

Prandtl number and thermoacoustic refrigerators

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From kinetic gas theory, it is known that the Prandtl number for hard-sphere monatomic gases is $2/3$. Lower values can be realized using gas mixtures of heavy and light monatomic gases. Prandtl numbers varying between 0.2 and 0.67 are obtained by using gas mixtures of helium–argon, helium–krypton, and helium–xenon. This paper presents the results of an experimental investigation into the effect of Prandtl number on the performance of a thermoacoustic refrigerator using gas mixtures. The measurements show that the performance of the refrigerator improves as the Prandtl number decreases. The lowest Prandtl number of 0.2, obtained with a mixture containing 30% xenon, leads to a coefficient of performance relative to Carnot which is 70% higher than with pure helium. © 2002 Acoustical Society of America. [DOI: 10.1121/1.1489451]

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I. INTRODUCTION

The basic understanding of the physical principles underlying the thermoacoustic effect is well established and has been discussed in many papers.^{1,2} However, a quantitative experimental investigation of the effect of some important parameters on the behavior of the thermoacoustic devices is still lacking. One of these parameters is the Prandtl number, a dimensionless parameter characterizing the ratio of kinematic viscosity to thermal diffusivity. Viscous friction has a negative effect on the performance of thermoacoustic systems.^{2,3} Decreasing the Prandtl number generally increases the performance of thermoacoustic devices. Kinetic gas theory has shown that the Prandtl number for hard-sphere monatomic gases is $2/3$. Lower Prandtl numbers can be realized using mixtures of heavy and light monatomic gases, for example, binary gas mixtures of helium and other noble gases.^{3–5}

The calculations of the Prandtl number shown in this paper use more rigorous expressions for the transport coefficients than those used previously.^{4,5} For all binary gas mixtures of helium with other noble gases, calculations show that the coefficient of performance of the refrigerator is maximized when the Prandtl number is near its minimum value.

This paper presents the results of an experimental investigation into the effect of Prandtl number on the performance of a thermoacoustic refrigerator using gas mixtures of helium with argon, krypton, and xenon. These combinations provided gas mixtures with Prandtl numbers varying between 0.2 and 0.67. These results are discussed below.

The thermoacoustic refrigerator used for the measurements is shown in Fig. 1. The acoustic resonator is filled with an inert gas at a pressure of 10 bar. A channeled stack is strategically located in the resonator to facilitate heat transfer. At both ends of the stack heat exchangers are installed. The temperature of the hot heat exchanger is fixed at room temperature by circulating water. At the cold heat exchanger cooling power is generated. A loudspeaker generates a standing wave in the resonance tube, causing the gas to oscillate

while compressing and expanding. The interaction of the moving gas in the stack with the stack surface generates heat transport.² A detailed description of the refrigerator can be found in the literature.^{3,6}

II. KINETIC THEORY OF BINARY GAS MIXTURES

The Prandtl number is given by

$$\sigma = \frac{\mu c_p}{K}, \quad (1)$$

where μ is the dynamic viscosity, K is the thermal conductivity, and c_p is the isobaric specific heat. The Prandtl number can also be written in terms of the thermal and viscous penetration depths δ_k and δ_v as

$$\sigma = \left(\frac{\delta_v}{\delta_k} \right)^2, \quad (2)$$

where δ_k and δ_v are given by

$$\delta_k = \sqrt{\frac{2K}{\rho c_p \omega}}, \quad (3)$$

and

$$\delta_v = \sqrt{\frac{2\mu}{\rho \omega}}. \quad (4)$$

Here, ω is the angular frequency of the sound wave and ρ is the density.

The viscosity has a negative effect on the performance of thermoacoustic devices, so a reduction of the effect of viscosity means an increase in efficiency. This can be accomplished by lowering the Prandtl number. In the Appendix a survey of the kinematic theory for pure and binary gas mixtures is given. The text of Hirschfelder *et al.*⁷ forms the principal source for this survey. By substituting the expressions of viscosity μ_{mix} , thermal conductivity K_{mix} , and the isobaric specific heat c_p for binary gas mixtures into Eq. (1), the Prandtl number of binary gas mixtures can be calculated. The

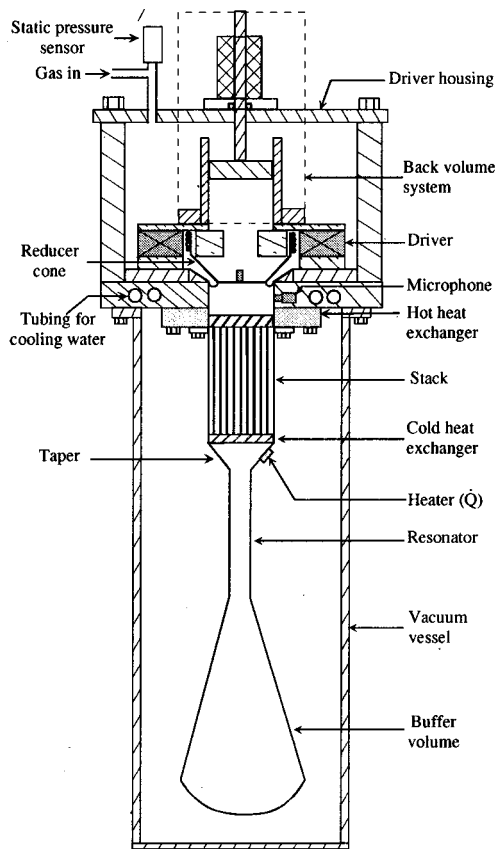


FIG. 1. Schematic diagram of the thermoacoustic refrigerator, showing the different parts.

resultant expression is too long to be given explicitly here. This was incorporated in a computer program to accomplish the calculations.

III. CALCULATION OF GAS MIXTURES PROPERTIES

In Fig. 2, some calculated properties of binary gas mixtures, consisting of He–Ne, He–Ar, He–Kr, and He–Xe, are plotted as functions of mole fraction, x , of the heavy component. The temperature and pressure used in the calculations are 250 K and 10 bar, respectively. These values apply to the experimental situation. Figure 2(a) shows that, for all mixtures, the density ρ increases linearly as a function of x , in accordance with Eq. (A14). From Figs. 2(b) and (c), it can be seen that the kinematic viscosity $\nu = \mu/\rho$ and thermal diffusivity $k = K/\rho c_p$ decrease as a function of x . This behavior is to be expected, since both properties are approximately inversely proportional to the square of the apparent mass of the mixture. But, in the range $0 < x < 0.4$ the rate of decrease for ν is larger than for k . This results in a decrease of the ratio of these two quantities which is the Prandtl number σ . For $x > 0.4$, k still decreases while ν remains nearly constant; this results in an increase of σ until the value for a pure noble gas is reached. From this behavior of the Prandtl number decreasing and then increasing as function of the increase of the mole fraction of the heavier component, a minimum value is to be expected as shown in Fig. 2(f). Similar calcu-

lation results are obtained by Giacobbe⁴ and Belcher *et al.*⁵ using approximative expressions for the viscosity and thermal conductivity.

The behavior of the sound velocity a as a function of x is illustrated in Fig. 2(d). It is also a decreasing function of x , in accordance with Eq. (A16), since the apparent mass of the mixture increases. The behavior of c_p is shown in Fig. 2(e).

As can be seen from Fig. 2(f), the Prandtl number has a minimum at $x \approx 0.38$ for all binary gas mixtures. The value of σ at the minimum is a function of the molecular weight of the added heavier component. The lowest value of 0.2 is reached with the heaviest noble gas xenon. The minimum Prandtl number is plotted in Fig. 3 as a function of the molar mass of the heavy component. Extrapolation of the data for radon shows that using a helium–radon mixture, a Prandtl number of about 0.1 can be reached which can be considered as the lowest Prandtl number for helium–noble gases mixtures. The Prandtl number is also calculated for different gas mixtures, at different temperatures, as shown in Fig. 4. The influence of the temperature on the Prandtl number is small. The effect of temperature is most pronounced for He–Xe mixture. At the helium mole fraction corresponding to the minimum Prandtl number, a change in temperature of 100 K results in a relative change in the value of the Prandtl of only 6.5% for a helium–xenon mixture.

Figures 4(a) and (b) show the thermal and viscous penetration depths δ_k and δ_ν , respectively. The behavior of δ_ν as a function of x can be illustrated by means of Eq. (4). Since ν varies approximately as $1/\sqrt{M}$, it follows that δ_ν will vary as $1/\sqrt{4M}$, thus a decreasing behavior as shown in Fig. 4(b). A similar analysis can be done to explain the behavior of δ_k . It is interesting to note that δ_k has a maximum for a binary mixture at about $x = 0.1$, as shown in Fig. 4(a). We note that for the calculations of δ_k and δ_ν , the wavelength is kept constant ($\lambda = 2.35$ m) and the frequency is allowed to vary as the sound velocity varies with the composition.

The product ρa is calculated for a temperature of 250 K and at a pressure of 10 bar, and it is plotted in Fig. 4(c). The behavior can be understood as follows: the density increases linearly and a decreases as $1/\sqrt{M}$; as a result the product ρa increases as x increases, as \sqrt{M} . The product ρa will be needed later for discussions concerning the cooling power of the thermoacoustic refrigerator.

IV. PERFORMANCE CALCULATIONS

The purpose of a refrigerator is to remove a heat (\dot{Q}_C) at a low temperature (T_C) and to reject heat (\dot{Q}_H) to the surroundings at a high temperature (T_H). To accomplish this, work (W) is required. The coefficient of performance (COP) is defined as

$$\text{COP} = \frac{\dot{Q}_C}{W}. \quad (5)$$

The quantity

$$\text{COPC} = \frac{T_C}{T_H - T_C}, \quad (6)$$

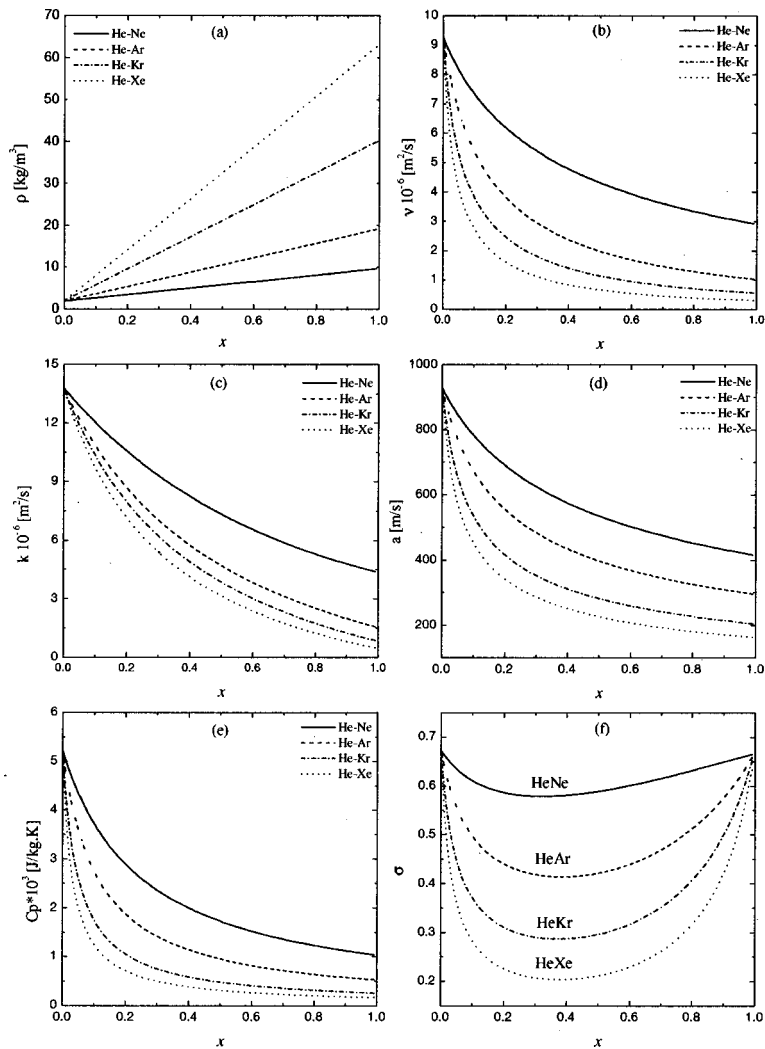


FIG. 2. Calculated properties of binary gas mixtures at a temperature of 250 K and a pressure of 10 bar. (a) Density. (b) Kinematic viscosity ν . (c) Thermal diffusivity k . (d) Speed of sound a . (e) The isobaric specific heat c_p . (f) Prandtl number σ .

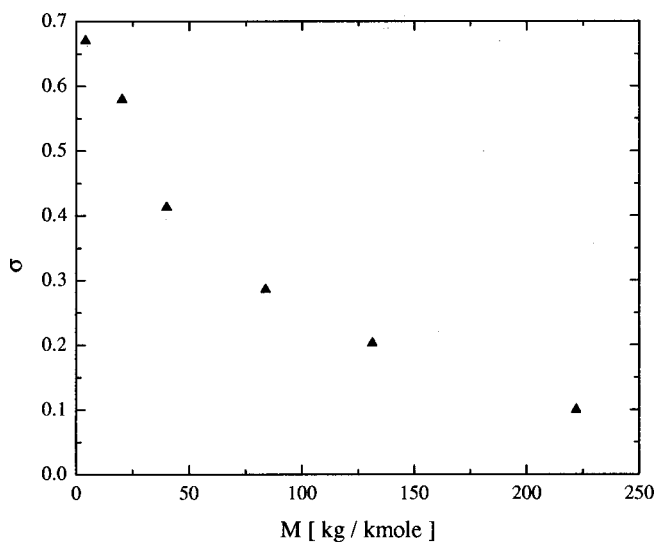


FIG. 3. Minimal Prandtl number, at a helium mole fraction of 0.62, as a function of the molecular weight of the heavy component. The first point corresponds to the Prandtl number of pure helium. The last point corresponds to the minimal Prandtl number of a helium–radon binary gas mixture obtained by extrapolation.

is called the Carnot coefficient of performance which defines the optimal performance for all refrigerators. The coefficient of performance relative to Carnot's coefficient of performance is defined as

$$\text{COPR} = \frac{\text{COP}}{\text{COPC}}. \quad (7)$$

The performance measurements for the refrigerator are presented in plots of COP, COPR, and ΔT given by

$$\Delta T = T_C - T_H, \quad (8)$$

as functions of the total heat load, which is the sum of the heat load applied by the heater and the heat leak. To understand the behavior of the cooler as a function of the varied parameters, the measured ΔT data will be fit with the equation

$$\Delta T = \Delta T_0 + \alpha \dot{Q}, \quad (9)$$

where \dot{Q} is the total heat load.³ During a given performance measurement, the drive ratio, defined as the ratio of the dynamic pressure to the average pressure, is kept constant

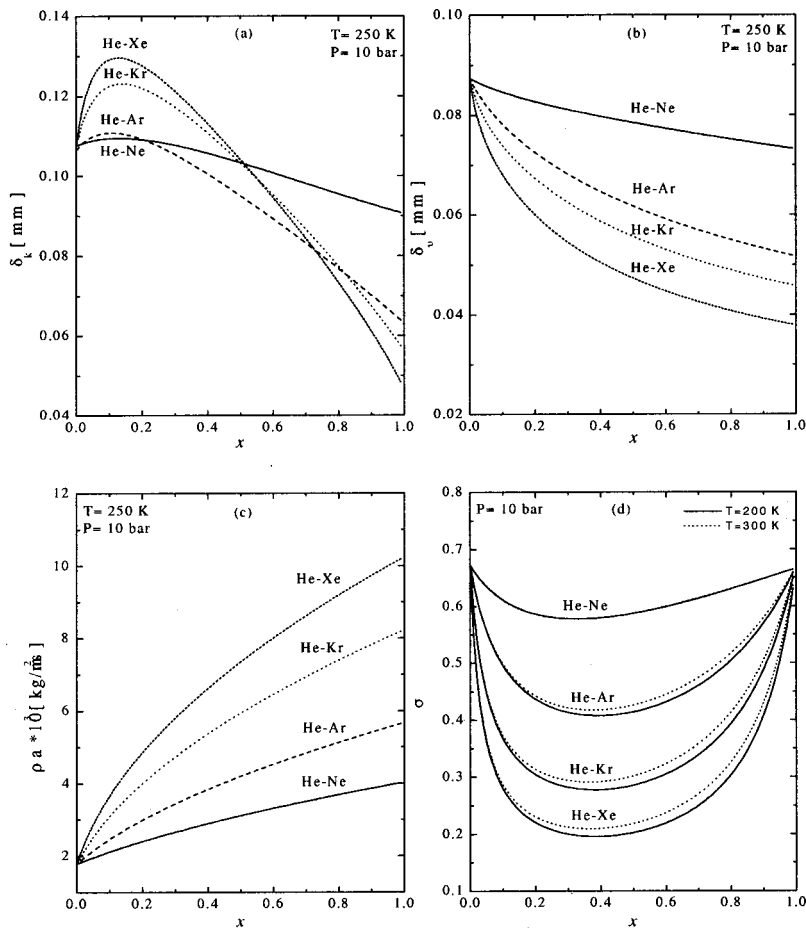


FIG. 4. The thermal and viscous penetration depths, δ_k and δ_v , the product ρa , and Prandtl number σ for helium–other noble gas binary mixtures. We note that, for the calculations δ_k and δ_v , the wavelength is kept constant ($\lambda = 2.35$ m) and the frequency is allowed to vary as the sound velocity varies with x . The temperature and pressure used in the calculations are also shown in the graphs. The Prandtl number has been calculated at two different temperatures.

while stepwise the heat load is applied to the cold heat exchanger and the temperature is allowed to stabilize. The heat load is applied by an electric heater placed at the cold heat exchanger. Two thermometers are used to monitor the temperatures at the hot heat exchanger and at the cold heat exchanger. Incorporating Eqs. (A5) and (A10) into the expressions for the energy flow and work flow in the stack,⁸ the COP has been calculated as function of x . Figure 5 shows the behavior of COP for helium–xenon, helium–krypton, and helium–argon mixtures. For all binary gas mixtures, as the Prandtl number decreases the performance improves. A maximum is reached nearly at the point where the Prandtl number is a minimum (cf. Fig. 2).

V. MIXTURES PREPARATION

A container is used to prepare the gas mixtures. The low mole fraction component is first filled in the container until the pressure fraction needed is reached. Then, the second component is filled up to 33 bar. The mixture is allowed to reach equilibrium prior to filling the refrigerator to a pressure of 10 bar. The composition of the mixture was checked by measuring the resonance frequency of the refrigerator. This frequency is then related to the resonance frequency of the system for pure helium by the expression

$$\frac{f_{\text{He}}}{f_{\text{mix}}} = \sqrt{\frac{xM_2 + (1-x)M_{\text{He}}}{M_{\text{He}}}}, \quad (10)$$

where f_{He} , and f_{mix} are the resonance frequencies for pure helium and mixture, respectively. M_2 and x are the molar mass and mole fraction of the heavy component, and M_{He} is the molar mass of helium.

VI. MEASUREMENTS

With our thermoacoustic refrigerator^{3,6} three different mixtures have been investigated: He–Ar, He–Kr, and He–Xe. The performance measurements were all made using a

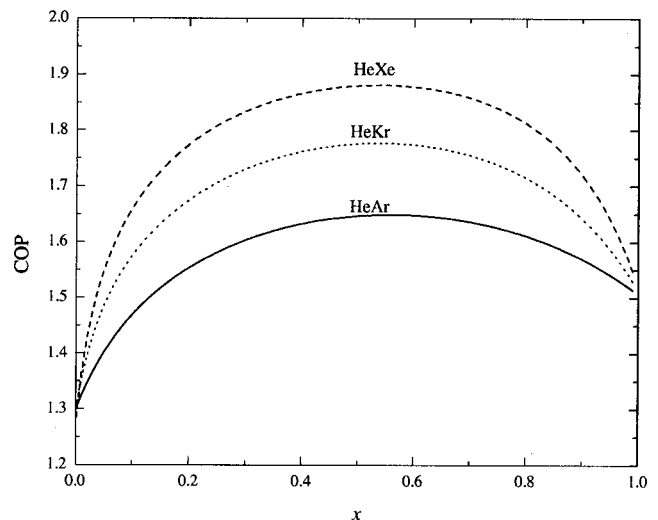


FIG. 5. Calculated COP as function of x .

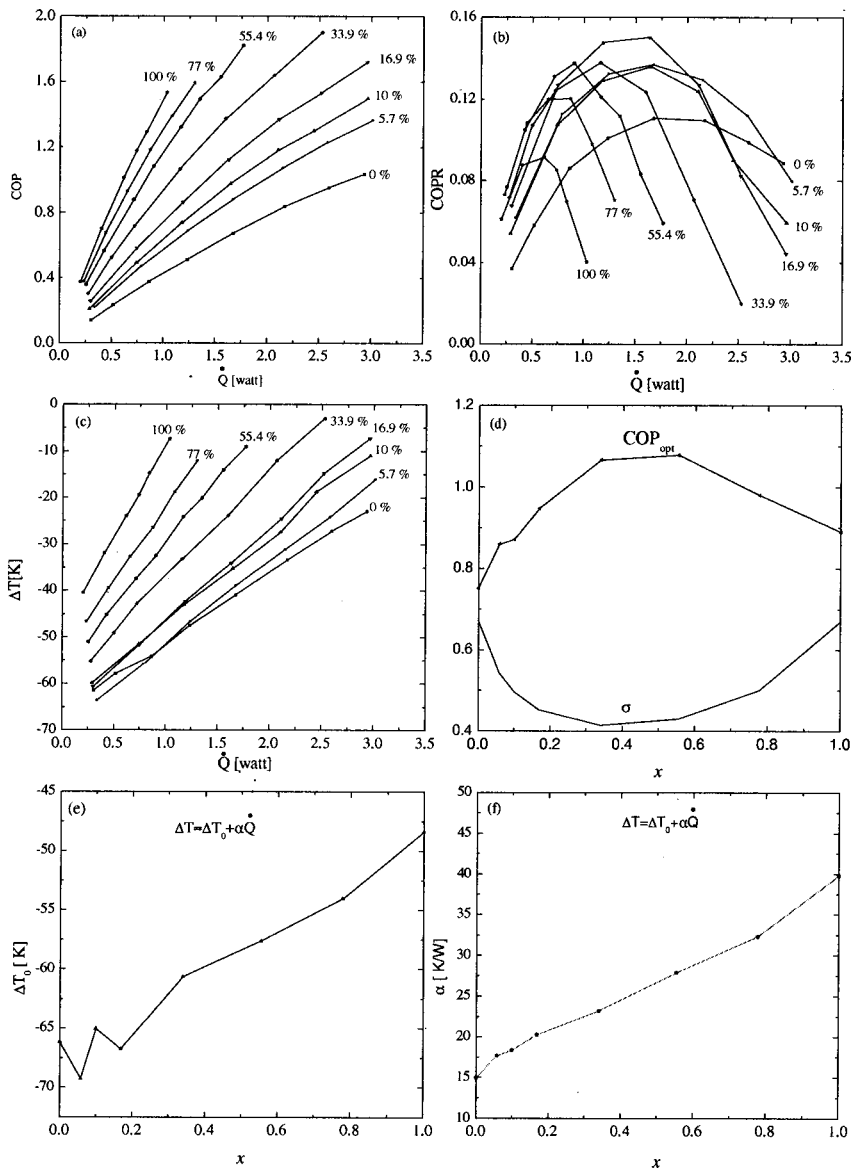


FIG. 6. Measurements with helium–argon mixtures. The average pressure is 10 bar, the drive ratio is 1.4%, and a stack with a plate spacing of 0.3 mm is used. In (a), (b), and (c) the values of x are indicated at the corresponding curves. (a) COP. (b) COPR. (c) ΔT . (d) Optimal COP corresponding to the peak COPR as function of the mole fraction argon x . (e) ΔT_0 as function of x . (f) α as function of x .

parallel-plate stack with spacing 0.3 mm, an average pressure of $p_m = 10$ bar, and a constant drive ratio of $D = p_1/p_m = 1.4\%$. The performance measurements for the refrigerator are presented in plots of COP, COPR, and ΔT as functions of the total heat load \dot{Q} .³ During a given performance measurement, the drive ratio is held constant while stepwise the heat load is applied to the cold heat exchanger by a heater and the temperature is allowed to stabilize.³

A. Helium–argon

Binary helium–argon mixtures have been used with an argon mole fraction, x , varying between zero and 100%. The results are shown in Fig. 6. The COP increases as function of the heat load, as can be seen from Fig. 6(a). The slope of COP is an increasing function of x . From the calculations (Fig. 5) we expect that the COP will increase as a function of x , reach a maximum around 40%, and then decrease. This behavior is difficult to see from Fig. 6(a), because the increase of the COP slope is a consequence of two effects: the Prandtl number decrease and the decrease of the cooling power. The energy and work flux equations⁸ show that the

cooling power is proportional to the inverse of the product of the density times the speed of sound if the length of the resonator is held constant [Fig. 4(c)]. This factor, which is also proportional to the speed of sound in the mixture, decreases as x increases. Therefore, the cooling power decreases as x increases. The COPR as function of the heat load is shown in Fig. 6(b). For all mixtures the COPR shows a parabolic behavior with a maximum.

In Fig. 6(c), the temperature differences as functions of the heat load are shown. For all gas mixtures, the dependence is linear. The slope increases as x increases. In Fig. 6(d), the COP_{opt} corresponding to the peak COPR (operation point) is plotted as function of x . This plot shows that the COP_{opt} has the expected trend dependent on the Prandtl number.

To understand the effect of the Prandtl number on ΔT , the temperature curves of Fig. 6(c) are fitted with Eq. (9). The parameters ΔT_0 and α are plotted as functions of x in Figs. 6(e) and (f), respectively. As can be seen, ΔT_0 increases with x , with the largest temperature difference obtained with pure helium. The slope α also increases with x .

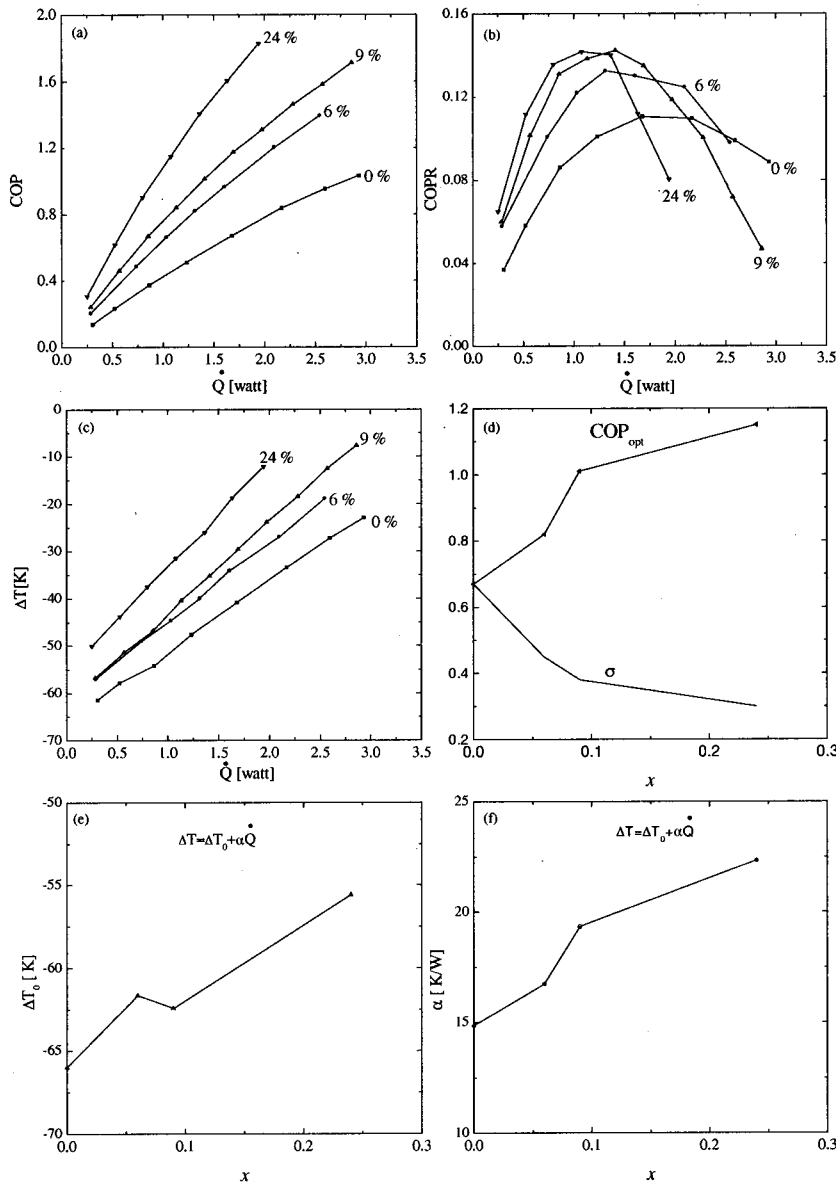


FIG. 7. Measurements with helium–krypton mixtures. The average pressure is 10 bar, the drive ratio is 1.4%, and a stack with a plate spacing of 0.3 mm is used. In (a), (b), and (c) the values of x are indicated at the corresponding curves. (a) COP. (b) COPR. (c) ΔT . (d) Optimal COP corresponding to the peak COPR as function of the mole fraction krypton x . (e) ΔT_0 as function of x . (f) α as function of x .

This means that the cooling power is a decreasing function of x . This is due to the increase of the density which increases as x increases.

In summary, the COP_{opt} has the reverse behavior of the Prandtl number as function of x . The COP_{opt} has a maximum when the Prandtl number has a minimum, since the viscous losses are minimum [Fig. 6(d)]. But, at the same time the addition of argon has a negative effect on the cooling power of the refrigerator, as the addition of argon increases the density which makes the cooling power decrease. The maximum COPR reached with helium–argon mixtures is 14.5% with a mixture containing 17% argon. This is a 40% improvement in comparisons with pure helium.

B. Helium–krypton

Measurements with helium–krypton mixtures are shown in Fig. 7. As can be seen from Fig. 7(d), the COP_{opt} increases as the Prandtl number σ decreases. The calculated Prandtl number for the different mixtures is also plotted in this figure. The ΔT curves from Fig. 7(c) are fitted with Eq. (9) and

the determined parameters ΔT_0 and α are plotted in Figs. 7(e) and (f). These parameters increase as functions of x , which means that the cooling power decreases. A maximal COPR of 14.3% is obtained with $x=24\%$, which has the lowest Prandtl number. The same general behavior as for helium–argon can be seen, except that the slopes of ΔT_0 and α are steeper in the case of helium–krypton. This means that the cooling power of the refrigerator is more heavily influenced by the addition of krypton than by the addition of argon. This is due to the higher density of krypton.

C. Helium–xenon

Figure 8 shows the results using helium–xenon mixtures. The general behavior of the COP and COPR is similar to those for He–Ar and He–Kr. As can be seen from Fig. 8(b), the magnitude of the maximum of COPR increases as the mole fraction xenon x increases and reaches a maximum of 17% for $x=0.3$. Figure 8(d) shows that the COP_{opt} increases when σ decreases. The parameters ΔT_0 and α are plotted in Figs. 8(e) and (f). Again, these parameters increase

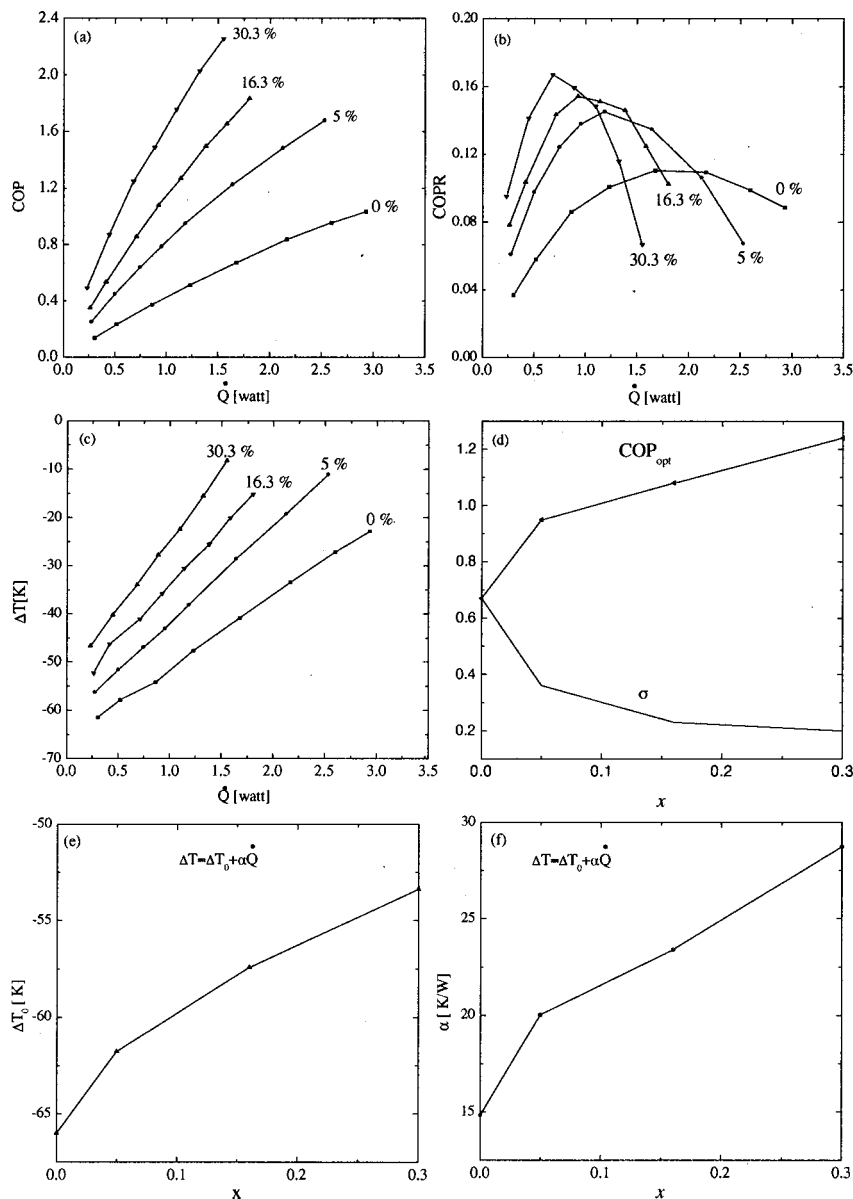


FIG. 8. Measurements with helium–xenon mixtures. The average pressure is 10 bar, the drive ratio is 1.4%, and a stack with a plate spacing of 0.3 mm is used. In (a), (b), and (c) the values of x are indicated at the corresponding curves. (a) COP. (b) COPR. (c) ΔT . (d) Optimal COP corresponding to the peak COPR as function of the mole fraction xenon x . (e) ΔT_0 as function of x . (f) α as function of x .

as functions of x . The slopes of ΔT_0 and α are steeper in the case of helium–xenon than in the case of helium–krypton. This indicates that the cooling power is more influenced by the addition of xenon than by the addition of krypton. This again is due to the density which is higher in the case of xenon.

A maximum COPR of 17% is reached with the mixture having the lowest Prandtl number and thus the minimal viscous losses in the system. This is equivalent to an improvement of 70% in comparison with pure helium. But, again the cooling power is smaller.

From the mixture measurements discussed above, we conclude that the Prandtl number can be decreased using helium and other noble-gas mixtures. This results in an improvement of the COPR of the refrigerator. However, at the same time the density increases which causes the cooling power to decrease. The behavior of the optimal performance COP_{opt} as function of the noble gas mole fraction is similar to the one predicted by the calculations (Fig. 5).

In Table I, the linear fit results of the parameters ΔT_0

and α as a function of the mole fraction x for the different gas mixtures are given, according to the formula $\Delta T_0 = a + bx$ and $\alpha = c + dx$. The parameter a , which corresponds to ΔT_0 for pure helium, is the same for all mixtures, as it should be. The slope b increases as function of the molecular weight of the noble-gas component. This indicates the negative effect of the density on the cooling power. Likewise, the parameter c , which corresponds to the pure helium case for all mixtures, is the same for the three mixtures. The slope d increases as a function of the molecular weight of the noble component, which indicates again the negative effect of the density on the cooling power. In conclusion, we can say that

TABLE I. Linearly fitted data for the different gas mixtures.

Gas	a	b	c	d
Ar	-68 ± 1	19 ± 2	$c = 16 \pm 0.5$	23 ± 1
Kr	-65 ± 1	41 ± 6	$c = 15 \pm 1$	31 ± 6
Xe	-65 ± 1	40 ± 6	$c = 16 \pm 1$	43 ± 7

TABLE II. Lennard-Jones potential parameters and collision integrals of noble gases.

Gas	ϵ/k (K)	ζ (Å)	$T=200$ K	$T=250$ K	$T=300$ K
			$\Omega^{(2,2)*}$	$\Omega^{(2,2)*}$	$\Omega^{(2,2)*}$
He	10.2	2.6	0.75	0.72	0.70
Ne	34.9	2.8	0.90	0.87	0.84
Ar	122	3.4	1.26	1.16	1.09
Kr	171	3.6	1.47	1.32	1.23
Xe	221	4.1	1.67	1.49	1.37

the best COPR is obtained with the heaviest noble-gas component, but the cooling power is the smallest.

VII. CONCLUSIONS

The influence of the Prandtl number on the performance of the refrigerator is studied systematically by using mixtures of helium and other, heavier noble gases. The theoretical and experimental results show that the coefficient of performance improves as the Prandtl number decreases. However, the cooling power decreases when the mole fraction of the heavy noble-gas component increases in the mixture. It is shown that this decrease is a consequence of the increase of the density. A maximum performance relative to Carnot of 17% is obtained by using a mixture of helium and xenon containing 30% xenon. This is an improvement of 70% in comparison with pure helium. This trade-off between the performance and the cooling power has to be considered in the design of a thermoacoustic refrigerator.

APPENDIX A: KINETIC THEORY

In order to calculate the Prandtl number, Eq. (1), for gas mixtures we need the kinetic expressions for the dynamic viscosity μ , and thermal conductivity K . We now summarize the formulas for μ , K to be used with the tabulations of $\Omega^{(1,s)}$ for nonpolar molecules. These are written in the forms most convenient for practical calculations. In the text of Hirschfelder *et al.*,⁷ it is indicated that the viscosity can be calculated to within 2% to 5% over a range of 400 K. But, the thermal conductivity may be in error as much as 10%, as this property is highly sensitive to the form of the potential function which is used.

1. Viscosity

The viscosity of a pure gas is given in Ref. 7, Eq. (8.2-18), as

$$\mu = 2.67 \cdot 10^{-6} \frac{\sqrt{MT}}{\zeta_{12}^2 \Omega^{(2,2)*}(T^*)}, \quad (\text{A1})$$

where M is the molecular weight in kg/kmol, T is the temperature in K, ζ is the collision diameter in Å, and $\Omega^{(2,2)*}(T^*)$ is a tabulated function of the dimensionless temperature $T^* = kT/\epsilon$. These functions $\Omega^{(2,2)*}(T^*)$ are given in Table II for noble gases at different temperatures. The unit of the viscosity is [Pas].

In order to discuss the viscosity of a binary gas mixtures, consisting of two components 1 and 2, it is convenient to define a quantity μ_{12} as

$$\mu_{12} = 2.67 \cdot 10^{-6} \frac{\sqrt{\frac{2M_1M_2}{M_1+M_2}} T}{\zeta_{12}^2 \Omega_{12}^{(2,2)*}(T_{12}^*)}. \quad (\text{A2})$$

This quantity can be regarded as the viscosity of a hypothetical pure substance, the molecules of which have molecular weight $2M_1M_2/(M_1+M_2)$, and interact according to the potential specified by the interaction parameters ζ_{12} and ϵ_{12} . The interaction of two nonpolar molecules of species 1 and 2 is described by the Lennard-Jones potential, where the parameters ζ and ϵ are given by the combining laws [Ref. 7, Eq. (3.6-8)]

$$\zeta_{12} = \frac{1}{2}(\zeta_1 + \zeta_2), \quad (\text{A3})$$

$$\epsilon_{12} = \sqrt{\epsilon_1 \epsilon_2}, \quad (\text{A4})$$

and are given in Table III. The appropriate parameters ζ_i and ϵ_i , for the molecules of the i th components, are given in Table II.

The viscosity μ_{mix} of a binary gas mixture, in terms of the quantities defined previously, is given by [Ref. 7, Eq. (8.2-22)]

$$\mu_{\text{mix}} = \frac{1 + Z_\mu}{X_\mu + Y_\mu}, \quad (\text{A5})$$

where X_μ , Y_μ , and Z_μ are given by

$$X_\mu = \frac{x_1^2}{\mu_1} + \frac{2x_1x_2}{\mu_{12}} + \frac{x_2^2}{\mu_2},$$

TABLE III. Lennard-Jones potential parameters and collision integrals for gas mixtures.

Gas	ϵ_{12}/k (K)	ζ_{12} (Å)	$T=200$ K	$T=250$ K	$T=300$ K
			$\Omega^{(2,2)*}$	$\Omega^{(2,2)*}$	$\Omega^{(2,2)*}$
He-Ne	18.89	2.67	0.82	0.79	0.77
He-Ar	35.31	2.98	0.91	0.87	0.84
He-Kr	41.80	3.08	0.93	0.89	0.87
He-Xe	47.52	3.33	0.96	0.92	0.89

TABLE IV. A^* and B^* parameters for binary gas mixtures at different temperatures.

Gas	$T=200$ K		$T=250$ K		$T=300$ K	
	A_{12}^*	B_{12}^*	A_{12}^*	B_{12}^*	A_{12}^*	B_{12}^*
He-Ne	1.1106	1.0950	1.1128	1.0944	1.1153	1.0909
He-Ar	1.1024	1.0909	1.1054	1.0916	1.1081	1.0912
He-Kr	1.1001	1.0941	1.1032	1.0903	1.1056	1.0911
He-Xe	1.0956	1.0978	1.1011	1.0916	1.1039	1.0918

$$Y_\mu = \frac{3}{5} A_{12}^* \left[\frac{x_1^2 M_1}{\mu_1 M_2} + \frac{2x_1 x_2 (M_1 + M_2)^2}{\mu_{12} 4M_1 M_2} \frac{\mu_{12}^2}{\mu_1 \mu_2} + \frac{x_2^2 M_2}{\mu_2 M_1} \right] \quad (\text{A6})$$

$$Z_\mu = \frac{3}{5} A_{12}^* \left[x_1^2 \frac{M_1}{M_2} + 2x_1 x_2 \left[\frac{(M_1 + M_2)^2}{4M_1 M_2} \left(\frac{\mu_{12}}{\mu_1} + \frac{\mu_{12}}{\mu_2} \right) - 1 \right] + x_2^2 \frac{M_2}{M_1} \right].$$

The parameter A_{12}^* is a function of kT/ϵ_{12} , and defined as follows:

$$A_{12}^* = \frac{\Omega^{(2,2)*}}{\Omega^{(1,1)*}}. \quad (\text{A7})$$

These parameters are given for helium–other noble gases mixtures in Table IV.

2. Thermal conductivity

The thermal conductivity for a pure gas is given by [Ref. 7, Eq. (8.2–31)]

$$K = 8.32 \cdot 10^{-2} \frac{\sqrt{T/M}}{\zeta^2 \Omega^{(2,2)*}(T^*)} = \frac{15}{4} \frac{R}{M} \mu, \quad (\text{A8})$$

where R is the universal gas constant. The other parameters are as defined for viscosity. The unit is [W/Km].

In order to derive the thermal conductivity of binary gas mixture, it is convenient to define a quantity K_{12} as follows:

$$K_{12} = 8.32 \cdot 10^{-2} \frac{\sqrt{\frac{M_1 + M_2}{2M_1 M_2}} T}{\zeta_{12}^2 \Omega_{12}^{(2,2)*}(T_{12}^*)}. \quad (\text{A9})$$

The thermal conductivity K_{mix} of a binary gas mixture is given by [Ref. 7, Eq. (8.2–36)]

$$K_{\text{mix}} = \frac{1 + Z_K}{X_K + Y_K}, \quad (\text{A10})$$

where X_K , Y_K , and Z_K are given by

$$X_K = \frac{x_1^2}{K_1} + \frac{2x_1 x_2}{K_{12}} + \frac{x_2^2}{K_2}$$

$$Y_K = \frac{x_1^2}{K_1} U^{(1)} + \frac{2x_1 x_2}{K_{12}} U^{(Y)} + \frac{x_2^2}{K_2} U^{(2)}$$

$$Z_K = x_1^2 U^{(1)} + 2x_1 x_2 U^{(Z)} + x_2^2 U^{(2)}$$

$$U^{(1)} = \frac{4}{15} A_{12}^* - \frac{1}{12} \left(\frac{12}{5} B_{12}^* + 1 \right) \frac{M_1}{M_2} + \frac{1}{2} \frac{(M_1 - M_2)^2}{M_1 M_2} \quad (\text{A11})$$

$$U^{(2)} = \frac{4}{15} A_{12}^* - \frac{1}{12} \left(\frac{12}{5} B_{12}^* + 1 \right) \frac{M_2}{M_1} + \frac{1}{2} \frac{(M_2 - M_1)^2}{M_1 M_2}$$

$$U^{(Y)} = \frac{4}{15} A_{12}^* \frac{(M_1 + M_2)^2}{4M_1 M_2} \frac{K_{12}^2}{K_1 K_2} - \frac{1}{12} \left(\frac{12}{5} B_{12}^* + 1 \right) - \frac{5}{32 A_{12}^*} \left(\frac{12}{5} B_{12}^* - 5 \right) \frac{(M_1 - M_2)^2}{M_1 M_2}$$

$$U^{(Z)} = \frac{4}{15} A_{12}^* \left[\frac{(M_1 + M_2)^2}{4M_1 M_2} \left(\frac{K_{12}}{K_1} + \frac{K_{12}}{K_2} \right) - 1 \right] - \frac{1}{12} \left(\frac{12}{5} B_{12}^* + 1 \right),$$

where the parameter B_{12}^* is a function of kT/ϵ_{12} , and it is defined in terms of the $\Omega^{(1,s)*}$ integrals as follows [Ref. 7, Eqs. (8.2–15)–(8.2–16)]:

$$B_{12}^* = \frac{5\Omega^{(1,2)*} - 4\Omega^{(1,3)*}}{\Omega^{(1,1)*}}. \quad (\text{A12})$$

These parameters are given for helium–noble gases mixtures in Table IV.

3. Specific heat and sound velocity

The last parameter which is needed to calculate the Prandtl number, using Eq. (1), for binary gas mixtures is c_p . Considering the binary gas mixture as an ideal gas, the specific heat at constant volume and constant pressure are given, respectively, by

$$c_v = \frac{3}{2} \frac{R}{M}, \quad c_p = \frac{5}{2} \frac{R}{M}, \quad (\text{A13})$$

where M is the apparent molecular mass of the mixture given by

$$M = x_1 M_1 + x_2 M_2. \quad (\text{A14})$$

The ratio of the isobaric to isochoric specific heats for the inert gas mixture is given by

$$\gamma = \frac{c_p}{c_v} = \frac{5}{3}. \quad (\text{A15})$$

The sound velocity a in the binary gas mixture is given by

$$a^2 = \gamma \frac{p}{\rho} = \gamma \frac{RT}{x_1 M_1 + x_2 M_2}, \quad (\text{A16})$$

where p is the pressure, ρ is the density, and T is the temperature of the gas mixture.

By substituting Eqs. (A5), (A10), and (A13) into Eq. (1), the Prandtl number of the binary gas mixtures can be calculated. The resultant expression is too long to be given explicitly here. This was incorporated in a computer program to accomplish the calculations.

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