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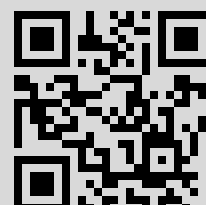
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PRACTICAL SCHEME OF REDUCTION TO GAUGE-INVARIANT VARIABLES

For systems with first-class constraints, the reduction scheme to gauge-invariant variables is considered. The method is based on an analysis of restricted 1-forms in gauge-invariant variables. This scheme is applied to the models of electrodynamics and Yang–Mills theory. For the finite-dimensional model with the $SU(2)$ gauge group of symmetry, the possible mechanism of confinement is obtained.

1. INTRODUCTION

Most of the interesting physical models and theories are described by gauge-invariant Lagrangians, which are singular, and in the Hamiltonian formulation, lead to constrained systems [1–5]. For constrained Hamiltonian systems, there are in principle two ways¹⁾ of quantization [1–6]:

- 1) first quantize and then reduce,
- 2) first reduce and then quantize.

In the present paper, we deal mostly with the latter type of reduction procedure. For gauge theories, this procedure is a restriction to the constraint surface \mathcal{M} with subsequent reduction to the physical phase space $\widetilde{\mathcal{M}} = \mathcal{M}/G$, i.e., to the space of gauge orbits.

If the action of the gauge group (G) on the constraint surface (\mathcal{M}) is regular, the manifold of orbits ($\widetilde{\mathcal{M}} = \mathcal{M}/G$) is well defined, and it possesses a symplectic structure. Coordinates on $\widetilde{\mathcal{M}}$ are gauge-invariant true physical degrees of freedom.

Quite often, in practical applications, this theoretical scheme of reduction encounters technical problems related to the explicit construction of $\widetilde{\mathcal{M}}$ supplied with the symplectic structure: apart from the mathematical difficulties, the physical content of the true degrees of freedom may well be unpredictable.

A commonly used reduction scheme is a gauge fixing procedure related to some constraints $\chi(p, q) = 0$ [1–5, 7]. In simple cases, the explicit form of the true physical variables

¹⁾In this paper, we do not consider the path integral approach.

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is obvious and this reduction scheme works perfectly well. But in general, as it was shown in [8], (namely, for the Yang–Mills theory), the space of gauge orbits (\mathcal{M}/G) cannot be obtained by “simple” gauge fixing. Obviously, the problem of gauge fixing is related to the above-mentioned possible nontrivial structure of the physical phase space [1–5, 9, 10].

Another reduction scheme can be based on the gauge-invariant variables (GIV’s) [1–6]. As a rule, the GIV’s are based on the structure of gauge transformations. If one can find the complete set of GIV’s, then it allows one to describe the physical phase space \mathcal{M}/G endowed with the symplectic structure. This paper deals with such a gauge-invariant approach in terms of restricted 1-forms. We also consider situations when only a part of the GIV’s is known. Analysis of the structure of restricted 1-forms helps one find the remaining part of the GIV’s.

Note that the reduction scheme with 1-forms for an arbitrary constrained system was proposed in [11]. In these papers, the elimination of extra variables was based on the Darboux theorem. But sometimes, the Darboux theorem is not effective in some applications, and the choice of GIV’s is just a practical way of realizing this reduction program in gauge theories.

Our paper is organized as follows: in Sec. 2, the reduction scheme for the GIV’s is introduced and, as an illustration, simple examples are considered. An additional example of $(2 + 1)$ -dimensional massive photodynamics is given in the Appendix. In Sec. 3, this scheme is applied to the finite-dimensional system with the $SU(2)$ gauge group of symmetry. This system can be considered as a toy model for the Yang–Mills theory with fermions. It is shown that there is an essential difference between the $SU(2)$ and the corresponding $U(1)$ model. The structure of GIV’s in the $SU(2)$ case can be interpreted as the confinement phenomenon. In Sec. 4, we study the infinite-dimensional model with a semisimple gauge group. The GIV’s are constructed and the complete reduction is accomplished. It is shown that the model is equivalent to the Yang–Mills theory with some boundary conditions. The last section contains remarks and conclusions.

2. REDUCTION SCHEME IN GAUGE-INVARIANT VARIABLES

Starting from the gauge-invariant Lagrangian $L = L(q_k, \dot{q}_k)$ ($k = 1, \dots, N$) and using the Dirac procedure [1] or the first order formalism [11], we arrive at the extended phase space Γ with coordinates (p_k, q_k) and the action

$$S = \int p_k dq_k - [H(p, q) + \lambda_a \phi_a(p, q)] dt, \quad (2.1)$$

$$k = 1, \dots, N; \quad a = 1, \dots, M \quad (N > M),$$

where $\phi_a(p, q)$ are constraints, $H(p, q)$ is a canonical Hamiltonian, and λ_a are Lagrange multipliers. The constraint surface \mathcal{M} is defined by

$$\phi_a(p, q) = 0, \quad (2.2)$$

and the following relations are fulfilled:

$$\{H, \phi_a\}_\Gamma = d_a^b \phi_b, \quad \{\phi_a, \phi_b\}_\Gamma = f_{ab}^c \phi_c. \quad (2.3)$$

Index Γ on the left hand side of the equations indicates that the Poisson brackets are calculated in the extended phase space.

A function $\xi = \xi(p, q)$ is called GIV [1–5] if $\xi|_{\mathcal{M}} \neq 0$ and

$$\{\xi, \phi_a\}_\Gamma = \tilde{d}_a^b \phi_b, \quad (2.4)$$

where $|_{\mathcal{M}}$ denotes the restriction to \mathcal{M} and the functions \tilde{d}_a^b (as well as d_a^b and f_{ab}^c in (2.3)) are assumed to be regular in the neighborhood of \mathcal{M} .

Each GIV ξ possesses the class $\{\xi\}$ of equivalent GIV's on Γ [1–5]. A gauge-invariant function $\tilde{\xi}$ is equivalent to ξ if $\tilde{\xi}|_{\mathcal{M}} = \xi|_{\mathcal{M}}$. On the other hand, the function $\xi|_{\mathcal{M}}$ is a constant along the gauge orbit (on \mathcal{M}) and it defines the function $\bar{\xi}$ in the physical space $\tilde{\mathcal{M}} = \mathcal{M}/G$. Thus, $\{\xi\}$, $\xi|_{\mathcal{M}}$ and $\bar{\xi}$ denote the GIV ξ in different contexts. If there is no ambiguity, we will use the notation ξ for all of them.

The maximum number of GIV's (2.4) that are functionally independent on the constraint surface \mathcal{M} is $2(N - M)$ [2]. Assume that $\{\xi^\alpha : \alpha = 1, \dots, 2(N - M)\}$ is such a complete set of GIV's. Then, one can prove [11] that

$$\begin{aligned} 1) \quad & p_k dq_k|_{\mathcal{M}} = \theta_1 + \theta_2, \text{ with} \\ & \text{a) } d\theta_1 = 0, \\ & \text{b) } \theta_2 = \theta_\alpha(\xi) d\xi^\alpha, \\ & \text{c) } \det \omega_{\alpha\beta} \neq 0, \text{ where } \omega_{\alpha\beta}(\xi) = \partial_\alpha \theta_\beta - \partial_\beta \theta_\alpha; \\ 2) \quad & H(p, q)|_{\mathcal{M}} = h(\xi). \end{aligned} \quad (2.5)$$

The main statement of (2.5) is that after restriction to the constraint surface \mathcal{M} , the dependence on extra (nonphysical) variables is presented only in the term θ_1 , which is a “total derivative”.

Since $d\theta_1 = 0$, it gives no contribution to the variation of a restricted action. We can neglect it, and for the reduced system we obtain,

$$S|_{\mathcal{M}} \equiv \tilde{S} = \int \theta_\alpha(\xi) d\xi^\alpha - h(\xi) dt. \quad (2.6)$$

Thus, the dynamics for the GIV's are described by the Hamilton equations

$$\xi^{\dot{\alpha}} = \omega^{\alpha\beta}(\xi) \partial_\beta h(\xi), \quad (2.7)$$

where $\omega^{\alpha\beta}(\xi)$ is the inverse of the symplectic matrix $\omega_{\alpha\beta} = \partial_\alpha \theta_\beta - \partial_\beta \theta_\alpha$; it defines the Poisson brackets for the reduced system

$$\{\xi^\alpha, \xi^\beta\}_{\tilde{\mathcal{M}}} = \omega^{\alpha\beta}(\xi). \quad (2.8)$$

So the reduced system (2.6)–(2.8) is an ordinary (nonconstrained) Hamiltonian system that can be quantized.

It should be noticed that, in the general case, any $2(N - M)$ number of GIV's play the role of local coordinates in the physical phase space $\widetilde{\mathcal{M}}$ and, respectively, (2.5)–(2.8) are defined locally. Global description can be achieved by the set of GIV's, which defines the global structure of the physical phase space $\widetilde{\mathcal{M}}$. The number of such GIV's is greater than $2(N - M)$, but on the constraint surface, there are relations among them. They are the relations that define the geometry of $\widetilde{\mathcal{M}}$. For illustration, let us consider the following example of (2.1)–(2.3) [12]:

$$S = \int \vec{p} \cdot d\vec{q} - [\lambda_1 \phi_1 + \lambda_2 \phi_2] dt. \quad (2.9)$$

Here, \vec{p} and \vec{q} are vectors in \mathbf{R}^3 , the canonical Hamiltonian is zero,

$$\phi_1 = \vec{p} \cdot \vec{q}, \quad \phi_2 = \vec{p}^2 \vec{q}^2 - (\vec{p} \cdot \vec{q})^2 - r^2,$$

and r is a parameter. These constraints are Abelian ($\{\phi_1, \phi_2\} = 0$) and the second constraint ϕ_2 can be written in the form

$$\phi_2 = \vec{J}^2 - r^2,$$

where $\vec{J} = \vec{q} \times \vec{p}$ is the angular momentum.

It is clear that the physical phase space is two-dimensional and the components of angular momentum \vec{J} are the GIV's (they commute with constraints, since constraints are $O(3)$ scalars). On the constraint surface, these three components are related by $\vec{J} \cdot \vec{J} = r^2$. Therefore, the physical phase space $\widetilde{\mathcal{M}}$ is a two-dimensional sphere. So any two coordinates (as well as the 1-forms θ_1 and θ_2) are only defined locally (on the phase space geometry of constrained systems, see [9]).

The reduction scheme described (2.5)–(2.8) can be used if all $2(N - M)$ GIV's are known. For practical application of the scheme, one can introduce arbitrary variables η^1, \dots, η^M , which are complementary to the GIV's, in order to complete the coordinate system

$$(\xi^1, \dots, \xi^{2(N-M)}, \eta^1, \dots, \eta^M)$$

on \mathcal{M} . Calculating the restricted 1-form $p_k dq_k|_{\mathcal{M}}$ in these coordinates and taking its differential, we can find the symplectic form $\omega = \omega_{\alpha\beta}(\xi) d\xi^\alpha \wedge d\xi^\beta$. Note that in actual calculations it is possible to select the 1-form $\theta_2 = \theta_\alpha(\xi) d\xi^\alpha$ and arrive at (2.6).

Application of this procedure to model (2.9) gives $\theta_2 = z d\varphi$, where z and φ are the cylindric coordinates on the sphere:

$$J_1 = \sqrt{r^2 - z^2} \cos \varphi, \quad J_2 = \sqrt{r^2 - z^2} \sin \varphi, \quad J_3 = z.$$

It is clear that although $z d\varphi$ is not a global 1-form, its differential can be continued to the well-defined symplectic form on the sphere [13]

$$\omega = -\frac{J_1(dJ_2 \wedge dJ_3) + J_2(dJ_3 \wedge dJ_1) + J_3(dJ_1 \wedge dJ_2)}{r^2}.$$

After this, the system can be quantized by means of geometric quantization [14] (see also [12, 15, 16]). A consistent quantum theory exists only for discrete values of the parameter r .

Generalization of the scheme to the infinite-dimensional case is straightforward (in the Appendix, we present the example of massive photodynamics in $(2+1)$ dimensions). If we use the Dirac observables [17]

$$\psi_{in} = e^{i\Delta^{-1}(\vec{\nabla}\vec{A})}\psi \quad (2.10)$$

in ordinary QED, we will easily obtain the photons in the Coulomb gauge and the “four-fermion interaction” for the “dressed fermions” (compare with the example in Sec. 3 and see [11, 18]).

Note that the commutation relations of the complete set of GIV’s (2.8) can be also derived by calculation of the Poisson brackets on the extended phase space [1–5]. This, more standard procedure is based on the fact that the Poisson bracket of any two GIV’s is again a GIV. This procedure and the scheme described in this paper (2.5)–(2.8) are almost equivalent. Sometimes, however, the calculation of differential forms is more reliable (especially, when the canonical quantization is not applicable [14]).

In general, from the structure of gauge transformations, one can easily find only part of the GIV’s, while the construction of the complete set (2.5) is troublesome. In many cases, our approach with differential forms can be effectively used for the solution of this problem, as well.

Let us consider the situation where we know the set of GIV’s $\{\xi^\mu : \mu = 1, \dots, K\}$, where $N - M \leq k \leq 2(N - M)$. We can add arbitrary variables $\eta^1, \dots, \eta^{2N-M-K}$ in order to complete the coordinate system on \mathcal{M} and calculate the restricted 1-form $p_k dq_k|_{\mathcal{M}}$. Assume that we can release the “total derivatives” and the differentials $d\xi^\mu$ from the form

$$p_k dq_k|_{\mathcal{M}} = dF(\xi, \eta) + \theta_\mu(\xi, \eta) d\xi^\mu. \quad (2.11)$$

Then, using (2.5), we can easily conclude that $\theta_\mu(\xi, \eta)$ will be the GIV’s. Note that passing to the GIV’s ξ^μ is helpful for obtaining the form (2.11). To illustrate this method, we apply it to a relativistic particle [3], where the 1-form $\theta = \vec{p}d\vec{q} - p_0 dq_0$, and the constraint surface $\mathcal{M}: p^2 - m^2 = 0$ ($p_0 > 0$). The momenta \vec{p} are gauge invariant, and after restriction on \mathcal{M} , we have

$$\theta|_{\mathcal{M}} = \vec{p}d\vec{q} - \sqrt{\vec{p}^2 + m^2} dq_0.$$

One can easily rewrite it in the following form:

$$\theta|_{\mathcal{M}} = d\left(\vec{p} \cdot \vec{q} - \sqrt{\vec{p}^2 + m^2} q_0\right) - \left(\vec{q} - \frac{\vec{p}}{\sqrt{\vec{p}^2 + m^2}} q_0\right) d\vec{p}.$$

Evidently, the coefficients of the differentials $-d\vec{p}$ are GIV's. They are canonically conjugate to \vec{p} :

$$\vec{Q} = \vec{q} - \frac{\vec{p}}{\sqrt{\vec{p}^2 + m^2}} q_0.$$

The gauge invariance of \vec{Q} can also be established from the relation

$$\vec{L} = \sqrt{\vec{p}^2 + m^2} \vec{Q}, \quad (2.12)$$

where \vec{L} are the generators of Lorentz transformations. Since all generators of the Poincaré group ($P_\mu, M_{\mu\nu}$) are GIV's, the same property holds for the coordinates \vec{Q} . On the constraint surface $p^2 - m^2 = 0$ ($p_0 > 0$), all of these coordinates are the functions only of the reduced variables \vec{p} and \vec{Q} .

The reduced system can be easily quantized in the momentum representation: $\widehat{\vec{p}} = \vec{p}$ and $\widehat{\vec{Q}} = i\hbar\vec{\nabla}$. An operator ordering problem arises only for the generators (2.12). Therefore, the standard Lorentz covariant measure of a scalar product,

$$\langle \Psi_2 | \Psi_1 \rangle = \int \frac{d^3\vec{p}}{\sqrt{\vec{p}^2 + m^2}} \bar{\Psi}_2(\vec{p}) \Psi_1(\vec{p})$$

corresponds to the ordering $\widehat{\vec{L}} = i\hbar\sqrt{\vec{p}^2 + m^2} \vec{\nabla}$.

3. FINITE-DIMENSIONAL MODELS WITH $U(1)$ AND $SU(2)$ GAUGE SYMMETRIES

In this section we consider the finite-dimensional model with the $SU(2)$ gauge group of symmetry. It is difficult to find all the GIV's at the beginning, and therefore, we use the method described at the end of Sec. 2. The obtained structure of the GIV's is quite unexpected. For comparison, we also present the corresponding $U(1)$ model. These $U(1)$ and $SU(2)$ models can be considered as the toy models of electrodynamics and the Yang–Mills theory (with matter), respectively. In the classical description, all “fields” are assumed to be c -numbers.

A. The model with $U(1)$ symmetry. Let us consider the action

$$S = \int dt \left[\frac{i}{2} (\bar{\psi}\dot{\psi} - \dot{\bar{\psi}}\psi) - m\bar{\psi}\psi + A_0(\bar{\psi}\psi - kE) + EA - \frac{1}{2}E^2 \right], \quad (3.1)$$

where all “fields” ($\bar{\psi}, \psi, A_0, A, E$) are functions only of time t ; m and k ($k \neq 0$) are parameters. The similarity to electrodynamics is apparent from the notations. At the same time, (3.1) has the form (2.1), where $A_0 \equiv \lambda(t)$ is a Lagrange multiplier, and $\phi \equiv \bar{\psi}\psi - kE$ is a constraint (we use the time derivatives instead of differential form where it is convenient).

The nonzero Poisson brackets are

$$\{\psi, \bar{\psi}\} = i, \quad \{E, A\} = 1,$$

and we have the gauge transformations

$$\begin{aligned}\psi(t) &\longrightarrow e^{+i\alpha(t)}\psi(t), & \bar{\psi}(t) &\longrightarrow e^{-i\alpha(t)}\bar{\psi}(t), \\ A(t) &\longrightarrow A(t) + k\alpha(t), & E(t) &\longrightarrow E(t).\end{aligned}\quad (3.2)$$

Then,

$$A_0(t) \longrightarrow A_0(t) + \dot{\alpha}(t)$$

leaves action (3.1) invariant.

The reduced system is two-dimensional, and two GIV's can be chosen as follows:

$$\Psi_{\text{inv}} = e^{\frac{-i}{k}A}\psi, \quad \bar{\Psi}_{\text{inv}} = e^{\frac{i}{k}A}\bar{\psi} \quad (3.3)$$

(compare with (2.10)). Here the reduction procedure (2.5) is trivial and we find

$$\tilde{S} = \int dt \left[\frac{i}{2}(\bar{\Psi}_{\text{inv}}\dot{\Psi}_{\text{inv}} - \dot{\bar{\Psi}}_{\text{inv}}\Psi_{\text{inv}}) - m\bar{\Psi}_{\text{inv}}\Psi_{\text{inv}} - \frac{1}{k^2}(\bar{\Psi}_{\text{inv}}\Psi_{\text{inv}})^2 \right]. \quad (3.4)$$

Thus, the ‘‘gauge field’’ A vanishes and the only physical excitations are the ‘‘dressed fields’’ Ψ_{inv} (with ‘‘four-fermion interactions’’).

This model has a simple generalization in the case of a multi-component gauge field \vec{A} with the gauge transformations

$$\vec{A} \longrightarrow \vec{A} + \vec{k}\alpha,$$

where \vec{k} are parameters ($\vec{k}^2 \neq 0$). The GIV Ψ_{inv} is constructed similar to (3.3) (or, to (2.10)). Then, after reduction, the ‘‘longitudinal’’ (to the \vec{k}) component of the gauge field \vec{A} vanishes and the only physical variables are the ‘‘transverse’’ ones, together with the constructed ‘‘dressed field’’ Ψ_{inv} . So, for these Abelian models, the structure of the GIV's is very similar to the physical observables in electrodynamics [11, 18].

B. The model with $SU(2)$ symmetry. For the model with $SU(2)$ gauge group of symmetry, we consider the action

$$S = \int dt \left[\frac{i}{2}(\bar{\psi}_\alpha\dot{\psi}_\alpha - \dot{\bar{\psi}}_\alpha\psi_\alpha) - m\bar{\psi}_\alpha\psi_\alpha + \vec{A}_0(\vec{j} + \vec{J}) + \vec{E}\dot{\vec{A}} - \frac{1}{2}\vec{E}^2 \right]. \quad (3.5)$$

Here ψ_α are the 2-component spinors ($\alpha = 1, 2$), m is a parameter, \vec{A} and \vec{E} are three-dimensional vectors, \vec{A}_0 are the Lagrange multipliers, and the angular momenta \vec{j} and \vec{J} are given by

$$\vec{j} = \bar{\psi}\frac{\vec{\sigma}}{2}\psi, \quad \vec{J} = \vec{A} \times \vec{E}, \quad (3.6)$$

where $\vec{\sigma}$ are the standard Pauli matrices.

The connection with the Yang–Mills theory is obvious.

The nonzero Poisson brackets are

$$\{\psi_\alpha, \bar{\psi}_\beta\} = i\delta_{\alpha\beta}, \quad \{E_m, A_n\} = \delta_{mn} \quad (m, n) = 1, 2, 3, \quad (3.7)$$

and the constraints $\vec{\phi} = \vec{j} - \vec{J}$ generate the gauge transformations:

$$\psi \rightarrow \omega\psi, \quad \bar{\psi} \rightarrow \bar{\psi}\omega^{-1}, \quad \mathbf{A} \rightarrow \omega\mathbf{A}\omega^{-1}, \quad \mathbf{E} \rightarrow \omega\mathbf{E}\omega^{-1},$$

where $\omega(t) \in SU(2)$ and

$$\mathbf{A} \equiv \frac{1}{2}\vec{A}\vec{\sigma}, \quad \mathbf{E} \equiv \frac{1}{2}\vec{E}\vec{\sigma}. \quad (3.8)$$

Then, for $\mathbf{A}_0 \equiv \frac{1}{2}\vec{A}_0\vec{\sigma}$, we get $\mathbf{A}_0 \rightarrow \omega\mathbf{A}_0\omega^{-1} - i\dot{\omega}\omega^{-1}$.

Any scalar product of the vectors $\vec{A}, \vec{E}, \vec{J}, \vec{j}$ will be a GIV. But on the constraint surface ($\vec{j} + \vec{J} = 0$), only three of them are functionally independent. If we choose these independent GIV's as:

$$l_0 = \frac{1}{4}(\vec{A}^2 + \vec{E}^2), \quad l_1 = \frac{1}{2}(\vec{E}\vec{A}), \quad l_2 = \frac{1}{4}(\vec{A}^2 - \vec{E}^2), \quad (3.9)$$

then from (3.7), we obtain the $SL(2, \mathbf{R})$ algebra:

$$\{l_\mu, l_\nu\} = \epsilon_{\mu\nu\sigma} g^{\sigma\rho} l_\rho, \quad \text{where } g^{\mu\nu} = \text{diag}(+, -, -). \quad (3.10)$$

Since there are three constraints, the physical phase space is four-dimensional. To construct the fourth GIV and find the complete symplectic structure, we use the method of Sec. 2 (see (2.11)).

For parameterization of the constraint surface, we introduce the new variables $(j, \Phi; h, \phi)$:

$$\begin{aligned} j &= \frac{1}{2}(h_1 + h_2), & h &= \frac{1}{2}(h_1 - h_2), \\ \Phi &= \varphi_1 + \varphi_2, & \varphi &= \varphi_1 - \varphi_2, \end{aligned} \quad (3.11)$$

where

$$\psi_\alpha = \sqrt{h_\alpha} e^{-i\varphi_\alpha}, \quad \bar{\psi}_\alpha = \sqrt{h_\alpha} e^{i\varphi_\alpha} \quad (\alpha = 1, 2).$$

Then, for the 1-form, we have

$$\frac{i}{2}(\bar{\psi}_\alpha d\psi_\alpha - \psi_\alpha d\bar{\psi}_\alpha) = j d\Phi + h d\varphi. \quad (3.12)$$

The vector \vec{j} (3.6) in these new coordinates takes the form

$$\vec{j} = \begin{pmatrix} \sqrt{j^2 - h^2} \cos \varphi \\ \sqrt{j^2 - h^2} \sin \varphi \\ h \end{pmatrix}$$

and $\vec{j}^2 = j^2$. Note that on the constraint surface, we have (see (3.9)) $l^\mu l_\mu = j^2/4$, and for fixed j , the commutation relations (3.10) define the well-known symplectic structure on this hyperboloid (see, e.g., [19]).

If we introduce the orthonormal basis ($\vec{e}_i \cdot \vec{e}_k = \delta_{ik}$, $\vec{e}_i \times \vec{e}_j = \epsilon_{ijk} \vec{e}_k$):

$$\vec{e}_1 = \begin{pmatrix} -\sin \varphi \\ \cos \varphi \\ 0 \end{pmatrix}, \quad \vec{e}_2 = -\frac{h}{j} \begin{pmatrix} \cos \varphi \\ \sin \varphi \\ -\frac{\sqrt{j^2 - h^2}}{h} \end{pmatrix}, \quad \vec{e}_3 = \frac{\vec{j}}{j},$$

then \vec{A} and \vec{E} can be parameterized as follows:

$$\vec{A} = \vec{e}_1 q_1 + \vec{e}_2 q_2, \quad \vec{E} = \vec{e}_1 p_1 + \vec{e}_2 p_2,$$

where

$$p_1 q_2 - p_2 q_1 = j. \quad (3.13)$$

Calculating the restricted 1-form $\vec{E} d\vec{A}|_{\mathcal{M}}$ in these new coordinates and using (3.13), we obtain

$$\vec{E} d\vec{A}|_{\mathcal{M}} = p_1 dq_1 + p_2 dq_2 - h d\varphi. \quad (3.14)$$

Comparing (3.12) and (3.14), we see that there is a cancellation of the 1-form $h d\varphi$. This means that the corresponding degrees of freedom vanish.

Now, it is convenient to introduce the polar coordinates for the two-vectors (q_1, q_2) and (p_1, p_2) :

$$\begin{aligned} q_1 &= r \cos \beta, & p_1 &= \rho \cos \gamma, \\ q_2 &= r \sin \beta, & p_2 &= \rho \sin \gamma. \end{aligned}$$

Then, three of them (r, ρ and $(\beta - \gamma)$) are connected by the GIV's (3.9):

$$r^2 = 2(l_0 + l_2) \equiv l_+, \quad \rho^2 = 2(l_0 - l_2) \equiv l_-, \quad r\rho \cos(\beta - \gamma) = 2l_1.$$

Using these relations, we finally get the reduced 1-form

$$\theta|_{\mathcal{M}} = j d\vartheta + l_1 \frac{dl_+}{l_+}, \quad \text{where } \vartheta = \Phi - \beta. \quad (3.15)$$

So the coordinate $\vartheta = \Phi - \beta$ is the fourth GIV. Respectively, the reduced Hamiltonian takes the form

$$H|_{\mathcal{M}} = 2mj + \frac{j^2 + 4l_1^2}{l_+}, \quad (3.16)$$

and this is the complete reduction.

Note that the second part of the reduced 1-form, $l_1 d(\ln l_+)$, defines the above-mentioned symplectic structure on the hyperboloid $l^\mu l_\mu = \frac{1}{4}j^2$ [19].

We see that the physical picture of this reduced system differs from the corresponding Abelian case. Here, after reduction, part of the degrees of freedom of the “gauge field” (A), as well as part of the “matter field” (ψ) degrees of freedom have vanished. Below, we shall see that in quantum theory, the vanishing of “matter field” degrees of freedom can be interpreted as the confinement phenomenon.

Geometric quantization [14] is a natural way of constructing the quantum theory of the reduced systems (3.15)–(3.16), but, in principle, one can use canonical quantization, as well. For this purpose, it is convenient to introduce (global) “creation” and “annihilation” variables

$$a^+ = \sqrt{j}e^{i\vartheta}, \quad a = \sqrt{j}e^{-i\vartheta}, \quad (3.17)$$

and in quantum theory, we get the discrete eigenvalues for $j = a^+a$. Then, quantization of the system with the canonical 1-form $l_1 d(\ln l_+)$ and the Hamiltonian (3.16) (for the obtained discrete eigenvalues of j), gives the irreducible representations of $SL(2, \mathbf{R})$ group (see, e.g., [19]).

Next, from (3.11), we have the relation $N \equiv \bar{\psi}_\alpha \psi_\alpha = 2j$. It is natural to interpret the corresponding operator ($\widehat{N} \equiv 2\widehat{j}$) as the ψ particle number operator. In quantum theory, we have

$$[\widehat{N}, \widehat{a}^+] = 2\widehat{a}^+$$

where \widehat{a}^+ is a physical creation operator (3.17). So among the physical excitations (created by the operator \widehat{a}^+) there are states with only even numbers of “fermions”. This fact can also be seen from the structure of the variable a^+ (see (3.17) and (3.11)). It has the phase factor $e^{i(\varphi_1 + \varphi_2)}$. Thus, in the quantum case, it creates (see [20]) pairs of “dressed” ψ -particles.

Note that for similar finite-dimensional constrained systems, such “confinement”-like phenomenon has been derived by the “first quantize and then reduce” method (see [21]). In that approach, the reduction of the extended “Hilbert” space by the conditions $\widehat{\phi}_\alpha |\Psi_{\text{phys}}\rangle = 0$ forbids states with certain quantum numbers.

4. FIELD THEORY MODELS WITH NON-ABELIAN GAUGE GROUP OF SYMMETRIES

For the finite-dimensional models of the previous section, the gauge group G acts on the configuration space of “gauge field” (A) and on the phase space of “matter field” (ψ). This is the standard situation for the Yang–Mills theories.

Using the notations of (3.8), we have

$$\vec{E} d\vec{A} = \langle \mathbf{E}, d\mathbf{A} \rangle, \quad (4.1)$$

where $\langle \cdot, \cdot \rangle$ is a scalar product in the corresponding Lie algebra \mathcal{A} . Thus, the Lie algebra \mathcal{A} can be interpreted as the configuration space of a “gauge field” \vec{A} and a trivial cotangent bundle as the phase space.

If one takes a manifold of a semi-simple Lie group (G) as the configuration space, then there are the natural actions (left and right) of G on this manifold. One can similarly construct the gauge theory where the phase space is a cotangent bundle [22] $T^*G = \{(g, R) \mid g \in G, R \in \mathcal{A}\}$. The symplectic form on T^*G is given by

$$\omega = d\theta \quad \text{with } \theta = \langle R, g^{-1} dg \rangle. \quad (4.2)$$

Generators of the left and right transformation ($g \longrightarrow \omega g, g \longrightarrow g\omega$) are, respectively, the left and right currents ($L \equiv gRg^{-1}, R$). Choosing gauge transformations as the right action, we find that the constraints are $\phi \equiv R = 0$. Thus, the ‘‘gauge field’’ part in the action takes the form

$$\int \langle R, g^{-1} dg \rangle - (\langle \Lambda, R \rangle + H(R, g)) dt, \quad (4.3)$$

where $\Lambda \in \mathcal{A}$ is a Lagrange multiplier and H is a gauge-invariant Hamiltonian.

The field theory generalization of (3.5) is the standard Yang–Mills theory. In this section, we consider corresponding generalization of (4.3) with the action

$$S = \int dt \left[\int d^{D-1} \vec{x} \left(\sum_{k=1}^{D-1} \langle R_k, g_k^{-1} \dot{g}_k \rangle + e \langle A_0, \phi \rangle \right) - H \right], \quad (4.4)$$

where $g_k(\vec{x}, t) \in G$ and $R_k(\vec{x}, t), A_0(\vec{x}, t) \in \mathcal{A}$; H is a gauge-invariant Hamiltonian, A_0 are Lagrange multipliers, $\phi(\vec{x}, t) \equiv e \sum_{k=1}^{D-1} R_k(\vec{x}, t)$ are constraints, and e is the coupling constant (see below).

The ‘‘1-form’’ $\sum_{k=1}^{D-1} \langle R_k, g_k^{-1} dg_k \rangle$ defines the equal-time Poisson brackets (see, e.g., [22]):

$$\begin{aligned} \{R_{k,a}(\vec{x}), R_{l,b}(\vec{y})\} &= \delta_{kl} \delta(\vec{x} - \vec{y}) f_{ab}^c R_{k,c}(\vec{x}), \\ \{g_k(\vec{x}), R_{l,a}(\vec{y})\} &= \delta_{kl} \delta(\vec{x} - \vec{y}) (g_k T_a(\vec{x})), \\ \{g_k(\vec{x}), g_l(\vec{y})\} &= 0, \end{aligned} \quad (4.5)$$

where the set $\{T_a \mid T_a \in \mathcal{A}\}$ forms a basis in the Lie algebra, $R_a \equiv \langle T_a, R \rangle$, and the last two relations are the matrix equalities [22]. Thus, for the constraints $\phi_a \equiv \langle T_a, \phi \rangle$, we have

$$\{\phi_a(\vec{x}), \phi_b(\vec{y})\} = \delta(\vec{x} - \vec{y}) f_{ab}^c \phi_c(\vec{x}). \quad (4.6)$$

The corresponding gauge transformations are

$$g_k \longrightarrow g_k \omega, \quad R_k \longrightarrow \omega^{-1} R_k \omega, \quad (4.7)$$

and one can easily construct the GIV’s such that

$$g_{kl} = g_k g_l^{-1} \quad \text{and} \quad L_k = g_k R_k g_k^{-1}. \quad (4.8)$$

The Hamiltonian H in (4.4) is an arbitrary functional of such GIV's.

Since Eq. (4.8) gives us the sufficient number of GIV's, we can use the scheme described in Sec. 2. The first nontrivial case is three-dimensional space-time. If we introduce $g = g_1 g_2^{-1}$ as the ξ^μ variables, and R_1, R_2 and g_2 as the η variables of the scheme (see (2.11)), then for the "1-form" $\theta = \langle R_1, g_1^{-1} dg_1 \rangle + \langle R_2, g_2^{-1} dg_2 \rangle$ (where integration over \mathbf{R}^2 is assumed), we immediately get

$$\langle R_1 + R_2, g_2^{-1} dg_2 \rangle + \langle g_2 R_1 g_2^{-1}, g^{-1} dg \rangle,$$

and after reduction we have

$$\theta|_{\mathcal{M}} = \langle r, g^{-1} dg \rangle, \quad (4.9)$$

where $r = g_2 R_1 g_2^{-1}$ is also GIV.

Thus, the structure of the 1-form is the same, with only the number of variables decreasing. One can verify this for other dimensions, as well.

It is clear that the phase spaces of the systems with 1-forms (4.1) and (4.2) are essentially different and they can not be transformed into each other. But in the field theory where an infinite number of such spaces exists, there is a nonlocal transformation (see [23]):

$$A_k = \frac{1}{e} g_k^{-1} \partial_k g_k, \quad E_k = -e g_k^{-1} \partial_k^{-1} (L_k) g_k, \quad (4.10)$$

which transforms system (4.4) into the Yang–Mills theory with the same gauge group G . Indeed, from (4.4) and (4.10), one can see that

$$\phi = \sum_{k=1}^{D-1} \partial_k E_k + e[A_k, E_k] \quad (\text{Gauss law})$$

and

$$\langle E_k, \dot{A}_k \rangle = \langle R_k, g_k^{-1} \dot{g}_k \rangle + (\text{total derivatives}). \quad (4.11)$$

To get the corresponding Hamiltonian of the Yang–Mills theory [23–25],

$$H = \frac{1}{2} \int d^{D-1} \vec{x} \left(\sum_{k=1}^{D-1} \langle E_k, E_k \rangle + \frac{1}{2} \sum_{k,l=1}^{D-1} \langle F_{kl}, F_{kl} \rangle \right),$$

with $F_{kl} = \partial_k A_l - \partial_l A_k + e[A_k, A_l]$, one must choose, in (4.4),

$$H = \frac{1}{2} \int d^{D-1} \left[e^2 \langle \partial_k^{-1} L_k, \partial_k^{-1} L_k \rangle + \frac{1}{e^2} \langle \partial_k (g_{kl} \partial_l g_{lk}), \partial_k (g_{kl} \partial_l g_{lk}) \rangle \right]. \quad (4.12)$$

Thus, one can assume that system (4.4) with Hamiltonian (4.12) is equivalent to the ordinary Yang–Mills theory with some boundary conditions (which allows us to invert (4.10) and to neglect the total derivatives in (4.11)).

The boundary behavior is a subtle problem even for simple models of the field theory (see, e.g., the Appendix). It is too complicated for the Yang–Mills theory and we do not consider it here.

Unfortunately the complicated form of the Hamiltonian (4.12) is not simplified after the reduction procedure. For example, in the three-dimensional case considered, the reduced Hamiltonian acquires the form

$$H = \frac{1}{2} \int d^2x \left[e^2 \langle \partial_1^{-1} r, \partial_1^{-1} r \rangle + e^2 \langle \partial_2^{-1} l, \partial_2^{-1} l \rangle + \frac{1}{e^2} \langle \partial_1 (g \partial_2 g^{-1}), \partial_1 (g \partial_2 g^{-1}) \rangle \right], \quad (4.13)$$

where $l = grg^{-1}$.

Gribov's ambiguity problem has stimulated many papers on the gauge-invariant description of the Yang–Mills theory and the reduced system. Equations (4.9), (4.13) give us one of the possible versions (for the literature and new results on this problem, see [5]). The main problem in such approaches is the complicated form of the Poincaré generators in terms of the GIV's [23–25]. For example, Hamiltonian (4.13) is nonlocal in fields and nonanalytical in the coupling constant. So the standard perturbative quantization is not applicable here.

Note that such a Hamiltonian with a corresponding symplectic form was obtained in [23] by the Dirac bracket formalism.

5. CONCLUSIONS

Of course, essential progress has been made in the study of constraint systems since the publication of [26], but from the point of view of practical applications, there is no universal approach, yet. The method presented in this paper is a one such possible practical step towards the quantization of gauge theories.

As mentioned in the Introduction, there is an alternate way of quantizing for the gauge-invariant systems when one “first quantizes and then reduces.” In general, there are two problems with this approach:

- a) the construction of physical states $|\Psi_{\text{phys}}\rangle$ as solutions of the equations $\hat{\phi}_a |\Psi_{\text{phys}}\rangle = 0$, where $\hat{\phi}$ are the constraint operators;
- b) the problem of the scalar product for the physical states.

Sometimes, the first problem is only a technical one (for the Yang–Mills theory see [26]), but in general, both of these problems are related and need further investigation [27].

In this paper we have not mentioned other important methods such as the path integral approach [2, 7] and the BRST quantization [28] (for a review see [29]). Quantization procedure is not unique even for the ordinary, nonconstrained systems [13, 30]. It depends on the choice of canonical variables (if they exist globally), the operator ordering, etc. Therefore, it is not surprising that different quantization procedures of constraint systems, generally speaking, lead to the nonequivalent quantum systems [7, 31].

As mentioned in Sec. 2, there are no global canonical coordinates for a reduced classical system and, therefore, the standard canonical quantization is not applicable. This, together with technical problems of classical reduction, was the main obstacle in the general formulation of the “first reduce and then quantize” approach.

Geometric quantization [14] and other “new” quantization schemes [12, 29, 32] allow one to quantize Hamiltonian systems without global canonical structures as well. At the same time, progress has been made in the construction of classical reduction schemes [11]. Therefore, for a wide class of constrained systems, the quantization method “first reduce and then quantize” appears to be technically preferable. Here a possible combination of the two quantization schemes should be mentioned: if a reduced classical system is complicated, then one can construct a new extended system with simple constraints on the cotangent bundle of the reduced phase space, and subsequently use the first method of quantization [12, 15, 29]. Of course, the question of which scheme gives the “correct” quantum description of a given classical system, remains open.

APPENDIX

The 2 + 1-dimensional massive photodynamics is described by the Lagrangian (see, e.g., [33])

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m}{4}\epsilon^{\mu\nu\sigma}F_{\mu\nu}A_\sigma. \quad (\text{A.1})$$

We choose $g_{\mu\nu} = \text{diag}(+, -, -)$, $\epsilon^{012} = 1$ and obtain in the first order formalism [11]:

$$S = \int dt \int_{\mathbb{R}^2} d^2x \left[\left(E_i - \frac{m}{2}\epsilon_{ij}A_j \right) \dot{A}_i - \frac{1}{2}(E_i E_i + B^2) + A_0(\partial_i E_i - mB) \right], \quad (\text{A.2})$$

where

$$E_i \equiv F_{0i} \equiv \dot{A}_i - \partial_i A_0, \quad B \equiv \frac{1}{2}\epsilon_{ij}F_{ij} \quad (\epsilon_{ij} \equiv \epsilon^{0ij}),$$

and we neglect the boundary term $\int_{\mathbb{R}^2} d^2x \partial_i [A_0(\frac{m}{2}\epsilon_{ij}A_j - E_i)]$.

If we use “1-forms” instead of time derivatives (see the comment following Eq. (3.1)), action (A.2) takes the form (2.1) with A_0 playing the role of a Lagrange multiplier.

For the reduction, we choose E_1 and E_2 to be the variables ξ^μ , and A_1 the additional variable η (see (2.11)). Then

$$\tilde{S} = \int dt \int_{\mathbb{R}^2} d^2x \left[\frac{1}{m}E_2 \dot{E}_1 - \frac{1}{2} \left[E_i E_i + \frac{1}{m^2}(\partial_k E_k)^2 \right] + \frac{d}{dt}\Theta \right], \quad (\text{A.3})$$

where

$$\Theta = \frac{1}{2} \left[E_1 A_1 + E_2 \widehat{K} \left(A_1 + \frac{1}{m} E_2 \right) \right]$$

and the operator $\widehat{K} \equiv \partial_1^{-1} \partial_2$ is assumed to be symmetrical due to the corresponding boundary conditions.

Neglecting the Θ term as the total derivative, we get the local Hamiltonian theory with the canonical commutation relations

$$\{E_2(x), E_1(y)\} = m\delta^{(2)}(x - y) \quad (\text{A.4})$$

and the quadratic Hamiltonian

$$\frac{1}{2} \int_{R^2} d^2x \left[E_i E_i + \frac{1}{m^2} (\partial_k E_k)^2 \right]. \quad (\text{A.5})$$

The energy-momentum tensor can also be expressed in terms of E_1 and E_2 alone:

$$T_{00} = \frac{1}{2} \left[E_i E_i + \frac{1}{m^2} (\partial_k E_k)^2 \right], \quad T_{0i} = \frac{1}{m} \epsilon_{ij} E_j (\partial_k E_k). \quad (\text{A.6})$$

Let us briefly discuss the boundary conditions. We can assume that the boundary behavior of the physical variables (E_1, E_2) should provide the Poincaré invariance of the reduced system (A.3)–(A.6), while the boundary behavior of the fields of the initial system (A.1) should allow the outlined reduction procedure.

Generators of the Poincaré group (constructed from the energy-momentum tensor (A.6)) generate transformations of E_1 and E_2 according to the Poisson brackets (A.4). The space of functions $E_1(x)$ and $E_2(x)$ should be invariant under these transformations. It is natural to choose the class of smooth functions rapidly vanishing at infinity.

For diagonalization of the Hamiltonian and momentum, let us make the Fourier transformation:

$$E_j(x) = i \int d^2p \frac{e^{-i(p \cdot x)}}{2\pi} \tilde{E}_j(p) \quad (\text{A.7})$$

and introduce the longitudinal and transverse components:

$$\tilde{E}_j(p) = \frac{p_j}{|p|} e_1(p) - \frac{\epsilon_{jl} p_l}{|p|} e_2(p), \quad (\text{A.8})$$

where $|p| = \sqrt{p_1^2 + p_2^2}$.

Then diagonalization of the energy and momentum occurs in the variables

$$\begin{aligned} a(p) &= \frac{\frac{\omega_p}{m} e_1(p) + i e_2(p)}{\sqrt{2\omega_p}} e^{-i\varphi(p)}, \\ a^*(p) &= \frac{\frac{\omega_p}{m} e_1(-p) - i e_2(-p)}{\sqrt{2\omega_p}} e^{i\varphi(p)}, \end{aligned} \quad (\text{A.9})$$

with

$$\omega_p = \sqrt{|p|^2 + m^2} \quad \text{and} \quad e^{\pm i\varphi(p)} = \frac{p_1 \pm i p_2}{|p|}.$$

Note that for the chosen class of $E_1(x)$ and $E_2(x)$, the longitudinal and transverse components of $\tilde{E}_j(p)$ have a singularity at the origin ($p = 0$), and we need to introduce the phase factor $e^{i\varphi(p)}$ to cancel it. On the other hand, one can easily check that the class of smooth functions $a(p), a^*(p)$ is Poincaré invariant. This phase factor was introduced in [33]

to avoid anomalies in the commutation relations of the Poincaré algebra of quantum operators. As we have seen, this phase factor is connected to the Poincaré invariance of the classical system, as well.

After describing the class of physical variables, one can go back and find the class of gauge potentials A_μ . One can show that these classes for massive and ordinary photodynamics in (2+1) dimensions are different.

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ПРАКТИЧЕСКАЯ СХЕМА РЕДУКЦИИ К
КАЛИБРОВОЧНО-ИНВАРИАНТНЫМ ПЕРЕМЕННЫМ

Для систем со связями первого рода рассмотрена схема редукции к калибровочно-инвариантным переменным. Метод основан на анализе ограниченных 1-форм в калибровочно-инвариантных переменных. Указанная схема применена к моделям электродинамики и теории Янга–Миллса. Для конечномерной модели с калибровочной группой симметрии $SU(2)$ получен возможный механизм конфайнмента.