Practical Development and Application of Fragility Functions

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INTRODUCTION

A second-generation performance-based earthquake engineering (PBEE-2) procedure has been developed by the Pacific Earthquake Engineering Research (PEER) Center and others that estimates the probabilistic future seismic performance of buildings and bridges in terms of system-level decision variables such as repair cost, casualties, and loss of use (dollars, deaths and downtime). The Applied Technology Council has undertaken to transfer the methodology to professional practice [ATC 2005]. The methodology involves four stages: hazard analysis, structural analysis, damage analysis, and loss analysis. This paper addresses the damage analysis, whose inputs are demand parameters calculated in a suite of multiple-degree-of-freedom nonlinear dynamic analyses, and whose output is the damage state of each damageable structural and nonstructural component in the facility. The analysis uses fragility functions, which in this context give the probability that a component, element or system will be damaged to a given or more severe damage state as a function of a single predictive demand parameter such as story drift or floor acceleration. One such fragility function is required for each component type and damage state. Many building-component fragility functions have been created in the past, but no comprehensive set of procedures exists on how to create them. This paper summarizes such a standard developed for ATC-58. See Porter et al. [2006] for more detail, examples, commentary, and alternative approaches.

FRAGILITY FUNCTION DEFINITION

Fragility functions are probability distributions that are used to indicate the probability that a component, element or system will be damaged to a given or more severe damage state as a function of a single predictive demand parameter such as story drift or floor acceleration. Here, fragility functions take the form of lognormal cumulative distribution functions, having a median value θ and logarithmic standard deviation, β. The mathematical form for such a fragility function is:

$$F_i(D) = \Phi\left(\frac{\ln(D/\theta_i)}{\beta_i}\right) \tag{1}$$

where: $F_i(D)$ is the conditional probability that the component will be damaged to damage state "i" or a more severe damage state as a function of demand parameter, D; Φ denotes the standard normal (Gaussian) cumulative distribution function, θ_i denotes the

median value of the probability distribution, and β_i denotes the logarithmic standard deviation. Both θ and β are established for each component type and damage state using one of the methods presented later. The probability that a component will be damaged to damage state "i" and not to a more or less severe level given that it experiences demand, D is given by:

$$P[i|D] = F_i(D) - F_{i+1}(D) \tag{2}$$

where $F_{i+1}(D)$ is the conditional probability that the component will be damaged to damage state "i+1" or a more severe state and $F_i(D)$ is as previously defined. Note that, when β_{i+1} is unequal to β_i, (2) can produce a meaningless negative probability at some levels of D. This case is addressed later.

The lognormal is used here because it fits a variety of structural component failure data well (e.g., [Beck et al. 2002], [Aslani 2005], [Pagni and Lowes 2006]), as well as nonstructural failure data ([Badillo-Almaraz et al. 2006], [Porter and Kiremidjian 2001], [Reed et al. 1991 Appendix J]), and building collapse by incremental dynamic analysis (e.g., [Cornell et al. 2005]). It has strong precedent in seismic risk analysis (e.g., [Kennedy and Short 1994]; [Kircher et al. 1997]). Finally, there is a strong theoretical reason to use the lognormal: it has zero probability density at and below zero EDP, is fully defined by measures of the first and second moments—$\ln(x_m)$ and β—and imposes the minimum information given these constraints, in the information-theory sense (Goodman 1985).

Figure 1(a) shows the form of a typical fragility function when plotted in the form of a cumulative distribution function; and (b), the calculation of the probability that a component will be in damage state "i" at a particular level of demand, d.
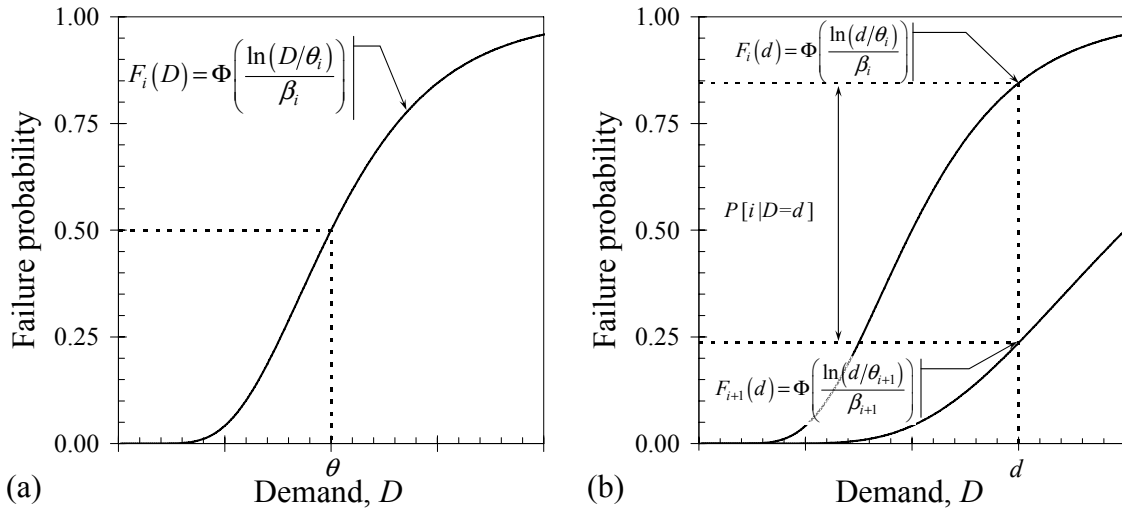

FIGURE 1
(A) EXAMPLE FRAGILITY FUNCTION, AND (B) EVALUATING INDIVIDUAL DAMAGE-STATE PROBABILITIES

The logarithmic standard deviation, β, represents uncertainty in the actual value of demand, D, at which a damage state is likely to initiate in a component. This uncertainty is a result of variability in the quality of construction and installation of the components in a building, as well as variability in the loading history that the component may

experience before it fails. When fragility parameters are determined on the basis of a limited set of laboratory test data, two components of the variance should be considered. The first of these, termed herein β_r, represents the random variability that is observed in the available data from which the fragility parameters are determined. The second portion, β_u, represents uncertainty that the tests represent the actual conditions of installation and loading that a real component in a building will experience. The logarithmic standard deviation parameter β, is computed as:

$$\beta = \sqrt{\beta_r^{\ 2} + \beta_u^{\ 2}} \qquad\qquad\qquad (3)$$

The ATC-58 guidelines recommend the following minimum values of the uncertainty parameter β_u. A minimum value of 0.25 should be used if any of the following apply:

- Test data are available for five (5) or fewer specimens.
- In an actual building, the component can be installed in a number of different configurations, however, all specimens examined for the fragility function had the same configuration.
- All test specimens were subjected to the same loading protocol.
- Actual behavior of the component is expected to be dependent on two or more demand parameters, e.g. simultaneous drift in two orthogonal directions, however, specimens were loaded with only one of these parameters.

If none of the above conditions apply, a value of β_u of 0.10 may be used.

DERIVATION METHODS

Fragility functions can best be derived when there is a large quantity of appropriate test data available on the behavior of the component of interest at varying levels of demand. *FEMA 461* provides recommended protocols for performing such tests and recording the data obtained. Since testing is expensive and time consuming, there is not a great body of test data presently available to serve as the basis for determining fragility functions for many building components. Therefore, these guidelines provide procedures for developing the median (θ) and logarithmic standard deviation (β) values for a fragility under five different conditions of data. These are:

A. *Actual demand data*: When test data is available from M number of specimens and each tested component actually experienced the damage state of interest at a known value of demand, D.

B. *Bounding demand data*: When test data or earthquake experience data are available from M number of specimens, however, the damage state of interest only occurred in some specimens. For the other specimens, testing was terminated before the damage state occurred or the earthquake did not damage the specimens. The value of the demand, D_i, to which each specimen was subjected is known for each specimen. (Need not be the demand at which the damage state initiated.)

C. *Capable demand data*: When test data or earthquake experience data are available from M number of specimens, however, the damage state of interest did not occur in any of the specimens. The maximum value of demand, D_i, to which each specimen was subjected is known.

D. *Derivation (analysis)*: When no laboratory or earthquake experience data are available, however, it is possible to model the behavior and estimate the level of demand at which the damage state of interest will occur.

E. *Expert opinion*: When no data are available and analysis of the behavior is not feasible, however, one or more knowledgeable individuals can offer an opinion as to level of demand at which damage is likely to occur, based either on experience or judgment.

U. *Updating*: Where an existing fragility function and new bounding data are available. Updating revises the existing fragility function using the new data.

Before providing guidelines for each condition, this section provides recommendations for documenting the basis for fragility function.

1. *Description of applicability.* Describe the type of component that the fragility function addresses including any limitations on the type of installation to which the fragility applies.

2. *Description of specimens.* Describe the specimens used to establish the fragility including identifying the number of specimens examined, their locations, and the specific details of the specimen fabrication/construction, mounting and installation.

3. *Demands and load application.* Detail the loading protocol or characteristics of earthquake motion applied to each specimen. Identify the demand parameters examined that might be most closely related to failure probability and define how demand is calculated or inferred from the loading protocol or excitation. Indicate whether the reported demand quantities are the value at which damage occurred (Method A data) or the maximum to which each specimen was subjected.

4. *Damage state.* Fully describe each damage state for which fragilities are developed including the kinds of physical damage observed and any force-deformation quantities recorded. Define damage states quantitatively in terms of the repairs required or potential downtime or casualty consequences.

5. *Observation summary, analysis method, and results.* Present a tabular or graphical listing of specimens, demand parameters, and damage states. Identify the method(s) used to derive the fragility parameters. Present resulting fragility function parameters θ and β and results of tests to establish fragility function quality (discussed below). Provide sample calculations.

FRAGILITY PARAMETER DERIVATION

Method A. Actual Demand Data

This section defines the procedures for deriving fragility parameters (θ, β) when data are available from a suitable series of tests and in each specimen, the damage state of interest was initiated at a known value of the demand. In this case, the median value of the demand at which the damage state is likely to initiate, θ, is given by the equation:

$$\theta = e^{\left(\frac{1}{M}\sum_{i=1}^{M}\ln d_i\right)}$$ (4)

Where:

> M = total number of specimens tested to at least the initiation of the damage state
> d_i = demand in test "i" at which the damage state was first observed to occur.

The value of the random logarithmic standard deviation, β_r, is given by:

$$\beta_r = \sqrt{\left(\frac{1}{M-1}\sum_{i=1}^{M}\left(\ln\left(\frac{d_i}{\theta}\right)\right)^2\right)^2}$$ (5)

where M, r_i and θ are as defined above.

If one or more of the r_i data appear to lie far from the bulk of the data, either above or below, apply the Pierce's Criterion for dealing with outliers (detailed later). Finally, test the resulting fragility parameters using the Lilliefors goodness-of-fit test (detailed later). If it passes at the 5% significance level, the fragility function may be deemed acceptable.

Example: Determine the parameters θ and β, from a series of 10 tests, all of which produced the damage state of interest. Demands at which the damage state initiated are respectively story drifts of: 0.9, 0.9, 1.0, 1.1, 1.1, 1.2, 1.3, 1.4, 1.7, and 2 percent.

Test #	Demand di	ln(di)	ln(di/θi)	ln(di/θi)^2
1	0.9	-0.10536	-0.30384	0.092321
2	0.9	-0.10536	-0.30384	0.092321
3	1	0	-0.19848	0.039396
4	1.1	0.09531	-0.10317	0.010645
5	1.1	0.09531	-0.10317	0.010645
6	1.2	0.182322	-0.01616	0.000261
7	1.3	0.262364	0.063881	0.004081
8	1.4	0.336472	0.137989	0.019041
9	1.7	0.530628	0.332145	0.11032
10	2	0.693147	0.494664	0.244692
Σ		1.984833		0.623723

$$\theta = e^{\left(\frac{1}{M}\sum_{i=1}^{M}\ln d_i\right)} = e^{\left(\frac{1}{10}(1.9848)\right)} = 1.22$$

$$\beta_r = \sqrt{\frac{1}{M-1}\sum_{i=1}^{M}\left(\ln\left(\frac{d_i}{\theta}\right)\right)^2} = \sqrt{\frac{1}{(10-1)}(0.6237)} = 0.26$$

Method B. Bounding Demand Data

This section defines the procedures for deriving fragility parameters (θ, β) when data are available from a suitable series of tests or earthquake experience, however, the damage state of interest was initiated in only some of the specimens. For the other specimens,

testing or the earthquake ended before the damage state of interest occurred. For each specimen "i", it is necessary to know the value of the demand, d_i to which the specimen was subjected, and whether or not the damage state did occur in the specimen.

Divide the data into a series of N bins, where N is taken as the largest integer that is less than or equal to the square root of M, and M is the total number of specimens available.

In order to divide the specimens into the several bins, sort the specimen data in order of ascending maximum demand value, d_i, for each test, then divide the list into N groups of approximately equal size. Each group "j" will have M_j specimens, where:

$$\sum_{i=1}^{N} M_j = M \tag{6}$$

Next, determine the average value of the maximum demand for each bin of specimens:

$$\bar{d}_j = \frac{1}{M_j} \sum_{k=1}^{M_j} d_k \tag{7}$$

and x_j, the natural logarithm of d_j (i.e., $\ln(d_j)$). Also determine the number of specimens within each bin, m_j, in which the damage state of interest was achieved and the inverse standard normal distribution, y_j, of the failed fraction specimens in the bin:

$$y_j = \Phi^{-1}\left(\frac{m_j + 1}{M_j + 1}\right) \tag{8}$$

That is, determine the number of standard deviations, above the mean that the stated fraction lies, assuming a mean value, $\mu=0$ and a standard deviation, $\sigma=1$. This can be determined using the "normsinv" function on a Microsoft Excel spreadsheet or by referring to standard tables of the normal distribution. Next, fit a straight line to the data points, x_j, y_j, using a least-squares approach. The straight line will have the form:

$$y = bx + c \tag{9}$$

where b is the slope of the line and c is the y intercept. The slope b is given by:

$$b = \frac{\sum_{i=1}^{M}(x_j - \bar{x})(y_j - \bar{y})}{\sum_{i=1}^{M}(x_j - \bar{x})^2} \tag{10}$$

$$\bar{x} = \frac{1}{M} \sum_{j=1}^{M} x_j \tag{11}$$

$$\bar{y} = \frac{1}{M} \sum_{j=1}^{M} y_j \tag{12}$$

Determine the value of the random logarithmic standard deviation, β_r as:

$$\beta_r = \frac{1}{b} = \frac{\sum_{i=1}^{M}(x_j - \bar{x})}{\sum_{i=1}^{M}(x_j - \bar{x})(y_j - \bar{y})} \tag{13}$$

The value of the median, θ, is taken as:

$$\theta = e^{-c\beta_r} = e^{(\bar{x} - \bar{y}\beta_r)} \tag{14}$$

Example: Consider the damage statistics shown in Figure 2. The figure depicts the hypothetical performance of motor control centers (MCCs) observed after various earthquakes in 45 facilities. Each box represents one specimen. Several damage states are represented. Crosshatched boxes represent MCCs that experienced a noticeable earthquake effect such as shifting but that remained operable. Black boxes represent those that were found to be inoperable following the earthquake. Each stack of boxes represents one facility. Calculate the fragility function using *PGA* as the demand parameter, binning between halfway points between *PGA* values shown in the figure.
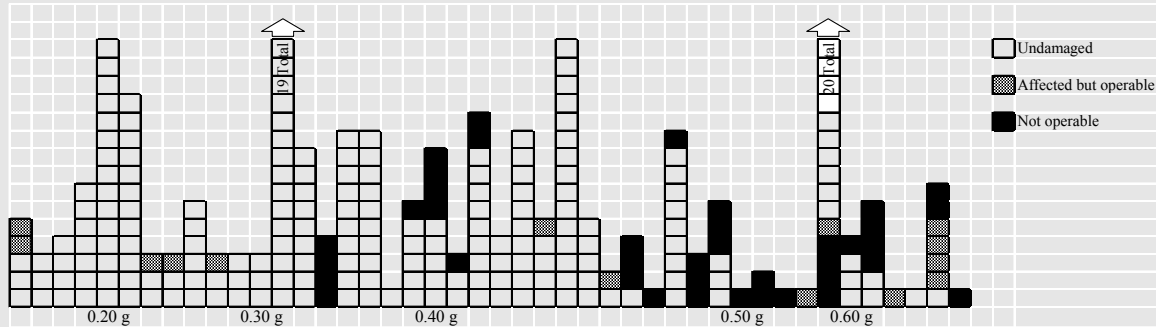

FIGURE 2

HYPOTHETICAL OBSERVED EARTHQUAKE DAMAGE DATA FOR MOTOR CONTROL CENTERS

The number of bins, *N*, and the lower demand bounds a_j, are dictated by the available data: *N* is taken as 5 with lower bounds, a_j of 0.15g, 0.25g, 0.35g, 0.45g, and 0.55g respectively. The damage state of interest is loss of post-earthquake functionality (black boxes in figure). The values of M_j and m_j are found by counting all boxes and black boxes, respectively, in the figure in each bin, and are shown in Table 1. The value of *M* is found by summing: $M = \Sigma M_j = 260$. Values x_j and y_j are calculated as $x_j = \ln(\bar{r}_j)$, and $y_j = \Phi^{-1}((m_j+1)/(M_j+1))$. Average values are calculated as shown: $\bar{x} = -0.99$, $\bar{y} = -1.05$, according to (11) and (12). For each bin, the values of $x_j - \bar{x}$ and $y_j - \bar{y}$ are calculated as shown.

j	a_j (g)	d_j (g)	M_j	m_j	x_j	y_j	$x_j - \bar{x}$	$y_j - \bar{y}$	$(x_j - \bar{x})^2$	$(x_j - \bar{x})(y_j - \bar{y})$
1	0.15	0.2	52	0	-1.61	-2.08	-0.623	-1.031	0.388	0.642
2	0.25	0.3	48	4	-1.20	-1.27	-0.217	-0.223	0.047	0.049
3	0.35	0.4	84	8	-0.92	-1.25	0.070	-0.202	0.005	-0.014
4	0.45	0.5	35	15	-0.69	-0.14	0.294	0.907	0.086	0.266
5	0.55	0.6	41	12	-0.51	-0.50	0.476	0.549	0.226	0.261
		$\Sigma =$	260		-4.93	-5.23			0.753	1.204
	Average =				-0.99	-1.05				

TABLE 1

EXAMPLE SOLUTION DATA

Then, β and θ are calculated as:

$$\beta_r = \frac{\sum_{j=1}^{N}(x_j - \bar{x})^2}{(x_j - \bar{x})(y_j - \bar{y})} = \frac{0.753}{1.204} = 0.63$$

$$\theta = e^{(\bar{x} - \bar{y}\beta_r)} = e^{(-0.99 + 1.05(0.63))} = e^{(-0.329)} = 0.72g$$

Method C. Capable Demand Data

This section defines the procedures for deriving fragility parameters (θ, β) when data are available from a suitable series of specimens, however, the damage state of interest was not initiated in any of the specimens. For each available specimen, "i," the maximum demand at which the specimen was loaded, "d_i", and whether or not the specimen experienced any distress or damage must be known.

From the data for M specimens, determine the maximum demand experienced by each specimen, d_{max}, and the minimum demand for any of the specimens that exhibited any distress or damage, d_{min}. Determine d_a as the smaller of d_{min} or $0.7d_{max}$. Determine M_A as the number of specimens that did not exhibit distress or damage, but that were loaded with demands, $d_i \geq d_a$; M_B as the number of specimens that exhibited distress or damage, but which did not appear to be initiating or on the verge of initiating the damage state of interest; and M_C as the number of specimens appeared to be on the verge of initiating the damage state of interest.

If none of the specimens in any of the tests exhibited any sign of distress or damage, take the value of d_m as d_{max}. If one or more of the specimens exhibited distress or damage of some type, take d_m as:

$$d_m = \frac{d_{max} + d_a}{2} \tag{15}$$

Determine the subjective failure probability S at d_m as:

$$S = \frac{0.5M_C + 0.1M_B}{M_A + M_B + M_C} \tag{16}$$

Take the logarithmic standard deviation, β, as having a value of 0.4. Determine the median, θ, as:

$$\theta = d_m e^{-0.4z} \tag{17}$$

where z is determined from Table 2 based on the value of M_A and S.

Conditions	Z
$M_A \geq 3$ and $S = 0$	-2.326
$M_A < 3$ and $S \leq 0.075$	-1.645
$0.075 < S \leq 0.15$	-1.282
$0.15 < S \leq 0.3$	-0.842
$S > 0.3$	-0.253

TABLE 2
VALUES OF Z

Example: Determine the parameters θ and β from tests of 10 specimens. Five of the specimens had maximum imposed drift demands of 1% with no observable signs of distress. Three of the specimens had maximum imposed drift demands of 1.5% and exhibited minor distress, but did not appear to be at or near the initiation of the damage state of interest. Two of the specimens had maximum imposed drift demands of 2%, did not enter the damage state of interest during the test, but appeared to be about to sustain such damage. From the given data determine: $d_{max} = 2\%$, $d_{min} = 1.5\%$. d_a is the smaller of $0.7d_{max}$ or d_{min} and therefore, is $0.7(2\%) = 1.4\%$. $M_A = 0$, $M_B=3$, and $M_C=2$.

$$d_m = \frac{d_{max} + d_a}{2} = \frac{2\% + 1.4\%}{2} = 1.7\%$$

$$S = \frac{0.5M_C + 0.1M_B}{M_A + M_B + M_C} = \frac{0.5(2) + 0.1(3)}{0 + 3 + 2} = 0.26$$

From Table 2, z is taken as -0.842. Therefore,

$\theta = d_m e^{-0.4z} = 0.017e^{-0.4(-0.842)} = 0.024$ or 2.4% drift

and β is taken as 0.4.

Method D. Derivation (Analysis)

There are two methods available for analytical derivation of fragility parameters. The first of these uses a single calculation of the probable capacity and a default value of the logarithmic standard deviation. The second method uses Monte Carlo analysis to explore the effect of variation in material strength, construction quality and other random variables.

Single calculation. Calculate the capacity of the component, Q in terms of a demand parameter, d, using average material properties and dimensions and estimates of workmanship. Resistance factors should be taken as unity and any conservative bias in code equations, if such equations are used, should be removed. The logarithmic standard deviation, β, is taken as having a value of 0.4. The median capacity θ is taken as:

$$\theta = 0.92Q \tag{18}$$

Monte Carlo simulation. Identify all those factors, important to predicting the capacity that are uncertain including material strength, cross section dimensions, member straightness, workmanship. Estimate a median value and variance for each of these random variables. Conduct sufficient analyses, randomly selecting the values of each of these random variables in accordance with their estimated distribution properties, each time calculating the capacity. Determine the median value of the capacity as that capacity exceeded in 50% of the calculations. Determine the random logarithmic standard deviation, β_r, as the standard deviation of the natural logarithm of the calculated capacity values. Use (3) to determine the total logarithmic standard deviation, β, assuming a value of β_u of 0.25.

Method E. Expert Opinion

Select one or more experts with professional experience in the design or post-earthquake damage observation of the component of interest. Solicit their advice using the format shown in Figure 3. Note the suggested inclusion of representative images, which should

be recorded with the responses. If an expert refuses to provide estimates or limits them to certain conditions, either narrow the component definition accordingly and iterate, or ignore that expert's response and analyze the remaining ones. Calculate the median value, θ, as:

$$\theta = \frac{\sum_{i=1}^{N} w_i^{1.5} \theta_i}{\sum_{i=1}^{N} w_i^{1.5}} \tag{19}$$

where N is the number of experts providing an opinion; θ_i is the opinion of expert i opinion as to the median value, and w_i is expert i's level of expertise, on a 1-5 scale. Calculate the lower bound value for the capacity as:

$$d_l = \frac{\sum_{i=1}^{N} w_i^{1.5} d_{li}}{\sum_{i=1}^{N} w_i^{1.5}} \tag{20}$$

where d_{li} is expert i's opinion as to the lower bound value and other terms are as previously defined. The value of the logarithmic standard deviation, β, is taken as:

$$\beta = \frac{\ln(\theta / d_l)}{1.28} \tag{21}$$

If this calculation produces an estimate of β that is less than 0.4, either justify the β, or take β as having a value of 0.4 and recalculate θ as:

$$\theta = 1.67 d_l \tag{22}$$

Objective. This form solicits your judgment about the values of a demand parameter (*D*) at which a particular damage state occurs to a particular building component. Judgment is needed because the component may contribute significantly to the future earthquake repair cost, fatality risk, or post-earthquake operability of a building, and because relevant empirical and analytical data are currently impractical to acquire. Your judgment is solicited because you have professional experience in the design or post-earthquake damage observation of the component of interest.

Definitions. Please provide judgment on the damageability of the following component and damage state. Images of a representative sample of the component and damage state may be attached. It is recognized that other demand parameters may correlate better with damage, but please consider only the one specified here.

Component name: _____

Component definition:_____

Damage state name: _____

Damage state definition: _____

_____

Demand parameter:_____

Definition of demand parameter:_____

_____

Uncertainty; no personal stake. Please provide judgment about this general class of components, not any particular instance, and not one that you personally designed, constructed, checked, or otherwise have any stake in. There is probably no precise threshold level of demand that causes damage, because of variability in design, construction, installation, inspection, age, maintenance, interaction with nearby components, etc. Even if there were such a precise level, nobody might know it with certainty. To account for these

uncertainties, **please provide two values of demand at which damage occurs: median and lower bound.**

Estimated median capacity _____ *Definition*. Damage would occur at this level of demand in 5 cases out of 10, or in a single instance, you judge there to be an equal chance that your median estimate is too low or too high.

Estimated lower-bound capacity _____ *Definition*. Damage would occur at this level of demand in 1 case in 10. In a single case, you judge there to be a 10% chance that your estimate is too high. *Judge the lower bound carefully*. Make an initial guess, then imagine all the conditions that might make the actual threshold demand lower, such as errors in design, construction or installation, substantial deterioration, poor maintenance, more interaction with nearby components, etc. Revise accordingly and record your revised estimate. Research shows that without careful thought, expert judgment of the lower bound tends to be too close to the median estimate, so think twice and do not be afraid of showing uncertainty.

On a 1-to-5 scale, please judge your expertise with this component and damage state, where 1 means "no experience or expertise" and 5 means "very familiar or highly experienced."

Your level of expertise:_____

Your name:_____   Date: _____

FIGURE 3
FORM FOR SOLICITING EXPERT JUDGMENT ON COMPONENT FRAGILITY

Method U. Updating

This section addresses procedures for re-evaluating fragility parameters for a building component as additional data become available. The pre-existing and updated fragility parameters are respectively termed θ, β, θ', and β'. The additional data are assumed to be a set of M specimens with known maximum demand and damage states. It is not necessary that any of the specimens experienced damage.

Calculate the revised median, θ' and logarithmic standard deviation β' as follows:

$$\theta' = e^{\sum_{j=1}^{5} w'_j \ln(d_j)} \tag{23}$$

$$\beta' = \sum_{j=1}^{5} w'_j \beta_j \tag{24}$$

where:

$$w'_j = \frac{w_j \prod_{i=1}^{M} L(i,j)}{\sum_{j=1}^{5} w_j \prod_{i=1}^{M} L(i,j)} \tag{25}$$

where Π denotes the product of the terms that come after it, and

$$L(i,j) = 1 - \Phi\left(\frac{\ln(d_i/x_j)}{0.707\beta_j}\right)$$

if specimen i did not experience the damage state of interest or

$$L(i, j) = \Phi\left(\frac{\ln\left(d_i/x_j\right)}{0.707\beta_j}\right)$$

if specimen i did experience the damage state of interest, or a more severe state, and

$$x_1 = x_4 = x_5 = \theta \qquad \beta_1 = \beta_2 = \beta_3 = \beta$$
$$x_2 = \theta e^{-1.22\beta} \qquad \beta_4 = 0.64\beta$$
$$x_3 = \theta e^{1.22\beta} \qquad \beta_5 = 1.36\beta$$

$$w_1 = 1/3$$
$$w_2 = w_3 = w_4 = w_5 = 1/6$$

ASSESSING FRAGILITY FUNCTION QUALITY

The previous section provided mathematical procedures for developing fragility parameters. This section provides procedures to assess the quality of those parameters.

Competing Demand Parameters

The behavior of some components may be dependent on several types of demands, for example in-plane and out-of-plane drift, or both drift and acceleration. It may not be clear which demand is the best single predictor of component damage. Assuming that data are available to create fragility functions for each possibly relevant demand, do so. Choose the fragility function that has the lowest β.

Dealing with Outliers using Peirce's Criterion

When fragilities are determine on the basis of actual demand data (i.e., Method A), it is possible that one or more tests reported spurious values of demand, d_i, and reflect experimental errors rather than the true demands at which the specimens failed. In cases where one or more d_i values in the data set are obvious outliers from the bulk of the data, investigate whether the data reflects real issues in the damage process that may recur, especially where $d_i \ll \theta$ for these outliers. If there is no indication that these data reflect a real recurring issue in the damage process, apply the following procedure (Peirce's criterion) to test and eliminate doubtful observations of d_i.

1. Calculate $\ln(\theta)$ and β of the complete data set.
2. Let D denote the number of doubtful observations, and let R denote the maximum distance of an observation from the body of the data, defined as:

$$R = \frac{\left|\ln(d) - \ln\theta\right|}{\beta} \qquad (26)$$

where θ, β, and M are as previously defined, d is a measured demand value, and R is as shown in Table 3. Assume $D = 1$ first, even if there appears to be more than one doubtful observation.
3. Calculate the maximum allowable deviation: $|\ln(d) - \ln(\theta)|_{max}$. Note that this can include $d \gg \theta$ and $d \ll \theta$.
4. For any suspicious measurement d_i, obtain $|\ln(d_i) - \ln(\theta)|$.
5. Eliminate the suspicious measurements if: $\left|\ln(d_i) - \ln(\theta)\right| > \left|\ln(d) - \ln(\theta)\right|_{max}$

6. If this results in the rejection of one measurement, assume D=2, keeping the original values of θ and β, and go to step 8.
7. If more than one measurement is rejected in the above test, assume the next highest value of doubtful observations. For example, if two measurements are rejected in step 5, assume the case of $D = 3$, keeping the original values of θ, and β, as the process is continued.
8. Repeat steps 2 – 5, sequentially increasing D until no more data measurements are eliminated.
9. Obtain θ and β of the reduced data set as for the original data.

M	D=1	D=2	D=3	D=4	D=5	D=6	D=7	D=8	D=9
3	1.1960								
4	1.3830	1.0780							
5	1.5090	1.2000							
6	1.6100	1.2990	1.0990						
7	1.6930	1.3820	1.1870	1.0220					
8	1.7630	1.4530	1.2610	1.1090					
9	1.8240	1.5150	1.3240	1.1780	1.0450				
10	1.8780	1.5700	1.3800	1.2370	1.1140				
11	1.9250	1.6190	1.4300	1.2890	1.1720	1.0590			
12	1.9690	1.6630	1.4750	1.3360	1.2210	1.1180	1.0090		
13	2.0070	1.7040	1.5160	1.3790	1.2660	1.1670	1.0700		
14	2.0430	1.7410	1.5540	1.4170	1.3070	1.2100	1.1200	1.0260	
15	2.0760	1.7750	1.5890	1.4530	1.3440	1.2490	1.1640	1.0780	
16	2.1060	1.8070	1.6220	1.4860	1.3780	1.2850	1.2020	1.1220	1.0390
17	2.1340	1.8360	1.6520	1.5170	1.4090	1.3180	1.2370	1.1610	1.0840
18	2.1610	1.8640	1.6800	1.5460	1.4380	1.3480	1.2680	1.1950	1.1230
19	2.1850	1.8900	1.7070	1.5730	1.4660	1.3770	1.2980	1.2260	1.1580
20	2.2090	1.9140	1.7320	1.5990	1.4920	1.4040	1.3260	1.2550	1.1900
>20					$a\ln M + b$				
a	0.4094	0.4393	0.4565	0.4680	0.4770	0.4842	0.4905	0.4973	0.5046
b	0.9910	0.6069	0.3725	0.2036	0.0701	-0.0401	-0.1358	-0.2242	-0.3079

TABLE 3
PARAMETERS FOR APPLYING PEIRCE'S CRITERION

Goodness of Fit Testing

Fragility parameters that are developed based on actual demand data (Section 2.1) should be tested for goodness of fit in accordance with this Section. Calculate
$$D = \max_x |F_i(d) - S_M(d)| \tag{27}$$
where $S_M(d)$ denotes the sample cumulative distribution function
$$S_M(d) = \frac{1}{M}\sum_{i=1}^{M} H(d_i - d) \tag{28}$$
and H is taken as:
    1.0 if $d_i - d$ is positive
    ½ if $d_i - d$ is zero
    0 if $d_i - d$ is negative.

If $D > D_{\text{crit}}$ from Table 4, the fragility function fails the goodness of fit test. This result is used in assigning a quality level to the fragility function. Use α = 0.05.

Significance Level	D_{crit}
α = 0.15	$0.775 / (M^{0.5} - 0.01 + 0.85M^{-0.5})$
α = 0.10	$0.819 / (M^{0.5} - 0.01 + 0.85M^{-0.5})$
α = 0.05	$0.895 / (M^{0.5} - 0.01 + 0.85M^{-0.5})$
α = 0.025	$0.995 / (M^{0.5} - 0.01 + 0.85M^{-0.5})$

TABLE 4
CRITICAL VALUES FOR THE LILLIEFORS TEST

Fragility Functions that Cross

Some components will have two or more possible damage states, with a defined fragility function for each. For any two (cumulative lognormal) fragility functions i and j with medians $\theta_j > \theta_i$ and logarithmic standard deviations $\beta_i \neq \beta_j$, the fragility functions will cross at extreme values. In such a case, adjust the fragility functions by one of the following two methods.

Method 1: adjust the fragility functions such that

$$F_i(D) = \max_j \left\{ \Phi\left(\frac{\ln(D/\theta_i)}{\beta_i} \right) \right\} \text{ for all } j \geq i \tag{29}$$

This has the effect that for the damage state with the higher median value, the probability of failure, $F_i(D)$ is never taken as less than the probability of failure for a damage state with a lower median value.

Method 2: First establish θ and β values for the various damage states independently. Next calculate the average of the variance values for each of the damage states with crossing fragility curves as:

$$\beta'_i = \frac{1}{N} \sum_{i=1}^{N} \beta_i \tag{30}$$

This average logarithmic standard deviation is used as a replacement for the independently calculated values. An adjusted median value must be calculated for each of the crossing fragilities as:

$$\theta'_i = e^{(1.28(\beta' - \beta_i) + \ln \theta_i)} \tag{31}$$

Assigning a Single Quality Level to a Fragility Function

Assign each fragility function a quality level of high, medium, or low, per Table 5.

Quality	Method	Peer reviewed*	Number of specimens	Other
High	A	Yes	≥ 5	Passes Lilliefors test at 5% significance level. Examine and justify (a) differences of greater than 20% in θ or β, compared with past estimates, and (b) any case of $\beta < 0.2$ or $\beta > 0.6$.
	B	Yes	≥ 20	Examine and justify (a) differences of greater than 20% in θ or β, compared with past estimates, and (b) any case of $\beta < 0.2$ or $\beta > 0.6$.
	U	Yes	≥ 6	Prior was at least moderate quality
Moderate	A		≥ 3	Examine and justify any case of $\beta < 0.2$ or $\beta > 0.6$.
	B		≥ 16	Examine and justify any case of $\beta < 0.2$ or $\beta > 0.6$.
	C	Yes	≥ 6	
	D	Yes		
	E	Yes		At least 3 experts with $w \geq 3$
	U		≥ 6	or prior was moderate quality
Low				All other cases
* Data and derivation published in a peer-reviewed archival journal.				

TABLE 5

FRAGILITY FUNCTION QUALITY LEVEL

CONCLUSIONS

Six methods for creating fragility functions were presented, including three new ones: one for dealing with cases where no failure has been observed, another for situations where one must rely on expert opinion, and a third for updating an existing fragility function with new damage observations. The procedures have been adopted for ATC-58, a technology-transfer project by the Applied Technology Council to bring a second-generation performance-based earthquake engineering methodology to practice. The procedures are intended for engineering professionals who will eventually use PBEE. Little unfamiliar math is involved, and no calculus. A larger document, [Porter et al. 2006], presents these procedures with more commentary, some alternative approaches, and more sample problems.

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