbf{v}_2 + \mathbf{C}_{23}\mathbf{v}_3 \geq \mathbf{f}_2 \end{aligned} \tag{4.8}$$

A problem like this is simple to solve and needs no special treatment. Each subproblem (Equations 4.6-4.8) is solved separately with the resulting solutions summed together to give an overall optimal solution to Equation 4.5. A problem arises if the original problem is defined as shown in Equation 4.9 where every constraint contains the $\mathbf{y}_1$ variable and hence cannot be decomposed into blocks.

$$\begin{aligned} &\text{minimise}\left[\mathbf{g}_1\mathbf{x}_1 + \mathbf{g}_2\mathbf{x}_2 + \mathbf{h}_1\mathbf{y}_1 + \mathbf{m}_1\mathbf{v}_1 + \mathbf{m}_2\mathbf{v}_2 + \mathbf{m}_3\mathbf{v}_3\right] \\ &\text{Subject to:} \\ &\qquad \mathbf{A}_{11}\mathbf{x}_1 + \mathbf{A}_{12}\mathbf{x}_2 + \mathbf{B}_{11}\mathbf{y}_1 \geq \mathbf{d}_1 \\ &\qquad \mathbf{A}_{21}\mathbf{x}_1 + \mathbf{A}_{22}\mathbf{x}_2 + \mathbf{B}_{21}\mathbf{y}_1 \geq \mathbf{d}_2 \\ &\qquad \mathbf{B}_{31}\mathbf{y}_1 \geq \mathbf{e}_1 \\ &\qquad \mathbf{B}_{41}\mathbf{y}_1 + \mathbf{C}_{11}\mathbf{v}_1 + \mathbf{C}_{12}\mathbf{v}_2 + \mathbf{C}_{13}\mathbf{v}_3 \geq \mathbf{f}_1 \\ &\qquad \mathbf{B}_{51}\mathbf{y}_1 + \mathbf{C}_{21}\mathbf{v}_1 + \mathbf{C}_{22}\mathbf{v}_2 + \mathbf{C}_{23}\mathbf{v}_3 \geq \mathbf{f}_2 \end{aligned} \tag{4.9}$$

In this example the variable $\mathbf{y}_1$ is considered the *complicating* variable as it is the one stopping the problem being decomposed into several separate subproblems.

Solving problems with complicating variables can be done by using Benders Decomposition. The principle of the decomposition technique is to take advantage of the structure of the problem, namely the complicating variable, and split the original optimisation problem into a master problem and a subproblem [69]. The master problem optimises with respect to the complicating variable while approximating the optimal solution value of the subproblem(s). The subproblem uses a 'trial' value of the complicating variable to optimise with respect to the non complicating variables. The main focus of the decomposition technique is the approximation of the subproblem by the use of linear constraints, also known as Benders Cuts. The decomposition iterates between solving the master problem and the subproblem where the solution of the subproblem provides dual variables that are used to calculate a single Benders Cut. The Benders Cut provides a linear approximation to the value of the subproblem and is used in solving the master problem. Each iteration of the algorithm adds another Benders Cut to the master problem until the approximation of the subproblem by Benders Cuts is a sufficiently accurate representation of the subproblem.

The success of Benders Decomposition hinges on the ability to approximate the subproblem via Benders Cuts. Each Benders Cut illustrates how the optimal value of the subproblem changes for different values of the complicating variables. The combined set of Benders Cuts is a linear piecewise representation of the optimal value of the subproblem where this subproblem function is equivalent to the future cost function of Dynamic Programming.

4.3.3 Benders Cuts

Benders Decomposition is used to solve optimisation problems that have complicating variables. The following derivation draws from both [4] and [69].

Consider the optimisation problem in Equation 4.10 that has $\mathbf{x}_1$ as a complicating variable.

$$
\begin{aligned}
f(\mathbf{x}_1, \mathbf{x}_2) = \min &\left[\mathbf{c}_1{}^{\mathbf{T}}\mathbf{x}_1 + \mathbf{c}_2{}^{\mathbf{T}}\mathbf{x}_2\right] \\
s.t. & \\
& \mathbf{A}\mathbf{x}_1 \geq \mathbf{b} \\
& \mathbf{F}\mathbf{x}_1 + \mathbf{E}\mathbf{x}_2 \geq \mathbf{h} \\
& \mathbf{x}_1 \geq 0, \mathbf{x}_2 \geq 0,
\end{aligned}
\tag{4.10}
$$

Where:

- $\mathbf{x}_1$ = Complicating Decision Variable
- $\mathbf{x}_2$ = Non complicating decision variables

The problem can be split into a master problem as in Equation 4.11 and a subproblem as in Equation 4.12. The master problem solves for the complicating variable and the subproblem the non complicating variables.

Master Problem

$$
\begin{aligned}
f(\mathbf{x}_1) = \min \left[\mathbf{c}_1{}^{\mathbf{T}} \mathbf{x}_1 + \alpha(\mathbf{x}_1) \right] & \\
s.t. & \\
\mathbf{A}\mathbf{x}_1 \geq \mathbf{b} & \\
\alpha(\mathbf{x}_1) \geq \alpha_{min} & \\
\mathbf{x}_1 \geq 0 & \qquad (4.11)
\end{aligned}
$$

Where:

- $\alpha(\mathbf{x}_1)$ = Value of approximated subproblem.
- α_{min} = Lower bound on the value of the subproblem.

Subproblem

$$
\begin{aligned}
\alpha(\mathbf{x}_1) = \min \left[\mathbf{c}_2{}^{\mathbf{T}} \mathbf{x}_2 \right] & \\
s.t. & \\
\mathbf{E}\mathbf{x}_2 \geq \mathbf{h} - \mathbf{F}\mathbf{x}_1 & \\
\mathbf{x}_2 \geq 0 & \qquad (4.12)
\end{aligned}
$$

Where:

- $\mathbf{x}_1$ = 'Trial' value of $\mathbf{x}_1$ found from solving the master problem.

To solve the master problem the function $\alpha(\mathbf{x}_1)$ must be known. Benders Decomposition calculates linear constraints, also known as Benders Cuts, to approximate the subproblem, $\alpha(\mathbf{x}_1)$. As detailed in Section 4.3.1, calculating Benders Cuts requires knowledge of the value of the dual variables of the problem that is being approximated. The first step in calculating the Benders Cuts is to construct the dual problem of the subproblem as in Equation 4.13.

Subproblem Dual

$$
\begin{aligned}
\alpha(\mathbf{x}_1) = \max[(\mathbf{h} - \mathbf{F}\mathbf{x}_1)^{\mathbf{T}} \boldsymbol{\lambda}] & \\
s.t. & \\
\mathbf{E}^{\mathbf{T}} \boldsymbol{\lambda} \leq \mathbf{c}_2 & \\
\boldsymbol{\lambda} \geq 0 & \qquad (4.13)
\end{aligned}
$$

Where:

- $\boldsymbol{\lambda}$ = Dual variables
- $\mathbf{x}_1$ = 'Trial' value of the complicating variable in the master problem.

The constraint set of a linear optimisation problem defines the set of vertices of the problem, one of which is the optimal solution. Note how in Equation 4.13 the constraints contain no occurrence of the complicating $\mathbf{x}_1$ variable. This results in the set of vertices (and hence the optimal solution) being known without any prior knowledge of the value of $\mathbf{x}_1$. Theoretically Equation 4.13 can be solved by enumeration of all the vertices given by the constraints in Equation 4.13 as shown in Equation 4.14.

$$\alpha(\mathbf{x}_1) = \max[(\mathbf{h} - \mathbf{F}\mathbf{x}_1)^{\mathbf{T}} \boldsymbol{\lambda}^i \text{ for all } i = 1, \ldots, r] \tag{4.14}$$

Where:

- $\boldsymbol{\lambda}^i$ = Dual variables that represent vertex i of the constraint set.
- r = Number of trial decisions from master problem that are used to construct Benders Cuts.

Equation 4.14 is equivalent to Equation 4.15.

$$\begin{aligned} \alpha(\mathbf{x}_1) &= \min \alpha \\ s.t.& \\ &\alpha \geq (\mathbf{h} - \mathbf{F}\mathbf{x}_1)^{\mathbf{T}} \boldsymbol{\lambda}^1 \\ &\alpha \geq (\mathbf{h} - \mathbf{F}\mathbf{x}_1)^{\mathbf{T}} \boldsymbol{\lambda}^i \\ &\vdots \\ &\alpha \geq (\mathbf{h} - \mathbf{F}\mathbf{x}_1)^{\mathbf{T}} \boldsymbol{\lambda}^r \end{aligned} \tag{4.15}$$

Where:

- α = A scalar variable representing the optimal value of the subproblem.
- $\alpha \geq (\mathbf{h} - \mathbf{F}\mathbf{x}_1)^{\mathbf{T}} \boldsymbol{\lambda}^1$ = Benders Cut.
- $\boldsymbol{\lambda}^i$ = Dual variables associated with vertex i of subproblem. The vertex is found by solving the dual of the subproblem using trial value $\mathbf{x}_1^i$ from the master problem.
- r = Number of trial decisions from master problem that are used to construct Benders Cuts.

The equivalence of Equations 4.14 and 4.15 is shown by noting that the constraints from Equation 4.15

$$\alpha \geq (\mathbf{h} - \mathbf{F}\mathbf{x}_1)^{\mathbf{T}}\boldsymbol{\lambda}^i, i = 1, \ldots, r$$

imply that α is greater than or equal to each $(\mathbf{h} - \mathbf{F}\mathbf{x}_1)^{\mathbf{T}}\boldsymbol{\lambda}^i$ and is in particular greater than or equal to

$$\max\left[(\mathbf{h} - \mathbf{F}\mathbf{x}_1)\right]^{\mathbf{T}}\boldsymbol{\lambda}^i$$

As the objective of Equation 4.15 is to minimise α then solution of Equation 4.15 will be equal to the constraint with the maximum value, i.e. Equation 4.14

Graphically Equation 4.15 can be shown as seen in Figure 4.4 where each constraint represents a single linear constraint or Benders Cut that defines the optimal value of the subproblem, $\alpha(\mathbf{x}_1)$. The subproblem or future cost function is approximated from below by the Benders Cuts where a greater number of cuts provides a more accurate approximation but at the expense of an increased number of calculations.

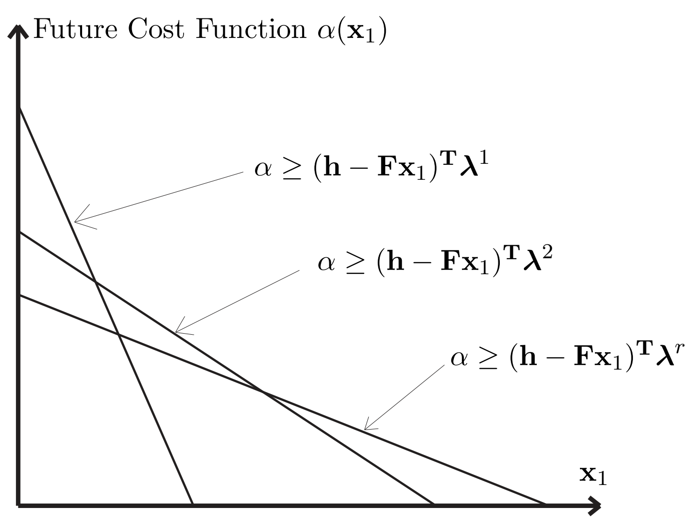


Figure 4.4 Dynamic Programming Discretisation

By substituting Equation 4.15 into the master problem (Equation 4.11) for $\alpha(\mathbf{x}_1)$, the master problem can now approximate the subproblem by the Benders Cuts of Equation 4.15. Equation 4.17 illustrates the new master problem.

Master Problem

$$f(\mathbf{x}_1) = \min(\mathbf{c}_1{}^{\mathbf{T}}\mathbf{x}_1 + \alpha)$$
$$s.t.$$
$$\mathbf{A}\mathbf{x}_1 \geq \mathbf{b}$$
$$\alpha \geq (\mathbf{h} - \mathbf{F}\mathbf{x}_1)^{\mathbf{T}}\boldsymbol{\lambda}^i \tag{4.16}$$
$$\mathbf{x} \geq 0 \tag{4.17}$$

Where:

- $\alpha \geq (\mathbf{h} - \mathbf{F}\mathbf{x}_1)^{\mathbf{T}}\boldsymbol{\lambda}^i$ = Benders Cut

The above form of the Benders Cut are not always useful from a computational standpoint so an alternative form of the Benders Cut can be derived that is more amenable to simple computation. Using a specific 'trial' value of $\mathbf{x}_1 = \mathbf{x}_1^i$ and solving Equation 4.14 results in Equation 4.18.

$$\alpha(\mathbf{x}_1^i) = (\mathbf{h} - \mathbf{F}\mathbf{x}_1^i)^{\mathbf{T}}\boldsymbol{\lambda}^i \tag{4.18}$$

Where:

- $\mathbf{x}_1^i$ = A specific trial decision from the master problem.
- $\boldsymbol{\lambda}^i$ = Dual variables from the subproblem that are associated with the vertex i.
- $\alpha(\mathbf{x}_1^i)$ = Subproblem optimal solution value for trial value $\mathbf{x}_1^i$ from the master problem.

Rearranging:

$$\mathbf{h}^{\mathbf{T}}\boldsymbol{\lambda}^i = \alpha(\mathbf{x}_1^i) + (\mathbf{F}\mathbf{x}_1^i)^T\boldsymbol{\lambda}^i \tag{4.19}$$

Substituting Equation 4.19 into the Benders Cut constraint 4.16, the alternative form of the Benders Cut can be obtained as shown in Equation 4.20.

$$\alpha \geq \alpha(\mathbf{x}_1^i) - (\mathbf{x}_1 - \mathbf{x}_1^i)^T\mathbf{F}^{\mathbf{T}}\boldsymbol{\lambda}^i \tag{4.20}$$

Where:

- $\alpha(\mathbf{x}_1^i)$ = The optimal value of the subproblem for the trial decision x_1^i
- $\mathbf{x}_1^i$ = Trial decision from master problem.
- $\boldsymbol{\lambda}^i$ = Dual variables associated with the solution to the subproblem, for a trial decision $\mathbf{x}_1^i$ from the master problem.

4.3.4 Benders Decomposition Algorithm

To solve an optimisation problem using Benders Decomposition an algorithm is used to iterate between the master problem and the subproblem. At each iteration an additional Benders Cut is calculated by solving the dual of the subproblem and is added as an additional constraint to the master problem. It is assumed the problem is constructed in such a way that only feasible solutions result. This is a not unreasonable assumption as a value of lost load generator can be used to 'catch' any load that can not be served. See Section 7.2.3 for further discussion on the value of lost load.

In order to assess whether the algorithm should continue a convergence test between a lower bound on the optimal solution and an upper bound on the optimal solution is performed. This test of the bounds checks if the current number of Benders Cuts provides an sufficiently accurate representation of the subproblem.

4.3.4.1 Lower Bound

The master problem of Equation 4.17 is a relaxed version of the original optimisation problem, Equation 4.10 [70]. It is 'relaxed' as the master problem uses a subset of the large number of potentially available Benders Cuts. The most 'relaxed' the master problem can be is to have no Benders Cuts representing the subproblem. In this situation the subproblem cost is assumed to be equal to zero. Solving the master problem with no Benders Cuts gives a lower bound to the optimal solution value of the original problem. As Benders Cuts are added to the master problem throughout the algorithm the subproblem approximation is built up from below [70]. The Benders Cuts require α to be $\geq$ to the value of the Benders Cut but the master problem is a minimisation problem, therefore the value of α will sit on the surface of the linear piecewise approximation. As cuts are added, the value of α increases, the approximation is improved and value of the lower bound increases. The lower bound is calculated by using the trial variable solution from the master problem ($\mathbf{x}_1^k$) in this iteration, k, and the current subset of Benders Cuts. Equation 4.21 shows the calculation of the lower bound of the optimisation problem.

$$z_{low_k} = \mathbf{c}_1{}^{\mathbf{T}}\mathbf{x}_1^k + \alpha(\mathbf{x}_1^k) \tag{4.21}$$

Where:

- z_{low_k} = Lower Bound of optimal solution of original problem

- $\mathbf{x}_1^k$ = Trial decision from master problem for this iteration.
- $\alpha(\mathbf{x}_1^k)$ = Value of subproblem approximation at $\mathbf{x}_1^k$,

4.3.4.2 Upper Bound

The subproblem is more constrained than the original optimisation problem as the complicating variable is fixed at a specific 'trial' value. This added restriction results in the optimal value of the *original* optimisation problem being an upper bound to the actual optimal solution value. The upper bound calculation in iteration k is given by Equation 4.22.

$$z_{up_k} = \mathbf{c_1}^{\mathbf{T}}\mathbf{x}_1^k + \mathbf{c_2}^{\mathbf{T}}\mathbf{x}_2^k \tag{4.22}$$

Where:

- z_{up_k} = Upper Bound of optimal solution of original problem
- $\mathbf{x}_1^k$ = Trial decision from master problem in this iteration.
- $\mathbf{x}_2^k$ = Value of decision variables from subproblem in this iteration.

4.3.4.3 Algorithm Structure

Benders Decomposition works by iteratively solving the master problem and subproblem, comparing the upper and lower bounds and terminating the algorithm when the bounds are considered close enough. The algorithm is presented next [70]:

Input:A linear programming problem with complicating variables.

Output: The solution to the linear programming problem with complicating variables using Benders Decomposition.

Step 1: Initialise the iteration counter $k = 1$, the lower bound to $z_{low_k} = 0$ and the upper bound to $z_{up_k} = +\infty$.

Step 2: Solve the master problem as shown in Equation 4.24.

$$f(\mathbf{x}_1) = \min\left[\mathbf{c_1}^{\mathbf{T}}\mathbf{x}_1 + \alpha\right]$$
$$s.t.$$
$$\mathbf{A}\mathbf{x}_1 \geq \mathbf{b}$$
$$\alpha \geq \alpha_{min}$$
$$\alpha \geq \alpha(\mathbf{x}_1^{k-1}) - (\mathbf{x}_1 - \mathbf{x}_1^{k-1})^{\mathbf{T}}\mathbf{F}^{\mathbf{T}}\boldsymbol{\lambda}^{\mathbf{x}_1^{k-1}} \tag{4.23}$$
$$0 \leq \mathbf{x} \leq \mathbf{x}_{up} \tag{4.24}$$

Where:

- α = Value of approximated future cost function
- $\mathbf{x}_{up}$ = The maximum allowed value for $\mathbf{x}$
- α_{min} = Predefined restriction on the lower bound of the optimal solution.

Store the optimal solution of $\mathbf{x}_1$ as $\mathbf{x}_1^k$.

Note that constraint 4.23 will not be present in the master problem if this iteration is $k = 1$. This is because at the beginning of the algorithm no Benders Cuts are known, they are calculated by solving the subproblem in Step 4 below.

Step 3: Check for convergence between the upper and lower bounds.

Calculate the lower bound by Equation 4.25

$$z_{low_k} = \mathbf{c_1}^{\mathbf{T}}\mathbf{x}_1^k + \alpha(\mathbf{x}_1^k) \tag{4.25}$$

Where:

- z_{low_k} = Lower Bound of optimal solution of original problem
- $\mathbf{x}_1^k$ = Trial decision from master problem for this iteration.
- $\alpha(\mathbf{x}_1^k)$ = Value of subproblem approximation for this iteration

If $k = 1$ then:

$$z_{up_1} = +\infty$$

Else calculate the upper bound by Equation 4.26.

$$z_{up_k} = \mathbf{c_1}^{\mathbf{T}}\mathbf{x}_1^k + \mathbf{c_2}^{\mathbf{T}}\mathbf{x}_2^k \tag{4.26}$$

Where:

- z_{up_k} = Upper Bound of optimal solution of original problem

- $\mathbf{x}_1^k$ = Trial decision from master problem for this iteration.
- $\mathbf{x}_2^k$ = Value of decision variables from subproblem

Compare the upper and lower bounds. If

$$z_{up_k} - z_{low_k} \leq \epsilon$$

then terminate the algorithm with the optimal solution being $\mathbf{x}_1^k$ and $\mathbf{x}_2^k$ and an optimal solution value of $z_{up_k} \simeq z_{low_k}$.

Otherwise:

$$z_{up_k} - z_{low_k} > \epsilon$$

continue to Step 4.

Step 4: Update the iteration counter, $k \leftarrow k+1$. Solve the subproblem, Equation 4.27

$$\begin{aligned} \alpha(\mathbf{x}_1^{k-1}) &= \min \left[\mathbf{c_2}^{\mathbf{T}}\mathbf{x}_2\right] \\ s.t.& \\ &\mathbf{E}\mathbf{x}_2 \geq \mathbf{h} - \mathbf{F}\mathbf{x}_1^{k-1} \\ &\mathbf{x}_2 \geq 0 \end{aligned} \tag{4.27}$$

Where:

- $\mathbf{x}_1^{k-1}$ = The value of $\mathbf{x}_1$ found from solving the master problem in iteration $k-1$.

The solution to this subproblem in this iteration k is $\mathbf{x}_2^k$ with associated dual variables $\boldsymbol{\lambda}^{\mathbf{x}_1^{k-1}}$. Calculate the Benders Cut

$$\alpha \geq \alpha(\mathbf{x}_1^{k-1}) - (\mathbf{x}_1 - \mathbf{x}_1^{k-1})^{\mathbf{T}}\mathbf{F}^{\mathbf{T}}\boldsymbol{\lambda}^{\mathbf{x}_1^{k-1}}$$

that approximates the function $\alpha(\mathbf{x}_1)$.

Step 5: The algorithm returns to Step 2.

4.3.5 Multistage Benders Decomposition

Benders Decomposition can be extended to solve multistage optimisation problems such as those posed by dynamic planning problems in power systems. The optimisation problem is split into a series of master problems and subproblems where an individual optimisation problem can represent both a master problem and a subproblem. An example of this is where the master problem at time t is also the subproblem of the master problem at time $t = t - 1$. For example, Equation 4.28 shows the optimisation problem for a 3 time period problem.

Original Problem

$$\begin{aligned} f(\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}) = \min \left[\mathbf{c_1}^{\mathbf{T}}\mathbf{x_1} + \mathbf{c_2}^{\mathbf{T}}\mathbf{x_2} + \mathbf{c_3}^{\mathbf{T}}\mathbf{x_3}\right] \\ s.t. \\ \mathbf{A_1x_1} \geq \mathbf{b_1} \\ \mathbf{A_2x_2} \geq \mathbf{b_2} \\ \mathbf{F_1x_1} + \mathbf{E_2x_2} \geq \mathbf{h_1} \\ \mathbf{A_3x_3} \geq \mathbf{b_3} \\ \mathbf{F_2x_2} + \mathbf{E_3x_3} \geq \mathbf{h_2} \\ \mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3} \geq 0 \end{aligned} \tag{4.28}$$

Where

- $\mathbf{x_1}$ = Decision variable for time period 1.
- $\mathbf{x_2}$ = Decision variable for time period 2.
- $\mathbf{x_3}$ = Decision variable for time period 3.
- $\mathbf{c_1}^{\mathbf{T}}$ = Coefficient of decision variable $\mathbf{x_1}$.
- $\mathbf{c_2}^{\mathbf{T}}$ = Coefficient of decision variable $\mathbf{x_2}$.
- $\mathbf{c_3}^{\mathbf{T}}$ = Coefficient of decision variable $\mathbf{x_3}$.
- $\mathbf{A_1}$ = Matrix of constraint coefficients for constraints that involve only $\mathbf{x_1}$.
- $\mathbf{A_2}$ = Matrix of constraint coefficients for constraints that involve only $\mathbf{x_2}$.
- $\mathbf{A_3}$ = Matrix of constraint coefficients for constraints that involve only $\mathbf{x_3}$.
- $\mathbf{F_1}$ = Matrix of constraint coefficients for variable $\mathbf{x_1}$ in constraint(s) that have decision variables $\mathbf{x_1}$ and $\mathbf{x_2}$.
- $\mathbf{E_2}$ = Matrix of constraint coefficients for variable $\mathbf{x_2}$ in constraint(s) that have decision variables $\mathbf{x_1}$ and $\mathbf{x_2}$.
- $\mathbf{F_2}$ = Matrix of constraint coefficients for variable $\mathbf{x_2}$ in constraint(s) that have decision variables $\mathbf{x_2}$ and $\mathbf{x_3}$.
- $\mathbf{E_3}$ = Matrix of constraint coefficients for variable $\mathbf{x_3}$ in constraint(s) that have decision variables $\mathbf{x_2}$ and $\mathbf{x_3}$.
- $\mathbf{b_1}$ = Constraint limitation for constraint(s) that involve only $\mathbf{x_1}$.
- $\mathbf{b_2}$ = Constraint limitation for constraint(s) that involve only $\mathbf{x_2}$.
- $\mathbf{b_3}$ = Constraint limitation for constraint(s) that involve only $\mathbf{x_3}$.
- $\mathbf{h_1}$ = Constraint limitation for constraint(s) that involve $\mathbf{x_1}$ and $\mathbf{x_2}$.
- $\mathbf{h_2}$ = Constraint limitation for constraint(s) that involve $\mathbf{x_2}$ and $\mathbf{x_3}$.

The original problem is split into a number of master problems and subproblems. Equation 4.29 illustrates the first master problem with Equation 4.30 showing the first subproblem.

Master Problem 1

$$\begin{aligned} f(\mathbf{x}_1) = \min \left[\mathbf{c}_1{}^{\mathbf{T}}\mathbf{x}_1 + \alpha_1\right] \\ s.t. \\ \mathbf{A}_1\mathbf{x}_1 \geq \mathbf{b}_1 \\ \alpha_1 \geq \alpha_1(\hat{\mathbf{x}}_1) - (\mathbf{x}_1 - \hat{\mathbf{x}}_1)^{\mathbf{T}}\mathbf{F}_1^{\mathbf{T}}\boldsymbol{\lambda}^{\hat{\mathbf{x}}_1} \\ \mathbf{x} \geq 0 \end{aligned} \tag{4.29}$$

Where:

- $f(\mathbf{x}_1)$ = Approximate value of optimal solution to whole optimisation problem.
- α_1 = Approximate solution of subproblem 1.
- $\boldsymbol{\lambda}^{\hat{\mathbf{x}}_1}$ = Dual variable found from the dual solution of subproblem 1 for the trial value $\hat{\mathbf{x}}_1$.

Where:

Subproblem 1

$$\begin{aligned} \alpha_1(\hat{\mathbf{x}}_1) = \min \left[\mathbf{c}_2{}^{\mathbf{T}}\mathbf{x}_2 + \mathbf{c}_3{}^{\mathbf{T}}\mathbf{x}_3\right] \\ s.t. \\ \mathbf{A}_2\mathbf{x}_2 \geq \mathbf{b}_2 \\ \mathbf{E}_2\mathbf{x}_2 \geq \mathbf{h}_1 - \mathbf{F}_1\hat{\mathbf{x}}_1 \\ \mathbf{A}_3\mathbf{x}_3 \geq \mathbf{b}_3 \\ \mathbf{F}_2\mathbf{x}_2 + \mathbf{E}_3\mathbf{x}_3 \geq \mathbf{h}_2 \\ \mathbf{x}_2, \mathbf{x}_3 \geq 0 \end{aligned} \tag{4.30}$$

Where:

- $\hat{\mathbf{x}}_1$ = The trial value of $\mathbf{x}_1$ found from the solution of master problem 1.

The first master problem is easy to solve via linear programming techniques but the subproblem $\alpha_1(\hat{\mathbf{x}}_1)$ is still complex. To overcome this, the first subproblem is split into another master problem and a subproblem as shown in Equations 4.31 and 4.32.

Master Problem 2

$$
\begin{aligned}
\alpha_1(\hat{\mathbf{x}}_1) = \min & \left[\mathbf{c}_2{}^{\mathbf{T}}\mathbf{x}_2 + \alpha_2\right] \\
s.t. & \\
& \mathbf{A}_2\mathbf{x}_2 \geq \mathbf{b}_2 \\
& \mathbf{E}_2\mathbf{x}_2 \geq \mathbf{h}_1 - \mathbf{F}_1\hat{\mathbf{x}}_1 \\
& \alpha_2 \geq \alpha_2(\hat{\mathbf{x}}_2) - (\mathbf{x}_2 - \hat{\mathbf{x}}_2)^{\mathbf{T}}\mathbf{F}_2^{\mathbf{T}}\boldsymbol{\lambda}^{\hat{\mathbf{x}}_2} \\
& \mathbf{x}_2 \geq 0
\end{aligned} \tag{4.31}
$$

Where:

- $\alpha_1(\hat{\mathbf{x}}_1)$ = Approximate value of optimal solution to master problem 2.
- $\hat{\mathbf{x}}_1$ = The trial value of $\mathbf{x}_1$ found from the solution of master problem 1.
- α_2 = Approximate solution of subproblem 2.
- $\boldsymbol{\lambda}^{\hat{\mathbf{x}}_2}$ = Dual variable found from the dual solution of subproblem 2 for the trial value $\hat{\mathbf{x}}_2$.

Subproblem 2

$$
\begin{aligned}
\alpha_2(\hat{\mathbf{x}}_2) = \min & \left[\mathbf{c}_3{}^{\mathbf{T}}\mathbf{x}_3 + \alpha_3\right] \\
s.t. & \\
& \mathbf{A}_3\mathbf{x}_3 \geq \mathbf{b}_3 \\
& \mathbf{E}_3\mathbf{x}_3 \geq \mathbf{h}_2 - \mathbf{F}_2\hat{\mathbf{x}}_3 \\
& \alpha_3 = 0 \\
& \mathbf{x}_3 \geq 0
\end{aligned} \tag{4.32}
$$

Where:

- $\hat{\mathbf{x}}_2$ = The trial value of $\mathbf{x}_2$ found from the solution of master problem 2.

Note how in Equation 4.32 $\alpha_3 = 0$. This is because subproblem 2 is the final time period of the optimisation problem. The function α_3 symbolically represents the cost past the end of the optimisation. If a real world problem has some estimated costs, or salvage value beyond the optimisation time period the function, α_3, can be assumed to take this value but if it is unknown the function is usually assumed to be equal to zero. In real world planning problems where a value of zero is assumed, there is potential for 'end effects' of the optimisation to be experienced [59]. This is where the solutions to the final time periods of the optimisation can be skewed in some way by the zero value assumption. To overcome this the optimisation is often undertaken over a slightly longer time frame than is required where the 'extra' time periods negate the end effects to some extent.

In Equation 4.29, α_1 approximates the value of the first subproblem. In Equation 4.30, α_2 approximates the value of the second subproblem. Graphically this concept can be illustrated as shown in Figure 4.5 where α_3 is not represented due to its zero value assumption. Any length multistage problem can be split into a series of master problems and subproblems. Each subproblem approximation is an approximation of optimal solution of all the remaining time periods.

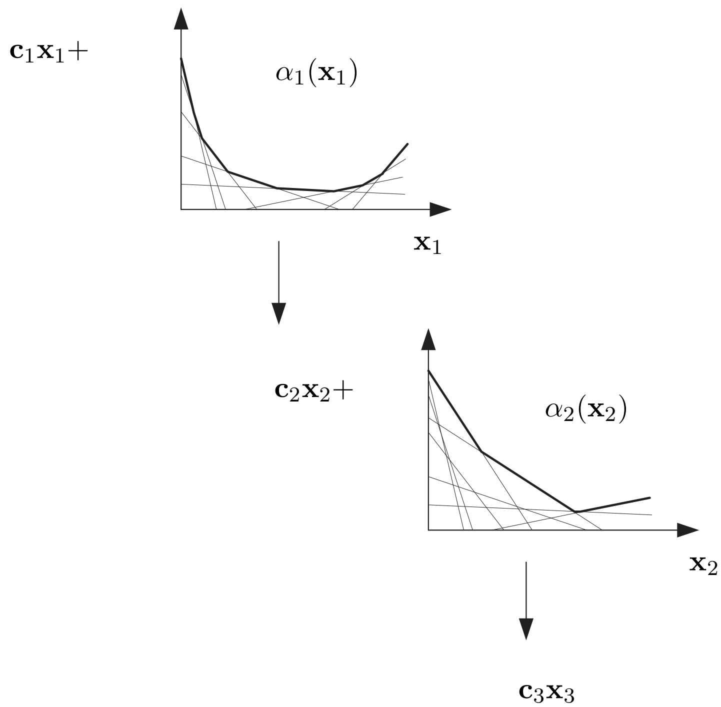


Figure 4.5 Multistage Master and Subproblems

4.3.6 Multistage Benders Decomposition Algorithm

The Benders Decomposition algorithm to solve multistage optimisation problems uses the same concept of using Benders Cuts to approximate the subproblems, adding additional cuts to improve the subproblem approximation and testing bounds on the optimal solution value to assess when the algorithm should exit. The main differences between the multistage algorithm and the single stage algorithm presented in Section 4.3.4.3 is the use of a forward pass to compute the 'trial' values of the master problem complicating variables and a backward pass to compute the dual variables and hence Benders Cuts of the subproblems. The algorithm is presented next [4]:

Input: A linear programming problem with complicating variables.

Output: The solution to the linear programming problem with complicating variables using Benders Decomposition.

Step 1: Initialise iteration counter $k = 1$,upper bound $z_{up_k} = +\infty$, lower bound $z_{low_k} = 0$,and $\alpha_t(\mathbf{x}_t) = 0$. Define the convergence tolerance ϵ.

Step 2: Calculate the solution to the first time period master problem to obtain $\mathbf{x}_1^k$. Use this trial value of $\mathbf{x}_1$ to calculate the lower bound as in Equation 4.33.

$$z_{low_k} = c_1{}^{\mathbf{T}}\mathbf{x}_1^k + \alpha_1(\mathbf{x}_1^k) \tag{4.33}$$

Check for convergence between the upper and lower bounds.

If:

$$z_{up_k} - z_{low_k} \leq \epsilon$$

then terminate the algorithm. The optimal solution has been found.

Else if:

$$z_{up_k} - z_{low_k} > \epsilon$$

continue to Step 3.

Step 3: Forward Pass.

For each time period $t = 1, \ldots, T$, solve the master problem as in Equation 4.35.

$$\begin{aligned}
\alpha_{t-1}(\mathbf{x}_{t-1}^k) = \min \left[\mathbf{c}_t{}^{\mathbf{T}}\mathbf{x}_t + \alpha_t\right] & \\
s.t. & \\
& \mathbf{A}_t\mathbf{x}_t \geq \mathbf{b}_t \\
& \mathbf{E}_t\mathbf{x}_t \geq \mathbf{h}_{t-1} - \mathbf{F}_{t-1}\mathbf{x}_{t-1}^k \\
& \alpha_t \geq \alpha_t(\mathbf{x}_t^{k-1}) - (\mathbf{x}_t - \mathbf{x}_t^{k-1})^{\mathbf{T}}\mathbf{F}_t^{\mathbf{T}}\boldsymbol{\lambda}^{\mathbf{x}_t^{k-1}} && (4.34) \\
& \mathbf{x}_t \geq 0 && (4.35)
\end{aligned}$$

Where:

- α_t = Value of approximated subproblem.
- $\alpha_t \geq \alpha_t(\mathbf{x}_t^{k-1}) - (\mathbf{x}_t - \mathbf{x}_t^{k-1})^{\mathbf{T}}\mathbf{F}_t^{\mathbf{T}}\boldsymbol{\lambda}^{\mathbf{x}_t^{k-1}}$ = Benders Cut that approximates the function $\alpha_t(\mathbf{x}_t)$. This cut is calculated during the previous backward pass.
- $\boldsymbol{\lambda}^{\mathbf{x}_t^{k-1}}$ = Dual variable(s) found by solving the dual of the subproblem to this master problem on the previous backward pass.
- $\mathbf{x}_{t-1}^k$ = Trial solution to the previous time period forward pass master problem.

Store the optimal value of the decision variables $\mathbf{x}_t$ as $\mathbf{x}_t^k$. These become the ‘trial’ variables used by the subproblem calculations.

Note that constraint 4.34 will not be present in the master problem if this iteration is $k = 1$. This is because at the beginning of the algorithm no Benders Cuts are known, they are calculated by solving the subproblem in Step 5 below.

Step 4: Calculate the new upper bound by Equation 4.36.

$$z_{up_k} = \sum_{t=1}^{T} \mathbf{c}_t{}^{\mathbf{T}} \mathbf{x}_t^k \tag{4.36}$$

The upper bound calculation uses the current optimal variable values to compute the value of the original problem objective function.

Step 5: Backward Pass.

Solve the **dual** of the subproblems at each time period $t = T, T-1, \ldots, 2$

Where the subproblem is given in Equation 4.37:

$$\begin{aligned} \alpha_{t-1}(\mathbf{x}_{t-1}^k) = \min &\left[\mathbf{c}_t{}^{\mathbf{T}} \mathbf{x}_t + \alpha_t\right] \\ s.t.& \\ &\mathbf{A}_t \mathbf{x}_t \geq \mathbf{b}_t \\ &\mathbf{E}_t \mathbf{x}_t \geq \mathbf{h}_{t-1} - \mathbf{F}_{t-1} \mathbf{x}_{t-1}^k \\ &\alpha_t \geq \alpha_t(\mathbf{x}_t^k) - (\mathbf{x}_t - \mathbf{x}_t^k)^{\mathbf{T}} \mathbf{F}_t^{\mathbf{T}} \boldsymbol{\lambda}^{\mathbf{x}_t^k} \\ &\mathbf{x}_t \geq 0 \end{aligned} \tag{4.37}$$

Where:

- α_t = Value of approximated subproblem.
- $\alpha_t \geq \alpha_t(\mathbf{x}_t^k) - (\mathbf{x}_t - \mathbf{x}_t^k)^{\mathbf{T}} \mathbf{F}_t^{\mathbf{T}} \boldsymbol{\lambda}^{\mathbf{x}_t^k}$ = Benders cut that approximates the function $\alpha_t(\mathbf{x}_t)$
- $\boldsymbol{\lambda}^{\mathbf{x}_t^k}$ = Dual variable(s) found by solving the dual of the subproblem at time $t = t+1$ on this backward pass.
- $\mathbf{x}_{t-1}^k$ = Trial solution found from forward pass master problem at time $t = t-1$, iteration k.

Let $\boldsymbol{\lambda}^{\mathbf{x}_{t-1}^k}$ be the dual variables associated with the constraints of the subproblem 4.37. These dual variables are used in calculating the Benders Cut

$$\alpha_{t-1} \geq \alpha_{t-1}(\mathbf{x}_{t-1}^k) - (\mathbf{x}_{t-1} - \mathbf{x}_{t-1}^k)^{\mathbf{T}} \mathbf{F}_{t-1}^{\mathbf{T}} \boldsymbol{\lambda}^{\mathbf{x}_{t-1}^k}$$

which is added to the correct time period forward pass master problem and correct time period backward pass subproblem.

Increment the iteration counter $k \leftarrow k + 1$

Step 6: Return to Step 2.

4.3.7 Multistage Benders Decomposition Example

To illustrate the multistage Benders Decomposition algorithm the planning problem of 3.2.4 is solved using the multistage or nested Benders Decomposition. The example is simplified and illustrative only and does not use realistic costing values as would be seen in a real world power system planning problem.

The problem: A power system currently has generating capacity equal to 800MW and peak demand of 750MW. Demand has been forecast to grow at 60MW per year for at least the next three years. New generation investments are costed at \$150/MW. Fixed costs for all installed capacity is \$3/MW per year and variable costs for generation are \$2/MWh. The planning problem is to minimise the cost of investment and operation of the power system for the next three years while ensuring capacity is always greater than demand. The optimisation problem is written mathematically as Equation 4.38.

$$
\begin{aligned}
f(\mathbf{c}_0) = min\big[&(150\mathbf{x}_1 + 3\mathbf{c}_1 + 17520\mathbf{d}_1)+\\
&(150\mathbf{x}_2 + 3\mathbf{c}_2 + 17520\mathbf{d}_2)+\\
&(150\mathbf{x}_3 + 3\mathbf{c}_3 + 17520\mathbf{d}_3)\big]
\end{aligned}
$$

Subject to:

$$
\begin{aligned}
&\mathbf{c}_0 = 800\\
&\mathbf{c}_0 + \mathbf{x}_1 = \mathbf{c}_1\\
&\mathbf{c}_1 \geq \mathbf{d}_1\\
&\mathbf{d}_1 = 810\\
&\mathbf{c}_1 + \mathbf{x}_2 = \mathbf{c}_2\\
&\mathbf{c}_2 \geq \mathbf{d}_2\\
&\mathbf{d}_2 = 870\\
&\mathbf{c}_2 + \mathbf{x}_3 = \mathbf{c}_3\\
&\mathbf{c}_3 \geq \mathbf{d}_3\\
&\mathbf{d}_3 = 930\\
&\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \geq 0 \qquad (4.38)
\end{aligned}
$$

Where:

- $\mathbf{c}_0, \mathbf{c}_1, \mathbf{c}_2$ = Value of system capacity at the **beginning** of time periods 1,2 and 3 respectively.
- $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ = Investment decision variable in time periods 1,2 and 3 respectively where investment capacity is added at the start of a time period.
- $\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3$ = Value of system demand at the **end** of each respective time period.
- 150 = Cost per MW for new generation investments.
- 3 = Fixed cost of capacity = \$3/MW
- 17520 = Variable cost of generation (hours in year x \$2/MWh)
- $f(\mathbf{c}_0)$ = Optimal installation and operation cost at time period 0 (the beginning of the optimisation) for a system with $\mathbf{c}_0$ amount of capacity.

The state variable for this problem is system capacity which is represented by the variables $\mathbf{c}_0, \mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3$ in each time period respectively. Splitting Equation 4.38 into master problems and subproblems gives Equation 4.39 as Master Problem 1, Equation 4.40 as Master Problem 2 and Subproblem 1 and Equation 4.41 as Subproblem 2 and Master Problem 3.

Master Problem 1

$$\begin{aligned} f(\mathbf{c}_0) = min\big[(150\mathbf{x}_1 + 3\mathbf{c}_1 + 17520\mathbf{d}_1) + \alpha_1\big] \\ \text{Subject to:} \\ \mathbf{c}_0 &= 800 \\ \mathbf{c}_0 + \mathbf{x}_1 &= \hat{\mathbf{c}}_1 \\ \hat{\mathbf{c}}_1 &\geq \mathbf{d}_1 \\ \mathbf{d}_1 &= 810 \\ \alpha_1 &\geq \alpha(\hat{\mathbf{c}}_1) - (\mathbf{c}_1 - \hat{\mathbf{c}}_1)\mathbf{F}_1^{\mathbf{T}}\boldsymbol{\lambda}^{\hat{\mathbf{c}}_1} \\ \mathbf{x}_1 &\geq 0 \end{aligned} \tag{4.39}$$

Master Problem 2 = Subproblem 1

$$\begin{aligned} \alpha_1(\hat{\mathbf{c}}_1) = min\big[(150\mathbf{x}_2 + 3\mathbf{c}_2 + 17520\mathbf{d}_2) + \alpha_2\big] \\ \text{Subject to:} \\ \hat{\mathbf{c}}_1 + \mathbf{x}_2 &= \hat{\mathbf{c}}_2 \\ \hat{\mathbf{c}}_2 &\geq \mathbf{d}_2 \\ \mathbf{d}_2 &= 870 \\ \alpha_2 &\geq \alpha(\hat{\mathbf{c}}_2) - (\mathbf{c}_2 - \hat{\mathbf{c}}_2)\mathbf{F}_2^{\mathbf{T}}\boldsymbol{\lambda}^{\hat{\mathbf{c}}_2} \\ \mathbf{x}_2 &\geq 0 \end{aligned} \tag{4.40}$$

Subproblem 2 = Master Problem 3

$$
\begin{aligned}
\alpha_2(\hat{\mathbf{c}}_2) = min\big[(150\mathbf{x}_3 + 3\mathbf{c}_3 + 17520\mathbf{d}_3) + \alpha_3\big] \\
\text{Subject to:} \\
\mathbf{c}_2 + \mathbf{x}_3 = \hat{\mathbf{c}}_3 \\
\hat{\mathbf{c}}_3 \geq \mathbf{d}_3 \\
\mathbf{d}_3 = 930 \\
\alpha_3 \geq 0 \\
\mathbf{x}_3 \geq 0
\end{aligned}
\tag{4.41}
$$

Using the Multistage Benders Decomposition Algorithm described in Section 4.3.6 the optimisation problem is solved as follows:

Step 1: Initialisations.

- Iteration counter $k = 1$.
- Upper bound $z_{up} = +\infty$.
- Lower bound $z_{low} = 0$.
- $\alpha_t(\hat{\mathbf{c}}_t) = 0$.
- $\epsilon = 1000$.

Step 2: Calculate the solution to the first time period master problem.

$$
\begin{aligned}
f(\mathbf{c}_0) = min\big[(150\mathbf{x}_1 + 3\mathbf{c}_1 + 17520\mathbf{d}_1) + \alpha_1\big] \\
\text{Subject to:} \\
\mathbf{c}_0 = 800 \\
\mathbf{c}_0 + \mathbf{x}_1 = \mathbf{c}_1^1 \\
\mathbf{c}_1^1 \geq \mathbf{d}_1 \\
\mathbf{d}_1 = 810 \\
\alpha_1 = 0 \\
\mathbf{x}_1 \geq 0
\end{aligned}
\tag{4.42}
$$

The optimal solution is $\mathbf{x}_1^1 = 10$.

Calculating the lower bound by Equation 4.33, $z_{low_k} = \$14,195,130$.

Check for convergence between the upper and lower bounds. Currently $z_{up_k} = +\infty$ and $z_{low_k} = \$14,195,130$ so

$$z_{up_k} - z_{low_k} \nleq \epsilon$$

and the algorithm moves to Step 3.

Step 3: Forward Pass

For each time period $t = 1, \ldots, T$, solve the master problem.

Time $t = 1$, Solve Equation 4.43 as Master Problem 1.

$$\begin{aligned}
f(\mathbf{c}_0) = min\big[(150\mathbf{x}_1 + 3\mathbf{c}_1 + 17520\mathbf{d}_1) + \alpha_1\big] \\
\text{Subject to:} \\
\mathbf{c}_0 &= 800 \\
\mathbf{c}_0 + \mathbf{x}_1 &= \mathbf{c}_1^1 \\
\mathbf{c}_1^1 &\geq \mathbf{d}_1 \\
\mathbf{d}_1 &= 810 \\
\alpha_1 &= 0 \\
\mathbf{x}_1 &\geq 0
\end{aligned} \tag{4.43}$$

The solution to this problem is: $f(\mathbf{c}_0) = \$14,195,130,\ \mathbf{x}_1^1 = 10MW,\ \mathbf{c}_1^1 = 810MW$

Time $t = 2$, Solve Equation 4.44 as Master Problem 2.

$$\begin{aligned}
\alpha_1(\mathbf{c}_1^1) = min\big[(150\mathbf{x}_2 + 3\mathbf{c}_2 + 17520\mathbf{d}_2) + \alpha_2\big] \\
\text{Subject to:} \\
\mathbf{c}_1^1 + \mathbf{x}_2 &= \mathbf{c}_2^1 \\
\mathbf{c}_2^1 &\geq \mathbf{d}_2 \\
\mathbf{d}_2 &= 870 \\
\alpha_2 &= 0 \\
\mathbf{x}_2 &\geq 0
\end{aligned} \tag{4.44}$$

The solution to this problem is: $\alpha_1(\mathbf{c}_1^1) = \$15,254,010,\ \mathbf{x}_2^1 = 60MW,\ \mathbf{c}_2^1 = 870MW$

Time $t = 3$, Solve Equation 4.45 as Master Problem 3.

$$
\begin{aligned}
\alpha_2(\mathbf{c}_2^1) = min\big[&(150\mathbf{x}_3 + 3\mathbf{c}_3 + 17520\mathbf{d}_3) + \alpha_3\big] \\
\text{Subject to:}& \\
&\mathbf{c}_2^1 + \mathbf{x}_3 = \mathbf{c}_3^1 \\
&\mathbf{c}_3^1 \geq \mathbf{d}_3 \\
&\mathbf{d}_3 = 930 \\
&\alpha_3 = 0 \\
&\mathbf{x}_3 \geq 0
\end{aligned} \tag{4.45}
$$

The solution to this problem is: $\alpha_2(\mathbf{c}_2^1) = \$16,305,390$, $\mathbf{x}_3^1 = 60MW$, $\mathbf{c}_3^1 = 930MW$

Step 4: Calculate the new upper bound by Equation 4.46.

$$
z_{up_k} = \big[(150\mathbf{x}_1^1 + 3\mathbf{c}_1^1 + 17520\mathbf{d}_1) + (150\mathbf{x}_2^1 + 3\mathbf{c}_2^1 + 17520\mathbf{d}_2) + (150\mathbf{x}_3^1 + 3\mathbf{c}_3^1 + 17520\mathbf{d}_3)\big] \tag{4.46}
$$

$$
\begin{aligned}
z_{up_k} = \big[&(150 \times 10 + 3 \times 810 + 17520 \times 810) \\
&+ (150 \times 60 + 3 \times 870 + 17520 \times 870) \\
&+ (150 \times 60 + 3 \times 930 + 17520 \times 930)\big] = \$45,754,530
\end{aligned} \tag{4.47}
$$

Step 5: **Backward Pass**.

Solve the **dual** of the subproblems for each time period $t = T, T-1, \ldots, 2$

Time $t = 3$, Solve Equation 4.48 as Subproblem 2.

$$\alpha_2(\mathbf{c}_2^1) = min\big[(150\mathbf{x}_3 + 3\mathbf{c}_3 + 17520\mathbf{d}_3) + \alpha_3\big]$$
$$\text{Subject to:}$$
$$\mathbf{c}_2^1 + \mathbf{x}_3 = \mathbf{c}_3$$
$$\mathbf{c}_3 \geq \mathbf{d}_3$$
$$\mathbf{d}_3 = 930$$
$$\alpha_3 = 0$$
$$\mathbf{x}_3 \geq 0 \tag{4.48}$$

Where:

- $\mathbf{c}_2^1 = 870MW$ as found from the master problem at time $t = 2$ on the previous forward pass.

The solution to this problem is: $\alpha_2(\mathbf{c}_2^1) = \$16,305,240$, $\mathbf{x}_3 = 60MW$, $\mathbf{c}_3 = 930MW$, $\boldsymbol{\lambda}^{\mathbf{c}_2^1} = 150$.

Calculating the Benders Cut: $\alpha_2 \geq \alpha_2(\mathbf{c}_2^1) - (\mathbf{c}_2 - \mathbf{c}_2^1)\mathbf{F_2}^{\mathbf{T}}\boldsymbol{\lambda}^{\mathbf{c}_2^1} \Rightarrow \alpha_2 \geq -150\mathbf{c}_2 + 16435890$

Time $t = 2$, Solve Equation 4.49 as Subproblem 1.

$$\alpha_1(\mathbf{c}_1^1) = min\big[(150\mathbf{x}_2 + 3\mathbf{c}_2 + 17520\mathbf{d}_2) + \alpha_2\big]$$
$$\text{Subject to:}$$
$$\mathbf{c}_1^1 + \mathbf{x}_2 = \mathbf{c}_2$$
$$\mathbf{c}_2 \geq \mathbf{d}_2$$
$$\mathbf{d}_2 = 870$$
$$\alpha_2 \geq -150\mathbf{c}_2 + 16435890$$
$$\mathbf{x}_2 \geq 0 \tag{4.49}$$

The solution to this problem is: $\alpha_1(\mathbf{c}_1^1) = \$31,559,400$, $\mathbf{x}_2 = 60MW$, $\mathbf{c}_2 = 870MW$, $\boldsymbol{\lambda}^{\mathbf{c}_1^1} = 150$.

Calculating the Benders Cut: $\alpha_1 \geq \alpha_1(\mathbf{c}_1^1) - (\mathbf{c}_1 - \mathbf{c}_1^1)\mathbf{F_1}^{\mathbf{T}}\boldsymbol{\lambda}^{\mathbf{c}_1^1} \Rightarrow \alpha_1 \geq -150\mathbf{c}_1 + 31680900$

Increment the iteration counter $k = 2$.

Step 6: Go back to Step 2.

Step 2: Calculate the solution to the Master problem 1, now with an updated constraint for $\alpha_1(\mathbf{c}_1)$.

$$f(\mathbf{c}_0) = min\big[(150\mathbf{x}_1 + 3\mathbf{c}_1 + 17520\mathbf{d}_1) + \alpha_1\big]$$

Subject to:

$$\begin{aligned} &\mathbf{c}_0 = 800 \\ &\mathbf{c}_0 + \mathbf{x}_1 = \mathbf{c}_1 \\ &\mathbf{c}_1 \geq \mathbf{d}_1 \\ &\mathbf{d}_1 = 810 \\ &\alpha_1 \geq -150\mathbf{x}_2 + 31680900 \\ &\mathbf{x}_1 \geq 0 \end{aligned} \tag{4.50}$$

The solution is $\mathbf{x}_1 = 10$. Calculate the new lower bound by Equation 4.51.

$$z_{low_2} = (150\mathbf{x}_1 + 3\mathbf{c}_1 + 17520\mathbf{d}_1) + (-150\mathbf{c}_2 + 31680900) \tag{4.51}$$

$$z_{low_2} = (150 \times 10 + 3 \times 810 + 17520 \times 810) + (-150 \times 810 + 31680900) = \$45,754,530 \tag{4.52}$$

Check for convergence between the upper and lower bounds. Currently $z_{up_2} = \$45,754,530$ and $z_{low_2} = \$45,754,530$ so

$$z_{up} - z_{low} \leq \epsilon$$

and the algorithm finishes.

The Solution: The solution of the multistage algorithm gives an optimal cost of $45,754,530. This is the minimum cost of investment and operation of the power system over the three year time period. The optimal investment path is given by $\mathbf{x}_1 = 10, \mathbf{x}_2 = 60, \mathbf{x}_3 = 60$.

Comparing the solution obtained from the multistage algorithm with the deterministic dynamic programming method of Section 3.2.4 for the same example problem, it can be seen that the same optimal cost and investment path is found in each solution method. The biggest difference is in the number of single stage linear programming problems that must be solved. In the example of Section 3.2.4 that used deterministic dynamic programming, 23 single stage linear programming optimisations were calculated over the three year optimisation problem. In the Multistage Benders Decomposition algorithm, 7 single stage linear programming optimisations were calculated over the three year optimisation problem. This is a large difference and illustrates the computational savings gained by approximating the future cost function via linear constraints as in Benders Decomposition.

The example problem solved using Multistage Benders Decomposition consisted of only one forward and one backward pass before the upper bound and lower bound were equal. This will not always be the case. In more complex problems, a number of forward and backward passes may be necessary before the bounds are close enough. Where multiple forward passes are required, the single stage problem in the forward pass is altered by adding in the relevant linear cut (as calculated on the previous backward pass)to approximate the future cost function.

4.4 STOCHASTIC DUAL DYNAMIC PROGRAMMING

The Deterministic Dual Dynamic Programming technique can be extended to handle stochastic variables. Planning problems inherently deal with forecasting future variables and assessment of the validity of the forecast values. Incorporating stochastic variables into a dynamic optimisation problem allows the problem solver to understand the effect that stochasticity has on the optimal solution to the problem.

Stochastic Dual Dynamic Programming or SDDP operates on the same principles as Deterministic Dual Dynamic Programming. The optimisation problem is split into a series of master problems and subproblems, the subproblems are approximated by Benders Cuts and the algorithm exits when the upper bound and lower bound meet within tolerance limits. The effect that stochasticity has on the decomposition process is to introduce uncertainty into the subproblem calculation. Each subproblem is represented by a number of potential subproblems, where each potential subproblem has a probability of occurrence. The probability of subproblem occurrence is the probability of the stochastic variable for that potential subproblem. To approximate the original subproblem with a single Benders Cut, the expected value of the dual variables from each potential subproblem is found and an expected Benders Cut is calculated. This cut is used to approximate the *expected* value of the original subproblem. By using an expected approximation to the subproblem the upper and lower bound become *expected* upper and lower bounds and the optimal solution is an *expected* optimal solution.

Periera and Pinto [4] consider an optimisation problem similar to Equation 4.53. They present the solution to this stochastic problem via Benders Decomposition. A similar exposition is presented below. Equation 4.53 is split into a single master problem that solves for $\mathbf{x}_1$ and n subproblems that occur with probability p_j.

$$
\begin{aligned}
z = min\big[\mathbf{c_1}^\mathbf{T}\mathbf{x} + p_1\mathbf{c_2}^\mathbf{T}\mathbf{x}_{21} + p_2\mathbf{c_2}^\mathbf{T}\mathbf{x}_{22} + \ldots + p_n\mathbf{c_2}^\mathbf{T}\mathbf{x}_{2n}\big]& \\
\text{Subject to:}& \\
\mathbf{A}\mathbf{x}_1 &\geq \mathbf{b}_1 \\
\mathbf{F}\mathbf{x}_1 + \mathbf{E}\mathbf{x}_{21} &\geq \mathbf{h}_{21} \\
\mathbf{F}\mathbf{x}_1 + \mathbf{E}\mathbf{x}_{22} &\geq \mathbf{h}_{22} \\
\ddots& \\
\mathbf{F}\mathbf{x}_1 + \mathbf{E}\mathbf{x}_{2n} &\geq \mathbf{h}_{2n} \\
\mathbf{x}_1 \geq 0, \mathbf{x}_{21}, \ldots, \mathbf{x}_{2n} &\geq 0
\end{aligned} \tag{4.53}
$$

Where:

- p_j represents the probability of subproblem j occurring. Note that $\sum_{j=1}^{n} p_j = 1$.

The master problem is shown in Equation 4.54 where the problem is to optimise the first stage complicating variables $\mathbf{x}_1$ plus the *expected* value of the approximation to the subproblem $\bar{\alpha}(\mathbf{x}_1)$.

Master Problem

$$
\begin{aligned}
z = \min\left[\mathbf{c_1}^\mathbf{T}\mathbf{x}_1 + \bar{\alpha}(\mathbf{x}_1)\right]& \\
\text{Subject to:}& \\
\mathbf{A}\mathbf{x}_1 &\geq \mathbf{b}_1 \\
\mathbf{x}_1 &\geq 0
\end{aligned} \tag{4.54}
$$

Where:

- $\bar{\alpha}(\mathbf{x}_1)$ = The expected value of the second stage subproblems.

There are n subproblems, each occurring with probability p_j. Equation 4.55 illustrates the subproblem j for a 'trial' variable $\hat{\mathbf{x}}_1$ from the master problem.

Subproblem

$$
\begin{aligned}
\alpha_j(\hat{\mathbf{x}}_1) = \min\left[\mathbf{c_2}^\mathbf{T}\mathbf{x}_{2j}\right]& \\
\text{Subject to:}& \\
\mathbf{E}\mathbf{x}_{2j} &\geq \mathbf{h}_{2j} - \mathbf{F}\hat{\mathbf{x}}_1 \\
\mathbf{x}_{2j} &\geq 0
\end{aligned} \tag{4.55}
$$

All the subproblems from $j = 1, \ldots, n$ are solved, the dual of each subproblem finding a dual variable $\boldsymbol{\lambda}_j^{\hat{\mathbf{x}}_1}$ and optimal solution cost $\alpha_j(\hat{\mathbf{x}}_1)$. The expected value of the dual variable is given by Equation 4.56 and the expected value of the optimal solution is given by Equation 4.57.

$$\bar{\boldsymbol{\lambda}}^{\hat{\mathbf{x}}_1} = \sum_{j=1}^{n} p_j \boldsymbol{\lambda}_j^{\hat{\mathbf{x}}_1} \tag{4.56}$$

$$\bar{\alpha}(\mathbf{x}_1) = \sum_{j=1}^{n} p_j \alpha_j(\hat{\mathbf{x}}_1) \tag{4.57}$$

The expected optimal value of the subproblem and the expected value of the dual variable is used to calculate a Benders Cut to add to the master problem.

4.4.1 Benders Cut Calculation

For a deterministic problem the Benders Cut is given by Equation 4.58, where $\boldsymbol{\lambda}^{\hat{\mathbf{x}}_1}$ is the dual variable found by solving the subproblem with a 'trial' variable $\hat{\mathbf{x}}_1$ from the master problem.

$$\alpha \geq \alpha(\hat{\mathbf{x}}_1) - (\mathbf{x}_1 - \hat{\mathbf{x}}_1)^T \mathbf{F}^{\mathbf{T}} \boldsymbol{\lambda}^{\hat{\mathbf{x}}_1} \tag{4.58}$$

The derivation and form of a Benders Cut for stochastic problems is the same as for deterministic problems with two exceptions. The expected value of the subproblem and the expected value of the dual variables are used. Equation 4.59 shows the form of the Benders Cuts for a stochastic optimisation problem.

$$\alpha \geq \bar{\alpha}(\mathbf{x}_1) - (\mathbf{x}_1 - \hat{\mathbf{x}}_1)^T \mathbf{F}^{\mathbf{T}} \bar{\boldsymbol{\lambda}}^{\hat{\mathbf{x}}_1} \tag{4.59}$$

4.4.2 Multistage Stochastic Dual Dynamic Programming Algorithm

The Stochastic Dual Dynamic Programming algorithm has the same structure as Deterministic Dual Dynamic Programming. An expected upper bound is compared to an expected lower bound, if they are within tolerance then the algorithm exits. The biggest change to the algorithm between a stochastic and deterministic problem is the introduction of sampling and Monte Carlo simulation, as well as changes to the way in which the expected upper and lower bounds are calculated. Details of these changes are presented in Sections 4.4.2.1,4.4.2.2 and 4.4.2.3 below.

4.4.2.1 Sampling and Monte Carlo Simulation

Forward Pass: Ideally, whenever a single stage problem is solved on the forward pass the problem would be solved repeated times, once for every realisation of the stochastic variable. This would become computationally intractable very quickly. An alternative is to use Monte Carlo simulation [4] [56]. The forward pass is solved a number of times, where each optimisation problem on each forward pass samples the value of the stochastic variable from either a discrete or continuous probability distribution. The 'trial' variables found from solving each problem are stored for use on the backward pass. If, for example, there are m forward passes, there will be m 'trial' variables at each time period of the optimisation.

Backward Pass: Due to the forward pass being run m times, there are m 'trial' variables from the master problem for each subproblem on the backward pass. The subproblem is solved for each value of the 'trial' variable and for each realisation of the stochastic variable. The stochastic variable many have infinitely many realisations if it is defined by a continuous probability distribution, therefore a certain number of realisations must be sampled from the distribution. At each 'trial' variable value, at each time period, the algorithm calculates an expected value of the subproblem and expected dual variable. The associated expected Benders Cut is calculated and added to the master problem. For an optimisation with m forward passes, m Benders Cuts will be calculated at each time period on the backward pass.

4.4.2.2 Upper Bound

The upper bound of the deterministic multistage Benders Decomposition is found by calculating the value of the objective function of the original optimisation problem using the current 'trial' variable values. The upper bound in SDDP is found in a similar fashion except that the upper bound is an expected value. Each time the forward pass is calculated, the 'trial' variables from each time period solution are used to calculate the value of the original objective function. As the forward pass is calculated a number of times during the Monte Carlo simulation there will be a number of values of the original objective function. The *expected* upper bound is given by the Equation 4.67 where the average of each forward pass original objective function value is calculated.

$$\bar{z}_{up} = \frac{1}{m} \sum_{i=1}^{m} z_i \tag{4.60}$$

Where:

- $z_i = \sum_{t=1}^{T} \mathbf{c}_t{}^{\mathbf{T}} \hat{\mathbf{x}}_t$.
- m = number of forward passes calculated.

4.4.2.3 Lower Bound

The calculation of the expected lower bound in SDDP is the same as for the deterministic problem. It is the solution of the first stage problem, Equation 4.61.

$$
\begin{aligned}
z = min\big[\mathbf{c}_1{}^{\mathbf{T}}\mathbf{x} + \bar{\alpha}(\mathbf{x})\big] \\
\text{Subject to:} \\
\mathbf{A}\mathbf{x} \geq \mathbf{b} \\
\mathbf{x} \geq 0
\end{aligned}
\tag{4.61}
$$

Where:

- $\bar{\alpha}(\mathbf{x})$ = The *expected* value of the second stage subproblems

4.4.2.4 Comparison of Bounds

For deterministic problems the upper bound and lower bound are compared directly and when the difference between the two is small enough the algorithm exits. For stochastic problems the upper bound is an expected value for which there is an uncertainty around the estimate of the upper bound. The uncertainty around the calculation of the expected upper bound can be given by the standard deviation of the expected upper bound as in Equation 4.62.

$$
\sigma = \frac{1}{m-1}\Big[\sum_{i=1}^{m}(z_i - \bar{z}_{up})^2\Big]^{\frac{1}{2}} \tag{4.62}
$$

Where:

- z_i = Upper bound calculated on forward pass i.
- m = number of forward passes calculated.
- $\bar{z}_{up}$ = Expected upper bound.

It is now possible to statistically quantify the ‘true’ value of the upper bound using confidence intervals. For example the 95% confidence interval given in 4.63 states that with 95% confidence the actual upper bound will be found within the range stated in 4.63.

$$z_{up} \in \left[(\bar{z}_{up} - 1.96\sigma), (\bar{z}_{up} + 1.96\sigma)\right] \tag{4.63}$$

Where:

- σ = Standard deviation of the upper bound calculated in Equation 4.62.
- $\bar{z}_{up}$ = Expected value of the upper bound as calculated in Equation 4.67.

The confidence interval provides a useful test for convergence of the SDDP algorithm such that if the expected lower bound falls within the confidence interval of the expected upper bound the algorithm is considered to have converged.

4.4.2.5 SDDP Algorithm

The Stochastic Dual Dynamic Programming Algorithm is detailed below:

Step 1: Initialise iteration counter $k = 1$, expected upper bound $\bar{z}_{up} = +\infty$, expected lower bound $\bar{z}_{low} = 0$,and $\bar{\alpha}_t(\mathbf{x}_t) = 0$.

Step 2: Calculate the solution to the first time period master problem to obtain $\mathbf{x}_1^k$. Use this trial value of $\mathbf{x}_1$ to calculate the lower bound as in Equation 4.64.

$$\bar{z}_{low_k} = {c_1}^{\mathbf{T}}\mathbf{x}_1^k + \bar{\alpha}_1(\mathbf{x}_1^k) \tag{4.64}$$

Check for convergence between the upper and lower bounds.

If

$$(\bar{z}_{up_k} - 1.96\sigma) \leq \bar{z}_{low_k} \leq (\bar{z}_{up_k} + 1.96\sigma)$$

then terminate the algorithm. The optimal solution has been found.

Else continue to Step 3.

Step 3: Forward Pass.

For each forward pass $i = 1, \ldots, m$

For each time period $t = 1, \ldots, T$, sample the stochastic variable from the stochastic variable distribution and solve the master problem as in Equation 4.66.

$$\bar{\alpha}_{t-1}(\mathbf{x}^k_{t-1,i}) = \min\left[\mathbf{c}_t{}^{\mathbf{T}}\mathbf{x}_{t,i} + \bar{\alpha}_t\right]$$

$$s.t.$$

$$\mathbf{A}_{t,i}\mathbf{x}_{t,i} \geq \mathbf{b}_{t,i}$$

$$\mathbf{E}_{t,i}\mathbf{x}_{t,i} \geq \mathbf{h}_{t-1,i} - \mathbf{F}_{t-1,i}\mathbf{x}^k_{t-1,i}$$

$$\bar{\alpha}_t \geq \bar{\alpha}(\mathbf{x}^{k-1}_{t,i}) - (\mathbf{x}_{t,i} - \mathbf{x}^{k-1}_{t,i})^T\mathbf{F}^{\mathbf{T}}_{t,i}\bar{\boldsymbol{\lambda}}^{\mathbf{x}^{k-1}_{t,i}} \tag{4.65}$$

$$\mathbf{x}_{t,i} \geq 0 \tag{4.66}$$

Where:

- $\bar{\alpha}_t$ = Expected value of approximated subproblem.
- $\bar{\alpha}_t \geq \bar{\alpha}(\mathbf{x}^{k-1}_{t,i}) - (\mathbf{x}_{t,i} - \mathbf{x}^{k-1}_{t,i})^T\mathbf{F}^{\mathbf{T}}_{t,i}\bar{\boldsymbol{\lambda}}^{\mathbf{x}^{k-1}_{t,i}}$ = linear cut that approximates the function $\bar{\alpha}_t(\mathbf{x}_t)$. This cut is calculated during the backward pass.
- $\boldsymbol{\lambda}^{\mathbf{x}^{k-1}_{t,i}}$ = Expected dual variable(s) found by solving the dual of the subproblem to this master problem on the backward pass.
- $\mathbf{x}^k_{t-1,i}$ = Trial solution to the previous time period master problem on this forward pass.

Store the optimal value of the decision variables $\mathbf{x}_t$ as $\mathbf{x}^k_{t,i}$. These become the 'trial' variables used by the subproblem calculations.

Note that constraint 4.65 will not be present in the master problem if this iteration is $k = 1$. This is because at the beginning of the algorithm no Benders Cuts are known, they are calculated by solving the subproblem in Step 5 below.

Step 4: Calculate the new upper bound by Equation 4.67.

$$\bar{z}_{up} = \frac{1}{m}\sum_{i=1}^{m} z_i \tag{4.67}$$

Where:

- $z_i = \sum_{t=1}^{T} \mathbf{c}_t{}^{\mathbf{T}}\mathbf{x}^k_{t,i}$
- m = number of forward passes calculated.

Step 5: Backward Pass.

For each time period $t = T, T-1, \ldots, 2$

For each value of $\mathbf{x}^k_{t-1,i}$

For each sampled value of the stochastic variable

Solve the **dual** of the subproblem given in Equation 4.68.

$$
\begin{aligned}
\bar{\alpha}_{t-1}(\mathbf{x}^k_{t-1,i}) = \min \left[\mathbf{c}_t{}^{\mathbf{T}}\mathbf{x}_{t,i} + \bar{\alpha}_t\right] & \\
s.t. \quad & \\
& \mathbf{A}_{t,i}\mathbf{x}_{t,i} \geq \mathbf{b}_{t,i} \\
& \mathbf{E}_{t,i}\mathbf{x}_{t,i} \geq \mathbf{h}_{t-1,i} - \mathbf{F}_{t-1,i}\mathbf{x}^k_{t-1,i} \\
& \bar{\alpha}_t \geq \bar{\alpha}_t(\mathbf{x}^k_{t,i}) - (\mathbf{x}_{t,i} - \mathbf{x}^k_{t,i})^{\mathbf{T}}\mathbf{F}^{\mathbf{T}}_{t,i}\bar{\boldsymbol{\lambda}}^{\mathbf{x}^k_{t,i}} \\
& \mathbf{x}_{t,i} \geq 0
\end{aligned} \tag{4.68}
$$

Where:

- $\bar{\alpha}_t(\mathbf{x}_t)$ = Expected value of approximated subproblem.
- $\bar{\alpha}_t \geq \bar{\alpha}_t(\mathbf{x}^k_{t,i}) - (\mathbf{x}_{t,i} - \mathbf{x}^k_{t,i})^{\mathbf{T}}\mathbf{F}^{\mathbf{T}}_{t,i}\bar{\boldsymbol{\lambda}}^{\mathbf{x}^k_{t,i}}$ = linear cut that approximates the function $\bar{\alpha}_t$
- $\bar{\boldsymbol{\lambda}}^{\mathbf{x}^k_{t,i}}$ = Dual variable(s) found by solving the dual of the subproblem at time $t = t + 1$ on the backward pass.
- $\mathbf{x}^k_{t-1,i}$ = Trial solution found from forward pass master problem at time $t = t - 1$ and forward pass i.

Let $\bar{\boldsymbol{\lambda}}^{\mathbf{x}^k_{t-1,i}}$ be the dual variables associated with the constraints of the subproblem 4.68. These dual variables are used in calculating the linear constraint,

$$\bar{\alpha}_{t-1} \geq \bar{\alpha}_{t-1}(\mathbf{x}^k_{t-1,i}) - (\mathbf{x}_{t-1,i} - \mathbf{x}^k_{t-1,i})^{\mathbf{T}}\mathbf{F}^{\mathbf{T}}_{t-1,i}\bar{\boldsymbol{\lambda}}^{\mathbf{x}^k_{t-1,i}}$$

The constraint is added to the correct time period forward pass master problem and correct time period backward pass subproblem.

Increment the iteration counter $k \leftarrow k + 1$

Step 6: Return to Step 2.

4.4.3 Independence of Stochastic Variable

The SDDP algorithm above makes the assumption that the stochastic variable at each time period is independent. That is, the value the stochastic variable takes in time period t is not dependent in any way on the realisations of the stochastic variable is time periods $t = t-1, \ldots, 1$. This allows the value of the stochastic variable to be sampled from the distribution without prior consideration for previous realisations of the stochastic variable.

If the stochastic variable not independent, its probabilities must be described as a Markov Chain or a continuous Markov Process [71]. The research described in this thesis will assume the use of independent stochastic variables and therefore further discussion of dependent stochastic variables is not undertaken.

4.5 SUMMARY

This chapter has described the necessity of approximations to Dynamic Programming due to the large numbers of computations required and the 'Curse of Dimensionality'. Duality of linear programming was introduced and it was detailed how the dual variables represent the marginal cost of resources in a linear programming problem. The link between the dual variables and the linear constraints used to approximate the subproblem was developed along with the connection of linear piecewise approximation to Benders Decomposition. Dual Dynamic Programming, an implementation of Benders Decomposition was presented, with derivations of the Benders Cuts calculation. The Benders Decomposition/Dual Dynamic Programming algorithm was presented and shown via example to be more computationally efficient than dynamic programming. The Dual Dynamic Programming algorithm was extended to multistage optimisation problems and the associated algorithm was detailed. Multistage Dual Dynamic Programming algorithm was extended to stochastic problems to give the Stochastic Dual Dynamic Programming or SDDP algorithm. SDDP sampling and Monte Carlo simulation was discussed along with stochastic variable probability representation.

The theory presented in this Chapter has focussed on a general primal minimisation problem. This has been done to fit with the research presented in this thesis that focusses on a cost minimisation optimisation problem. More generally, this may not be the case. The DDP and SDDP algorithm are equally applicable to primal maximisation problems so long as the future cost function has a concave structure. In a maximisation problem, the linear piecewise representation of the subproblem will be approximated from above with the value of α still sitting on the surface (albeit 'underneath') of the approximation. Whether or not a problem can be solved using DDP or SDDP and development of the associated optimisation problem is the subject of Chapter 5.

Chapter 5

DEFINING AND SOLVING OPTIMISATION PROBLEMS

Optimisation problems exist in many forms and must be carefully studied and understood before they can be solved. Every problem is unique with different objectives and constraints. The process for taking a real world optimisation problem and solving it centres around constructing a mathematical representation of the problem. The mathematical representation identifies the purpose of the optimisation through the objective function and constrains the optimal solution through the use of constraints.

Solving an optimisation problem can be broken into four steps. The first is to identify the purpose of the optimisation; this is done by constructing the objective function of the mathematical representation. The second step involves identifying where the limitations of the real world problem sets bounds on the optimal solution, this is done through constructing the mathematical constraints of the problem. The third step involves deciding how to solve the problem i.e. selection of a solution technique. The final step is solution and analysis of the problem.

Solving optimisation problems by Stochastic Dual Dynamic Programming requires all four steps of the process identified above. This chapter focuses on the first three steps of the solution process as well as the flow of data through the SDDP algorithm.

5.1 OBJECTIVE FUNCTION

An optimisation problem is a tradeoff between competing entities or variables in order to achieve an optimal solution to the problem. The objective function identifies the goal of the optimisation, e.g. maximise profit, minimise cost, and states mathematically how the variables of the problem contribute to the goal of the optimisation. The optimal solution of an optimisation problem is the value of the variables in the objective function that give an optimal value of the objective function.

The first step in constructing the objective function is to identify which variables should be included in the objective function. The variables required in the objective function are those that contribute to the goal of the optimisation problem. For example, for an optimisation problem whose goal is to minimise system costs, the objective function variables would be all those that contribute a cost to the problem. The second step in constructing the optimisation problem is to match each variable in the objective function with a coefficient. The coefficient states how the variable contributes to the objective function e.g. how much does each unit of variable $\mathbf{x}$ contribute to the cost of the problem. This information may be available from the problem directly or may need to be estimated or assumed.

Once the variables and their corresponding coefficients are known, the problem solver must then construct a mathematical formula that is the objective function. This formula states how the variables of the objective function combine to represent the value of the objective function. If each variable individually contributes to the cost of the problem the objective function is simply the summation of the variables and is linear. A linear optimisation technique can be used to solve this type of problem. Where a non linear combination of variables defines the objective function the problem must be solved using non linear optimisation techniques. The objective function is the first indicator as to the type of solution technique that can be used to solve the problem. This is discussed further in Section 5.4.1.

The completed objective function comprises of the mathematical formula for contributing variables plus the identifier of a maximisation or minimisation problem.

5.2 CONSTRAINTS

Once the objective function is defined the second step of solving the optimisation problem is undertaken. This step involves constructing the constraints of the problem. The constraints limit the values of the variables in the objective function in order to ensure that physical, financial and system restrictions are met. Constraints exist in two broad categories, the first group are constraints that restrict some function of the problem variables to within system restrictions, the second group are constraints that act directly on the variables and are often referred to as 'limits'.

The first group of constraints outlined above are constraints that restrict some function of the problem variables. The size and type of the optimisation problem reflects how many of these constraints may be required, with real world problems potentially having hundreds or thousands of constraints. Constraints are normally written with the function of unknown variables on the left hand side and the known restriction on the right hand side. Each constraint may be either an equality constraint, where the left hand side function must equal the known right hand side restriction, or an inequality constraint, where the left hand side function must be either $\geq$ or $\leq$ the known right hand side restriction. Most optimisation problems will have a combination of both constraint types[1]. An optimisation problem usually involves some sort

[1] Constraints can be changed from inequality to equality constraints through the use of slack variables. Equality constraints can be changed to inequality constraints by introducing two opposite (e.g. one $\geq$ and one $\leq$ constraint) inequality constraints to replace the one equality constraint. These conversion techniques allow opti-

of system or process e.g. a manufacturing process, a transport network or technical system (electricity, water, communications etc.). The operation, limits and constraints of the process or system define the constraints of the optimisation problem. When constructing constraints it is useful to select each problem variable in the objective function and analyse it individually or as part of a group of similar variables. This helps ensure that constraints are not missed or formed incorrectly[2]. The assessment identifies how the system or process constrains or limits the value of the variable or limits some function of several variables. An individual variable may be involved in numerous constraints, a single constraint, or no constraint if it is completely free to vary. Every constraint is represented as a mathematical equation and becomes part of the mathematical representation of the optimisation problem.

The second type of constraint is known as a 'limit' constraint. They usually define the outer boundaries of the solution space (as known as the feasible region) such as not allowing negative variable values. This second group of constraints is usually easily identified and are often given by the type of problem being optimised. A variable that represents a physical quantity will usually be required to be positive so a 'limit' on the value of this quantity would require it to be ≥ 0 at all times. A limit may be dictated by the problem itself such as restricting a resource to be $\leq$ a limiting factor where the limiting factor may be related to physical limitations such storage space, safety limits or transportation abilities.

Defining the mathematical representation of the optimisation problem can be considered to be part art and part science [73]. The science is in understanding the problem and identifying the goals of the optimisation. The art is in constructing the constraints. While each constraint relates to a defined limitation or restriction of the problem there may be numerous ways to define the constraint. The best form of the constraint is dependent on the type of optimisation problem, the optimisation technique being used to solve the problem and the experience of the problem solver. Where the optimisation technique can affect the construction of constraints or the structure of the mathematical formulation, then constraint definition and choice of solution technique may become an iterative process.

5.3 CHOICE OF OPTIMISATION TECHNIQUE

The choice of optimisation solution technique is governed by a number of factors generated by both the problem itself and the resources available to the problem solver.

The problem influences the choice of optimisation technique through its size and number of variables, whether it is linear or non linear and what type of problem is it e.g. static, dynamic,

---

misation problems to be written in a standardised form. The use of a standardised form is important for solution techniques to work properly. Many commercial solvers can convert an optimisation problem into a standardised form automatically saving the problem solver time and potential conversion errors. [72].

[2] Though no system is foolproof.

integer, continuous, mixed integer etc. Optimisation problems that exhibit special structure such as complicating variables or complicating constraints may be able to utilise decomposition techniques like Benders Decomposition and Dantzig Wolfe Decomposition [74] [75]. Each feature of the optimisation problem such as linearity, non linearity, continuous, integer etc. reduces the number of suitable solution techniques available to the problem solver. It is not usually possible to identify the correct solution technique in advance of solving a problem as choice of solution technique is heavily dependent on the problem type and mathematical representation.

External factors can also influence the choice of solution technique. If the solution is required in real time, a fast solution technique is necessary, whereas a planning problem can sacrifice speed for problem detail. The computing power, both software and hardware, available to solve the problem can influence the choice of solution technique. The problem solver must check that the solution technique can solve the desired problem with the technology they have available.

Choosing an optimisation solution technique can be difficult and often there is no single correct answer. There is often an iterative process between constructing the mathematical representation and optimisation technique choice as optimisation techniques can require specific structure within the mathematical representation. If the mathematical representation can't be adjusted to fit the optimisation technique the search for a suitable optimisation technique continues.

5.4 POWER SYSTEM INVESTMENT PLANNING

The power system investment planning problem is to minimise the investment and operational costs of the power system over a specific planning horizon. From knowledge of the real world problem and that investments are made sequentially over time, the power system investment planning problem is a dynamic optimisation problem. The problem is also stochastic as system demand is uncertain. The uncertainty of demand may alter the optimal investment choice. The optimal solution of a dynamic planning problem must be flexible to adjust to uncertainty in variables such as demand, therefore the power system investment problem is a stochastic dynamic optimisation problem.

The mathematical model of the investment problem is developed in an iterative process with the choice of solution technique. The solution technique chosen may require a specific form of mathematical representation leading to the mathematical representation being re-developed.

5.4.1 Objective Function

To achieve the goal of minimisation of investment and operating costs, the optimisation must make a tradeoff between investing in capital projects in the current time period or to wait until a future time period. This tradeoff involves not only the capital cost of investment but the resulting effects on the costs of system operation. Installed capacity attracts not only variable operation and maintenance costs but also fixed costs for maintenance that are related to the installed capacity. The objective function must contain all the variables and their associated coefficients that contribute a cost to the power system.

The first variables considered are those that represent capital investments e.g generation and transmission. These variables represent a cost to the system in the form of an investment cost. Each time period of the optimisation will have variables that represent the range of available capital investment options. The form these variables take must also be decided. There are two options, the first is to represent each investment as a binary variable where the variable value 0 indicates the investment *is not* made in this time period and the value 1 indicates the investment *is* made this time period. The alternative representation is to have each investment variable representing the MW size of the capital investment e.g. the size of the generator or the capacity of the transmission line. Where a binary representation is used the optimisation problem must be told in some other way (usually via constraints) what the size of the chosen investment(s) are. The model used in this thesis uses the second investment representation, where each variable represents the MW size of the capital investment. This decision was taken only after an iterative process between constraint construction and objective function definition showed that some constraints were simpler to write and understand when the investment variables represented a MW investment size.

The second set of variables that contribute to the overall cost of system investment and operation are those that affect the operational costs of the power system. For the power system investment planning problem these variables include the variable and fixed costs of operation. The variable and fixed costs are not restricted to fuel, operation and maintenance costs but could also include other charges such as carbon taxes or governmental subsidies. The variables associated with these operational costs are associated with the MWh of generation from each generator and the total installed capacity of each generator, transmission line or demand reduction technology.

The next step in defining the objective function is to identify the coefficients that match each variable in the objective function. These coefficients represent how each variable contributes to the overall cost of system investment and operation. The objective function minimises cost and therefore each term in the objective function must be represented in term of a monetary ($) amount. The investment variables have the unit MW, so to make each term a $ value the coefficient must represent $/MW i.e. the cost per MW of installing that particular investment type. The variables associated with variable operating costs are have the unit MWh. The coefficients for these variables must be a $/MWh value i.e. the cost per MWh of running that particular generator/transmission line/demand reduction technology. This $/MWh value may be a combination of costs like fuel, maintenance and carbon taxes and income such as government subsidies. The variables associated with the fixed costs of the system are defined by generation capacity, measured in MW, so the coefficients for these variables must be defined as a $/MW value. Similar to the variables representing the variable operation costs, the coefficients for the fixed operating costs may represent a joint value of contributing costs and income such as maintenance, taxes, and subsidies. The values of all coefficients must be found by the problem solver. Some values might be easily obtainable such as costs of new investment whereas operating cost information may be harder to find especially for existing plant or if it is commercially sensitive information. Where information is difficult to obtain the problem solver must make a decision between solution accuracy (spending time and resources to find a correct value) and using an approximate value. This decision is influenced by the impact the particular coefficient may have on the overall optimal solution and the size of contribution it makes to the final optimal cost.

The power system investment problem is a multistage sequential investment problem. The decision variables and cost coefficients in each single stage decision objective function are shown in Equation 5.1.

$$\min\left[c_1\mathbf{x}_{invest} + c_2\mathbf{x}_{genMWH} + c_3\mathbf{x}_{cap}\right] \tag{5.1}$$

Where:

- $\min$ = Represents that the optimisation problem is a minimisation problem.
- c_1 = Coefficients representing cost per MW of investment.
- $\mathbf{x}_{invest}$ = Variables representing MW size of investment options.
- c_2 = Coefficients representing cost per MWh of operation of generation, transmission and demand reduction technology.
- $\mathbf{x}_{genMWH}$ = Variables representing the MWh of operation of generation and transmission.
- c_3 = Coefficients representing the cost per MW of installed capacity or transmission or demand reduction technology.
- $\mathbf{x}_{cap}$ = Variables representing the installed MW of generation and transmission.

The single stage investment problem is repeated in each time period of the optimisation problem therefore the single stage objective functions are combined to create Equation 5.2, the objective function for the entire planning horizon of the power system investment problem.

$$\min\left[c_{1_{t1}}\mathbf{x}_{invest_1} + c_{2_1}\mathbf{x}_{genMWH_1} + c_{3_1}\mathbf{x}_{cap_2} + \ldots + c_{1_T}\mathbf{x}_{invest_T} + c_{2_T}\mathbf{x}_{genMWH_T} + c_{3_T}\mathbf{x}_{cap_{T+1}}\right] \tag{5.2}$$

Where:

- 1 = First time period.
- T = Final time period.

5.4.2 Constraints

5.4.2.1 Functional Constraints

The functional constraints of the power system investment planning problem describe how the system restrictions and limitations define the variables of the problem and how the variables of the problem interact. Constructing the constraints can be done by considering them in groups. The initial constraint groups are identified by working systematically through the variables present in the objective function. The final group of constraints are given by the physical restrictions of the real world problem.

The first group of constraints relate to the investment variables. The constraint(s) for this group describes how the value of system capacity changes every time an investment decision is made. Equation 5.3 shows the form of this constraint and indicates that it is valid in all time periods and hence single stage optimisations.

$$
\begin{aligned}
&\mathbf{x}_{cap_1} + \mathbf{x}_{invest_1} = \mathbf{x}_{cap_2} \\
&\vdots \\
&\mathbf{x}_{cap_T} + \mathbf{x}_{invest_T} = \mathbf{x}_{cap_{T+1}}
\end{aligned}
\tag{5.3}
$$

Equation 5.3 relates the investment variables from the objective function to the problem variable of system capacity but doesn't specify any restrictions on the size of the resulting investment. In the power system investment planning optimisation the investment sizes are restricted to being large integer sizes and so the constraint in Equation 5.4 is required to restrict the values of the investment variables in each time period of the problem.

$$
\begin{aligned}
&\mathbf{x}_{invest_1} - (InvestSize * \mathbf{x}_{binary_1}) = 0 \\
&\mathbf{x}_{invest_T} - (InvestSize * \mathbf{x}_{binary_T}) = 0 \\
&\mathbf{x}_{binary_1}, \mathbf{x}_{binary_T} \leq 1, \in \mathbb{Z}
\end{aligned}
\tag{5.4}
$$

Where:

- $InvestSize$ = Large integer investment size specified to the optimisation by the problem solver.
- $\mathbf{x}_{binary_1}$ = Binary integer variable for time period 1 that can only take the values 1 or 0.

The second group of variables in the objective function are the fixed operational costs of the system. This group of variables represent the total installed capacity of the system being optimised. This installed capacity value is given by the variable(s) $\mathbf{x}_{cap_{t+1}}$. Equation 5.3 already specifies how the value of $\mathbf{x}_{cap_{t+1}}$ is found and no further constraints are necessary.

The third group of variables in the objective function represent the variable cost of system operation. These variables represent the MWh generated by each generator in the power system. Defining the constraints for this group of objective function variables requires a good understanding of the physical system being modelled as it is necessary to understand how generators meet system demand and hence how many MWh each generator provides. For the power system investment optimisation problem the demand throughout a time period is given by a load duration curve, such that the cheapest generators are dispatched first, working up to the most expensive generator resulting in a merit order dispatch. The first constraint of the set ensures that the total MWh of each generator sum together to give the value of system demand at a time point i.e. system generation equals system demand. For a power system operating in a market environment it is inappropriate to use a constraint that ensure capacity will always supply 100% of demand. To overcome this problem the model includes a Value of Lost Load (VoLL) generator. Dispatch of the VoLL generator indicates that the system has insufficient capacity and that load is dropped until capacity meets demand. Dropping load is a politically and socially undesirable outcome and should be avoided. The VoLL generator is treated similarly to existing capacity types but has a high variable operating cost. It does not have a fixed operating cost per MW of capacity as the capacity of the VoLL generator is very large, at a minimum it must represent the size of the load, potentially thousands of MW. Instead, the VoLL generator has a fixed penalty cost that applies for any reserve cover that is supplied by the VoLL generator. Further discussion of modelling of the VoLL generator is presented in Section 7.2.3.

Equation 5.5 shows the general form of the first constraint. Note how the system demand value on the right hand side of Equation 5.5 is multiplied by the variable *time*. This is because the generation variables are in MWh and system demand is specified in MW.

$$\begin{aligned}
&\mathbf{x}_{genMWH_1} = (\mathbf{x}_{dem_2} * time_1) \\
&\vdots \\
&\mathbf{x}_{genMWH_T} = (\mathbf{x}_{dem_{T+1}} * time_T)
\end{aligned} \tag{5.5}$$

Where:

- $\mathbf{x}_{genMWH_1}$ = Number of MWh generated in time period 1.
- $\mathbf{x}_{dem_2}$ = Demand to be met in time period 1. This demand at the beginning of time period 1 plus demand growth for this time period.
- $time$ = Number of hours in the time period.

This constraint has a hidden usefulness because as the optimisation problem is a minimisation problem, the optimisation, via this constraint, will automatically schedule the cheapest generator first. This is due to the coefficients of the relevant objective function variables and the minimisation goal of the optimisation problem. Due to this effect a separate constraint to ensure the cheapest generator is scheduled first is not required. As part of the modelling for the investment problem a 'dummy' generator representing the Value of Lost Load or VoLL is included as a potential generator.

Two more constraints are required to support Equation 5.5, one to define the value of demand in the each time period and one to restrict the number of MWh generated by each generator to the capacity of the generation plant. Equation 5.6 illustrates the demand constraints.

$$\begin{aligned}
&\mathbf{x}_{dem_1} + \mathbf{x}_{demInc_1} = \mathbf{x}_{dem2} \\
&\vdots \\
&\mathbf{x}_{dem_T} + \mathbf{x}_{demInc_T} = \mathbf{x}_{demT+1}
\end{aligned} \tag{5.6}$$

Where:

- $\mathbf{x}_{dem_1}$ = Demand at time period 1
- $\mathbf{x}_{demInc_1}$ = Peak demand growth in time period 1.
- $\mathbf{x}_{dem_2}$ = System demand after peak demand growth is added to the current system demand. It is the maximum peak demand value occurring in this time period (coincident with the beginning of the following time period)

Equation 5.7 illustrates the energy generated restriction constraint. Note how the constraint is a $\leq$ inequality constraint. This allows the optimisation problem the flexibility to schedule a particular generator to generate nothing if the generator is not required.

$$\begin{aligned}
&\mathbf{x}_{genMWH_1} \leq (\mathbf{x}_{cap_2} \times time_1) \\
&\vdots \\
&\mathbf{x}_{genMWH_T} \leq (\mathbf{x}_{cap_{T+1}} \times time_T)
\end{aligned} \tag{5.7}$$

Where:

- m = Total number of generators in power system.
- $\mathbf{x}_{genMWH_1}$ = MWh generated by in time period 1.
- $\mathbf{x}_{cap_1}$ = Generation capacity in time period 1.
- $time_T$ = Number of hours in time period T.

The constraints described in this section are very general as the system description is very basic. They are only an illustration of the constraint types in the power system investment problem. A full set of constraints can only be fully defined for a specific problem (Refer to Chapters 7, 8 and 9 for full examples).

5.4.2.2 Limits

The limit constraints of an optimisation problem are often easily identified. Every variable in the optimisation will have an upper and lower limit but it may not be necessary to explicitly specify every limit for every variable. The optimisation software used to solve the mathematical formulation may specify a default lower bound of 0 and hence only require exceptions to this to be explicitly defined. Limit constraints are also necessary to produce a bounded optimisation problem. For minimisation problems this means not allowing the objective function to reach negative infinity and for maximisation problems, not allowing the objective function to reach positive infinity.

The power system investment planning problem has a lower bound of 0 for all variables as it doesn't make sense for any of them to be negative. A new investment cannot be a negative MW size, likewise the MWh generated cannot be negative. There is no concern with the objective function reaching positive infinity as the aim is to minimise the cost of investment and operation. This means the upper bound on the variables can be left undefined. The exception to this is given in Equation 5.7 where the MWh of each generator has an upper bound given by the capacity of the generation plant multiplied by the number of hours of the time period.

5.4.2.3 Mathematical Representation

An example mathematical representation for the power system investment planning problem is shown in Equation 5.8. The general system modelled has $\mathbf{k}$ generating units, and the problem is optimised over two time periods. The two individual time period optimisation problems are tied together through the $\mathbf{x}_{dem_2}$ and $\mathbf{x}_{cap_2}$ variables which are present in the constraints of each time period. These are complicating variables and prevent the single stage investment problems from being solved independently.

$$\min\left[c_{1_1}\mathbf{x}_{invest_1} + c_{2_1}\mathbf{x}_{genMWH_1} + c_{3_1}\mathbf{x}_{cap_2} + c_{1_2}\mathbf{x}_{invest_2} + c_{2_2}\mathbf{x}_{genMWH_2} + c_{3_2}\mathbf{x}_{cap_3}\right]$$

$$\text{Subject To}$$

$$\underbrace{\begin{aligned} \mathbf{x}_{cap_1} + \mathbf{x}_{invest_1} &= \mathbf{x}_{cap_2} \\ \mathbf{x}_{dem_1} + \mathbf{x}_{demInc_1} &= \mathbf{x}_{dem2} \\ \mathbf{x}_{invest_1} - (InvestSize * \mathbf{x}_{binary_1}) &= 0 \\ \mathbf{x}_{genMWH_1} &= (\mathbf{x}_{dem_2} * time_1) \\ \mathbf{x}_{genMWH_1} &\leq (\mathbf{x}_{cap_2} \times time_1) \end{aligned}}_{\text{Time Period 1 Constraints}}$$

$$\underbrace{\begin{aligned} \mathbf{x}_{cap_2} + \mathbf{x}_{invest_2} &= \mathbf{x}_{cap_3} \\ \mathbf{x}_{dem_2} + \mathbf{x}_{demInc_2} &= \mathbf{x}_{dem3} \\ \mathbf{x}_{invest_2} - (InvestSize * \mathbf{x}_{binary_2}) &= 0 \\ \mathbf{x}_{genMWH_2} &= (\mathbf{x}_{dem_3} * time_2) \\ \mathbf{x}_{genMWH_2} &\leq (\mathbf{x}_{cap_3} \times time_2) \end{aligned}}_{\text{Time Period 2 Constraints}}$$

$$\underbrace{\begin{aligned} \mathbf{x}_{binary_1}, \mathbf{x}_{binary_2} &\leq 1, \in \mathbb{Z} \\ \mathbf{x}_{invest_1}, \mathbf{x}_{invest_2} &\geq 0 \\ \mathbf{x}_{genMWH_1}, \mathbf{x}_{genMWH_2} &\geq 0 \\ \mathbf{x}_{cap_1}, \mathbf{x}_{cap_2}, \mathbf{x}_{cap_3} &\geq 0 \\ \mathbf{x}_{demInc_1}, \mathbf{x}_{demInc_2} &\geq 0 \\ \mathbf{x}_{dem_1}, \mathbf{x}_{dem_2}, \mathbf{x}_{dem_3} &\geq 0 \end{aligned}}_{\text{Limits}} \qquad (5.8)$$

5.4.3 Optimisation Solution Technique

The investment problem is a stochastic dynamic problem therefore the solution technique chosen to solve the problem must be a stochastic dynamic optimisation technique. Dynamic programming and stochastic dynamic programming fit this criteria but as discussed in Chapter 4 both dynamic and stochastic dynamic programming suffer from computation intractability for real world sized problems. Stochastic Dual Dynamic Programming (SDDP) provides a computationally tractable approximation to stochastic dynamic programming and is identified as a possible solution technique for the power system investment problem.

Before confirming the choice of SDDP as a solution technique the problem must satisfy a number of requirements pertaining to the use of SDDP. These requirements are discussed in the following sections.

5.4.3.1 Optimisation Model Structure

The first requirement is that the problem must be able to be represented in a suitable form. SDDP solves a series of sequential single stage optimisations that are linked together through complicating variables. Each single stage optimisation represents a single time period of the multistage investment problem. The mathematical formulation of the investment problem must be able to be represented as a series of master problems and subproblems.

The power system investment planning problem fulfils this model structure requirement as it is a series of sequential investments over time. The mathematical model can be split into a series of single stage optimisation problems where the complicating variables of system capacity and system demand link the single stage optimisations together. Using the example mathematical representation of Equation 5.8, the power system investment planning problem can be represented as a master problem in Equation 5.9 and a subproblem 5.10. Each single stage optimisation is deterministic with the stochastic values of $\mathbf{x}_{demInc_1}$ and $\mathbf{x}_{demInc_2}$ being sampled before the respective single stage optimisation is solved.

Master

$$
\begin{aligned}
\min \big[c_{1_1}\mathbf{x}_{invest_1} &+ c_{2_1}\mathbf{x}_{genMWH_1} + c_{3_1}\mathbf{x}_{cap_2} + \alpha(\mathbf{x}_{dem2}, \mathbf{x}_{cap_2})\big] \\
&\text{Subject To} \\
&\qquad \mathbf{x}_{cap_1} + \mathbf{x}_{invest_1} = \mathbf{x}_{cap_2} \\
&\qquad \mathbf{x}_{dem_1} + \mathbf{x}_{demInc_1} = \mathbf{x}_{dem2} \\
&\qquad \mathbf{x}_{invest_1} - (InvestSize * \mathbf{x}_{binary_1}) = 0 \\
&\qquad \mathbf{x}_{genMWH_1} = (\mathbf{x}_{dem_2} * time_1) \\
&\qquad \mathbf{x}_{genMWH_1} \leq (\mathbf{x}_{cap_2} \times time_1) \\
&\qquad \mathbf{x}_{binary_1} \leq 1, \in \mathbb{Z} \\
&\qquad \mathbf{x}_{invest_1}, \alpha(\mathbf{x}_{dem2}, \mathbf{x}_{cap_2}), \mathbf{x}_{genMWH_1}, \mathbf{x}_{demInc_1} \geq 0 \\
&\qquad \mathbf{x}_{cap_1}, \mathbf{x}_{cap_2}, \mathbf{x}_{dem_1}, \mathbf{x}_{dem_2} \geq 0
\end{aligned} \tag{5.9}
$$

Where $\alpha(\mathbf{x}_{dem2}, \mathbf{x}_{cap_2})$ is the minimisation of the subproblem.

Subproblem

$$\alpha(\mathbf{x}_{dem2}, \mathbf{x}_{cap_2}) = \min \left[c_{1_2}\mathbf{x}_{invest_2} + c_{2_2}\mathbf{x}_{genMWH_2} + c_{3_2}\mathbf{x}_{cap_3} \right]$$

Subject To

$$\begin{aligned}
&\mathbf{x}_{cap_2} + \mathbf{x}_{invest_2} = \mathbf{x}_{cap_3} \\
&\mathbf{x}_{dem_2} + \mathbf{x}_{demInc_2} = \mathbf{x}_{dem3} \\
&\mathbf{x}_{invest_2} - (InvestSize * \mathbf{x}_{binary_2}) = 0 \\
&\mathbf{x}_{genMWH_2} = (\mathbf{x}_{dem_3} * time_2) \\
&\mathbf{x}_{genMWH_2} \leq (\mathbf{x}_{cap_3} \times time_2) \\
&\mathbf{x}_{binary_2} \leq 1, \in \mathbb{Z} \\
&\mathbf{x}_{invest_2}, \mathbf{x}_{genMWH_2}, \mathbf{x}_{cap_2}, \mathbf{x}_{cap_3} \geq 0 \\
&\mathbf{x}_{demInc_2}, \mathbf{x}_{dem_2}, \mathbf{x}_{dem_3} \geq 0
\end{aligned} \tag{5.10}$$

5.4.3.2 Convexity of Feasible Region

The second requirement that must be met is that the feasible region for each single stage optimisation problem must be convex. A strictly convex function has the property that any local optimum is also the global optimum. This is a necessary requirement for SDDP else there is no guarantee of convergence for the algorithm. Section 4.3 illustrated how a non convex function approximated with linear constraints can incorrectly restrict the feasible region of an optimisation problem. A function is considered to be strictly convex if for any two distinct points $\mathbf{X}_1$ and $\mathbf{X}_2$ Equation 5.11 is true [76].

$$f(\lambda\mathbf{X}_1 + (1-\lambda)\mathbf{X}_2) < \lambda f(\mathbf{X}_1) + (1-\lambda)f(\mathbf{X}_2) \tag{5.11}$$

where $0 < \lambda < 1$. This is represented graphically in Figure 5.1. Equation 5.11 states that the function f at any point x, a distance $\lambda\mathbf{X}_1 + (1-\lambda)\mathbf{X}_2$ between points $\mathbf{X}_1$ and $\mathbf{X}_2$, must be less than the value of a straight line connecting points $\mathbf{X}_1$ and $\mathbf{X}_2$. The left hand side of Equation 5.11 represents the point b in Figure 5.1 and the right hand side of Equation 5.11 represents the point a.

Simply put, a convex function is one where all points on a straight line joining any two distinct points of the function lie *above* the function. A concave function is the opposite, i.e. $-f(x)$ and hence all points on a straight line joining any two distinct points of the function lie *below* the function.

Convexity of the feasible region is necessary as SDDP relies on the use of dual variables and associated Benders Cuts to approximate the subproblem optimal solution. Each subproblem is a primal problem that is approximated by Benders Cuts where the Benders Cuts use the dual

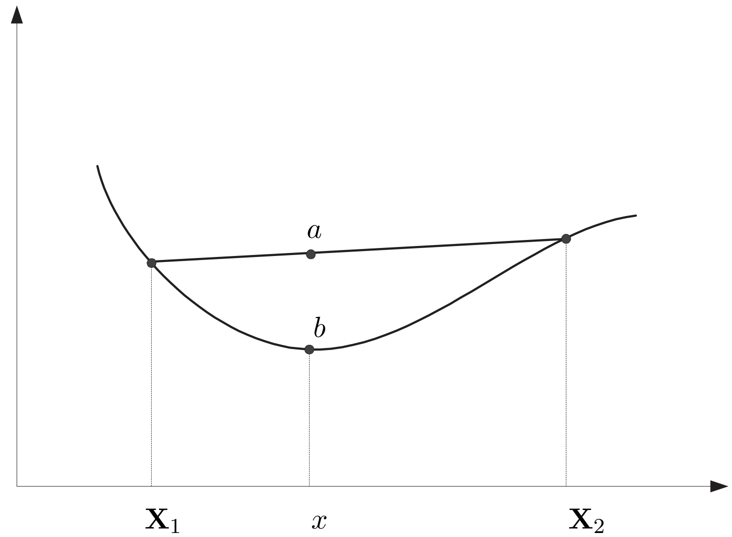


Figure 5.1 Convex Function

variables from the dual of the subproblem. For the Benders Cuts to accurately approximate the primal subproblem, the optimal solution of the primal and dual subproblems must be equal. For the optimal dual solution to equal the optimal primal solution the strong duality theorem (refer Appendix B) must hold. The strong duality theorem states that if an optimal solution exists for the primal problem there also exists an optimal solution to the dual problem and the primal and dual solutions are equal. The strong duality theorem holds for most convex problems [77] therefore if the feasible region of the optimisation problem is convex, the strong duality theorem will (most likely) hold, the optimal primal solution value will equal the optimal dual solution value and the Benders Cuts will accurately represent the subproblem being solved.

Convexity and Power System Planning Determining convexity of the feasible region of the investment planning problem requires knowledge of the shape of the feasible region. The feasible region is determined by objective function and the solution variables. The objective function of the power system investment planning problem can be considered in two parts; the immediate costs of the system and the future costs of the system.

The optimisation problem performs a tradeoff between costs in the immediate time period and those in future time periods. When the immediate costs are high due to large amounts of investment, the future costs are likely to be low as generation investment is not required. The immediate costs are given by the investment costs and operating costs, both variable and fixed, of the single stage investment problem. The general shape of the objective function for immediate costs is shown in Figure 5.2. At low levels of capacity the immediate cost is high as the optimisation is likely to add generation to prevent the expensive VoLL generator from operating. As capacity increases, additional generation is not required and the immediate cost to the system becomes the costs of operation. Figure 5.2 illustrates the general form of the future costs. At low levels of capacity in the immediate time period, the future costs are likely to be low as investment will take place in the immediate time period rather than future time periods. Both the immediate costs and future cost functions illustrated are very simplified but both can be characterised by a step function. This is due to investment options being characterised by large integer sizes resulting in large integer jumps in cost.

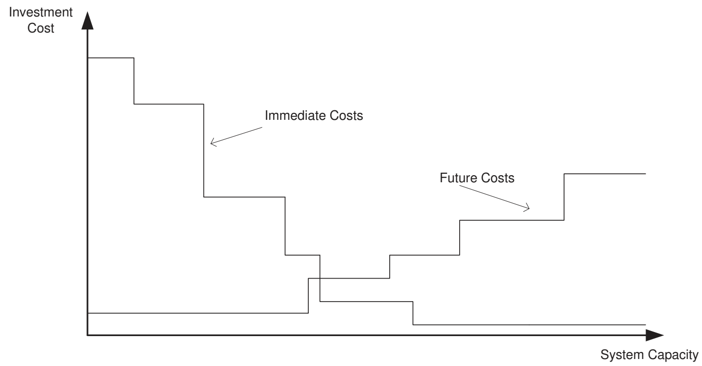


Figure 5.2 Immediate and Future Costs

The objective function feasible region is the combination of the immediate cost and future cost functions. These functions represent the lower bound of the optimal solution, hence the optimal solution sits on the function. This function is not convex as straight line drawn between two points does not sit above the function at all times. Figure 5.3 illustrates how the step function is non convex.

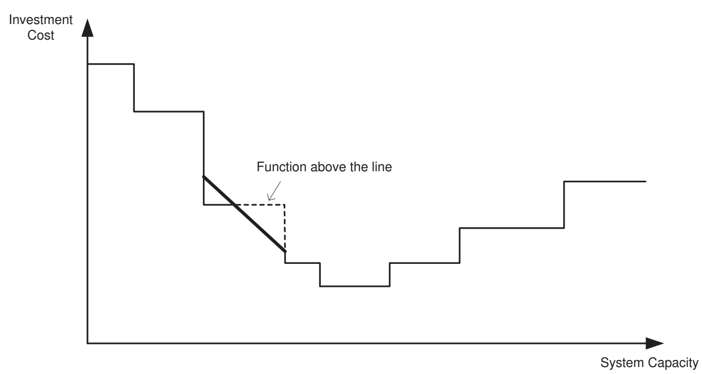


Figure 5.3 Non Convex Illustration of Objective Function

As the feasible region of the objective function of the power system investment planning problem is non convex some extensions to the SDDP algorithm will be required to allow the problem to be solved using SDDP. These extensions are discussed in Chapter 6.

5.4.3.3 Continuous Problem Variables

The final requirement of SDDP is that the optimisation problem variables are continuous. This requirement ensures that dual variables of the constraints with complicating variables can be found and used in the calculation of Benders Cuts. Requiring the solution to be continuous is in direct opposition to the goal of this research, to optimise large integer investments in power systems. This contrast is discussed in detail in Chapter 6 and is one of the extensions to SDDP offered by this research.

5.5 DATA REQUIREMENTS OF SDDP

To solve an optimisation problem using SDDP the algorithm requires a number of data inputs to be provided by the problem solver. The information and data required consists of four categories; existing generation and transmission data, investment options data, demand growth data and algorithm data.

The data required for existing generation and transmission capacity includes:

- Size of capacity (MW)
- Variable Operating Costs($/MWh)
- Fixed Operating Costs($/MW)

The data required for new investment options includes:

- Size of investment (MW)
- Capital investment cost($/MW)
- Variable Operating Costs($/MWh)
- Fixed Operating Costs($/MW)
- Payback Period of investment(years)

Demand growth within the system is a stochastic variable but each single stage problem is deterministic. This requires the value of demand growth to be sampled from a probability distribution. The data required by the algorithm is the data describing the probability distribution of demand growth. The sampled value of peak demand growth is used to construct a load duration curve representing the MWh of load over the time period.

- Expected value of peak demand growth (MW)
- Standard deviation of expected peak demand growth

- Load duration curve description including maximum and minimum load values and shape of the load duration curve.

Algorithm data includes any information required by the SDDP algorithm to run including:

- Length of Planning Period (years)
- Discount Rate

This data is supplied to the algorithm and is used in the objective function coefficients and constraints. Figure 5.4 illustrates the input data, the SDDP algorithm structure and the outputs of the algorithm. The outputs of SDDP are the optimal policy of functions at each time point of the algorithm, convergence information and results of the monte carlo simulation. The optimal policy functions and monte carlo simulation results indicate the optimal cost and likely optimal investments under different demand growth scenarios. The convergence information identifies how far apart the expected upper and lower bounds are and the confidence interval surrounding the expected upper bound.

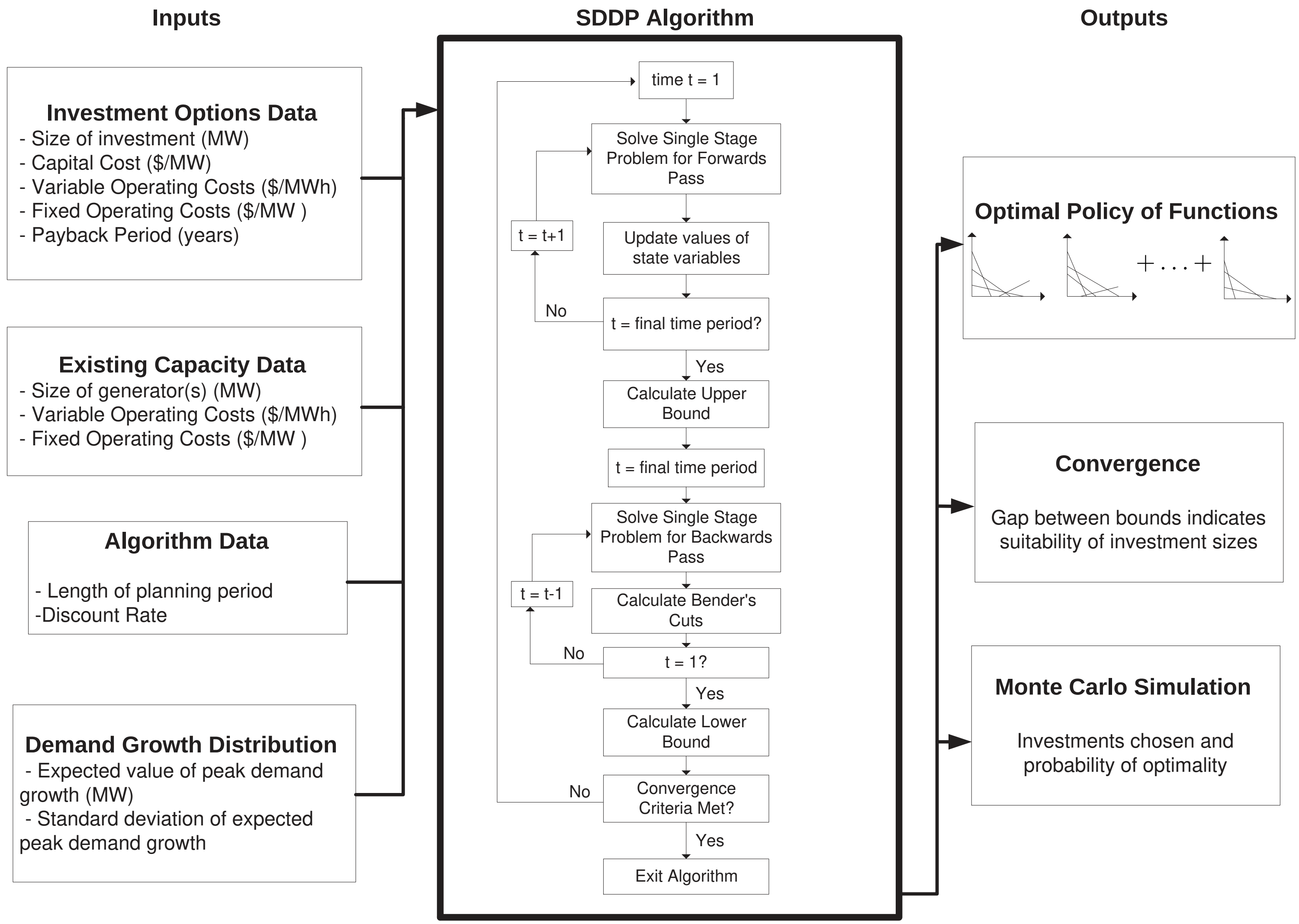


Figure 5.4 Data Flow for SDDP Algorithm

5.6 SUMMARY

This chapter has presented the process to construct a mathematical representation of a real world optimisation problem and choose an optimisation solution technique to solve the problem.

The process described has 4 steps, the first is to define the objective function of the problem. This states the goal of the optimisation problem, e.g. to minimise costs, and contains all problem variables that contribute to this goal.

The second step of the process is to construct the constraints of the optimisation problem. The constraints restrict the solution space of the problem to ensure the solution space represents the actual physical situation. Constructing constraints can be difficult and time consuming as there are many ways that constraints can be represented. Choosing the correct representation for the current problem requires the skill and knowledge of the problem solver.

The third step in the process is choosing a suitable solution technique. The choice of solution technique is guided by the type of optimisation being solved e.g. dynamic, stochastic, linear or non linear etc.. The power system investment planning problem is mathematically represented as a series of sequential investment decisions so it fits the model structure required by SDDP. Other requirements of SDDP such as convexity of the feasible region and continuous problem variables are not met by the investment planning problem. These challenges motivate the extensions to SDDP presented in the thesis and are necessary to undertake the fourth step which is to solve the optimisation problem. The extensions to SDDP to solve the power system investment problem are discussed further in Chapter 6.

Chapter 6

SDDP AND POWER SYSTEM PLANNING

6.1 SDDP AND INVESTMENT PLANNING

SDDP has been widely applied to hydro scheduling optimisation problems around the world where it is used to optimise the operational costs of the power system. The optimisation must trade off the cost of immediately releasing water to generate cheap power versus using expensive thermal generation and storing water for future time periods. Hydro scheduling is a dynamic planning problem as the optimal water release decision changes over time depending on the current system state. The problem is also stochastic as water inflows into storage lakes are uncertain. This situation is very similar to the power system investment planning problem where the optimal investment decision is dependent on the current system state. The investment planning problem is also stochastic as demand growth is uncertain. The similarities between the two types of planning problems indicate that SDDP is a good choice of solution technique for both.

While the power system investment planning problem is similar to hydro scheduling in that it is a stochastic dynamic planning problem, it also differs from hydro scheduling in that investment planning involves decision variables that are integer in value and large in size. Hydro scheduling may be approximated as a continuous variable optimisation problem where each optimal water release decision is a continuous value. Investment planning is a mixed integer optimisation problem where the optimal solution is the minimum total cost of investment and operation of the power system. Investment decisions are large integers and the operational decision variables are continuous. These differences necessitate the development and extension of the SDDP algorithm to allow the power system investment planning problem to be solved using SDDP.

The research presented in this thesis has developed three extensions to allow mixed integer problems to be solved via SDDP. The first extension is to relax the mixed integer problems on the backward pass of the SDDP algorithm so they are continuous problems. This allows the dual variables to be found and Benders Cuts to be calculated. The second extension is to introduce a dynamic constraint that restricts already selected investment options from being reused. This dynamic constraint is used on both the forward and backward pass and reduces the state space over time. The third extension to SDDP results from the first; by relaxing the optimisations on the backward pass, the convergence criteria of the algorithm must be changed. Each of these SDDP extensions are discussed in the following sections.

6.2 INTEGER CAPACITY INVESTMENTS

The first extension to SDDP is developed to allow mixed integer optimisation problems to be solved using SDDP. SDDP relies on calculating dual variables for the subproblems but dual variables are only defined for continuous optimisation problems. The mixed integer investment problem motivated the extension to SDDP of allowing the subproblem optimisations on the backward pass to be relaxed and become continuous optimisation problems.

The power system investment problem chooses capacity investments from a pool of investment options that optimise the system investment and operational costs over the planning horizon. The optimisation may choose either no investment or one or more investments while simultaneously ensuring that all system constraints are met. The investment choices available to the problem reflect real world capacity investment opportunities and as such each investment option has a specific capacity size. The sizes of investments are a reflection of many interacting factors such as plant location, environmental constraints, manufacturing limitations, transport restrictions, fuel supply, available labour and personnel skills, government regulations and capital available for investment. Due to these influencing factors capacity investments for a particular location are generally considered to be large integer sized. If a potential investment can be selected in a range of integer sizes, each different size option is represented as a separate investment opportunity in the optimisation problem.

Representing the investment options of the investment planning optimisation as large integers turns both the master and the subproblem optimisations of the SDDP algorithm into mixed integer optimisation problems[1]. The introduction of mixed integer master and subproblems creates a difficulty for solution via SDDP as the dual variables of an integer or mixed integer problem are not easily interpreted compared with those for a continuous problem. Dual variables from a integer or mixed integer problem create incorrect Benders Cuts resulting in an inaccurate representation of the future cost function. This issue motivates the first extension to SDDP of relaxing the subproblem optimisations to become continuous optimisation problems.

[1]Benders Decomposition was originally developed to optimise mixed integer optimisation problems [68] but this technique requires all integer variables to be present in the master problem and all continuous variables to be in the subproblem. This decomposition of integer and continuous variables does not accurately reflect the dynamic investment planning problem and therefore is not considered a valid solution.

6.2.1 Dual Variables

Dual variables represent the marginal cost of additional capacity in the power system investment problem. They represent the change in overall problem solution cost for a unit change in the state variables. For example, Equation 6.1 represents a simplistic power system optimisation where (unlike a market system) sufficient capacity is required to supply demand. The state variables are capacity and demand and the only capacity investments available are 75MW and 100MW integer investments i.e $\mathbf{x}_{inv} = 75$ or $\mathbf{x}_{inv} = 100$. .

$$
\begin{aligned}
f(\mathbf{x}_{inv}) =& min\left[30\mathbf{x}_{inv}\right] \\
\text{Subject to:} & \\
& \mathbf{x}_{cap_{t+1}} \geq \mathbf{x}_{dem_{t+1}} \\
& \mathbf{x}_{cap_t} + \mathbf{x}_{inv} = \mathbf{x}_{cap_{t+1}} \\
& \mathbf{x}_{dem_t} + \mathbf{x}_{dem_{growth}} = \mathbf{x}_{dem_{t+1}} \\
& \mathbf{x}_{dem_{growth}} = 50, \mathbf{x}_{cap_t} = 150, \mathbf{x}_{dem_t} = 120 \\
& \mathbf{x} \geq 0
\end{aligned}
\tag{6.1}
$$

Where:

- 30 = Cost per MW of investment.
- $\mathbf{x}_{inv}$ = Number of MW invested
- $\mathbf{x}_{dem_t}$= System demand in MW at time t - before demand growth.
- $\mathbf{x}_{cap_t}$= System capacity in MW at time t - before investment.
- $\mathbf{x}_{cap_{t+1}}$= System capacity in MW at time $t+1$ - after investment.
- $\mathbf{x}_{dem_{t+1}}$= System demand in MW at time $t+1$ - after demand growth.
- $\mathbf{x}_{dem_{growth}}$ = Demand growth in MW

In continuous problems a unit change in the state variables results in a corresponding change in the optimal decision variable values and hence a change in optimal solution cost. For a large integer optimisation problem, a unit change in the state variables of the problem constraints may not result in a change to the optimal decision variables although this is not universally true. In this situation the optimal solution cost will not alter and the dual variables will equal zero. As discussed in Section 4.3.1 the dual variables (for a two dimensional problem) represent the slope of a linear constraint or Benders Cut. Where a Benders Cut resulting from an integer problem results in a zero slope, the cut may remove valid sections of the feasible region of the optimisation hence give an incorrect approximation to the future cost function.

The solution of Equation 6.1 is trivial with $\mathbf{x}_{inv} = 75$ and the optimal solution cost $f(\mathbf{x}_{inv}) = 2250$. The dual variables of the problem represent the change in optimal solution cost for a unit change in the state variables i.e. if $\mathbf{x}_{cap_t}$ was increased to 151MW or if $\mathbf{x}_{dem_t}$ was increased to 121MW. In either situation the dual variables equal zero because a unit change in either demand or capacity has no effect on the choice of investment option i.e. the optimal solution cost is unchanged. If the optimisation problem had been continuous with $\mathbf{x}_{inv}$ able to take any value, the optimal solution variable would be $\mathbf{x}_{inv} = 20$ with an optimal solution cost of $f(\mathbf{x}_{inv}) = 600$. The dual variables for this continuous problem are $\boldsymbol{\lambda}_{demand} = 30$ and $\boldsymbol{\lambda}_{capacity} = -30$. This shows if $\mathbf{x}_{cap_t}$ increased by 1 the optimal solution cost will drop by 30 and if $\mathbf{x}_{dem_t}$ increased by 1 the optimal solution cost will increase by 30.

Figure 6.1(a) illustrates the actual cost of the *continuous* optimisation problem of Equation 6.1 and the Benders Cut calculated from the dual variable for capacity ($\boldsymbol{\lambda}_{capacity} = -30$). Figure 6.1(b) illustrates the actual cost of the *integer* optimisation problem of Equation 6.1 shown as a stepped function. The Benders Cut calculated for the state variable $\mathbf{x}_{cap_t} = 150$MW has zero slope due to the zero value of the dual variable. Note that demand for these graphs is $\mathbf{x}_{dem_{t+1}} = 120 + 50 = 170$MW.

The shape of the actual cost function in Figure 6.1(b) is found by knowledge of the investment options available ($\mathbf{x}_{inv} = 75$ or $\mathbf{x}_{inv} = 100$) and the values of capacity at which each investment would be chosen. If capacity is less than 70MW the cost is undefined as there is no investment available that will satisfy the constraint of $\mathbf{x}_{cap_{t+1}} \geq \mathbf{x}_{dem_{t+1}}$. The feasible region of the cost function sits above the stepped function but due to the minimisation of the optimisation problem, the optimal solution sits on the line. The zero valued Benders Cut in Figure 6.1(b) approximates the value of the cost function but removes part of the feasible region. The shaded area below the Benders Cut in Figure 6.1(b) shows the area of the feasible region of the optimisation problem that the Benders Cut removes. This illustrates how a zero value dual variable cannot be used to calculate a Benders Cut that accurately approximates the optimal cost of an optimisation problem.

Integer and mixed integer optimisation problems may provide non zero dual variables for certain values of the state variables. Where the cost function transitions from one step to another, i.e. at the corner points, the dual variables will be non zero. For example, in the integer optimisation problem of Equation 6.1, if the initial value of capacity in the system $\mathbf{x}_{cap_t} = 95$MW, the optimal solution is $f(\mathbf{x}_{inv}) = 2250$ with dual variables $\boldsymbol{\lambda}_{demand} = 30$ and $\boldsymbol{\lambda}_{capacity} = -30$. A Benders Cut constructed from these dual variables results in the constraint shown in Figure 6.2. This constraint does not remove any section of the feasible region from the problem. Identifying the corner points across vast and multidimensional state spaces is not realistic for real world problems therefore relying on finding such points is not a suitable solution to approximating mixed integer subproblems in SDDP.

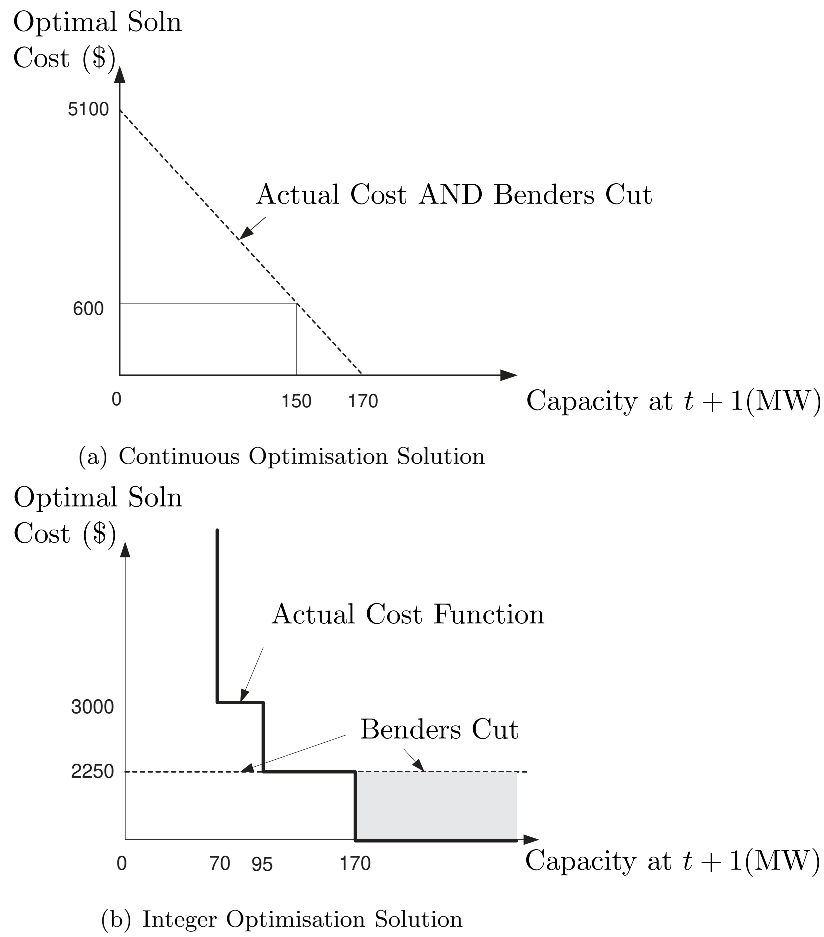


Figure 6.1 Optimal Solution Costs for Continuous and Integer Problems

6.2.2 SDDP Extension

To overcome the dual variable issue, this research has developed an extension to SDDP that allows optimisation problems to be solved where both the master and subproblems contain integer variables. The extension is to relax the subproblem optimisations on the backward pass of the algorithm and allow the optimisation problems to be continuous. This allows dual variables and Benders Cuts to be calculated for an equivalent continuous investment problem with these cuts approximating the future cost functions at each time period. The backward pass optimisations continue to the use the state variable values found on the forward pass, where the forward pass optimises the original mixed integer problems.

This extension to SDDP is similar to the work of Cerisola et al. [78] and Cerisola and Ramos [79], who suggest the use of Lagrangian relaxation to find Lagrangian dual variables in a multistage nested Benders Decomposition. The Lagrangian approach removes complicating constraints of the problem and replaces them with a penalty term in the objective function. The penalty

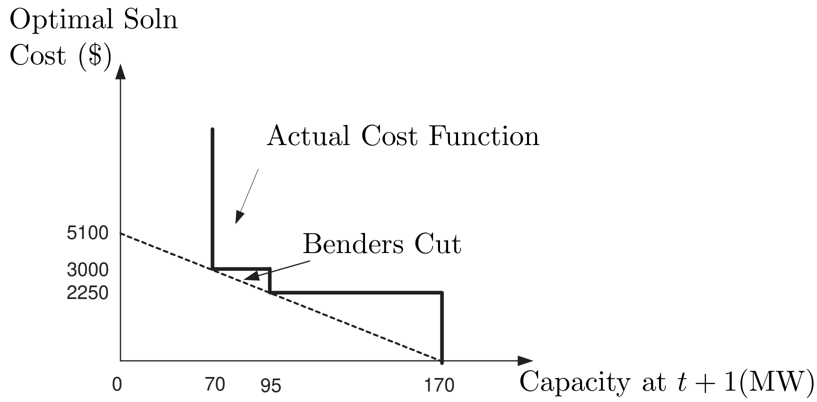


Figure 6.2 Benders Cut from Integer Problem with Non Zero Dual Variables

term represents the level of violation of the complicating constraints which in turn provides Lagrangian dual variables [80]. The work undertaken by Cerisola et al. in [78] studies the computational burden of Lagrangean relaxation on the unit commitment problem in power systems. They conclude that while the Lagrangean relaxation algorithm can provide better information about the quality of the solution compared to other methods such as continuous relaxation and stochastic optimisation, it doesn't guarantee a better solution and has increased computational burden.

The additional computational burden of Lagrangian relaxation suggests this technique is not a good choice for the power system investment planning problem. One of the features of the extended SDDP algorithm is its solution speed and improved computational ability over traditional dynamic programming techniques. An increase in computational burden cannot be justified without an increase in solution quality, something not shown by the work of Cerisola et al. The choice of continuous relaxation of subproblems for the extended SDDP model is the best option to balance solution speed with solution accuracy.

Implications of SDDP Extension

There are two major implications of this extension, the first being that the future cost functions will no longer represent the future cost function exactly. The optimal solution value of the continuous optimisation problems on the backward pass will usually be less than the equivalent integer problem. Using the example optimisation from Equation 6.1 and the graph of Figure 6.2, Figure 6.3 (not to scale) shows the Benders Cut from the continuous problem approximating the actual cost function from the integer problem. The Benders Cut shows the Optimal Solution Cost at $\mathbf{x}_{cap_{t+1}} = 120$MW as $600 whereas the actual cost from the integer problem is $2250.

The result of this is that the approximation of the mixed integer problems future cost functions by Benders Cuts, found from the equivalent continuous or relaxed problem, will never be an exact approximation of the system investment and operational costs for the mixed integer problem. The lower bound, given by the value of the future cost function at the initial values of the state

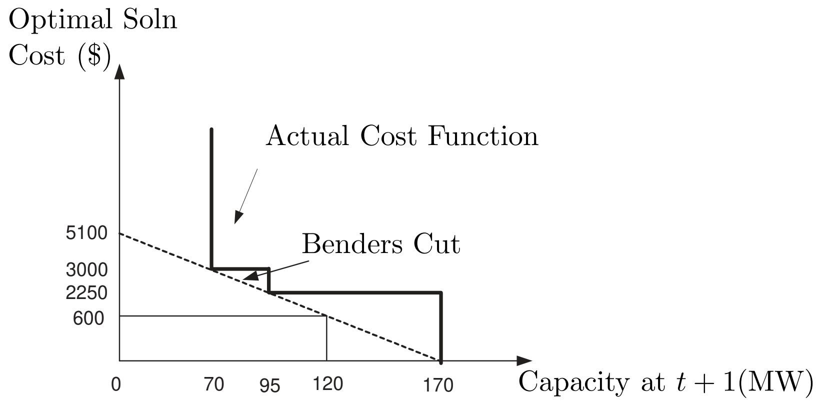


Figure 6.3 Continuous Benders Cut Approximating Integer Optimisation Problem

variables, will never exactly equal the upper bound that is found from the mixed integer solutions on the forward pass. Having bounds that do not meet each other means there is no way to tell when the algorithm has converged as the final gap between the upper and lower bounds cannot be calculated in advance. To overcome this issue the convergence criteria of the algorithm has been changed. This is an extension to SDDP in itself and is discussed in Section 6.4 below.

The gap between the upper and lower bounds also gives useful information to the problem solver. The upper bound gives the cost of actually investing in the capacity options that are available whereas the lower bound gives the cost of investing in the minimum capacity required to meet the system constraints. The gap between the bounds therefore illustrates the difference in system cost between investing in only the required capacity and investing in the capacity options that are currently available. Where the gap is considered large the integer capacity investment options available are much larger than the capacity actually required by the system. This can signal to the problem solver that the capacity options available are not a good match for the system requirements. The problem solver may be able to find a better range of investment choices that are closer to system requirements and will achieve a lower cost optimal policy of investment decisions.

6.3 INVESTMENT RE-USE

The power system investment planning problem has another constraint that differentiates it from the current applications of SDDP. For the investment problem, each investment option is available for selection only once. This differs from hydro scheduling where there are no restrictions on selecting the same size water release in different time periods. The constraint for the investment planning problem can be considered an investment re-use constraint, that is, once an investment option has been chosen it cannot be reselected for investment in subsequent time period. This constraint originates from the real world problem where each capacity investment in a power system is unique in size, cost and location.

To incorporate this system constraint into SDDP a new mathematical constraint that has been termed a ***dynamic constraint*** is introduced to the SDDP algorithm. This constraint differs from the single stage optimisation problem constraints in that its form changes from one time period to the next. The constraint tracks which investments have already been used and ensures the current time period optimisation problem cannot re-use them. The constraint is calculated post optimisation in each time period on the forward pass for use in the following time period optimisation. The investment re-use constraint used in the current time period optimisation is altered to include the investment option(s) just selected. The constraint is then passed to the next time period and is appended to the optimisation problem constraint set. The investment re-use constraint used at each time period is also stored for use on the backward pass.

The form of the constraint is shown in Equation 6.2.

$$\mathbf{x}_{inv_i} + \mathbf{x}_{inv_j} + \mathbf{x}_{inv_k} = 0 \tag{6.2}$$

Where:

- $\mathbf{x}_{inv_i}$ = Investment i in MW
- $\mathbf{x}_{inv_k}$ = Investment k in MW

The variables on the left hand side of the constraint indicate the investments that have been previously selected and have the units of MW. In this example 3 investments have already been selected in previous time periods. Requiring the constraint to be equal to 0 ensures that all variables on the left side of the constraint are also forced to be 0, where 0MW indicates the particular investment is not selected in the current time period.

Introduction of the dynamic constraint to the optimisation has an effect on the state space of the optimisation. By removing investment options from the pool of available opportunities over time, the state space of the problem reduces. The state space at each time period on the forward pass must also match the state space of each time period optimisation on the backward pass. If this doesn't happen there is no guarantee the algorithm will converge as the backward pass could repeatedly select a cheap investment option at each time period. The forward pass would never be able to match this due to the investment re-use constraint. Ensuring that each time period on both the forward and backward passes has the same state space requires the dynamic re-use constraint from the forward pass be stored and re-used at the corresponding time period on the backward pass. Introducing the dynamic constraint has no effect on the convergence of the SDDP algorithm as the state space at each time period on the forward and backward passes are identical.

6.4 CONVERGENCE

The SDDP extension that allows for mixed integer optimisation problems to be solved by using a relaxed continuous optimisation on the backward pass results in an upper bound and a lower bound that may never be equal (Refer Section 6.2). The existing convergence criteria for SDDP differ depending on whether the problem is stochastic or deterministic but both rely on equality between bounds being tested. The mixed integer investment problem cannot be assured of having equal bounds, resulting in the current convergence criteria for SDDP being invalid. This motivates an extension to SDDP in the form of altered convergence criteria. The changes utilise knowledge from both the existing deterministic and stochastic convergence information.

The current convergence criteria for both deterministic and stochastic problems are described in the following sections. The new convergence criteria for mixed integer problems is then described.

6.4.1 Deterministic Dual Dynamic Programming Convergence

For continuous deterministic problems the DDP algorithm terminates when the lower bound and the upper bound are equal. Benders Decomposition theory [70] shows that if enough Benders Cuts are used to approximate the future cost function the function will be exactly equal to the value of the subproblem. In this situation the lower bound will equal the upper bound.

When the future cost function accurately represents the subproblem each iteration of the forward pass will achieve the same optimal solution in each time period as in the previous iteration. When this occurs the upper bound will no longer change in value and the trial values of the state variables do not change. The trial values are used on the backward pass to solve each time period optimisation and calculate Benders Cuts. If the trial values do not change between iterations, the backward pass optimal solutions do not change, therefore the calculated Benders Cuts are replications of previously calculated Benders Cuts. The future cost functions are no longer improving in accuracy and consequently the lower bound will not improve further. At this point convergence of the continuous deterministic optimisation problem has been reached.

6.4.2 Stochastic Dual Dynamic Programming Convergence

For continuous stochastic problems the original SDDP algorithm terminates when the expected lower bound falls within a predetermined confidence interval of the expected upper bound.

The introduction the confidence interval convergence criteria is necessitated by the upper bound being calculated by Monte Carlo simulation. The forward pass is simulated a number of times with the upper bound from each simulation being used to calculate the expected upper bound and a confidence interval describing the range in which the upper bound is expected to fall. The lower bound cannot be expected to equal the upper bound as the exact value of the upper bound is unknown. The algorithm therefore requires that the expected lower bound fall within the confidence interval of the expected upper bound to achieve convergence. At this point the future cost functions are deemed sufficiently accurate (though may not be exact) that any changes in optimal solutions between iterations are attributed to the change in the sampled stochastic variable and not an inaccurate future cost function approximation.

6.4.3 **Deterministic** Dual Dynamic Programming Convergence for Mixed Integer Problems

For a deterministic investment problem where the forward pass invests in large integer investments and the backward pass optimises a continuous relaxed equivalent problem, the upper and lower bounds may never be equal. This research has developed an extension to DDP that overcomes this issue by creating new convergence criteria.

Continuous deterministic problems use an absolute convergence criteria of the lower bound being equal to the upper bound. This occurs when the future cost functions no longer improve in value. The new convergence criteria for a deterministic mixed integer problem uses a rate of change convergence criteria to identify when the future cost functions and hence lower bound are no longer improving in accuracy. At this point the value of the lower bound becomes static. This convergence test does not compare the upper and lower bounds at all and hence is applicable for mixed integer problems.

6.4.4 **Stochastic** Dual Dynamic Programming Convergence for Mixed Integer Problems

For stochastic mixed integer problems the new convergence criteria are more difficult to define. The problem is stochastic suggesting that a confidence interval convergence criteria should be used but the gap between the upper and lower bounds due to the relaxed backward pass of the extended SDDP algorithm results in this convergence criteria being invalid. Due to the gap between the bounds, the lower bound may never fall within the confidence interval of the expected upper bound. This doesn't indicate that the problem never converges, only that the gap between the bounds is larger than the confidence interval of the upper bound. For stochastic problems, the lower bound may not become static due to the effect of sampling resulting in a continually changing Benders Cut being calculated. A rate of change convergence criterion is therefore inappropriate.

Convergence of SDDP can also be decided based on the future cost function approximations. When these function no longer improve in accuracy the algorithm should exit as the optimal solutions at each time period will not change. The new convergence criteria uses the original convergence criterion for stochastic problems (using a confidence interval) but applies it the lower bound given by the relaxed continuous problems from the backward pass and an expected upper bound found from solving *continuous* optimisation problems on the forward pass. This new criterion requires relaxed continuous optimisation problems to be solved in addition to the mixed integer optimisation problems at each time period on the forward pass. The relaxed continuous problems use the same constraints and initial state variable values as the mixed integer optimisations. The additional continuous optimisation problems replicate the corresponding time period optimisation problem on the backward pass. When the lower bound falls within the predetermined confidence interval of the expected *continuous* upper bound, the future cost functions are considered to no longer be improving in value and the algorithm is considered to have converged.

The extension to the convergence criteria of SDDP is valid for the mixed integer optimisation problem due to the same future cost functions being used for both the mixed integer and continuous forward pass optimisations. When the future cost functions for the continuous problems are no longer improving in accuracy the future cost functions used in the mixed integer problems are no longer improving in accuracy. Any change in the mixed integer optimal solution can therefore be attributed to stochastic sampling and not to improvements in the future cost functions.

Continuous Upper Bound Two issues result from the continuous upper bound using the state variable values from the associated mixed integer optimisation. The first is the continuous upper bound may be greater than the mixed integer upper bound. The second is the continuous upper bound may never equal the (continuous) lower bound even though the single stage optimisation problems are identical.

The first issue occurs because the expected upper bound is the summation of the *immediate* time period costs across the planning horizon. An unrestricted continuous problem may invest heavily in the immediate time period if this reduces the expected future costs of the system. The associated mixed integer problem is restricted to investments of specific sizes so may not be able to reduce the future costs of the system to the same extent as the continuous problem. Due to the continuous optimisation problem using the state variable values from the mixed integer problem, the continuous problem may not be able to tradeoff high immediate costs for low future costs. The result is the potential for future time periods to also invest heavily and incur high immediate time period costs. In this situation the expected continuous upper bound may be greater than the expected integer upper bound. This is not a deterrent to using the new convergence criteria as the criteria still identifies when the future cost functions are no longer improving in accuracy, regardless of whether the expected integer or expected continuous upper bound is larger.

The second issue is that the continuous upper bound may never equal the lower bound, even if the problem is deterministic. The continuous upper bound is the summation of the first time period immediate costs plus the immediate costs of each remaining time period in the planning horizon. Each future time period continuous optimisation is solved at the mixed integer state variable values. In comparison, the lower bound is the value of the continuous future cost function at the initial state variable values. The future cost functions are continuous approximations where the continuous optimisation state variable values are used to find the future costs of the system. The potential for the bounds to be unequal does not render the convergence criteria unsuitable as the algorithm can still identify when the future cost functions are no longer improving in accuracy. For deterministic problems the rate of change convergence criteria of Section 6.4.3 is still valid. For stochastic problems the discrepancy between bounds is greatest when the optimal continuous solution is very different to the optimal mixed integer solution. If the optimal solutions of the two problems are similar, the discrepancy between the continuous upper bound and lower bound will be absorbed within the confidence interval of the expected continuous upper bound. Where the two solutions vary greatly, the large integer investment choices available to the system are very different to the optimal continuous solution. This situation is similar to that discussed in Section 6.2.2 where the problem solver can use the large gap between bounds to assess suitability of available investment options.

6.4.5 Optimality of Extended SDDP Algorithm

Where the future cost function can be represented precisely using Bender's Cuts, such as in a continuous variable problem, the solution found to the problem will be globally optimal. The future cost function will stop improving in accuracy when the applicable region of the state space is precisely represented. The approximation of a non convex function by convex linear constraints, as in the power system investment problem, means the future cost function can never be represented with full precision. The inaccuracy of the approximation is undesirable but is unavoidable in problems that contain integer variables. The inaccuracy of the future cost function may result in a suboptimal investment solution being found. While a suboptimal solution is not the desired outcome it may not have a material effect on the solution if the investment choices do not deviate too far from optimality. To identify whether the inaccuracy inherent in the future cost function approximation is material, the investment choices made by the extended SDDP algorithm must be at least locally optimal or 'near optimal'. It is only reasonable to attest whether local or near optimality of various investment choices applies as a globally optimal solution may be impossible to find for a non convex problem.

To investigate whether the extended SDDP algorithm provides local or near optimal solutions to the investment problem there must be a benchmark optimal solution with which to compare it. One way to provide a benchmark solution is to use a deterministic investment problem and compare a static mixed integer optimisation to the extended SDDP algorithm. If the extended SDDP algorithm provides a precisely accurate representation of the future cost function the static optimisation solve and the SDDP solve will give the same investment choices at the same time period. Where the approximation is less accurate, the solutions of the two optimisation problems will give differing solutions. In this situation the investment choices made by the SDDP algorithm must be locally optimal or near optimal in order for the SDDP algorithm to be

Investment Num	Capacity (MW)	Investment Cost ($/MW))	Variable Operating Cost ($/MWh)	Fixed Operating Cost ($/MW)
1	80	65,000	$100	$25,000
2	50	80,000	$100	$25,000
3	50	100,000	$100	$25,000
4	85	110,000	$100	$25,000
5	45	150,000	$100	$25,000
6	65	175,000	$100	$25,000
7	70	200,000	$100	$25,000
8	55	220,000	$100	$25,000
9	150	225,000	$100	$25,000
10	100	250,000	$100	$25,000

Table 6.1 Investment Problem Data - New Investments

Existing Demand (MW)	2500
Existing Capacity (MW)	2500

Table 6.2 Investment Problem Data - Existing System

considered suitable for solving the power system investment problem. Identifying if a solution is 'near optimal' is challenging for mixed integer problems as the closest solution to the optimal solution may result in a slightly different timing and choice of investments. This is illustrated by the following example:

Problem: The problem is to chose a series of optimal investments over an 8 year period to ensure that capacity is always greater than or equal to demand. Tables 6.1, 6.2 and 6.3 detail the system and investment specifications.

The mathematical representation of the problem for each stage (year) is shown in Equation 6.3.

Year	1	2	3	4	5	6	7	8
Demand Increases By (MW)	80	50	50	85	45	65	70	55

Table 6.3 Demand Forecast

$$
\begin{aligned}
f(\mathbf{x}) = & min\big[65,000\mathbf{x}_1 + 80,000\mathbf{x}_2 + 100,000\mathbf{x}_3 + 110,000\mathbf{x}_4 + \dots \\
& 150,000\mathbf{x}_5 + 175,000\mathbf{x}_6 + 200,000\mathbf{x}_7 + \dots \\
& 220,000\mathbf{x}_8 + 225,000\mathbf{x}_9 + 250,000\mathbf{x}_{10} + \dots \\
& 100\mathbf{x}_{op} + 25,000\mathbf{x}_{fix} + \alpha\big]
\end{aligned}
$$

Subject to:

$$
\begin{aligned}
& \mathbf{x}_1 + \dots + \mathbf{x}_{10} + \mathbf{x}_{cap_t} + = \mathbf{x}_{cap_{t+1}} \\
& \mathbf{x}_{cap_{t+1}} \geq \mathbf{x}_{dem_{t+1}} \\
& \mathbf{x}_{dem_t} + \mathbf{x}_{deminc_t} = \mathbf{x}_{dem_{t+1}} \\
& \mathbf{x}_{op} = 8760\mathbf{x}_{dem_t} \\
& \mathbf{x}_1 - 80\mathbf{y}_1 = 0 \qquad \mathbf{x}_2 - 50\mathbf{y}_2 = 0 \\
& \mathbf{x}_3 - 50\mathbf{y}_3 = 0 \qquad \mathbf{x}_4 - 85\mathbf{y}_4 = 0 \\
& \mathbf{x}_5 - 45\mathbf{y}_5 = 0 \qquad \mathbf{x}_6 - 65\mathbf{y}_6 = 0 \\
& \mathbf{x}_7 - 70\mathbf{y}_7 = 0 \qquad \mathbf{x}_8 - 55\mathbf{y}_8 = 0 \\
& \mathbf{x}_9 - 150\mathbf{y}_9 = 0 \qquad \mathbf{x}_{10} - 100\mathbf{y}_{10} = 0 \\
& \mathbf{x}_{fix} = \mathbf{x}_{cap_{t+1}} \\
& \mathbf{y}_1, \dots, \mathbf{y}_{10} \in \mathbb{Z}
\end{aligned}
\tag{6.3}
$$

Where:

- $\mathbf{x}_1, \dots, \mathbf{x}_{10}$ = Investment number
- $\mathbf{x}_{cap_{t+1}}$= System demand in MW at time t - before demand growth.
- $\mathbf{x}_{cap_t}$= System capacity in MW at time t - before investment.
- $\mathbf{x}_{cap_{t+1}}$= System capacity in MW at time $t+1$ - after investment.
- $\mathbf{x}_{dem_{t+1}}$= System demand in MW at time $t+1$ - after demand growth.
- $\mathbf{x}_{deminc_t}$ = Demand growth in MW for time period t.
- $\mathbf{y}_1, \dots, \mathbf{y}_{10}$ = Binary variable to restrict investments to integer sizes.
- $\mathbf{x}_{fix}$ = Fixed operating costs in MW.
- $\mathbf{x}_{op}$ = Variable operating costs in MWh
- α = Represents future costs of investment and operation, is represented by Bender's cuts.

Note: The investments have an monotonically increasing investment cost from investment number 1 though investment 10 only to make it obvious which investment should be chosen first i.e. investment one should be chosen before investment two etc.

The example shown has perfectly matching demand increases and integer sized investment options. This is equivalent to relaxing the integer investment constraints of the problem and solving

Year	Inv 1	Inv 2	Inv 3	Inv 4	Inv 5	Inv 6	Inv 7	Inv 8	Inv 9	Inv 10
1	80	0	0	0	0	0	0	0	0	0
2	0	50	0	0	0	0	0	0	0	0
3	0	0	50	0	0	0	0	0	0	0
4	0	0	0	85	0	0	0	0	0	0
5	0	0	0	0	45	0	0	0	0	0
6	0	0	0	0	0	65	0	0	0	0
7	0	0	0	0	0	0	70	0	0	0
8	0	0	0	0	0	0	0	55	0	0
Optimal Solution Cost $20146560000										

Table 6.4 Static and SDDP solution: Continuous Variables

a continuous variable problem. In this situation both a static optimisation solve and the SDDP solve should provide the same optimal solution. These results for both optimisations are the same and are shown in Table 6.4.

The result of the example above is expected as continuous variable problems will provide a precisely represented future cost function in the applicable region of the state space. For the particular example above, the definition of a future cost function does not change the investment choices from the first forward pass where the future cost function is equal to zero. This does not indicate that the SDDP algorithm has failed, rather it is an artefact of the particular simplistic problem demonstrated, where the optimisation gains no benefit from knowing the future cost of the decision. A more complex problem would be highly unlikely to exhibit this simplistic behaviour and the resulting future cost functions would be defined through greater numbers of linear constraints. Overall the same result would be seen, where the precisely represented future cost function provides the optimal solution to the investment problem, and matches the investments chosen by the equivalent static problem.

When the investment sizes available do not match the demand increases perfectly the future cost function approximation will be inaccurate due to the non convex future cost function shape. This may result in only a 'near optimal' set of investment decisions being found. For example, in the preceding problem suppose investment option 1 is now changed to be 150MW rather than the original 80MW. The solution of the static optimisation is shown in Table 6.5 and the SDDP optimisation in Table 6.6.

Whilst the two optimisation problems give different optimal solution paths the difference between the optimal solution costs is only 0.018% or $39.4mill. The example shows that the investments in years 3, 7 and 8 are the same for the two optimisation problems and the 50MW investment in year 6 of the static problem only moves by one year to year 5 in the SDDP problem. Comparing the cumulative investment profile of the static and SDDP solution illustrates where the SDDP solution deviates from the static solution. This direct comparison is only valid because the fixed

Year	Inv 1	Inv 2	Inv 3	Inv 4	Inv 5	Inv 6	Inv 7	Inv 8	Inv 9	Inv 10
1	0	0	0	85	0	0	0	0	0	0
2	0	0	0	0	45	0	0	0	0	0
3	0	50	0	0	0	0	0	0	0	0
4	150	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	50	0	0	0	0	0	0	0
7	0	0	0	0	0	65	0	0	0	0
8	0	0	0	0	0	0	0	55	0	0
Optimal Solution Cost $20139485000										

Table 6.5 Static Solution: Investment 1 = 150MW

Year	Inv 1	Inv 2	Inv 3	Inv 4	Inv 5	Inv 6	Inv 7	Inv 8	Inv 9	Inv 10
1	150	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	0	50	0	0	0	0	0	0	0	0
4	0	0	0	85	0	0	0	0	0	0
5	0	0	50	0	0	0	0	0	0	0
6	0	0	0	0	45	0	0	0	0	0
7	0	0	0	0	0	65	0	0	0	0
8	0	0	0	0	0	0	0	55	0	0
Optimal Solution Cost $20141110000										

Table 6.6 SDDP Solution: Investment 1 = 150MW

and variable operating costs are the same for all investments. Table 6.7 shows this comparison.

In solving the SDDP problem the optimisation changed the optimal choice of solution after the first iteration due to the addition of the linear constraints that define the future cost function approximation. In the first forward pass the optimal investment choices were those shown in Table 6.8

With the addition of linear cuts representing the future cost function, in subsequent forward passes the optimal investment choices change to those shown in Table 6.6. These are the final optimal investment choices that provide convergence of the algorithm in 9 iterations. The large number of iterations reflect the need for the algorithm to promulgate the effect of the linear constraints from the last time period through to the first time period, the time point where the lower bound is calculated.

Overall the SDDP solution is choosing to invest in larger amounts earlier in the time horizon as this minimises the future costs of the system i.e. fewer investment costs in subsequent time periods. The earlier timing of investment choices is a balance between the future costs of

Year	Static (MW)	SDDP (MW)
1	85	150
2	130	150
3	180	200
4	330	285
5	330	335
6	380	380
7	445	445
8	500	500

Table 6.7 Comparison of Cumulative *Investment* Capacity at each Year

Year	Inv 1	Inv 2	Inv 3	Inv 4	Inv 5	Inv 6	Inv 7	Inv 8	Inv 9	Inv 10
1	0	0	0	85	0	0	0	0	0	0
2	0	50	0	0	0	0	0	0	0	0
3	0	0	50	0	0	0	0	0	0	0
4	150	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	45	0	0	0	0	0
7	0	0	0	0	0	65	0	0	0	0
8	0	0	0	0	0	0	0	55	0	0

Table 6.8 Optimal Investment Choice, First Forward Path, Future Cost = 0

investment that are saved by investing in the current time period, and the additional fixed costs of capacity that are incurred by having surplus capacity in the system at earlier years. The $39.4mill difference between the static and SDDP solutions could be considered large in comparison to even the most expensive investment option, investment 10 at $25mill investment and $2.5mill fixed operating costs per year but a direct comparison of various investments based on cost is not a valid approach. The choice of one investment over another and the resulting effect on the overall solution cost is dependent not only on the investment cost but the cumulative effect on fixed operational costs over the planning horizon.

Cost alone does not give a good indication of local optimality but the comparison of cumulative investment capacity alongside the difference in investment costs may suggest the likelihood of local or near optimality of the solution. For the example above the very small difference in optimal solution cost, the similar investment choice and timing for the majority of the years and the similarity between the cumulative investment capacity suggests that the SDDP optimisation can be considered to provide a 'near optimal' solution. This is the best result that can be hoped for considering the future cost function will never be approximated precisely. The integer nature of the problem makes it difficult to identify whether local optimality of the SDDP solutions exists as more than one investment choice and/or timing may change for a single perturbation of an investment size. As large parts of the investment path are the same between the optimisation problems and the optimal solution cost is very similar the resulting 'near optimal' solution from the SDDP algorithm suggests that the approximation of the non convex future cost function by linear constraints provides a reasonable solution to the investment problem. Despite the challenges of assessing local optimality of the SDDP solution, the results illustrated above give sufficient confidence in the 'near optimality' of the solution to move forward with developing a real world optimisation model and problem.

The example problem above is very simplistic and small in size. It is highly constrained problem that requires investment in nearly every time period. Real world problems are very different in scope and have many more inputs such as unique operational costs for each investment type. The scale of real world problems mean that the vast majority of system costs consist of operational costs that are represented by continuous variables. This suggests that much of the future cost function can be very accurately represented by linear constraints. The inaccuracies introduced to the future cost function by the relaxation of integer variables will be much smaller relative to the cost contribution of the operational costs. This effect suggests for a certain size and structure of power system investment problem that a relaxed future cost function will result in a locally or near optimal solution via the extended SDDP algorithm.

Further consideration of the local or near optimality and the associated approximate future cost function is considered in Section 11.2.1.

6.5 EXTENDED SDDP ALGORITHM STRUCTURE

Figure 6.4 presents a flow diagram of the extended SDDP algorithm.

6.6 SUMMARY

This chapter has shown that the SDDP algorithm must be extended to allow the solution of mixed integer problems such as the power system investment planning problem. Three extensions to SDDP have been developed.

The first extension overcomes the problem of being unable to use dual variables to approximate the future cost functions for mixed integer problems via Benders Cuts. The extension requires the algorithm to solve mixed integer optimisations on the forward pass and a relaxed continuous optimisations on the backward pass. This allows Benders Cuts to be used to approximate the subproblems. The difficulty this extension introduces is that the future cost functions now approximate a continuous rather than mixed integer subproblem and the lower bound may never equal the upper bound. The gap between bounds necessitates changes to the convergence criteria of the SDDP algorithm but also gives useful information to the problem solver about the suitability of investment options for the planning problem being optimised.

The second extension to SDDP is the introduction of a dynamic constraint that restricts investment options from being re-used. In power system capacity planning each investment option is available only once so re-use of investments is not allowed. The constraint tracks the investments used over the forward pass of the algorithm and is updated post optimisation in each time period. The constraint is also stored for use on the backward pass. The effect of introducing a constraint such as this is to reduce the state space of the problem over time. It is important that the corresponding time period optimisations in the forward and backward pass have the same state space else the SDDP algorithm may never achieve convergence.

The third extension to SDDP is the altering of the convergence criteria of the algorithm. By implementing the first SDDP extension, relaxing the optimisations problems on the backward pass, the upper and lower bounds may never be equal. The new convergence criteria requires that a mixed integer and equivalent continuous optimisation problem is solved at each time period of the forward pass. An expected continuous upper bound can be calculated along with a confidence interval for the value of the continuous upper bound. When the lower bound falls within the confidence interval for the continuous upper bound the future cost functions are considered approximate the subproblems with sufficient accuracy that the problem has converged.

The difficulties in identifying optimality or 'near optimality' of solutions obtained using the extended SDDP algorithm is discussed. The approximation of the non convex future cost function of the power system investment problem can not be perfectly represented using linear constraints but the size of real world investment problems suggests that the extended SDDP algorithm is suitable for solving the power system investment problem.

The extensions to SDDP allow the power system investment planning problem to be solved using SDDP. Before the algorithm can be implemented the system must be modeled in detail. Chapters 7 and 8 discuss mathematical modelling of power systems and investments.

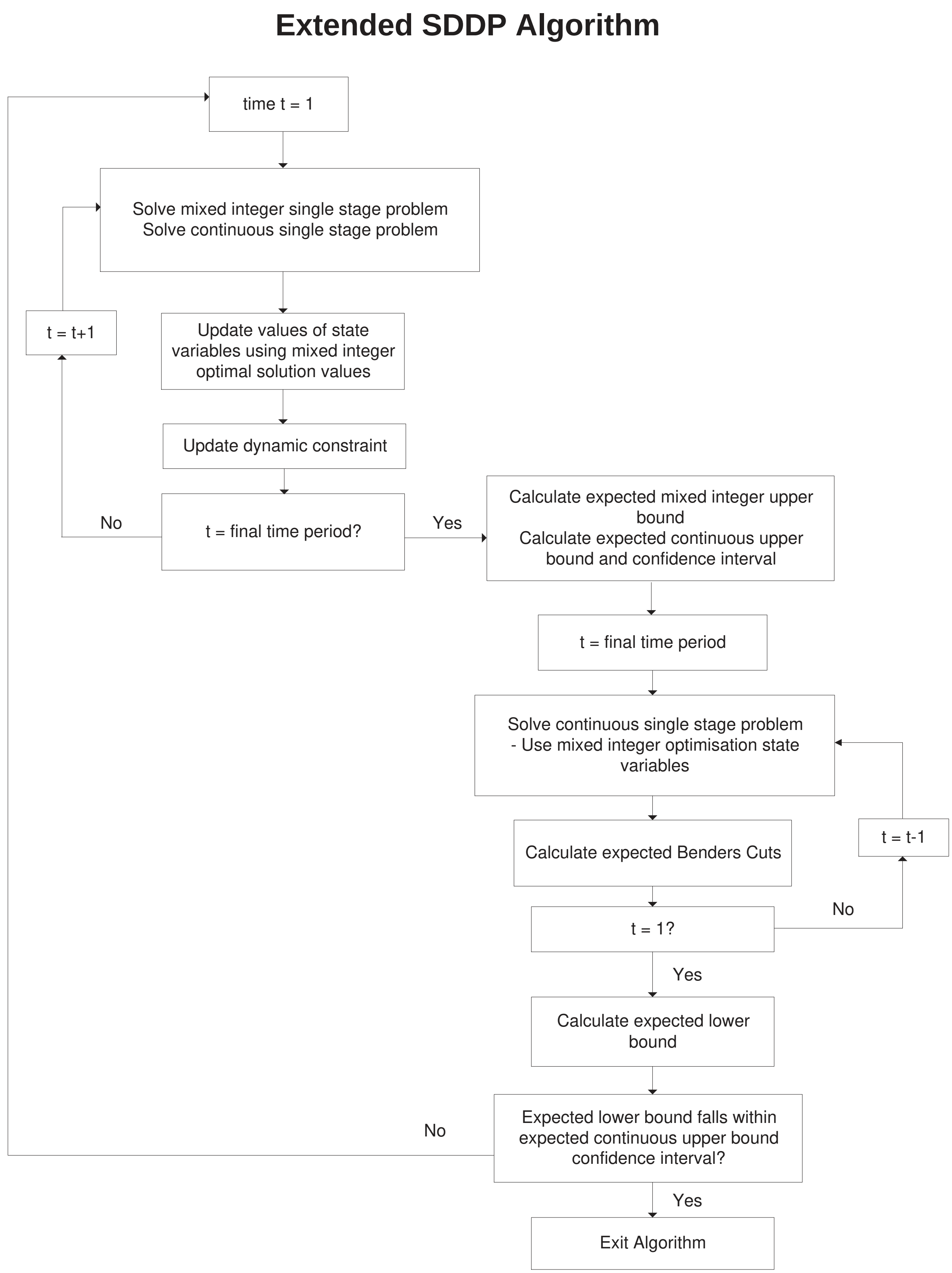


Figure 6.4 Flow Diagram of Extended SDDP Algorithm

Chapter 7

GENERATION INVESTMENT OPTIMISATION

7.1 INTRODUCTION

The power system investment planning problem is to decide the type and timing of investments in a power system in order to ensure it is optimal at all times within a defined planning horizon. As each single stage optimisation problem is solved within the SDDP algorithm, the optimisation selects the combination of investments that gives the optimal solution to the planning problem. The optimal solution is defined as the system state that minimises the costs of system investment and operation for the current time period plus the remainder of the planning horizon.

The problem solver must take the real world planning problem and construct a mathematical representation that can be solved at each time period of the planning horizon. The construction of the mathematical model must account for both the objective function and the constraints of the problem. This chapter discusses the mathematical modelling of the generation investment planning problem and presents the constraints applicable to a general problem.

7.2 MODELLING DECISIONS

A number of modelling decisions must be made in advance of constructing the mathematical representation of the model. The necessary decisions for the generation investment problem include the representation of state variables within the model, how demand fluctuations within a time period are modelled, timing of investments within a time period and how the Value of Lost Load or VoLL is represented within the model.

7.2.1 Capacity Representation

The first modelling decision made is the representation of capacity. There are three options, the first is to represent total system capacity as a single value, the second is to represent each distinct capacity type, e.g. wind, hydro, thermal etc., as separate capacity variables, the third option is to represent every individual generation investment and existing plant as individual variables.

The first option is very simplistic and gives very little information to the problem solver about the composition of generation within the system. With system capacity represented as a single variable it is not possible to calculate the costs of system operation using differing costs for separate generation types.

The second option of representing each capacity type as a separate variable is also unsuitable for the model. This improves on option one as each capacity type can be costed separately but it doesn't account for operational cost differences between large and small generation plants of the same capacity type.

The third option of representing all existing generation plants and every generation investment as a separate variables in the problem has been chosen. This option allows every generation plant and investment to have a unique cost of operation that results in an accurate calculation of system operation costs. The drawback is an increase in the state space of the model as each generator is considered a individual state variable. It is more important to have an accurate solution than a fast solution for planning problems so the increase in state space is acceptable.

Selecting option three for representing system capacity has an effect on the future cost function where each separate capacity variable is now a dimension of the function. This results in the future cost function being a better approximation of the future system costs and shows how the future system costs will change for specific investment opportunities.

7.2.2 Demand Representation

7.2.2.1 Modelling Demand

System demand can be modelled in two ways, the first is as a single variable representing peak system demand and the second is to represent each demand type as a separate variable. The generation investment planning model uses the first option of a single variable to represent peak system demand. This option has been chosen for simplicity of modelling and to allow the focus of the optimisation model to be on generation investments. Option two is a valid modelling choice if detailed information regarding system demand composition (i.e. residential, industrial, commercial, rural etc.) is available and the optimisation is to focus on demand types and the effects on generation investments.

7.2.2.2 Load Duration Curve

Another modelling decision to be made is how to represent the changes to system demand within a time period. Demand within a time period fluctuates due to daily and seasonal variation. This is usually represented by a Load Duration Curve or LDC. The LDC shows the number of hours the load can be expected to be above a certain level. Figure 7.1 shows a representation of a typical LDC where the load MW are expected to be above the level y for x hours of the year. The curve is constructed from the cumulative probability distribution associated with the expected load parameters. The x axis hour values are found by multiplying the number of hours in the time period (in this case, 1 year) by the cumulative probability value.

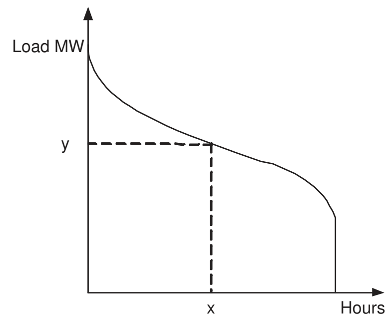


Figure 7.1 Typical Load Duration Curve

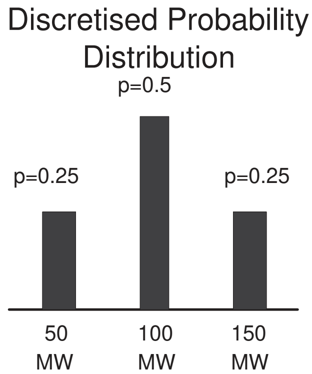
The expected MWh during the year is the area under the curve but it is easier, computationally, to discretise this curve rather than integrate it. Figure 7.2(a) shows how the load distribution probability curve is discretised which allows the equivalent LDC to be discretised as shown in Figure 7.2(b)

A coarse discretisation of the load probability distribution results in the discrete LDC being a poor representation of the actual LDC. Increasing the number of discretisations of the probability distribution would improve the calculation of expected MWh but at the cost of solution speed.

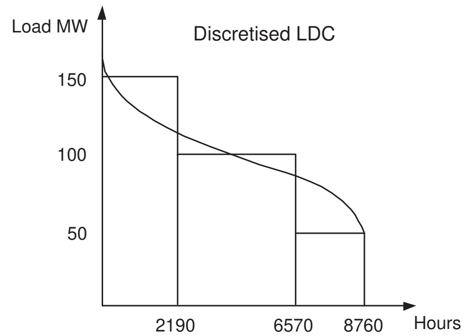
The generation investment planning model uses a discretised LDC to represent system demand within a time period. The curve is used to calculate the required MWh of generation from the system and hence the operational costs of the system for the time period.

7.2.3 Value of Loss Load

For power systems that operate within a market environment there is no compulsion for investors to ensure that the system has enough capacity to supply demand. The threat of re-regulation indirectly motivates market participants to ensure adequate generation capacity but no one is charged with ensuring capacity provision. The investment model therefore has no explicit constraints that require generation capacity to be greater than peak demand. Without such constraints the system can potentially have more demand than capacity but this non supplied demand has an associated cost to the system. This cost, called the Value of Lost Load, or VoLL, is usually attributed a single monetary value in terms of \$/MWh not supplied. Attributing a single monetary value to all non supplied load may be considered unwise as undoubtedly non supply situations are valued by customers differently, nevertheless, this is the traditional



(a) Discretised Probability Distribution



(b) Discretised Load Duration Curve

Figure 7.2 Discretised Load Probability and Load Duration Curve

approach. In this model a high variable operating cost is assigned to VoLL to reflect the political and social costs of failing to meet demand.

For the investment problem, the demand not supplied is modelled using a 'dummy' or VoLL generator. This VoLL generator is added to model as an additional capacity type. The size of the VoLL generator is given by the problem solver who has the option of either simply specifying a very large MW value, eg. 10,000MW, or using the peak demand value of the system. Using the peak demand value of the system intuitively makes sense as this is maximum amount of demand that may potentially be not supplied by generation. The peak value of demand should be the smallest capacity size used for the VoLL generator else in the worst case situation, where the system has no generating capacity, the VoLL generator will be operating at full capacity but there will still be non supplied load.

There are two options for modelling the costs of the VoLL generator. The first is to assign both variable and fixed operating costs at values greater than zero. The second is to assign only a variable operating cost. The first option allows the VoLL generator to be compared on an equal basis with other existing generators that have both variable and fixed operating costs. The second option reflects the market information of many systems that define only a single value, a variable operating cost, to the Value of Lost Load. The second approach assumes that consumers value their security of supply at a constant value regardless of the number of times they experience outages or the length of time an outage occurs. While markets and national level grid systems often use a single MWh variable cost to represent the value of security of supply, distribution level systems measure the reliability of supply in three different ways. The first is the System Average Interruption Duration Index (SAIDI) that measures the total time in minutes an average customer is without supply during a year, the second is the System Average Interruption Frequency Index (SAIFI) that indicates the number of times an average customer will experience an outage over a year, the third measurement is Customer Average Interruption Duration Index (CAIDI) that measures the average duration in minutes of any single outage. These three measurements indicate that the value placed on security of supply could be better modelled if the costs of VoLL represented both length of time of outages and number of MW lost. This type of comprehensive approach to modelling VoLL is consistent with the first modelling option of including both variable and fixed operating costs for the VoLL generator. Real world market systems have not adopted this level of model detail therefore the modelling undertaken in this research uses the second option of assigning a single variable operational cost to the VoLL generator.

It is acknowledged that more detailed modelling of VoLL costs would create a more complete model but assigning a single MWh cost for VoLL is not unreasonable. VoLL costs are designed to reflect the high value consumers place on security of supply therefore a suitably high single value for VoLL will achieve the goal of ensuring that dispatch of the VoLL generator is a last resort.

7.2.4 Reserves

To maintain power system security during generator outages reserve capacity is required by the system. The amount of reserve capacity required varies depending a number of factors including system design, levels of interconnection and generation types. Many systems base their reserve requirements around the largest generating risk to the system[1]. Providing sufficient reserve capacity to cover the entire capacity of the largest generator is likely to overestimate the amount of reserve necessary but is the approach taken in this modelling due to its simplicity.

The cost of providing reserve capacity is the additional investment and fixed costs of capacity that may be utilised infrequently. Any constraint that requires capacity to be sufficient to meet demand is contrary to market design where there is no requirement of investors to ensure demand is supplied. To overcome this issue the VoLL generator is used in calculations of total capacity. Equation 7.1 illustrates the type of constraint required to ensure that sufficient reserve capacity is available. Due to the inclusion of the large VoLL generator, any capacity constraint such as Equation 7.1 becomes trivial in that the constraint will never be binding.

$$\mathbf{cap}_{t+1} + \mathbf{cap}_{VoLL_{t+1}} - \mathbf{MW}_{risk_{t+1}} \geq \mathbf{dem}_{t+1} \tag{7.1}$$

[1]The New Zealand market provides enough reserve to cover a portion of the largest generating risk. The portion is described as a reserve adjustment factor or RAF [81]. This RAF value is calculated by an iterative dispatch and simulation process where the level of reserve dispatched is used in the simulation of a generator outage. Where the system frequency standards are not maintained by the level of reserve in the system, the simulation finds the optimal level of reserve that will maintain system frequency standards. This new reserve level is entered into the scheduling pricing and dispatch optimisation software and the optimal system dispatch and reserves are recalculated.

Where:

- $\mathbf{cap}_{t+1}$ = Total capacity excluding the VoLL generator capacity.
- $\mathbf{MW}_{risk_{t+1}}$ = Capacity of largest system generation risk.

The generation investment model does not represent actual generator outages, it only calculates the cost of providing reserve capacity cover should an outage occur. The VoLL generator has no investment or fixed costs of operation therefore Equation 7.1 will therefore always allow the VoLL generator to provide reserves as no costs are incurred by doing so. This situation is equivalent to the system dropping load any time there is a generator outage. Dropping load as a first resort is not a politically or socially sustainable solution and therefore must be discouraged in the optimisation problem. To discourage the use of VoLL for reserves a penalty cost is associated with the VoLL used to provide reserve capacity. The penalty cost is included in the objective function. Determining how many MW of VoLL are used requires the constraint in Equation 7.1 to be reconstructed into Equation 7.2.

$$\mathbf{dem}_{t+1} - (\mathbf{cap}_{t+1} - \mathbf{MW}_{risk_{t+1}}) \leq \mathbf{VoLL}_{pen_t} \qquad (7.2)$$
$$\mathbf{VoLL}_{pen_t} \geq 0$$

Where:

- $\mathbf{VoLL}_{pen_t}$ = MW of VoLL used to provide reserve capacity.

The variable $\mathbf{VoLL}_{pen_t}$ is included in the objective function and assigned a penalty cost. The penalty cost value should reflect the investment and fixed costs of capacity that consumers are willing to pay to avoid an outage. The value a consumer places on an outage (or security of supply) will depend on how they use electricity. Where one consumer may be prepared to pay extra to ensure a high security of supply, another may value their security at a much lower level. Similar to the discussion in Section 7.2.3 a single value representing the penalty cost of using VoLL to supply reserves does not accurately model the costs of outages to consumers but a high penalty cost will deter the optimisation from utilising VoLL capacity for reserves. The size of the penalty cost should be more than the investment and fixed costs (per MW) of any investment options available as even the most expensive investment option will be built before load is dropped under normal operating conditions.

7.2.5 Investment Timing

A modelling decision must also be made regarding the timing of investments and how demand growth is modelled. This thesis has adopted the convention of adding capacity investments at

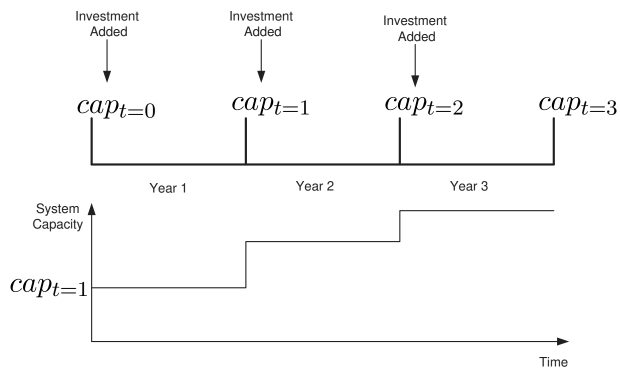


Figure 7.3 Investment Timing Beginning of Time Period

the beginning of the time period. Figure 7.3 illustrates the how system capacity grows when capacity is added at the beginning of the time period. Demand growth is modelled as a integer increase at the beginning of the time period and assumes that peak demand doesn't change within the time period.

7.3 OBJECTIVE FUNCTION

The capacity investment problem is optimised over a predefined planning horizon that is characterised by a series of sequential capacity investment decisions. For the problem studied in this research each time period is one year. As shown in Section 4.3.5 the multistage planning problem has an objective function that is the summation of each individual time period's objective function. Equation 7.3 illustrates the general form of the objective function for single time period. An important feature of the single stage objective function is the inclusion of the term $\bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap_1}, \ldots, \mathbf{cap_{numGen_{t+1}}}, \mathbf{dem_{t+1}})$ that represents the future costs of system investment and operation. The value of this function is dependent on the investment decisions made in the current single stage optimisation.

$$
\begin{aligned}
\min\big[&(\mathbf{c_{1}}_t \times \mathbf{cap_{1inv}}_t) + \ldots + (\mathbf{c_{capOp}}_t \times \mathbf{cap_{capOp}}_t) + \ldots \\
&(\mathbf{vo_1}_t \times \mathbf{MWhTotal_{cap_{1_t}}}) + \ldots + (\mathbf{vo_{numGen}}_t \times \mathbf{MWhTotal_{cap_{numGen_t}}}) + \ldots \\
&(\mathbf{fo_1}_t \times \mathbf{cap_1}_{t+1}) + \ldots + (\mathbf{fo_{numGen}}_t \times \mathbf{cap_{numGen}}_{t+1}) + \ldots \\
&(\mathbf{res}_{pen_t} \times \mathbf{VoLL}_{pen_t}) + \ldots \\
&\boldsymbol{\beta}_t \times \bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap_1}, \ldots, \mathbf{cap_{numGen}}_{t+1}, \mathbf{dem}_{t+1})\big]
\end{aligned}
\tag{7.3}
$$

Where:

- $\mathbf{capOp}$ = Number of capacity investment options.
- $\mathbf{c_{1}}_t$ = Installation cost of investment option 1 (\$/MW).
- $\mathbf{c_{capOp}}_t$ = Installation cost of investment option **capOp** (\$/MW).
- $\mathbf{cap_{1inv}}_t$ = Decision variable associated with investment option **1** (MW).
- $\mathbf{cap_{capOp}}_t$ = Decision variable associated with investment option **capOp** (MW).
- **numGen** = Total number of generation capacity types in each respective load region. This *includes* VoLL generators.
- $\mathbf{vo_{1}}_t$ = Variable operating costs of capacity type **1** (\$/MWh).
- $\mathbf{vo_{numGen}}_t$ =Variable operating costs of capacity type **numGen**(\$/MWh).
- $\mathbf{MWhTotal_{cap_{1}}}_t$ = Decision variable associated with variable operating costs for capacity type **1**.
- $\mathbf{MWhTotal_{cap_{numGen}}}_t$ = Decision variable associated with variable operating costs for capacity type **numGen**.
- $\mathbf{fo_{1}}_t$ = Fixed operating costs of capacity type 1 (\$/MWh).
- $\mathbf{fo_{numGen}}_t$ = Fixed operating costs of capacity type **numGen** (\$/MWh).
- $\mathbf{cap_{1}}_{t+1}$ = Decision variable associated with fixed operation costs for capacity type **1**. This is the value of capacity type **1** after investment in this time period.
- $\mathbf{cap_{numGen}}_{t+1}$ = Decision variable associated with fixed operation costs for capacity type **numGen**. This is the value of capacity type **numGen** after investment in this time period.
- $\mathbf{res}_{pen_t}$ = Penalty cost for using VoLL to provide reserves.
- $\mathbf{VoLL}_{pen_t}$ = MW of VoLL used for providing reserves.
- $\bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap_1}, \ldots, \mathbf{cap_{numGen}}_{t+1}, \mathbf{dem}_{t+1})$ = Future cost function value for this time period as a function of the state variable values.
- t = Time period for the optimisation.
- $\boldsymbol{\beta}_t$ = Time value of money associated with risk adjusted discount rate.

Equation 7.3 can be split into five sections, each of which contribute to the costs of the system. The first section is the cost of investment,the second is the variable operating costs of the system, the third section is the fixed operating costs of the system, the fourth section is reserve penalty costs and finally the future costs of the system that are dependent on the investments made in the current time period. Each of the costs of the problem has a cost coefficient and associated decision variable where the cost coefficient is supplied by the problem solver and can reflect a number of costs to the system. For example, the variable cost coefficients may include the costs of fuel, maintenance, government subsidies and environmental taxes. The fixed cost coefficients can also include a number of system costs such as maintenance, subsidies, capacity payments and taxes. The future cost term is found through the construction of Bender's Cuts during the SDDP algorithm and has the coefficient of $\boldsymbol{\beta}_t$ to represent the time value of money associated with a risk adjusted discount rate.

7.4 CONSTRAINTS

The constraints of the optimisation problem restrict the optimal solution to conform to real world system specifications and restrictions. They also tell the model how the system state changes over time. The system here can be considered to be over determined, that is there are more constraints than variables, but this is preferable from an ease of exposition and understanding of the model. Commercial optimisation solvers will usually reduce an over determined problem down in order to efficiently solve the problem. While it can be useful for the problem solver to do this themselves and hence increase solution speed, to an outside reader the resulting system constraints would be hard to interpret.

7.4.1 Investment Constraints

Investment constraints relate directly to the investment of new generation capacity and demand side reduction technologies. The constraints include changes to the state variables, ensuring large integer investments are undertaken, that investment options are not re-used and that demand side investment capacity totals are tracked.

7.4.1.1 State Equations

The state equations of the optimisation problem describe how the state variables that represent the system state change due to the decision variables. The power system investment model has two state variables and therefore two state equations. Equation 7.4 shows the capacity state equation.

$$\mathbf{cap}_{t+1} = \mathbf{cap}_t + \mathbf{cap}_{\mathbf{1inv}_t} + \ldots + \mathbf{cap}_{\mathbf{capOpinv}_t} \tag{7.4}$$

Where:

- $\mathbf{cap}_{t+1}$ = Total system capacity after investment in this time period.
- $\mathbf{cap}_t$ = Total system capacity before investment at the beginning of this time period.
- $\mathbf{cap}_{\mathbf{1inv}_t}$ = Investment in generation capacity of type 1 in time period t.
- $\mathbf{cap}_{\mathbf{capOpinv}_{t_\mathbf{A}}}$ = Investment in generation capacity of type **capOp** in time period t.
- **capOp** = Number of capacity investment options.

Equation 7.4 shows the state equation for the multidimensional variable of system capacity but the model must also know how each individual generator capacity changes over time. As investment options are chosen individual generator capacities change based on the investment capacity size. This information is used in other constraints such as those that calculate fixed operating costs and those that restrict the number of MWh each generator can provide. Equation 7.5 shows the form of the individual capacity state equations.

$$\begin{aligned}\mathbf{cap}_{\mathbf{1}_{t+1}} &= \mathbf{cap}_{\mathbf{1}_t} + \mathbf{cap}_{\mathbf{1inv}} \\ \mathbf{cap}_{\mathbf{2}_{t+1}} &= \mathbf{cap}_{\mathbf{2}_t} + \mathbf{cap}_{\mathbf{2inv}_t} \\ &\vdots \\ \mathbf{cap}_{\mathbf{numGen}_{t+1}} &= \mathbf{cap}_{\mathbf{numGen}_t} + \mathbf{cap}_{\mathbf{numGeninv}_t}\end{aligned} \tag{7.5}$$

Where:

- $\mathbf{cap}_{\mathbf{1}_{t+1}}$ = Capacity of type 1 after investment in this time period.
- $\mathbf{cap}_{\mathbf{1}_t}$ = Capacity of type 1 before investment at the beginning of this time period.
- $\mathbf{cap}_{\mathbf{1inv}_t}$ = Investment in capacity of type 1 given by the decision variables for investment options in this time period.
- **numGen** = Total number of generation capacity types.

Demand is also a state variable with Equation 7.6 representing the demand state equation.

$$\mathbf{dem}_{t+1} = \mathbf{dem}_t + \mathbf{dem}_{\mathbf{inc}_t} \tag{7.6}$$

Where:

- $\mathbf{dem}_{t+1}$ = System demand after demand growth in this time period.
- $\mathbf{dem}_t$ = System demand before demand growth at the beginning of this time period.
- $\mathbf{dem}_{\mathbf{inc}_t}$ = Demand growth given by the sampled value of demand for this time period.

Both the capacity and demand state equations are included as constraints in the mathematical optimisation model. These constraints are important to the model as they contain the complicating variables of the model. Where an optimisation problem must use the results of a previous optimisation e.g. $\mathbf{dem}_t$ in this time period is equal to $\mathbf{dem}_{t+1}$ from the previous time period, to construct a constraint, the constraint has complicating variables. These constraints give the dual variables that are used in the construction of Bender's Cuts.

7.4.1.2 Integer Investment

To force each capacity investment decision to be a large integer, a constraint is used to force the investment decision to be either zero or the required investment size. The constraint uses a binary variable $\mathbf{y}$ to force the result to be one value or the other.

$$
\begin{aligned}
&\mathbf{cap_{1inv}}_t - InvestSize_{\mathbf{1}}\mathbf{y_1}_t = 0 \\
&\vdots \\
&\mathbf{cap_{capOp}}_t - InvestSize_{\mathbf{capOp}}\mathbf{y_{capOp}}_t = 0 \\
&\mathbf{y_1}_t \leq 1, \ldots, \mathbf{y_{capOp}}_t \leq 1, \\
&\mathbf{y_1}_t, \mathbf{y_{capOp}}_t \epsilon \mathbb{Z}
\end{aligned} \tag{7.7}
$$

Where:

- $\mathbf{cap_{1inv}}_t$ = Investment in capacity of type 1 given by the decision variables for investment options in this time period.
- $InvestSize_{\mathbf{1}}$ = Large investment size in MW for capacity investment option 1.
- $\mathbf{y_1}_t$ = Binary variable for investment type 1 in time period t.
- $\mathbf{capOp}$ = Total number of generation capacity investment options.

Binary variables may be used as decision variables where the cost coefficient of the objective function is equal to the total investment cost for a particular investment option. This approach was not taken in this research because it was desired to use the megawatt size of the capacity investment decision in other constraints. If a binary variable had been used to represent a capacity investment decision, a second set of variables would have been necessary to represent the actual megawatt size of the investment. The method chosen reduces the number of variables in the optimisation model making it easier to solve and construct.

7.4.1.3 Dynamic Constraints

Section 6.3 discusses the extension of SDDP to include a dynamic constraint that is re-formed after each time period optimisation. This constraint restricts previously used investment options from being re-used. The constraint's form is dependent on the outcome of the previous time period optimal decision so cannot be constructed in advance. For the first time period the constraint is not used at all. An example of how the constraint would change over the sequential time periods of the planning horizon where the first time period optimal solution is to invest in generating capacity type 2 and the second time period optimal solution is to invest in generation capacity type 5 follows:

Dynamic Constraint - Time Period 1 No constraint is used.

Dynamic Constraint - Time Period 2

$$\mathbf{cap}_{\mathbf{2}_{\mathbf{inv}_t}} = 0 \tag{7.8}$$

Dynamic Constraint - Time Period 3

$$\mathbf{cap}_{\mathbf{2}_{\mathbf{inv}_t}} + \mathbf{cap}_{\mathbf{5}_{\mathbf{inv}_t}} = 0 \tag{7.9}$$

7.4.2 Variable Operational Costs

Variable operational costs track the costs incurred by the system that are directly related to utilisation of capacity e.g. generation of energy or use of a demand reduction technology. Calculating the variable operational costs requires the optimisation problem to calculate the number of MWh of operation of both generation and demand technologies. The total costs are calculated by multiplying the number of MWh of operation by the cost coefficients from the objective function.

7.4.2.1 Variable Operation Costs of Generation

The variable costs relating to generation require the MWh of energy generated to be calculated. The energy generated by all generation types (including VoLL) must equal system demand where the system demand value is given by the discretised load duration curve.

The constraints constructed to calculate the energy generated define a offer stack that dispatches generators to meet demand for each discrete block of the LDC. The cheapest generator, based on its variable operating cost, is dispatched first followed by the second cheapest then the third etc. until the peak demand level of the LDC block is reached. The final dispatched generator (or marginal generator) may not be fully dispatched, using only a portion of its potential capacity. Figure 7.4 illustrates the offer stack where the variable operating cost of each generator is ordered as:

$$genCost_3 \leq genCost_1 \leq genCost_5 \leq genCost_2 \leq genCost_4$$

Figure 7.4 shows that the marginal generator in each dispatch block may not be fully dispatched,

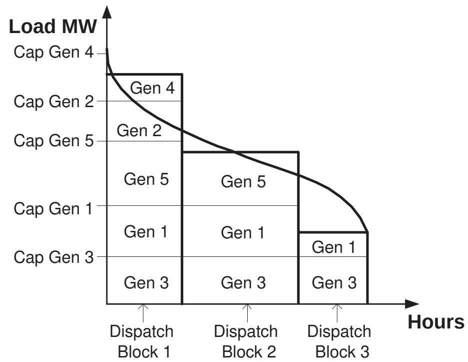


Figure 7.4 Offer stacks for Each LDC Curve Discrete Block

for example, in dispatch block 3 the marginal generator is Gen 1 which is only partially dispatched. The same situation occurs with Gen 5 in dispatch block 2 and Gen 4 in dispatch block 1. The levels marked on the y axis represent the maximum capacity of each generator in megawatts. The MWh of operation of each generator for the time period are calculated by adding together the MWh of operation from each LDC discrete block, based on the offer stack.

Each generator is assigned a problem decision variable for each discrete LDC dispatch block. These decision variables have the units of MWh. For the optimisation model to calculate the MWh generated by each generator three sets of constraints are required.

The first set of constraints restrict the number of MWh each generator can generate to ensure that maximum capacity of the generation plant is not exceeded. There is a constraint for each dispatch block and for each generator.

$$\begin{array}{l}\dfrac{\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk_1}}}}{hrs_{t_{blk_1}}} \leq \mathbf{MaxCap}_{\mathbf{cap_1}_{t+1}}, \quad \ldots \quad , \dfrac{\mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blk_1}}}}{hrs_{t_{blk_1}}} \leq \mathbf{MaxCap}_{\mathbf{cap_{numGen}}_{t+1}} \\ \quad \vdots \\ \dfrac{\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk_d}}}}{hrs_{t_{blk_d}}} \leq \mathbf{MaxCap}_{\mathbf{cap_1}_{t+1}}, \quad \ldots \quad , \dfrac{\mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blk_d}}}}{hrs_{t_{blk_d}}} \leq \mathbf{MaxCap}_{\mathbf{cap_{numGen}}_{t+1}}\end{array} \tag{7.10}$$

Where:

- $\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk_1}}}$ = The decision variable associated with the MWh generated by capacity type 1 in dispatch block 1 (MWh) in time period t.
- $hrs_{t_{blk_1}}$ = Number of hours of dispatch block 1 (hours) in time period t.
- $\mathbf{MaxCap}_{\mathbf{cap_1}_{t+1}}$ = Maximum installed plant capacity of capacity type 1 after investment

in time period t (MW).

- **numGen** = Number of different capacity types including the VoLL generator.
- d = Number of dispatch blocks in the LDC.

The second set of constraints needed to calculate the variable operational costs require the sum of all generated MWh in a dispatch block to be equal to the system demand of the dispatch block. Equation 7.11 details the constraints where **numGen** represents all capacity types including the VoLL generator. This prevents the constraints in Equation 7.11 from compelling the model to provide sufficient generating capacity to supply demand.

$$\begin{aligned}
\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk_1}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blk_1}}} &= hrs_{t_{blk_1}}(\mathbf{dem}_{t+1_{blk_1}}) \\
\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk_d}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blk_d}}} &= hrs_{t_{blk_d}}(\mathbf{dem}_{t+1_{blk_d}})
\end{aligned} \tag{7.11}$$

Where:

- $\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk_1}}}$ = The decision variable associated with the MWh generated by capacity type 1 in dispatch block 1 in time period t (MWh).
- $hrs_{t_{blk_1}}$ = Number of hours of dispatch block 1 (hours) in time period t.
- $\mathbf{dem}_{t+1_{blk_1}}$ = Number of MW of demand in dispatch block 1 after investment and growth in time period t
- **numGen** = Number of different capacity types including the VoLL generator.
- d = Number of dispatch blocks in the LDC.

The third set of constraints for the variable operational costs sum together the MWh generated by a single generator from each dispatch block to give the total MWh of operation for the whole optimisation time period.

$$\begin{aligned}
&\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk_1}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_1}_{t_{blk_d}}} = \mathbf{MWhTotal}_{\mathbf{cap_1}_t} \\
&\vdots \\
&\mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blk_1}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blk_d}}} = \mathbf{MWhTotal}_{\mathbf{cap_{numGen}}_t}
\end{aligned} \tag{7.12}$$

Where:

- $\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk_1}}}$ = The decision variable associated with the MWh generated by capacity type 1 in dispatch block 1 in time period t (MWh).
- $\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk_d}}}$ = The decision variable associated with the MWh generated by capacity type 1 in dispatch block d in time period t (MWh)
- $\mathbf{MWhTotal}_{\mathbf{cap_1}_t}$ = Total MWh of operation for capacity type 1 in this time period.
- $\mathbf{MWhTotal}_{\mathbf{cap_{numGen}}_t}$ = Total MWh of operation for capacity type **numGen** in this time period.
- **numGen** = Number of different capacity types including the VoLL generator.
- d = Number of dispatch blocks in the LDC.

7.4.3 Fixed Costs of Generation

The fixed operational costs of generation capacity are given by the number of MW of installed capacity multiplied by the fixed cost coefficient in the objective function. The MW of installed capacity is known to the optimisation model through Equation 7.5 where the variables $\mathbf{cap_1}_{t+1}, \ldots, \mathbf{cap_n}_{t+1}$ represent installed capacity after optimisation in the current time period. The optimisation model does not require further constraints to calculate the fixed operational cost of generation.

7.4.4 Reserves

Two constraints are required to calculate the penalty cost of using VoLL for reserves. The first constraint is shown in Equation 7.13 and calculates the MW capacity of the largest generator and hence largest risk in the system.

$$\begin{aligned} \mathbf{MW}_{risk_{t+1}} &\geq \mathbf{cap_1}_{t+1} \\ &\vdots \\ \mathbf{MW}_{risk_{t+1}} &\geq \mathbf{cap_{numGens}}_{t+1} \end{aligned} \tag{7.13}$$

Where:

- $\mathbf{MW}_{risk_{t+1}}$ = Variable representing capacity of largest generator in the system.
- $\mathbf{cap_1}_{t+1}$ = Generation capacity of type 1 in time period t after investment in this time period.
- **numGens** = Total number of generators.

The value of the variable $\mathbf{MW}_{risk_{t+1}}$ is driven to the lowest possible value, that is equal to the largest generator capacity. This occurs because $\mathbf{MW}_{risk_{t+1}}$ is used in the penalty constraint in Equation 7.14 where a larger value of $\mathbf{MW}_{risk_{t+1}}$ will create a greater value of $\mathbf{VoLL}_{pen_t}$ and hence additional cost in the solution.

$$\begin{aligned} &\mathbf{dem}_{t+1} - (\mathbf{cap}_{t+1} - \mathbf{MW}_{risk_{t+1}}) \leq \mathbf{VoLL}_{pen_t} \\ &\mathbf{VoLL}_{pen_t} \geq 0 \end{aligned} \tag{7.14}$$

Where:

- $\mathbf{VoLL}_{pen_t}$ = MW of VoLL used to provide reserves.

7.4.5 Future Costs

The objective function includes the term $\bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap_1}, \ldots, \mathbf{cap_{numGen_{\mathit{t}+1}}}, \mathbf{dem}_{t+1})$ which represents the value of the future cost function for the next time period as a function of the state variable values found after optimisation in the current time period.

The future cost function is built from Bender's Cuts through iteration of the SDDP algorithm. At the beginning of the algorithm these cuts are not defined so the optimisation model uses the constraint shown in Equation 7.15 to restrict the future cost function values to be positive.

$$\bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap_1}, \ldots, \mathbf{cap_{numGen_{\mathit{t}+1}}}, \mathbf{dem}_{t+1}) \geq 0 \tag{7.15}$$

Where:

- $\mathbf{cap_1}, \ldots, \mathbf{cap_{numGen_{\mathit{t}+1}}}, \mathbf{dem}_{t+1}$ = Value of the capacity state variables (including VoLL) after optimisation in this time period.
- $\mathbf{dem}_{t+1}$ = Value of the demand state variable after optimisation in this time period.
- $\bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}$ = Future cost function for time period t

As SDDP iterates additional Bender's Cuts are calculated and added to the constraint set of the problem (Refer Section 4.4.2).

7.5 EXAMPLE PROBLEM - MATHEMATICAL REPRESENTATION

The following example problem illustrates the mathematical representation of a simple optimisation problem. The problem itself is not solved but it should be noted that the initial system

Existing Generator	Capacity (MW)	Variable Operating Cost ($/MWh)	Fixed Operating Cost ($/MW)
Generator A	200	$55	$75,000
Generator B	50	$85	$60,000
VoLL Generator	1000	$20,000	$0
Reserve Penalty VoLL			$10,000,000

Table 7.1 Existing Generation Data

New Investment Type	Capacity (MW)	Installation Cost ($/MW)	Variable Operating Cost ($/MWh)	Fixed Operating Cost ($/MW)
Generation C	100	$1.5 million	$80	$100,000
Generation D	150	$1 million	$70	$85,000
Generation E	40	$750,000	$105	$60,000

Table 7.2 New Generation and Demand Investment Data

data represents a system that would use VoLL to provide reserve. That is, the system does not have sufficient spare capacity to cover the capacity of the largest generator. The optimisation is likely to rectify this issue in the first time period by investing in a high level of new generation.

A small power system has two existing generators. The system has a current peak demand level of 190MW that is expected to grow by 40MW per year. Three generation investment options are available that range in size from 40MW maximum capacity to 150MW maximum capacity. The problem is to find the optimal policy of investments over a three year period using SDDP. Each year of the planning problem is a single stage optimisation problem. The mathematical representation of the investment problem is essentially the same for each year of the optimisation problem. The only exception is the dynamic constraint that evolves over time. The example problem shown here represents the mathematical representation of the investment problem for the first year of the planning problem. Where a constraint may alter in subsequent time period additional information is supplied.

Data

Table 7.1 shows the information for fixed and variable operating costs for the existing generators. The final row of the table shows the penalty cost assigned to any MW of VoLL used for providing reserves. Table 7.2 shows the data for the new generation and demand investment options. The Load Duration Curve is discretised into three blocks as shown in Figure 7.5.

The objective function of the problem is shown in Equation 7.16. The objective of the optimisation is to minimise immediate system investment and operation costs plus future system costs subject to the constraints detailed below. This objective function is valid for all three time

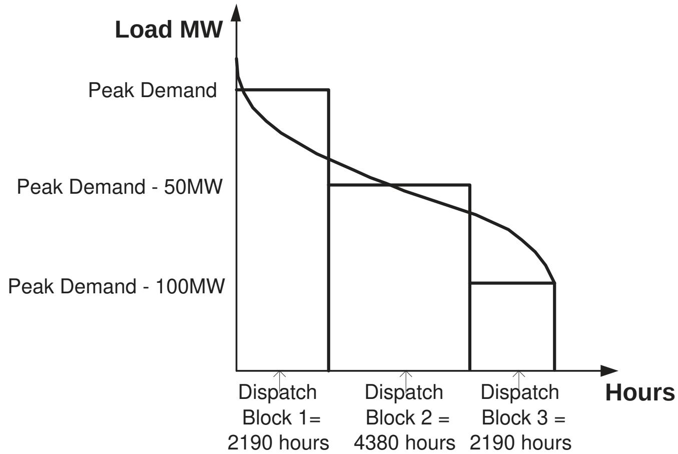


Figure 7.5 Load Duration Curve Discretisation

periods of the planning horizon.

Objective Function

$$\begin{aligned}
\min\big[&\$1,500,000\mathbf{x}_{\mathbf{C}_t} + \$1,000,000\mathbf{x}_{\mathbf{D}_t} + \$750,000\mathbf{x}_{\mathbf{E}_t} + \\
&\$55\mathbf{MWhTot}_{\mathbf{A}_t} + \$85\mathbf{MWhTot}_{\mathbf{B}_t} + \$80\mathbf{MWhTot}_{\mathbf{C}_t} + \\
&\$70\mathbf{MWhTot}_{\mathbf{D}_t} + \$105\mathbf{MWhTot}_{\mathbf{E}_t} + \$20,000\mathbf{MWhTot}_{\mathbf{VoLL}_t} \\
&+ \$75,000\mathbf{cap}_{\mathbf{A}_{t+1}} + \$60,000\mathbf{cap}_{\mathbf{B}_{t+1}} + \$100,000\mathbf{cap}_{\mathbf{C}_{t+1}} + \\
&\$85,000\mathbf{cap}_{\mathbf{D}_{t+1}} + \$60,000\mathbf{cap}_{\mathbf{E}_{t+1}} + \\
&\$10,000,000\mathbf{VoLL}_{pen_t} + \\
&\bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap}_{\mathbf{A}_{t+1}}, \mathbf{cap}_{\mathbf{B}_{t+1}}, \mathbf{cap}_{\mathbf{C}_{t+1}}, \mathbf{cap}_{\mathbf{D}_{t+1}}, \mathbf{cap}_{\mathbf{E}_{t+1}}, \mathbf{allDem}_{t+1})
\end{aligned} \tag{7.16}$$

Where:

- $\mathbf{x}_{\mathbf{C}_t}$ = The decision variable associated with the generation investment C.
- $\mathbf{x}_{\mathbf{D}_t}$ = The decision variable associated with the generation investment D.
- $\mathbf{x}_{\mathbf{E}_t}$ = The decision variable associated with the generation investment E.
- $\mathbf{MWhTot}_{\mathbf{A}_t}$ = Total number of MWh of operation by generation type A.
- $\mathbf{MWhTot}_{\mathbf{B}_t}$ = Total number of MWh of operation by generation type B.
- $\mathbf{MWhTot}_{\mathbf{C}_t}$ = Total number of MWh of operation by generation type C.
- $\mathbf{MWhTot}_{\mathbf{D}_t}$ = Total number of MWh of operation by generation type D.
- $\mathbf{MWhTot}_{\mathbf{E}_t}$ = Total number of MWh of operation by generation type E.
- $\mathbf{MWhTot}_{\mathbf{VoLL}_t}$ = Total number of MWh of operation by the VoLL generator.
- $\mathbf{cap}_{\mathbf{A}_{t+1}}$ = Total installed MW of generation type A at the end of the time period.
- $\mathbf{cap}_{\mathbf{B}_{t+1}}$ = Total installed MW of generation type B at the end of the time period.

- $\mathbf{cap}_{\mathbf{C}_{t+1}}$ = Total installed MW of generation type C at the end of the time period.
- $\mathbf{cap}_{\mathbf{D}_{t+1}}$ = Total installed MW of generation type D at the end of the time period.
- $\mathbf{cap}_{\mathbf{E}_{t+1}}$ = Total installed MW of generation type E at the end of the time period.
- $\mathbf{VoLL}_{pen}$ = MW of VoLL used to provide reserve capacity.
- $\bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap}_{\mathbf{A}_{t+1}}, \mathbf{cap}_{\mathbf{B}_{t+1}}, \mathbf{cap}_{\mathbf{C}_{t+1}}, \mathbf{cap}_{\mathbf{D}_{t+1}}, \mathbf{cap}_{\mathbf{E}_{t+1}}, \mathbf{allDem}_{t+1})$ = The future cost of investment and operation of the system from time period $t+1$ to the end of the planning horizon.

The constraints of the optimisation problem are separated into four categories, investment constraints, variable operating costs, fixed operational costs and the future costs of investment and operation.

Generation Investment Constraints

The following generation investment constraints are dependent on the time period of the optimisation.

$$\mathbf{allCap}_t = 250 \tag{7.17}$$
$$\mathbf{cap}_{\mathbf{A}_t} = 200 \tag{7.18}$$
$$\mathbf{cap}_{\mathbf{B}_t} = 50 \tag{7.19}$$
$$\mathbf{cap}_{\mathbf{C}_t} = 0 \tag{7.20}$$
$$\mathbf{cap}_{\mathbf{D}_t} = 0 \tag{7.21}$$
$$\mathbf{cap}_{\mathbf{E}_t} = 0 \tag{7.22}$$

The right hand side value of constraint 7.17 gives the total system generation capacity at the beginning of the time period. The right hand side values of constraints (refer to constraints 7.18 to 7.22) represent the individual capacity type totals. These capacity values change as investments are made, therefore subsequent time period optimisations may have differing values on the right hand side of the constraints.

The following constraints are time independent, that is they are constructed in exactly the same form for each time period optimisation.

$$\mathbf{allCap}_t + \mathbf{x}_{\mathbf{C}_t} + \mathbf{x}_{\mathbf{D}_t} + \mathbf{x}_{\mathbf{E}_t} = \mathbf{allCap}_{t+1} \tag{7.23}$$

$$\mathbf{cap}_{\mathbf{A}_t} = \mathbf{cap}_{\mathbf{A}_{t+1}} \text{ \textit{As there is no investment of type A available}} \tag{7.24}$$

$$\mathbf{cap}_{\mathbf{B}_t} = \mathbf{cap}_{\mathbf{B}_{t+1}} \text{ \textit{As there is no investment of type B available}} \tag{7.25}$$

$$\mathbf{cap}_{\mathbf{C}_t} + \mathbf{x}_{\mathbf{C}_t} = \mathbf{cap}_{\mathbf{C}_{t+1}} \tag{7.26}$$

$$\mathbf{cap}_{\mathbf{D}_t} + \mathbf{x}_{\mathbf{D}_t} = \mathbf{cap}_{\mathbf{D}_{t+1}} \tag{7.27}$$

$$\mathbf{cap}_{\mathbf{E}_t} + \mathbf{x}_{\mathbf{E}_t} = \mathbf{cap}_{\mathbf{E}_{t+1}} \tag{7.28}$$

$$\mathbf{cap}_{\mathbf{VoLL}_t} = \mathbf{cap}_{\mathbf{VoLL}_{t+1}} \text{ \textit{As there is no increase of the VoLL generator capacity}} \tag{7.29}$$

$$\mathbf{x}_{\mathbf{C}_t} - 100\mathbf{y}_{\mathbf{1}_t} = 0 \tag{7.30}$$

$$\mathbf{x}_{\mathbf{D}_t} - 150\mathbf{y}_{\mathbf{2}_t} = 0 \tag{7.31}$$

$$\mathbf{x}_{\mathbf{E}_t} - 40\mathbf{y}_{\mathbf{3}_t} = 0 \tag{7.32}$$

$$\mathbf{y}_{\mathbf{1}_t} \leq 1 \tag{7.33}$$

$$\mathbf{y}_{\mathbf{2}_t} \leq 1 \tag{7.34}$$

$$\mathbf{y}_{\mathbf{3}_t} \leq 1 \tag{7.35}$$

$$\mathbf{y}_{\mathbf{1}_t}, \mathbf{y}_{\mathbf{2}_t}, \mathbf{y}_{\mathbf{3}_t} \in \mathbb{Z} \tag{7.36}$$

Constraint 7.23 calculates the total generation capacity from all capacity types in the system including new investments made in this time period. Constraints 7.24 to 7.28 calculate the number of installed MW of each individual generation capacity type in the system. The variables $\mathbf{cap}_{\mathbf{A}_{t+1}}, \mathbf{cap}_{\mathbf{B}_{t+1}}, \mathbf{cap}_{\mathbf{C}_{t+1}}, \mathbf{cap}_{\mathbf{D}_{t+1}}, \mathbf{cap}_{\mathbf{E}_{t+1}}$ and $\mathbf{VoLL}_{t+1}$ are used in the objective function to find the fixed costs of operation for the installed generation capacity. Constraints 7.30 to 7.32 along with 7.33 to 7.35 restrict the investment of new generation to be a large integer size. There are no dynamic constraints in the first time period mathematical representation of the problem. Subsequent time period optimisations will have a dynamic constraint to restrict investment re-use, the form of which depends on the investment opportunities used in previous time periods.

Demand Constraints

Constraint 7.37 is dependent on the time period of the optimisation. The right hand side value of the constraint represents the system demand at the beginning of the time period. This value changes over the planning horizon of the problem due to the increase in demand over time.

$$\mathbf{allDem}_t = 190 \tag{7.37}$$

The following demand investment constraints are time independent and are valid in the following form for every time period of the planning horizon.

$$\mathbf{dem}_{\mathbf{inc}_t} = 40 \tag{7.38}$$

$$\mathbf{allDem}_t + \mathbf{dem}_{\mathbf{inc}_t} = \mathbf{allDem}_{t+1} \tag{7.39}$$

Constraint 7.38 states the size of the peak demand increase during the current time period. For the planning problem modelled here the right hand side value is static over the entire planning horizon. For a stochastic problem this value will be sampled from a known probability distribution of the demand growth. Constraint 7.39 shows how overall system demand changes in the current time period due to demand growth.

Variable Operating Costs for Generation Capacity

All constraints in this section are time independent and apply to every time period optimisation problem.

$$\mathbf{MWh}_{\mathbf{A}_{t_{blk_1}}} + \mathbf{MWh}_{\mathbf{A}_{t_{blk_2}}} + \mathbf{MWh}_{\mathbf{A}_{t_{blk_3}}} = \mathbf{MWhTot}_{t_\mathbf{A}} \tag{7.40}$$

$$\mathbf{MWh}_{\mathbf{B}_{t_{blk_1}}} + \mathbf{MWh}_{\mathbf{B}_{t_{blk_2}}} + \mathbf{MWh}_{\mathbf{B}_{t_{blk_3}}} = \mathbf{MWhTot}_{t_\mathbf{B}} \tag{7.41}$$

$$\mathbf{MWh}_{\mathbf{C}_{t_{blk_1}}} + \mathbf{MWh}_{\mathbf{C}_{t_{blk_2}}} + \mathbf{MWh}_{\mathbf{C}_{t_{blk_3}}} = \mathbf{MWhTot}_{t_\mathbf{C}} \tag{7.42}$$

$$\mathbf{MWh}_{\mathbf{D}_{t_{blk_1}}} + \mathbf{MWh}_{\mathbf{D}_{t_{blk_2}}} + \mathbf{MWh}_{\mathbf{D}_{t_{blk_3}}} = \mathbf{MWhTot}_{t_\mathbf{D}} \tag{7.43}$$

$$\mathbf{MWh}_{\mathbf{E}_{t_{blk_1}}} + \mathbf{MWh}_{\mathbf{E}_{t_{blk_2}}} + \mathbf{MWh}_{\mathbf{E}_{t_{blk_3}}} = \mathbf{MWhTot}_{t_\mathbf{E}} \tag{7.44}$$

Constraints 7.40 to 7.44 sum together the individual MWh of operation from each generation type and each LDC block to give the total number of MWh of operation for each generator. The variables $\mathbf{MWhTot}_{t_\mathbf{A}}, \mathbf{MWhTot}_{t_\mathbf{B}}, \mathbf{MWhTot}_{t_\mathbf{C}}, \mathbf{MWhTot}_{t_\mathbf{D}}$ and $\mathbf{MWhTot}_{t_\mathbf{E}}$ are present in the objective function and represent the total MWh of operation for each generation type.

$$\frac{\mathbf{MWh}_{\mathbf{A}_{t_{blk_1}}}}{2190hrs} \leq \mathbf{cap}_{\mathbf{A}_{t+1}} \tag{7.45}$$

$$\frac{\mathbf{MWh}_{\mathbf{A}_{t_{blk_2}}}}{4380hrs} \leq \mathbf{cap}_{\mathbf{A}_{t+1}} \tag{7.46}$$

$$\frac{\mathbf{MWh}_{\mathbf{A}_{t_{blk_3}}}}{2190hrs} \leq \mathbf{cap}_{\mathbf{A}_{t+1}} \tag{7.47}$$

$$\frac{\mathbf{MWh}_{\mathbf{B}_{t_{blk_1}}}}{2190hrs} \leq \mathbf{cap}_{\mathbf{B}_{t+1}} \tag{7.48}$$

$$\frac{\mathbf{MWh}_{\mathbf{B}_{t_{blk_2}}}}{4380hrs} \leq \mathbf{cap}_{\mathbf{B}_{t+1}} \tag{7.49}$$

$$\frac{\mathbf{MWh}_{\mathbf{B}_{t_{blk_3}}}}{2190hrs} \leq \mathbf{cap}_{\mathbf{B}_{t+1}} \tag{7.50}$$

$$\frac{\mathbf{MWh}_{\mathbf{C}_{t_{blk_1}}}}{2190hrs} \leq \mathbf{cap}_{\mathbf{C}_{t+1}} \tag{7.51}$$

$$\frac{\mathbf{MWh}_{\mathbf{C}_{t_{blk_2}}}}{4380hrs} \leq \mathbf{cap}_{\mathbf{C}_{t+1}} \tag{7.52}$$

$$\frac{\mathbf{MWh}_{\mathbf{C}_{t_b lk_3}}}{2190hrs} \leq \mathbf{cap}_{\mathbf{C}_{t+1}} \tag{7.53}$$

$$\frac{\mathbf{MWh}_{\mathbf{D}_{t_{blk_1}}}}{2190hrs} \leq \mathbf{cap}_{\mathbf{D}_{t+1}} \tag{7.54}$$

$$\frac{\mathbf{MWh}_{\mathbf{D}_{t_{blk_2}}}}{4380hrs} \leq \mathbf{cap}_{\mathbf{D}_{t+1}} \tag{7.55}$$

$$\frac{\mathbf{MWh}_{\mathbf{D}_{t_{blk_3}}}}{2190hrs} \leq \mathbf{cap}_{\mathbf{D}_{t+1}} \tag{7.56}$$

$$\frac{\mathbf{MWh}_{\mathbf{E}_{t_{blk_1}}}}{2190hrs} \leq \mathbf{cap}_{\mathbf{E}_{t+1}} \tag{7.57}$$

$$\frac{\mathbf{MWh}_{\mathbf{E}_{t_{blk_2}}}}{4380hrs} \leq \mathbf{cap}_{\mathbf{E}_{t+1}} \tag{7.58}$$

$$\frac{\mathbf{MWh}_{\mathbf{E}_{t_{blk_3}}}}{2190hrs} \leq \mathbf{cap}_{\mathbf{E}_{t+1}} \tag{7.59}$$

Constraints 7.45 to 7.59 restrict the MWh of operation of each generation capacity type in each LDC block to be less than or equal to the total installed capacity of the respective generation type.

$$\mathbf{MWh}_{\mathbf{A}_{t_{blk_1}}} + \mathbf{MWh}_{\mathbf{B}_{t_{blk_1}}} \mathbf{MWh}_{\mathbf{C}_{t_{blk_1}}} + \mathbf{MWh}_{\mathbf{D}_{t_{blk_1}}} + \mathbf{MWh}_{\mathbf{E}_{t_{blk_1}}} = 2190(\mathbf{allDem}_{t+1}) \tag{7.60}$$

$$\mathbf{MWh}_{\mathbf{A}_{t_{blk_2}}} + \mathbf{MWh}_{\mathbf{B}_{t_{blk_2}}} + \mathbf{MWh}_{\mathbf{C}_{t_{blk_2}}} + \mathbf{MWh}_{\mathbf{D}_{t_{blk_2}}} + \mathbf{MWh}_{\mathbf{E}_{t_{blk_2}}} = 4380(\mathbf{allDem}_{t+1} - 50) \tag{7.61}$$

$$\mathbf{MWh}_{\mathbf{A}_{t_{blk_3}}} + \mathbf{MWh}_{\mathbf{B}_{t_{blk_3}}} + \mathbf{MWh}_{\mathbf{C}_{t_{blk_3}}} + \mathbf{MWh}_{\mathbf{D}_{t_{blk_3}}} + \mathbf{MWh}_{\mathbf{E}_{t_{blk_3}}} = 2190(\mathbf{allDem}_{t+1} - 100) \tag{7.62}$$

Constraints 7.60 to 7.62 ensure the sum of MWh of operation from each generator in each LDC block adds up to the demand level of the respective block. Note how constraint 7.61 has the term ($\mathbf{allDem}_{t+1} - 50$) on the right hand side of the constraint. The -50 reflects the reduction in peak demand in the second block of the LDC. Constraint 7.62 operates in the same way with -100 used to reflect the lowest peak demand value of the LDC (Refer 7.5).

Fixed Costs of Generation Capacity

There are no additional constraints used to find the fixed costs of generation capacity. The variables $\mathbf{cap}_{\mathbf{A}_{t+1}}, \mathbf{cap}_{\mathbf{B}_{t+1}}, \mathbf{cap}_{\mathbf{C}_{t+1}}, \mathbf{cap}_{\mathbf{D}_{t+1}}, \mathbf{cap}_{\mathbf{E}_{t+1}}$ and $\mathbf{VoLL}_{t+1}$ provide the data required, namely the total installed capacity in MW of each generation type, to calculate the fixed costs of generation capacity.

Reserve Constraints

All reserve constraints are time independent and apply in all time periods. Constraints 7.63 to 7.67 calculate the MW capacity of the largest installed generator. If a generator has not yet been installed, its capacity is 0MW and constraint will still hold.

$$\mathbf{MW}_{risk_{t+1}} \geq \mathbf{cap}_{\mathbf{A}_{t+1}} \tag{7.63}$$

$$\mathbf{MW}_{risk_{t+1}} \geq \mathbf{cap}_{\mathbf{B}_{t+1}} \tag{7.64}$$

$$\mathbf{MW}_{risk_{t+1}} \geq \mathbf{cap}_{\mathbf{C}_{t+1}} \tag{7.65}$$

$$\mathbf{MW}_{risk_{t+1}} \geq \mathbf{cap}_{\mathbf{D}_{t+1}} \tag{7.66}$$

$$\mathbf{MW}_{risk_{t+1}} \geq \mathbf{cap}_{\mathbf{E}_{t+1}} \tag{7.67}$$

(7.68)

Constraint 7.69 calculates the MW of VoLL used for providing reserves.

$$\mathbf{allDem}_{t+1} - ((\mathbf{cap}_{\mathbf{A}_{t+1}} + \mathbf{cap}_{\mathbf{B}_{t+1}} + \mathbf{cap}_{\mathbf{C}_{t+1}} + \mathbf{cap}_{\mathbf{D}_{t+1}} + \mathbf{cap}_{\mathbf{E}_{t+1}}) - \mathbf{MW}_{risk_{t+1}}) \leq \mathbf{VoLL}_{pen_t} \tag{7.69}$$

$$\mathbf{VoLL}_{pen_t} \geq 0 \tag{7.70}$$

Where:

- $\mathbf{VoLL}_{pen_t}$ = MW of VoLL used to provide reserves.

Future Cost Function

Each single stage problem optimises both the current time period costs of investment and operation plus the future costs of system investment and operation. The future costs are represented through the future cost function that is in turn approximated by Bender's Cuts, found through the SDDP algorithm. As this example is illustrating the mathematical representation for the first time period of the optimisation the future cost function is restricted to be greater than 0, as shown in Constraint 7.71.

$$\bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap}_{\mathbf{A}_{t+1}}, \mathbf{cap}_{\mathbf{B}_{t+1}}, \mathbf{cap}_{\mathbf{C}_{t+1}}, \mathbf{cap}_{\mathbf{D}_{t+1}}, \mathbf{cap}_{\mathbf{E}_{t+1}}, \mathbf{allDem}_{t+1}) \geq 0 \tag{7.71}$$

7.6 SUMMARY

This chapter has presented the mathematical representation of a power system generation investment optimisation problem that optimises both generation and demand side investments. To produce a mathematical model of a the planning problem modelling decisions had to made regarding the representation of capacity and demand. Capacity is modelled as a separate variable for each generation type whereas demand is modelled as a single variable regardless of the type of demand. Varying demand within a time period (one year) is modelled through the use of a discretised load duration curve. Any demand not supplied is modelled as a 'dummy' value of lost load (VoLL) generator that is added to the model as an additional generation type. The VoLL generator has the highest operating cost of all generators so that it is dispatched last. For the power system to remain stable during unforseen generator outages reserve capacity must be available. To deter the optimisation from selecting the VoLL generator to supply reserve capacity a penalty cost is imposed. The cost applies to any MW of VoLL used to provide reserve capacity. The final modelling decision made was to assume all investments and demand growth occur at the beginning of the time period.

The mathematical model of the planning problem consists of the objective function and constraints. The objective function was outlined where the relevant costs to the system that must be included are the cost of investment, the variable costs of system operation, the fixed costs of system operation, reserve penalty costs and the costs of system operation in future time periods.

The constraints are described in five categories, investment, variable operating costs, fixed operating costs, reserve penalty constraints and future costs. A small example generation investment problem has been used to illustrate the objective function and constraints of the mathematical representation. This example while useful for illustrative purposes is very general and doesn't represent real world systems. Real systems are more complex with more constraints being used to create a more realistic model. Alternative systems, additional constraints and mathematical representations are the subjects of the following chapters.

Chapter 8

TRANSMISSION INVESTMENT OPTIMISATION

8.1 INTRODUCTION

Chapter 7 described the mathematical representation of a generation investment planning problem but acknowledged that many planning problems are more complex, involving factors other than just generation investments. This chapter extends the mathematical representation of Chapter 7 to include transmission investments.

The introduction of transmission investments in the power system planning problem extends the system being modelled to include two load regions, one at either end of the transmission line. The mathematical representation is extended to include the new features of the system by modelling the transmission line as generators.

The following sections in this chapter detail the specific modelling decisions associated with the transmission investment extension, the form of the objective function and the construction of constraints for the transmission investment problem.

8.2 MODELLING DECISIONS

The modelling relevant to the generation investment problem and mathematical representation of Chapter 7 is valid for the extended transmission investment problem. Generation capacity is modelled by separate variables for each generator, system demand is modelled by a single variable and demand growth is assumed to occur instantaneously at the beginning of the time period. Demand within a time period is modelled using a discretised load duration curve.

Further modelling is required for the extension to transmission investment which includes representation of transmission lines and transmission investments, representation of two load regions and modelling of VoLL.

8.2.1 Transmission Representation

A transmission line connects two load regions for the purpose of transferring power from one region to the other. The sending end of the line is located in the region that is transferring power out and the receiving end is located in the region that is importing power from the transmission line. The transfer of power across the line is equivalent to adding load in the region at the sending end and generation in the region at the receiving end. This allows the transmission line to be modelled using two generators, one at either end of the line. The two generators are required to have mutually exclusive operation, that is, they cannot both operate at the same time hence bidirectional power flow through the line is disallowed. The transmission line model requires two generators as the direction of power transfer across the line may change between time periods and both load regions must have the ability to send and receive power.

The two generators used to represent the transmission line are represented as an additional capacity type and hence additional variables in the mathematical representation. They are treated the same way as generation investments albeit with some specific constraints to ensure the investment and operational characteristics of the transmission line are retained.

Defining the transmission generators as sending end or receiving end is a dynamic process that is done within the single stage optimisation. During every time period of the SDDP algorithm the single stage optimisation is re-solved, allowing the opportunity for power flow in the transmission line to be altered between time periods. This results in a dynamic allocation of the labels ‘sending end’ and ‘receiving end’ of the line.

8.2.1.1 Sending End

The sending end of the transmission line is transferring energy out of a load region and is considered to be an additional load in that region. The generator that represents the transmission line in the sending end region is restricted to generate 0MWh as power cannot be imported and exported simultaneously.

8.2.1.2 Receiving End

The receiving end of a transmission line is transferring power into a load region and is treated as a generator. This generator has a maximum capacity that is equal to the maximum capacity of the transmission line. The receiving end generator is included in the offer stack of the receiving end load region and is dispatched accordingly. The value of this dispatch is the amount of power being transferred into the region.

8.2.1.3 Mutually Exclusive Operation of Transmission Generators

To restrict the transmission generators to mutually exclusive operation there are two options; the first is to define binary variables that represent operation of each transmission generator. The sum of the binary variables is constrained to 1, restricting operation to a single generator. The second option is to create a set of variables known as a Special Ordered Set of type 1, or SOS1. This special set only allows one member of the set to be non zero at any one time.

Both these representations are suitable to solve the generation and transmission investment problem but the SOS1 representation can offer some benefits in terms of solution time. The transmission investment mathematical representation uses SOS1 constraints to restrict the transmission generator's operation as it is more efficient than using binary variables. Both options are described in the following sections.

8.2.1.4 Binary Variables

A binary variable can only take the value 1 or 0. It is effectively an integer variable with an upper bound of 1. With reference to the investment planning problem, a binary variable can be used to indicate if a transmission generator is operational (e.g. generating MWh), where the value 1 indicates operation and 0 non operation. The constraints for each transmission generator can be [82]:

$$MWh_i - \mathbf{M}y \leq 0 \tag{8.1}$$

$$MWh_i - \mathbf{r}y \geq 0 \tag{8.2}$$

$$y \in \{1, 0\}$$

$$\mathbf{r} \leq MWh_i \leq \mathbf{M}$$

Where:

- MWh_i =The MWh generated by transmission generator i
- y =Binary integer variable
- $\mathbf{M}$ =Arbitrary value greater than MWh_i
- $\mathbf{r}$= Arbitrary value, greater than zero but less than MWh_i

These constraints show that if $MWh_i > 0$ then Equation 8.1 forces the value of y to be 1 whereas Equation 8.2 will allow y to be 1 or 0. Conversely, if $MWh_i = 0$ then Equation 8.1 will allow y to be 1 or 0 and Equation 8.2 will force y to 0. Combined, these two equations force the binary variable to either 0 or 1 depending on the value of MWh_i.

Once the value of the binary variables has been found it is possible to restrict the transmission generators operation to be mutually exclusive. This is done by requiring that only one binary variable be equal to 1 at any one time. Equation 8.3 implements this.

$$\begin{aligned} y_1 + y_2 &= 1 \\ y_k &\geq 0, \in \mathbb{Z} \end{aligned} \tag{8.3}$$

Where:

- y_1=Binary variable associated with transmission generator 1
- y_2=Binary variable associated with transmission generator 2

This representation is useful in small simple problems but can be inefficient in terms of solution speed for larger problems due the additional variables and constraints. For efficient solutions in large scale problems it is better to use Special Ordered Sets. This is discussed in the following section.

8.2.1.5 Special Ordered Sets

Special Ordered Sets type 1, SOS1, [1] are groupings of variables so that at most a single variable is greater than zero at any one time. The SOS1 constraint represents a multiple choice problem of choosing a single variable from the set. For the generation and transmission investment problem the restricted variables are the MWh of generation of the transmission generators.

A group of constraints make up the definition of SOS1 as shown in Equation 8.4.

$$\begin{aligned} f(y) &= f(\hat{y_1})x_1 + f(\hat{y_2})x_2 + \ldots + f(\hat{y_k})x_k \\ \hat{y_1}x_1 + \hat{y_2}x_2 + \ldots + \hat{y_k}x_k - y &= 0, y \geq 0 \\ x_1 + x_2 + \ldots + x_k &= 1, x \geq 0 \end{aligned} \tag{8.4}$$

Where:

- $f(y)$ =Function representing the objective function cost associated with the variables in the set.
- $\hat{y_1} \ldots \hat{y_k}$ = Value of variable after solution.

[1]SOS constraints of type 2 or 3 or 4 etc are also defined by a set of variables where at most 2 can be non zero at any one time (for a type 2 set) or at most 3 variables can be non zero at any one time (for a type 3 set) etc. These higher level SOS variable sets are used in linear programming for modelling non-convex functions.

- $x_1 \ldots x_k$ =Variables of which only one can be non zero and that one must be equal to one.
- k=Number of variables in the set.

Molding these constraints to the generation and transmission investment problem results in Equations 8.5 to 8.7.

$$f(y) = f(\hat{y_1})x_1 + f(\hat{y_2})x_2 \tag{8.5}$$

$$\hat{y_1}x_1 + \hat{y_2}x_2 - y = 0, y \geq 0 \tag{8.6}$$

$$x_1 + x_2 = 1, x \geq 0 \tag{8.7}$$

- $f(\hat{y_1})$ = Operational cost multiplied by the MWh of generation for transmission generator 1
- $f(\hat{y_2})$ = Operational cost multiplied by the MWh of generation for transmission generator 2
- $\hat{y_1}$= Number of Mwh generated by transmission generator 1
- $\hat{y_2}$= Number of Mwh generated by transmission generator 2
- x_1 = Binary variable 1
- x_2 = Binary variable 2

The discrete function described by Equation 8.5 can take only one possible value that is weighted by the one x variable that is equal to 1. Equation 8.5 is called the function row, Equation 8.6 the reference row and Equation 8.7 the convexity row. The y variables are known as the SOS1 variables. It is easy to see the similarity between the definition of SOS1 variables and binary variables. The reasons for using a SOS1 representation rather than binary variables is to enable a more efficient Branch and Bound solution process.

SOS constraints were initially developed by Beale and Tomlin [83] as a special method for making branching decisions in the Branch and Bound optimisation method for integer and mixed integer problems (Refer A for Branch and Bound Description). The special ordered set definition allows for better branching decisions to be made as a penalty can be calculated for the SOS1 variables that enable set partitioning and ultimately a more efficient solution process. Traditional Branch and Bound that contains binary variables defined with a constraint such as $\sum y_k = 1$, where y_k are the binary variables, must make a branching decision that takes a single binary variable and forces it to 0 or 1. Forcing to 0 doesn't facilitate a fast solution as this has only dealt with 1 variable and there are still $k - 1$ decisions to be made on the binary variable values. If a variable is set to 1, this facilitates a fast solution as it forces all the other binary variables to 0. Beale and Tomlin note that while forcing a binary variable to 1 can give a fast solution, it can also be too drastic in reality. There is no consideration of other potential options and it ties the solution process in to fixing a variable value very early on. Special Ordered Sets allow for a penalty calculation to be made that helps guide the solution process without removing potential solutions early in the algorithm. Further details of this penalty calculation can be found in [83] and [84].

8.2.2 Load Regions

The extension of the generation investment model of Chapter 7 to include transmission investments results in the system being modelled consisting of two load regions. Each region has an independent peak demand growth variable and independent load duration curve. Each load duration curve may have differing discretisations, peak demand values and base load levels. The two LDC models increase the number of constraints as the model now requires two offer stacks, one in each load region. The assumption of independence between the demand of each load region is used for its modelling simplicity.

Two load regions requires the definition of two stochastic variables, one for system demand in each region. The introduction of additional stochastic variables affects the SDDP algorithm. The forward pass must sample two variables at each time period of the optimisation. The backward pass must also sample two variables at each time period but the greatest effect is on the number of times each single stage problem is solved on the backward pass. With one stochastic variable the problem may be solved, for example, 5 times with 5 different samples of the stochastic variable. With two stochastic variables the number of times the single stage problem is solved increases to 5 times 5 i.e. 25 solutions, an exponential increase.

8.2.3 Value of Lost Load

The Value of Lost Load is modelled in the same way as for the generation investment problem, as a ‘dummy’ or VoLL generator. The only change for the transmission investment problem is that each load region has a VoLL generator. The problem now has two ‘dummy’ generators defined each with a very high variable operating cost and a $0/MW fixed operating cost.

8.2.4 Reserves

The reserve penalty cost constraints change to reflect the introduction of two load areas and the transmission line. Three reserve penalty constraints are required; one for both load regions, and one each for the separate load regions. Equation 8.8 details the joint region reserve constraint that does not include the transmission generators as these generators do not add additional capacity to the system.

$$\mathbf{cap}_{t+1_\mathbf{A}} + \mathbf{cap}_{VoLL_{t+1_\mathbf{A}}} \mathbf{cap}_{t+1_\mathbf{B}} + \mathbf{cap}_{VoLL_{t+1_\mathbf{B}}} - \mathbf{MW}_{risk_{t+1_{\mathbf{A+B}}}} \geq \mathbf{dem}_{t+1_\mathbf{A}} + \mathbf{dem}_{t+1_\mathbf{B}} \tag{8.8}$$

Where:

- $\mathbf{cap}_{t+1_\mathbf{A}}$ = Total capacity of load region A excluding the transmission and VoLL generators.
- $\mathbf{cap}_{t+1_\mathbf{B}}$ = Total capacity of load region B excluding the transmission and VoLL generators.
- $\mathbf{MW}_{risk_{t+1_{\mathbf{A+B}}}}$ = Largest generator risk in whole system.

Equation 8.9 shows the reserve constraint for load region A and Equation 8.10 the reserve constraint for load region B. The variables $\mathbf{cap}_{t+1_\mathbf{A}}$ and $\mathbf{cap}_{t+1_\mathbf{B}}$ include the transmission generators in these equations as for an individual load region the transmission generator can supply additional capacity. If both Equations 8.9 and 8.10 are satisfied then Equation 8.8 will also be satisfied. Equation 8.8 will not affect the feasible region and could be removed though it has been modelled here for completeness.

$$\mathbf{cap}_{t+1_\mathbf{A}} + \mathbf{cap}_{VoLL_{t+1_\mathbf{A}}} - \mathbf{MW}_{risk_{t+1_\mathbf{A}}} \geq \mathbf{dem}_{t+1_\mathbf{A}} \tag{8.9}$$

$$\mathbf{cap}_{t+1_\mathbf{B}} + \mathbf{cap}_{VoLL_{t+1_\mathbf{B}}} - \mathbf{MW}_{risk_{t+1_\mathbf{B}}} \geq \mathbf{dem}_{t+1_\mathbf{B}} \tag{8.10}$$

Where:

- $\mathbf{cap}_{t+1_\mathbf{A}}$ = Total capacity of load region A including the transmission generator but excluding the VoLL generator.
- $\mathbf{cap}_{t+1_\mathbf{B}}$ = Total capacity of load region B including the transmission generator but excluding the VoLL generator.
- $\mathbf{MW}_{risk_{t+1_\mathbf{A}}}$ = Largest generator risk in load region A.
- $\mathbf{MW}_{risk_{t+1_\mathbf{A}}}$ = Largest generator risk in load region B.

Similarly to Equation 7.1, Equations 8.8 - 8.10 are not used directly in the model as they are trivial due to the inclusion of the VoLL generator. These constraints are used to generate penalty constraints in Equations 8.11, 8.12 and 8.13.

$$\begin{aligned}(\mathbf{dem}_{t+1_\mathbf{A}} + \mathbf{dem}_{t+1_\mathbf{B}}) - (\mathbf{cap}_{t+1_\mathbf{A}} + \mathbf{cap}_{t+1_\mathbf{B}} - \mathbf{MW}_{risk_{t+1_{\mathbf{A+B}}}}) \leq \mathbf{VoLL}_{pen_{t_\mathbf{A}}} + \mathbf{VoLL}_{pen_{t_\mathbf{B}}} \\ \mathbf{VoLL}_{pen_{t_\mathbf{A}}}, \mathbf{VoLL}_{pen_{t_\mathbf{B}}} \geq 0\end{aligned} \tag{8.11}$$

Where:

- $\mathbf{cap}_{t+1_\mathbf{A}}$ = Total capacity of load region A excluding the transmission and VoLL generators.
- $\mathbf{cap}_{t+1_\mathbf{B}}$ = Total capacity of load region B excluding the transmission and VoLL generators.
- $\mathbf{VoLL}_{pent_\mathbf{A}}$ = Variable representing the number of MW of VoLL used to supply reserves in region A.
- $\mathbf{VoLL}_{pent_\mathbf{B}}$ = Variable representing the number of MW of VoLL used to supply reserves in region B.

$$\mathbf{dem}_{t+1_\mathbf{A}} - (\mathbf{cap}_{t+1_\mathbf{A}} - \mathbf{MW}_{risk_{t+1_\mathbf{A}}}) \leq \mathbf{VoLL}_{pent_\mathbf{A}} \tag{8.12}$$

$$\mathbf{dem}_{t+1_\mathbf{B}} - (\mathbf{cap}_{t+1_\mathbf{B}} - \mathbf{MW}_{risk_{t+1_\mathbf{B}}}) \leq \mathbf{VoLL}_{pent_\mathbf{B}} \tag{8.13}$$

$$\mathbf{VoLL}_{pent_\mathbf{A}}, \mathbf{VoLL}_{pent_\mathbf{B}} \geq 0$$

Where:

- $\mathbf{cap}_{t+1_\mathbf{A}}$ = Total capacity of load region A including the transmission generator but excluding the VoLL generator.
- $\mathbf{cap}_{t+1_\mathbf{B}}$ = Total capacity of load region B including the transmission generator but excluding the VoLL generator.
- $\mathbf{VoLL}_{pent_\mathbf{A}}$ = Variable representing the number of MW of VoLL used to supply reserves for load region A.
- $\mathbf{VoLL}_{pent_\mathbf{B}}$ = Variable representing the number of MW of VoLL used to supply reserves for load region B.

Each VoLL penalty variable is represented in the objective function and is associated with a penalty cost. Where the penalty costs associated with $\mathbf{VoLL}_{pent_\mathbf{A}}$ and $\mathbf{VoLL}_{pent_\mathbf{B}}$ are the same, the optimisation will be indifferent to the level of reserve cover from each generator. If the penalty costs differ the combination of penalty constraints will ensure the lowest cost solution will be found.

8.3 OBJECTIVE FUNCTION

The objective function for the transmission line investment model is very similar to the generation investment model. The costs of the system, both investment and operational, are the focus of the objective function where the overall goal is to minimise the costs of system investment and operation over the planning horizon of the problem. The costs of the system for the transmission investment problem are the same as those for the generation investment problem, namely, investment costs, variable operational costs, fixed operational costs and future costs of the current investment decisions. Each cost component is described by a cost coefficient and a decision variable. The future cost term is a scalar value given by the value of the future cost function at the state variables. The transmission line capacity is modelled as system capacity and is therefore considered a state variable and contributes to the future cost function. The objective function for the transmission investment problem is shown in Equation 8.14.

$$
\begin{aligned}
\min\big[&(\mathbf{c_1}_{t_\mathbf{A}} \times \mathbf{cap_{1inv}}_{t_\mathbf{A}}) + \ldots + (\mathbf{c_{totInvOp}}_{t_\mathbf{A}} \times \mathbf{cap_{totInvOpinv}}_{t_\mathbf{A}}) + \ldots \\
&(\mathbf{c_1}_{t_\mathbf{B}} \times \mathbf{cap_{1inv}}_{t_\mathbf{B}}) + \ldots + (\mathbf{c_{totInvOp}}_{t_\mathbf{B}} \times \mathbf{cap_{totInvOpinv}}_{t_\mathbf{B}}) + \ldots \\
&(\mathbf{vo_1}_{t_\mathbf{A}} \times \mathbf{MWhTot}_{\mathbf{cap_1}_{t_\mathbf{A}}}) + \ldots + (\mathbf{vo_{totUnits}}_{t_\mathbf{A}} \times \mathbf{MWhTot}_{\mathbf{cap_{totUnits}}_{t_\mathbf{A}}}) + \ldots \\
&(\mathbf{vo_1}_{t_\mathbf{B}} \times \mathbf{MWhTot}_{\mathbf{cap_1}_{t_\mathbf{B}}}) + \ldots + (\mathbf{vo_{totUnits}}_{t_\mathbf{B}} \times \mathbf{MWhTot}_{\mathbf{cap_{totUnits}}_{t_\mathbf{B}}}) + \ldots \\
&(\mathbf{fo_1}_{t_\mathbf{A}} \times \mathbf{cap_1}_{t+1_\mathbf{A}}) + \ldots + (\mathbf{fo_{totUnits}}_{t_\mathbf{A}} \times \mathbf{cap_{totUnits}}_{t+1_\mathbf{A}}) + \ldots \\
&(\mathbf{fo_1}_{t_\mathbf{B}} \times \mathbf{cap_1}_{t+1_\mathbf{B}}) + \ldots + (\mathbf{fo_{totUnits}}_{t_\mathbf{B}} \times \mathbf{cap_{totUnits}}_{t+1_\mathbf{B}}) + \ldots \\
&(\mathbf{res}_{pen_{t_\mathbf{A}}} \times \mathbf{VoLL}_{pen_{t_\mathbf{A}}}) + (\mathbf{res}_{pen_{t_\mathbf{B}}} \times \mathbf{VoLL}_{pen_{t_\mathbf{B}}}) + \ldots \\
&\boldsymbol{\beta}_t \times \bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap_1}_{t+1_\mathbf{A}}, \ldots, \mathbf{cap_{numGen}}_{t+1_\mathbf{A}}, \mathbf{cap_{tx}}_{t+1_\mathbf{A}}, \ldots \\
&\qquad\qquad \mathbf{cap_1}_{t+1_\mathbf{B}}, \ldots, \mathbf{cap_{numGen}}_{t+1_\mathbf{B}}, \mathbf{cap_{tx}}_{t+1_\mathbf{B}}, \ldots \\
&\qquad\qquad\quad \mathbf{dem}_{t+1_\mathbf{A}}, \mathbf{dem}_{t+1_\mathbf{B}})\big]
\end{aligned}
\tag{8.14}
$$

Where:

- $\mathbf{totInvOp}$ = Total number of investment options = $\mathbf{capOp} + \mathbf{txOp}$
- $\mathbf{capOp}$ = Number of capacity investment options.
- $\mathbf{txOp}$ = Number of transmission investment options.
- $\mathbf{tx}$ = Number of transmission generators in each load region.
- $\mathbf{c_1}_{t_\mathbf{A}}$ = Installation cost of investment option 1 in load region A($/MW).
- $\mathbf{c_{totInvOp}}_{t_\mathbf{A}}$ = Installation cost of investment option **totInvOp** in load region A($/MW).
- $\mathbf{cap_{1inv}}_{t_\mathbf{A}}$ = Decision variable associated with investment option **1** in load region A (MW).
- $\mathbf{cap_{totInvOp}}_{t_\mathbf{A}}$ = Decision variable associated with investment option **totInvOp** in load region A (MW).
- $\mathbf{totUnits}$ = Sum of all generation and transmission types = $\mathbf{numGen} + \mathbf{tx}$.
- $\mathbf{numGen}$ = Total number of generation capacity types in each respective load region. This *excludes* transmission capacity and *includes* VoLL generators.
- $\mathbf{vo_1}_{t_\mathbf{A}}$ = Variable operating costs of capacity type **1** in load region A ($/MWh).
- $\mathbf{vo_{totUnits}}_{t_\mathbf{A}}$ =Variable operating costs of capacity type **totUnits** in load region A($/MWh).
- $\mathbf{MWhTot}_{\mathbf{cap_1}_{t_\mathbf{A}}}$ = Decision variable associated with variable operating costs for capacity

type **1** in load region A.

- $\mathbf{MWhTot}_{\mathbf{cap}_{\mathbf{totUnits}_{t_\mathbf{A}}}}$ = Decision variable associated with variable operating costs for capacity type **totUnits** in load region A.
- $\mathbf{fo}_{\mathbf{1}_{t_\mathbf{A}}}$ = Fixed operating costs of capacity type 1 in load region A ($/MWh).
- $\mathbf{fo}_{\mathbf{totUnits}_{t_\mathbf{A}}}$ = Fixed operating costs of capacity type **totUnits** in load region A($/MWh).
- $\mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{A}}}$ = Decision variable associated with fixed operation costs for capacity type **1**. This is the value of capacity type **1** after investment in this time period.
- $\mathbf{cap}_{\mathbf{totUnits}_{t+1_\mathbf{A}}}$ = Decision variable associated with fixed operation costs for capacity type **totUnits**. This is the value of capacity type **totUnits** after investment in this time period.
- $\mathbf{dem}_{t+1_\mathbf{A}}$ = Variable representing system demand in load region A after peak demand growth in this time period.
- $\mathbf{dem}_{t+1_\mathbf{B}}$ = Variable representing system demand in load region B after peak demand growth in this time period.
- $\mathbf{res}_{pen_{t_\mathbf{A}}}$ = Reserve penalty cost associated with region A reserve penalty variable.
- $\mathbf{VoLL}_{pen_{t_\mathbf{A}}}$ = Reserve penalty variable from region A reserve constraint.
- $\mathbf{res}_{pen_{t_\mathbf{B}}}$ = Reserve penalty cost associated with region B reserve penalty variable.
- $\mathbf{VoLL}_{pen_{t_\mathbf{B}}}$ = Reserve penalty variable from region B reserve constraint.
- $\bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{A}}}, \ldots, \mathbf{cap}_{\mathbf{numGen}_{t+1_\mathbf{A}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_\mathbf{A}}}, \ldots$ $\mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{B}}}, \ldots, \mathbf{cap}_{\mathbf{numGen}_{t+1_\mathbf{B}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_\mathbf{B}}}, \mathbf{dem}_{t+1_\mathbf{A}}, \mathbf{dem}_{t+1_\mathbf{B}})$ = Future cost function value for this time period as a function of the state variable values.
- t = Time period for the optimisation.
- $\boldsymbol{\beta}_t$ = Time value of money associated with risk adjusted discount rate.

8.4 CONSTRAINTS

The constraints of the transmission investment problem restrict the solution of the investment problem to ensure the optimisation problem restrictions are enforced. The constraints are split into four sections, investment constraints, variable operating cost constraints, fixed operational cost constraints and future cost constraints. Each constraint type consists of a range of constraints that include both generation and demand constraints, single load region constraints and constraints that restrict interaction between the load regions.

The following constraints are similar in form to those described in Section 7.4 but the inclusion of a transmission investment option creates a new variable that must be included in the constraints. The additional variable is explicitly detailed to highlight the differences from the constraints of Section 7.4.

8.4.1 Investment Constraints

Investment constraints for the transmission investment problem cover the same restrictions as the generation investment problem, namely restrictions on generation and transmission investments, restrictions on investment size and re-use and changes to state variables over time.

The state variables of the transmission investment problem are system capacity and system demand. Each load region has separate capacity state variable equations that describe how the optimal decision variables of the problem affect the value of the system capacity. Equation 8.15 shows the state variable constraints for system capacity for each load region.

$$\begin{aligned}\mathbf{cap}_{t+1_\mathbf{A}} &= \mathbf{cap}_{t_\mathbf{A}} + \mathbf{cap}_{\mathbf{1inv}_{t_\mathbf{A}}} + \ldots + \mathbf{cap}_{\mathbf{capOpinv}_{t_\mathbf{A}}} + \mathbf{cap}_{\mathbf{txinv}_{t_\mathbf{A}}} \\ \mathbf{cap}_{t+1_\mathbf{B}} &= \mathbf{cap}_{t_\mathbf{B}} + \mathbf{cap}_{\mathbf{1inv}_{t_\mathbf{B}}} + \ldots + \mathbf{cap}_{\mathbf{capOpinv}_{t_\mathbf{B}}} + \mathbf{cap}_{\mathbf{txinv}_{t_\mathbf{B}}}\end{aligned} \tag{8.15}$$

Where:

- $\mathbf{cap}_{t+1_\mathbf{A}}$ = System capacity in Load Region A after investment in this time period.
- $\mathbf{cap}_{t_\mathbf{A}}$ = System capacity in Load Region A before investment at the beginning of this time period.
- $\mathbf{cap}_{\mathbf{1inv}_{t_\mathbf{A}}}$ = Investment in generation capacity of type 1 in Load Region A in time period t.
- $\mathbf{cap}_{\mathbf{capOpinv}_{t_\mathbf{A}}}$ = Investment in generation capacity of type **capOp** in Load Region A in time period t.
- $\mathbf{cap}_{\mathbf{txOp}_{t_\mathbf{A}}}$ = Investment in transmission capacity in load region A in time period t.
- **capOp** = Number of capacity investment options in each respective load region.

The capacity state equation of Equation 8.15 incorporates the transmission investment decision variable due to the transmission line being modelled as a capacity investment. Every generation capacity investment option is restricted to being installed in a specific load region but a transmission investment applies to both load regions and is therefore part of both load regions capacity state variable equation. Every individual capacity type also has a capacity state equation that describes how each capacity type changes based on the optimal decision variables of the problem. Equation 8.16 shows these constraints in general form. Note how the transmission capacity type investment changes the transmission capacity of both regions.

$$
\begin{aligned}
&\mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{A}}} = \mathbf{cap}_{\mathbf{1}_{t_\mathbf{A}}} + \mathbf{cap}_{\mathbf{1inv}_{t_\mathbf{A}}} \\
&\vdots \\
&\mathbf{cap}_{\mathbf{numGen}_{t+1_\mathbf{A}}} = \mathbf{cap}_{\mathbf{numGen}_{t_\mathbf{A}}} + \mathbf{cap}_{\mathbf{numGenInv}_{t_\mathbf{A}}} \\
&\mathbf{cap}_{\mathbf{tx}_{t+1_\mathbf{A}}} = \mathbf{cap}_{\mathbf{tx}_{t_\mathbf{A}}} + \mathbf{cap}_{\mathbf{txInv}_t} \\
\\
&\mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{B}}} = \mathbf{cap}_{\mathbf{1}_{t_\mathbf{B}}} + \mathbf{cap}_{\mathbf{1inv}_{t_\mathbf{B}}} \\
&\vdots \\
&\mathbf{cap}_{\mathbf{numGen}_{t+1_\mathbf{B}}} = \mathbf{cap}_{\mathbf{numGen}_{t_\mathbf{B}}} + \mathbf{cap}_{\mathbf{numGenInv}_{t_\mathbf{B}}} \\
&\mathbf{cap}_{\mathbf{tx}_{t+1_\mathbf{B}}} = \mathbf{cap}_{\mathbf{tx}_{t_\mathbf{B}}} + \mathbf{cap}_{\mathbf{txInv}_t}
\end{aligned}
\tag{8.16}
$$

Where:

- $\mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{A}}}$ = System capacity of type 1, in Region A, after investment in this time period.
- $\mathbf{cap}_{\mathbf{1}_{t_\mathbf{A}}}$ = System capacity of type 1, in Region A, before investment at the beginning of this time period.
- $\mathbf{cap}_{\mathbf{1inv}_{t_\mathbf{A}}}$ = Investment in capacity of type 1, in Region A, given by the decision variables for investment options in this time period.
- $\mathbf{cap}_{\mathbf{numGenInv}_{t_\mathbf{A}}}$ = Investment in capacity of type *numGen*, in Region A, given by the decision variables for investment options in this time period.
- $\mathbf{cap}_{\mathbf{tx}_{t+1_\mathbf{A}}}$ = Transmission system capacity, in Region A, after investment in this time period.
- $\mathbf{cap}_{\mathbf{txInv}_t}$ = Investment in transmission capacity that ties load regions A and B, given by the decision variables for investment options in this time period.
- **numGen** = Total number of generation capacity types in each respective load region. This *excludes* the transmission capacity.

The state variable constraint for system demand is shown in Equation 8.17 where each load region has an individual demand state equation constraint.

$$
\begin{aligned}
\mathbf{dem}_{t+1_\mathbf{A}} = \mathbf{dem}_{t_\mathbf{A}} + \mathbf{dem}_{\mathbf{inc}_{t_\mathbf{A}}} \\
\mathbf{dem}_{t+1_\mathbf{B}} = \mathbf{dem}_{t_\mathbf{B}} + \mathbf{dem}_{\mathbf{inc}_{t_\mathbf{B}}}
\end{aligned}
\tag{8.17}
$$

Where:

- $\mathbf{dem}_{t+1_\mathbf{A}}$ = System demand in Region A after demand increase in this time period.
- $\mathbf{dem}_{t_\mathbf{A}}$ = System demand in Region A before demand increase at the beginning of this time period.
- $\mathbf{dem}_{\mathbf{inc}_{t_\mathbf{A}}}$ = Increase in demand in Region A given by the sampled value of demand for this time period.
- $\mathbf{dem}_{t+1_\mathbf{B}}$ = System demand in Region B after demand increase in this time period.
- $\mathbf{dem}_{t_\mathbf{B}}$ = System demand in Region B before demand increase at the beginning of this time period.
- $\mathbf{dem}_{\mathbf{inc}_{t_\mathbf{B}}}$ = Increase in demand in Region B given by the sampled value of demand for this time period.

8.4.2 Integer Investment

Similarly to the generation investment model all generation and transmission investments are restricted to be of a large integer capacity size. Equation 8.18 shows the general form of the integer investment constraints for both load regions.

$$
\begin{aligned}
&\mathbf{cap}_{\mathbf{1inv}_{t_\mathbf{A}}} - InvestSize_{\mathbf{1_A}}\mathbf{y}_{\mathbf{1}_{t_\mathbf{A}}} = 0\\
&\vdots\\
&\mathbf{cap}_{\mathbf{capOp}_{t_\mathbf{A}}} - InvestSize_{\mathbf{capOp_A}}\mathbf{y}_{\mathbf{capOp}_{t_\mathbf{A}}} = 0\\
&\mathbf{cap}_{\mathbf{txOp}_{t_\mathbf{A}}} - InvestSize_{\mathbf{txOp_A}}\mathbf{y}_{\mathbf{txOp}_{t_\mathbf{A}}} = 0\\
&\mathbf{y}_{\mathbf{1}_{t_\mathbf{A}}} \leq 1, \ldots, \mathbf{y}_{\mathbf{capOp}_{t_\mathbf{A}}} \leq 1, \mathbf{y}_{\mathbf{txOp}_{t_\mathbf{A}}} \leq 1\\
&\mathbf{y}_{\mathbf{1}_{t_\mathbf{A}}}, \mathbf{y}_{\mathbf{capOp}_{t_\mathbf{A}}}, \mathbf{y}_{\mathbf{txOp}_{t_\mathbf{A}}} \epsilon \mathbb{Z}\\
\\
&\mathbf{cap}_{\mathbf{1inv}_{t_\mathbf{B}}} - InvestSize_{\mathbf{1_B}}\mathbf{y}_{\mathbf{1}_{t_\mathbf{B}}} = 0\\
&\vdots\\
&\mathbf{cap}_{\mathbf{capOp}_{t_\mathbf{B}}} - InvestSize_{\mathbf{capOp_B}}\mathbf{y}_{\mathbf{capOp}_{t_\mathbf{B}}} = 0\\
&\mathbf{cap}_{\mathbf{txOp}_{t_\mathbf{B}}} - InvestSize_{\mathbf{txOp_B}}\mathbf{y}_{\mathbf{txOp}_{t_\mathbf{B}}} = 0\\
&\mathbf{y}_{\mathbf{1}_{t_\mathbf{B}}} \leq 1, \ldots, \mathbf{y}_{\mathbf{capOp}_{t_\mathbf{B}}} \leq 1, \mathbf{y}_{\mathbf{txOp}_{t_\mathbf{B}}} \leq 1\\
&\mathbf{y}_{\mathbf{1}_{t_\mathbf{B}}}, \mathbf{y}_{\mathbf{capOp}_{t_\mathbf{B}}}, \mathbf{y}_{\mathbf{txOp}_{t_\mathbf{B}}} \epsilon \mathbb{Z}
\end{aligned}
\tag{8.18}
$$

Where:

- $\mathbf{cap_{1inv_{t_A}}}$ = Investment in capacity of type 1, in Region A, given by the decision variables for investment options in this time period.
- $InvestSize_{\mathbf{1_A}}$ = Large investment size in MW for capacity investment option 1 in Region A.
- $\mathbf{y_{1_{t_A}}}$ = Binary variable for investment type 1 in Region A in time period t.
- **capOp** = Total number of generation capacity investment options in each respective load region.
- **txOp** = Number of transmission capacity investments.

8.4.3 Variable Operational Cost Constraints

8.4.3.1 Variable Operation Costs of Generation

The variable operational costs of the combined load regions and transmission line are given by the number of MWh each capacity type generates multiplied by the variable cost coefficient in the objective function. The constraints used to calculate the number of MWh generated by each capacity type are similar to those detailed in Section 7.4.2 but each load region has its own Load Duration Curve and hence its own constraint set. As the transmission line is modelled as two generators, one in each load area, the transmission generators are treated the same way as other generating plant with regard to building a offer stack.

There are three sets of constraints for each load region, the first restricts the number of MWh each capacity type can generate to ensure the maximum capacity of the plant is not exceeded. Equations 8.19 and 8.20 illustrates these constraints. There is a constraint for each dispatch block (Refer Section 7.4.2.1) and for each generator in each load region.

$$\begin{gathered}
\frac{\mathbf{MWh_{cap_{1_{t_{blk_{1_A}}}}}}}{hrs_{t_{blk_{1_A}}}} \leq \mathbf{MaxCap_{cap_{1_{t+1_A}}}}, \ldots, \frac{\mathbf{MWh_{cap_{numGen+tx_{t_{blk_{1_A}}}}}}}{hrs_{t_{blk_{1_A}}}} \leq \mathbf{MaxCap_{cap_{numGen+tx_{t+1_A}}}} \\
\vdots \\
\frac{\mathbf{MWh_{cap_{t_{1_{blk_{d_A}}}}}}}{hrs_{t_{blk_{d_A}}}} \leq \mathbf{MaxCap_{cap_{1_{t+1_A}}}}, \ldots, \frac{\mathbf{MWh_{cap_{numGen+tx_{t_{blk_{d_A}}}}}}}{hrs_{t_{blk_{d_A}}}} \leq \mathbf{MaxCap_{cap_{numGen+tx_{t+1_A}}}}
\end{gathered} \tag{8.19}$$

$$\frac{\mathbf{MWh_{cap_{1_{t_{blk_{1_B}}}}}}}{hrs_{t_{blk_{1_B}}}} \leq \mathbf{MaxCap_{cap_{1_{t+1_B}}}}, \ldots, \frac{\mathbf{MWh_{cap_{numGen+tx_{t_{blk_{1_B}}}}}}}{hrs_{t_{blk_{1_B}}}} \leq \mathbf{MaxCap_{cap_{numGen+tx_{t+1_B}}}}$$
$$\vdots$$
$$\frac{\mathbf{MWh_{cap_{1_{t_{blk_{h_B}}}}}}}{hrs_{t_{blk_{h_B}}}} \leq \mathbf{MaxCap_{cap_{1_{t+1_B}}}}, \ldots, \frac{\mathbf{MWh_{cap_{numGen+tx_{t_{blk_{h_B}}}}}}}{hrs_{t_{blk_{h_B}}}} \leq \mathbf{MaxCap_{cap_{numGen+tx_{t+1_B}}}} \tag{8.20}$$

Where:

- $\mathbf{MWh_{cap_{1_{t_{blk_{1_A}}}}}}$ = The decision variable associated with the MWh generated by capacity type 1 in dispatch block 1 (MWh), in Region A in time period t.
- $hrs_{t_{blk_{1_A}}}$ = Number of hours of dispatch block 1 (hours) for the LDC for load region A in time period t.
- $\mathbf{MaxCap}_{cap_{\mathbf{1}_{t+1_A}}}$ = Maximum installed plant capacity of capacity type 1 (MW), in Region A, *after* investment in time period t.
- **numGen+tx** = Number of different capacity types in each respective load region plus the transmission generator.
- d = Number of dispatch blocks in the LDC for Region A.
- h = Number of dispatch blocks in the LDC for Region B.

The second set of constraints require that the sum of the MWh generated by all capacity types in a dispatch block is equal to the system demand of the respective dispatch block. The introduction of the transmission line changes this constraint from that detailed in Equation 7.11 to Equation 8.21 where the total energy generated in a region plus imported energy across the transmission line must equal the regional demand plus any energy exported out of the region across the transmission line. Note how each constraint contains both variables representing the sending and receiving ends of the transmission lines, ($\mathbf{MWh_{cap_{tx_{t_{blk_{1_A}}}}}}$ and $\mathbf{MWh_{cap_{tx_{t_{blk_{1_B}}}}}}$). Due to the constraints detailed in Section 8.2.1.5 the transmission line generators are restricted from operating simultaneously, preventing bi-directional flow, resulting in only one of the variables $\mathbf{MWh_{cap_{tx_{t_{blk_{1_A}}}}}}$ or $\mathbf{MWh_{cap_{tx_{t_{blk_{1_B}}}}}}$ being non zero at any point in time. The generators considered in these constraints include the VoLL generator to prevent the constraints from enforcing investment to ensure demand is met.

$$\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk1_{\mathbf{A}}}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blk1_{\mathbf{A}}}}} + \mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blk1_{\mathbf{A}}}}} = \ldots \quad (8.21)$$
$$hrs_{t_{blk1_{\mathbf{A}}}} (\mathbf{dem}_{t+1_{blk1_{\mathbf{A}}}}) + \mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blk1_{\mathbf{B}}}}}$$

$$\vdots$$

$$\mathbf{MWh}_{\mathbf{cap_1}_{t_{blkd_{\mathbf{A}}}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blkd_{\mathbf{A}}}}} + \mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blkd_{\mathbf{A}}}}} = \ldots \quad (8.22)$$
$$hrs_{t_{blkd_{\mathbf{A}}}} (\mathbf{dem}_{t+1_{blkd_{\mathbf{A}}}}) + \mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blkd_{\mathbf{B}}}}}$$

$$\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk1_{\mathbf{B}}}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blk1_{\mathbf{B}}}}} + \mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blk1_{\mathbf{B}}}}} = \ldots \quad (8.23)$$
$$hrs_{t_{blk1_{\mathbf{B}}}} (\mathbf{dem}_{t+1_{blk1_{\mathbf{B}}}}) + \mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blk1_{\mathbf{A}}}}}$$

$$\vdots$$

$$\mathbf{MWh}_{\mathbf{cap_1}_{t_{blkh_{\mathbf{B}}}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blkh_{\mathbf{B}}}}} + \mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blkh_{\mathbf{B}}}}} = \ldots \quad (8.24)$$
$$hrs_{t_{blkh_{\mathbf{B}}}} (\mathbf{dem}_{t+1_{blkh_{\mathbf{B}}}}) + \mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blkh_{\mathbf{A}}}}}$$

$$(8.25)$$

Where:

- $\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk1_{\mathbf{A}}}}}$ = The decision variable in time period t associated with the MWh generated by capacity type 1 in dispatch block 1 (MWh) in load region A.
- $hrs_{t_{blk1_{\mathbf{A}}}}$ = Number of hours of dispatch block 1 (hours) for the LDC of load region A in time period t.
- $\mathbf{dem}_{t+1_{blk1_{\mathbf{A}}}}$ = Number of MW of demand in dispatch block 1 in load region A after demand increase in time period t.
- **numGen** = Number of different capacity types in each respective load region *excluding* transmission capacity but *including* VoLL generator.
- d = Number of dispatch blocks in the LDC for load region A.
- h = Number of dispatch blocks in the LDC for Region B.
- $\mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blk1_{\mathbf{A}}}}}$ = The decision variable associated with the MWh generated by the transmission capacity type in dispatch block 1 (MWh) in load region A in time period t.
- $\mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blk1_{\mathbf{B}}}}}$ = The decision variable associated with the MWh generated by the transmission capacity type in dispatch block 1 (MWh) in load region B in time period t.

The third set of constraints calculates the total MWh generated by each capacity type across all

LDC blocks. The transmission line generators are treated exactly the same as all other capacity types and are included in the constraint set. Equation 8.26 describes these constraints.

$$
\begin{aligned}
&\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk1}}_{\mathbf{A}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_1}_{t_{blkd}}_{\mathbf{A}}} = \mathbf{MWhTot}_{\mathbf{cap_1}_{t}_{\mathbf{A}}} \\
&\vdots \\
&\mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blk1}}_{\mathbf{A}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blkd}}_{\mathbf{A}}} = \mathbf{MWhTot}_{\mathbf{cap_{numGen}}_{t}_{\mathbf{A}}} \\
&\mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blk1}}_{\mathbf{A}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blkd}}_{\mathbf{A}}} = \mathbf{MWhTot}_{\mathbf{cap_{tx}}_{t}_{\mathbf{A}}} \\
\\
&\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk1}}_{\mathbf{B}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_1}_{t_{blkh}}_{\mathbf{B}}} = \mathbf{MWhTot}_{\mathbf{cap_1}_{t}_{\mathbf{B}}} \\
&\vdots \\
&\mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blk1}}_{\mathbf{B}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_{numGen}}_{t_{blkh}}_{\mathbf{B}}} = \mathbf{MWhTot}_{\mathbf{cap_{numGen}}_{t}_{\mathbf{B}}} \\
&\mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blk1}}_{\mathbf{B}}} + \ldots + \mathbf{MWh}_{\mathbf{cap_{tx}}_{t_{blkh}}_{\mathbf{B}}} = \mathbf{MWhTot}_{\mathbf{cap_{tx}}_{t}_{\mathbf{B}}}
\end{aligned}
\tag{8.26}
$$

Where:

- $\mathbf{MWh}_{\mathbf{cap_1}_{t_{blk1}}_{\mathbf{A}}}$ = The decision variable associated with the MWh generated by capacity type 1 in dispatch block 1 (MWh), in load region A, in time period t.
- $\mathbf{MWh}_{\mathbf{cap_1}_{t_{blkd}}_{\mathbf{A}}}$ = The decision variable associated with the MWh generated by capacity type 1 in dispatch block d (MWh), in load region A, in time period t.
- $\mathbf{MWhTot}_{\mathbf{cap_1}_{t}_{\mathbf{A}}}$ = Total MWh of operation for capacity type **1** in load region A in this time period.
- $\mathbf{MWhTot}_{\mathbf{cap_{numGen}}_{t}_{\mathbf{A}}}$ = Total MWh of operation for capacity type **numGen** in load region A in this time period.
- **numGen** = Number of different capacity types in each respective load region plus the transmission generator.
- **tx** = Transmission generator.
- d = Number of dispatch blocks in the LDC for load region A.
- h = Number of dispatch blocks in the LDC for Region B.

8.4.4 Reserve Constraints

The reserve penalty constraints first require a group of constraints to identify the largest generating risk in both load regions. The constraints in 8.27 find the largest generator in region A, the constraints in 8.28 find the largest generator in region B and the constraints in 8.29 find the largest generator of the two regions.

$$\begin{aligned}
&\mathbf{MW}_{risk_{t+1_{\mathbf{A}}}} \geq \mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{A}}}}\\
&\quad\vdots\\
&\mathbf{MW}_{risk_{t+1_{\mathbf{A}}}} \geq \mathbf{cap}_{\mathbf{numGens}_{t+1_{\mathbf{A}}}}
\end{aligned} \tag{8.27}$$

$$\begin{aligned}
&\mathbf{MW}_{risk_{t+1_{\mathbf{B}}}} \geq \mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{B}}}}\\
&\quad\vdots\\
&\mathbf{MW}_{risk_{t+1_{\mathbf{B}}}} \geq \mathbf{cap}_{\mathbf{numGens}_{t+1_{\mathbf{B}}}}
\end{aligned} \tag{8.28}$$

$$\begin{aligned}
&\mathbf{MW}_{risk_{t+1_{\mathbf{A+B}}}} \geq \mathbf{MW}_{risk_{t+1_{\mathbf{A}}}}\\
&\mathbf{MW}_{risk_{t+1_{\mathbf{A+B}}}} \geq \mathbf{MW}_{risk_{t+1_{\mathbf{B}}}}
\end{aligned} \tag{8.29}$$

Where:

- $\mathbf{MW}_{risk_{t+1_{\mathbf{A}}}}$ = Variable representing capacity of largest generator in the load region A.
- $\mathbf{MW}_{risk_{t+1_{\mathbf{A}}}}$ = Variable representing capacity of largest generator in the load region B.
- $\mathbf{MW}_{risk_{t+1_{\mathbf{A+B}}}}$ = Variable representing the larger of the largest capacity from both regions.
- **numGens** = Total number of generators in each respective load region. This *excludes* both the transmission and VoLL generators.

The values of these largest generators are then used in the reserve penalty constraints that were developed in Section 8.2.4.

$$\begin{aligned}
&(\mathbf{dem}_{t+1_{\mathbf{A}}} + \mathbf{dem}_{t+1_{\mathbf{B}}}) - (\mathbf{cap}_{t+1_{\mathbf{A}}} + \mathbf{cap}_{t+1_{\mathbf{B}}} - \mathbf{MW}_{risk_{t+1_{\mathbf{A+B}}}}) \leq \mathbf{VoLL}_{pen_{t_{\mathbf{A}}}} + \mathbf{VoLL}_{pen_{t_{\mathbf{B}}}}\\
&\mathbf{VoLL}_{pen_{t_{\mathbf{A}}}}, \mathbf{VoLL}_{pen_{t_{\mathbf{B}}}} \geq 0
\end{aligned} \tag{8.30}$$

Where:

- $\mathbf{cap}_{t+1_{\mathbf{A}}}$ = Total capacity of load region A excluding the transmission and VoLL generators.

- $\mathbf{cap}_{t+1_\mathbf{B}}$ = Total capacity of load region B excluding the transmission and VoLL generators.
- $\mathbf{VoLL}_{pen_{t_\mathbf{A}}}$ = Variable representing the number of MW of VoLL used to supply reserves in region A.
- $\mathbf{VoLL}_{pen_{t_\mathbf{B}}}$ = Variable representing the number of MW of VoLL used to supply reserves in region B.

$$\mathbf{dem}_{t+1_\mathbf{A}} - (\mathbf{cap}_{t+1_\mathbf{A}} - \mathbf{MW}_{risk_{t+1_\mathbf{A}}}) \leq \mathbf{VoLL}_{pen_{t_\mathbf{A}}} \tag{8.31}$$

$$\mathbf{dem}_{t+1_\mathbf{B}} - (\mathbf{cap}_{t+1_\mathbf{B}} - \mathbf{MW}_{risk_{t+1_\mathbf{B}}}) \leq \mathbf{VoLL}_{pen_{t_\mathbf{B}}} \tag{8.32}$$

$$\mathbf{VoLL}_{pen_{t_\mathbf{A}}}, \mathbf{VoLL}_{pen_{t_\mathbf{B}}} \geq 0$$

8.4.4.1 Transmission Operation and Special Ordered Sets

The software used to solve the optimisation problem has a large bearing on how the SOS constraints are constructed. This research has used lp_solve [85], an open source software package, and Matlab [86] to model the transmission investment problem. Lp_solve provides a mixed integer linear programming solver that is able to be used in many programming platforms. This software does not require explicit definition of the SOS constraints. Defining a set of SOS variables and the type of the SOS (e.g. type 1, type2) is sufficient for the solver to build the SOS constraints itself.

The transmission variables that are defined in the SOS1 set are shown in Equation 8.33.

$$\begin{gathered} \mathbf{MWhTot}_{\mathbf{cap}_{\mathbf{tx}_{t_\mathbf{A}}}} \\ \mathbf{MWhTot}_{\mathbf{cap}_{\mathbf{tx}_{t_\mathbf{B}}}} \end{gathered} \tag{8.33}$$

Where:

- $\mathbf{MWhTot}_{\mathbf{cap}_{\mathbf{tx}_{t_\mathbf{A}}}}$ = Total MWh of operation of transmission generator located in region A.
- $\mathbf{MWhTot}_{\mathbf{cap}_{\mathbf{tx}_{t_\mathbf{B}}}}$ = Total MWh of operation of transmission generator located in region B.

8.4.5 Fixed Costs of Capacity

The fixed operational costs are found by multiplying the MW of installed capacity of each capacity type by the respective coefficients in the objective function. The MW of installed capacity are given by the variables $\mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{A}}}, \ldots, \mathbf{cap}_{\mathbf{n}_{t+1_\mathbf{A}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_\mathbf{B}}}$ and $\mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{B}}}, \ldots, \mathbf{cap}_{\mathbf{n}_{t+1_\mathbf{B}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_\mathbf{B}}}$ from Equation 8.16. No further constraints are required.

8.4.6 Dynamic Constraints

The dynamic constraints to restrict investment re-use in the transmission problem have the same properties as those used in the generation investment problem. The constraint's form is dependent on the outcome of the previous time period's optimal decision so cannot be constructed in advance. For the first time period the constraint is not used at all.

8.4.7 Future Costs

The objective function includes the term $\bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{A}}}, \ldots, \mathbf{cap}_{\mathbf{numGen}_{t+1_\mathbf{A}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_\mathbf{A}}}, \ldots$ $\mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{B}}}, \ldots, \mathbf{cap}_{\mathbf{numGen}_{t+1_\mathbf{B}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_\mathbf{B}}}, \mathbf{dem}_{t+1_\mathbf{A}}, \mathbf{dem}_{t+1_\mathbf{B}})$ which represents the value of the future cost function for the next time period as a function of the state variable values found after optimisation in the current time period. The future cost function for the transmission investment problem includes the installed transmission capacity as a state variable.

The future cost function is built from Bender's Cuts through iteration of the SDDP algorithm. At the beginning of the algorithm these cuts are not defined so the optimisation model uses the constraint shown in Equation 8.34 to restrict the future cost function values to be positive.

$$\begin{aligned} \bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{A}}}, \ldots, \mathbf{cap}_{\mathbf{numGen}_{t+1_\mathbf{A}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_\mathbf{A}}}, \ldots \\ \mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{B}}}, \ldots, \mathbf{cap}_{\mathbf{numGen}_{t+1_\mathbf{B}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_\mathbf{B}}}, \mathbf{dem}_{t+1_\mathbf{A}}, \mathbf{dem}_{t+1_\mathbf{B}}) \geq 0 \end{aligned} \tag{8.34}$$

Where:

- $\mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{A}}}, \ldots, \mathbf{cap}_{\mathbf{numGen}_{t+1_\mathbf{A}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_\mathbf{A}}}, \ldots$ $\mathbf{cap}_{\mathbf{1}_{t+1_\mathbf{B}}}, \ldots, \mathbf{cap}_{\mathbf{numGen}_{t+1_\mathbf{B}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_\mathbf{B}}}, \mathbf{dem}_{t+1_\mathbf{A}}, \mathbf{dem}_{t+1_\mathbf{B}}$ = Value of the capacity state variables (including transmission and VoLL) after optimisation in this time period.
- $\mathbf{dem}_{t+1_\mathbf{A}}, \mathbf{dem}_{t+1_\mathbf{B}}$ = Value of the demand state variable after optimisation in this time period.
- $\bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}$ = Future cost function for time period t

As SDDP iterates additional Bender's Cuts are calculated and added to the constraint set of the problem (Refer Section 4.4.2).

8.5 SUMMARY

This chapter has extended the generation investment modelling of Chapter 7 to include transmission investments. The inclusion of transmission investments introduces a second load region to the model. Four major modelling decisions specific to the transmission investment problem were made. The first was to represent the transmission line as two generators, one in each load region. The region exporting power across the transmission line has a transmission generator that generates 0MWh and the region importing power has a transmission generator that can generate MWh, the number of which is restricted by the maximum capacity of the transmission line. To ensure unidirectional energy flow through the transmission line the two generators are not allowed to generate energy simultaneously. The second modelling decision made was to use independent load duration curves for each load region. The assumption of independence of load was made for simplicity of modelling. The third modelling decision involved choosing between binary variables and SOS constraints to ensure mutually exclusive operation of the two transmission generators. SOS constraints were chosen for their efficiency in the Branch and Bound optimisation technique. Finally the reserves penalty constraints were extended to three constraints to consider generator outages in either load region. The penalty costs associated with each region's VoLL generator are included in the objective function.

The objective function of the transmission investment problem is very similar to the generation investment problem where investment costs, variable operational costs, fixed operational costs, reserve penalty costs and future costs all contribute to the overall system costs. The constraints are also similar to the generation problem but there are more constraints than previously because each load region requires an individual offer stack. The transfer of power through the transmission line is limited by constraints that are also part of calculating the MWh of operation of the generating plant and transmission line.

The mathematical model described in this chapter is very general and doesn't consider transmission line losses. Any real world problem will have unique modelling issues that must be incorporated in the mathematical model. An example of a real world problem being represented mathematically and solved used SDDP is the subject of Chapter 9.

Chapter 9

CASE STUDY: HVDC UPGRADE, MODEL CONSTRUCTION

9.1 INTRODUCTION

The development of the New Zealand power system has been strongly influenced by the geography of the country and the location of resources used to produce energy. The South Island contains much of the hydro resource of the country with the majority of load located in the North Island, in particular, the Auckland region. The geographical distance between generation and load motivated the development of the interisland HVDC link that was commissioned in 1965 [87]. The original mercury arc valve link was upgraded in 1992 to operate in a bipole arrangement with a new thyristor convertor. The original mercury arc valve technology, operating as Pole 1, was decommissioned in late 2007 [88]. The owner and operator of the national grid, including the HVDC link, Transpower New Zealand Ltd., has identified a range of investment options to replace the decommissioned Pole 1. The two options investigated in the following case study are either a 'do nothing' option requiring no investment or to install a second 700MW thyristor convertor. The resultant HVDC link would be a 1400MW bipole arrangement.

The HVDC investment problem can be studied using the extended SDDP algorithm presented in this thesis as it is a dynamic investment planning problem involving large integer capacity investments. The problem resembles the transmission investment problem presented in Chapter 8 with the North Island and South Island representing two individual load regions connected by a transmission line, the HVDC link. The extended SDDP algorithm is used to solve the HVDC investment problem for three different future generation scenarios. Each scenario represents a potential future for generation mix in New Zealand where each scenario may be influenced by governmental policy, future economic climate and resource values. The data required for each SDDP optimisation is detailed in Section 9.4.1.

The following sections discuss the HVDC investment options and detail the modelling considerations. The mathematical representation of the optimisation problem is presented followed by the data inputs and scenarios for the SDDP algorithm.

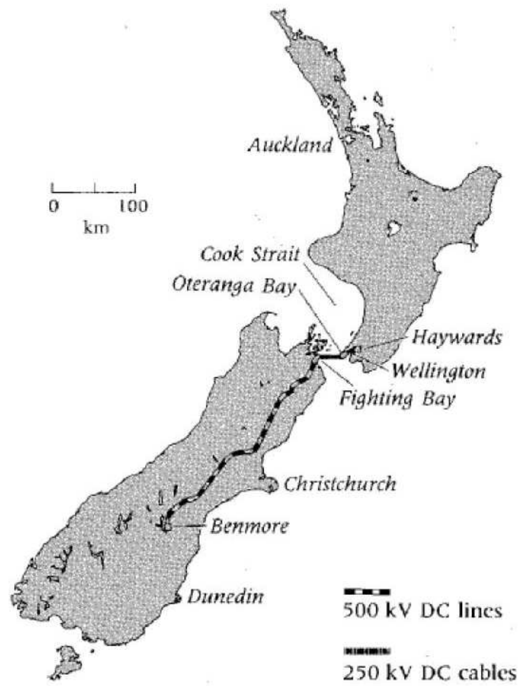


Figure 9.1 HVDC Link [87]

9.2 NEW ZEALAND POWER SYSTEM AND HVDC LINK

The majority of the population and associated load growth of New Zealand is situated in the North Island and in particular the north of the North Island. The South Island has significant hydro resources for power generation, far in excess of the load in the South Island. The geographical distance between the cheap hydro resources in the South Island and the load centre of the North Island motivated the development a HVDC link connecting the two islands. The link runs from Benmore in the South Island (refer Figure 9.1 [87]) overland to Fighting Bay in the Marlborough Sounds. Between Fighting Bay and Oteranga Bay in the North Island undersea cables run through a legally protected corridor of seabed. The link then runs overland from Oteranga Bay to Haywards Substation in the Hutt Valley. Haywards and Benmore are the two ends of link where convertor stations are situated, transferring power from ac to dc and vice versa.

The link was commissioned in 1965 and consisted of two poles using mercury arc technology. It operated at ± 250 kV with a capacity of 600 MW. In 1992 Transpower upgraded the link by installing a 700MW thyristor convertor (known as Pole 2) and additional undersea cables to increase the capacity of the link to 1240MW. At the same time the original link was reconfigured to operate as part of a bipole hybrid link with the new convertor technology and was known as Pole 1. In late 2007 Pole 1 was decommissioned resulting in the link being downgraded to the capacity of Pole 2, 700MW.

9.2.1 HVDC Link Upgrade and Investment Options

As the transmission grid owner and operator, Transpower is in a monopoly position within the New Zealand electricity market. Their investments and charges are regulated by the Electricity Commission, the regulatory body for the New Zealand electricity market. This regulation ensures that any investment undertaken by Transpower is the least cost investment solution that considers both additional generation and HVDC investments. The regulatory environment that Transpower operates in fits well with the extended SDDP algorithm where the algorithm compares both generation and transmission to find a least cost solution.

The two investment options studied in this case study are detailed in [89]. The first investment option is to do nothing, i.e. remove the old Pole 1 equipment and operate the HVDC link via Pole 2 as a single 700MW thyristor convertor. This option obviously requires no capital outlay as the equipment is currently installed. The second investment option is to replace Pole 1 with a 700MW thyristor convertor similar to the existing Pole 2. This would result in a bipole link of 1400MW. The costs for each of these investment options is detailed in Section 9.4.3.

Both investment options include the dismantling and removal of the Pole 1 mercury arc technology. As this pole is currently out of service the cost of dismantling is considered an unavoidable cost and is not included in the optimisation problem.

9.3 OPTIMISATION PROBLEM

The HVDC investment problem can be re-stated as the problem of optimising the investment and operational costs of the New Zealand power system in both islands over the planning period. It is a dynamic planning problem and the optimal solution must be flexible to cope with changes in demand forecasts and varying investment decisions from generation companies. Every investment, both generation or HVDC, is a large integer capacity size. The dynamic nature of the problem combined with the integer nature of investments allows the problem to be solved using the extended SDDP algorithm.

The first step in solving the problem via SDDP is to construct the mathematical optimisation problem that represents the real world situation. This requires the system representation and objective function to be defined and the constraints to be developed. Each of these are described in the following sections.

9.3.1 Modelling Decisions

The HVDC investment problem model can be based on the transmission investment problem of Chapter 8. The modelling decisions (e.g. Defining load areas and the LDC's, investment timing, transmission representation, VoLL) of the transmission investment problem all apply to the HVDC investment problem. Two additional decisions must be made in order to reflect the status of the HVDC with regard to influencing reserve prices in New Zealand and to represent the losses on the HVDC.

9.3.1.1 Load Regions and LDC's

The North Island and South Island represent the two load regions of the investment problem. Each region has a unique LDC and load growth distribution that is described further in Section 9.4.4. The two regions are considered to be independent with respect to demand growth and usage patterns.

9.3.1.2 Transmission Representation

The HVDC link is modelled by two generators, one in each island. The generators may generate if the island is importing power across the link and is restricted to generating 0MWh if the island is exporting power. The mutually exclusive operation of the generators is facilitated through the use of SOS constraints within the optimisation model.

9.3.1.3 VoLL

Each island is assigned a VoLL generator. If the generator is dispatched this indicates the island is not able to meet demand requirements. The energy available for dispatch includes any that is imported across the HVDC link. Demand includes demand within the island plus any exported power across the HVDC link.

9.3.1.4 Investment Timing

The timing of investments is at the beginning of the time period. That is, the optimisation problem is solved and any investments occur immediately at the beginning of the time period. Demand growth occurs at the same time as investments so that new investments are meeting an increase in peak demand value.

9.3.1.5 Reserves and System Security

The cost of providing reserve capacity is the cost of having generation capacity installed and not generating energy. The additional generation above that which is required for energy generation contributes additional investment and fixed operating costs to the total system costs. These costs must be included in the objective function of the optimisation problem. The level of the additional generation capacity required is considered to be the capacity of the largest generation risk to the system. Additional capacity may be supplied by the VoLL generator but this is penalised in the objective function with an additional cost.

The modelling of reserves for the HVDC model is based on the modelling developed in Chapters 7 and 8 with penalty constraints being used to identify if the VoLL generators are used for providing reserves. The New Zealand market considers the largest generating risk to be either the largest currently installed generator *or* a single pole tripping of the HVDC link. This definition of a contingent event requires the HVDC model to explicitly consider the risk of the HVDC even if it is not the largest generating risk therefore an additional reserves penalty constraint is required.

The modelling of reserves used in this research is similar to the system security constraints of the New Zealand Electricity Commission Generation Expansion Model (GEM) [1]. The GEM model considers the largest generating risk for the whole system, the largest generating risk in the North Island and the risk of a single pole HVDC tripping. The HVDC problem also considers these risks but due to its simplified structure does not consider the contribution of capacity to supply *peak* demand as the GEM model does. The HVDC model does not separate capacity into base load or peaking provision and therefore uses total capacity values rather than contribution to peak demand in reserve and penalty constraints. The effect of this is to assume that all capacity in the HVDC model has the ability to serve peak demand and hence provide reserves, resulting in the model underestimating the capacity required in the system for reserves provision.

The HVDC reserve and penalty constraints offer some improvements over those in the GEM model as GEM assumes capacity values and timing of upgrades to the HVDC as predefined static values. The HVDC model allows the SDDP algorithm to find the values of HVDC capacity and timing of investment and use these values directly in the penalty constraints. GEM assumes a value of 400MW for the capacity of the largest unit in the system. The HVDC model will calculate this value as investments over time may result in the largest unit title being transferred to new investments. GEM also uses a static peak demand forecast value whereas SDDP samples this value from the peak demand growth distribution.

9.3.2 HVDC Losses

The HVDC experiences significant losses, the size of which are dependent on the amount of power being transferred and the configuration of the link i.e. monopole or bipole. One of the benefits of operating the link in a bipole arrangement is that the losses experienced by a bipole are approximately half the losses experienced by a monopole operating at an equivalent power rating. This is because the conductors of a bipole carry half the current of an equivalent monopole.

The loss model used in this thesis uses representative data from the loss model used in the New

[1] GEM is a mixed integer static optimisation model that uses a static demand forecast and generation investment scenarios to provide an optimal generation expansion plan for New Zealand [90].

Total Sent (MW)	Total Received (MW)	Segment (MW)	MW Lost	Incremental Loss (%)	Average Loss (%)
198	193	198	5	3	2.5
312	305	114	11	4	3.5
437	417	125	20	7.2	4.6
572	537	135	35	11	6
700	650	128	50	12	7

Table 9.1 HVDC Loss Data (For Single 700MW Pole)

Total Sent (MW)	Total Received (MW)	Segment (MW)	MW Lost	Incremental Loss (%)	Average Loss (%)
396	391	396	5	1.5	1.25
624	613	228	11	2	1.75
874	854	250	20	3.6	2.3
1144	1109	270	35	5.5	3
1400	1350	256	50	6	3.5

Table 9.2 HVDC Loss Data (For 1400MW Bipole)

Zealand electricity market scheduling and dispatch software (SPD). The loss data available from SPD pertains to a single pole, the existing 700MW Pole 2. The representative loss data used in this research is shown in Table 9.1 [91].

The loss data is split into five tranches as the actual losses are quadratic and are approximated here by a linear piecewise approximation. The first column represents the upper MW limit of each of the five tranches of the sending end capacity of the Pole. The second column represents the upper MW limit in each tranche at the receiving end of the link. The third column represents the number of MW in each tranche. The fourth column represents the cumulative number of MW lost in each tranche. The fifth column represents the incremental % losses i.e. the % of MW lost increases as power transfer across the link increases. The sixth column represents the average power losses based on the tranche of the loss model.

For a bipole operation the MW loss values do not change but the size of each loss tranche doubles. For example, the first tranche sending end value will double to 396MW but the MW lost remains at 5MW. This means both the incremental and average % losses halve. Table 9.2 represents the loss data for a bipole configuration.

The HVDC losses must be calculated by the optimisation as a function of both power transfer across the link and link capacity. The losses are calculated via a set of constraints that are discussed in Section 9.3.5.1.

9.3.3 Reserve Constraints for the HVDC Model

The reserve constraints for the HVDC investment problem consider the largest risk of the whole country, the largest generating risk in the North and South Islands and the risk of a single pole HVDC tripping. As in Chapters 7 and 8 the reserve constraints are trivial as they include the VoLL generators. This is necessary to reflect market operations and ensure the model does not enforce investment to meet demand. The reserve constraints are included for completeness and will not constraint the optimal solution but serve as a basis for developing penalty constraints.

There are five reserve constraints; a nationwide reserve constraint, a north and south island generating risk constraint, an HVDC northward transfer risk constraint and an HVDC southward transfer risk constraint.

Nationwide Reserve Constraint:

This constraint is considers the effect on the whole country if the largest generator trips. The losses of the HVDC are significant and must be considered within the constraint.

$$\mathbf{x}_{cap_{\mathbf{NZ}}} - \mathbf{x}_{NZ_{large}} - \mathbf{x}_{HVDC_{losses}} \geq \mathbf{x}_{demand_{\mathbf{NZ}}} \tag{9.1}$$

Where:

- $\mathbf{x}_{cap_{\mathbf{NZ}}}$ = National capacity including both NI and SI VoLL generators.
- $\mathbf{x}_{NZ_{large}}$ = Largest installed generator in the country.
- $\mathbf{x}_{HVDC_{losses}}$ = Maximum losses the HVDC will exhibit i.e. losses when running at maximum capacity.
- $\mathbf{x}_{demand_{\mathbf{NZ}}}$ = National peak demand.

Island Reserve Constraints:

These two constraints ensures that should the largest unit each load region trip, the remaining capacity including the HVDC link capacity is able to meet peak demand.

$$\mathbf{x}_{cap_{\mathbf{NI}}} + \mathbf{x}_{HVDC_{ReceiveCap}} - \mathbf{x}_{NI_{large}} \geq \mathbf{x}_{demand_{\mathbf{NI}}} \tag{9.2}$$

$$\mathbf{x}_{cap_{\mathbf{SI}}} + \mathbf{x}_{HVDC_{ReceiveCap}} - \mathbf{x}_{SI_{large}} \geq \mathbf{x}_{demand_{\mathbf{SI}}} \tag{9.3}$$

Where:

- $\mathbf{x}_{cap_{\mathbf{NI}}}$ = North Island capacity including the VoLL generator.
- $\mathbf{x}_{cap_{\mathbf{SI}}}$ = South Island capacity including the VoLL generator.

- $\mathbf{x}_{NI_{large}}$ = Largest generator in the North Island.
- $\mathbf{x}_{SI_{large}}$ = Largest generator in the South Island.
- $\mathbf{x}_{HVDC_{ReceiveCap}}$ = The maximum receiving end capacity of the HVDC link (note this is less than the capacity of the HVDC due to losses).
- $\mathbf{x}_{demand_{\mathbf{NI}}}$ = North Island peak demand.
- $\mathbf{x}_{demand_{\mathbf{SI}}}$ = South Island peak demand.

HVDC Reserve Constraints:

The HVDC reserve constraints ensure that should a single pole tripping of the HVDC occur, the capacity of the load region plus the remaining pole of the HVDC (assuming the HVDC has a bipole arrangement) is able to meet peak demand of the region.

$$\mathbf{x}_{HVDC_{RCTotal}} - \mathbf{x}_{HVDC_{RCSP}} + \mathbf{x}_{cap_{\mathbf{NI}}} \geq \mathbf{x}_{demand_{\mathbf{NI}}} \tag{9.4}$$

$$\mathbf{x}_{HVDC_{RCTotal}} - \mathbf{x}_{HVDC_{RCSP}} + \mathbf{x}_{cap_{\mathbf{SI}}} \geq \mathbf{x}_{demand_{\mathbf{SI}}} \tag{9.5}$$

Where:

- $\mathbf{x}_{cap_{\mathbf{NI}}}$ =North Island capacity including the VoLL generator.
- $\mathbf{x}_{cap_{\mathbf{SI}}}$ =South Island capacity including the VoLL generator.
- $\mathbf{x}_{HVDC_{RCTotal}}$ = Receiving end capacity of maximum installed capacity of the HVDC.
- $\mathbf{x}_{HVDC_{RCSP}}$ = Receiving end capacity of a single pole of the HVDC.
- $\mathbf{x}_{demand_{\mathbf{NI}}}$ = North Island peak demand.
- $\mathbf{x}_{demand_{\mathbf{SI}}}$ = South Island peak demand.

Note how the term $\mathbf{x}_{HVDC_{RCTotal}} - \mathbf{x}_{HVDC_{RCSP}}$ is valid under either a single or bipole configuration of the HVDC. Where a single pole trips under a single pole configuration, this term equals zero. Under a bipole configuration a single pole trip will allow this term to represent the remaining pole's receiving end capacity. This constraint relies on a bipole configuration having equal capacity on each pole, a valid assumption under the HVDC investment options described in Section 9.2.1.

The reserve constraints are used to develop reserve penalty constraints in Section 9.3.5.2. The penalty constraints define variable penalty variables $\mathbf{VoLL}_{t_{\mathbf{NI}}}$ and $\mathbf{VoLL}_{t_{\mathbf{SI}}}$. The variables are included in the objective function with an associated VoLL penalty cost.

9.3.4 Objective Function

The objective function of the HVDC investment problem is based on the objective function described in Section 8.3. It comprises the costs of the problem. The costs are in five groups, investment costs, variable operating costs, fixed operating costs, reserve costs and future costs of investment and operation of the system.

Investment Costs: The investment costs of the system are those associated with each potential investment option, including the transmission investment that is modelled as additional capacity. The coefficients and decision variables relating to investment costs in the objective function are shown in Equation 9.6.

$$\begin{aligned}\mathbf{Invest}_t =&(\mathbf{c_{1}}_{t_{\mathbf{NI}}} \times \mathbf{cap_{1inv}}_{t_{\mathbf{NI}}}) + \ldots + (\mathbf{c_{totInvOp}}_{t_{\mathbf{NI}}} \times \mathbf{cap_{totInvOpinv}}_{t_{\mathbf{NI}}}) + \ldots \\ &(\mathbf{c_{1}}_{t_{\mathbf{SI}}} \times \mathbf{cap_{1inv}}_{t_{\mathbf{SI}}}) + \ldots + (\mathbf{c_{totInvOp}}_{t_{\mathbf{SI}}} \times \mathbf{cap_{totInvOpinv}}_{t_{\mathbf{SI}}})\end{aligned} \tag{9.6}$$

Where:

- $\mathbf{totInvOp}$ = Total number of investment options = $\mathbf{capOp} + \mathbf{txOp}$
- $\mathbf{capOp}$ = Number of capacity investment options.
- $\mathbf{txOp}$ = Number of transmission investment options.
- $\mathbf{c_{1}}_{t_{\mathbf{NI}}}$ = Installation cost of investment option 1 in load region NI ($/MW).
- $\mathbf{c_{totInvOp}}_{t_{\mathbf{NI}}}$ = Installation cost of investment option **totInvOp** in load region NI ($/MW).
- $\mathbf{cap_{1inv}}_{t_{\mathbf{NI}}}$ = Decision variable associated with investment option **1** in load region NI (MW).
- $\mathbf{cap_{totInvOp}}_{t_{\mathbf{NI}}}$ = Decision variable associated with investment option **totInvOp** in load region NI (MW).

Variable Operational Costs: The variable operational costs are associated with the MWh of operation of each generator installed in the system. The generation technologies include the transmission system due to the modelling of transmission as generation. Equation 9.7 describes the coefficients and variables relating to variable operational costs in the objective function.

$$\begin{aligned}\mathbf{VariableOp}_t =&(\mathbf{vo_{1}}_{t_{\mathbf{NI}}} \times \mathbf{MWhTot}_{\mathbf{cap_{1}}_{t_{\mathbf{NI}}}}) + \ldots + (\mathbf{vo_{totUnits}}_{t_{\mathbf{NI}}} \times \mathbf{MWhTot}_{\mathbf{cap_{totUnits}}_{t_{\mathbf{NI}}}}) + \ldots \\ &(\mathbf{vo_{1}}_{t_{\mathbf{SI}}} \times \mathbf{MWhTot}_{\mathbf{cap_{1}}_{t_{\mathbf{SI}}}}) + \ldots + (\mathbf{vo_{totUnits}}_{t_{\mathbf{SI}}} \times \mathbf{MWhTot}_{\mathbf{cap_{totUnits}}_{t_{\mathbf{SI}}}})\end{aligned} \tag{9.7}$$

Where:

- $\mathbf{totUnits}$ = Sum of all generation and transmission types = $\mathbf{numGen} + \mathbf{tx}$.
- $\mathbf{numGen}$ = Total number of generation capacity types in each respective load region. This *excludes* transmission capacity and *includes* VoLL generators.
- $\mathbf{tx}$ = Number of transmission generators in each load region.
- $\mathbf{vo_{1_{t_{NI}}}}$ = Variable operating costs of capacity type **1** in load region NI ($/MWh).
- $\mathbf{vo_{totUnits_{t_{NI}}}}$ =Variable operating costs of capacity type **totUnits** in load region NI ($/MWh).
- $\mathbf{MWhTot_{cap_{1_{t_{NI}}}}}$ = Decision variable associated with variable operating costs for capacity type **1** in load region NI.
- $\mathbf{MWhTot_{cap_{totUnits_{t_{NI}}}}}$ = Decision variable associated with variable operating costs for capacity type **totUnits** in load region NI.

Where the generation type is transmission, the variables representing the MWh of operation of the transmission line pseudo generators are $\mathbf{MWhTot_{cap_{tx_{t_{NI}}}}}$ and $\mathbf{MWhTot_{cap_{tx_{t_{SI}}}}}$ One variable represents transfer in one direction and the other variable represents transfer in the reverse direction. The variable cost of operation applies regardless of the direction of transfer hence the variable cost coefficient for transmission applied to both $\mathbf{MWhTot_{cap_{tx_{t_{NI}}}}}$ and $\mathbf{MWhTot_{cap_{tx_{t_{SI}}}}}$.

Fixed Operational Costs: The fixed operational costs are associated with the number of MW of installed generation and transmission. Equation 9.8 describes the coefficients and variables relating to fixed operational costs in the objective function.

$$\begin{aligned}\mathbf{FixedOp}_t =&(\mathbf{fo_{1_{t_{NI}}}} \times \mathbf{cap_{1_{t+1_{NI}}}}) + \ldots + (\mathbf{fo_{totUnits_{t_{NI}}}} \times \mathbf{cap_{totUnits_{t+1_{NI}}}}) + \ldots \\ &(\mathbf{fo_{1_{t_{SI}}}} \times \mathbf{cap_{1_{t+1_{SI}}}}) + \ldots + (\mathbf{fo_{1_{t_{SI}}}} \times \mathbf{cap_{totUnits_{t+1_{SI}}}})\end{aligned} \tag{9.8}$$

Where:

- $\mathbf{fo_{1_{t_{NI}}}}$ = Fixed operating costs of capacity type 1 in load region NI ($/MW).
- $\mathbf{fo_{totUnits_{t_{NI}}}}$ = Fixed operating costs of capacity type **totUnits** in load region NI($/MW).
- $\mathbf{cap_{1_{t+1_{NI}}}}$ = Decision variable associated with fixed operation costs for capacity type **1**. This is the value of capacity type **1** after investment in this time period.
- $\mathbf{cap_{totUnits_{t+1_{NI}}}}$ = Decision variable associated with fixed operation costs for capacity type **totUnits**. This is the value of capacity type **totUnits** after investment in this time period.

Where the capacity type is transmission, the variables representing the capacity of the transmission line in each load region are $\mathbf{cap_{tx_{t+1_{NI}}}}$ and $\mathbf{cap_{tx_{t+1_{SI}}}}$. As the total capacity of the line is represented in both load regions and the fixed costs of operation are based on installed capacity, the fixed costs of operation must be split evenly between the two variables to prevent the costs being calculated twice. This is achieved by dividing the fixed cost of operation of the transmission line ($/MW) by two and assigning the result to the coefficients of $\mathbf{cap}_{\mathbf{tx}_{t+1_{\mathbf{NI}}}}$ and $\mathbf{cap}_{\mathbf{tx}_{t+1_{\mathbf{SI}}}}$ in the objective function.

Reserve Costs: Reserve costs are the sum of costs associated with extra capacity required for system security and penalty costs if VoLL is utilised in the provision of reserves. Where additional capacity is installed for reserves the investment costs and fixed operating costs will be included in the total cost through the investment and fixed costs sections of the objective function. The contribution of penalty costs from the use of VoLL for reserve capacity is outlined in Equation 9.9.

$$\mathbf{Reserves}_t = (\mathbf{res}_{pen_{t_{\mathbf{NI}}}} \times \mathbf{VoLL}_{t_{\mathbf{NI}}}) + (\mathbf{res}_{pen_{t_{\mathbf{SI}}}} \times \mathbf{VoLL}_{t_{\mathbf{SI}}}) \tag{9.9}$$

Where:

- $\mathbf{res}_{pen_{t_{\mathbf{NI}}}}$ = Penalty cost of utilising either North Island VoLL for reserve capacity($/MW).
- $\mathbf{res}_{pen_{t_{\mathbf{SI}}}}$ = Penalty cost of utilising either South Island VoLL for reserve capacity($/MW).
- $\mathbf{VoLL}_{t_{\mathbf{NI}}}$ = Decision variable associated with the number of MW of North Island VoLL used for reserve capacity.
- $\mathbf{VoLL}_{t_{\mathbf{SI}}}$ = Decision variable associated with the number of MW of South Island VoLL used for reserve capacity.

Future Investment and Operational Costs: The future costs of system investment and operation are given by Bender's Cuts calculated during the SDDP algorithm. The value of the future cost function is dependent on the values of state variables where the state variables for the HVDC investment problem are; all capacity types (including transmission) and demand in each load region. The coefficient of the future cost function, $\boldsymbol{\beta}_t$, represents the time value of money using the risk adjusted discount rate (Refer Section 3.4). The future costs contribution to the objective function is shown in Equation 9.10.

$$\begin{aligned}\mathbf{Future}_{t+1} =& \boldsymbol{\beta}_t \times \bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{NI}}}}, \dots, \mathbf{cap}_{\mathbf{numGen}_{t+1_{\mathbf{NI}}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_{\mathbf{NI}}}}, \dots \\ & \mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{SI}}}}, \dots, \mathbf{cap}_{\mathbf{numGen}_{t+1_{\mathbf{SI}}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_{\mathbf{SI}}}}, \dots \\ & \mathbf{dem}_{t+1_{\mathbf{NI}}}, \mathbf{dem}_{t+1_{\mathbf{SI}}})\end{aligned} \tag{9.10}$$

Where:

- $\mathbf{dem}_{t+1_\mathbf{A}}$ = Variable representing system demand in load region A after investment and growth in this time period.
- $\mathbf{dem}_{t+1_\mathbf{B}}$ = Variable representing system demand in load region B after investment and growth in this time period.
- $\mathbf{cap_{1}}_{t+1_\mathbf{A}}$ = Variable representing capacity of type 1 in load region A, after investment in this time period.
- **numGen** = Number of generation types, including transmission, in each load region.
- $\boldsymbol{\beta}_t$ = Time value of money associated with risk adjusted discount rate.

Objective Function for Time Period t:

$$\min\left[\mathbf{Invest}_t + \mathbf{FixedOp}_t + \mathbf{VariableOp}_t + \mathbf{Reserves}_t + \mathbf{Future}_{t+1}\right] \tag{9.11}$$

The cost coefficients are dependent on the time period of the optimisation. Costs may change over time due inflation, altered fuel costs, changes to subsidies, changes to investment costs, increases in maintenance costs etc. If the nature of the changing costs is known or can be estimated the objective function cost coefficients can reflect the dynamic cost coefficients.

9.3.4.1 Investment Costs

Large capital investments are usually either financed via debt or equity. Debt financing in its simplest form is by borrowing money from a lending institution. Equity financing is achieved by raising funds from investors. Assuming that debt financing is used, a loan plus interest must be paid back by the borrower in a predetermined period of time. The simplest form of repayment is an annuity cost where the total cost payable (principle plus interest) is evenly divided across the term of the loan. The HVDC investment model uses a repayment period (Refer Section 9.4.5) that defines how many years the borrower has to pay back the loan. This is used to calculate the annual cost, known as the levelized capital cost, as shown in Equation 9.12 [92].

$$\text{Yearly cost} = \text{total borrowed} \times \left(\frac{(\text{interest rate} - 1)(\text{interest rate})^{\text{payback period}}}{(\text{interest rate})^{\text{payback period}} - 1} \right) \tag{9.12}$$

SDDP cannot handle investment costs spread across time periods without significantly increasing the state space [2]. To find an equivalent single cost that reflects the equal annual costs, the present

[2] If the state variables included both the current and previous time period(s) (as many time periods as the number of years required to pay back the loan) state variables it is theoretically possible to spread payments out over a number of years. This would dramatically increase the model complexity and associated solution time and has not been tested in this research.

value of the equal annual costs is found. This present value can be considered as the lump sum of money that must be invested now at a specific interest rate, so that withdrawals of the annuity or yearly loan repayments each period completely exhausts the original principle plus accumulated interest at the end of the loan repayment period.

If an investment is made close to the end of the planning time period, the investment may have a loan repayment time frame that extends past the end of the planning period. The optimisation problem is only concerned with costs that occur *inside* the planning horizon therefore any costs or loan repayments made outside the planning horizon must be discarded. This is achieved by discounting the yearly investment costs back to the present time using the minimum of either; the years left in the planning horizon or the payback period of the loan. Equation 9.13 illustrates the present value calculation.

$$\text{Present Value of Investment Cost} = \frac{yearlycost}{(1 + \text{discount rate})^{-1}} + \ldots + \frac{yearlycost}{(1 + \text{discount rate})^{-T}} \tag{9.13}$$

Where:

- T = The smaller of the payback period or years remaining in planning horizon.

The present value of the investment cost is used as the investment cost coefficient in the objective function. As the value of T (i.e. the minimum of payback period or years left in planning horizon) is likely to change from one time period to the next, the present value of investment and hence investment cost coefficient is recalculated for each time period optimisation.

9.3.5 Constraints

The constraints of the HVDC investment problem are the same as the constraints of the transmission investment problem of Chapter 8 but with the addition of HVDC loss constraints and the additional reserve constraints pertaining to the HVDC reserve risk. The constraints of the transmission investment problem are not restated here but are included in the complete optimisation problem definition in Section 9.3.6.

9.3.5.1 HVDC Loss Constraints

The power transfer losses on the HVDC link are dependent on both the configuration of the HVDC, either monopole or bipole, and the level of transfer across the link. While the actual MW of power lost in each tranche is the same for both monopole and bipole operation, the size of the tranche is dependent on the total link capacity. Losses represent the difference between power sent and power received. A constraint that takes the power at each receiving end tranche and calculates the losses depending on power transfer would be of the form shown in Equation 9.14.

$$\begin{aligned}
\mathbf{SendingEndMW} = \ldots \\
[1 + (\frac{\mathbf{HVDC_{singleCap}}_{send}}{\mathbf{HVDC_{totCap}}_{send}})(\mathbf{incLoss}_{tr1})](\mathbf{MWtranche_1}) + \ldots \\
[1 + (\frac{\mathbf{HVDC_{singleCap}}_{send}}{\mathbf{HVDC_{totCap}}_{send}})(\mathbf{incLoss}_{tr2})](\mathbf{MWtranche_2}) + \ldots \\
[1 + (\frac{\mathbf{HVDC_{singleCap}}_{send}}{\mathbf{HVDC_{totCap}}_{send}})(\mathbf{incLoss}_{tr3})](\mathbf{MWtranche_3}) + \ldots \\
[1 + (\frac{\mathbf{HVDC_{singleCap}}_{send}}{\mathbf{HVDC_{totCap}}_{send}})(\mathbf{incLoss}_{tr4})](\mathbf{MWtranche_4}) + \ldots \\
[1 + (\frac{\mathbf{HVDC_{singleCap}}_{send}}{\mathbf{HVDC_{totCap}}_{send}})(\mathbf{incLoss}_{tr5})](\mathbf{MWtranche_5}) + \ldots
\end{aligned} \tag{9.14}$$

Where:

- $\mathbf{HVDC_{totCap}}_{send}$ = Total sending end capacity of the HVDC, wither 700 or 1400 MW.
- $\mathbf{HVDC_{singleCap}}_{send}$ = Total sending end capacity of a single pole. Is always equal to 700MW.
- $\frac{\mathbf{HVDC_{singleCap}}_{send}}{\mathbf{HVDC_{totCap}}_{send}}$ = Fraction used to represent whether the HVDC investment has been selected. As losses are halved with a bipole, using this fraction, the single pole incremental loss % is halved when the $\mathbf{HVDC_{totCap}}_{send}$ = 1400MW.
- $\mathbf{incLoss}_{tr1}$ = Incremental loss percentage for a single pole i.e. column 5 of Table 9.1.
- $\mathbf{MWtranche_t}$ = *Receiving End* capacity of tranche **t**. This value is calculated by the optimisation.

The constraint shown in Equation 9.14 is non linear in that the values of both $\mathbf{HVDC_{totCap}}_{send}$ and $\mathbf{MWtranche_t}$ are decision variables of the optimisation where the size of the tranche is also dependent on the capacity of the HVDC. Due to non linearity this constraint and hence loss model cannot be used in the HVDC investment problem.

If a bipole configuration experienced the same percentage losses as a monopole, the constraint would become linear as the term $\frac{\mathbf{HVDC_{singleCap}}_{send}}{\mathbf{HVDC_{totCap}}_{send}}$ would not be required. To overcome this difficulty a modelling decision was made that assumes both poles of a bipole configuration have the same losses as a monopole i.e. that a bipole link has twice as much loss as a monopole. This modelling decision is not taken lightly as the optimisation can no longer take advantage of the lower losses experienced by a bipole link and in fact suffers from greater losses than would actually be experienced by the system. The effects of this modelling decision on the optimal solution are to reduce the capacity of the link and misrepresent the ability of the system to transfer power from one load region to another. The effects on the optimal solution and ways in which a better model may be produced are discussed in Section 10.6.2.7. In light of the decision to assume a bipole link experiences double the losses of a monopole link, the loss data used in the HVDC investment problem is given in Table 9.1 and is applied to each pole of the bipole configuration. The result is that the incremental and average % losses for a bipole are the same as those for a monopole, effectively doubling the MW losses.

Example Loss Calculations: The first example calculation is for a single 700MW pole. The receiving end generator is generating 2,190,000MWh during a year, the power delivered to the receiving end are 2,190,000MWh/8760 (hrs per year) = 250MW. The receiving end MW loss tranches (based on Table 9.1) are shown in Table 9.3.

Receiving End Loss Tranche (MW)	Icremental Loss (%)	Receiving End (MW Transferred)
193	3 (0.03)	193.5
305	4 (0.04)	56.5
417	7 (0.07)	0
537	11 (0.11)	0
650	12 (0.12)	0

Table 9.3 HVDC Loss Data Tranches - Monopole

The sending end capacity is equal to:

$$1.03(193.5) + 1.04(56.5) = 258.1MW$$

The HVDC link in this example must send 258.1MW, on average for the year in order for the receiving end of the link to be supplied with 2,190,000MWh. The losses of the link are therefore 8.1MW or 70,956MWh over the year.

The second example is for a 1400MW bipole link. For the receiving end of the link to receive 1100MW, the link must send 1165.74MW. The losses in this example are therefore 65.74MW. Table 9.4 illustrates the receiving end loss tranches and the incremental losses for this example.

The sending end capacity is equal to:

$$1.03(386) + 1.04(224) + 1.07(224) + 1.11(240) + 1.12(26) = 1165.74MW$$

The first set of constraints relating to HVDC losses creates a separate variable for each loss tranche and restricts the maximum number of MW of each tranche to the receiving end value dictated in Table 9.1. These constraints are valid for both a single pole or bipole arrangement. For a bipole arrangement, the maximum capacity of each loss tranche is doubled. The set of

Receiving End Loss Tranche (MW)	Icremental Loss (%)	Receiving End (MW Transferred)
386	3 (0.03)	386
610	4 (0.04)	224
834	7 (0.07)	224
1074	11 (0.11)	240
1300	12 (0.12)	26

Table 9.4 HVDC Loss Data Tranches - Bipole

constraints in Equation 9.15 is repeated for each LDC discrete block and each load region.

$$
\begin{aligned}
&\mathbf{tranche1}_{t_{blk1_{\mathbf{NI}}}} - \mathbf{seg}_1\Big(\frac{\mathbf{cap}_{\mathbf{HVDC}_{t+1}}}{\mathbf{cap}_{singlePole}}\Big) \leq 0, \ldots, \mathbf{tranche1}_{t_{blkd_{\mathbf{NI}}}} - \mathbf{seg}_1\Big(\frac{\mathbf{cap}_{\mathbf{HVDC}_{t+1}}}{\mathbf{cap}_{singlePole}}\Big) \leq 0 \\
&\vdots \\
&\mathbf{tranche5}_{t_{blk1_{\mathbf{NI}}}} - \mathbf{seg}_5\Big(\frac{\mathbf{cap}_{\mathbf{HVDC}_{t+1}}}{\mathbf{cap}_{singlePole}}\Big) \leq 0, \ldots, \mathbf{tranche5}_{d_{blk1_{\mathbf{NI}}}} - \mathbf{seg}_5\Big(\frac{\mathbf{cap}_{\mathbf{HVDC}_{t+1}}}{\mathbf{cap}_{singlePole}}\Big) \leq 0 \\
\\
&\mathbf{tranche1}_{t_{blk1_{\mathbf{SI}}}} - \mathbf{seg}_1\Big(\frac{\mathbf{cap}_{\mathbf{HVDC}_{t+1}}}{\mathbf{cap}_{singlePole}}\Big) \leq 0, \ldots, \mathbf{tranche1}_{t_{blkd_{\mathbf{SI}}}} - \mathbf{seg}_1\Big(\frac{\mathbf{cap}_{\mathbf{HVDC}_{t+1}}}{\mathbf{cap}_{singlePole}}\Big) \leq 0 \\
&\vdots \\
&\mathbf{tranche5}_{t_{blk1_{\mathbf{NI}}}} - \mathbf{seg}_5\Big(\frac{\mathbf{cap}_{\mathbf{HVDC}_{t+1}}}{\mathbf{cap}_{singlePole}}\Big) \leq 0, \ldots, \mathbf{tranche5}_{t_{blkd_{\mathbf{SI}}}} - \mathbf{seg}_5\Big(\frac{\mathbf{cap}_{\mathbf{HVDC}_{t+1}}}{\mathbf{cap}_{singlePole}}\Big) \leq 0
\end{aligned}
\tag{9.15}
$$

Where:

- $\mathbf{tranche1}_{t_{blk1_{\mathbf{NI}}}}$ = HVDC loss tranche **1** for LDC dispatch block 1 in time period t in load region NI. These values represent the HVDC loss tranches at the receiving end.
- $\mathbf{cap}_{\mathbf{HVDC}_{t+1}}$ = Total capacity of the HVDC link after optimisation in the current time period.
- $\mathbf{seg}_1$ = Maximum size of receiving end of tranche 1 = 193 MW
- $\mathbf{seg}_2$ = Maximum size of receiving end of tranche 2 = 112 MW
- $\mathbf{seg}_3$ = Maximum size of receiving end of tranche 3 = 112 MW
- $\mathbf{seg}_4$ = Maximum size of receiving end of tranche 4 = 120 MW
- $\mathbf{seg}_5$ = Maximum size of receiving end of tranche 5 = 113 MW
- $\mathbf{cap}_{singlePole}$ = MW capacity of a single pole = 700MW.
- d = Number of dispatch blocks in the LDC for load region NI.
- h = Number of dispatch blocks in the LDC for load region SI.

The second set of HVDC loss constraints allocate the MW at the receiving end to the respective loss tranches. The MWh of energy generated by the HVDC receiving end generator is used to calculate how many loss tranches are 'filled' by the energy transfer across the link. The constraint in Equation 9.16 is repeated each discrete dispatch block in both load regions. Due to the mutually exclusive operation of the transmission generators, only one load region will have a non zero value for $\mathbf{receiveEnd}^{t}_{\mathbf{MWh}_{d_{\mathbf{X}}}}$.

$$
\begin{aligned}
\frac{\mathbf{receiveEnd}_{\mathbf{MWh}_{t_{blk_{1_{\mathbf{NI}}}}}}}{hrs_{t_{blk_{1_{\mathbf{NI}}}}}} &= \mathbf{tranche_1}_{t_{blk_{1_{\mathbf{NI}}}}} + \mathbf{tranche_2}_{t_{blk_{1_{\mathbf{NI}}}}} + \dots \\
&\qquad \mathbf{tranche_3}_{t_{blk_{1_{\mathbf{NI}}}}} + \mathbf{tranche_4}_{t_{blk_{1_{\mathbf{NI}}}}} + \mathbf{tranche_5}_{t_{blk_{1_{\mathbf{NI}}}}} \\
&\vdots \\
\frac{\mathbf{receiveEnd}_{\mathbf{MWh}_{t_{blk_{d_{\mathbf{NI}}}}}}}{hrs_{t_{blk_{d_{\mathbf{NI}}}}}} &= \mathbf{tranche_1}_{t_{blk_{d_{\mathbf{NI}}}}} + \mathbf{tranche_2}_{t_{blk_{d_{\mathbf{NI}}}}} + \dots \\
&\qquad \mathbf{tranche_3}_{t_{blk_{d_{\mathbf{NI}}}}} + \mathbf{tranche_4}_{t_{blk_{d_{\mathbf{NI}}}}} + \mathbf{tranche_5}_{t_{blk_{d_{\mathbf{NI}}}}} \\
\frac{\mathbf{receiveEnd}_{\mathbf{MWh}_{t_{blk_{1_{\mathbf{SI}}}}}}}{hrs_{t_{blk_{1_{\mathbf{SI}}}}}} &= \mathbf{tranche_1}_{t_{blk_{1_{\mathbf{SI}}}}} + \mathbf{tranche_2}_{t_{blk_{1_{\mathbf{SI}}}}} + \dots \\
&\qquad \mathbf{tranche_3}_{t_{blk_{1_{\mathbf{SI}}}}} + \mathbf{tranche_4}_{t_{blk_{1_{\mathbf{SI}}}}} + \mathbf{tranche_5}_{t_{blk_{1_{\mathbf{SI}}}}} \\
&\vdots \\
\frac{\mathbf{receiveEnd}_{\mathbf{MWh}_{t_{blk_{h_{\mathbf{SI}}}}}}}{hrs_{t_{blk_{h_{\mathbf{SI}}}}}} &= \mathbf{tranche_1}_{t_{blk_{h_{\mathbf{SI}}}}} + \mathbf{tranche_2}_{t_{blk_{h_{\mathbf{SI}}}}} + \dots \\
&\qquad \mathbf{tranche_3}_{t_{blk_{h_{\mathbf{SI}}}}} + \mathbf{tranche_4}_{t_{blk_{h_{\mathbf{SI}}}}} + \mathbf{tranche_5}_{t_{blk_{h_{\mathbf{SI}}}}}
\end{aligned}
\tag{9.16}
$$

Where:

- $\mathbf{receiveEnd}_{\mathbf{MWh}_{t_{blk_{1_{\mathbf{NI}}}}}}$ = MWh of generation from the receiving end generator in load region NI in LDC dispatch block 1.
- $hrs_{t_{blk_{1_{\mathbf{NI}}}}}$ = Number of hours in LDC dispatch block 1 in time period t for load region NI.
- $\mathbf{tranche_1}_{t_{blk_{1_{\mathbf{NI}}}}}$ = HVDC loss tranche 1 for LDC dispatch block 1 in load region NI.
- d = Number of dispatch blocks in the LDC for load region NI.
- h = Number of dispatch blocks in the LDC for load region SI.

The final constraint set relating to HVDC losses is a calculation of the sending end HVDC generator capacity based on the receiving end capacity and the loss model. The constraint in

Equation 9.17 is repeated for each LDC dispatch block in each load region.

$$\begin{aligned}
\mathbf{sendEnd_{MWh}}_{t_{blk_{1_{NI}}}} &= 1.03(\mathbf{tranche_1}_{t_{blk_{1_{NI}}}}) + 1.04(\mathbf{tranche_2}_{t_{blk_{1_{NI}}}}) + \ldots \\
&\quad 1.07(\mathbf{tranche_3}_{t_{blk_{1_{NI}}}}) + 1.11(\mathbf{tranche_4}_{t_{blk_{1_{NI}}}}) + 1.12(\mathbf{tranche_5}_{t_{blk_{1_{NI}}}}) \\
&\vdots \\
\mathbf{sendEnd_{MWh}}_{t_{blk_{d_{NI}}}} &= 1.03(\mathbf{tranche_1}_{t_{blk_{d_{NI}}}}) + 1.04(\mathbf{tranche_2}_{t_{blk_{d_{NI}}}}) + \ldots \\
&\quad 1.07(\mathbf{tranche_3}_{t_{blk_{d_{NI}}}}) + 1.11(\mathbf{tranche_4}_{t_{blk_{d_{NI}}}}) + 1.12(\mathbf{tranche_5}_{t_{blk_{d_{NI}}}}) \\
\mathbf{sendEnd_{MWh}}_{t_{blk_{1_{SI}}}} &= 1.03(\mathbf{tranche_1}_{t_{blk_{1_{SI}}}}) + 1.04(\mathbf{tranche_2}_{t_{blk_{1_{SI}}}}) + \ldots \\
&\quad 1.07(\mathbf{tranche_3}_{t_{blk_{1_{SI}}}}) + 1.11(\mathbf{tranche_4}_{t_{blk_{1_{SI}}}}) + 1.12(\mathbf{tranche_5}_{t_{blk_{1_{SI}}}}) \\
&\vdots \\
\mathbf{sendEnd_{MWh}}_{t_{blk_{d_{SI}}}} &= 1.03(\mathbf{tranche_1}_{t_{blk_{h_{SI}}}}) + 1.04(\mathbf{tranche_2}_{t_{blk_{h_{SI}}}}) + \ldots \\
&\quad 1.07(\mathbf{tranche_3}_{t_{blk_{h_{SI}}}}) + 1.11(\mathbf{tranche_4}_{t_{blk_{h_{SI}}}}) + 1.12(\mathbf{tranche_5}_{t_{blk_{h_{SI}}}})
\end{aligned} \tag{9.17}$$

Where:

- $\mathbf{sendEnd_{MWh}}_{t_{blk_{1_{NI}}}}$ = Capacity of sending end HVDC generator in LDC dispatch block 1 for time period t in load region NI.
- $\mathbf{tranche_1}_{t_{blk_{1_{NI}}}}$ = HVDC loss tranche 1 for LDC dispatch block 1 in load region NI.
- $1.03, 1.04$ etc. = The multiplication factor representing the percentage loss in each tranche of the HVDC loss model.

9.3.5.2 Reserve Penalty Constraints

The reserve constraints discussed in Section 9.3.3 must be converted into penalty constraints to identify if the VoLL generators are used for providing reserve capacity. The penalty constraints need to know the capacity value of the largest generator in each island and the largest generator in the country. The constraints in Equations 9.18, 9.19 and 9.20 identify these variables. These constraints exclude both the HVDC generators and VoLL generators as the HVDC is treated as a special case with an individual reserve penalty constraint and the VoLL generators do not add additional generating capacity to the system.

$$\begin{aligned}
\mathbf{MW}_{risk_{t+1_{\mathbf{NI}}}} &\geq \mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{NI}}}} \\
&\vdots \\
\mathbf{MW}_{risk_{t+1_{\mathbf{NI}}}} &\geq \mathbf{cap}_{\mathbf{n}_{t+1_{\mathbf{NI}}}}
\end{aligned} \tag{9.18}$$

$$\begin{aligned}
\mathbf{MW}_{risk_{t+1_{\mathbf{SI}}}} &\geq \mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{SI}}}} \\
&\vdots \\
\mathbf{MW}_{risk_{t+1_{\mathbf{SI}}}} &\geq \mathbf{cap}_{\mathbf{m}_{t+1_{\mathbf{SI}}}}
\end{aligned} \tag{9.19}$$

$$\begin{aligned}
\mathbf{MW}_{risk_{t+1_{\mathbf{SI+NI}}}} &\geq \mathbf{MW}_{risk_{t+1_{\mathbf{NI}}}} \\
\mathbf{MW}_{risk_{t+1_{\mathbf{SI+NI}}}} &\geq \mathbf{MW}_{risk_{t+1_{\mathbf{SI}}}}
\end{aligned} \tag{9.20}$$

Where:

- $\mathbf{MW}_{risk_{t+1_{\mathbf{NI}}}}$ = Variable representing capacity of largest generator in the North Island.
- $\mathbf{MW}_{risk_{t+1_{\mathbf{SI}}}}$ = Variable representing capacity of largest generator in the South Island.
- $\mathbf{MW}_{risk_{t+1_{\mathbf{A+B}}}}$ = Variable representing the larger of the largest capacity from both regions.
- $\mathbf{n}$ = Number of generators in the North Island. This *excludes* both the VoLL generator and the HVDC generator.
- $\mathbf{m}$ = Number of generators in the South Island. This *excludes* both the VoLL generator and the HVDC generator.

National Penalty Constraint:

The national penalty reserve constraint identifies the total number of MW of VoLL used for reserve capacity but does not identify which island (or combination of islands) the VoLL reserve is provided from. The island and HVDC penalty constraints combine with the national penalty constraint to limit the lower values of VoLL reserve from each island and ensure the HVDC link capacity is not exceeded when transferring VoLL capacity from one island to another. The optimisation will chose the lowest cost combination of VoLL capacity that meets these constraints in order to minimise the overall system costs.

$$\begin{aligned}
&(\mathbf{dem}_{t+1_{\mathbf{NI}}} + \mathbf{dem}_{t+1_{\mathbf{SI}}} + \mathbf{HVDC}_{\mathbf{maxLoss}}) - \ldots \\
&\qquad (\mathbf{cap}_{t+1_{\mathbf{NI}}} + \mathbf{cap}_{t+1_{\mathbf{SI}}} - \mathbf{MW}_{risk_{t+1_{\mathbf{SI+NI}}}}) \leq \mathbf{VoLL}_{t_{\mathbf{NI}}} + \mathbf{VoLL}_{t_{\mathbf{SI}}}
\end{aligned} \tag{9.21}$$

Where:

- $\mathbf{dem}_{t+1_{\mathbf{NI}}}$ = North Island peak demand.
- $\mathbf{HVDC}_{\mathbf{maxLoss}}$ = Maximum losses on the HVDC when operating at maximum transfer.
- $\mathbf{cap}_{t+1_{\mathbf{NI}}}$ = North Island capacity excluding both the HVDC and VoLL generators.
- $\mathbf{MW}_{risk_{t+1_{\mathbf{SI+NI}}}}$ = Largest generating risk in the country.
- $\mathbf{VoLL}_{t_{\mathbf{NI}}}$ = Variable representing MW of North Island VoLL used for reserve capacity.
- $\mathbf{VoLL}_{t_{\mathbf{SI}}}$ = Variable representing MW of South Island VoLL used for reserve capacity.

Island Penalty Constraints:

These constraints identify if VoLL is used as reserve capacity is either load region for a contingent event involving the largest generator that isn't the HVDC. The HVDC losses are implicitly included in these constraints through the use of the receiving end capacity of the link rather than installed capacity.

North Island

$$\mathbf{dem}_{t+1_{\mathbf{NI}}} - \dots$$
$$\left(\mathbf{cap}_{t+1_{\mathbf{NI}}} - \mathbf{MW}_{risk_{t+1_{\mathbf{NI}}}} + \underbrace{\frac{\mathbf{HVDC}_{t+1_{\mathbf{totCap}_{send}}}}{\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{send}}}}(\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}})}_{\text{Receiving end capacity of HVDC at time } t+1}\right) \leq \mathbf{VoLL}_{t_{\mathbf{NI}}}$$
$$\mathbf{Voll}_{t_{\mathbf{NIpen}}} \geq 0 \tag{9.22}$$

Where:

- $\mathbf{cap}_{t+1_{\mathbf{NI}}}$ = North Island capacity *excluding* the HVDC and VoLL generators.
- $\mathbf{MW}_{risk_{t+1_{\mathbf{NI}}}}$ = Capacity of largest generator in the North Island.
- $\mathbf{HVDC}_{t+1_{\mathbf{totCap}_{send}}}$ = Installed capacity of the HVDC link after optimisation in the current time period. This value represents the maximum capacity of the sending end of the link.
- $\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{send}}}$ = Maximum sending capacity of a single pole. In this optimisation problem this is 700MW.
- $\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}}$ = Maximum available capacity at the receiving end of the link for a single pole. In this optimisation this is 649.6MW.
- $\mathbf{dem}_{t+1_{\mathbf{NI}}}$ = North Island peak demand.
- $\mathbf{VoLL}_{t_{\mathbf{NI}}}$ = MW of North Island VoLL used for reserve capacity.

South Island

$$\mathbf{dem}_{t+1_{\mathbf{SI}}} - \dots \left(\mathbf{cap}_{t+1_{\mathbf{SI}}} - \mathbf{MW}_{risk_{t+1_{\mathbf{SI}}}} + \underbrace{\frac{\mathbf{HVDC}_{t+1_{\mathbf{totCap}_{send}}}}{\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{send}}}}(\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}})}_{\text{Receiving end capacity of HVDC at time } t+1}\right) \leq \mathbf{VoLL}_{t_{\mathbf{SI}}}$$

$$\mathbf{Voll}_{t_{\mathbf{SIpen}}} \geq 0 \tag{9.23}$$

Where:

- $\mathbf{cap}_{t+1_{\mathbf{SI}}}$ = South Island capacity *excluding* the HVDC and VoLL generators.
- $\mathbf{MW}_{risk_{t+1_{\mathbf{SI}}}}$ = Capacity of largest generator in the South Island.
- $\mathbf{HVDC}_{t+1_{\mathbf{totCap}_{send}}}$ = Installed capacity of the HVDC link after optimisation in the current time period. This value represents the maximum capacity of the sending end of the link.
- $\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{send}}}$ = Maximum sending capacity of a single pole. In this optimisation problem this is 700MW.
- $\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}}$ = Maximum available capacity at the receiving end of the link for a single pole. In this optimisation this is 649.6MW.
- $\mathbf{dem}_{t+1_{\mathbf{SI}}}$ = South Island peak demand.
- $\mathbf{VoLL}_{t_{\mathbf{SI}}}$ = MW of South Island VoLL used for reserve capacity.

HVDC Penalty Constraints:

A single pole outage of the HVDC link is treated as a special case and has a separate penalty constraint. The constraint is consistent with those used for the island penalty constraints where the loss of a single pole is considered instead of the largest generating unit[3]. In New Zealand the HVDC link is, for the foreseeable future, the largest risk to either island as the single pole capacity is 700MW. In this situation the HVDC penalty constraints will always constrain the VoLL penalty variables more than the island penalty constraints.

North Island

[3] The remaining pole has a ramp up ability to transfer greater than its rated capacity for short periods of time, i.e. in a contingent event. This facility has not been modelled in the HVDC optimisation problem due to its complexity. With more detailed modelling the ramp up ability of the pole(s) could be included.

$$\mathbf{dem}_{t+1_{\mathbf{NI}}} - \ldots$$
$$\left[\mathbf{cap}_{t+1_{\mathbf{NI}}} + \left(\underbrace{\underbrace{\frac{\mathbf{HVDC}_{t+1_{\mathbf{totCap}_{send}}}}{\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{send}}}}(\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}})}_{\substack{\text{Maximum receiving end capacity}\\ \text{of HVDC link at } t+1}} - (\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}})}_{\text{Maximum receiving end capacity of HVDC after loss of single pole}}\right)\right] \leq \mathbf{VoLL}_{t_{\mathbf{NI}}}$$
$$\mathbf{Voll}_{t_{\mathbf{NIpen}}} \geq 0 \tag{9.24}$$

Where:

- $\mathbf{cap}_{t+1_{\mathbf{NI}}}$ =North Island capacity *excluding* both the VoLL and HVDC generators.
- $\mathbf{HVDC}_{t+1_{\mathbf{totCap}_{send}}}$ = Installed capacity of the HVDC link after optimisation in the current time period. This value represents the maximum capacity of the sending end of the link.
- $\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{send}}}$ = Maximum sending capacity of a single pole. In this optimisation problem this is 700MW.
- $\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}}$ = Maximum available capacity at the receiving end of the link for a single pole. In this optimisation this is 649.6MW.
- $\mathbf{dem}_{t+1_{\mathbf{NI}}}$ = North Island peak demand.
- $\mathbf{VoLL}_{t_{\mathbf{NI}}}$ = MW of North Island VoLL used for reserve capacity.

South Island

$$\mathbf{dem}_{t+1_{\mathbf{SI}}} - \ldots$$
$$\left[\mathbf{cap}_{t+1_{\mathbf{SI}}} + \left(\underbrace{\underbrace{\frac{\mathbf{HVDC}_{t+1_{\mathbf{totCap}_{send}}}}{\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{send}}}}(\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}})}_{\substack{\text{Maximum receiving end capacity}\\ \text{of HVDC link at } t+1}} - (\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}})}_{\text{Maximum receiving end capacity of HVDC after loss of single pole}}\right)\right] \leq \mathbf{VoLL}_{t_{\mathbf{SI}}}$$
$$\mathbf{Voll}_{t_{\mathbf{SIpen}}} \geq 0 \tag{9.25}$$

Where:

- $\mathbf{cap}_{t+1_{\mathbf{SI}}}$ = South Island capacity *excluding* both the VoLL and HVDC generators.
- $\mathbf{HVDC}_{t+1_{\mathbf{totCap}_{send}}}$ = Installed capacity of the HVDC link after optimisation in the current time period. This value represents the maximum capacity of the sending end of the link.

- $\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{send}}}$ = Maximum sending capacity of a single pole. In this optimisation problem this is 700MW.
- $\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}}$ = Maximum available capacity at the receiving end of the link for a single pole. In this optimisation this is 649.6MW.
- $\mathbf{dem}_{t+1_{\mathbf{SI}}}$ = South Island peak demand.
- $\mathbf{VoLL}_{t_{\mathbf{SI}}}$ = MW of South Island VoLL used for reserve capacity.

Both the individual South Island reserve penalty constraints are unlikely to ever result in the $\mathbf{VoLL}_{t_{\mathbf{SI}}}$ value being a value greater than 0. This is due to the very high levels of capacity in the South Island in comparison to the load. The constraints are included here for completeness and incase the unlikely situation of constrained South Island reserve capacity is encountered under the optimisation and simulation.

9.3.6 Overall Mathematical Representation

Presented here is the overall mathematical representation of the HVDC investment planning problem.

Objective Function

$$
\begin{aligned}
&\min \Big[(\mathbf{c1}_{t_{\mathbf{NI}}} \times \mathbf{cap}_{\mathbf{1inv}_{t_{\mathbf{NI}}}}) + \ldots + (\mathbf{c}_{\mathbf{totInvOp}_{t_{\mathbf{NI}}}} \times \mathbf{cap}_{\mathbf{totInvOpinv}_{t_{\mathbf{NI}}}}) + \ldots \\
&(\mathbf{c1}_{t_{\mathbf{SI}}} \times \mathbf{cap}_{\mathbf{1inv}_{t_{\mathbf{SI}}}}) + \ldots + (\mathbf{c}_{\mathbf{totInvOp}_{t_{\mathbf{SI}}}} \times \mathbf{cap}_{\mathbf{totInvOpinv}_{t_{\mathbf{SI}}}}) + \ldots \\
&(\mathbf{vo}^{\mathbf{NI}}_{\mathbf{1}_t} \times \mathbf{MWhTotal}^{\mathbf{NI}}_{\mathbf{cap}_{\mathbf{1}_t}}) + \ldots + (\mathbf{vo}^{\mathbf{NI}}_{\mathbf{totUnits}_t} \times \mathbf{MWhTotal}^{\mathbf{NI}}_{\mathbf{cap}_{\mathbf{totUnits}_t}}) + \ldots \\
&(\mathbf{vo}^{\mathbf{SI}}_{\mathbf{1}_t} \times \mathbf{MWhTotal}^{\mathbf{SI}}_{\mathbf{cap}_{\mathbf{1}_t}}) + \ldots + (\mathbf{vo}^{\mathbf{SI}}_{\mathbf{totUnits}_t} \times \mathbf{MWhTotal}^{\mathbf{SI}}_{\mathbf{cap}_{\mathbf{totUnits}_t}}) + \ldots \\
&(\mathbf{fo}^{\mathbf{NI}}_{\mathbf{1}_t} \times \mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{NI}}}}) + \ldots + (\mathbf{fo}^{\mathbf{NI}}_{\mathbf{totUnits}_t} \times \mathbf{cap}_{\mathbf{totUnits}_{t+1_{\mathbf{NI}}}}) + \ldots \\
&(\mathbf{fo}^{\mathbf{SI}}_{\mathbf{1}_t} \times \mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{SI}}}}) + \ldots + (\mathbf{fo}^{\mathbf{SI}}_{\mathbf{totUnits}_t} \times \mathbf{cap}_{\mathbf{totUnits}_{t+1_{\mathbf{SI}}}}) + \ldots \\
&(\mathbf{res}_{pen_{t_{\mathbf{NI}}}} \times \mathbf{VoLL}_{t_{\mathbf{NI}}}) + (\mathbf{res}_{pen_{t_{\mathbf{SI}}}} \times \mathbf{VoLL}_{t_{\mathbf{SI}}}) + \ldots \\
&\boldsymbol{\beta}_t \times \bar{\boldsymbol{\alpha}}_{\boldsymbol{t+1}}(\mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{NI}}}}, \ldots, \mathbf{cap}_{\mathbf{numGen}_{t+1_{\mathbf{NI}}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_{\mathbf{NI}}}}, \ldots \\
&\qquad \mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{SI}}}}, \ldots, \mathbf{cap}_{\mathbf{numGen}_{t+1_{\mathbf{SI}}}}, \mathbf{cap}_{\mathbf{tx}_{t+1_{\mathbf{SI}}}}, \ldots \\
&\qquad\qquad \mathbf{dem}_{t+1_{\mathbf{NI}}}, \mathbf{dem}_{t+1_{\mathbf{SI}}})\Big]
\end{aligned}
\tag{9.26}
$$

Subject To:

Table 9.5: Constraints for the HVDC Investment Problem

Constraint	**Constraint Type**
$\mathbf{cap}_{t+1_{\mathbf{NI}}} = \mathbf{cap}_{t_{\mathbf{NI}}} + \mathbf{cap}_{\mathbf{1inv}_{t_{\mathbf{NI}}}} + \ldots + \mathbf{cap}_{\mathbf{capOpinv}_{t_{\mathbf{NI}}}} + \mathbf{cap}_{\mathbf{txinv}_{t_{\mathbf{NI}}}}$	Capacity Investment Constraint
$\mathbf{cap}_{t+1_{\mathbf{SI}}} = \mathbf{cap}_{t_{\mathbf{SI}}} + \mathbf{cap}_{\mathbf{1inv}_{t_{\mathbf{SI}}}} + \ldots + \mathbf{cap}_{\mathbf{capOpinv}_{t_{\mathbf{SI}}}} + \mathbf{cap}_{\mathbf{txinv}_{t_{\mathbf{SI}}}}$	
$\mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{NI}}}} = \mathbf{cap}_{\mathbf{1}_{t_{\mathbf{NI}}}} + \mathbf{cap}_{\mathbf{1inv}_{t_{\mathbf{NI}}}}$ $\vdots$ $\mathbf{cap}_{\mathbf{numGen}_{t+1_{\mathbf{NI}}}} = \mathbf{cap}_{\mathbf{numGen}_{t_{\mathbf{NI}}}} + \mathbf{cap}_{\mathbf{numGenInv}_{t_{\mathbf{NI}}}}$ $\mathbf{cap}_{\mathbf{tx}_{t+1_{\mathbf{NI}}}} = \mathbf{cap}_{\mathbf{tx}_{t_{\mathbf{NI}}}} + \mathbf{cap}_{\mathbf{txOp}_{t}}$	Individual Capacity Investment Constraints
$\mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{SI}}}} = \mathbf{cap}_{\mathbf{1}_{t_{\mathbf{SI}}}} + \mathbf{cap}_{\mathbf{1inv}_{t_{\mathbf{SI}}}}$ $\vdots$ $\mathbf{cap}_{\mathbf{numGen}_{t+1_{\mathbf{SI}}}} = \mathbf{cap}_{\mathbf{numGen}_{t_{\mathbf{SI}}}} + \mathbf{cap}_{\mathbf{numGenInv}_{t_{\mathbf{SI}}}}$ $\mathbf{cap}_{\mathbf{tx}_{t+1_{\mathbf{SI}}}} = \mathbf{cap}_{\mathbf{tx}_{t_{\mathbf{SI}}}} + \mathbf{cap}_{\mathbf{txInv}_{t}}$	
$\mathbf{cap}_{\mathbf{1inv}_{t_{\mathbf{NI}}}} - InvestSize_{\mathbf{1}_{\mathbf{NI}}}\mathbf{y}_{\mathbf{1}_{t_{\mathbf{NI}}}} = 0$ $\vdots$ $\mathbf{cap}_{\mathbf{capOp}_{t_{\mathbf{NI}}}} - InvestSize_{\mathbf{capOp}_{\mathbf{NI}}}\mathbf{y}_{\mathbf{capOp}_{t_{\mathbf{NI}}}} = 0$ $\mathbf{cap}_{\mathbf{txOp}_{t_{\mathbf{NI}}}} - InvestSize_{\mathbf{txOp}_{\mathbf{NI}}}\mathbf{y}_{\mathbf{txOp}_{t_{\mathbf{NI}}}} = 0$ $\mathbf{y}_{\mathbf{1}_{t_{\mathbf{NI}}}} \leq 1, \ldots, \mathbf{y}_{\mathbf{capOp}_{t_{\mathbf{NI}}}} \leq 1, \mathbf{y}_{\mathbf{txOp}_{t_{\mathbf{NI}}}} \leq 1$ $\mathbf{y}_{\mathbf{1}_{t_{\mathbf{NI}}}}, \mathbf{y}_{\mathbf{capOp}_{t_{\mathbf{NI}}}}, \mathbf{y}_{\mathbf{txOp}_{t_{\mathbf{NI}}}} \epsilon \mathbb{Z}$	Large Integer Investment Constraints
$\mathbf{cap}_{\mathbf{1inv}_{t_{\mathbf{SI}}}} - InvestSize_{\mathbf{1}_{\mathbf{SI}}}\mathbf{y}_{\mathbf{1}_{t_{\mathbf{SI}}}} = 0$ $\vdots$ $\mathbf{cap}_{\mathbf{capOp}_{t_{\mathbf{SI}}}} - InvestSize_{\mathbf{capOp}_{\mathbf{SI}}}\mathbf{y}_{\mathbf{capOp}_{t_{\mathbf{SI}}}} = 0$ $\mathbf{cap}_{\mathbf{txOp}_{t_{\mathbf{SI}}}} - InvestSize_{\mathbf{txOp}_{\mathbf{SI}}}\mathbf{y}_{\mathbf{txOp}_{t_{\mathbf{SI}}}} = 0$ $\mathbf{y}_{\mathbf{1}_{t_{\mathbf{SI}}}} \leq 1, \ldots, \mathbf{y}_{\mathbf{capOp}_{t_{\mathbf{SI}}}} \leq 1, \mathbf{y}_{\mathbf{txOp}_{t_{\mathbf{SI}}}} \leq 1$ $\mathbf{y}_{\mathbf{1}_{t_{\mathbf{SI}}}}, \mathbf{y}_{\mathbf{capOp}_{t_{\mathbf{SI}}}}, \mathbf{y}_{\mathbf{txOp}_{t_{\mathbf{SI}}}} \epsilon \mathbb{Z}$	
$\frac{\mathbf{MWh}_{\mathbf{cap}_{\mathbf{1}_{t_{blk_{1}}}}}_{\mathbf{NI}}}{hrs_{t_{blk_{1}}}_{\mathbf{NI}}} \leq \mathbf{MaxCap}_{\mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{NI}}}}}, \ldots, \frac{\mathbf{MWh}_{\mathbf{cap}_{\mathbf{numGen+tx}_{t_{blk_{1}}}}}_{\mathbf{NI}}}{hrs_{t_{blk_{1}}}_{\mathbf{NI}}} \leq \mathbf{MaxCap}_{\mathbf{cap}_{\mathbf{numGen+tx}_{t+1_{\mathbf{NI}}}}}$ $\vdots$ $\frac{\mathbf{MWh}_{\mathbf{cap}_{\mathbf{1}_{t_{blk_{d}}}}}_{\mathbf{NI}}}{hrs_{t_{blk_{d}}}_{\mathbf{NI}}} \leq \mathbf{MaxCap}_{\mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{NI}}}}}, \ldots, \frac{\mathbf{MWh}_{\mathbf{cap}_{\mathbf{numGen+tx}_{t_{blk_{d}}}}}_{\mathbf{NI}}}{hrs_{t_{blk_{d}}}_{\mathbf{NI}}} \leq \mathbf{MaxCap}_{\mathbf{cap}_{\mathbf{numGen+tx}_{t+1_{\mathbf{NI}}}}}$	Restriction of MWh of Operation to Capacity of Generator
$\frac{\mathbf{MWh}_{\mathbf{cap}_{\mathbf{1}_{t_{blk_{1}}}}}_{\mathbf{SI}}}{hrs_{t_{blk_{1}}}_{\mathbf{SI}}} \leq \mathbf{MaxCap}_{\mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{SI}}}}}, \ldots, \frac{\mathbf{MWh}_{\mathbf{cap}_{\mathbf{numGen+tx}_{t_{blk_{1}}}}}_{\mathbf{SI}}}{hrs_{t_{blk_{1}}}_{\mathbf{SI}}} \leq \mathbf{MaxCap}_{\mathbf{cap}_{\mathbf{numGen+tx}_{t+1_{\mathbf{SI}}}}}$ $\vdots$ $\frac{\mathbf{MWh}_{\mathbf{cap}_{\mathbf{1}_{t_{blk_{h}}}}}_{\mathbf{SI}}}{hrs_{t_{blk_{h}}}_{\mathbf{SI}}} \leq \mathbf{MaxCap}_{\mathbf{cap}_{\mathbf{1}_{t+1_{\mathbf{SI}}}}}, \ldots, \frac{\mathbf{MWh}_{\mathbf{cap}_{\mathbf{numGen+tx}_{t_{blk_{h}}}}}_{\mathbf{SI}}}{hrs_{t_{blk_{h}}}_{\mathbf{SI}}} \leq \mathbf{MaxCap}_{\mathbf{cap}_{\mathbf{numGen+tx}_{t+1_{\mathbf{SI}}}}}$	
$\mathbf{MWh}_{\mathbf{cap}_{\mathbf{1}_{t_{blk_{1}}}}}_{\mathbf{NI}} + \ldots + \mathbf{MWh}_{\mathbf{cap}_{\mathbf{numGen}_{t_{blk_{1}}}}}_{\mathbf{NI}} + \mathbf{MWh}_{\mathbf{cap}_{\mathbf{tx}_{t_{blk_{1}}}}}_{\mathbf{NI}} = \ldots$ $hrs_{t_{blk_{1}}}_{\mathbf{NI}} (\mathbf{dem}_{t+1_{blk_{1}}}_{\mathbf{NI}}) + \mathbf{MWh}_{\mathbf{cap}_{\mathbf{tx}_{t_{blk_{1}}}}}_{\mathbf{SI}}$ $\vdots$ $\mathbf{MWh}_{\mathbf{cap}_{\mathbf{1}_{t_{blk_{d}}}}}_{\mathbf{NI}} + \ldots + \mathbf{MWh}_{\mathbf{cap}_{\mathbf{numGen}_{t_{blk_{d}}}}}_{\mathbf{NI}} + \mathbf{MWh}_{\mathbf{cap}_{\mathbf{tx}_{t_{blk_{d}}}}}_{\mathbf{NI}} = \ldots$ $hrs_{t_{blk_{d}}}_{\mathbf{NI}} (\mathbf{dem}_{t+1_{blk_{d}}}_{\mathbf{NI}}) + \mathbf{MWh}_{\mathbf{cap}_{\mathbf{tx}_{t_{blk_{d}}}}}_{\mathbf{SI}}$	Generation Demand Balance
$\mathbf{MWh}_{\mathbf{cap}_{\mathbf{1}_{t_{blk_{1}}}}}_{\mathbf{SI}} + \ldots + \mathbf{MWh}_{\mathbf{cap}_{\mathbf{numGen}_{t_{blk_{1}}}}}_{\mathbf{SI}} + \mathbf{MWh}_{\mathbf{cap}_{\mathbf{tx}_{t_{blk_{1}}}}}_{\mathbf{SI}} = \ldots$ $hrs_{t_{blk_{1}}}_{\mathbf{SI}} (\mathbf{dem}_{t+1_{blk_{1}}}_{\mathbf{SI}}) + \mathbf{MWh}_{\mathbf{cap}_{\mathbf{tx}_{t_{blk_{1}}}}}_{\mathbf{NI}}$	

Constraint	**Constraint Type**
⋮ $\mathbf{MWh}_{{\mathbf{cap_1}}_{t_{blk_{h_{\mathbf{SI}}}}}} + \dots + \mathbf{MWh}_{{\mathbf{cap_{numGen}}}_{t_{blk_{h_{\mathbf{SI}}}}}} + \mathbf{MWh}_{{\mathbf{cap_{tx}}}_{t_{blk_{h_{\mathbf{SI}}}}}} = \dots$ $hrs_{t_{blk_{h_{\mathbf{SI}}}}}(\mathbf{dem}_{t+1_{blk_{h_{\mathbf{SI}}}}}) + \mathbf{MWh}_{{\mathbf{cap_{tx}}}_{t_{blk_{h_{\mathbf{NI}}}}}}$	
$\mathbf{MWh}_{{\mathbf{cap_1}}_{t_{blk_{1_{\mathbf{NI}}}}}} + \dots + \mathbf{MWh}_{{\mathbf{cap_1}}_{t_{blk_{d_{\mathbf{NI}}}}}} = \mathbf{MWhTot}_{{\mathbf{cap_1}}_{t_{\mathbf{NI}}}}$ ⋮ $\mathbf{MWh}_{{\mathbf{cap_{numGen}}}_{t_{blk_{1_{\mathbf{NI}}}}}} + \dots + \mathbf{MWh}_{{\mathbf{cap_{numGen}}}_{t_{blk_{d_{\mathbf{NI}}}}}} = \mathbf{MWhTot}_{{\mathbf{cap_{numGen}}}_{t_{\mathbf{NI}}}}$ $\mathbf{MWh}_{{\mathbf{cap_{tx}}}_{t_{blk_{1_{\mathbf{NI}}}}}} + \dots + \mathbf{MWh}_{{\mathbf{cap_{tx}}}_{t_{blk_{d_{\mathbf{NI}}}}}} = \mathbf{MWhTot}_{{\mathbf{cap_{tx}}}_{t_{\mathbf{NI}}}}$	MWh Sum for Individual Capacity
$\mathbf{MWh}_{{\mathbf{cap_1}}_{t_{blk_{1_{\mathbf{SI}}}}}} + \dots + \mathbf{MWh}_{{\mathbf{cap_1}}_{t_{blk_{h_{\mathbf{SI}}}}}} = \mathbf{MWhTot}_{{\mathbf{cap_1}}_{t_{\mathbf{SI}}}}$ ⋮ $\mathbf{MWh}_{{\mathbf{cap_{numGen}}}_{t_{blk_{1_{\mathbf{SI}}}}}} + \dots + \mathbf{MWh}_{{\mathbf{cap_{numGen}}}_{t_{blk_{h_{\mathbf{SI}}}}}} = \mathbf{MWhTot}_{{\mathbf{cap_{numGen}}}_{t_{\mathbf{SI}}}}$ $\mathbf{MWh}_{{\mathbf{cap_{tx}}}_{t_{blk_{1_{\mathbf{SI}}}}}} + \dots + \mathbf{MWh}_{{\mathbf{cap_{tx}}}_{t_{blk_{h_{\mathbf{SI}}}}}} = \mathbf{MWhTot}_{{\mathbf{cap_{tx}}}_{t_{\mathbf{SI}}}}$	
${\mathbf{tranche_1}}_{t_{blk_{1_{\mathbf{NI}}}}} - \mathbf{seg}_1(\frac{\mathbf{cap_{HVDC}}_{t+1}}{\mathbf{cap}_{singlePole}}) \leq 0, \dots, {\mathbf{tranche_1}}_{t_{blk_{d_{\mathbf{NI}}}}} - \mathbf{seg}_1(\frac{\mathbf{cap_{HVDC}}_{t+1}}{\mathbf{cap}_{singlePole}}) \leq 0$ ⋮ ${\mathbf{tranche_5}}_{t_{blk_{1_{\mathbf{NI}}}}} - \mathbf{seg}_5(\frac{\mathbf{cap_{HVDC}}_{t+1}}{\mathbf{cap}_{singlePole}}) \leq 0, \dots, {\mathbf{tranche_5}}_{d_{blk_{1_{\mathbf{NI}}}}} - \mathbf{seg}_5(\frac{\mathbf{cap_{HVDC}}_{t+1}}{\mathbf{cap}_{singlePole}}) \leq 0$	Restricting Loss Tranche MW Capacity
${\mathbf{tranche_1}}_{t_{blk_{1_{\mathbf{SI}}}}} - \mathbf{seg}_1(\frac{\mathbf{cap_{HVDC}}_{t+1}}{\mathbf{cap}_{singlePole}}) \leq 0, \dots, {\mathbf{tranche_1}}_{t_{blk_{d_{\mathbf{SI}}}}} - \mathbf{seg}_1(\frac{\mathbf{cap_{HVDC}}_{t+1}}{\mathbf{cap}_{singlePole}}) \leq 0$ ⋮ ${\mathbf{tranche_5}}_{t_{blk_{1_{\mathbf{NI}}}}} - \mathbf{seg}_5(\frac{\mathbf{cap_{HVDC}}_{t+1}}{\mathbf{cap}_{singlePole}}) \leq 0, \dots, {\mathbf{tranche_5}}_{t_{blk_{d_{\mathbf{SI}}}}} - \mathbf{seg}_5(\frac{\mathbf{cap_{HVDC}}_{t+1}}{\mathbf{cap}_{singlePole}}) \leq 0$	
$\frac{\mathbf{receiveEnd}_{\mathbf{MWh}_{t_{blk_{1_{\mathbf{NI}}}}}}}{hrs_{t_{blk_{1_{\mathbf{NI}}}}}} = {\mathbf{tranche_1}}_{t_{blk_{1_{\mathbf{NI}}}}} + {\mathbf{tranche_2}}_{t_{blk_{1_{\mathbf{NI}}}}} + \dots$ ${\mathbf{tranche_3}}_{t_{blk_{1_{\mathbf{NI}}}}} + {\mathbf{tranche_4}}_{t_{blk_{1_{\mathbf{NI}}}}} + {\mathbf{tranche_5}}_{t_{blk_{1_{\mathbf{NI}}}}}$ ⋮ $\frac{\mathbf{receiveEnd}_{\mathbf{MWh}_{t_{blk_{d_{\mathbf{NI}}}}}}}{hrs_{t_{blk_{d_{\mathbf{NI}}}}}} = {\mathbf{tranche_1}}_{t_{blk_{d_{\mathbf{NI}}}}} + {\mathbf{tranche_2}}_{t_{blk_{d_{\mathbf{NI}}}}} + \dots$ ${\mathbf{tranche_3}}_{t_{blk_{d_{\mathbf{NI}}}}} + {\mathbf{tranche_4}}_{t_{blk_{d_{\mathbf{NI}}}}} + {\mathbf{tranche_5}}_{t_{blk_{d_{\mathbf{NI}}}}}$	HVDC Receiving End MW
$\frac{\mathbf{receiveEnd}_{\mathbf{MWh}_{t_{blk_{1_{\mathbf{SI}}}}}}}{hrs_{t_{blk_{1_{\mathbf{SI}}}}}} = {\mathbf{tranche_1}}_{t_{blk_{1_{\mathbf{SI}}}}} + {\mathbf{tranche_2}}_{t_{blk_{1_{\mathbf{SI}}}}} + \dots$ ${\mathbf{tranche_3}}_{t_{blk_{1_{\mathbf{SI}}}}} + {\mathbf{tranche_4}}_{t_{blk_{1_{\mathbf{SI}}}}} + {\mathbf{tranche_5}}_{t_{blk_{1_{\mathbf{SI}}}}}$ ⋮ $\frac{\mathbf{receiveEnd}_{\mathbf{MWh}_{t_{blk_{h_{\mathbf{SI}}}}}}}{hrs_{t_{blk_{h_{\mathbf{SI}}}}}} = {\mathbf{tranche_1}}_{t_{blk_{h_{\mathbf{SI}}}}} + {\mathbf{tranche_2}}_{t_{blk_{h_{\mathbf{SI}}}}} + \dots$ ${\mathbf{tranche_3}}_{t_{blk_{h_{\mathbf{SI}}}}} + {\mathbf{tranche_4}}_{t_{blk_{h_{\mathbf{SI}}}}} + {\mathbf{tranche_5}}_{t_{blk_{h_{\mathbf{SI}}}}}$	
$\mathbf{sendEnd}_{\mathbf{MWh}_{t_{blk_{1_{\mathbf{NI}}}}}} = 1.03({\mathbf{tranche_1}}_{t_{blk_{1_{\mathbf{NI}}}}}) + 1.04({\mathbf{tranche_2}}_{t_{blk_{1_{\mathbf{NI}}}}}) + \dots$ $1.07({\mathbf{tranche_3}}_{t_{blk_{1_{\mathbf{NI}}}}}) + 1.11({\mathbf{tranche_4}}_{t_{blk_{1_{\mathbf{NI}}}}}) + 1.12({\mathbf{tranche_5}}_{t_{blk_{1_{\mathbf{NI}}}}})$ ⋮ $\mathbf{sendEnd}_{\mathbf{MWh}_{t_{blk_{d_{\mathbf{NI}}}}}} = 1.03({\mathbf{tranche_1}}_{t_{blk_{d_{\mathbf{NI}}}}}) + 1.04({\mathbf{tranche_2}}_{t_{blk_{d_{\mathbf{NI}}}}}) + \dots$ $1.07({\mathbf{tranche_3}}_{t_{blk_{d_{\mathbf{NI}}}}}) + 1.11({\mathbf{tranche_4}}_{t_{blk_{d_{\mathbf{NI}}}}}) + 1.12({\mathbf{tranche_5}}_{t_{blk_{d_{\mathbf{NI}}}}})$	HVDC Sending End MW
$\mathbf{sendEnd}_{\mathbf{MWh}_{t_{blk_{1_{\mathbf{SI}}}}}} = 1.03({\mathbf{tranche_1}}_{t_{blk_{1_{\mathbf{SI}}}}}) + 1.04({\mathbf{tranche_2}}_{t_{blk_{1_{\mathbf{SI}}}}}) + \dots$ $1.07({\mathbf{tranche_3}}_{t_{blk_{1_{\mathbf{SI}}}}}) + 1.11({\mathbf{tranche_4}}_{t_{blk_{1_{\mathbf{SI}}}}}) + 1.12({\mathbf{tranche_5}}_{t_{blk_{1_{\mathbf{SI}}}}})$ ⋮ $\mathbf{sendEnd}_{\mathbf{MWh}_{t_{blk_{d_{\mathbf{SI}}}}}} = 1.03({\mathbf{tranche_1}}_{t_{blk_{h_{\mathbf{SI}}}}}) + 1.04({\mathbf{tranche_2}}_{t_{blk_{h_{\mathbf{SI}}}}}) + \dots$	

Constraint	Constraint Type
$1.07(\mathbf{tranche_3}_{t_{blk}h_{SI}}) + 1.11(\mathbf{tranche_4}_{t_{blk}h_{SI}}) + 1.12(\mathbf{tranche_5}_{t_{blk}h_{SI}})$	
$\mathbf{MW}_{risk_{t+1_{NI}}} \geq \mathbf{cap_1}_{t+1_{NI}}$ $\vdots$ $\mathbf{MW}_{risk_{t+1_{NI}}} \geq \mathbf{cap_n}_{t+1_{NI}}$	Largest Generating Unit in North Island
$\mathbf{MW}_{risk_{t+1_{SI}}} \geq \mathbf{cap_1}_{t+1_{SI}}$ $\vdots$ $\mathbf{MW}_{risk_{t+1_{SI}}} \geq \mathbf{cap_n}_{t+1_{SI}}$	Largest Generating Unit in South Island
$\mathbf{MW}_{risk_{t+1_{SI+NI}}} \geq \mathbf{MW}_{risk_{t+1_{NI}}}$ $\mathbf{MW}_{risk_{t+1_{SI+NI}}} \geq \mathbf{MW}_{risk_{t+1_{SI}}}$	Largest Generating Unit in New Zealand
$(\mathbf{dem}_{t+1_{NI}} + \mathbf{dem}_{t+1_{SI}} + \mathbf{HVDC_{maxLoss}}) - \ldots$ $(\mathbf{cap}_{t+1_{NI}} + \mathbf{cap}_{t+1_{SI}} - \mathbf{MW}_{risk_{t+1_{SI+NI}}}) \geq \mathbf{VoLL}_{t_{NI}} + \mathbf{VoLL}_{t_{SI}}$	National Penalty Constraint
$\mathbf{dem}_{t+1_{NI}} - \ldots$ $\left(\mathbf{cap}_{t+1_{NI}} - \mathbf{MW}_{risk_{t+1_{NI}}} + \frac{\mathbf{HVDC}_{t+1_{\mathbf{totCap}_{send}}}}{\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{send}}}}(\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}})\right) \leq \mathbf{VoLL}_{t_{NI}}$ $\mathbf{Voll}_{t_{NIpen}} \geq 0$	North Island Reserve Penalty Constraint
$\mathbf{dem}_{t+1_{SI}} - \ldots$ $\left(\mathbf{cap}_{t+1_{SI}} - \mathbf{MW}_{risk_{t+1_{SI}}} + \frac{\mathbf{HVDC}_{t+1_{\mathbf{totCap}_{send}}}}{\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{send}}}}(\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}})\right) \leq \mathbf{VoLL}_{t_{SI}}$ $\mathbf{Voll}_{t_{SIpen}} \geq 0$	South Island Reserve Penalty Constraint
$\mathbf{dem}_{t+1_{NI}} - \ldots$ $\left[\mathbf{cap}_{t+1_{NI}} + \left(\frac{\mathbf{HVDC}_{t+1_{\mathbf{totCap}_{send}}}}{\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{send}}}}(\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}}) - (\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}})\right)\right] \leq \mathbf{VoLL}_{t_{NI}}$ $\mathbf{Voll}_{t_{NIpen}} \geq 0$	North Island HVDC Reserve Penalty Constraint
$\mathbf{dem}_{t+1_{SI}} - \ldots$ $\left[\mathbf{cap}_{t+1_{SI}} + \left(\frac{\mathbf{HVDC}_{t+1_{\mathbf{totCap}_{send}}}}{\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{send}}}}(\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}}) - (\mathbf{HVDC}_{t+1_{\mathbf{singleCap}_{rec}}})\right)\right] \leq \mathbf{VoLL}_{t_{SI}}$ $\mathbf{Voll}_{t_{SIpen}} \geq 0$	South Island HVDC Reserve Penalty Constraint
$\bar{\boldsymbol{\alpha}}_{t+1}(\mathbf{cap_1}_{t+1_{NI}}, \ldots, \mathbf{cap_{numGen}}_{t+1_{NI}}, \mathbf{cap_{tx}}_{t+1_{NI}}, \ldots$ $\mathbf{cap_1}_{t+1_{SI}}, \ldots, \mathbf{cap_{numGen}}_{t+1_{SI}}, \mathbf{cap_{tx}}_{t+1_{SI}}, \ldots$ $\mathbf{dem}_{t+1_{NI}}, \mathbf{dem}_{t+1_{SI}})$	Future Cost of Investment and Operation
$f(y) = (\mathbf{vo}_{\mathbf{cap_{tx}}_{t_{NI}}} \times \mathbf{MWhTotal}_{\mathbf{cap_{tx}}_{t_{NI}}})x_1 + (\mathbf{vo}_{\mathbf{cap_{tx}}_{t_{SI}}} \times \mathbf{MWhTotal}_{\mathbf{cap_{tx}}_{t_{SI}}})x_2$ $\mathbf{MWhTotal}_{\mathbf{cap_{tx}}_{t_{NI}}} x_1 + \mathbf{MWhTotal}_{\mathbf{cap_{tx}}_{t_{SI}}} x_2 - y = 0, y \geq 0$ $x_1 + x_2 = 1, x_k \geq 0$	SOS Constraint
Sum of previously used investments = 0	Dynamic Investment Constraint

9.4 SDDP MODEL

The mathematical representation of the problem is only one of the inputs required by the SDDP model. Other inputs include investment options data, demand growth and LDC data, existing generation plant data, length of planning period, discount rate and finance payback period. This data must be sourced and manipulated into the correct format by the problem solver before the

SDDP algorithm can be run. The following sections describe the data inputs required by the HVDC SDDP model. The majority of the data used has been drawn from publicly available modelling data distributed by the New Zealand Electricity Commission.

9.4.1 Generation Scenarios

The Electricity Commission generates a document titled "Statement of Opportunities" (SOO) [93] that details the Commissions view of the future of demand growth and generation investment for New Zealand. As part of the development process of the SOO the Commission details a series of Grid Planning Assumptions that include key inputs, forecasts and scenarios. The HVDC investment optimisation model will utilise three of the generation scenarios from the SOO along with data used by the Commission in developing demand forecasts[4]. The generation scenarios reflect a potential view of the future for the New Zealand electricity and energy landscape. Each scenario is based on a combination of factors including political influence, available fuels, industrial growth and decline and economic investment options.

The three generation scenarios as described by the Electricity Commission are [95]:

Primary renewables: Government policies strongly discourage the development of fossil-fuel-based generation. Combined with a constrained gas supply, this leads to the development of renewable options. Geothermal, hydro and wind generation all feature strongly, with the majority of the new hydro projects being located in the South Island, but all the geothermal and most of the wind in the North Island. In the later part of the scenario, both renewable and thermal projects are added to provide peaking capacity in the North Island (including pumped and peaking hydro schemes, and gas- or oil-fired thermal units). Demand-side measures also contribute to peak management.

Mixed Technologies: Government policies provide more moderate support to renewable generation. A mixture of generation technologies is the result, with a substantial amount of new coal-fired generation (black coal in the North Island and lignite in the South), as well as moderate amounts of geothermal, wind, and hydro. Towards the end of the scenario, expensive but plentiful imported LNG is available, allowing the development of more gas generation. Demand-side measures also contribute to peak management.

High Gas Discovery: Timely and extensive exploration for gas leads to a relatively unrestricted supply of natural gas at prices similar to today's. Major new gas-fired power stations are constructed, with five new CCGTs installed by 2026 (including Huntly e3p and Otahuhu C). Government policies supporting renewable generation also encourage the development of

[4]The South Island Surplus Renewables generation scenario has been omitted from the HVDC investment case study as this scenario was developed to specifically investigate the effect of Tiwai Point Aluminium Smelter withdrawing from New Zealand. This scenario is no longer valid as the owners of the smelter have recently signed a long term electricity supply agreement with Meridian Energy Limited - confirming their business will stay in New Zealand until at least 2030 [94]

	No New Investment	**700MW Pole**
Capital Cost	\$0	$\$788,000,000 = \$1,126,000/MW$

Table 9.6 HVDC Investment Costs

geothermal resources, and, to a lesser extent, wind and hydro power. Demand-side measures contribute to peak management. This scenario would imply the lowest power prices for consumers.

The generation scenarios are used as a basis for developing sets of potential generation investment options where each set reflects the investment options most likely to be available and/or installed under each scenario. The details of investments including size, installation costs and operational costs can be found in Appendices D, E and F.

9.4.2 Existing Generation System

The existing generation system data consists of the currently installed generation plants and generation projects currently under construction. This data, obtained from the Electricity Commission [96], details the capacity of the existing generators and the fixed and variable operating costs. The existing system data set is used for each generation scenario as it represents the state of the system at the beginning of the planning period. Appendix C presents the existing generation system data in tabular form.

9.4.3 HVDC Investment Data

The data pertaining to the HVDC investment options was obtained from Transpower New Zealand Ltd [89]. As detailed in Section 9.2.1, there are two investment HVDC options, the first is to do nothing except decommission Pole 1. The second option is to install a second 700MW pole as well as decommission Pole 1. Table 9.6 details the costs associated with the two HVDC investment options. The decommissioning of Pole 1 occurs under both scenarios so the cost is excluded from the optimisation.

The HVDC has no variable operating costs but does have fixed operating costs. These costs are different depending on the installed capacity of the HVDC. Table 9.7 details the fixed operating cost data,

9.4.4 Demand Modelling

Demand modelling for the HVDC investment project comprises of two categories. The first is the modelling of peak demand growth over time for each load region. The second category is the modelling and shape of the load duration curve for both the North and South Island.

	No Investment	**700MW Investment**
HVDC Total Capacity	700MW	1400MW
Fixed Costs Per Pole	$0.8mill	$1.6mill
Fixed Costs Submarine Cables	$1.7mill	$1.7mill
Fixed Costs Substations	$1.6mill	$1.6mill
Fixed Costs HVDC Lines	$0.6mill	$0.6mill
Total Fixed Costs	$4.7mill = $6714/MW	$5.5mill = $3928/MW

Table 9.7 HVDC Fixed Operating Costs

9.4.4.1 Peak Demand Growth

The peak demand growth is modelled separately for both the North and South Island. The distribution used for the HVDC investment model is a normal distribution, given by an expected value and standard deviation. The data used to calculate the normal distribution was obtained from the Electricity Commission [97]. This data supplied 30 years of peak demand forecasts along with 30 years of prudent peak demand forecasts. The prudent peak demand forecast indicates a maximum likely value of peak demand in each year. The Electricity Commission interprets the prudent forecast using a 10% probability of exceedence (POE) criterion. The 10% POE value indicates there is a 10% chance of peak demand exceeding the prudent forecast value and can be interpreted as a 90% confidence interval around the expected peak value.

The forecasts produced by the Electricity Commission are net of embedded generation, that is, the forecasts are for expected demand minus that which is supplied by embedded generation. The author of [97] notes that the forecast is constructed as a 'business as usual' forecast that doesn't allow for changes in consumer behaviour, technology investment or increases in demand side reduction. The same assumption is used in the HVDC SDDP modelling. The peak demand forecast is assumed to be net of any embedded generation where the percentage of demand served by embedded generation is constant over time.

The peak demand at the beginning of the planning horizon for the HVDC investment problem is shown in Table 9.8. The values represent the observed half hourly peak demand in each island for the 2006-2007 year. The expected peak growth values were calculated by finding the average peak growth value of all years of the forecast. The average probability of exceedence value, over all forecast years, was used to find the 90% confidence interval and hence standard deviation of the growth distribution.

The forecast data from the Electricity Commission could be used to calculate an individual demand growth distribution for each year of the planning horizon in contrast to the averaged approach undertaken in the HVDC modelling. This would require the SDDP algorithm to sample a particular demand growth distribution at each year rather than using the same distribution in all years. This alternative approach would allow for specific changes in demand modelling, such

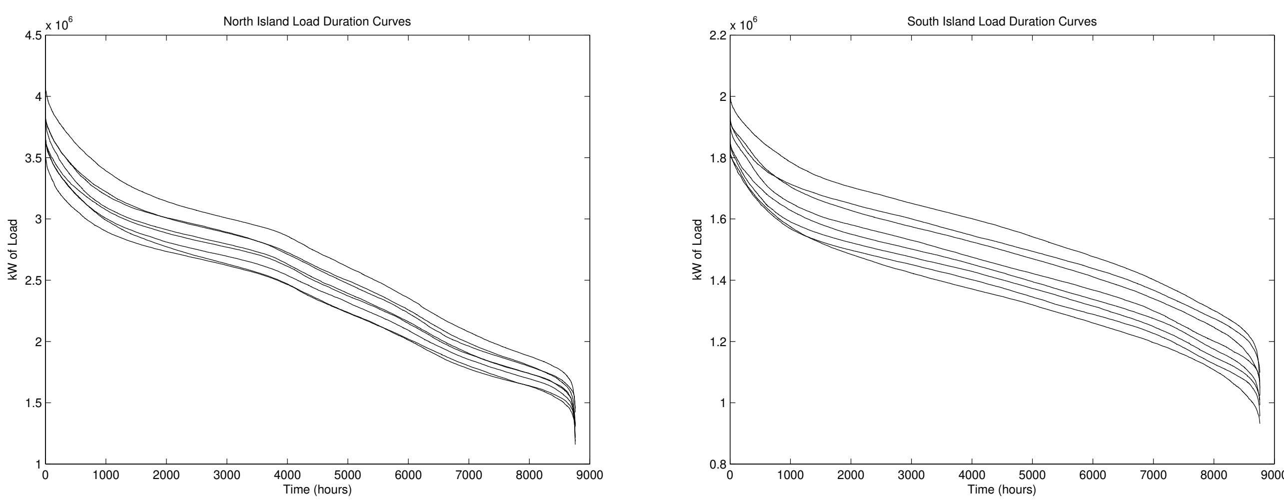
	North Island	South Island
Initial Peak Demand	4307 (MW)	2118.6 (MW)
Expected Peak Demand Growth	91 (MW)	19.3 (MW)
POE Value (refer 9.4.4.1)	127(MW)	28.7 (MW)

Table 9.8 Peak Demand Growth Distribution Parameters

as industrial changes, e.g. withdrawal of Tiwai, or rapid uptake of demand reduction technology to be modelled. Such a change to the sampling of the stochastic variable would extend the uses of SDDP but is not considered here.

9.4.4.2 Load Duration Curve

Each island has a separate load duration curve (LDC) that represents the variation in demand over a year. The shape of the LDC reflects the amount of time load in a region exceeds a specific MW value. To derive a discrete LDC for each load region the continuous LDC must first be found. The continuous LDC has two important data points, the peak demand value and the lowest demand value. The peak demand value records the highest system demand level and the lowest demand level records the lowest level of demand that is always present in the system. Between these two points the LDC can take any shape but in general represents a form of cumulative distribution function. Using half hourly demand data [98] the LDC for the North and South Islands for the years 1997-2004 are illustrated in Figure 9.2.



(a) North Island Historical Load Duration Curve (b) South Island Historical Load Duration Curve

Figure 9.2 Load Duration Curves from Historical Data (1997-2004)

For the HVDC investment problem the LDC for both the North and South Island is individual to each time period. The peak value of each time period LDC is sampled from the demand growth distribution. The lowest demand value is found by using historical demand data to calculate the average difference between peak and lowest demand values recorded for 1997-2004. This average difference is then subtracted from the sampled peak value to give the lowest demand value of the LDC. The LDC curve between the highest and lowest demand values is modelled by a normal cumulative distribution function. The normal cumulative distribution function is described by the mean value μ and the standard deviation σ. For a normal distribution the mean is found by calculating the mean value of the highest and lowest values, i.e. the peak and lowest demand values. The standard deviation is found by using the knowledge that 100% of all demand values are greater than the lowest demand value to find the standard z value for the distribution. The standard z value is then used in Equation 9.27 to find the standard deviation of the distribution.

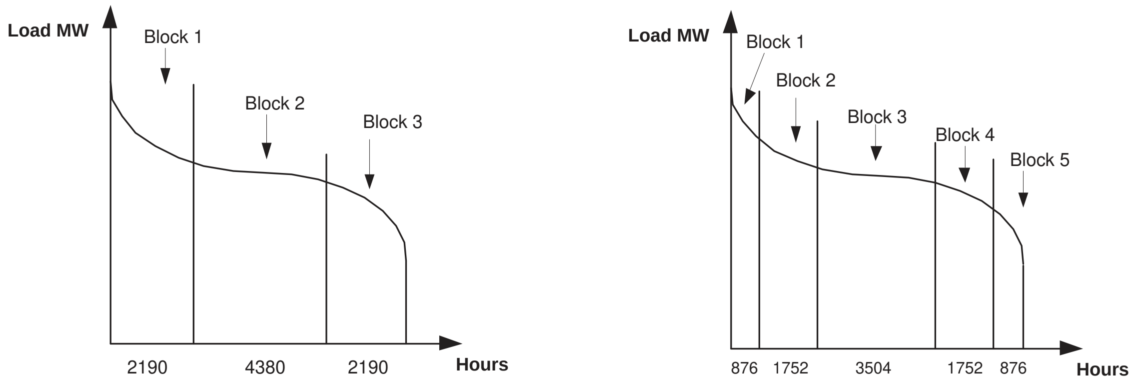
$$\sigma = \frac{X - \mu}{z} \tag{9.27}$$

Where:

- σ =Standard deviation.
- X = Lowest demand value.
- μ = Mean.
- z = Standard normal z value.

The continuous LDC function is now discretised into a desired number of blocks. Each block is described by two values, the width or number of hours of the block and the height or demand value the block. The width of the blocks is decided first. The HVDC model uses a graduated block width with the peak and lowest demand block being the thinnest and each successive block, towards the centre, doubling in width. Figure 9.3 illustrates the block widths for three and five discrete blocks. Using graduated block widths allows the extreme points of the LDC to be modelled whilst maintaining a balance between modelling accuracy and solution speed. The discretised LDC will slightly overestimate the MWh of the continuous LDC as the discrete block representing peak demand is always greater than the continuous curve. The overestimation will reduced with increased numbers of discrete blocks as each block represents fewer hours.

The height of each block represents the demand value of the block. The far left block containing the peak demand value is treated separately as it contains the peak demand value. The peak demand value is responsible for deciding on the most expensive or marginal generator in a year and hence it is important this point is included in the model to ensure accurate representation of operational costs. The demand value of the far left block is therefore modelled as the sampled peak demand value for the time period. All other blocks use the midpoint of the block width to calculate the demand value given by the continuous cumulative distribution function. The



(a) Three Discrete Blocks (b) Five Discrete Blocks

Figure 9.3 Discrete Block Widths

continuous demand value becomes the height of the block. The midpoint is used to help offset the block demand value that will sometimes over or under represent the continuous LDC. Figure 9.4 illustrates a discrete LDC with 5 blocks and the associated block heights. Larger numbers of discrete blocks give a more accurate representation of the continuous LDC but at the expense of solution speed.

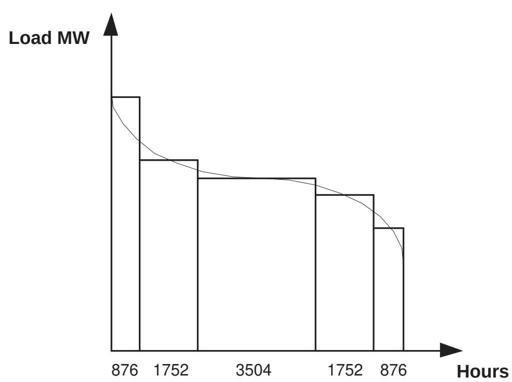
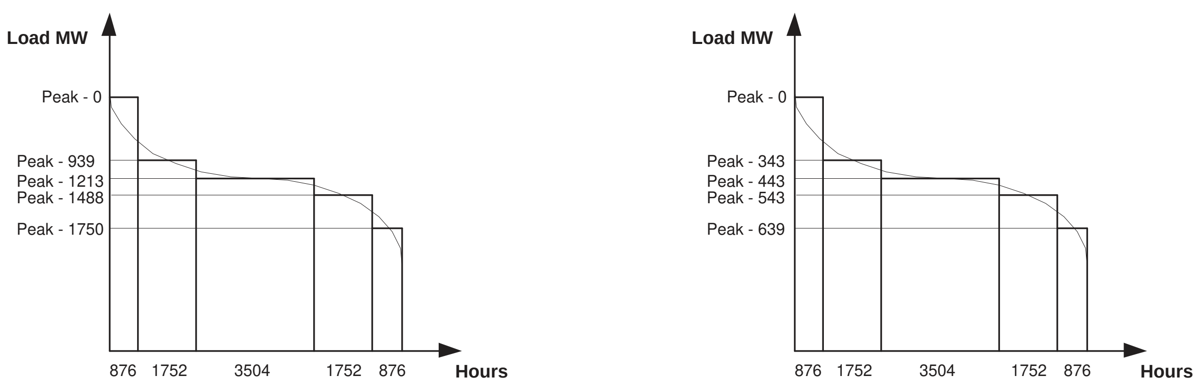


Figure 9.4 Discrete LDC with Five Blocks

The LDC curves for the North and South Island using 5 discrete blocks are described in Table 9.9. Peak demand values are sampled at every time period requiring that the discrete block heights be described in terms of subtractions from the peak demand value. The graphical representation of the LDC's for the North and South Island are illustrated in Figure 9.5.

Load Region	Time	Demand Reduction From Peak (MW)
North Island	876 hours	0
	1752hours	−939
	3504 hours	−1213
	1752hours	−1488
	876 hours	−1750
South Island	876 hours	0
	1752hours	−343
	3504 hours	−443
	1752hours	−543
	876 hours	−639

Table 9.9 Load Duration Curve Parameters



(a) North Island Discretised Load Duration Curve (b) South Island Discretised Load Duration Curve

Figure 9.5 Discretised Load Duration Curve for the HVDC Investment Model

9.4.5 Payback Period

The exact payback period for each investment is a value known only by the individual investor but it is common for investors and companies to structure their finances to achieve an optimal debt equity ratio that is constant over time [99]. This means that regardless of the specific initial financing arrangements made, the investor is likely to restructure their financing over the lifetime of the investment to maintain a constant debt equity ratio. The payback period of the investment therefore becomes the lifetime of the asset. The lifetimes of investments used in the HVDC SDDP model are detailed in Appendices D, E and F.

9.4.6 Remaining Input Data

Simulation Length: The simulation length reflects the number of years in the planning horizon under consideration. The HVDC planning problem has a planning horizon of 20 years. The simulation length used is 25 years, where the final 5 years of data are discarded. This reduces any end effects created by the assumption of zero system investment and operation costs after the planning period under consideration.

Operational Cost of VoLL: A single cost value is attributed to all energy not supplied (VoLL). The Electricity Commission and Transpower New Zealand Ltd. currently use a value of $20,000/MWh and this value has been adopted in the HVDC investment problem.

Capacity of VoLL: The HVDC investment problem models VoLL as generation, the capacity of which is given either by the amount of demand in the system or a suitably large value. The HVDC investment problem uses a value of 10,000MW for the capacity of the VoLL modelling.

Penalty Price for VoLL used for Reserves: Where the optimisation utilises VoLL to provide reserves during a contingent event, the physical system equivalent is unscheduled dropping of load. While the system has mechanisms such as Automatic Under Frequency Load Shedding (AUFLS) and interruptible load that drops load in emergency situations this is contracted for and can be costed appropriately[5]. Any load dropped over and above that specifically contracted for is a highly undesirable situation. No data currently exists to suggest what this cost should be. The HVDC model assumes this situation would be so undesirable as to put an extremely high price on unscheduled load drop during a contingent event. The fixed operating cost used in the model is $1,000,000/MW.

Discount Rate: The discount rate applied to investments reflects the value of money over time. The rate is used to find the present value of future investment and operational costs. The rate used in the HVDC model is 7%, inline with that used by Transpower [89].

Fuel Costs: The costs of fuel for gas, coal, diesel and lignite power plants are detailed in Appendix G. All other generation plant is assumed to have a $0/MWh fuel cost.

9.4.7 Lpsolve Optimisation Software

To solve the mixed integer optimisations at each time period the open source solver Lpsolve is used in conjunction with Matlab. Lpsolve [85] is a mixed integer linear programming solver that uses Branch and Bound (Refer Appendix A). The solver can handle continuous, integer and semi continuous variables. Lpsolve can be called from a number of different programming languages and software programmes of which Matlab is one. The reason for using Lpsolve was

[5]Though neither is explicitly modelled in the HVDC investment model so as to reduce complexity of the system representation.

the necessity for a simple and robust mixed integer solver that could be utilised from available software.

9.5 SUMMARY

This chapter has presented the model and associated data requirements for a case study regarding investment in the HVDC link in New Zealand. The model utilises the extended SDDP algorithm to optimise the investment and operation of the New Zealand power system. The case study specifically investigates the potential investments associated with the inter-island HVDC link that connects the North and South Island of New Zealand. The model used to represent the investment problem is largely based on the transmission investment model presented in Chapter 8 where the HVDC link is the transmission line.

The HVDC investment problem requires an additional group of extra constraints to be added to the mathematical representation to calculate the transmission losses on the HVDC. The reserve constraints were modified slightly to reflect the definition of a contingent event in New Zealand, being the loss of the largest generator or a single pole of the HVDC. An extra reserves penalty constraint is added to explicitly model the HVDC loss criteria.

The complete mathematical representation of the HVDC investment problem was presented and incorporated into the SDDP algorithm. Other input data for the algorithm was detailed including generation scenarios, demand growth distributions, LDC derivation, HVDC investment options, generation investment options, existing system data, payback period calculations, planning horizon length, operational costs of VoLL and the discount rate.

The HVDC optimisation model and extended SDDP algorithm are used to optimise the investment and operation of the New Zealand power system with specific focus on the investment options for the HVDC link. The results of this optimisation and associated discussion are presented in Chapter 10.

Chapter 10

CASE STUDY RESULTS AND MODELLING DISCUSSION

10.1 INTRODUCTION

The HVDC investment problem is to minimise the investment and operational costs of the New Zealand power system over a 20 year period. The investment of most interest is the upgrade of the HVDC link where the options are to either retain the existing link capacity at 700MW or to add an additional 700MW pole to increase the capacity to 1400MW. Additional to the HVDC investment decision the optimisation model also considers a range of alternative generation investments. These generation investment options are split into three different scenarios that reflect a potential future with regards to available generation investments, governmental policy and economic environment.

The optimisation model and associated data are detailed in Chapter 9 with the results of the optimisation presented in the following sections. Firstly the high gas scenario results are presented, followed by the mixed technologies scenario and finally the primary renewables scenario. For each scenario the convergence results, investment selections, HVDC investment information and a selection of future cost functions are presented. The chapter concludes with a discussion of results and implications for investment decisions and the investment planning process.

10.2 OPTIMISATION PARAMETERS

Each scenario is optimised over a 25 year period but the last five years of results are discarded to reduce the end effects of the optimisation. The optimisation suffers from end effects due to the assumption of zero investment or operational cost past the end of the planning horizon. This assumption may distort the optimal decision made near the end of the simulated planning horizon. By discarding the final five years of the optimisation the end effects of the simulation should be minimised.

The number of Monte Carlo simulations undertaken on the forward pass of the SDDP algorithm is set to ten. This value has been chosen as it is large enough to provide a range of samples and while being small enough to obtain solutions within reasonable time frame.

The single stage optimisations, at each state variable combination (found from the forward pass), at each time period is solved 25 times. This value originates from the each stochastic variable being sampled 5 times at each other value of state variable i.e. $5^2 = 25$.

10.3 HIGH GAS SCENARIO

The high gas scenario assumes the exploration and discovery of significant natural gas resources that are available at prices similar to today's. Governmental policies supporting renewable generation encourage the development of geothermal generation with a lesser emphasis on wind and hydro power. This scenario is expected to deliver the lowest power prices for consumers.

10.3.1 Convergence

This scenario converged with two iterations of the forward pass. The values for the integer and continuous upper bounds, the limits of the 95% confidence interval of the expected continuous upper bound and the lower bound are shown in Table 10.1. The first iteration value for the upper bound is defined before the algorithm starts. The first iteration lower bound value is 0 as no Bender's Cuts have been defined yet. The actual bound values defined by the optimisation are not of particular interest to the problem solver. The investment results are more important. The bound values therefore indicate convergence of the algorithm but nothing more.

Iteration	**1**	**2**
Integer Expected Upper Bound	$\$1 \times 10^{15}$	$\$2.003 \times 10^{10}$
Continuous Upper Bound + 95% Confidence Interval	N/A	$\$2.2224 \times 10^{10}$
Continuous Expected Upper Bound	$\$1 \times 10^{15}$	$\$1.9901 \times 10^{10}$
Continuous Upper Bound - 95% Confidence Interval	N/A	$\$1.7579 \times 10^{10}$
Lower Bound	\$0	$\$1.8876 \times 10^{10}$

Table 10.1 High Gas Scenario Convergence Values

For the algorithm to successfully converge the lower bound must fall within the 95% confidence interval of the continuous upper bound. The results of this scenario show the lower bound sitting just within the lower 95% confidence interval limit and therefore the optimisation has successfully converged. The gap between the expected integer upper bound and the lower bound can indicate the suitability of investment sizes to the optimisation problem. For the high gas scenario this gap is $\$1.1538 \times 10^9$. As a comparison, the existing Taranaki Combined Cycle (CC) generator has a capacity of 377MW, fixed operating costs of \$75,000/MW and variable operating costs of \$54/MWh. The value of the gap in terms of the Taranaki CC generator is 48,919 hours of operation or 5.6 years.

$$(377 \times \$75,000 \times 5.6\text{years}) + (377 \times \$54 \times 48,919) \simeq \$1.1538 \times 10^9$$

Considering the length of time of the planning period, 20 years, a gap that represents only 5.6 years of operation of a single generator, is not large. The convergence gap is not large but when compared to the mixed technologies and primary renewables scenarios below, it is the highest value. This indicates that this scenario has the least suitably sized investment options compared to the minimum required investment. The high gas scenario has the greatest selection of large investment options so the largest gap between bounds is expected.

10.3.2 Investment Options

Presented in Table 10.2 are the investment options selected during the final forward pass, directly prior to achieving convergence, of the extended SDDP algorithm. The values in the table represent the number of times a particular investment option is chosen in each year. As the forward pass Monte Carlo simulation runs ten times, each investment option may be selected a maximum of ten times across all the time periods of the optimisation. Investment options that are never selected have not been displayed. The years five to nine have no investments selected and are not displayed.

The results for the high gas scenario are consistent with those expected from the HVDC investment problem. The low capital cost, low operating cost gas plants of the high gas scenario are selected first followed by the higher capital cost wind and geothermal investments. The first investment made is the Otoi Waiau hydro plant. This investment has very low installation and operating costs in comparison to other hydro investments. Its low costs allow this plant to be commissioned and dispatched in preference to an existing, more costly, generator in the North Island.

The next set of investments chosen do not occur until year eleven. All investment options chosen in this time period are gas generators reflecting the scenario's cheap gas fuel costs. Where an investment is chosen in all ten Monte Carlo simulations in a single year, such as Huntly e3p, Otahuhu C and the HVDC, the investment is optimal regardless of the system state at the time of commissioning. Investments decisions are made far in advance of commissioning (i.e. generators cannot be built instantly) therefore the system state at the time of commissioning is uncertain when an investment decision is finalised. Having an investment option that is optimal under a wide range of potential system states gives planners and investors certainty regarding selection of an optimal investment. The risk of investing in one of these options and it resulting in a suboptimal power system is low. The investments of OCGT 3 and Taranaki CC 2 are selected as optimal in only one or two Monte Carlo simulations indicating that specific system states are required for these investments to be optimal choices. The risk of investing in one of these generators and the power system expanding in a suboptimal fashion, is high.

Year→ Investment Option↓	**MW**	1	2	3	4→10	11	12	13	14	15	16	17	18	19	20
North Island															
Otoi Waiau (Hydro)	16.5	3		7											
Huntly e3p (Gas)	365					10									
Generic OCGT 1 (Gas)	150					5					2				
Generic OCGT 3 (Gas)	150					2									
Generic OCGT 4 (Gas)	150					4									2
Otahuhu C (Gas)	407					10									
Taranaki CC 2 (Gas)	380					1									
Generic gas 2 Taranaki (Gas)	410					2				6	2				
Unspecified Wind (Wind)	200									2	4		4		
Mokairau (Wind)	16										7		1	2	
Tararua 3 (Wind)	93									1			4	4	1
West Wind (Wind)	210										2		2		3
Committed Geo (Geothermal)	17												3	3	
Kawerau (Geothermal)	80												2	3	3
Generic Geo 4 (Geothermal)	80														2
Transmission															
HVDC Investment	700												10		

Table 10.2 High Gas Scenario Investment Choices

The group of investments most commonly chosen after the gas options are wind power. These investments, while having higher variable operating costs than the geothermal options, have a much lower installation cost than the geothermal investments. No wind investment is selected in all ten Monte Carlo simulations in a single year but selections tend to be grouped together in blocks of time. For example, the unspecified wind investment is selected between years fifteen and eighteen. After year eighteen the investment is optimal in all Monte Carlo simulations giving an investor greater certainty in regarding this investment as optimal. The investor or planner may choose to wait until year eighteen to invest to ensure optimality of the investment or they may invest earlier and accept the risk of the investment resulting in a suboptimal system. Wind power is assumed to be dispatchable and no variability in output is considered. The effect on the optimal solution is to invest in less capacity than the system really requires. This issue is discussed further in Section 10.6.2.2.

The geothermal investment options have high installation and fixed operational costs in comparison to both wind and gas investment types and as a result are selected as investments of last choice. None of the geothermal investments are selected in every Monte Carlo simulation but the Kawerau investment is the most likely to be optimal by year twenty. Geothermal investments are often subject to Resource Management Act [100] restrictions on steam usage. This can reduce the capacity available from investments therefore the optimal solution will invest in less capacity than is actually required.

The HVDC investment is selected in year eighteen in all Monte Carlo simulations. This investment is likely to be optimal under the most probable system states in year eighteen and is discussed further in Section 10.3.3

10.3.3 HVDC Investment

The HVDC investment is selected in year eighteen of the planning horizon. Figure 10.1 illustrates the MWh of energy transferred across the HVDC link. Transfer occurs only in the South to North direction which reflects the low cost of hydro generation in the South Island. The link is fully utilised at its lower capacity level of 700MW. The maximum transfer level illustrated in Figure 10.1 is the receiving end capacity of the link. This accounts for losses on the HVDC. After investment in year eighteen the level of transfer does not reach the maximum transfer capability of the HVDC and drops off slightly in subsequent years.

The HVDC investment is not chosen until year eighteen due to the high installation costs associated with the large 700MW investment. The alternative investment options for the North Island such as gas and wind provide a lower cost solution than the HVDC investment. After the cheap North Island investment options are used the investment cost of the HVDC is comparable to the remaining geothermal investments and is therefore chosen as optimal by the algorithm.

The high utilisation of the HVDC in years one to seventeen illustrates the low cost of South Island hydro generation in comparison to North Island existing generation and potential investments. The utilisation of the HVDC after investment does not reach full capacity due to the level of capacity available in the South Island. Table 10.3 details the expected levels of South Island capacity, South Island demand and HVDC capacity in years eighteen to twenty. The surplus South Island capacity available to generate power to send north is less than the capacity of the

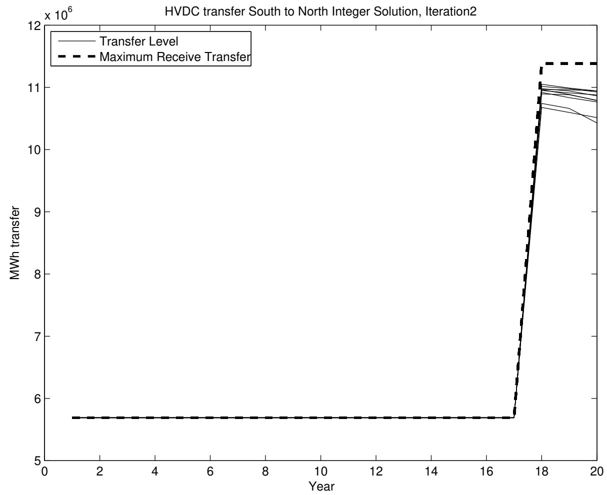


Figure 10.1 HVDC Link Transfer South to North, Final SDDP Iteration

HVDC. As South Island demand grows the capacity available in the South Island to provide power for the North Island is reduced. To enable the link to be fully utilised at the higher capacity further South Island generation would need to be built. The hydro investments in the South Island are limited in size and have high investment costs. The optimal solution therefore is to choose larger sized and consequently lower cost North Island investments in conjunction with the under-utilised HVDC link.

	Year 18	**Year 19**	**Year 20**
South Island Capacity (MW)	3536	3536	3536
South Island Demand (MW)	2461.7	2478.6	2498.8
SI Capacity - SI Demand (MW)	1074.3	1057.4	1037.2
HVDC Capacity (MW)	1400	1400	1400

Table 10.3 South Island Capacity and Demand, Years Eighteen to Twenty - High Gas Scenario

10.3.4 Future Cost Function Example

The future cost functions created by the extended SDDP algorithm are multidimensional and impossible to view without isolating particular variables of interest. Figures 10.2 and 10.3 illustrate the future cost of investment and operation from the beginning of year two until the end of the planning horizon. The demand value range on each plot illustrates those values most likely to occur at the beginning of year two. The actual value of optimal solution cost is less important than the overall trend in costs illustrated by the function. The approximate future cost function lies on the surface of the linear constraints and is an upward trend.

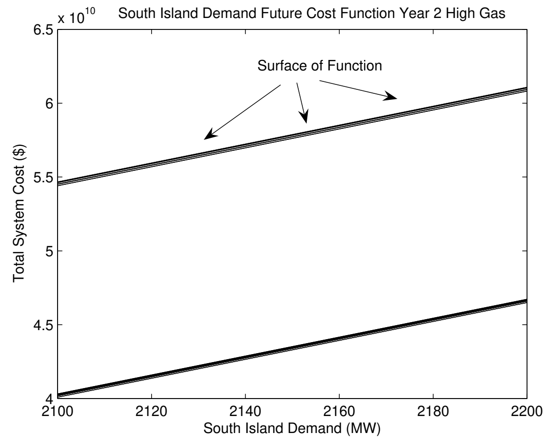


Figure 10.2 High Gas Future Cost Function for Year 2 - South Island Demand

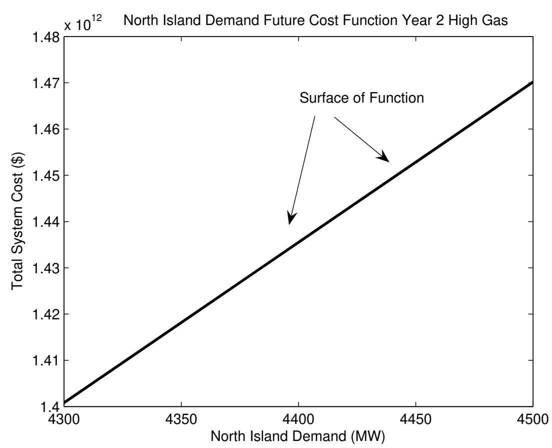
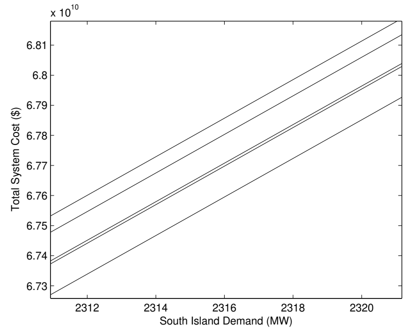


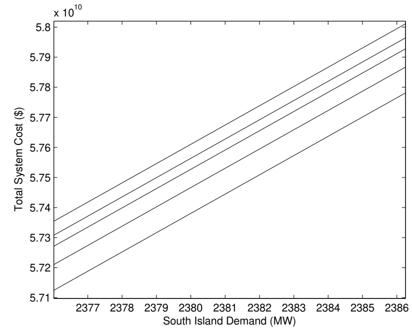
Figure 10.3 High Gas Future Cost Function for Year 2 - North Island Demand

The future cost functions in Figures 10.2 and 10.3 consist of ten linear constraints each. Each constraint is linearised around a demand value after demand growth in year one on the backward pass. The constraints are shown in expanded form in Figure 10.4. Figure 10.4(a) shows the set of constraints that define the surface of the function for the South Island demand future cost function, Figure 10.4(b) shows the lower set of constraints of the South Island demand future cost function and Figure 10.4(c) shows the expanded constraints of the North Island demand future cost function.

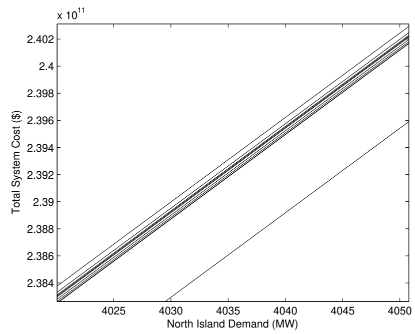
Figure 10.5 shows the future cost function for year nineteen until the end of the planning horizon for a range of HVDC capacity values. The constraints are linearised around 1400MW, the capacity of the HVDC after investment in year 18. Figure 10.5 is the future cost function used



(a) Expanded South Island Demand - Upper Constraints



(b) Expanded South Island Demand - Lower Constraints



(c) Expanded North Island Demand Constraints

Figure 10.4 Expanded Future Cost Functions for Demand - High Gas

in the single stage optimisation at year eighteen, the time period where the HVDC investment is selected in the original HVDC problem. The function illustrates that the future cost of system investment and operation from year nineteen onwards drops as the capacity of the HVDC increases. When the optimisation chooses the HVDC in year eighteen as an optimal investment, this problem is optimising the cost of investment and operation in year eighteen plus the value of the future cost function illustrated in Figure 10.5. If the additional costs involved in installing extra HVDC capacity in year eighteen are not offset by the savings gained from the future cost function, the HVDC investment would not be optimal.

The validity of this future cost function could be questioned as it is late in the planning horizon. It is possible that the optimisation may still experience end effects at this late time period. As discussed in Section 10.6.1, the future costs of the system do not significantly influence the choice of optimal solution in the immediate time period due to the operational costs being significantly higher than investment costs. In this situation the end effects of the optimisation due to the assumption of zero costs past the end of the optimisation will be minimal and the future cost function of Figure 10.5 can be considered valid.

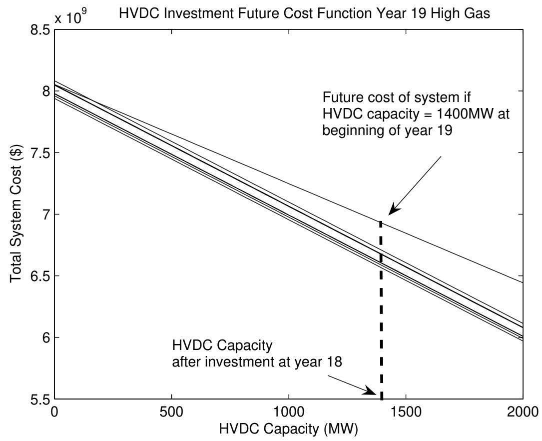


Figure 10.5 High Gas Future Cost Function for Year 19 - HVDC Capacity

10.4 MIXED TECHNOLOGIES SCENARIO

The mixed technologies scenario assumes moderate governmental support for renewable generation resulting in a mix of generation technologies being available. More coal fired generation is available along with medium levels of geothermal, wind and hydro.

10.4.1 Convergence

This scenario converged with two iterations of the forward pass. This integer and continuous upper bounds, the 95% confidence interval limits of the continuous upper bound and the lower bound are shown in Table 10.4.

Iteration	**1**	**2**
Integer Expected Upper Bound	$\$1 \times 10^{15}$	$\$2.0808 \times 10^{10}$
Continuous Upper Bound + 95% Confidence Interval	N/A	$\$2.3221 \times 10^{10}$
Continuous Expected Upper Bound	$\$1 \times 10^{15}$	$\$2.0707 \times 10^{10}$
Continuous Upper Bound - 95% Confidence Interval	N/A	$\$1.8192 \times 10^{10}$
Lower Bound	\$0	$\$2.0572 \times 10^{10}$

Table 10.4 Mixed Technologies Scenario Convergence Values

Convergence is reached as the lower bound sits within the 95% confidence interval of the continuous upper bound. The gap between the integer upper bound and the lower bound is 2.3651×10^8. For comparison the value of the convergence gap in terms of operating costs of the Taranaki CC generator is 10,027 hours or 1.15 years of operation.

$$(377 \times \$75,000 \times 1.15\text{years}) + (377 \times \$54 \times 10,027) \simeq \$2.3651 \times 10^8$$

The value of the convergence gap represents a very short time period of operation of a single generator. This indicates that the gap value is very small compared to the overall contribution of costs from individual generators. This gap is much smaller than that seen in the high gas scenario illustrating that investment option sizes in the mixed technologies scenario are more suited to the system demand growth characteristics.

10.4.2 Investment Options

Table 10.5 presents the investment options investment options selected during the final forward pass, directly prior to achieving convergence, of the extended SDDP algorithm. Each value in the table represents the number of times the particular investment option is chosen in each year. Investments that are never selected are not displayed, similarly for years one to eleven when no investment occurs.

The extended SDDP algorithm gives good results for the mixed technologies scenario. They are logical and expected when considering the types, sizes and costs of available investment options. Even with higher gas prices in the mixed technologies scenario the gas investment options are the optimal investment choices in years twelve and thirteen. The investment costs for the gas generators are smaller than any other investment type and while the variable operational costs are high, the fixed operational costs are low. The high variable operating costs of gas investments puts the generator at the top of the offer stack where it may not be fully dispatched initially. The MWh of generation from these generators is low resulting in a low overall cost. The majority of the gas investment options selected are optimal for a wide range of system states by year thirteen. The risk of the system expanding sub-optimally is low if these investment options are chosen in years twelve or thirteen.

The Mangawhero to Wanganui Hydro Diversion is optimal between years twelve and sixteen. This investment has a higher installation cost than the wind investment options but a lower variable operational cost. The low operational costs make this a preferential investment over the wind investments.

The wind investments are chosen toward the end of the planning horizon in years seventeen to twenty. Similarly to the high gas scenario the investor or planner can chose to wait until later in the planning horizon before choosing these investments to lower the risk of a sub optimal system or they may take the risk of sub-optimality and invest earlier.

The two geothermal investments are chosen in year twenty but only a limited range of system states. These investments carry a high risk of expanding the system sub-optimally if they are

Year→ **Investment Option↓**	**MW**	12	13	14	15	16	17	18	19	20
North Island										
Huntly e3p (Gas)	365	1	9							
Otahuhu C (Gas)	407	10								
Generic OCGT 1 (Gas)	150	5	5							
Generic OCGT 2 (Gas)	150		8							
Generic OCGT 3 (Gas)	150		10							
Generic OCGT 4 (Gas)	150	4	6							
Mangawhero to Wanganui Diversions (Hydro)	60	2	4			4				
Turitea (Wind)	150						9			
Titiokura (Wind)	48						4	4	2	
Te Waka (Wind)	111						2	4	3	
Puketiro (Wind)	120						8		2	1
Hawkes Bay Wind Farm (Wind)	225						3		2	1
Tararua 3 (Wind)	93							3	4	
Committed Geo (Geothermal)	17									3
Kawerau (Geothermal)	80									2
Transmission										
HVDC Investment	700								10	

Table 10.5 Mixed Technologies Scenario Investment Choices

chosen for commissioning in year twenty. They may become a less risky option in future time periods that could be studied with a longer planning horizon.

The HVDC is chosen as an optimal investment in all Monte Carlo simulations in year nineteen. At this time point all cheap gas and wind investment options have been chosen previously so the large and consequently expensive HVDC investment is now an optimal investment choice. The HVDC link is chosen as optimal a year later in this scenario in comparison to the high gas scenario. This is a result of the greater number of cheap wind investment options available.

10.4.3 HVDC Investment

The HVDC investment is selected in all Monte Carlo simulations at year nineteen in the mixed technologies scenario. Figure 10.6 illustrates the MWh of energy transferred across the HVDC link. Similarly to the High Gas scenario, transfer only occurs in a South to North Direction reflecting the surplus of low cost hydro generation in the South Island. The initial 700MW capacity of the link is fully utilised in years one to eighteen with investment occurring at year nineteen.

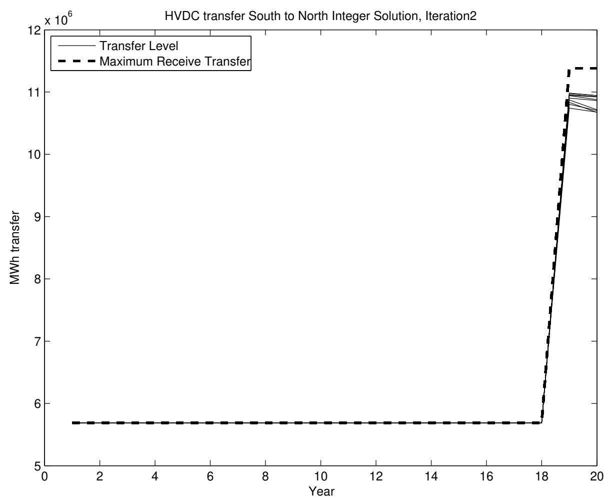


Figure 10.6 HVDC Link Transfer South to North, Final SDDP Iteration

Similarly to the high gas scenario the HVDC capacity is not fully utilised after the investment is chosen as the surplus capacity available for generating power to send north is less than the capacity of the link. Table 10.6 details the expected levels of South Island capacity and demand and the HVDC capacity. The drop in surplus generation given by the difference between capacity and demand is the reason for the drop in power transferred between years nineteen and twenty. Additional South Island generation is required to fully utilise the capacity of the HVDC but the investment options available are either very small (less than 20MW) or very large (greater than 200MW). The alternative North Island geothermal investments, while having a greater costs per MW are overall a lower cost option as the investment sizes are smaller (less than 100MW).

10.4.4 Future Cost Function Example

Figures 10.7 and 10.8 illustrate the future cost of investment and operation from the beginning of year two until the end of the planning horizon, for a probable range of demand values. The approximated future cost function lies on the surface of the constraints and illustrates an upward trend in cost.

	Year 19	Year 20
South Island Capacity (MW)	3536	3536
South Island Demand (MW)	2471.5	2490.3
SI Capacity - SI Demand (MW)	1064.5	1045.7
HVDC Capacity (MW)	1400	1400

Table 10.6 South Island Capacity and Demand, Years Nineteen and Twenty - Mixed Technologies Scenario

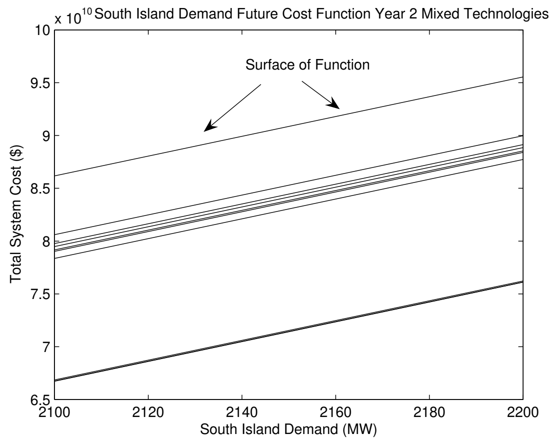


Figure 10.7 Mixed Technologies Future Cost Function for Year 2 - South Island Demand

The future cost functions in Figures 10.7 and 10.8 consist of ten linear constraints each. Each constraint is linearised, using dual variables, around the stochastic end of year one demand value on the backward pass. These constraints are shown in expanded form in Figure 10.9. Figure 10.9(a) shows expanded constraints of the South Island demand future cost function and Figure 10.9(b) shows the expanded constraints of the North Island demand future cost function.

Figure 10.10 shows the future cost function for year twenty for a range of HVDC capacity values. The constraints are linearised around 1400MW, the new HVDC capacity value. Figure 10.10 is the future cost function used in the HVDC optimisation problem at year nineteen, where the HVDC investment is chosen. The function trends downwards as HVDC capacity increases but the future cost value is limited by the HVDC capacity after investment, 1400MW.

Similarly to the HVDC future cost function Figure 10.5 from the high gas scenario, the validity of the HVDC future cost function shown in Figure 10.10 could be questioned due to being year twenty. While it is possible the function underestimates the future costs of the system due to the end of simulation zero future cost assumption, the removal of the final five years of simulation helps to minimise this effect.

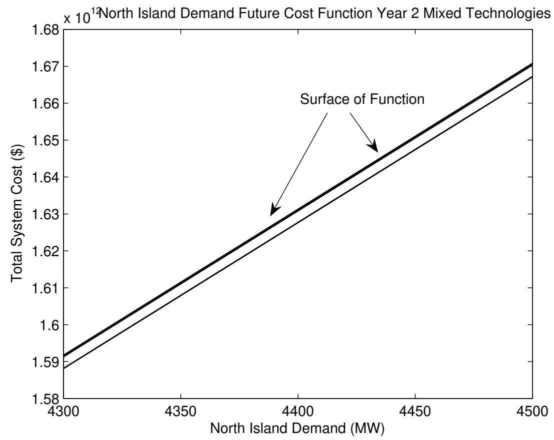
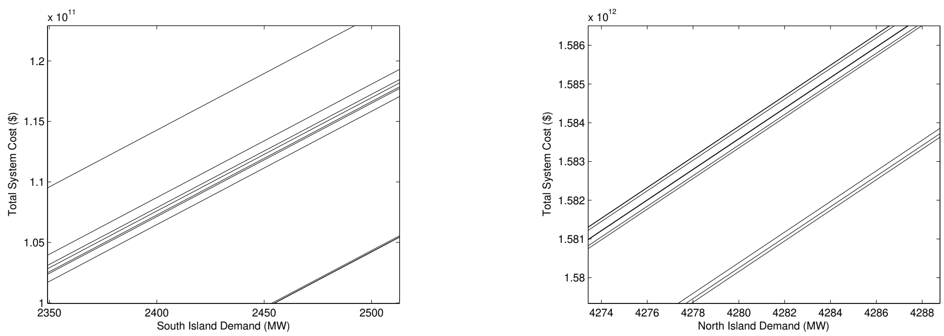


Figure 10.8 Mixed Technologies Future Cost Function for Year 2 - North Island Demand



(a) Expanded South Island Demand Constraints

(b) Expanded North Island Demand Constraints

Figure 10.9 Expanded Future Cost Functions for Demand - Mixed Technologies

10.5 PRIMARY RENEWABLES SCENARIO

The primary renewables scenario assumes strong governmental policy discouraging the development of fossil fueled generation. Combined with limited natural gas supplies the resulting generation is mostly renewables. Hydro, wind and geothermal all feature strongly with all geothermal and most wind located in the North Island. All hydro is located in the South Island.

10.5.1 Convergence

This scenario converged with two iterations of the forward pass. The integer and continuous upper bounds, the 95% confidence interval of the continuous upper bound and the lower bound are shown in Table 10.7.

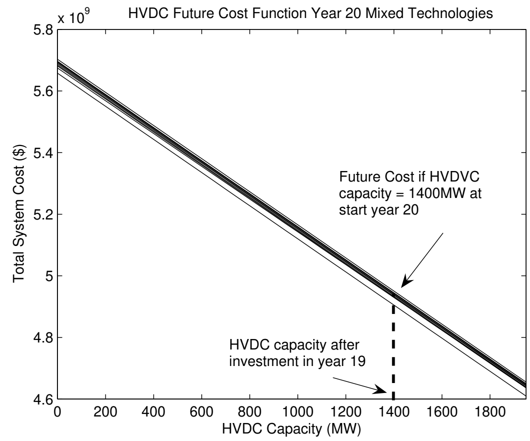


Figure 10.10 Mixed Technologies Future Cost Function for Year 20 - HVDC Capacity

Iteration	1	2
Integer Expected Upper Bound	1×10^{15}	2.0565×10^{10}
Continuous Upper Bound + 95% Confidence Interval	N/A	2.2411×10^{10}
Continuous Expected Upper Bound	1×10^{15}	2.0486×10^{10}
Continuous Upper Bound - 95% Confidence Interval	N/A	1.8561×10^{10}
Lower Bound	0	1.9811×10^{10}

Table 10.7 Primary Renewables Scenario Convergence Values

The lower bound sits within the 95% confidence interval of the continuous upper bound indicating convergence is reached. The gap between the integer upper bound and the lower bound is 6.7555×10^8 or 3.3% of the integer upper bound. Using the operating costs from the Taranaki CC generator for comparison, the value of the convergence gap represents 28,642 hours or 3.3 years of operation.

$$(377 \times \$75,000 \times 3.3\text{years}) + (377 \times \$54 \times 28,642) \simeq \$6.7555 \times 10^8$$

The value of the convergence gap represents a short period of operation of a single generator relative to the length of the planning period. The small gap shows that the investments options available match the minimum system investment requirements well.

10.5.2 Investment Options

Table 10.8 presents the investments chosen in the Primary Renewables scenario. All investments not selected and years with no investment have been removed.

Year→ Investment Option↓	**MW**	5	6	→	11	12	13	14	15	16	17	18	19	20
North Island														
Otoi Waiau (Hydro)	16.5	4	6											
Waimarino (Hydro)	5.9				1	5					1			1
Tarawera at Lake Outlet (Hydro)	14				1		5		3				1	
Whakapapanui Papamanuka (Hydro)	16				1				4		1		3	1
Marokopa Falls (Hydro)	6.7					9	1							
Waikato upgrade (Hydro)	200					10								
Generic pumped hydro (Hydro)	300					10								
Mohaka (Hydro)	44						10							
Mokairau (Wind)	16						4	5			1			
Red Hill (Wind)	20							4	1		3		1	1
Huntly e3p (Gas)	365								6					
Tarawera at Falls (Hydro)	7								3					2
Mangawhero to Wanganui Diversions (Hydro)	60								4		6			
Long Gully (Wind)	70								4			1	1	3
Rangitaiki at Kiorenui (Hydro)	8.5										3		1	3
Unspecified wind (Wind)	200										7	3		
Te Rere Hau (Wind)	48.5										1			7
Titiokura (Wind)	48										1			9
Motorimu (Wind)	80										1	3	1	5
Puketiro (Wind)	120										1	5	1	3
Wharauroa Plateau (Wind)	72											1	2	7
Belmont Hills (Wind)	80											3	1	5

Table 10.8 Primary Renewables Scenario Investment Choices

The investment pattern for the primary renewables scenario show a much larger selection of investments are chosen over the planning period. This is consistent with the small size of investment options available. The investment selections are not as clear cut in this scenario in comparison to the high gas and mixed technologies scenarios. The first investment chosen is the Otoi Waiau hydro investment in years five and six. No further investments are chosen until year eleven.

The Waikato upgrade, Mohaka and Generic pumped hydro investments are optimal investments, in all Monte Carlo simulations in year twelve or thirteen. These investments are optimal for a wide range of system states and hence the risk of these investments resulting in a suboptimal system is low. Hydro and wind investments feature in the majority of investment choices with no particular investment type favoured over another. Hydro investments have low operating costs but high installation costs whereas wind investments have lower installation costs and higher variable operating costs. The majority of investments are selected as optimal within a small block of adjoining time periods. The three exceptions to this are the Waimarino, Tarawera at lake outlet and Whakapapanui Papamanuka hydro investments. These three investments are selected as optimal in a number of different years between year eleven and twenty. These investments carry a far greater risk of resulting in a suboptimal system when the system state, at the time of investment commissioning, is uncertain at the time the investment decision is made. The only gas investment chosen is Huntly e3p in year fifteen. It is an optimal investment choice in six of ten Monte Carlo simulations so carries some risk of resulting in a suboptimal system. The investment options of Rotokawa Expansion, Waitangi Falls, Whangaehu, Te Waka, Tararua 3, Turitea and West Wind are selected for the first time in year 20. These have been omitted from Table 10.8 due to their late selection having little bearing on the overall investment timing.

The HVDC investment is not selected at any time point in this scenario. This is due to the large numbers of low cost investment options, both hydro and wind, available in the North Island. The large size and associated expensive cost of the HVDC investments makes it an unattractive investment when a large number of cheaper alternatives are available.

10.5.3 HVDC Investment

The HVDC investment is not selected in this scenario. Similarly to previous scenarios transfer only occurs in a South to North Direction reflecting the surplus of low cost hydro generation in the South Island. The link is fully utilised but does not increase in capacity.

This scenario does not chose investment in the HVDC due to high availability of low cost North Island investment options. The number of small scale, low cost, investment options in the North Island are more attractive than the large HVDC capacity upgrade and associated investment costs.

10.5.4 Future Cost Function Example

Figures 10.11 and 10.12 illustrate the future cost of investment and operation from the beginning of year two until the end of the planning horizon. The demand values illustrated represent a range of the most likely stochastic demand value at the end of year one. The constraints of each function are found by linearising the optimal solution cost, using dual variables, around a stochastic demand value from year one of the backward pass. The approximated future cost function lies on the upper surface of the constraints and illustrates an upward trend in cost.

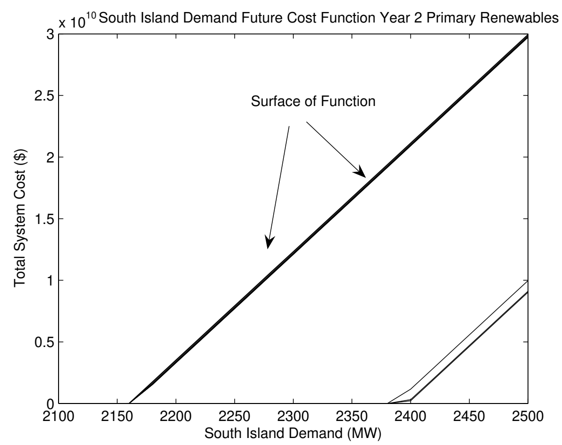


Figure 10.11 Primary Renewables Future Cost Function for Year 2 - South Island Demand

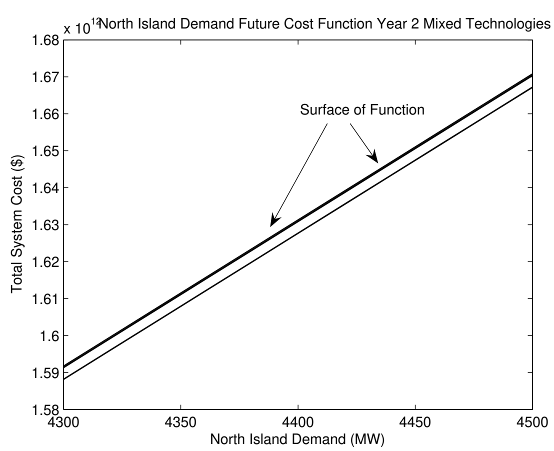


Figure 10.12 Primary Renewables Future Cost Function for Year 2 - North Island Demand

The future cost functions in Figures 10.11 and 10.12 consist of ten linear constraints each. The constraints for the North Island demand function are expanded and shown in Figure 10.13

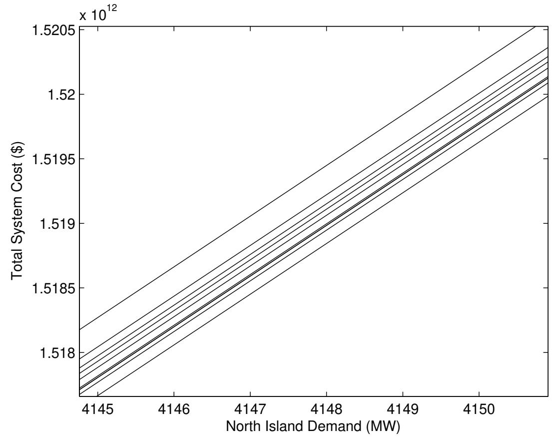


Figure 10.13 North Island Demand Expanded Constraints

Figure 10.14 shows the future cost function for year twenty for a range of Huntly e3p capacity values. This future cost function is used in the single stage optimisation at year fifteen, where the Huntly e3p investment is chosen in six of the Monte Carlo simulations. The function shows that a lower value for the future cost of system could be obtained if the capacity of Hnutly e3p was reduced to approximately 300MW. The integer nature of the investment size prevents this from occurring but the future costs are still lower with 365MW of Huntlye3p than 0MW.

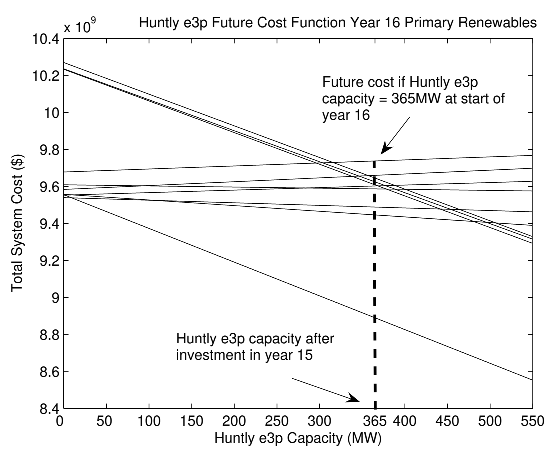


Figure 10.14 Primary Renewables Future Cost Function for Year 16 - Huntly e3p Capacity

10.6 DISCUSSION

10.6.1 Number of Iterations

All three scenario optimisations converged very quickly, within two iterations of the forward pass. The small number of iterations of the algorithm suggests that the choice of optimal investment in the current time period does little to affect the total solution cost.

The total optimal solution cost comprises of two variables, investment costs and operational costs. Operational costs contribute the largest portion of total system costs as detailed in Table 10.9 and are dictated by the level of demand in the system. New generation or transmission investments are able to alter operational costs in two ways; the first is by removing existing expensive generators from the top of the offer stack, the second is by adding themselves to the top of the offer stack. In either situation the new investment capacity will only be a small percentage of the total system capacity therefore its effect on the total operational costs is small. As the majority of total system costs are operational costs and the influence of investment choice over operational costs is small, the total solution cost changes little with investment choice.

This does not mean the choice of investment is irrelevant as there is always an optimal investment choice. It means that the sensitivity of the solution to similarly costed investments is low.

		Expected Total ($)	% of Total Costs
High Gas	Investment Costs	5434549891	17
	Operational Costs	25974206488	83
Mixed Technologies	Investment Costs	5162827139	15
	Operational Costs	28540805282	85
Primary Renewables	Investment Costs	6354221867	17
	Operational Costs	29885357229	83

Table 10.9 Expected Investment and Operational Costs

10.6.2 Modelling Assumptions and Implications

Some of the modelling decisions, simplifications and assumptions made in constructing the mathematical representation of the HVDC model and data inputs to the extended SDDP algorithm do not represent the real world situation accurately and affect the validity of the results obtained in the SDDP optimisations. These issues are discussed in the following sections.

10.6.2.1 Hydro Inflow Variability

The HVDC model assumes all hydro plant can operate at maximum capacity at all times. This is vastly different to reality where the stochasticity of hydro inflows and operating restrictions result in variable hydro output. The hydro scheduling optimisation problem is itself a stochastic dynamic planning problem with a large state space. SDDP was originally applied to this problem as discussed in Section 4.1.1.

Consideration of hydro inflow variability could be achieved by restricting the energy generated by hydro stations to the average expected energy production per year. This data represents the variation of inflows and related energy generated over a period of time. The average expected energy generation data may be available on a per station basis or for a combination of stations in a river chain. Using an average value does not assess extreme cases of variability as may be seen in dry years but does acknowledge that hydro stations cannot continually operate at full capacity.

To implement a hydro generation restriction in the investment model another group of constraints would be required. For hydro generators that have individual expected energy data, a constraint can be written that would require the summation of MWh from each LDC block in a year to be less than or equal to the expected energy value. For example, for hydro generator 1 a LDC with 5 time blocks would result in a constraint of:

$$\mathbf{MWh}_{\mathbf{hydro1}_{blk_1}} + \mathbf{MWh}_{\mathbf{hydro1}_{blk_2}} + \ldots + \mathbf{MWh}_{\mathbf{hydro1}_{blk_5}} \leq \mathbf{AvgGen}_{MWh}$$

For groups of generators connected along a river, the average expected energy generated may be given as a total for the entire river chain. In this situation the new constraint would require the total generation from all stations in the chain to be restricted to the average energy generated value. The MWh from each involved station, in each LDC block is included in the constraint. For example where hydro stations 3, 4 and 5 are connected in a river chain:

$$\begin{aligned}
&\mathbf{MWh}_{\mathbf{hydro3}_{blk_1}} + \ldots + \mathbf{MWh}_{\mathbf{hydro3}_{blk_5}} + \ldots \\
&\qquad \mathbf{MWh}_{\mathbf{hydro4}_{blk_1}} + \ldots + \mathbf{MWh}_{\mathbf{hydro4}_{blk_5}} + \ldots \\
&\qquad\qquad \mathbf{MWh}_{\mathbf{hydro5}_{blk_1}} + \ldots + \mathbf{MWh}_{\mathbf{hydro5}_{blk_5}} \leq \mathbf{AvgGenChain}_{MWh}
\end{aligned}$$

The new constraints do not supersede the constraints limiting individual generators to only generate within their plant capacity. They also do not change the way in which the optimisation will dispatch the cheapest generator first. This is particularly pertinent to river chains where the optimisation does not consider if a particular station operation and dispatch is connected to dispatch of other generators within the chain. More detailed modelling may be able to accommodate such operational requirements. The suggested constraints for implementing restrictions on hydro generators based on inflows and available storage do not attempt to fully model the variability of hydro generation in real time but provide a method to consider the availability of water and storage on energy generated over a year.

If energy restriction constraints were introduced to the investment model the solution of the

HVDC model would likely alter. Due to the restriction on total energy generated by hydro stations the optimisation will utilise the cheap hydro MWh at peak load times. In the current model hydro stations are used as base load generators due to their low operating costs. Where the energy available from hydro is restricted the optimisation will choose to use more expensive generation during low load periods to 'save' the cheap hydro generation for high load times. The outcome is that hydro generators are used for peaking and some base load, a more realistic modelling situation as hydro generators can quickly ramp up production during peak times.

Including energy restrictions and the associated shift of hydro towards peaking provision would impact on the solution cost and investment timing of the HVDC investment problem. Restricting energy generation ultimately reduces the generation capacity of the country, particularly in the South Island, and this may require more generation investment at earlier time periods to meet load therefore increasing the optimal solution cost. The shift of hydro generation toward peak load times may make the HVDC upgrade investment optimal at earlier time periods. With an increased HVDC link capacity the South Island is able to supply peak North Island loads therefore negating the need to invest in North Island generation plant. An earlier HVDC investment will only occur when the benefits gained from using South Island hydro to supply peak North Island load, outweigh the HVDC investment costs.

Restricting the energy generated from hydro plants is effectively a capacity restriction meaning that the investment cost of a hydro plant per MW of *available* capacity increases. This may make smaller gas or diesel investments more attractive as the investment costs are low and the total capacity of the plant is (usually) available. This is no different to the current situation in the HVDC model but modelling hydro variability will make large hydro investments even less attractive due to effectively increasing investment costs relative to other investment options.

10.6.2.2 Variable Wind Power Output

All wind generation both new and existing is assumed to be dispatchable, that is, the full capacity of the installation is available on demand. Real world systems do not operate in this fashion as wind generation output is completely dependent on wind speed at any particular time. All three HVDC generation scenarios select wind generation investment options under the assumption that the full capacity of the investment is available at any time. The current optimisation therefore underestimates the actual investment and optimal solution costs of the system.

The majority of wind investment selections occurred in the North Island. If the variability of wind generation was considered in the model the generating capacity of the North Island would be reduced as full wind capacity is not available at all times. The reduction in capacity would require additional generation investments to be made at earlier time periods. As the cheapest generation options are utilised earlier than previously, the more expensive HVDC investment option may become optimal at an earlier time period as it is compared with the remaining more expensive generation investments.

Accounting for wind variability could be achieved similarly to hydro variability, by introducing constraints that limit the total energy generated by wind generating plant. The constraints would operate in a similar fashion, by restricting the summation of MWh generated in each LDC block to be less than or equal to an average expected energy generated value. Wind generators situated in the same locale may be treated similarly to river chains where groups of generators are restricted by the average expected energy generation of the group. An issue with using these constraints is that the optimisation will 'save' the cheap wind generation that is available for peak load times (as for hydro variability) but wind generation cannot be considered completely dispatchable as it is not always available. The restrictions on energy generated are designed to consider that wind generation is not always available but wind cannot be stored like water and therefore may not be available in peak load times. This restricts the effectiveness of simple constraints in modelling wind variability.

Wind does not have guaranteed availability and therefore cannot be considered to be fully dispatchable. Including wind generators in the offer stack is misleading as the energy may not be available when required. An interesting modelling idea to overcome this issue is to consider hydro and wind as a combined generation type. When wind generation is available hydro generation is likely to be restricted, allowing for additional hydro generation when wind is not available. This modelling approach is equivalent to considering wind generation not as dispatchable capacity but as providing additional generation from hydro. This will result in smoothing the variability of the two generation types. Where wind and hydro are modelled separately the system must supply extra capacity to cover the generation variability. A combined modelling approach reduces variability therefore reducing the amount of additional capacity required to cover generation variability and giving a lower cost optimal solution. Extending the HVDC investment model to consider a combined wind and hydro generation type is likely to make the optimisation problem non linear as hydro generation output becomes dependent on wind availability and amount of installed wind capacity.

10.6.2.3 Transmission Constraints

The model does not consider transmission constraints within the two defined load regions and allows generation investments to be developed anywhere within the load region. This modelling decision disregards the effects transmission constraints may have on the ability to transport the generated power to the load. The effect on the optimal solution is to underestimate its cost as the cheapest investments may not be a feasible investment solution. If transmission constraints were considered within the model, cheap generation investments that are constrained by transmission may become expensive investment options when transmission upgrades are required as part of the generation investment.

10.6.2.4 Discount Rates

The discount rate used to calculate the present value of loan repayments in the HVDC scenario's is 7%. This is default value set by the Electricity Commission for use in the Grid Investment Test [101].

Applying the same discount rate to all investments could be considered inappropriate as different investors will have differing financial arrangements. Where investors face higher investment risks their cost of financing may be higher resulting in a higher discount rate. The effect of changing the discount rate is to change the present value of the loan repayment annuities where this present value represents the cost of investment. Where varying discount rates are used between private and public investments or technology types, optimal investment choices may differ from those obtained in the HVDC scenarios. Riskier investment types such as peaking plant investment or newer technology types may have higher discount rates making them more expensive. This allows capital intensive investments such as hydro and the HVDC link to compete will the riskier investment options and may be the investment of preference.

A discount rate of 7% is low for commercial ventures, higher discount rates effectively increase the investment cost. Higher investment costs will alter the investment to operational cost ratio discussed in Section 10.6.1. This will increase the optimal solution cost and may result in the optimisation requiring more iterations to reach convergence.

10.6.2.5 Generator Outages and Value of Lost Load

The HVDC model considers generator outages by calculating the cost of reserve capacity for outages but the model does not consider the operating costs incurred during outages. This is because actual generation outages are random occurrences and generally short in duration. Inclusion would dramatically increase the complexity of the model.

The costs of reserve capacity are given by the additional fixed costs of surplus capacity plus the penalty costs where VoLL is used for reserve capacity. None of the scenarios utilised the VoLL generators indicating that the system had sufficient capacity and reserve margin to supply demand at all times, even during outage situations. The high penalty cost for using VoLL as reserve capacity motivates the optimisation to invest in additional generation capacity to provide reserve cover. This extra generation capacity is always available even when the largest generation risk to the system is still connected . The extra capacity in the system results in the VoLL generator never being dispatched under normal operating conditions.

Situations when the VoLL generator may be dispatched are not considered by the model as they represent two or more large generators tripping. The very small risk of this type of situation occurring must be weighed against the cost of additional capacity investment that may never be required to generate. In this environment the optimal solution may be to dispatch the VoLL generator as the penalty costs of doing so are less than the long term fixed costs of under-utilised additional generation capacity.

The HVDC investment problem models the costs of providing reserve capacity well with the optimisation results showing that no VoLL capacity was required, an expected outcome under normal operating conditions. The model does not consider the operational costs of dispatching VoLL to supply energy during a generator outage. The model will therefore underestimate the operational costs of the model. The underestimation in costs is likely to be very small due to the short time period and random nature of most generator outages compared with the long term nature of the planning horizon.

10.6.2.6 Load Duration Curve

The load duration curve distribution is assumed to be static between time periods within a single generation scenario. While the expected value of demand changes, the probability distribution around this value doesn't. Load growth comes from different areas where residential growth will create a peakier load profile and industrial growth may add to base load. The model does not consider where load growth comes from, only that the peak increases. This may not reflect reality where much demand growth may come from the industrial sector that may increase base load demand rather than peak demand. The peaky LDC that results from not considering the types of load growth affects the optimisation results by investing in peaking plant that may not be optimal in reality. If the demand peak is high, the marginal generator may be utilised for a short period of time, allowing low investment cost and high operational cost plant to be chosen over high investment cost, low operational cost plant. The optimal investment choices and resulting optimal solution cost will therefore overestimate the system cost.

To overcome this problem more detailed modelling of demand within the system is necessary. This would require splitting demand into groups defined as residential, industrial, commercial, rural etc.. This is also required for modelling of demand side investments and is discussed further in Section 11.2.3.

10.6.2.7 HVDC Loss Modelling

As discussed in Section 9.3.5.1 the HVDC loss model used by the optimisation is acknowledged to be inaccurate as the correct loss model representation requires the use of a non linear constraint. The inaccuracies of the loss model only affect a bipole configuration, before the HVDC investment is chosen the losses on the link are modelled correctly.

The model assumes that losses on a bipole link are twice the losses of a monopole configuration as this allows a linear constraint to be constructed. Accurate modelling of losses would show that incremental % losses for a bipole are half those of a monopole. The effect of the incorrect loss model on the optimisation is to reduce the number of received MW of the link, resulting in less usable energy for the same number of sending end MWh. The investment and operating costs of a bipole link are therefore greater per MW of received energy than would be experienced with correct loss modelling. As the received energy by the North Island is reduced but the operating and investment costs are the same, the HVDC investment becomes a less attractive investment option. The HVDC upgrade to a bipole configuration will be delayed within the planning period due to the increased losses and hence increased costs, allowing additional North Island generation to be constructed. Correct modelling of losses could bring forward the selection of the HVDC upgrade as an optimal investment choice.

One method of improving the loss modelling would be to consider the HVDC upgrade as a separate lossless transmission line. Each half of the bipole arrangement would be considered as an individual transmission line between the two islands but only one line would model losses. The power to be transferred across the link would be split evenly at the sending end of each line. This allows one line to calculate all the losses for the combined bipole arrangement and the other to be lossless. The receiving end power of the link would become the combined receiving end power of the two individual transmission lines. This modelling approach is only valid for a bipole upgrade option for the HVDC link. Other operating configurations would need to be considered on an individual basis. The ability of the extended SDDP model to consider multiple transmission lines between regions is discussed further in Section 11.2.2.

10.6.3 Results Validity and Conclusions

The discussion points raised in Section 10.6.2 question the validity of the HVDC optimisation results. The main issues affecting validity are the lack of consideration of hydro stochasticity, wind variability and transmission constraints. In all cases the effect of not modelling these factors results in the HVDC optimisation underestimating the amount of investment required and consequently underestimating the optimal solution cost.

Where the problem requires greater investment than is determined by the extended SDDP algorithm the timing of investment selection from the optimisation results will alter. Investments will be required earlier in the planning horizon resulting in a greater investment cost contribution to the total optimal solution cost. A higher proportion of investment costs may result in the extended SDDP algorithm requiring more iterations to reach reach convergence. Where significant amounts of new investment are required, a larger pool of available investment options may be necessary. If a limited number of investments are available the optimisation may be forced to select investments that differ greatly from the minimum requirement, given by the continuous optimisation problems. This may result a large gap between the lower bound and mixed integer upper bound and potentially could lead to non convergence of the algorithm. A possible solution to this situation would be to shorten the planning horizon so long as this didn't negate the value of the investment plan.

The model considers all investments to have the same discount rate, an unlikely situation in reality. Investments with more uncertainty in profits are riskier investment prospects and financing such investments is likely to be more expensive. The HVDC model compares investments under the assumption they all have the same risk profile. This effectively results in investment risk being ignored by the optimisation. Where differing discount rates are used by the optimisation risker investments will have greater capital costs potentially delaying their selection as optimal investments. Other investments that are selected later in the planning horizon such as geothermal or the HVDC may then be selected earlier.

The discussion points outlined above suggest the optimal investment results of the extended SDDP algorithm are underestimated but overall the optimisation results do show that the extended SDDP algorithm can be successfully applied to a large mixed integer stochastic dynamic planning problem. The optimal investments illustrated in each generation scenario are sensible with regard to the modelling decisions made and the mathematical representation of the HVDC problem. The investment selection tables give a broad understanding of the types of generation that are optimal either early or late within the planning horizon. The selection of the HVDC investment in two generation scenarios are logical with respect to the numbers, sizes and costs of alternative generation investment options.

10.7 SUMMARY

The extended SDDP algorithm has been used to optimise the HVDC investment problem for three different generation investment scenarios. All three scenarios achieved convergence within two iterations of the algorithm.

The results of the high gas scenario optimisation are characterised by initial selection of gas investment options due to their low investment costs. This was followed by wind and geothermal. The HVDC investment option was selected in year eighteen in all Monte Carlo simulations. The HVDC investment was not selected until late in the planning horizon due to its large capacity and associated high investment costs. Once the HVDC was selected the higher link capacity was not fully utilised due to lack of surplus generation in the South Island.

The mixed technologies scenario results are similar to the high gas scenario with gas investments chosen first followed by wind then geothermal. The HVDC investment is also chosen in this scenario but later than in the high gas scenario. This is due to the increased number of wind investment options available in this scenario. These are selected in advance of the HVDC investment due to their low investment and operation costs.

The primary renewables scenario has large numbers of hydro and wind investment options. These investments are lower cost options than the HVDC and consequently the HVDC investment is never chosen.

The output of the extended SDDP algorithm, and in particular the Monte Carlo simulation, allows investors and planners to identify the investment risk with respect to suboptimal development of the system. Investment decisions are made in advance of actual plant commissioning therefore there is uncertainty regarding the system state at the time when the investment is commissioned. Using the Monte Carlo simulation output can help identify the time periods and range of system states when investments become optimal.

Many of the modelling decisions, assumptions and simplifications have resulted in the optimal solution underestimating the number of investments required and therefore the optimal solution cost is also underestimated. The main causes of this are the lack of consideration of stochasticity in wind and hydro generation and omission of transmission constraints within the load regions. Despite these modelling issues the results of the HVDC optimisation are valid and give a broad indication of the sequence and sizes of optimal investments.

Chapter 11

CONCLUSIONS AND FUTURE WORK

11.1 CONCLUSIONS

Power system investment planning has undergone significant changes as markets have been restructured and become deregulated. The new planning environment no longer guarantees investment returns, information sharing or cooperation between market participants. These changes have introduced greater uncertainty into the planning process. Detailed and widely varying research has been undertaken to investigate new planning techniques in deregulated systems, though the majority has focused on a single issues such as gaming or income uncertainty in generation investment, and competition issues associated with transmission investment. There is great need to understand how uncertainty influences both generation and transmission investments over the long term. The research presented in this thesis was undertaken with the aim of developing a new optimisation tool that aids in understanding of uncertainty with respect to large integer investment in restructured power systems.

The dynamic nature of power system investment planning in deregulated markets led to the investigation of dynamic programming as a potential optimisation technique. Dynamic programming has been used in power system planning previously but suffers from the 'Curse of Dimensionality' for real world problems. Large, multi-dimensional problems quickly become computationally intractable making dynamic programming in its traditional form unsuitable for solving the investment planning problem. Dynamic programming approximations retain the benefits of dynamic programming while improving computation times. One dynamic programming approximation technique that has been successfully applied in the power systems area is Stochastic Dual Dynamic Programming or SDDP. This approximation has been widely and successfully applied to the hydrothermal scheduling problem.

The power system investment planning problem is to optimise the immediate cost of system investment and operation plus the future costs of system investment and operation. The optimisation problem is therefore a mixed integer optimisation due to the large integer nature of investment options. This differs from the hydrothermal scheduling problem where the decision variables are continuous variables resulting in a continuous optimisation problem. The differences between the two type of optimisation problem led to the development of extensions the the SDDP algorithm to accommodate mixed integer problems.

The first extension to the SDDP algorithm overcomes the inability to use dual variables from mixed integer problems to define Benders Cuts. The extension to SDDP requires mixed integer optimisations to be solved on the forward pass and relaxed continuous optimisations on the backward pass. Relaxing the integrality requirements for the backward pass optimisations allows dual variables to be found and linear constraints to represent the future cost functions. The linear piecewise representation of the future cost functions now represents the relaxed optimisation problem possibly resulting in the upper and lower bounds never being equal.

The second extension to SDDP alters the convergence criteria by introducing a second continuous relaxed optimisation problem at each time period on the forward pass. This new optimisation problem uses the same future cost function approximation and state variable values as the *mixed integer* forward pass optimisations. The additional continuous forward pass optimisations are equivalent to the continuous backward pass optimisations. When the lower bound from the backward pass falls within the confidence interval of the expected *continuous* upper bound, the future cost function approximations no longer improve in accuracy and the algorithm exits.

The final extension to SDDP is the introduction of a dynamic constraint that tracks the previous investments selected. This constraint reflects that a generation or transmission investment is only available once and cannot be reselected. The effect of introducing a constraint such as this it to reduce the state space over time.

Extension of the SDDP algorithm to allow mixed integer stochastic dynamic multistage decision process to be solved is the first research contribution made by this thesis.

Modelling of the power system investment problem using the extended SDDP algorithm requires a mathematical representation of the problem be defined. To define the mathematical representation a number of modelling decisions must be made. The most important of these are the representation of generators, transmission lines and system load. Generators, both new and existing, are represented as individual state variables as they have individual variable and operational costs. Transmission lines are represented as two generators, one at either end of the line. The transmission generators have mutually exclusive operation where if a region is exporting power across the line, the transmission generator is constrained to zero generation. If a region is importing power across the line, the transmission generator is dispatched according to the offer stack. System load is represented as a discretised load duration curve. The variable operational costs of the system are calculated by constructing an offer stack of generation based on the demand of the load duration curve. The number of MWh of each dispatched generator is used to calculate variable operational costs.

The extended SDDP model was applied to a case study investigating upgrading of the interisland HVDC link in New Zealand. The model considered whether to upgrade the existing link or remain with the current 700MW configuration. The model of the investment planning problem treats the HVDC link as a transmission line and the North and South Islands as the load regions at each end of the line. Successful optimisation of the HVDC investment problem was undertaken using the extended SDDP algorithm. The results of the optimisation show that some investments are likely to be optimal under widely varying system states. These investments have a lower risk of developing a suboptimal system. Other investments are selected only under specific system configurations resulting in a high risk of the system developing sub-optimally. The optimisation results, while not providing a definitive investment path, indicate the time frame when particular generators may become optimal investments and consequently how these affect the decision (or not) to upgrade the HVDC link. While a number of modelling decisions and simplifications have resulted in the HVDC model underestimating the amount of investment required, the extended SDDP algorithm has provided timely results for the model presented. Issues such as variability in wind and hydro inflows, transmission constraints and varying discount rates may potentially be considered in a more detailed model representation and optimised using the same extended SDDP formulation.

The detailed modelling undertaken and resulting solution via the extended SDDP algorithm illustrates the second significant contribution of this research, the development of a tool for dynamically co-optimising generation and transmission investments in deregulated power systems. The mathematical representation of the problem combined with the extended SDDP algorithm presented can be used as a tool to enhance and inform the investment planning process.

The The new SDDP formulation and power system representation achieves the overarching goal of this research to develop a planning tool that can study the effects of uncertainty within the planning process while remaining adaptable to uncertain system states over the planning horizon.

11.2 FUTURE WORK

A number of areas have been identified to progress this research further and allow for more detailed and complex power systems to be modelled. Further investigation of the future cost function approximation and it's influence on local and global optimality of the solution would be beneficial. Other areas for development could include wind and hydro variability, transmission constraints, demand side investments and improvements in loss modelling. Necessary developments before further work can be undertaken include using an optimisation focused software platform and introducing more advanced Branch and Bound solution techniques.

11.2.1 Solution Optimality

The simple example presented in Chapter 6 showed that the extended SDDP algorithm gives reasonable 'near optimal' solutions to the investment problem. The study and assessment of local optimality and the quality of the solution obtained from the extended SDDP algorithm requires further development.

A known issue in mixed integer power system optimisation is that changes in optimal investment choice and timing often result in only very small changes in optimal solution cost. This makes local optimality of solutions difficult to identify. Further work is required to extend this assessment of local optimality and the associated accuracy of the future cost function approximation to large real world problems. Identification of the structure and size of problems that can be successfully solved to local optimality via the extended SDDP algorithm would be a useful first step.

11.2.2 Transmission Constraints and Multiple Load Areas

Development of the model to represent multiple load areas and multiple connecting transmission lines is a potential development of the extended SDDP model. A multi load area model would better represent transmission constraints within a large load region and improve the modelling of locational aspects of investment options. The resulting multi area model would be more complex and detailed than the existing model offering more detailed and realistic solutions to the optimisation problem. The tradeoff is the increased complexity of the mathematical representation and the computational effort required to obtain the optimal solution.

11.2.3 Demand Side Investments

An extension to the power system investment planning model is to introduce demand side investments into the optimisation model. Demand side investments are likely to increase in number in the future as the technology becomes more accessible and consumers become aware of rising electricity prices and environmental concerns. Demand side investments will alter the shape of the load duration curve in different ways depending on the technology type of the investment. Some investments may reduce peak loads, others may only be available during specific parts of the year due to climatic conditions and some investments may simply replace old outdated equipment with newer more energy efficient technology. Identifying how the load duration curve will change based on the demand side investment requires detailed knowledge of the technology type and the likely level of investment capacity.

One main difference between demand side and supply side investments is the small size of each demand side investment. Including numerous individual demand side investments in an extended SDDP model is unrealistic. Investment types could be aggregated to give a total capacity value for the respective investment type. Due to aggregation of the demand side investments the SDDP model could treat any such investments as continuous investment variables that have an upper limit set by the aggregated value.

An extended SDDP model that includes demand side investments would potentially be of most use those who wish to test out particular policy options with regard to increasing demand side investment. For example, building regulations may change to require all new homeowners to install solar hot water heating and the expected uptake of solar demand side investment could be included as an investment option within the model. Optimisation of the model would then show how this new demand side investment may affect the size and timing of generation and transmission investments. Inclusion of demand side investments will work best where the investment can be specified explicitly in terms of size, technology type and resulting effect on the load duration curve.

11.2.4 Modelling Language

The extended SDDP model presented in this thesis was developed in Matlab in conjunction with the mixed integer solver Lpsolve. These software and optimisation tools allowed for successful initial developmental work on the extension of SDDP but the matrix structure of Matlab makes development and alterations to the mathematical representation very difficult and time consuming. Using software such as GAMS (General Algebraic Modelling System) [102] or Xpress-MP [103] that is specifically designed for solving large mixed integer stochastic problems would make model development and solution more manageable. A detailed investigation of suitable tools has not been undertaken at this point, further research may provide other alternatives.

11.2.5 Modelling Variability of Hydro Generation

Implementing constraints that restrict energy generation from hydro generators as discussed in Section 10.6.2.1 would create a more detailed and accurate investment model. The restriction of energy generation is a simple method of acknowledging the variability of hydro inflows where that variability prevents generators from generating at full capacity all the time. The theoretical development of constraints is not complex but the construction of these constraints in the current Matlab optimisation model would be very time consuming. The additional modelling detail of energy restriction and inflow variability for hydro generators is an interesting area for further development should the model be redeveloped in a more suitable modelling language and software package.

11.2.6 Mixed Integer Solution Techniques

Using Branch and Bound as a solution technique for the extended SDDP model has not been completely straight forward. The primary renewables generation scenario in the HVDC investment case study had sufficient investment variables that Branch and Bound occasionally struggled to find an optimal solution in a timely manner. The Branch and Bound technique has been improved upon in recent times with the introduction of cutting plane techniques. Bixby et al. [104] [105] discuss how numerous techniques that appear in the literature have not, until recently, been implemented in commercial solvers. Solvers available today are beginning to incorporate these improvements. Bixby et al. extensively tested a range of solvers and found the introduction of cutting plane techniques gave significant improvements in solution speed over traditional Branch and Bound. Of the cutting plane techniques tested the greatest improvements were seen from Gomory mixed integer cuts [106].

While no testing has been undertaken it is probable that using cutting plane techniques such as Gomory mixed integer cuts would improve solution speeds for the extended SDDP optimisation model.

11.2.7 Improving the HVDC Loss Model

The main factor preventing correct modelling of HVDC bipole losses is the way in which the model treats the HVDC investment option. The model currently only allows for one transmission line to connect two load regions therefore the capacity of the existing line is simply increased by the size of the HVDC investment. This modelling decision requires that the loss characteristics (i.e. incremental losses) of a single pole configuration also apply to a bipole configuration. If the model was changed so that the HVDC investment was considered to be a second transmission line between the North and South Islands, the bipole configuration could have different loss characteristics on each line allowing the correct loss model to be modelled.

Appendix A

BRANCH AND BOUND OPTIMISATION

A.1 BRANCH AND BOUND

Integer and mixed integer programming problems require special algorithms to solve them, with experience showing Branch and Bound to be the most successful. The Branch and Bound algorithm involves three steps [107]:

1. Relax the integer programming problem by removing the integer constraints on all variables and replacing any binary variables with the continuous constraint $0 \leq y \leq 1$. The resulting relaxed problem is an ordinary linear program.
2. Solve the relaxed linear program and obtain the continuous optimum solution.
3. Using the continuous optimum as a starting point, iteratively add constraints to the relaxed problem so that eventually an optimum is obtained that satisfies the integer constraints.

The Branch and Bound algorithm iteratively solves first the relaxed problem as described in point 1 above followed by more and more constrained problems that eliminate sections of the solution space that lead to non-integer optimal solutions. The algorithm itself is best described by way of example [107].

The original problem is:

$$
\begin{aligned}
\text{Minimise } z = 3x_1 + 2x_2 \\
\text{Subject to:} \\
2x_1 + x_2 \geq 8 \\
6x_1 + 12x_2 \geq 72 \\
x_1, x_2 \geq 0, \in \mathbb{Z}
\end{aligned}
\tag{A.1}
$$

The graphical representation of this problem is shown in Figure A.1. The dots represent the possible feasible integer solutions in the solution space (The solution space extends towards infinity though this is not shown in the diagram). The optimal solution point is shown as the point where the two constraints cross over. The optimum integer solution to the problem will be one of the dots shown, the Branch and Bound algorithm will identify this point.

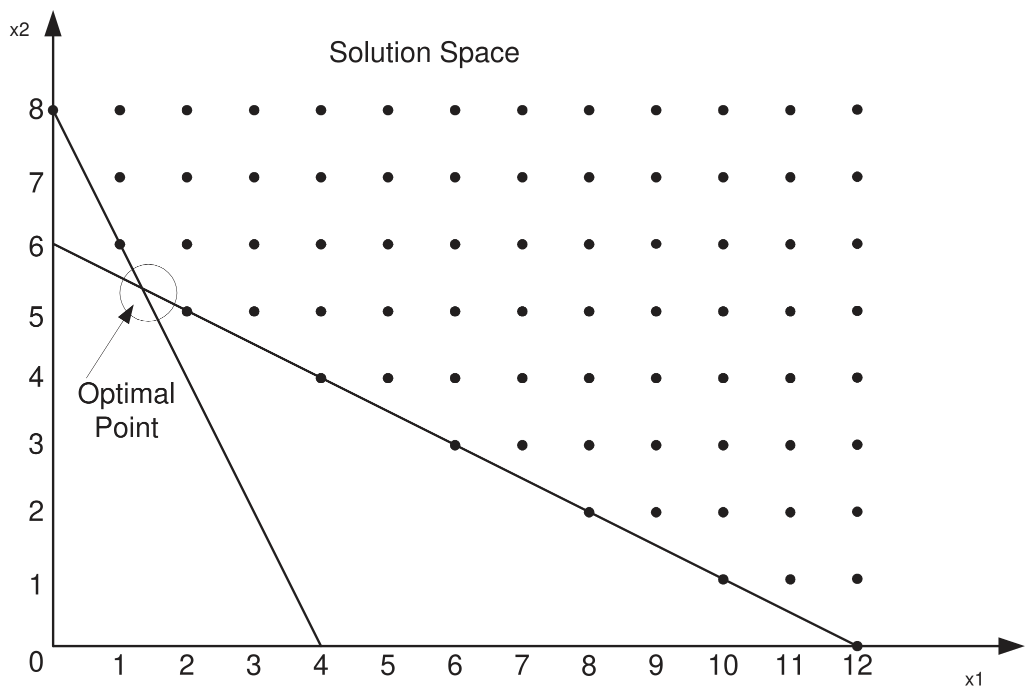


Figure A.1 Feasible Region for Equation A.1

To solve Equation A.1, the first step is to relax all integer requirements of the problem and solve relaxed optimisation of Equation A.2 to obtain the result of $x_1 = 1.3333$, $x_2 = 5.3333$ and $z = 14.6667$.

$$\begin{aligned} \text{Minimise } z &= 3x_1 + 2x_2 \\ \text{Subject to:} \\ 2x_1 + x_2 &\geq 8 \\ 6x_1 + 12x_2 &\geq 72 \\ x_1, x_2 &\geq 0 \end{aligned} \tag{A.2}$$

As the optimal solution to Equation A.2 doesn't satisfy the integrality requirements of the original problem the Branch and Bound algorithm will modify the solution space so the optimal integer solution will eventually be found. In order to do this one of the integer variables in the problem whose value in Equation A.2 is not integer is selected. Selecting x_2 arbitrarily, the region $5 < x_2 < 6$ contains no integer values of x_2 and can be excluded from the feasible solution space. Removing this section of the solution space is equivalent to creating two new optimisations, Equations A.3 and A.4

$$\begin{aligned} \text{Minimise } z &= 3x_1 + 2x_2 \\ \text{Subject to:} \\ 2x_1 + x_2 &\geq 8 \\ 6x_1 + 12x_2 &\geq 72 \\ x_2 &\leq 5 \\ x_1, x_2 &\geq 0 \end{aligned} \tag{A.3}$$

$$\begin{aligned} \text{Minimise } z &= 3x_1 + 2x_2 \\ \text{Subject to:} \\ 2x_1 + x_2 &\geq 8 \\ 6x_1 + 12x_2 &\geq 72 \\ x_2 &\geq 6 \\ x_1, x_2 &\geq 0 \end{aligned} \tag{A.4}$$

Graphical this is represented in Figure A.2

The two new feasible solution spaces still contain all the feasible integer solution points as for Equation A.1 so combined, the new solution spaces can be considered equivalent to Equation A.1. The new restrictions of $x_2 \leq 5$ and $x_2 \geq 6$ are mutually exclusive and cannot be solved together. This leads to the idea of *branching*, where each new problem is solved separately. In this example x_2 is the *branching variable.*

It is not known whether the optimal solution lies in the feasible region for Equation A.3 or A.4 so both must be solved. The decision as to which to solve first is arbitrary but can have a large impact on the speed of the solution (refer Section A.1.1). Choosing Equation A.3 as the first subproblem to examine, the solution to this equation is:$x_1 = 2$, $x_2 = 5$ and $z = 16$. This solution obviously fulfills the integer requirements of Equation A.1 as x_1 and x_2 both have integer solutions. This branch of x_2 is said to be *fathomed* and doesn't need to be investigated any further as there is no better integer solution. There is no way of knowing, at this point, if this solution is the optimal solution of Equation A.1 as Equation A.4 is still to be examined. What is known though is that the optimal solution to Equation A.3 of $z = 16$ is an upper bound on the optimal solution of the original problem. Now that an upper bound has been

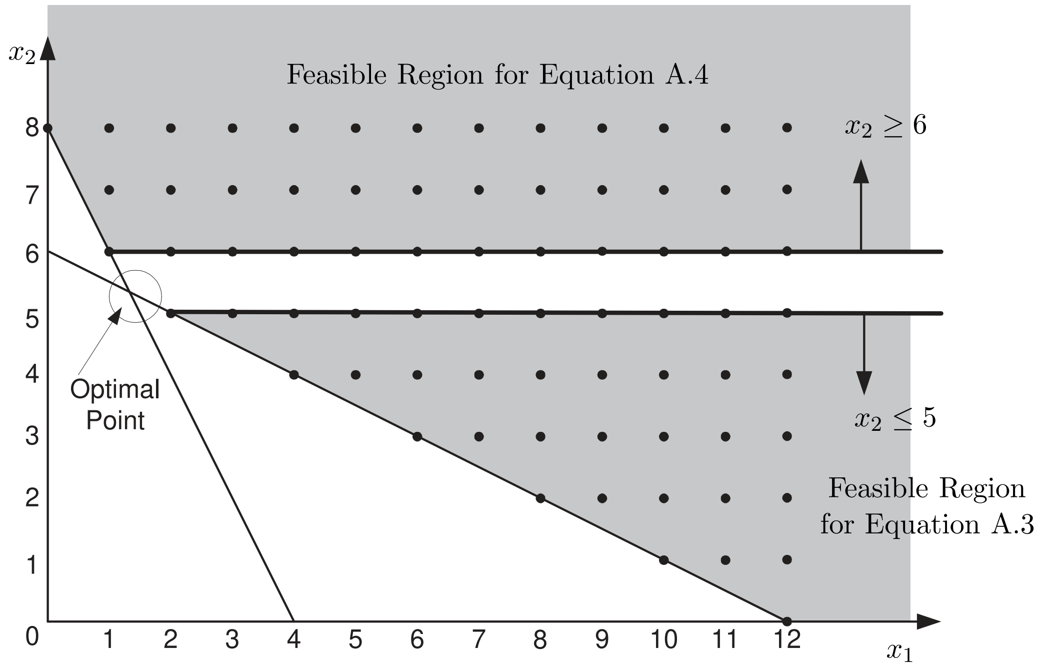


Figure A.2 Feasible Regions for Equations A.3 and A.4

identified, if further investigations of other subproblems do not produce a smaller upper bound, those subproblems can be discarded as they are less optimal than the solution already obtained.

The next step is to solve Equation A.4 to give $x_1 = 1$, $x_2 = 6$ and $z = 15$. This subproblem is also considered to be fathomed as both x_1 and x_2 are integers. The optimal solution to this subproblem is smaller than the current upper bound of 16 obtained previously, therefore the upper bound is updated to the new value of 15. Both subproblems (Equations A.3 and A.4) have now been fathomed with the resulting optimal solution to Equation A.1 being $x_1 = 1$, $x_2 = 6$ and $z = 15$.

This example is a very short Branch and Bound process, other more complicated problems may branch may more times before the solution is found. Should the solution to a subproblem contain non integer variables then this variable would have to branch again until a suitable solution is found.

A.1.1 Branching Variables

The question of which variable to branch on is very valid and can have a large impact on the solution speed of the branch and bound process. In the example above, the branching variable could easily have chosen to be x_1 instead of x_2. This would have changed the feasible regions to those shown in Figure A.3. Regardless of the branching variable chosen the optimal solution to Equation A.1 would be found to be $x_1 = 1$, $x_2 = 6$ and $z = 15$.

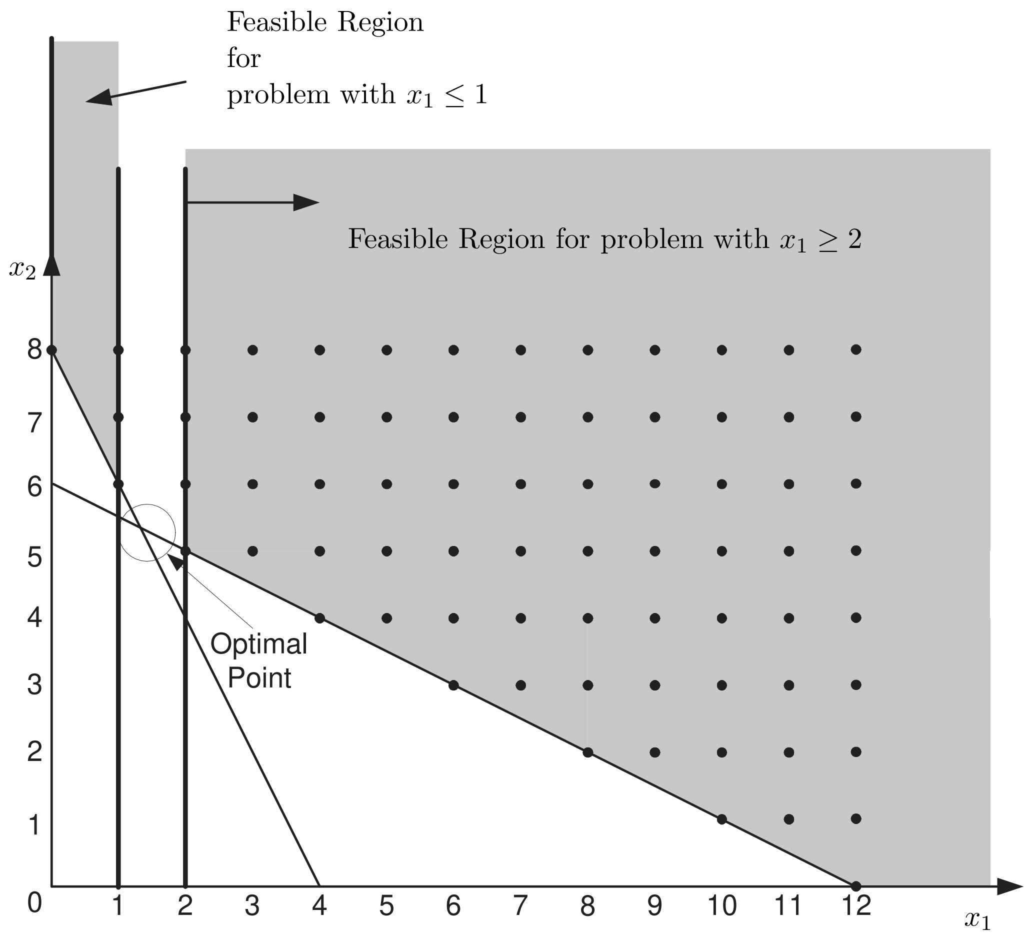


Figure A.3 Feasible Regions when x_1 is the Branching Variable

Choosing which variable to branch on is often done by means of penalty calculations. The basic idea of a penalty calculation is to estimate the change in value of the objective function when the new constraints, that define the new feasible solution spaces, are added to the problem. There are many different ways to calculate the penalty function with many being problem specific [108]. Once the penalty is found the decision on which variable to branch is based on which is more likely to give the most optimal solution. This may be the variable with the smallest penalty in the case of a minimisation problem or the subproblem that indicates a better upper bound (or lower bound in the case of a maximisation problem). Branching decisions based on penalties is reasonably successful but it still has limitations especially for large problems where calculating the penalty can be very time consuming. More recent work has tried to produce improved methods for deciding which variable to branch on but many are still heuristic and application specific [107].

Appendix B

LINEAR PROGRAMMING AND DUALITY

B.1 DUALITY THEORY

Duality theory is used to understand and prove the relationship between the primal and dual problems that states for feasible optimal solution in both the primal and dual problems, the optimal solution of the primal and dual problems are the same.

The proof of equality between the primal and dual problems utilises both the Weak Duality and Strong Duality Theorems.

B.1.1 Weak Duality

The Weak Duality Theorem states that given a primal problem such as Equation B.1, and Dual Problem such as Equation B.2, there is weak duality between the two problems if a feasible solution[1] for both problems is known.

$$\begin{aligned} z &= \min[\mathbf{c^T x}] \\ s.t.& \\ &\mathbf{Ax} \geq \mathbf{b} \\ &\mathbf{x} \geq 0 \end{aligned} \tag{B.1}$$

[1] A feasible solution is one that fulfills the constraints of the problem, it is not necessarily the optimal solution, though the optimal solution is itself a feasible solution.

$$
\begin{aligned}
w = & \max[\mathbf{b^T y}] \\
s.t. & \\
& \mathbf{A^T y} \leq \mathbf{c} \\
& \mathbf{y} \geq 0
\end{aligned} \tag{B.2}
$$

The dual problem provides a bound on the optimal solution of the primal problem. Where the primal is a minimisation problem, the dual provides a lower bound to the optimal primal problem solution. The reverse is true for a maximisation primal problem, the dual provides an upper bound to the optimal primal solution. Let $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ be feasible solutions to the primal and dual problems in Equations B.1 and B.2 respectively. The Weak Duality Theorem is then [109]:

Weak Duality:

$$\mathbf{b^T}\bar{\mathbf{y}} \leq \mathbf{c^T}\bar{\mathbf{x}}$$

Proof:

$$
\begin{aligned}
&\mathbf{b^T}\bar{\mathbf{y}} \leq (\mathbf{A}\bar{\mathbf{x}})^\mathbf{T}\bar{\mathbf{y}} && \textit{Constraint of Equation B.1 multiplied by } \bar{y} \\
&\mathbf{b^T}\bar{\mathbf{y}} \leq \bar{\mathbf{x}}^\mathbf{T}\mathbf{A^T}\bar{\mathbf{y}} \\
&\mathbf{b^T}\bar{\mathbf{y}} \leq \bar{\mathbf{x}}^\mathbf{T}(\mathbf{A^T}\bar{\mathbf{y}}) \\
&\mathbf{b^T}\bar{\mathbf{y}} \leq \bar{\mathbf{x}}^\mathbf{T}\mathbf{c} \\
&\mathbf{b^T}\bar{\mathbf{y}} \leq \mathbf{c^T}\bar{\mathbf{x}}
\end{aligned} \tag{B.3}
$$

The Weak Duality Theorem provides bounds on the optimal solution where the difference between the solutions, $\mathbf{c^T}\bar{\mathbf{x}} - \mathbf{b^T}\bar{\mathbf{y}}$, is called the duality gap [66]. Where the solution found is suboptimal, the duality gap is greater than zero.

B.1.2 Strong Duality

Where the duality gap between primal and dual problems is zero, the Strong Duality Theorem holds.

Strong Duality:

If the primal problem has an optimal solution $\hat{\mathbf{x}}$ then the dual also has an optimal solution $\hat{\mathbf{y}}$ such that [66] [67]:

$$\mathbf{c^T}\hat{\mathbf{x}} = \mathbf{b^T}\hat{\mathbf{y}}$$

The proof of the Strong Duality Theorem can be found in many linear programming textbooks. The general concept of the proof shows that the solution process in obtaining the primal optimal solution simultaneously provides the optimal dual solution therefore you cannot have one without the other. Similarly, the Strong Duality Theorem also shows that if one problem has an unbounded solution, the other problem has an empty feasible region [109].

Appendix C

EXISTING GENERATION DATA

This appendix contains details of the existing system generation data that is used in the HVDC investment problem optimisation of Chapter 9. This data was sourced from the New Zealand Electricity Commission data complied for producing the Statement of Opportunities [96] and documents commissioned by the Electricity Commission from Parsons Brinckerhoff Associates regarding costs of generation [110].

Table C.1: Existing Generation Data

Station Name	**Plant Type**	**Installed Capacity (MW)**	**Island**	**Heat Rate (GJ/GWh)**	**Heat Rate (GJ/MWh)**	**Fuel Cost ($/GWh)**	**Variable Fuel Operating Cost ($/MWh)**	**Variable O&M Operating Cost ($/MWh)**	**Variable Operating Cost (Fuel plus O&M) ($/MWh)**	**Fixed Operating Cost ($/MW)**
Southdown	Gas	122	North	8250	8.25	7	57.75	4.3	62.05	75000
Huntly	Coal	972	North	10500	10.5	4	42	9	51	70000
Huntly - p40	Gas	50	North	9500	9.5	7	66.5	6.4	72.9	75000
Taranaki CC	Gas	377	North	7100	7.1	7	49.7	4.3	54	75000
New Plymouth	Gas	300	North	11000	11	7	77	9.6	86.6	104000
Otahuhu A	Gas	40	North	14500	14.5	7	101.5	6.4	107.9	104000
Otahuhu B	Gas	365	North	7050	7.05	7	49.35	4.3	53.65	75000
Whirinaki	Oil	155	North	11000	11	7	77	6	83	66000
Te Awamutu	Cogen (gas)	20	North	13800	13.8	7	96.6	6.4	103	104000
Kinleith	Cogen (process)	30	North	11000	11	7	77	11.75	88.75	104000
Kapuni	Cogen (gas)	16	North	9300	9.3	7	65.1	6.4	71.5	75000
Glenbrook	Cogen (process)	50	North	0	0	7	0	9.6	9.6	75000
Kiwi Cogen	Cogen (gas)	40	North	9300	9.3	7	65.1	6.4	71.5	75000
Te Rapa	Cogen (gas)	10	North	12600	12.6	7	88.2	6.4	94.6	75000
Wairakei	Geothermal	75	North	0	0	0	0	5.4	5.4	83000
Ohaaki	Geothermal	42	North	0	0	0	0	5.4	5.4	83000
Rotokawa	Geothermal	27	North	0	0	0	0	6.5	6.5	99000
Pohipi	Geothermal	25	North	0	0	0	0	3.8	3.8	83000
Mokai	Geothermal	98	North	0	0	0	0	10	10	83000
Te Apiti	Wind	90	North	0	0	0	0	16	16	0
Tararua I, II	Wind	68	North	0	0	0	0	16	16	0
Arapuni	Hydro	192	North	0	0	0	0	0	0	15000
Aratiati	Hydro	90	North	0	0	0	0	0	0	15000
Atiamuri	Hydro	84	North	0	0	0	0	0	0	15000
Kaitawa	Hydro	37	North	0	0	0	0	0	0	15000
Karapiro	Hydro	96	North	0	0	0	0	0	0	15000
Mangahao	Hydro	37	North	0	0	0	0	0	0	15000
Maraetai	Hydro	352	North	0	0	0	0	0	0	15000
Matahina	Hydro	72	North	0	0	0	0	0	0	15000
Ohakuri	Hydro	112	North	0	0	0	0	0	0	15000
Piripaua	Hydro	44	North	0	0	0	0	0	0	15000

Station Name	Plant Type	Installed Capacity (MW)	Island	Heat Rate (GJ/GWh)	Heat Rate (GJ/MWh)	Fuel Cost ($/GWh)	Variable Fuel Operating Cost ($/MWh)	Variable O&M Operating Cost ($/MWh)	Variable Operating Cost (Fuel plus O&M) ($/MWh)	Fixed Operating Cost ($/MW)
Rangipo	Hydro	120	North	0	0	0	0	0	0	15000
Tokaanu	Hydro	240	North	0	0	0	0	0	0	15000
Tuai	Hydro	60	North	0	0	0	0	0	0	15000
Waipapa	Hydro	58	North	0	0	0	0	0	0	15000
Whakamaru	Hydro	100	North	0	0	0	0	0	0	15000
Patea	Hydro	31	North	0	0	0	0	0	0	15000
Wheo	Hydro	27	North	0	0	0	0	0	0	15000
Kaimai Scheme	Hydro	34	North	0	0	0	0	0	0	15000
White Hill	Wind	58	South	0	0	0	0	16	16	0
Aviemore	Hydro	220	South	0	0	0	0	0	0	15000
Benmore	Hydro	540	South	0	0	0	0	0	0	15000
Clyde	Hydro	432	South	0	0	0	0	0	0	15000
Cobb	Hydro	32	South	0	0	0	0	0	0	15000
Coleridge	Hydro	53	South	0	0	0	0	0	0	15000
Manapouri	Hydro	710	South	0	0	0	0	0	0	15000
Ohau A	Hydro	264	South	0	0	0	0	0	0	15000
Ohau B	Hydro	212	South	0	0	0	0	0	0	15000
Ohau C	Hydro	212	South	0	0	0	0	0	0	15000
Roxburgh	Hydro	320	South	0	0	0	0	0	0	15000
Tekapo A	Hydro	25	South	0	0	0	0	0	0	15000
Tekapo B	Hydro	146	South	0	0	0	0	0	0	15000
Waitaki	Hydro	90	South	0	0	0	0	0	0	15000
Waipori	Hydro	81	South	0	0	0	0	0	0	15000
Highbank	Hydro	25	South	0	0	0	0	0	0	15000
Manapouri Improvement 2	Hydro	16	South	0	0	0	0	0	0	15000

Appendix D

PRIMARY RENEWABLES SCENARIO INVESTMENT DATA

This appendix contains the generation investment data associated with the Primary Renewables generation scenario detailed in Chapter 9. This data was sourced from the New Zealand Electricity Commission data complied for producing the Statement of Opportunities [111] and documents commissioned by the Electricity Commission from Parsons Brinckerhoff Associates regarding costs of generation [110].

Table D.1: Primary Renewables New Generation Data

Station Name	**Plant Type**	**Installed Capacity (MW)**	**Island**	**Capital Cost ($/MW)**	**Heat Rate (GJ/ GWh)**	**Heat Rate (GJ/ MWh)**	**Fuel Cost ($/GWh)**	**Variable Fuel Operating Cost ($/MWh)**	**Variable O&M Operating Cost ($/MWh)**	**Variable Operating Cost (Fuel plus O&M) ($/MWh)**	**Fixed Operating Cost ($/MW)**	**Payback Period (years)**
Generic OCGT 1	Gas	150	North	800000	10000	10	10	100	4.25	104.25	66667	20
Generic OCGT 2	Gas	150	North	800000	10000	10	10	100	4.25	104.25	66667	20
Generic OCGT 3	Gas	150	North	800000	10000	10	10	100	4.25	104.25	66667	20
Generic OCGT 4	Gas	150	North	800000	10000	10	10	100	4.25	104.25	66667	20
Huntly e3p	Gas	365	North	1230000	7080	7.08	10	70.8	4.25	75.05	40603	25
Committed geo	Geothermal	17	North	3824000	0	0	0	0	0	0	102941	25
Kawerau	Geothermal	80	North	3624000	0	0	0	0	0	0	84375	25
Rotokawa expansion	Geothermal	40	North	3500000	0	0	0	0	0	0	87500	25
Generic geo 1	Geothermal	80	North	3750000	0	0	0	0	0	0	84375	25
Generic geo 3	Geothermal	80	North	3750000	0	0	0	0	0	0	84375	25
Tauhara	Geothermal	15	North	4633000	0	0	0	0	0	0	106000	25
Generic geo 4	Geothermal	80	North	4000000	0	0	0	0	0	0	84375	25
Generic geo 5	Geothermal	80	North	4250000	0	0	0	0	0	0	84375	25
Generic geo 6	Geothermal	80	North	4500000	0	0	0	0	0	0	84375	25
Generic geo 7	Geothermal	80	North	4500000	0	0	0	0	0	0	84375	25
Generic geo 9	Geothermal	80	North	5250000	0	0	0	0	0	0	84375	25

Station Name	**Plant Type**	**Installed Capacity (MW)**	**Island**	**Capital Cost ($/MW)**	**Heat Rate (GJ/ GWh)**	**Heat Rate (GJ/ MWh)**	**Fuel Cost ($/GWh)**	**Variable Fuel Operating Cost ($/MWh)**	**Variable O&M Operating Cost ($/MWh)**	**Variable Operating Cost (Fuel plus O&M) ($/MWh)**	**Fixed Operating Cost ($/MW)**	**Payback Period (years)**
Generic geo 8	Geothermal	80	North	5000000	0	0	0	0	0	0	84375	25
Generic geo 11	Geothermal	80	North	5750000	0	0	0	0	0	0	84375	25
Generic geo 10	Geothermal	80	North	5500000	0	0	0	0	0	0	84375	25
Generic geo 12	Geothermal	80	North	6000000	0	0	0	0	0	0	84375	25
Generic geo 13	Geothermal	80	North	6250000	0	0	0	0	0	0	84375	25
Turitea	Wind	150	North	2600000	0	0	0	0	16	16	0	20
Long Gully	Wind	70	North	2600000	0	0	0	0	16	16	0	20
West Wind	Wind	210	North	2650000	0	0	0	0	16	16	0	20
Mokairau	Wind	16	North	2600000	0	0	0	0	16	16	0	20
Puketiro	Wind	120	North	2600000	0	0	0	0	16	16	0	20
Otoi Waiau	Hydro	16.5	North	1677000	0	0	0	0	0	0	33333	80
Mohaka	Hydro	44	North	3409000	0	0	0	0	0	0	34091	80
Mangawhero to Wanganui Diversions	Hydro	60	North	3942000	0	0	0	0	0	0	19667	80
Tarawera at Lake Outlet	Hydro	14	North	3864000	0	0	0	0	0	0	77143	80
Marokopa Falls	Hydro	6.7	North	3678000	0	0	0	0	0	0	73134	80
Rangitaiki at Kiorenui	Hydro	8.5	North	4133000	0	0	0	0	0	0	82353	80

Station Name	**Plant Type**	**Installed Capacity (MW)**	**Island**	**Capital Cost ($/MW)**	**Heat Rate (GJ/ GWh)**	**Heat Rate (GJ/ MWh)**	**Fuel Cost ($/GWh)**	**Variable Fuel Operating Cost ($/MWh)**	**Variable O&M Operating Cost ($/MWh)**	**Variable Operating Cost (Fuel plus O&M) ($/MWh)**	**Fixed Operating Cost ($/MW)**	**Payback Period (years)**
Tarawera at Falls	Hydro	7	North	4139000	0	0	0	0	0	0	82857	80
Whakapapanui Papamanuka	Hydro	16	North	3906000	0	0	0	0	0	0	78125	80
Whangaehu	Hydro	19.6	North	4367000	0	0	0	0	0	0	87245	80
Waitangi Falls Ruakiteri	Hydro	16	North	4306000	0	0	0	0	0	0	86250	80
Tarawera at Te Matae Road	Hydro	10	North	4947000	0	0	0	0	0	0	99000	80
Wairehu Canal	Hydro	11.3	North	4661000	0	0	0	0	0	0	92920	80
Waimarino	Hydro	5.9	North	4461000	0	0	0	0	0	0	89831	80
Hautapu at Turangarere	Hydro	9.4	North	4917000	0	0	0	0	0	0	97872	80
Pohangina	Hydro	9.5	North	4974000	0	0	0	0	0	0	100000	80
Waikato upgrade	Hydro	200	North	3000000	0	0	0	0	0	0	15000	80
Generic pumped hydro	Hydro	300	North	3000000	0	0	0	0	0	0	15000	80
Tararua 3	Wind	93	North	2783000	0	0	0	0	16	16	0	20
Te Waka	Wind	111	North	2650000	0	0	0	0	16	16	0	20
Titiokura	Wind	48	North	2650000	0	0	0	0	16	16	0	20
Unspecified wind	Wind	200	North	2500000	0	0	0	0	16	16	0	20
Motorimu	Wind	80	North	2600000	0	0	0	0	16	16	0	20
Red Hill	Wind	20	North	2600000	0	0	0	0	16	16	0	20
Te Rere Hau	Wind	48.5	North	2650000	0	0	0	0	16	16	0	20

Station Name	Plant Type	Installed Capacity (MW)	Island	Capital Cost ($/MW)	Heat Rate (GJ/ GWh)	Heat Rate (GJ/ MWh)	Fuel Cost ($/GWh)	Variable Fuel Operating Cost ($/MWh)	Variable O&M Operating Cost ($/MWh)	Variable Operating Cost (Fuel plus O&M) ($/MWh)	Fixed Operating Cost ($/MW)	Payback Period (years)
Belmont Hills	Wind	80	North	2600000	0	0	0	0	16	16	0	20
Wharauroa Plateau	Wind	72	North	2600000	0	0	0	0	16	16	0	20
Deep Stream	Hydro	5	South	3500000	0	0	0	0	0	0	70000	80
Toaroha	Hydro	25	South	2954000	0	0	0	0	0	0	29600	80
Wairau	Hydro	70	South	3143000	0	0	0	0	0	0	15714	80
Kakapotahi	Hydro	17	South	3044000	0	0	0	0	0	0	61176	80
Manuherikia	Hydro	6.75	South	2806000	0	0	0	0	0	0	56296	80
Clarence to Waiau Diversions	Hydro	70	South	3543000	0	0	0	0	0	0	17714	80
Dobson	Hydro	50	South	3600000	0	0	0	0	0	0	18000	80
Cobb Scheme Supplement	Hydro	4.2	South	1542000	0	0	0	0	0	0	30952	80
Hawea Control Gate Retrofit	Hydro	16	South	2500000	0	0	0	0	0	0	50000	80
Pukaki Control Gate Retrofit	Hydro	44	South	3023000	0	0	0	0	0	0	30227	80
North Bank Tunnel	Hydro	280	South	3571000	0	0	0	0	0	0	17857	80

Station Name	Plant Type	Installed Capacity (MW)	Island	Capital Cost ($/MW)	Heat Rate (GJ/GWh)	Heat Rate (GJ/MWh)	Fuel Cost ($/GWh)	Variable Fuel Operating Cost ($/MWh)	Variable O&M Operating Cost ($/MWh)	Variable Operating Cost (Fuel plus O&M) ($/MWh)	Fixed Operating Cost ($/MW)	Payback Period (years)
Te Anau Control Gate Retrofit	Hydro	40	South	3750000	0	0	0	0	0	0	37500	80
Clutha River Queensberry	Hydro	180	South	4500000	0	0	0	0	0	0	22500	80
Lake Mahinerangi	Wind	200	South	2500000	0	0	0	0	16	16	0	20
Generic OCGT 5	Gas	150	South	800000	10000	10	10	100	4.25	104.25	66667	20

Appendix E

HIGH GAS SCENARIO INVESTMENT DATA

This appendix contains the generation investment data associated with the high gas generation scenario detailed in Chapter 9. This data was sourced from the New Zealand Electricity Commission data complied for producing the Statement of Opportunities [111] and documents commissioned by the Electricity Commission from Parsons Brinckerhoff Associates regarding costs of generation [110].

Table E.1: High Gas New Generation Data

Station Name	Plant Type	Installed Capacity (MW)	Island	Capital Cost ($/MW)	Heat Rate (GJ/GWh)	Heat Rate (GJ/MWh)	Fuel Cost ($/GWh)	Variable Fuel Operating Cost ($/MWh)	Variable O&M Operating Cost ($/MWh)	Variable Operating Cost (Fuel plus O&M) ($/MWh)	Fixed Operating Cost ($/MW)	Payback Period (years)
Huntly e3p	Gas	365	North	1035000	7080	7.08	7	49.56	4	53.56	75000	25
Committed geo	Geothermal	17	North	3824000	1000	1	0	0	0	0	102941	25
Tararua 3	Wind	93	North	2783000		0	0	0	16	16	0	20
Kawerau	Geothermal	80	North	3624000	1000	1	0	0	0	0	84375	25
Unspecified wind	Wind	200	North	2500000		0	0	0	16	16	0	20
Generic geo 1	Geothermal	80	North	3750000	1000	1	0	0	0	0	168750	25
Generic geo 2	Geothermal	80	North	3750000	1000	1	0	0	0	0	168750	25
Otoi Waiau	Hydro	16.5	North	1677000	1000	1	0	0	0	0	33333	80
Ngawha 2	Geothermal	15	North	4633000	1000	1	0	0	0	0	106000	25
Tauhara	Geothermal	15	North	4633000	1000	1	0	0	0	0	106000	25
Otahuhu C	Gas	407	North	1035000	7050	7.05	7	49.35	4.3	53.65	75000	25
Generic geo 4	Geothermal	80	North	4000000	1000	1	0	0	0	0	84375	25
West Wind	Wind	210	North	2650000		0	0	0	16	16	0	20
Mokairau	Wind	16	North	2600000		0	0	0	16	16	0	20
Generic OCGT 1	Gas	150	North	800000	10000	10	7	70	4.25	74.25	66667	20
Generic gas 2 Taranaki	Gas	410	North	1035000	10000	10	7	70	4.25	74.25	40585	25
Generic geo 5	Geothermal	80	North	4250000	1000	1	0	0	0	0	84375	25

Station Name	**Plant Type**	**Installed Capacity (MW)**	**Island**	**Capital Cost ($/MW)**	**Heat Rate (GJ/ GWh)**	**Heat Rate (GJ/ MWh)**	**Fuel Cost ($/GWh)**	**Variable Fuel Operating Cost ($/MWh)**	**Variable O&M Operating Cost ($/MWh)**	**Variable Operating Cost (Fuel plus O&M) ($/MWh)**	**Fixed Operating Cost ($/MW)**	**Lifetime (years)**
Taranaki CC 2	Gas	380	North	1035000	7000	7	7	49	4.25	53.25	41342	25
Generic geo 6	Geothermal	80	North	4500000	1000	1	0	0	0	0	84375	25
Generic gas 1 Auckland	Gas	410	North	1035000	7050	7.05	7	49.35	4.25	53.6	40585	25
Generic geo 7	Geothermal	80	North	4500000	1000	1	0	0	0	0	84375	25
Generic OCGT 4	Gas	150	North	800000	10000	10	7	70	4.25	74.25	66667	20
Generic OCGT 3	Gas	150	North	800000	10000	10	7	70	4.25	74.25	66667	20
Generic OCGT 2	Gas	150	North	800000	10000	10	7	70	4.25	74.25	66667	20
Generic coal 4 Tauranga	Coal	300	North	2400000	10000	10	4	40	9	49	2667	30
Deep Stream	Hydro	5	South	3500000		0	0	0	0	0	70000	80
Cobb Scheme Supplement	Hydro	4.2	South	1542000		0	0	0	0	0	30952	80
Wairau	Hydro	70	South	3143000		0	0	0	0	0	15714	80
Hawea Control Gate Retrofit	Hydro	16	South	2500000		0	0	0	0	0	50000	80

Station Name	**Plant Type**	**Installed Capacity (MW)**	**Island**	**Capital Cost ($/MW)**	**Heat Rate (GJ/ GWh)**	**Heat Rate (GJ/ MWh)**	**Fuel Cost ($/GWh)**	**Variable Fuel Operating Cost ($/MWh)**	**Variable O&M Operating Cost ($/MWh)**	**Variable Operating Cost (Fuel plus O&M) ($/MWh)**	**Fixed Operating Cost ($/MW)**	**Lifetime (years)**
Pukaki Control Gate Retrofit	Hydro	44	South	2050000		0	0	0	0	0	15000	80
Toaroha	Hydro	25	South	2954000		0	0	0	0	0	29600	80
Generic lignite 1 Southland	Lignite	400	South	2460000	10800	10.8	1.8	19.44	11.25	30.69	19850	30

Appendix F

MIXED TECHNOLOGIES SCENARIO INVESTMENT DATA

This appendix contains the generation investment data associated with the mixed Technologies generation scenario detailed in Chapter 9. This data was sourced from the New Zealand Electricity Commission data complied for producing the Statement of Opportunities [111] and documents commissioned by the Electricity Commission from Parsons Brinckerhoff Associates regarding costs of generation [110].

Table F.1: Mixed Technologies New Generation Data

Station Name	**Plant Type**	**Installed Capacity (MW)**	**Island**	**Capital Cost ($/MW)**	**Heat Rate (GJ/GWh)**	**Heat Rate (GJ/MWh)**	**Fuel Cost ($/GWh)**	**Variable Fuel Operating Cost ($/MWh)**	**Variable O&M Operating Cost ($/MWh)**	**Variable Operating Cost (Fuel plus O&M) ($/MWh)**	**Fixed Operating Cost ($/MW)**	**Payback Period (years)**
Generic coal 4 Tauranga	Coal	300	North	2400000	10000	10	4	40	9	49	2667	30
Marsden Coal	Coal	320	North	2125000	10800	10.8	4	43.2	9.6	52.8	32188	30
Generic coal 1 Glenbrook	Coal	400	North	2400000	10000	10	4	40	9	49	2675	30
Generic OCGT 1	Gas	150	North	800000	10000	10	10	100	4.25	104.25	66667	20
Generic OCGT 4	Gas	150	North	800000	10000	10	10	100	4.25	104.25	66667	20
Generic OCGT 2	Gas	150	North	800000	10000	10	10	100	4.25	104.25	66667	20
Generic OCGT 3	Gas	150	North	800000	10000	10	10	100	4.25	104.25	66667	20
Huntly e3p	Gas	365	North	1035000	7080	7.08	10	70.8	4	74.8	75000	25
Otahuhu C	Gas	407	North	1007000	7050	7.05	10	70.5	4.25	74.75	40590	25
Committed geo	Geothermal	17	North	3824000	0	0	0	0	0	0	102941	25
Kawerau	Geothermal	80	North	3624000	0	0	0	0	0	0	84375	25
Generic geo 2	Geothermal	80	North	3750000	0	0	0	0	0	0	84375	25
Generic geo 3	Geothermal	80	North	3750000	0	0	0	0	0	0	84375	25
Ngawha 2	Geothermal	15	North	4633000	0	0	0	0	0	0	106000	25
Tauhara	Geothermal	15	North	4633000	0	0	0	0	0	0	106000	25
Generic geo 4	Geothermal	80	North	4000000	0	0	0	0	0	0	84375	25

Station Name	Plant Type	Installed Capacity (MW)	Island	Capital Cost ($/MW)	Heat Rate (GJ/ GWh)	Heat Rate (GJ/ MWh)	Fuel Cost ($/GWh)	Variable Fuel Operating Cost ($/MWh)	Variable O&M Operating Cost ($/MWh)	Variable Operating Cost (Fuel plus O&M) ($/MWh)	Fixed Operating Cost ($/MW)	Payback Period (years)
Generic geo 5	Geothermal	80	North	4250000	0	0	0	0	0	0	84375	25
Generic geo 6	Geothermal	80	North	4500000	0	0	0	0	0	0	84375	25
Mangawhero to Wanganui Diversions	Hydro	60	North	3942000	0	0	0	0	0	0	19667	80
Tararua 3	Wind	93	North	2783000	0	0	0	0	16	16	0	20
Te Waka	Wind	111	North	2650000	0	0	0	0	16	16	0	20
Titiokura	Wind	48	North	2650000	0	0	0	0	16	16	0	20
Turitea	Wind	150	North	2600000	0	0	0	0	16	16	0	20
Puketiro	Wind	120	North	2600000	0	0	0	0	16	16	0	20
Hawkes Bay Wind Farm	Wind	225	North	2650000	0	0	0	0	16	16	0	20
Generic OCGT 5	Diesel	150	South	800000	10000	10	7	70	4.25	74.25	66667	20
Deep Stream	Hydro	5	South	3500000	0	0	0	0	0	0	70000	80
Manuherikia	Hydro	6.75	South	2806000	0	0	0	0	0	0	56296	80
Kakapotahi	Hydro	17	South	3044000	0	0	0	0	0	0	61176	80
Meg River Renewal	Hydro	7.5	South	3494000	0	0	0	0	0	0	69333	80
Teviot River Diversion	Hydro	3.9	South	3565000	0	0	0	0	0	0	71795	80
Wye Creek Renewal	Hydro	3.8	South	3347000	0	0	0	0	0	0	65789	80
North Bank Tunnel	Hydro	280	South	3571000	0	0	0	0	0	0	17857	80

Station Name	**Plant Type**	**Installed Capacity (MW)**	**Island**	**Capital Cost ($/MW)**	**Heat Rate (GJ/ GWh)**	**Heat Rate (GJ/ MWh)**	**Fuel Cost ($/GWh)**	**Variable Fuel Operating Cost ($/MWh)**	**Variable O&M Operating Cost ($/MWh)**	**Variable Operating Cost (Fuel plus O&M) ($/MWh)**	**Fixed Operating Cost ($/MW)**	**Payback Period (years)**
Generic lignite 1 Southland	Lignite	400	South	2460000	10800	10.8	1.8	19.44	11.25	30.69	19850	30

Appendix G

FUEL COST DATA

This appendix contains the fuel cost data associated with all generation scenarios detailed in Chapter 9. The fuel prices used in the SDDP algorithm are those given by the Grid Planning assumptions from the Electricity Commission [112]. The table is reproduced here:

Fuel	Primary Renewables	SI Surplus	Mixed Tech	High Gas
Gas ($/GJ)	10	10	10	7
Diesel ($/GJ)	25	25	25	25
Coal ($/GJ)	4	4	4	4
Lignite ($/GJ)	1.8	1.8	1.8	1.8

Table G.1 Fuel Costs

The data supplied indicates the expected costs of various fuels in the year 2020. This data is used for all years within the HVDC investment problem and as such can be considered a worst case scenario of fuel costs.

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