

Practical Formulation of the Relation Between Filter Specifications and the Requirements for Integrator Circuits

W. J. A. DE HEIJ, E. SEEVINCK, AND K. HOEN

Abstract—The design of integrated high-frequency continuous-time filters has made considerable progress in the past few years. As the signal frequencies increase the design of the integrator circuits used in most of these filters becomes more critical. To give direction to the circuit design minimum specifications for the gain and phase of the integrator circuits would be helpful. In this paper a practical method of finding these integrator specifications from the filter specifications is developed. An example of application of the method to a sixth-order Chebyshev band-pass filter is given and the result is verified by computer simulation.

I. INTRODUCTION

During the past few years there have been a number of publications on fully integrated continuous-time filters operating at high frequencies [1]–[8]. Except for [8] the basic building block of these filters is the integrator. The gyrator resonance circuits used in [5] and [6] can be considered as a pair of cross-coupled integrators.

An important problem in high-frequency filtering is the accurate realization of the specified poles and/or zeros of the filter. The quality factor Q of poles and zeros is highly sensitive to the phase of the integrators at the pole and zero frequencies. Therefore, the circuit designer should have accurate specifications for the gain and phase of integrator circuits.

In this paper a method to find the integrator specifications from filter specifications is developed from an analysis of the influence of nonideal integrators on the positions of poles and zeros of a filter. In Section II a simple model for the behavior of practical integrators is formulated. In Section III it is shown that the influence of nonideal integrators on the filter transfer function is independent of the filter topology. In Section IV the integrator model is used to study the influence of the non-ideal integrator on biquad transfer functions. In Section V the theory is applied to an example. Conclusions are given in Section VI.

II. THE INTEGRATOR MODEL

For the analysis of the effect of nonideal integrators on filter transfer functions we need a model of the nonideal integrator. In Fig. 1 the magnitude and phase of an ideal integrator $H_{id}(s)$ and a practical (nonideal) integrator $H_{ni}(s)$ are shown.

The ideal integrator in Fig. 1 has the following transfer function:

$$H_{id}(s) = \frac{\omega_0}{s}. \quad (1)$$

For angular frequencies up to $\approx 0.2 \cdot \omega_{p2}$ the transfer function

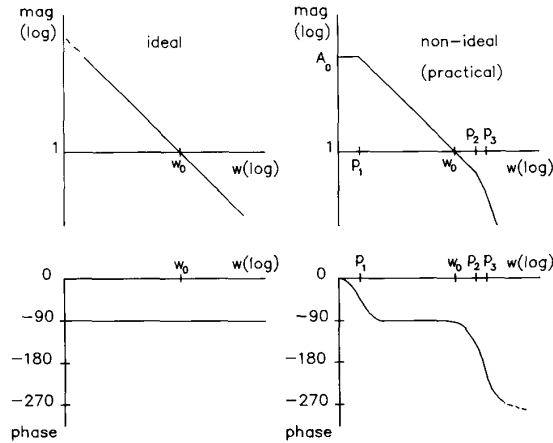


Fig. 1. Gain and phase of an ideal integrator and a non-ideal integrator.

$H_{ni}(s)$ of the practical integrator can be approximated by

$$H_{ni}(s) = \frac{A_0}{(1 + s\tau_1)(1 + s\tau_2)}. \quad (2)$$

Time constant $\tau_1 = \tau_{p1}$ belongs to the dominant pole p_1 . The dc-gain is equal to A_0 . The unity gain frequency ω_0 is well approximated by $\omega_0 \approx A_0/\tau_1$ if $1/\tau_1 \ll \omega_0 \ll 1/\tau_2$. Time constant τ_2 is used to approximate the phase shift due to the parasitic poles and zeros which are usually at frequencies much higher than the signal frequencies processed by the filter. Therefore, in our analysis $|s\tau_2| \ll 1$ will hold and we can write (2) as

$$H_{ni}(s) = A_0 \frac{1 - s\tau_2}{1 + s\tau_1}. \quad (3)$$

Model (3) is used in Section IV because it is mathematically easier to handle than model (2) [10]. To find an appropriate value for τ_2 we use the quality factor of an integrator which is easily defined if we write its transfer function ($s = j\omega$) as follows:

$$H_{int}(j\omega) = \frac{1}{R(\omega) + jX(\omega)}. \quad (4)$$

Then the quality factor of the integrator can be defined as [11]:

$$Q_{int}(\omega) = \frac{X(\omega)}{R(\omega)} = \tan(-\arg(H_{int}(j\omega))). \quad (5)$$

For the integrator models (2) and (3) the reciprocal of the quality factor ($\tau_1 \gg \tau_2$) can be written as

$$\frac{1}{Q_{int}(\omega)} = \frac{1 - \omega^2\tau_1\tau_2}{\omega(\tau_1 + \tau_2)} \approx \frac{1}{\omega\tau_1} - \omega\tau_2. \quad (6)$$

To model a practical integrator we first calculate τ_1 from the dominant pole frequency. Next the quality factor $Q_{int}(\omega_0)$ is found from the phase at the unity-gain frequency ω_0 using (5). Finally, τ_2 is calculated from $Q_{int}(\omega_0)$ and τ_1 using (6). This approximation by model (3) will be useful up to $\approx 0.2 \cdot \omega_{p2}$ (Fig. 1). Usually this frequency range covers the signal frequencies where the performance of the filter employing the integrators is critical.

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III. THE INDEPENDENCE OF THE EFFECT OF NONIDEAL INTEGRATORS OF THE FILTER TOPOLOGY

In this section it will be shown that the effect of nonideal integrators on the transfer function of a filter is independent of the filter topology in which the integrators are used.

Let the following normalized filter transfer function be specified:

$$H(S) = \frac{a_m S^m + a_{m-1} S^{m-1} + \cdots + a_1 S + a_0}{b_n S^n + b_{n-1} S^{n-1} + \cdots + b_1 S + b_0}, \quad S = \frac{s}{\omega_0} \quad (7)$$

The specified $H(S)$ can be realized by many filter topologies. In these topologies ideal integrators $H_{id}(S) = 1/S$ are interconnected in different ways via multiplication of signals by constants K_1, K_2, \dots, K_N and summation of signals, to form a network [9]. The constants are functions of the filter coefficients a_i and b_j :

$$K_h = K_h(a_0, a_1, \dots, a_m, b_0, b_1, \dots, b_n), \quad h = 1, 2, \dots, N. \quad (8)$$

The form of the functions and the number of constants N is topology dependent, causing for example the difference in sensitivity between filter topologies.

If the numerator and the denominator of (7) are divided by the highest power of S and the ideal integrator given by (1) is inserted with unity-gain frequency ω_0 we obtain the following expression for the case $m \leq n$ (the case $m > n$ is not essentially different):

$$H(s) = \frac{a_m H_{id}^{n-m}(s) + a_{m-1} H_{id}^{n-m+1}(s) + \cdots + a_1 H_{id}^{n-1}(s) + a_0 H_{id}^n(s)}{b_n + b_{n-1} H_{id}(s) + \cdots + b_1 H_{id}^{n-1}(s) + b_0 H_{id}^n(s)} \quad (9)$$

Note that in (9) identical integrators are assumed.

If different filter topologies realize $H(s)$ (9) with ideal integrators $H_{id}(s)$ they must produce the same set of filter coefficients a_i and b_j each through the *inverse* of its own set of functions (8). We can take any one of these filter topologies or networks and replace $H_{id}(s)$ with a nonideal integrator $H_{ni}(s)$ modeled by (3) having unity-gain frequency ω_0 *without* changing the network structure nor the constants K_1, K_2, \dots, K_N . Then the replacement results in a modified transfer function given by

$$H'(s) = \frac{a_m H_{ni}^{n-m}(s) + a_{m-1} H_{ni}^{n-m+1}(s) + \cdots + a_1 H_{ni}^{n-1}(s) + a_0 H_{ni}^n(s)}{b_n + b_{n-1} H_{ni}(s) + \cdots + b_1 H_{ni}^{n-1}(s) + b_0 H_{ni}^n(s)} \quad (10)$$

Because the network structure and the constants K_1, K_2, \dots, K_N remain unchanged and $H_{id}(s)$ and $H_{ni}(s)$ have the same unity gain frequency the difference between $H(s)$ and $H'(s)$ is only due to the nonideal behavior of $H_{ni}(s)$ compared to $H_{id}(s)$. We have to design our integrator circuits (which will always be non-ideal) in such a way that the difference between $H'(s)$ and the specified transfer function $H(s)$ is kept within acceptable limits.

By the argument above it is intuitively shown that $H'(s)$ is found from the specified filter transfer function $H(S)$ *independently* of the filter topology. So we can conclude that the effect of nonideal integrators can be analyzed independently of the filter topology. This result will be used in the next section to simplify the analysis.

IV. INFLUENCE OF NONIDEAL INTEGRATORS ON BIQUAD TRANSFER FUNCTIONS

In this section the influence of nonideal integrators on first- and second-order transfer functions will be analyzed. It is the

TABLE I
COEFFICIENTS OF BIQUAD TRANSFER FUNCTIONS

Function	a_2	a_1	a_0	b_1	b_0
Low pass	0	0	$K\Omega_p^2$	Ω_p/Q_p	Ω_p^2
Band pass	0	$H_B\Omega_p/Q_p$	0	Ω_p/Q_p	Ω_p^2
High pass	K	0	0	Ω_p/Q_p	Ω_p^2
Notch	K	0	$K\Omega_p^2$	Ω_p/Q_p	Ω_p^2
All pass	K	$-K\Omega_p/Q_p$	$K\Omega_p^2$	Ω_p/Q_p	Ω_p^2

most simple approach because complex pole and zero pairs can not be described with transfer functions with a degree lower than two. High order filters can be considered as a cascade of first- and second-order blocks enabling the analysis of the effect of nonideal integrators on the filter transfer function to be broken up into smaller problems. As a consequence of the argument in Section III the results will be valid independently of the actual topology used to realize the filter transfer function $H(S)$.

A general normalized biquad transfer function with complex poles and zeros is given by

$$H_b(S) = \frac{N(S)}{D(S)} = \frac{a_2 S^2 + a_1 S + a_0}{S^2 + b_1 S + b_0}, \quad S = \frac{s}{\omega_0} \quad (11)$$

There are five types of biquad transfer functions: low pass, band pass, high pass, notch or band reject, and all pass. The coefficients a_i and b_j of (11) for these transfer functions are given in Table I.

In Table I $\Omega_z = \omega_z/\omega_0$ and $\Omega_p = \omega_p/\omega_0$ are normalized frequencies. K and H_B are gain constants.

In the following analysis the ideal integrators $H_{id}(S)$ are replaced by nonideal integrators $H_{ni}(S)$ as described by (3)

having the same unity-gain frequency $\Omega_0 = 1$:

$$H_{ni}(S) = A_0 \frac{1 - ST_2}{1 + ST_1} \quad (12)$$

In (12) the time constants τ_1 and τ_2 of (3) are normalized with respect to ω_0 :

$$T_1 = \tau_1 \omega_0 = A_0 \quad \text{and} \quad T_2 = \tau_2 \omega_0 = \frac{A_0}{\tau_1} \tau_2 \quad (13)$$

The replacement results in a modified transfer function $H'_b(S)$:

$$H'_b(S) = \frac{N'(S)}{D'(S)} = \frac{a'_2 S^2 + a'_1 S + a'_0}{S^2 + b'_1 S + b'_0} \quad (14)$$

The modified coefficients a'_i and b'_j are given in Appendix A. They are derived under the assumption that Ω_0 is well approximated by $\Omega_0 = A_0/T_1$ (Section II).

The influence of nonideal integrators on any type of biquad transfer function can be analyzed by substitution of the coeffi-

cients from Table I into (A1)–(A5) in Appendix A. The case of the first-order transfer function is treated in Appendix A.

All biquad transfer functions have the same denominator polynomial $D(S)$. For the coefficients b'_j we find

$$b'_1 = \frac{\frac{2}{T_1} + \left(1 - \frac{T_2}{T_1}\right) \frac{\Omega_p}{Q_p} - 2T_2\Omega_p^2}{1 - \frac{\Omega_p}{Q_p} T_2 + \Omega_p^2 T_2^2} \quad (15)$$

$$b'_0 = \frac{\frac{1}{T_1^2} + \frac{\Omega_p}{Q_p T_1} + \Omega_p^2}{1 - \frac{\Omega_p}{Q_p} T_2 + \Omega_p^2 T_2^2} \quad (16)$$

Using Table I we can calculate the biquad parameters Ω'_p and Q'_p for nonideal integrators from b'_0 and b'_1 and simplify the result using the fact that for most known integrator designs $T_1 \gg 1$ and $T_2 \ll 1$. Using (6) and (13) the result can be expressed in the quality factor $Q_{\text{int}}(\Omega)$ of the integrator:

$$\Omega'_p = \sqrt{b'_0} \approx \Omega_p \quad (17)$$

$$Q'_p = \frac{\sqrt{b'_0}}{b'_1} \approx \frac{\Omega_p}{\frac{2}{T_1} + \frac{\Omega_p}{Q_p} - 2T_2\Omega_p^2} = Q_p \frac{1}{1 + \frac{2Q_p}{Q_{\text{int}}(\Omega_p)}} \quad (18)$$

At a frequency $\Omega = (1 + \alpha)\Omega_p$ the ratio of $|D'(j\Omega)|$ and $|D(j\Omega)|$ becomes equal to

$$\frac{|D'(j(1 + \alpha)\Omega_p)|}{|D(j(1 + \alpha)\Omega_p)|} = \frac{\sqrt{\alpha^2(\alpha + 2)^2 + \frac{(1 + \alpha)^2}{Q_p'^2}}}{\sqrt{\alpha^2(\alpha + 2)^2 + \frac{(1 + \alpha)^2}{Q_p^2}}} \quad (19)$$

Using (18) we find from (19) for $\Omega = \Omega_p$:

$$\frac{|D'(j\Omega_p)|}{|D(j\Omega_p)|} = 1 + \frac{2Q_p}{Q_{\text{int}}(\Omega_p)} \quad (20)$$

Equations (18) and (20) are important from a practical point of view because $Q_{\text{int}}(\Omega_p)$ is simply related to the phase of the integrator at Ω_p by (5). The biquad transfer functions in Table I have different numerators $N(S)$. They will be treated briefly below.

- *Low pass:* $N(S) = K\Omega_p^2$

$$K' = a'_0/b'_0 \approx K. \quad (21)$$

- *Band pass:* $N(S) = H_B(\Omega_p/Q_p)S$

$$H'_B = a'_1/b'_1 \approx H_B \frac{1}{1 + \frac{2Q_p}{Q_{\text{int}}(\Omega_p)}} \quad (22)$$

Magnitude at dc:

$$|H_b(j0)| = a'_0/b'_0 > 0. \quad (23)$$

- *All pass:* $N(S) = K(S^2 + (\Omega_z/Q_z)S + \Omega_z^2)$ with $Q_z = -Q_p$ and $\Omega_z = \Omega_p$

$$K' = a'_2 \approx K \quad (24)$$

$$Q'_z = \frac{\sqrt{a'_0 a'_2}}{a'_1} \approx Q_z \frac{1}{1 + \frac{2Q_z}{Q_{\text{int}}(\Omega_z)}} \quad (25)$$

$$\Omega'_z = \sqrt{a'_0/a'_2} \approx \Omega_z. \quad (26)$$

- *High pass:* $N(S) = KS^2$

K' is given by (24).

$$|H_b(j0)| = a'_0/b'_0 > 0. \quad (27)$$

- *Notch:* $N(S) = K(S^2 + \Omega_z^2)$

K' and Ω'_z are given by (24) and (26).

According to (25) Q'_z becomes equal to

$$Q'_z \approx 0.5 * Q_{\text{int}}(\Omega_z). \quad (28)$$

Magnitude at Ω_z :

$$|H'_b(j\Omega_z)| \approx \frac{\left| \frac{2K\Omega_z^2}{Q_{\text{int}}(\Omega_z)} \right|}{\sqrt{(\Omega_p^2 - \Omega_z^2)^2 + \Omega_p^2 \Omega_z^2 (1/Q_p + 2/Q_{\text{int}}(\Omega_p))^2}} \quad (29)$$

From (21) to (29) we can find for low-pass, band-pass, and high-pass biquads:

$$\frac{|H_b(j\Omega)|}{|H'_b(j\Omega)|} \approx \frac{|D'(j\Omega)|}{|D(j\Omega)|} \quad (30)$$

The right-hand side of (30) is given by (19) and (20). For the notch and all-pass biquads also changes of the numerator have to be taken into account, using (19) with $\Omega = (1 + \beta)\Omega_z$, Q_z and Q'_z . We can conclude that with use of model (3) for nonideal integrators simple and practically useful expressions for the effect of nonideal integrators on the magnitude of second-order transfer functions can be derived. The effect on phase can also be studied using the general equations of Appendix A. In the expressions the quality factor of the integrator which is related to the phase of the integrator by (5) is the key parameter, making the expressions particularly useful for practical calculations. The theory in this section can be used to find integrator specifications from a desired filter response. In the next section an example will be given.

V. APPLICATION TO A SIXTH-ORDER CHEBYSHEV BAND-PASS FILTER

In this section it is demonstrated how the results of the previous section can be used to find integrator specifications from the filter specifications. Assume we have the following normalized transfer function of a sixth-order Chebyshev band-pass filter with 0.5-dB ripple:

$$H(S) = \frac{KS^3}{(S^2 + 0.05627S + 0.8154)(S^2 + 0.1253S + 1)(S^2 + 0.06902S + 1.226)} \quad (31)$$

TABLE II
BIQUAD PARAMETERS ($j = 1, 2, 3$)

	H_{Bj}	Q_{pj}	Ω_{pj}
$H_{b1}(S)$	3.1793	16.04	0.9030
$H_{b2}(S)$	1.4279	7.98	1.0000
$H_{b3}(S)$	2.5920	16.04	1.1072

TABLE III
RESULTS FOR $Q_{int}(\Omega_{pj})$ FROM (32)

	0 ... +0.5 dB dev.	-0.5 ... 0 dB dev.
	$Q_{int}(\Omega_{pj})$	$Q_{int}(\Omega_{pj})$
$H_{b1}(\Omega)$	$-\infty \dots -573.5$	$541.4 \dots \infty$
$H_{b2}(\Omega)$	$-\infty \dots -285.3$	$269.4 \dots \infty$
$H_{b3}(\Omega)$	$-\infty \dots -573.5$	$541.4 \dots \infty$

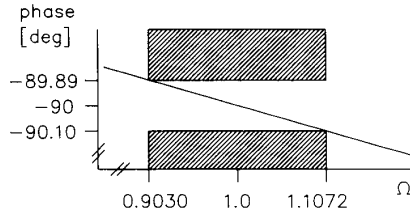


Fig. 2. Integrator phase specification for the sixth-order Chebyshev band-pass filter.

We want to find integrator specifications if the deviation from the ideal response may not exceed 0.5 dB over the passband. We can treat $H(S)$ as a cascade of three band-pass biquads $H_{b1}(S) * H_{b2}(S) * H_{b3}(S)$. The parameters of the biquads are given in Table II. We will use (17)–(20). Because band-pass biquads satisfy (30) we can use (20) to find that the response deviation of ± 0.5 dB at the pole frequency of a single biquad results in

$$Q_{int}(\Omega_{pj}) = \frac{2Q_{pj}}{10^{\pm 0.025} - 1}. \quad (32)$$

This equation gives two minimum Q -factors (considering absolute values) for each biquad. The interpretation of the result is given in Table III. Using (5) we find that the phase of the integrator over the passband must be between -89.89° and -90.10° . A useful phase specification is shown in Fig. 2.

The quality factor of the integrator is given by (6) and (13):

$$\frac{1}{Q_{int}(\Omega)} = \frac{1}{\Omega T_1} - \Omega T_2. \quad (33)$$

We can solve for T_1 and T_2 at frequencies $\Omega_{p1} = 0.9030$ and $\Omega_{p3} = 1.1072$ considering the maximum allowable phase error as shown in Fig. 2. We then find

$$T_1 = 113.4 \quad \text{and} \quad T_2 = 8.769 * 10^{-3}. \quad (34)$$

These are minimum specifications. Larger T_1 and smaller T_2 result in better integrators. Let the actual unity gain frequency ω_0 in our filter realization be $2\pi * 500 * 10^3$ rad/s. Then we find the

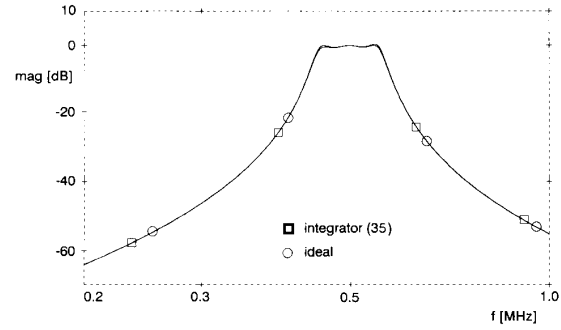


Fig. 3. Magnitude response of bandpass filter with ideal integrators and with integrators specified by (35).

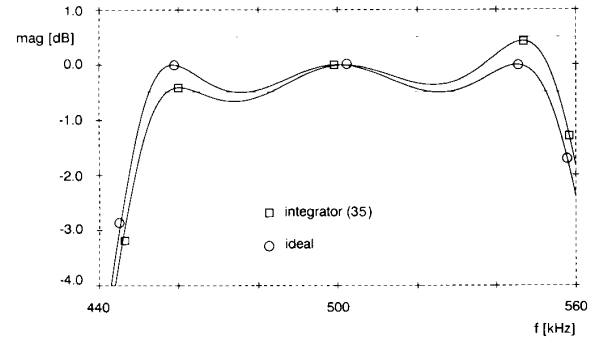


Fig. 4. Detail of passband response from Fig. 3.

following integrator specifications from (34) using (13):

dc-gain: 113.4

Dominant pole:

$$f_{p1} = \frac{1}{2\pi\tau_1} = 4409 \text{ Hz}. \quad (35)$$

High frequency pole:

$$f_{p2} = \frac{1}{2\pi\tau_2} = 57.02 \text{ MHz}.$$

In practice one must design the integrator circuits to match the gain and phase following from (35) using passive or active compensation or a Q -control loop [1], [4], [7] if necessary.

If we insert the integrator with these specifications in for example a leapfrog topology we obtain a filter response shown in Figs. 3 and 4. Similar simulations for other topologies result in exactly the same transfer functions as was expected from the argument in Section III.

Fig. 3 shows that the stopband response of the filter with integrators as specified by (35) does not significantly differ from the ideal response. Fig. 4 shows that the difference in the passband is within the required 0.5 dB. These results were obtained by PSPICE simulation with integrator models consisting of dependent sources, resistors and capacitors, allowing full control of the parasitic effects being studied.

In the calculation above it is assumed that the deviation of one of the biquads is not noticeable at the pole frequencies of the others. For bandpass biquads this can be checked with (19) and (30). Values of Q_p are found in Table II. Values of Q'_p can be calculated with (18) from (33) and (34). Substituting these values in (19) together with appropriate values of α one can verify that

the assumption made above indeed results in a very small error in this case.

VI. CONCLUSIONS

The influence of non-ideal integrators on a specified filter transfer function can be analyzed independently of the filter topology using only simple equations in which the quality factor of the integrator is the key parameter. Because the quality factor follows directly from the phase of the integrator as a function of frequency we obtain a practically useful formulation. The example of application demonstrates the possibility to find integrator specifications from the filter specifications. Especially for high frequency continuous-time filters where integrator design is critical it is important to have accurate specifications for the integrator circuits to match.

APPENDIX A

The modified biquad coefficients in (14):

$$a'_2 = \frac{a_2 - a_1 T_2 + a_0 T_2^2}{1 - b_1 T_2 + b_0 T_2^2} \quad (A1)$$

$$a'_1 = \frac{\frac{2a_2}{T_1} + \left(1 - \frac{T_2}{T_1}\right) a_1 - 2T_2 a_0}{1 - b_1 T_2 + b_0 T_2^2} \quad (A2)$$

$$a'_0 = \frac{\frac{a_2}{T_1^2} + \frac{a_1}{T_1} + a_0}{1 - b_1 T_2 + b_0 T_2^2} \quad (A3)$$

$$b'_1 = \frac{\frac{2}{T_1} + \left(1 - \frac{T_2}{T_1}\right) b_1 - 2T_2 b_0}{1 - b_1 T_2 + b_0 T_2^2} \quad (A4)$$

$$b'_0 = \frac{\frac{1}{T_1^2} + \frac{b_1}{T_1} + b_0}{1 - b_1 T_2 + b_0 T_2^2} \quad (A5)$$

Case of a First-Order Section:

With $S = s/\omega_0$ we write for a first-order section:

$$H(S) = \frac{a_1 S + a_0}{S + b_0} \quad (A6)$$

Replacing $H_{id}(S)$ by $H_{ni}(S)$ from (12) we obtain

$$H'(S) = \frac{a'_1 S + a'_0}{S + b'_0} \quad (A7)$$

Using the same conventions as for the biquad case we find

$$a'_1 = \frac{a_1 - a_0 T_2}{1 - b_0 T_2}; \quad a'_0 = \frac{a_0 + \frac{a_1}{T_1}}{1 - b_0 T_2}; \quad b'_0 = \frac{b_0 + \frac{1}{T_1}}{1 - b_0 T_2} \quad (A8)$$

For $T_1 \gg 1$ and $T_2 \ll 1$ we find small sensitivities of pole and zero frequencies and dc-gain with respect to T_1 and T_2 .

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High-Frequency and High-Q Tunable Active Filters

YANNICK DEVILLE

Abstract—An investigation of the performance of classical second-order bandpass active filters shows that since they use low-gain elements, they can be divided into two classes at high frequencies: low-selectivity stable filters and low-stability selective filters. This paper presents structures that realize a good compromise between both classes, so that high Q can be achieved with good stability. Verifications have been carried out with experimental Si hybrid high-frequency filters and with computer simulations of GaAs microwave integrated filters. The center frequency of the GaAs filters is tuned with MESFET's used as voltage-controlled resistors and can exceed 2 GHz.

I. INTRODUCTION

At low frequencies, continuous-time active filters can combine high selectivity and stability by using very high-gain operational amplifiers (op amps). However, the gain of high-frequency and microwave amplifiers is limited by stability considerations that are developed hereafter. We will show that second-order bandpass filters using finite-gain amplifiers, which are our main concern here, can be divided into two classes, according to their quality factor Q defined as the ratio of the center frequency to the 3-dB bandwidth:

- 1) *Class A*: filters that are stable ($Q > 0$) whatever their element values are, but that achieve low selectivity when the use limited values for the voltage gain K :

$$Q_{\max} = \frac{1}{2} \cdot \sqrt{K+1} \quad (1)$$

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