

Practical Cryptographic Civil GPS Signal Authentication

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Abstract—A practical technique is proposed to authenticate civil GPS signals. The technique combines cryptographic authentication of the GPS navigation message with signal timing authentication based on statistical hypothesis tests to secure civil GPS receivers against spoofing attacks. The notion of GNSS signal authentication is defined in probabilistic terms. Candidate GPS signal authentication schemes are evaluated in terms of effectiveness and practicality leading to a proposal for incorporating digital signatures into the extensible GPS civil navigation (CNAV) message. The proposal is sufficiently detailed to facilitate near-term implementation of security-hardened civil GPS.

INTRODUCTION

In the decade since Selective Availability was discontinued in 2000, civil technologies based on the Global Positioning System (GPS) have become ubiquitous and the GPS service has easily achieved the stated goal of the new policy regime, which is to “encourage acceptance and integration of GPS into peaceful civil, commercial, and scientific applications worldwide; and to encourage private sector investment in and use of U.S. GPS technologies and services” [1]. Also over the past decade, the concept of national security has evolved from a focus on protecting military and critical government resources to a broader ambit that includes the protection of vital elements of civilian and commercial infrastructure. Civil GPS is a critical component of the national infrastructure; hence, GPS security is a matter of national security.

In 2001, the U.S. Department of Transportation published a report assessing the vulnerability of the U.S. transportation infrastructure to disruption of civil GPS [2]. Known as the Volpe report, it highlighted the threats posed by spoofing and meaconing attacks—methods by which a victim GPS receiver is deceived into tracking counterfeit GPS signals. At the time, the open literature contained little research on such attacks and possible countermeasures. Accordingly, the report recommended further study of GPS spoofing and development of civil GPS anti-spoofing techniques. Global Navigation Satellite System (GNSS) security research over the past decade has made much progress toward these goals [3]–[11].

It is convenient to distinguish cryptographic spoofing defenses, which rely on secret keys that encrypt or digitally sign components of the broadcast signals, from non-cryptographic

defenses, which do not depend on encryption or digital signatures. Among non-cryptographic defenses, the multi-antenna defense [10], [12] appears to be one of the strongest, although it remains vulnerable to the coordinated spoofing attack explored in [9]. This defense requires two or more antennas spaced by an appreciable fraction of the approximately 20-cm GPS signal wavelength, which would tend to increase receiver cost, weight, and size. As a result, the multi-antenna defense is unlikely to be widely adopted by commercial GPS manufacturers. This is also true of other non-cryptographic defenses involving inertial measurement units or other hardware, which would exceed the cost, mass, or size constraints of a broad range of applications.

Cryptographic spoofing defenses are attractive because they offer significant protection against spoofing relative to the additional cost and bulk required for implementation. While it must be conceded that no anti-spoofing technique is impervious to the most sophisticated attacks, a cryptographic defense significantly raises the bar for a successful attack and can be combined with non-cryptographic spoofing defenses for better security than either category could offer separately.

Several civil GPS cryptographic spoofing defenses have been proposed whose implementation would require fundamental changes to the legacy GPS signal structure (e.g., [3], [4], [7]). These defenses are unlikely to be implemented over the next decade given the static nature of GPS signal definitions [13].

A growing literature suggests navigation message authentication (NMA) is a practical basis for civil GPS signal authentication [3], [6], [7], [14]. In NMA, the low-rate navigation message is encrypted or digitally signed, allowing a receiver to verify that the GPS Control Segment generated the data. NMA could be implemented without fundamental changes to the GPS Interface Specification by exploiting the extensibility of the modern GPS civil navigation (CNAV) messaging format. Moreover, NMA has been proposed for implementation in the European Galileo GNSS [5], [15].

Previous papers have pointed out that signal authentication based on NMA may be vulnerable to replay-type spoofing attacks [3], [7]. Thus, whereas it is clear that NMA authenticates the origin of the navigation data, there has been uncertainty regarding whether NMA can be used to authenticate the underlying GPS signal, which demands resistance against replay-type spoofing attacks. The combination of this paper and the statistical test recently developed in Ref. [16] clears up this uncertainty by demonstrating that NMA can in fact offer integrated civil GPS signal authentication—that is, combined data and signal authentication—if it is paired with timing

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authentication based on statistical hypothesis tests.

The present work offers four main contributions beyond those given in [3], [5]–[7], [14], [15]. First, it develops a general probabilistic interpretation of GNSS signal authentication that combines cryptographic code origin authentication with code timing authentication based on statistical hypothesis tests. Second, it identifies sensible design criteria for civil GPS signal authentication and, third, applies this framework to evaluate several proposed candidate authentication strategies. Finally, it proposes a specific cryptographic signal authentication implementation for civil GPS that meets the design criteria and is packaged for immediate adoption. The following sections are organized around these contributions, followed by conclusions.

A PROBABILISTIC INTERPRETATION OF GNSS SIGNAL AUTHENTICATION

Signal authentication, the topic of this paper, and message authentication, such as is used to sign data transmitted across the Internet, can be distinguished from one another by the models employed to describe their security. Message authentication security is predicated on the computational infeasibility of performing a brute-force search for the secret key used to sign the original message, or of reversing a so-called one-way function to discover the key [17]. While it is true that this assumed computational infeasibility can be couched in probabilistic terms (e.g., the probability that over the next 30 years a weakness will be found in a certain one-way hash function), such language is seldom used, either because the probabilities involved are too subjective or too small to be meaningful.

In contrast to message authentication, the security of signal authentication is much weaker and demands a probabilistic model, as described in this section.

Generalized Model for Security-Enhanced GNSS Signals

Current and proposed security-enhanced GNSS signals can be represented by a simple model from the perspective of a GNSS receiver. Let the signal exiting the radio frequency (RF) front-end of a GNSS receiver after having been downmixed and sampled be modeled as:

$$Y_k = w_k c_k \cos(2\pi f_{IF} t_k + \theta_k) + N_k \quad (1a)$$

$$= w_k s_k + N_k \quad (1b)$$

Here, at sample index k , w_k is a ± 1 -valued security code with chip length T_w , c_k is a known ± 1 -valued spreading (ranging) code with chip length T_c , f_{IF} is the intermediate value of the downmixed carrier frequency, θ_k is the beat carrier phase, and N_k is a sequence of independent, identically distributed zero-mean Gaussian noise samples with variance σ^2 that models the effects of thermal noise in the RF front end. The signal and noise have been normalized so that the modeled signal amplitude is unity. For convenience, $s_k = c_k \cos(2\pi f_{IF} t_k + \theta_k)$ is used to represent the deterministic signal components. Also for convenience, and without loss of generality, the receiver time t_k is assumed to be equivalent to true time with a uniform sampling interval $T_s = t_k - t_{k-1}$.

The model's security code w_k is a generalization of a binary modulating sequence that is either fully encrypted or contains periodic authentication codes. The defining feature of w_k is that some or all of its symbols are unpredictable to a would-be spoofer prior to broadcast from a legitimate GNSS source. The unpredictable symbols of w_k serve two related functions: they enable verification of w_k as originating from a GNSS Control Segment (standard message authentication) and they make possible a hypothesis test for a security code estimation and replay attack [16]. Various security code implementations will be considered in a later section.

Attacks against Security-Enhanced GNSS Signals

GNSS spoofing is the transmission of counterfeit GNSS signals with the intent to manipulate the position, velocity, and timing (PVT) readout of a GNSS receiver. A spoofer matches its counterfeit signal structure to that of the authentic signals, as modeled by Eq. (1). To circumvent the security afforded by the unpredictable security code w_k , the spoofer may attempt one of the following specialized spoofing attacks.

Meaconing: The recording and playback of an entire block of RF spectrum containing an ensemble of GNSS signals [2]. Constituent GNSS signals are not typically separated during record and playback, which implies that a meaconing attack cannot arbitrarily manipulate the PVT of target receivers; rather, target receivers will display the position and velocity of the meaconer and a time in arrears of true time. For a single GNSS signal corresponding to a particular satellite, the combined meaconed and authentic received signals can be modeled as

$$Y_k = \alpha w_{k-d} s_{k-d} + N_{m,k} + w_k s_k + N_k \quad (2)$$

where $N_{m,k}$ is the noise introduced by the meaconer's RF front end, N_k is the noise introduced by the target receiver's RF front end, and $d > 0$ is the number of samples of meaconing delay, such that the meaconed signal $\alpha w_{k-d} s_{k-d}$ arrives at the target receiver with a delay of d samples relative to the authentic signal $w_k s_k$. The coefficient α is the meaconed signal's amplitude advantage factor.

High performance digital signal processing hardware permits a meaconer located close to its intended target to drive the delay d to ever smaller values. In the limit as d approaches zero the attack becomes a zero-delay meaconing attack with the meaconed signals code-phase-aligned with their authentic counterparts. Such alignment enables a seamless liftoff of the target receiver's tracking loops, following which a meaconer can increase d at a rate that is consistent with the target receiver clock drift and gradually impose a significant timing delay.

Security Code Estimation and Replay (SCER) Attack: Allows greater flexibility than a meaconing attack in manipulating the target receiver's PVT solution. In a SCER attack, a spoofer receives and tracks individual authentic signals and attempts to estimate the values of each signal's unpredictable security code chips on-the-fly. It then reconstitutes a consistent ensemble of GNSS signals, with the security code chip estimates taking the place of the authentic codes, and rebroadcasts these with some delay. For a single GNSS signal

corresponding to a particular satellite, the combined SCER-spoofed and authentic received signals can be modeled as

$$Y_k = \alpha \hat{w}_{k-d} s_{k-d} + w_k s_k + N_k \quad (3)$$

where \hat{w}_{k-d} represents the security code estimate arriving with a delay of d samples relative to the authentic security code w_k and other quantities are as described previously. The delay d can be modeled as the sum $d = p + e$ of a processing and transmission delay p , which represents the required signal processing and propagation time and which does not contribute to better estimates of the security code chips, and an estimation and control delay e , which represents an additional delay imposed by the spoofer to improve its estimate of the security code chip values and to control the relative phasing of the spoofed signals so as to impose spoofer-defined position and timing offsets on the target receiver. If the initial delay d exceeds the spreading code chip interval (i.e., if $dT_s > T_c$), then the spoofer will be unable to dislodge the target receiver's tracking loops without forcing re-acquisition. Thus, if the spoofer has an irreducible delay $dT_s > T_c$ then it must first jam or obstruct the incoming GNSS signals to force the target receiver to perform re-acquisition. Attacks in which the spoofer avoids this condition by transmitting the counterfeit signals at a power level such that the sidelobe power is sufficient to disrupt tracking at the victim receiver would trigger the J/N detector under typical received signal strength conditions and in cases where the attacker is unable to physically block the victim antenna. Therefore, such attacks are excluded from consideration.

The success of a SCER attack depends on the accuracy of the security code estimate. Let k_l be the index of the first sample within the l th authentic security code chip. Then for the received sample Y_{k+d} , with $k_l \leq k < k_{l+1}$, a maximum of $\min(e + k - k_l + 1, \lfloor T_w/T_s \rfloor)$ security code samples will have been summed within the spoofer to produce the security code estimate $\hat{w}_{k+d-d} = \hat{w}_k$, where $\lfloor x \rfloor$ is the floor of x (the largest integer not greater than x). The accuracy of the chip estimates improves with increasing number of participating samples. For example, the probability of error for hard-decision chip estimates is $p_e = \text{erfc}(\sqrt{mT_s(C/N_0)_s})/2$ where m is the number of participating samples at sampling interval T_s , $(C/N_0)_s$ is the spoofer's carrier-to-noise ratio, and $\text{erfc}(\cdot)$ is the complementary error function. Thus, because $m \leq \lfloor T_w/T_s \rfloor$, small T_w severely limits the accuracy of the security code estimates. Consider that a spoofer receiving the legacy Y-code GPS signal, for which $T_w \approx 2 \mu\text{s}$ (i.e., W-code period) [18], at a nominal carrier-to-noise ratio of 48 dB-Hz, generates hard-decision chip estimates with a 30 percent probability of error. A detection strategy for short-delay SCER attacks is detailed in [16].

Long security code chips (e.g., $T_w = 20$ ms for NMA) allow the spoofer to increase e and thereby generate highly accurate chip estimates. A large delay $d = p + e$, however, is itself a liability for the spoofer. The signal denial prelude to a SCER attack must be made long enough that d is consistent with the target receiver's clock drift during the denial interval; otherwise, d will lead to a suspicious increment in the target receiver's pseudorange measurements. Thus, the spoofer finds

itself vulnerable to detection at low d due to poor security code chip estimates and at high d due to timing anomalies. This is suggestive of the probabilistic nature of signal authentication, which is further elucidated in the following section.

Components of an Integrated Probabilistic GNSS Signal Authentication Strategy

In simplest terms, GNSS signal authentication means certifying that a received signal is not counterfeit, that it originates from a GNSS satellite and not a spoofer. As opposed to data authentication, however, GNSS signal authentication is far from absolute; rather, it involves a set of hypothesis tests each with a probability of false alarm. In the formulation adopted here, the tests are designed to detect a spoofing attack under the assumption that a spoofer will either (1) generate a falsified security code that does not match the authentic security code, (2) attempt a non-zero-delay meaconing attack, or (3) attempt a SCER attack. Framed by these assumptions, GNSS signal authentication can be interpreted as involving two authentication sub-types: (1) code origin authentication, a certification that the security code originates with the GNSS Control Segment, and (2) code timing authentication, a certification that the security code arrives promptly and intact.

In the sections that follow, the functional components that support code origin authentication and code timing authentication are described. As a guide to the discussion, the components and their interconnections are presented schematically in Fig. 1 for a security code based on NMA. Adaptations to Fig. 1 for other types of security codes (e.g., GPS Y-code-type encryption or spread spectrum security codes [3]) are discussed further on.

For simplicity of presentation, Fig. 1 represents the authentication process for a single GNSS signal, i.e., a signal identified by a unique combination of spreading code and carrier frequency. An entire ensemble of GNSS signals is assumed to be downmixed and sampled in the RF front end to produce the sampled signal output Y_k , which is routed to the signal tracking and navigation processor where the raw digital output of the RF front end is correlated against receiver-generated signal replicas to acquire and track multiple constituent GNSS signals. However, from the perspective of downstream components, which are associated with a single GNSS signal, Y_k can be modeled as in Eq. (1) for unspoofed signals and in Eqs. (2) and (3) for meaconed and SCER-spoofed signals, respectively.

Code Origin Authentication: In the case of a security code based on NMA, the signal tracking and navigation processor produces a sequence W'_l of received navigation message symbol estimates. In most cases, these symbols are an error-correction-encoded version of the navigation message data (e.g., the GPS CNAV message is convolutionally encoded before transmission [19]). As the sequence W'_l passes through the error correction decoder, errors introduced by noise in the transmission channel are corrected and the navigation message symbols b_j are recovered. At low carrier-to-noise (C/N_0) ratios some errors may remain in b_j . The code integrity check exploits redundant symbols in b_j (e.g., cyclic redundancy

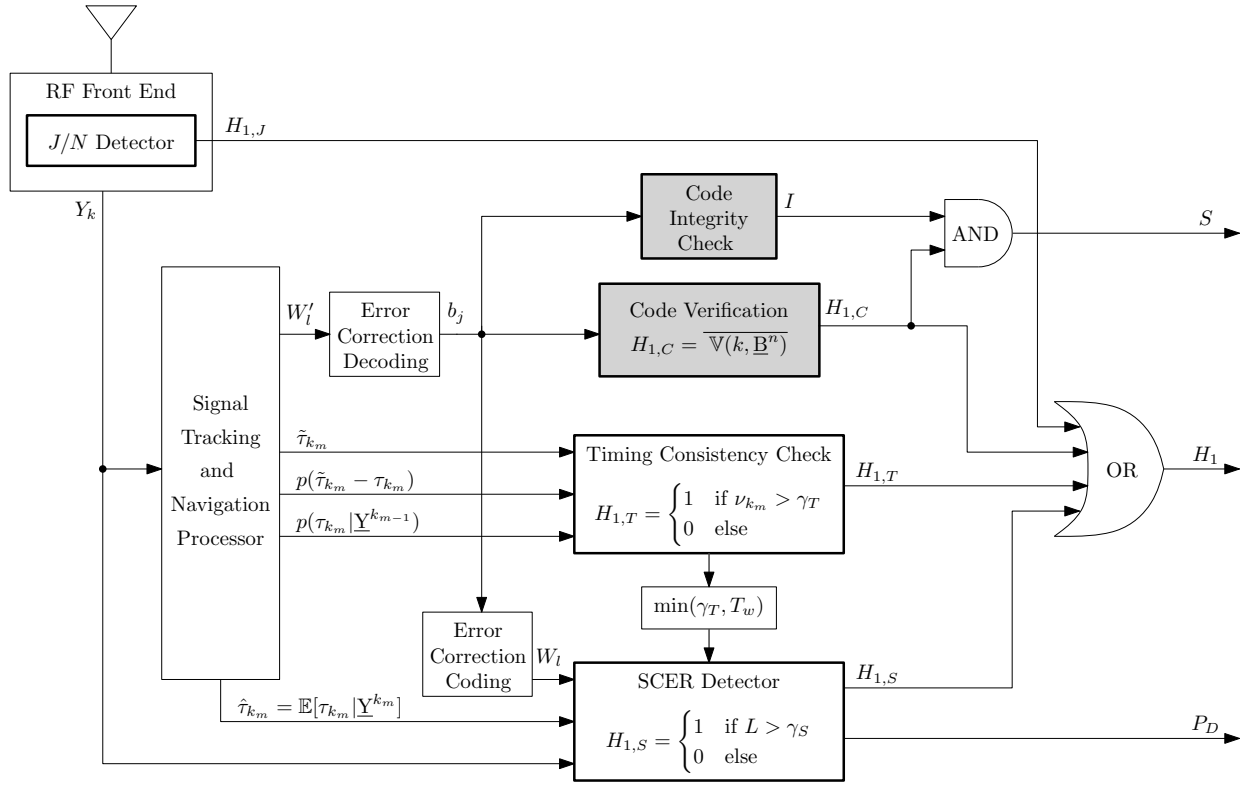


Fig. 1. Schematic showing GNSS receiver components required for GNSS signal authentication. Components that support code origin authentication are outlined in bold and have a gray fill, whereas components that support code timing authentication are outlined in bold and have no fill. The schematic assumes a security code based on navigation message authentication.

check codes in the GPS CNAV message [19]) to determine whether errors remain. Upon success, the code integrity check sets its logical output I high. For practical purposes, a successful integrity check indicates that the navigation message is correct as received.

The n th block of N_b navigation message symbols $\underline{\mathbf{B}}^n \equiv [b_{j_n}, b_{j_n+1}, \dots, b_{j_n+N_b-1}]^T$, which in an NMA scheme includes both navigation data and a digital signature, is passed to a code verification algorithm $\mathbb{V}(k, \underline{\mathbf{B}}^n)$ that verifies $\underline{\mathbf{B}}^n$ against a cryptographic key k . If the verification check passes, then $\underline{\mathbf{B}}^n$ can be safely assumed to originate with the GNSS Control Segment. In this case, the logical output signal $H_{1,C}$ remains low. Otherwise, if the verification fails, $H_{1,C}$ is asserted; however this does not necessarily indicate a spoofing attack.

Despite error correction, there may yet remain errors in the symbol stream b_j . A single error within the block $\underline{\mathbf{B}}^n$ would cause the code verification to fail. Because of this possibility, and by analogy with other hypothesis tests to be introduced shortly, it is convenient to view the code verification as a statistical hypothesis test. The probability of false alarm for the n th verification is $P_{F,C} = 1 - (1 - p_{e,j})^{N_b}$, with $p_{e,j}$ being the probability that b_j is wrong, which depends on C/N_0 over the j th symbol, where $j_n \leq j < j_n + N_b$.

To get a sense for the size of $P_{F,C}$, consider a conservative scenario in which the satellites broadcast a block of $N_b = 10,000$ non-error-correction-encoded navigation message symbols $\underline{\mathbf{B}}^n$. In this case, the probability that b_j is wrong is $p_{e,j} = \text{erfc}(\sqrt{T_w(C/N_0)_r})/2$. For $(C/N_0)_r \approx 29$ dB-Hz

and $T_w = 20$ ms, $P_{F,C} \approx 0.0001$. If error correction were employed, $P_{F,C}$ would be smaller for a given $(C/N_0)_r$. To ensure that $P_{F,C}$ remains negligible relative to $P_{F,J}$, $P_{F,T}$ and $P_{F,S}$, a receiver can ignore signals whose $(C/N_0)_r < 30$ dB-Hz.

The output $H_{1,C}$ is combined in a logical ‘OR’ operation with outputs from other hypothesis tests to produce H_1 . If the code verification fails ($H_{1,C}$ high) but the code integrity check passes (I high), then, with a very high likelihood, the code verification failure cannot be attributed to symbol errors caused by noise. In this case, the output S is asserted, indicating a nearly certain spoofing attack. As opposed to $H_{1,C}$, which goes high with false alarm rate probability $P_{F,C}$ even under normal unspoofed conditions, the infinitesimal probability of false alarm associated with output S suggests that S need not be viewed probabilistically.

One might ask why $H_{1,C}$ should be considered independently from S . The answer is that if only S is considered then a would-be spoofer could always maintain S low by injecting a symbol stream b_j that repeatedly fails the code integrity check. Thus, the outputs S and $H_{1,C}$ are monitored independently both to prevent this type of an attack and in recognition of the clear certainty of a spoofed condition when S goes high.

Code Timing Authentication: The following functional blocks are involved in code timing authentication: the timing consistency check, the SCER detector, and the jamming-to-noise (J/N) detector.

The timing consistency check is a hypothesis test on the

timing of the received spreading code c_k . It amounts to a consistency check on the code phase measurement innovation, or the difference between the measured and predicted code phase, and is essentially a special case of so-called receiver autonomous integrity monitoring [20]. The check takes three inputs from the signal tracking and navigation processor:

$\tilde{\tau}_{k_m}$: the receiver's m th measurement of code phase, expressed as the arrival time of some feature of the incoming signal and defined at receiver time t_{k_m} .

$p(\tilde{\tau}_{k_m} - \tau_{k_m})$: the probability distribution of the code phase measurement noise error.

$p(\tau_{k_m} | \underline{\mathbf{Y}}^{k_{m-1}})$: the *a priori* probability distribution of the code phase τ_{k_m} given all input data $\underline{\mathbf{Y}}^{k_{m-1}} \equiv [Y_1, Y_2, \dots, Y_{k_{m-1}}]^T$ up to the $(m-1)$ th code phase measurement.

In the consistency check, the difference, or innovation, between the measured code phase $\tilde{\tau}_{k_m}$ and the predicted code phase $\bar{\tau}_{k_m} = \mathbb{E}[\tau_{k_m} | \underline{\mathbf{Y}}^{k_{m-1}}]$ is compared against a threshold γ_T . Let $\nu_{k_m} = \tilde{\tau}_{k_m} - \bar{\tau}_{k_m}$ be the innovation. Then the output $H_{1,T}$ is asserted if $\nu_{k_m} > \gamma_T$; otherwise, $H_{1,T}$ remains low. The value of γ_T , which in general varies with time, depends on a pre-selected false alarm probability $P_{F,T}$ for the timing consistency check and on the innovation's conditional distribution, $p(\nu_{k_m} | \underline{\mathbf{Y}}^{k_{m-1}})$, which is derived from $p(\tilde{\tau}_{k_m} - \tau_{k_m})$ and $p(\tau_{k_m} | \underline{\mathbf{Y}}^{k_{m-1}})$. Commonly, the distributions involved can be modeled as Gaussian, in which case $p(\nu_{k_m} | \underline{\mathbf{Y}}^{k_{m-1}})$ can be summarized by its mean $\mathbb{E}[\nu_{k_m} | \underline{\mathbf{Y}}^{k_{m-1}}] = 0$ (assuming an unbiased estimator and unbiased measurements) and variance $\sigma_\nu^2 = \sigma_{\Delta\bar{\tau}}^2 + \sigma_{\Delta\tilde{\tau}}^2 + \sigma_m^2$, where $\sigma_{\Delta\bar{\tau}}^2 = \mathbb{E}[(\tau_{k_m} - \bar{\tau}_{k_m})^2 | \underline{\mathbf{Y}}^{k_{m-1}}]$, $\sigma_{\Delta\tilde{\tau}}^2 = \mathbb{E}[(\tilde{\tau}_{k_m} - \tau_{k_m})^2]$, and σ_m^2 is the pseudorange error due to multipath. The threshold γ_T is the value of γ for which

$$P_{F,T} = \int_{\gamma}^{\infty} p(\nu_{k_m} | \underline{\mathbf{Y}}^{k_{m-1}}) d\nu_{k_m} \quad (4)$$

Note that by comparing ν_{k_m} , not $|\nu_{k_m}|$, against the threshold, the consistency check doubles its sensitivity by making the implicit assumption that the spoofer can only delay the code phase (increase τ_{k_m}).

Another interpretation of γ_T is as the “window of acceptance” referred to in [3]. Between code phase measurement updates, the innovation's conditional distribution $p(\nu_{k_m} | \underline{\mathbf{Y}}^{k_{m-1}})$ widens as receiver clock drift and position uncertainty cause the *a priori* code phase estimate $\bar{\tau}_k$ to become less certain. The distribution can become especially wide if the receiver has a poor clock and is subjected to prolonged jamming or signal blockage. If, after re-acquisition, the innovations remains below γ_T , then the timing of the re-acquired signal is within the window of acceptance; i.e., it is consistent with the assumed uncertainty in $\bar{\tau}_k$.

It should be noted that $p(\nu_{k_m} | \underline{\mathbf{Y}}^{k_{m-1}})$ depends on all signals being tracked by the receiver, not only on the individual signal whose code phase measurement is $\tilde{\tau}_{k_m}$. This is because the *a priori* distribution $p(\tau_{k_m} | \underline{\mathbf{Y}}^{k_{m-1}})$, from which $p(\nu_{k_m} | \underline{\mathbf{Y}}^{k_{m-1}})$ is derived, is a complete summary of what the receiver knows about τ_{k_m} based on all the raw samples in $\underline{\mathbf{Y}}^{k_{m-1}}$. When a particular signal is acquired or re-acquired, its authentication depends on the time aiding provided by other signals. Vector tracking algorithms [21] are particularly well

suited for GNSS signal authentication because they combine timing information from all signals and can be designed to produce $p(\nu_{k_m} | \underline{\mathbf{Y}}^{k_{m-1}})$ as part of their routine processing.

To give a better understanding of factors that affect γ_T , two scenarios are considered. The top panel of Fig. 2 shows γ_T in a static scenario as a function of $(C/N_0)_r$ for $P_{F,T} = 0.0001$. Under H_0 (no spoofing), the analysis assumes that $p(\nu_{k_m} | H_0) = \mathcal{N}(0, \sigma_{\Delta\bar{\tau}}^2 + \sigma_{\Delta\tilde{\tau}}^2 + \sigma_m^2)$ where

- $\sigma_{\Delta\bar{\tau}}^2$ is the predicted code phase measurement error variance—a function of satellite geometry and $(C/N_0)_r$, which, for the purposes of this analysis, corresponds to a particular, but fairly typical 8-satellite arraignment and assumes every satellite has the same $(C/N_0)_r$;
- $\sigma_{\Delta\tilde{\tau}}^2 = dB_{DLL} T_c^2 / (4(C/N_0)_r)$ is the measured code phase measurement error with correlator spacing $d = 1/2$ chip, $T_c \approx 1 \mu\text{s}$, and phase-lock-loop-aided delay locked loop (DLL) bandwidth $B_{DLL} = 0.05 \text{ Hz}$; and,
- σ_m^2 is a conservative estimate of the assumed multipath error variance within a receiver that implements a multipath mitigation scheme; it is calculated by multiplying by 3 the maximum root mean square pseudorange multipath error for a typical (Fig. 5 [24]).

The plot shows how the window of acceptance must widen as $(C/N_0)_r$ decreases to maintain $P_{F,T} = 0.0001$.

The bottom plot of Fig. 2 corresponds to a scenario in which a stationary receiver falls victim to a complete satellite signal outage (e.g., via jamming or blockage) when driven by a temperature-compensated crystal oscillator (TCXO) with short-term stability $\sigma_{\text{TCXO}} = 10^{-8}$ or an oven-controlled crystal oscillator (OCXO) with short-term stability $\sigma_{\text{OCXO}} = 10^{-11}$ [22]. The plot assumes that the final tracking $(C/N_0)_r$ before the outage was 40 dB-Hz and that the outage lasts for duration T_{outage} . Clearly, the longer the interval T_{outage} , the greater γ_T must be to maintain $P_{F,T} = 0.0001$. As one might expect, OCXO-driven receivers maintain a lower γ_T for a given T_{outage} than their TCXO-driven counterparts. The bend in the OCXO plot marks a transition from an innovation distribution in which the measurement noise and initial timing uncertainty dominate to one in which the uncertainty contributed by the OCXO's frequency instability dominates.

The remaining two functional units involved in timing authentication are the J/N detector and the SCER detector. Their operation is summarized only briefly here; a fuller discussion is found in [16] and [23]. The SCER detector is a hypothesis test that decides whether the security code in the incoming samples Y_k arrives (1) intact and (2) near the *a posteriori* code phase estimate $\hat{\tau}_{k_m} = \mathbb{E}[\tau_{k_m} | \underline{\mathbf{Y}}^{k_m}]$ produced by the signal tracking and navigation processor. At least one of these two conditions is violated if a SCER attack is underway. The SCER detector performs time-weighted correlations with Y_k over the l th unpredictable security chip interval to produce a single-chip statistic S_l , which is derived in Sec. IV. of [16]. These correlations involve the error correction encoded symbols W_l , which are identical to the raw received symbols W'_l if no symbol errors are present in W_l , but, in general, include corrections to W_l made possible by the operation of error correction decoding and subsequent re-encoding.

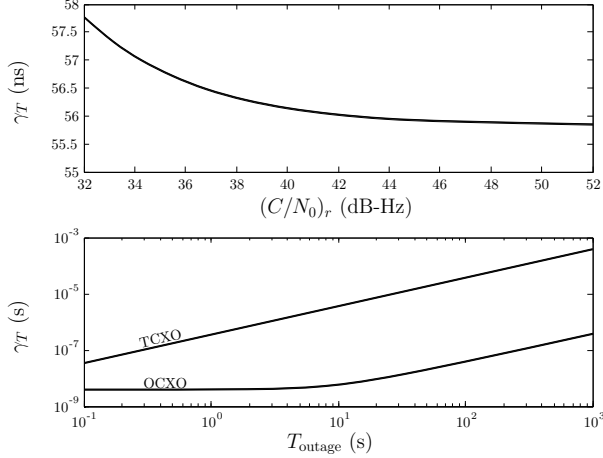


Fig. 2. Sensitivity analysis for γ_T under two static scenarios with $P_{F,T} = 0.0001$. Top panel: γ_T versus $(C/N_0)_r$ for a particular 8 satellite constellation each with $(C/N_0)_r$. Bottom panel: γ_T versus T_{outage} for a TCXO- and an OCXO-driven receiver.

The SCER detector combines a set of N single-chip correlations S_i into a detection statistic L , which it compares with a threshold γ_S that is set by a pre-selected probability of false alarm, $P_{F,S}$. If a SCER attack is underway, and if the estimation delay e is sufficiently small, then L will rise above γ_S , causing $H_{1,S}$ to assert. The SCER detector assumes that the spoofer's C/N_0 advantage over the target receiver's is limited to approximately 3 dB (i.e., $(C/N_0)_s \leq (C/N_0)_r + 3$ dB). This assumes the spoofer and defender are physically close and both use a commercially-available antenna with similar gain patterns. The at-most-3-dB advantage accounts for a scenario in which the spoofer's antenna may have a better noise figure or a better line-of-sight to the satellite, but not scenarios in which the spoofer employs a high-gain antenna array. The SCER detector further assumes that a J/N detector is monitoring the incoming in-band power so that the power advantage of the received spoofing signal ensemble is limited to approximately 4 dB above the authentic signal ensemble. Attacks in which the spoofer broadcasts its counterfeit signals with a power advantages greater than 4 dB fall outside the range of applicability of the SCER detector (Sec. VI.B. in [16]) and are easily detected at a low false alarm rate by a properly configured J/N detector [23]. This is why a J/N detector is a necessary component of an integrated signal authentication strategy. The J/N detector threshold is governed by a pre-determined false alarm probability $P_{F,J}$ [23].

In a typical application, the SCER detector performs a hypothesis test just after each code verification $\mathbb{V}(K, \underline{\mathbf{B}}^n)$. There is little point in performing the test more frequently, since the authenticity of the symbols b_j , and by extension the encoded symbols W_i used in the SCER detector correlations, cannot be guaranteed until the code verification has been performed.

The SCER detector outputs a probability of detection P_D that depends on the detector's model for the statistics of a SCER spoofing attack, which in turn depend on the possible

estimation delay e (Sec VI.C. in [16]). In setting P_D , the SCER detector pessimistically assumes that the total estimation delay in seconds eT_s could be as large as γ_T , which means that at each security code chip transition the spoofer could already have an estimate based on as much as $\min(\gamma_T, T_w)$ seconds into the upcoming chip. A degraded P_D reflects the penalty paid, in terms of ability to detect spoofing, for uncertainty in ν_{k_m} , which could be caused by an extended period of GNSS jamming or blockage. As $p(\nu_{k_m} | \underline{\mathbf{Y}}^{k_m-1})$ widens and γ_T increases, the limitations on spoofing delay d become less stringent. Knowing this, a SCER-attack spoofer can increase the estimation time e , thereby improving the reliability of its security code chip estimates. When the spoofer's $(C/N_0)_s$ is high and γ_T is large (e.g., $(C/N_0)_s > 50$ dB-Hz and $\gamma_T > 300$ μ s), then the null and spoof hypotheses become virtually indistinguishable within the SCER detector and P_D drops. Even though γ_T may subsequently contract and P_D increase, a low P_D creates a window of vulnerability after which signal authentication assurance is permanently degraded.

Other Security Code Implementations: The above components of a GNSS signal authentication system are specific to a security code based on NMA. The components are also valid for the civil public spreading code authentication technique introduced in [3] except that in this case the symbols b_j are routed directly to the SCER detector where they are used to seed a pseudorandom spreading code generator a segment of whose output gets inserted into the local spreading code replica.

For private spreading code authentication schemes such as the civil level-3 technique introduced in [3] and military GPS Y- and M-code security, the code verification block in Fig. 1 is unnecessary. The figure can be adapted to these cases by setting $H_{1,C}$ permanently low and by routing the symbols b_j directly to the SCER detector. These private-key techniques rely on storage of a secure "red key" in tamper-resistant hardware within the receiver. Segments of the symbol stream b_j are coupled with the red key in the SCER detector to produce a seed for a pseudorandom spreading code generator. Only segments of the generated code are used in the civil private-key technique of [3], whereas the continuous output of the generator constitutes the security code for GPS Y- and M-code security.

Operational Definition of GNSS Signal Authentication

With the authentication components and their interactions specified, an operational definition of GNSS signal authentication—in other words, how signals are declared authentic in practice—can now be formulated. A GNSS signal is declared authentic at a given moment if and only if, during the time elapsed since some initialization event at which the receiver was known to be tracking only genuine GNSS signals, (1) the logical output S has remained low, (2) the logical output H_1 has remained low, and (3) the real-valued output P_D has remained above an acceptable threshold (e.g., 0.9).

Some comments about this operational definition are in order. First, although there may be reasonable alternatives to this definition, they cannot be substantially different. Aside

from the variations that occur when implementing other security codes as discussed previously, the components of the proposed definition are each unique and necessary. Second, although a GNSS signal may be pronounced authentic by the above operational definition, it may in fact be counterfeit. Practical constraints of hypothesis testing prevent P_D from reaching unity. For example, for the NMA-based security codes discussed later on, nominal P_D may drop as low as 0.97. Moreover, jamming or signal blockage can temporarily reduce P_D . Inversely, even though a signal may be declared unauthentic, it may actually be authentic. In the case that S is asserted, the incoming signal is certainly unauthentic; on the other hand, H_1 will at times assert even under unspoofed conditions. It has a false alarm probability

$$P_F = 1 - (1 - P_{F,J})(1 - P_{F,C})(1 - P_{F,T})(1 - P_{F,S})$$

which is greater than any of the false alarm probabilities for the individual tests that can trigger H_1 . Third, movement of P_D below the acceptable threshold does not necessarily indicate a SCER spoofing attack, it only indicates that the SCER detector's probability of detecting a SCER attack has been compromised, and thus the currently tracked signal cannot be considered authentic.

Remarks

It is easy to appreciate the advantage of short over long security code chips given the authentication architecture proposed in Fig. 1. Short chips such as the $T_w \approx 2 \mu\text{s}$ chip of the legacy GPS Y code keep $\min(\gamma_T, T_w)$ to less than a few microseconds and thereby prevent significant degradation in P_D even during a prolonged signal blackout, whereas long chips such as $T_w \approx 20 \text{ ms}$ for NMA allow significant degradation in P_D for the same outage. This weakness of NMA-based GNSS signal authentication has been noted—although not in these formal terms—in [3] and [7]. Practically, the weakness translates into the following additional requirements for NMA-based GNSS security: For a static receiver in a known location, maintaining P_D high requires either continuous tracking of at least one strong GNSS signal or a clock that does not drift significantly during whatever complete signal outages occur. For a receiver mounted on a dynamic platform, either continuous tracking of at least 4 strong GNSS signals or a clock and inertial measurement unit (IMU) combination that does not drift significantly are required.

Given these requirements, one may question whether NMA-based GNSS security will be useful in practice. One should bear in mind that for many applications of interest the prolonged signal denial required to significantly degrade P_D would be highly suspicious. For example, consider a static receiver with a TCXO having short-term stability 10^{-8} . A spoofer would be forced to preface a spoofing attack with a 150-second complete signal denial interval in order to increase γ_T beyond $5 \mu\text{s}$ (assuming $P_{F,T} < 0.002$) and thereby cause a significant reduction in P_D [16]. If the complete signal denial is done via jamming, then the J/N detector will trigger; if done by obstructing the target receiver's antenna, this requires close physical access. In any case, the signal outage will appear suspicious.

Also, it is worth noting that security code alternatives to NMA are not foolproof and are likely to be less practical. Indeed, it appears that no exclusively cryptographic defense, no matter how short the security chip interval T_w , can detect a well-executed near-zero-delay meaconing attack. (This is why such an attack is excluded from the attack model in the discussion on components of signal authentication.) Universal vulnerability to near-zero meaconing suggests the need for a layered approach that combines cryptographic signal authentication with non-cryptographic techniques such as the vestigial signal defense [24]. It also suggests that expectations for GNSS signal authentication must be modest: the goal should not be preventing a successful attack at all cost, but making one difficult. Furthermore, a GNSS signal authentication scheme's potency must be weighed against its practicality. This tradeoff is the subject of the next section.

DESIGN AND EVALUATION OF CRYPTOGRAPHIC SIGNAL AUTHENTICATION STRATEGIES

The previous section considered general GNSS signal authentication, which relies in part on some or all of the security code w_k being unpredictable to a would-be spoofer. This section considers candidate signal authentication strategies (i.e., the design of w_k) specifically for civil GPS. These strategies are evaluated based on their:

effectiveness: how difficult they make it for a spoofer to carry off a successful spoofing attack; and their

practicality: how likely they are to be implemented.

In practice, a tradeoff emerges between effectiveness and practicality with the most effective approaches being impractical. This section elucidates this tradeoff and selects the most effective strategy from the set of practical ones.

Selecting T_w

The security code chip length T_w is fundamental to the design of a signal authentication strategy. To evaluate potential choices of T_w , the notions of effectiveness and practicality can be refined as follows. Effective strategies enable frequent signal authentication and offer receivers a high probability of detecting an attack. Such strategies significantly raise the bar for a successful spoofing attack but are not necessarily impervious to the most sophisticated attacks. Additionally, practical strategies (1) remain backward compatible, meaning legacy equipment will function correctly without modification if the approach is implemented (e.g., GPS L1 C/A remains unaltered) and (2) avoid fundamental modifications to the GPS Interface Specification (IS). The GPS Control Segment is less likely to support implementation of a civil GPS signal authentication strategy that fundamentally alters the GPS IS [13].

As noted earlier, a short T_w has the advantage that it prevents significant degradation of P_D due to timing uncertainty. Although one could define a new signal definition to support an arbitrarily small T_w , this approach is impractical because it fundamentally modifies the GPS IS. A more practical approach is to leverage one of the fundamental intervals defined for civil GPS signals in the GPS IS: the spreading code chip interval

(approximately 100 ns for L5 and approximately 1 μ s for L1 and L2) or the navigation data bit interval (20 ms for all civil frequencies).

In terms of effectiveness, setting T_w equal to the spreading code chip interval, a strategy known as spreading code authentication (SCA), is best. SCA meets the first criteria for practicality: backward-compatible SCA strategies have been proposed for GPS L5 [3], [7]. However, SCA does not satisfy the second requirement for practicality: it requires modification of the civil spreading codes, which must be considered a fundamental—and therefore unlikely—alteration of the GPS IS.

Consider instead setting T_w equal to the navigation data bit interval. This is the navigation message authentication (NMA) approach. In other words, $w_k = d_k$ where d_k are samples from the ± 1 -valued navigation message and $T_w = 20$ ms. One can either make all or part of the navigation message unpredictable to generate w_k . A possible approach encrypts all or nearly all of the navigation message with a cryptographic cipher (e.g., message recovery mode [6] or hybrid message recovery mode). This approach generates a high average rate of unpredictable navigation data bits, which reduces the required interval between signal authentication tests, but it is ultimately impractical since complete or nearly complete navigation message encryption would not be backward compatible and would require a fundamental alteration of the GPS IS.

The only practical strategy, then, is to form w_k by introducing periodic randomness into the navigation message. This NMA-based approach is assumed hereafter.

Generating Periodic Unpredictability

The previous discussion settled on a strategy of forming w_k by transmitting a periodically unpredictable navigation message. Unpredictable, however, does not mean unverifiable. A receiver can verify the origin of w_k —that is, who generated the security code—to prevent being spoofed with a forged w_k . Cryptographic digital signature protocols would enable receivers to verify the origin of signed messages. By their very nature, the signatures that enable this authentication are unpredictable prior to broadcast. The unpredictability of digital signatures allows receivers to treat the digital signature as a security code.

Before comparing various digital signature protocols for NMA, refined definitions of effectiveness and practicality with respect to digital signature protocols are offered to guide the selection process. A digital signature protocol is considered effective for signal authentication if it is standardized, is cryptographically secure, and offers a high P_D for a low P_F :

- Standardization indicates that the protocol has been well-studied by the cryptography community and is thought to be secure against even the strongest cryptographic attacks such as those described in [6] and [25]. Standardized protocols also facilitate adoption: verified open-source implementations often exist and certification programs can validate proper operation of cryptographic modules.
- The equivalent symmetric-key strength b_s in units of bits is a useful measure of the strength of a cryptographic

protocol. The U.S. National Institute of Standards and Technology (NIST) considers $b_s \geq 112$ secure for the years 2011–2030 [26]. To meet NIST guidelines, a cryptographic civil GPS signal authentication strategy must therefore set $b_s \geq 112$.

- A high P_D means that the receiver has a high likelihood of detecting a spoofing attack. The number N of chip-level correlations S_l that are combined to generate each SCER-attack detection statistic L increases with the length of the digital signature. Since a larger N tends to increase SCER-attack P_D , a longer signature leads to a higher P_D for a fixed probability of false alarm $P_{F,S}$ and threat model [16]. It is reasonable to define strategies offering $P_D \geq 0.95$ for $P_{F,S} = 0.0001$ as effective. For NMA, a digital signature that produces a signature length of approximately 400 bits will exceed this requirement in typical scenarios [16].

A practical digital signature protocol is one that does not burden the limited resources of the Control, User, or Space Segment. This paper considers a protocol to be practical if:

- its implementation does not adversely affect a standard receiver's ability to determine its position from the broadcast ephemeris;
- the percentage of the GPS navigation message required to transmit the digital signature is low (e.g., 10 percent or less);
- the computational resources of the receiver that are devoted to authentication are a small fraction of those devoted to standard GPS signal processing;
- it requires no additional receiver hardware, which would increase receiver cost, size, or weight; and,
- it allows feasible key management.

Protocols that have a short signature length for a given b_s , that have a low computational burden, and that can be implemented entirely in software are practical.

Given the foregoing definitions of the terms effective and practical as applied to digital signature protocols, the following discussion settles on a protocol appropriate for civil GPS signal authentication.

Public vs. Private Key Protocols: The primary categorization for digital signatures is their classification as either public key (i.e., asymmetric) or private key (i.e., symmetric) [17]. Private key algorithms are generally more computationally efficient and offer shorter signature length than public key protocols, but they require a shared and secure private key. This requirement makes private key digital signatures, however effective, impractical for civil GPS signal authentication because securely storing a private key requires tamper-proof receiver hardware [27]. Furthermore, key management for symmetric protocols would be complicated: if any one of the private keys were disclosed, then every receiver would need to securely update the private key. Thus, private key protocols are impractical for civil GPS signal authentication.

On the other hand, public key protocols are practical because the public key k_{public} can be stored unsecured in receiver memory. Despite the fact that the cryptographic key may be widely known, public key protocols offer as much security

as private key methods for a given b_s and are believed to be secure against even the strongest cryptographic attacks, such as those described in [6] and [25]. Although public key protocols generally have a higher computational burden than private key protocols, some public key digital signature protocols still have a low computational burden relative to the standard GPS signal processing, which makes them practical for this application. Moreover, public key techniques allow feasible key management in the form of a public key infrastructure.

Public Key Management: Key management for a civil GPS authentication scheme based on public key digital signatures would be fairly straightforward. The GPS Control Segment would publish a unique public key $k_{\text{public},i}$ for each satellite i (i.e., for each unique pseudorandom spreading code), hereafter referred to as k_{public} for convenience. A unique k_{public} for each satellite offers an additional layer of defense against cryptographic attacks. GPS receivers would then store k_{public} in local (potentially unsecured) memory. Although some proposals have suggested transmitting k_{public} over the GPS navigation message, this creates a new spoofing attack possibility whereby a spoofer broadcasts a counterfeit key to the receiver. Instead, the Control Segment should leverage the key management techniques already developed to facilitate public key protocol implementation. In general, k_{public} should be distributed through a secure secondary channel, such as over the Internet, with the guarantee of a mutually trusted third party. One frequently employed framework is called public key infrastructure (PKI) [17], [28]. In this framework, trusted Certificate Authorities would certify (i.e., sign) the Control Segment public key thereby binding k_{public} to the identity of the Control Segment and preventing a spoofer from publishing a forged key. The certified Control Segment public key would then be stored on the receiver for signature verification. Because public keys can have a valid lifetime (i.e., cryptoperiod) of several years, a receiver's stored public key can be updated infrequently [26]. Thus, a receiver need not be continuously connected to the Internet to take advantage of public key signature methods. In the case of a security breach, PKI also offers key revocation techniques upon which the Control Segment can rely [29].

Public Key Digital Signature Generation and Validation:

An overview of public key digital signatures will clarify the code verification block of Fig. 1. To digitally sign d_k and embed the signature in the navigation message thereby forming w_k , the GPS Control Segment would compute a private key k_{private} that remains secret and a public key k_{public} that is distributed to users and stored in receiver memory. To sign a message m , the Control Segment would compute a digital signature s based on m and k_{private} with a signing algorithm \mathbb{S} :

$$\mathbb{S}(k_{\text{private}}, m) = s. \quad (5)$$

The Control Segment would then transmit the signed message $\{m, s\}$ over d_k , thereby forming w_k . Public key cryptography assures that even with precise knowledge of k_{public} and of m , there is no computationally feasible method for a spoofer to predict s ; or, once s is known, to infer k_{private} . Once the receiver obtains an unauthenticated signed message

$\mathbb{B}^n = \{m', s'\}$, it runs a code verification protocol \mathbb{V} as in Fig. 1 to validate the message origin:

$$\mathbb{V}(k_{\text{public}}, m', s') = \{\text{true}, \text{false}\}. \quad (6)$$

If \mathbb{V} asserts, then $H_{1,C}$ remains low and the receiver can trust that the Control Segment generated $\{m', s'\}$ (i.e., $\{m', s'\} = \{m, s\}$).

EVALUATING DIGITAL SIGNATURE PROTOCOLS

By focusing on high-level design criteria, the discussion of cryptographic signal authentication thus far has settled on a NMA technique whereby a public key digital signature is embedded in the navigation message. This section evaluates four potential digital signature protocols that could generate the signed navigation message: a delayed-disclosure symmetric-key protocol called TESLA and three public key protocols called RSA, DSA, and ECDSA. The most effective and practical protocol for civil GPS signal authentication is sought.

TESLA

The Timed Efficient Stream Loss-Tolerant Authentication (TESLA) protocol, described in [30] and adapted for radionavigation authentication in [14] and [31], is similar to the S/KEY protocol from [25], in that it uses a one-way chain of symmetric keys k_n . A chain of intermediate keys is generated by applying a secure hash function H iteratively N times to a seed key k_0 to yield $N - 1$ intermediate keys such that for $m \leq n$, $H^{n-m}(k_m) = k_n$, along with a base key k_N that can be used to authenticate any intermediate key [e.g., $H^2(k) = H(H(k))$]. Intermediate keys are broadcast in reverse order $\{k_N, k_{N-1}, k_{N-2}, \dots\}$. Verification can be achieved by comparison to any previously-released key: if k_{n+m} has already been validated and $H^m(k_n) = k_{n+m}$, then k_n must also be valid. Intermediate keys are broadcast as part of the navigation message, and because they are generated using a one-way function, they are unpredictable in advance but verifiable afterward.

To authenticate the navigation message, an unreleased intermediate key k_i is used to compute a message authentication code (MAC) for part of the navigation message. MAC_i corresponding to k_i is then broadcast over the data bits. According to the key-release schedule, k_i is broadcast after MAC_i is broadcast. When k_i is received, MAC_i can be validated. Since MACs are based on private-key algorithms that do not provide data non-repudiation (i.e., a valid MAC can be generated by any user with knowledge of the private key), only received MACs corresponding to keys not yet broadcast can be considered suitable for authentication. When used for both timing and navigation message origin authentication, keys and MACs need verification; each of these tasks is independent and could be computationally intensive.

Although TESLA is a novel approach, it does not meet all of the design criteria discussed in the previous section. Foremost, TESLA is not standardized. The protocol was designed for broadcast authentication and has been tested and studied only in that context, including a trial implementation on an

eLORAN system. A concrete suggestion for implementation is given in [5], [31].

In addition, TESLA may not be effective in the sense defined above because of its low equivalent symmetric key strength b_s . Various proposals suggest that sufficient cryptographic strength can be achieved with keys that are 160 bits or shorter, which implies that the output of the secure hash function that generates the keys is also 160 bits (i.e., $b_s = 80$). But, hash functions used in signal authentication cannot have an output less than 224 bits as this is the minimum length necessary to achieve $b_s = 112$. Becker *et al.* suggest that the short cryptoperiod of individual keys and frequency of key updates dispels this concern [31]. However, if the hash protocol were broken off-the-air because $b_s < 112$, then the short cryptoperiod may no longer assure their security: all keys could potentially be computed before their release. If TESLA were designed for a $b_s \geq 112$, then the computational burden to support TESLA would likely increase.

Another concern for TESLA is the computational burden of key management. The public key k_N , distributed to receivers over a PKI scheme and then stored to the receiver, can authenticate any intermediate key. One proposal suggests intermediate keys are generated once per second and the public key k_N would be valid for several years [31]. If this were the case and a receiver obtained a one-year-old k_N from the PKI, then it would need approximately 2^{25} computations of H in order to generate the current intermediate key. This would impose a large computational burden on the receiver relative to standard GNSS signal processing. Although k_N could be published more often, frequent key updates discourage adoption.

RSA

The Rivest, Shamir, and Adleman (RSA) algorithm has become a *de facto* standard for data security [17], [25]. It was one of the first public key algorithms and can be applied for pure encryption and signature generation. It is believed that the only way to defeat RSA is to factor a number with large prime factors. As factoring has become faster, the length of RSA keys needed to preserve security has increased. RSA requires a 2048-bit modulus to achieve $b_s = 112$ and would therefore occupy a significant portion of the low-data-rate navigation message (i.e., the RSA digital signature is too long to be practically broadcast over the navigation message). Thus, RSA is impractical according to the earlier discussion of practical digital signatures.

DSA

The Digital Signature Algorithm (DSA) belongs to a class of algorithms that rely on the difficulty of finding logarithms in finite groups [17], [25]. It was developed by the U.S. National Security Agency (NSA) for NIST and adopted for use in U.S. government applications in 1993. Its widespread use indicates that it is cryptographically valid and strong, as it has been implemented in a variety of critical applications.

DSA has two domain parameters that determine the strength of the algorithm. In order to achieve $b_s = 112$, it is necessary to use a 2048-bit prime p and a 224-bit prime q . Verification

of digital signatures relies on p , making DSA comparable in computational complexity to RSA. Yet, DSA signatures are only twice as long as q (i.e., 448-bit signature for $b_s = 112$). Despite having signature length shorter than RSA, DSA is still not practical enough for cryptographic signal authentication because of its computational complexity.

ECDSA

Based on DSA, the Elliptic Curve Digital Signature Algorithm (ECDSA) operates on groups associated with an elliptic curve space [17]. For a given b_s , ECDSA signatures are the same length as DSA signatures. But by operating on a more complicated underlying elliptic curve space, ECDSA has smaller domain parameters and more efficient verification algorithms [32]–[34]. Furthermore, NSA recommends that systems built after 2010 implement ECDSA, which has been standardized by NIST [35].

Selecting the Appropriate Signature

With short signatures, efficient verification, and standardization, ECDSA appears to be both an effective and a practical digital signature protocol for NMA-based civil GPS signal authentication. Given the discussion above, ECDSA appears to be the best among current options although other signature schemes could be used if weaknesses in ECDSA were found. NIST offers several choices of standardized ECDSA domain parameters for a key strength $b_s \geq 112$ [35]. Among these, the standardized ECDSA 233-bit Koblitz curve (K-233) is attractive because it generates a short 466-bit signature amenable to optimized software-defined verification routines [36].

To sign messages, ECDSA first applies a secure hash function to generate a digital fingerprint of the message, which is typically shorter than the message itself, and then signs the fingerprint rather than the whole message. For proper implementation the following two conditions must be met: (1) the length of the signed navigation message must be at least as long as the output of the hash function (i.e., $2b_s$), and (2) each signed navigation message must vary in at least a single bit from previous messages to generate an unpredictable signature. These conditions are easily satisfied. The randomness introduced by the hash function along with the additional randomness introduced by the so-called salt, described in the next section, causes the signature to remain unpredictable even with knowledge of previous signed navigation messages.

In selecting the appropriate hash function for GPS signal authentication, NIST offers a standardized cryptographic hash family named SHA-2 [37]. Setting $b_s \geq 112$ implies implementing SHA-2 with at least a 224-bit key (i.e., SHA-224). Since there is no computational difference between SHA-224 and the stronger SHA-256, SHA-256 is proposed for implementation. Although the SHA-256 fingerprint is longer than the SHA-224 fingerprint, the digital signature length remains the length of the ECDSA signature, which is 466 bits long.

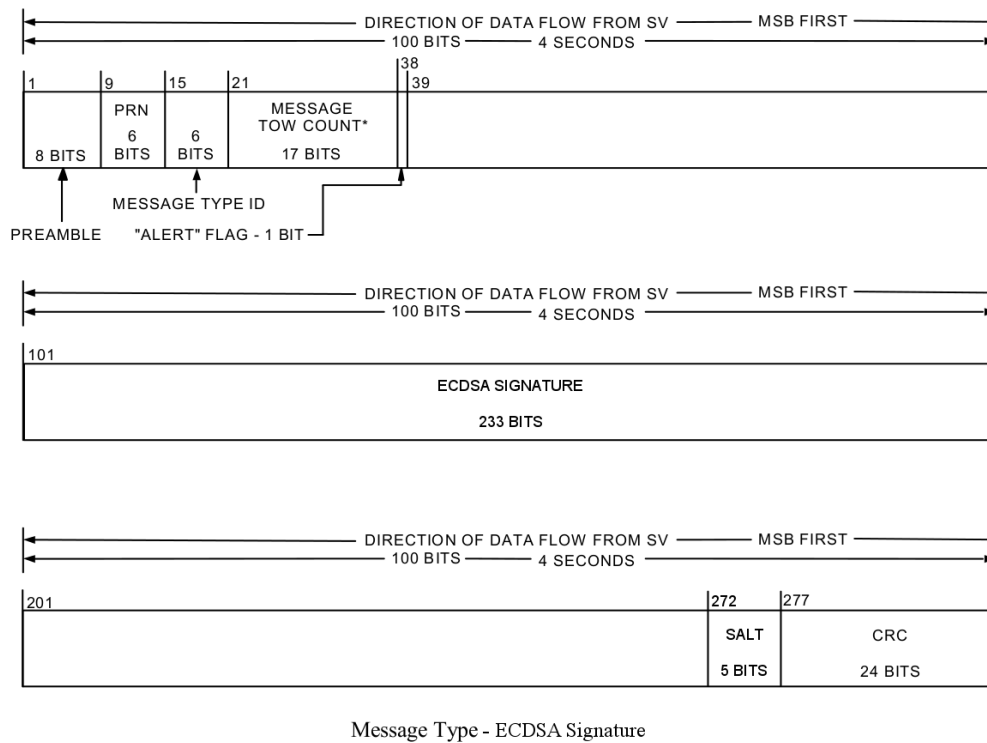


Fig. 3. Diagram showing the format of the proposed CNAV ECDSA signature message, which delivers the first or second half of the 466-bit ECDSA signature and a 5-bit salt in the 238-bit payload field (figure adapted from [19]).

A CRYPTOGRAPHIC CIVIL GPS SIGNAL AUTHENTICATION PROPOSAL

This section proposes a concrete strategy for cryptographic civil GPS signal authentication. Consistent with the conclusions of the previous two sections, the strategy is based on public key elliptic curve cryptographic signatures inserted periodically into the flexible GPS civil navigation (CNAV) message. Specific details of the strategy, offered here, facilitate near-term adoption by the GPS Control Segment. The proposed strategy enables civil GPS signal authentication as described in the second section and diagrammed in Fig. 1 with the following properties: (1) a probability of detection of $P_D > 0.97$ for $P_{F,S} = 0.0001$, (2) a cryptographic strength of $b_s = 112$ bits, and (3) authentication every five minutes per channel.

Digital Signature Conveyance via CNAV

The flexible CNAV message format that modulates modernized GPS signals offers a convenient conveyance for a digital signature. The CNAV format was designed to be extensible so that new messages can be defined within the framework of the GPS IS. The CNAV message format is broadcast from Block IIR-M GPS spacecraft at the L2 frequency and Block IIF GPS spacecraft at the L2 and L5 frequencies [19]. Plans call for CNAV to be broadcast from Block IIIA GPS spacecraft at the L2 and L5 frequencies and additionally at L1. Thus, future single-frequency receivers can benefit from the extension to the CNAV message proposed in this section.

Every 12 seconds, a CNAV message delivers a 300-bit packet, which includes a 38-bit header, a 238-bit payload, and a 24-bit cyclic redundancy check (CRC). The flexibility of CNAV is due in part to the information broadcast over the header, which delivers a 6-bit message type identification field identifying up to 64 unique message types. The current GPS IS defines only 15 of these messages, reserving the others for future applications [19].

The following proposal defines two new CNAV messages to deliver an ECDSA signature. This is not a fundamental change to the GPS IS, but rather an extension to CNAV. Thus, this extension to CNAV can be considered practical in the sense defined earlier.

CNAV Message Signature Type Definition

Since the CNAV structure does not support payloads larger than 238 bits, the 466-bit ECDSA signature selected at the end of the last section must be broadcast across two CNAV messages. It is proposed to define two CNAV messages that deliver the 466-bit ECDSA signature, each message having the format shown in Fig. 3. The first ECDSA CNAV message type contains the first 233-bit half of the signature and the second message type contains the second half of the signature.

A 466-bit signature broadcast over two 238-bit payloads leaves 10 bits undefined. It is proposed to uniquely and randomly generate these bits for each instance of a signed message with a standardized pseudorandom number generator [38]. This technique is known as adding cryptographic “salt.” Since the 10 salt bits are unpredictable prior to broadcast,

they contribute to the total number of unpredictable w_k symbols available to a receiver to perform SCER detection tests. However, they do not increase b_s since they are not part of the digital signature. Like other components of the navigation message, they are digitally signed and can therefore be authenticated as originating from the Control Segment. Together, the two CNAV signature messages transmit 476 unpredictable bits.

Signing the CNAV Message

The frequency at which the CNAV navigation message can broadcast signatures requires consideration of several factors. First, although the CNAV message format is flexible, it is not without constraints. Ephemeris message types 10 and 11 and a timing message of type 30–39 must be broadcast at least every 48 seconds to ensure accurate GPS receiver operation [5], [19]. Since a practical signal authentication strategy cannot adversely affect a receiver’s position solution, the CNAV signature must respect these requirements. Given these constraints, the smallest block of data in which a complete signature can be embedded is the 96-second signature block such as the one shown in Fig. 4. In this structure, the two CNAV signature messages are interleaved between the ephemeris and clock data to meet the broadcast requirements.

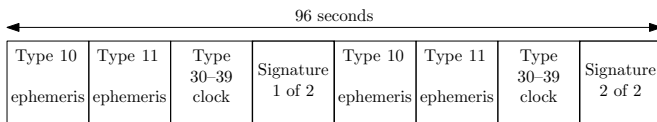


Fig. 4. Schematic illustrating the shortest broadcast signature block that does not violate the CNAV ephemeris and timing broadcast requirements. To meet the required broadcast interval of 48 seconds for message types 10, 11, and one of 30–39, the ECDSA signature is broadcast over a 96-second signature block that is composed of eight CNAV messages.

A second consideration when signing the CNAV message is the duration between signature blocks. This choice involves a tradeoff between effectiveness (i.e., offering frequent authentication) and practicality (i.e., imposing a low computational burden relative to standard GPS signal processing and maintaining a low percentage of the CNAV message reserved for the digital signature). The maximum rate at which the CNAV message can be signed corresponds to a scenario in which the 96-second signature block in Fig. 4 is broadcast continuously back-to-back. However, this strategy is not practical: besides the high percentage of the navigation message reserved for the signature (i.e., 25 percent), this back-to-back configuration would eliminate the possibility of sending any other message types than 10, 11, 30–39, and the signature. Instead, a reasonable approach would be to sign every 336 seconds (about every five minutes). In this case, one signature block would authenticate every 28 CNAV messages as illustrated in Fig. 5. This means the percentage of the navigation message devoted to the digital signature is a more practical 7.5 percent.

To broadcast a signature every five minutes, the Control Segment would first compute the next five minutes worth of CNAV navigation message including the salt. It would then concatenate signable navigation message bits in order—that

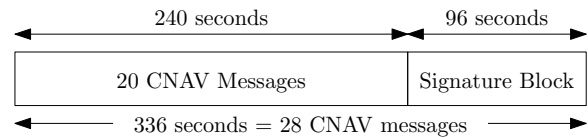


Fig. 5. Schematic illustrating a signed 336 second broadcast. The proposed strategy signs every 28 CNAV messages with a signature broadcast over two CNAV messages on each broadcast channel.

is the first 23 CNAV messages (i.e., the 20 CNAV message in Fig. 5 and the first three in Fig. 4), the first signature header, the first five bits of the salt, the 5th through 7th CNAV messages from Fig. 4, the second signature header, and remaining five salt bits—and then generate the SHA-256 fingerprint. After generating the ECDSA signature from the fingerprint, the Control Segment would break the signature into two parts and insert each part into a ECDSA signature message shown in Fig. 3. These two signature messages would then be transmitted at the appropriate times as part of the CNAV message signature block as seen in Fig. 4.

Note that the signature and corresponding CRC are not themselves signed. This is because neither is known until after signature generation. Unlike the signature field, which is entirely unpredictable, the CRC can be deterministically computed by a receiver immediately upon receiving the last unpredictable bit of any CNAV message. Thus, the CRC symbols cannot be used for SCER detection.

It is worth noting that a single uncorrected bit error would cause the verification algorithm to fail. CNAV has the option of being broadcast with forward error correction enabled. As described in the second section, FEC would enhance the robustness of NMA-based signal authentication. It is therefore recommended that FEC be enabled to support civil GPS signal authentication.

Constellation-Wide Signature Scheduling

Under the proposed strategy, each channel is authenticated every five minutes. However, the per-channel signature block could be offset from other channels (i.e., other satellites in the GPS constellation) such that a receiver tracking several satellites would see signatures more frequently. This offset strategy would substantially constrain the degrees-of-freedom that a spoofer could manipulate. An optimal offset strategy would minimize the maximum time between authentications T_{ba} [i.e., $\min(\max(T_{ba}))$] that a receiver at any point on earth between a certain upper and lower latitude would observe based on the current constellation spatial arrangement. The optimal satellite offset assignment problem can be reduced to a directional graph coloring problem [39] that is likely best solved via a genetic algorithm similar to the one proposed for use in future optimization of the GPS constellation itself [40]. A sub-optimal solution computed through a greedy algorithm for the constellation in August 2011 computed that $\min(\max(T_{ba})) = 144$ seconds was possible between $\pm 70^\circ$ latitude. Thus, even with a simple sub-optimal signature offset assignment, a receiver could receive signatures with a T_{ba} of at most about two minutes and a T_{ba} on average of about one minute.

Authentication Performance

The proposed civil GPS signal authentication strategy broadcasts 476 unpredictable symbols approximately every five minutes. Given this, the P_D output in Fig. 1 can now be computed for a given threat model based on the statistical tests in [16]. To appreciate the effectiveness of the proposed authentication strategy, consider the following challenging scenario from the target receiver's perspective:

- the spoofer has a 3 dB carrier-to-noise ratio advantage over the receiver (i.e., $(C/N_0)_s = (C/N_0)_r + 3$ dB);
- the received spoofed signals are 1.1 times stronger than the received authentic signals;
- the spoofer has introduced a timing error of $1 \mu\text{s}$ in the receiver through jamming or other means and exploits this entire delay to improve its estimates of the security code chip values (i.e., the quantity e from the discussion of the SCER attack is equal to $1 \mu\text{s}$); and,
- the false alarm probability for the SCER detector in Fig. 1 is $P_{F,S} = 0.0001$.

The statistics developed in [16] can be used to show that, under this scenario, the output P_D in Fig. 1 will be maintained above 0.97 over the range 34–51 dB-Hz of authentic signal carrier-to-noise ratio $(C/N_0)_r$ values as seen in Fig. 6. This indicates that the proposed NMA-based strategy enables effective anti-spoofing.

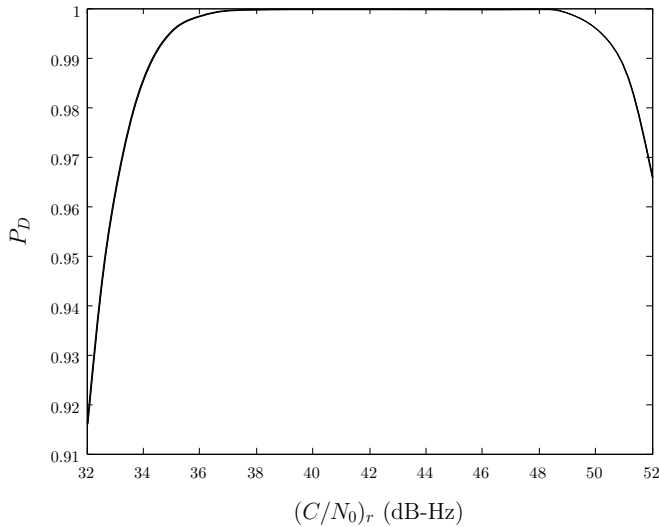


Fig. 6. P_D as a function of $(C/N_0)_r$ for a challenging spoofing attack scenario. The proposed civil GPS signal authentication strategy maintains $P_D > 0.97$ for $P_{F,S} = 0.0001$ over 34–51 dB-Hz $(C/N_0)_r$ as shown.

Implementation Details

The receiver modifications required to exploit the proposed civil GPS signal authentication strategy can be readily implemented on a software-defined receiver such as those presented in [41], [42] and [43]. A traditional receiver with application-specific correlation hardware would require some redesign to take advantage of the proposal. First, the correlation hardware would need to be modified to accommodate the new correlations needed for SCER detection [16]. Second, a traditional

receiver would need to monitor J/N . This could be a natural extension of the GNSS spectrum monitoring that some GNSS receivers already offer [44], [45]. Third, the traditional receiver would need to implement the remaining elements of Fig. 1 such as signature verification and the timing consistency check in its baseband processor, which is typically a general-purpose processor that is modifiable via firmware updates.

Although software receivers can be immediately modified to exploit the proposed authentication strategy and traditional receivers can be replaced as next-generation receivers are manufactured, there is a large number of receivers installed in critical applications that are not easily upgradeable. The GPS Assimilator introduced in [46] could be employed to protect such receivers by monitoring and sanitizing the incoming RF signals before they are ingested by the receiver.

The computational burden of verifying an ECDSA digital signature has been compared in a laboratory experiment to the computational burden of tracking GPS satellites. For this, P-256 ECDSA (i.e., a prime-curve-based ECDSA with a 256-bit key) was implemented in C++ with the GNU Multiple Precision Arithmetic Library (GMPlib). The code design was not optimized for implementation in a secure application. P-256 was implemented instead of K-233, the algorithm proposed earlier, because a reference design and test vectors were available to verify P-256. An actual ECDSA implementation of K-233 is likely even faster than P-256 because of optimizations that could be applied to Koblitz-based curve calculations [36], [47]; thus, if P-256 is shown to be computationally acceptable, then so will K-233 [48]. The computational expense of verifying a P-256 signature under this implementation was compared to the signal processing burden of the routine signal tracking in the post-processing software-defined GPS receiver presented in [42]. Over a 336-second authentication segment on one channel, the CPU time spent on routine signal processing was approximately two seconds. By comparison, the CPU time spent verifying the ECDSA signature was approximately 10 milliseconds. Thus, the expected verification burden is roughly 0.5 percent of the overall signal processing burden per channel.

It should be noted that one drawback of ECDSA is the intellectual property landscape. A company called Certicom holds 130 elliptic-curve-related patents. Although NSA purchased a license to allow ECDSA use in national security applications, the license only covers prime-curve ECDSA signatures with key sizes of 256, 384, or 512 bits [49]. A civil GPS signal authentication strategy that implemented ECDSA signatures would likely be included under the purview of the NSA license. However, the smallest key size among NSA-licensed curves is 256 bits, which would generate a 512-bit signature requiring three CNAV messages for broadcast.

Finally, as discussed in [16] and [23], the cryptographic anti-spoofing techniques proposed here can be augmented with a software-defined non-cryptographic technique such as the vestigial signal defense [24] for additional protection during the initial stages of a code-phase-aligned spoofing attack when the SCER detector P_D can drop to around 0.5.

CONCLUSIONS

This paper refines the meaning of GPS signal authentication and offers a practical technique to authenticate civil GPS signals. The proposed technique embeds digital signatures in the GPS civil navigation (CNAV) message and exploits a recently-developed statistical hypothesis test to secure civil GPS receivers against replay-type spoofing attacks. In a challenging example scenario, the technique was shown to detect a replay-type spoofing attack with probability of detection greater than 0.97 for a false alarm probability of 0.0001. The proposed strategy enables receivers to authenticate each individual civil GPS signal every five minutes.

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