CORRECTION: Practical Implementation of the Double Linear Damage Rule and Damage Curve Approach for Treating Cumulative Fatigue Damage*

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The paper* on implementation of the double linear damage rule for cumulative fatigue damage included Appendices for treatment of two major factors that enter into the analysis - inversion of the fatigue life relationship so that life fractions can more easily be calculated, and treatment of mean stress effects. The discussion of these two subjects represented the state of development at the time of the initial paper preparation - late 1979. Because of publication schedules the paper was actually not published until early 1981. During the interim period the authors continued working on the two subjects contained in the Appendices, resulting in new developments. In the case of the inversion problem, a new approach was formulated which could be expressed by a single formula over the entire life range, rather than requiring two formulas, each limited to a part of the life range. In the case of mean stress treatment we concluded that the formula cited, common in engineering use for more than 15 years, was indeed inaccurate and could lead to erroneous results in some cases.

The present report describes these new developments and can be regarded essentially as a replacement for the two Appendices of the original paper*. Fortunately, for the illustrative problems used therein, the mean stress formulation does not produce large differences from those that would be obtained in the revision presented herein. But problems can readily be envisioned for which the two can differ appreciably. It is our belief that the revised formulation would be the more accurate of the two.

The inversion procedure is described in detail in [1]. Basically, the life relation is given in the form originally proposed by Manson [2], using the notation later revised by Morrow [3], which is

$$\Delta\varepsilon/2 = \varepsilon_f' \, (2N_f)^c + (\sigma_f'/E)(2N_f)^b \tag{1}$$

A simpler form of the same relation has been proposed in [4],

$$\Delta\varepsilon/\Delta\varepsilon_T = (N_f/N_t)^c + (N_f/N_t)^b \tag{2}$$

where $\Delta\varepsilon_T$ and N_T are the coordinates of the well known transition point, the intersection of the elastic and plastic strain range lines.

**International Journal of Fracture* 17 (1981) 169-192.

The relationships linking (1) and (2) are

$$\Delta\varepsilon_T = 2\,\varepsilon_f'^{\,b/(b-c)}\,(\sigma_f'/E)^{c/(c-b)} \tag{3}$$

and

$$N_T = \frac{1}{2}\,[E\varepsilon_f'/\sigma_f']^{1/(b-c)} \tag{4}$$

A number of methods for inverting (1), that is, solving for N_f in terms of $\Delta\varepsilon$, have recently been developed, among them those described in [5]. However, the simplest procedure is that developed in [1] based on (2). The inversion formula is

$$N_f/N_t = [(\Delta\varepsilon/\Delta\varepsilon_T)^{z/c} + (\Delta\varepsilon/\Delta\varepsilon_T)^{z/b}]^{1/z} \tag{5}$$

where

$$\ell n\ z = p\ (\ell n\ \Delta\varepsilon/\Delta\varepsilon_T)^2 + Q[\ell n\ \Delta\varepsilon/\Delta\varepsilon_T] + \ell n[-0.889c(c/b)^{-0.36}] \tag{6}$$

and

$$P = -0.001277(c/b)^2 + 0.03893(c/b) - 0.0927 \tag{7}$$

$$Q = +0.004176(c/b)^2 - 0.135(c/b) + 0.2309 \tag{8}$$

Thus, N_f is expressed directly in terms of the material constants and strain range.

As an illustration, consider the alloy Ti-6Al-4V which was recently studied [6] at 170deg F in relation to the cumulative fatigue of a small turbojet compressor disk. The basic equation as given in [6] is

$$\Delta\varepsilon/2 = 2.852(2N_f)^{-0.9034} + 0.01987(2N_f)^{-0.1229} \tag{9}$$

Here, using (3) and (4),

$$\Delta\varepsilon_T = 0.0181$$

and

$$N_T = 291 \text{ cycles} \tag{10}$$

Thus, the basic life relation from (2) is

$$\Delta\varepsilon/0.0181 = (N_f/291)^{-0.9034} + (N_f/291)^{-0.1229} \tag{11}$$

Using the inversion relation (5)

$$N_f/291 = [(\Delta\varepsilon/0.0181)^{z/-0.9034} + (\Delta\varepsilon/0.0181)^{z/-0.1229}]^{1/z} \tag{12}$$

when plastic strain is small, (14) has been extensively cited and used with considerable success in the long life nominally elastic region when plastic strain range is of little practical significance. However, conditions do occur wherein mean stresses are present together with appreciable plastic strain. One such case is illustrated in Fig. 2. For the complex loading history shown, the loop 7-8 develops which combines mean stress and plastic strain, even though a loop 7-8 could not alone be sustained without cyclic mean stress relaxation. Since cumulative fatigue damage analysis is frequently concerned with complex loading histories, the damaging effect of a loop such as 7-8 must be determined. Although there are equations other than (14) that can be used for life determination, (14) is frequently used, e.g., [8]. For this reason it is appropriate to scrutinize its validity more carefully than has been done in the past.

A major problem associated with (14) is related to its treatment of compressive stress relative to tensile stress, which can best be illustrated through Fig. 1(b). For simplicity, consider a case in which the stress range is that at the transition point T. With a tensile mean stress and equal stress range, operation is at point T_2 on the new elastic line A_2B_2, while if the mean stress is compressive, operation is at point T_1. If the plastic line CD is unaffected by mean stress, tensile mean stress will result in a plastic strain range at point F_2. For compressive mean stress operation is at F_1, where plastic strain range is significantly different from that at F_2. The question is whether it is logical to assume that mean stress has such a drastic effect on plastic strain range for a given stress range.

To provide some experimental guidelines, three tests were conducted on a specimen of 316 stainless steel at room temperature, as shown in Fig. 3. For all three tests the stress range was 43.2×10^3 lb/in^2. Hysteresis loop (b) was formed when completely reversed loading was applied, no mean stress. Loop (a) was formed by cycling from zero to a negative 43.2×10^3 lb/in^2, with compressive mean stress of 21.6×10^3 lb/in^2. For all three cases the widths of the hysteresis loops were nearly the same. If (14) were valid, the plastic strain ranges would have been significantly different, as shown in Fig. 4. Here, in order to focus on plastic strain range, the elastic strains have been deleted, and the hysteresis loops show only plastic strain range *vs* stress. As seen in the figure, a plastic strain range of 1×10^{-4} should result with completely reversed loading, while (14) implies it should be 0.3×10^{-4} when mean stress is compressive, and 4.1×10^{-4} when mean stress is tensile. Reversal of sign of mean stress, in this illustration, implies that the plastic strain range be altered by a factor of more than 12, whereas the actual effect was negligible.

It is thus clear that the use of (14) for situations involving appreciable plastic strain can result in considerable error. Closer agreement with experimental fact can be obtained by requiring the cyclic stress-strain curve to remain unaltered by mean stress. This can be accomplished by retaining the same transition strain range, and moving only the transition life horizontally according to the Goodman, Morrow, or Manson models. Therefore, for a tensile mean where

$$z = P[\ell n(\Delta\varepsilon/0.0181)]^2 + Q[\ell n(\Delta\varepsilon/0.0181)] + S \tag{13}$$

and

$P = 0.1245$

$Q = -0.5358$

$S = [-0.889c(c/b)^{-0.36}] = -0.9374$

Eq. (12) is extremely easy to program, even on currently available and relatively inexpensive hand-held calculators. For example, at a value of $\Delta\varepsilon = 0.01$, (12) yields the value $N_f = 42776$ cycles. Using $N_f = 42776$ in (1) or (2), the corresponding "exact" value of $\Delta\varepsilon$ is 0.010001, which is essentially indistinguishable from the assumed valu of 0.01. The corresponding values of $\Delta\varepsilon$ and N_f obtained from (12) and (2) are as identical as can reasonably be required for engineering analysis over the entire practical range of strain range or fatigue life.

It is interesting that the same inversion relation (5) is used when mean stress is present. The only parameter affected is the numerical value of N_T, although for some materials $\Delta\varepsilon_T$ may be affected, as discussed below.

Eq. (1) is valid only when loading is completely reversed and mean stress is zero. When a mean stress σ_o is present, Morrow [7] has proposed that the appropriate relation is

$$\Delta\varepsilon/2 = \varepsilon_f'(2N_f)^c + [(\sigma_f' - \sigma_o)/E](2N_f)^b \tag{14}$$

This equation is based on the concept shown in Fig. 1, that if the elastic line for completely reversed loading intersects the vertical $2N_f = 1$ at an ordinate of σ_f'/E, then the elastic line with tensile mean stress intersects at an ordinate $(\sigma_f' - \sigma_o)/E$; with a compressive mean stress it is $(\sigma_f' + \sigma_o)/E$. Analytically, displacement of the elastic line to take account of mean stress is the same as constructii straight lines on the generalized Goodman Diagram, Fig. 1(a). While Goodman suggested that lines for various constant life values radiate from a point on the mean stress axis equal to the ultimate tensile strength, σ_u, Morrow [3] concluded that the value σ_f' is a better poin for steels he has studied. Manson [6] has noted that for other materials, such as Ti-6Al-4V, σ_f' is too high a value, and an intermediate value $k_m\sigma_f'$ can be determined by experiment. For such a material σ_f' in (14) is replaced by $k_m\sigma_f'$. This generality embraces the Goodman case by taking $k_m = \sigma_u/\sigma_f'$, and produces an intercept on the elastic line of $k_m\sigma_f'/E$ at $2N_f = 1.0$. For purposes of the present discussion, we shall take $k_m = 1$, although generalization can easily be extended if the value of k_m is known.

Since mean stress is usually an important consideration only stress the transition point in Fig. 1 would become point T_2, while for compression it would be T_1. Plastic lines thus also become translated by mean stress, as well as elastic lines. When the transition strain range is the same, the ratio of elastic strain range to plastic strain range is dependent only on strain range; thus, the cyclic stress-strain curve is unchanged. The sketch of Fig. 5 shows schematically the more acceptable assumption regarding movement of the plastic line in the presence of mean stress than that of (14).

To obtain the new transition life N_T', we observed that since the transition point must lie on the elastic lines for both completely reversed loading and in the presence of mean stress,

$$\Delta\varepsilon_T/2 = (\sigma_f'/E)(2N_T)^b = [(\sigma_f' - k_m\sigma_o)/E](2N_T')^b \tag{15}$$

from which it easily follows that

$$N_T' = (1/2)[(2N_T)^{-b} - 2k_m\sigma_o/E\Delta\varepsilon_T]^{-1/b} \tag{16}$$

where σ_o is the mean stress, positive if tension, negative if compression and $\Delta\varepsilon_T$ and N_T are the transition strain range and life of the basic material under completely reversed loading, and E is the elastic modulus.

Eq. (16) is especially convenient for use in the general life relation [5],

$$\Delta\varepsilon/\Delta\varepsilon_T = (N_f/N'_T)^b + (N_f/N'_T)^c \tag{17}$$

which is valid with or without mean stress if N'_T is determined from (16). For $\sigma_o = 0$, N'_T properly degenerates to N_T.

While (17) is probably valid for many materials, it should not be overlooked that for some materials or conditions the stress-strain curve may be affected to some extent by mean stress. Analytically, this effect can be accomplished by assuming that with mean stress the transition strain range changes from $\Delta\varepsilon_T$ to $k'\Delta\varepsilon_T$. However, if we assume that symmetry is still retained between the stress-strain curve with tensile mean stress and that with compressive mean stress, then we must assume the k' depends only on absolute value of mean stress, not algebraic sign. Then, the relation between elastic and plastic strain range will still depend only on strain range, and the cyclic stress-strain curves will be the same for equal values of both tension and compression, although different from the basic cylic stress-strain curve for zero mean stress. Until more experimental information is available on the nature of k', it is appropriate to assume it is unity. If known, however, the effect of mean stress on the cyclic stress-strain curve and on the life relation can be determined from (15) and (17) by replacing $\Delta\varepsilon_T$ by $k'\Delta\varepsilon_T$. If it should, for some reason, be preferable to express the life relation in the notation of the original (14), we can use the relations for N_T and $\Delta\varepsilon_T$ from (3) and (4), resulting in

$$\Delta\varepsilon/2 = [(\sigma_f' - k_m\sigma_o)/E](2N_f)^b + \{[(\sigma_f' - k_m\sigma_o/\sigma_f']k'^{[(b-c)/b]}\} \times \varepsilon_f'(2N_f)^c \qquad (18)$$

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10 June 1981

REPRESENTATION OF MEAN STRESS EFFECT BY DISPLACEMENT OF ONLY THE ELASTIC LINE

Figure 1.

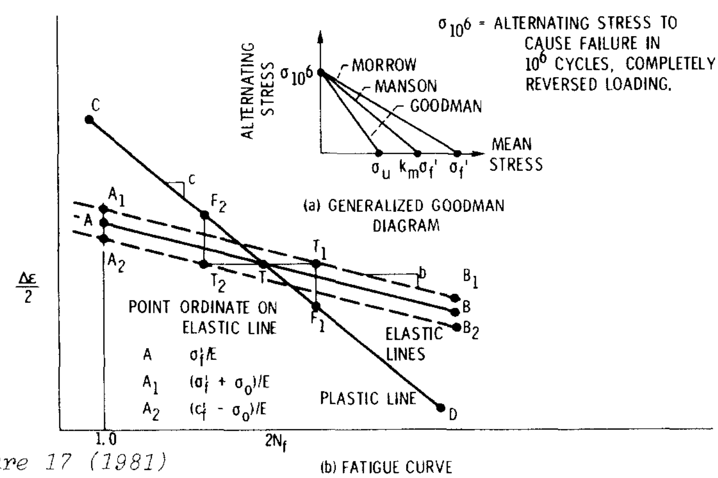


Figure 2.

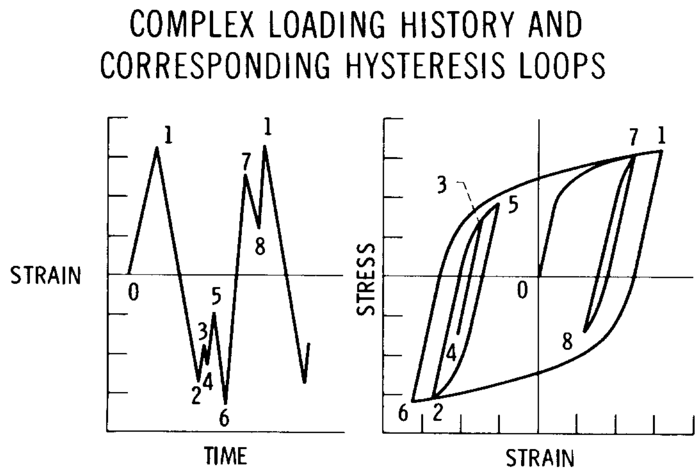
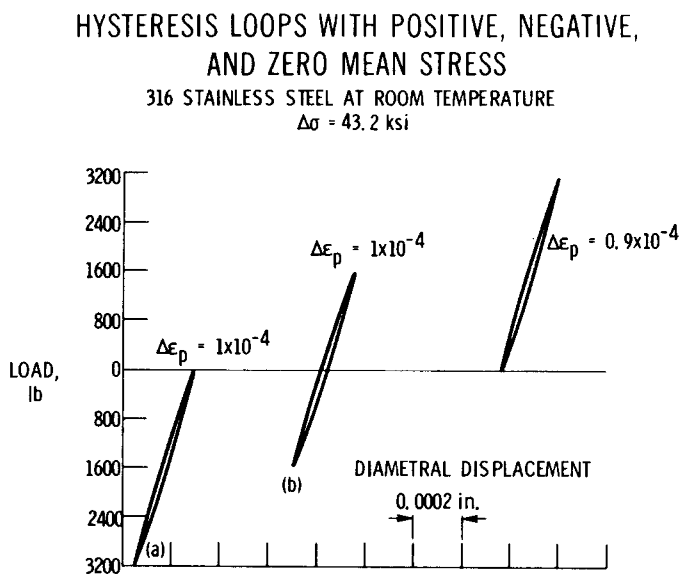


Figure 3.



HYSTERESIS LOOPS REQUIRED BY MORROW EQUATION FOR TENSILE, COMPRESSIVE, AND ZERO MEAN STRESS

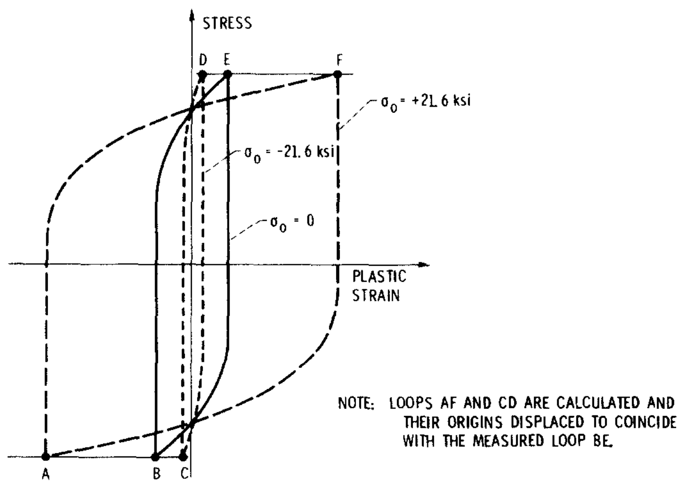


Figure 4.

DISPLACEMENT REQUIREMENTS OF ELASTIC AND PLASTIC LINES TO PRESERVE CYCLIC STRESS-STRAIN CURVE IN PRESENCE OF MEAN STRESS

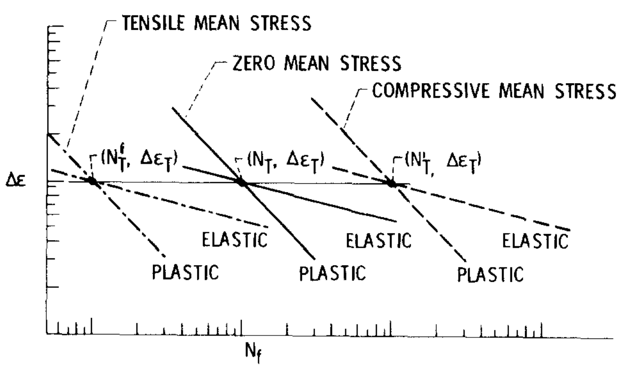


Figure 5.