Fig. 3 shows the mean square error as a function of the term $\alpha N/\sigma_w$, parametrized in ϵ. The curve is expected to give a reasonable prediction of the best achievable MSE for low values of the ratio $\alpha N/\sigma_w$, that is, in the low-power region. Further, when we let ϵ grow by moving from the lowermost curve (purely Gaussian case with $\sigma_1 = 1$) to the uppermost one (purely Gaussian case with $\sigma_2 = 3$), the MSE increases monotonically.

V. Conclusion

There have recently been a number of results on decentralized estimation in which the inherent additive fusion rule of a MAC is exploited. The results in [7] are particularly interesting, in that they show that the SNR-maximizing data-forwarding rule is proportional to the local MMSE, as opposed to the amplify-and-forward procedure that one might expect. Here that result is shown to apply directly to minimum fused-MSE estimation. By basing our analysis on an assumption, similar to that in locally-optimized detection, of vanishing received signal power, we are able to show that the optimal sensor-level forwarding rule is proportional to the local MMSE estimator even for *exchangeable* sensors. We also provide the optimal fusion post-processing rule, which, unlike cases in which the asymptotics relate to arbitrarily large numbers of sensors, depends strongly on the channel noise statistics.

References

[1] T. Cover and J. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.

[2] M. Gastpar, "To code or not to code," Ph.D. dissertation, Ecole Polytechnique Fédérale (EPFL), Lausanne, Switzerland, 2002.

[3] M. Gastpar and M. Vetterli, "On the capacity of large Gaussian relay networks," *IEEE Trans. Inf. Theory*, vol. IT-51, no. 3, Mar. 2005.

[4] G. Mergen and L. Tong, "Type based estimation over multiaccess channels," *IEEE Trans. Signal Process.*, vol. 54, no. 2, pp. 613–626, Feb. 2006.

[5] K. Liu and A. Sayeed, "Asymptotically optimal decentralized type-based detection in wireless sensor networks," in *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing (ICASSP)*, Montreal, QC, Canada, May 17–21, 2004, vol. 3, pp. 873–876.

[6] S. Marano, V. Matta, L. Tong, and P. Willett, "A likelihood based multiple access for estimation in sensor networks," *IEEE Trans. Signal Process.*, vol. 55, no. 11, pp. 5155–5166, Nov. 2007.

[7] K. S. Gomadam and S. A. Jafar, "Optimal relay functionality for SNR maximization in memoryless relay networks," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 390–401, Feb. 2007.

[8] N. Khajehnouri and A. Sayed, "Distributed mmse relay strategies for wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 55, no. 7, pp. 3336–3348, Jul. 2007.

[9] H. V. Poor, "Fine quantization in signal detection and estimation," *IEEE Trans. Inf. Theory*, vol. IT-34, no. 5, pp. 960–972, Sep. 1988.

[10] S. Marano, V. Matta, and P. Willett, "Asymptotic design of quantizers for decentralized MMSE estimation," *IEEE Trans. Signal Process.*, vol. 55, no. 11, pp. 5485–5496, Nov. 2007.

[11] S. Marano, V. Matta, and P. Willett, "Distributed estimation in large wireless sensor networks via a locally optimum approach," *IEEE Trans. Signal Process.*, vol. 56, no. 2, pp. 748–756, Feb. 2008.

Preamble-Based Channel Estimation for CP-OFDM and OFDM/OQAM Systems: A Comparative Study

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***Abstract*—In this correspondence, preamble-based least squares (LS) channel estimation in orthogonal frequency division multiplexing (OFDM) systems of the QAM and offset QAM (OQAM) types is considered. The construction of optimal (in the mean squared error (MSE) sense) preambles is investigated, for sparse (a subset of pilot tones, surrounded by nulls) preambles. The two OFDM systems are compared for the same transmit power, which, for cyclic prefix (CP) based OFDM/QAM, also includes the power spent for CP transmission. OFDM/OQAM, with a sparse preamble consisting of equipowered and equispaced pilots embedded in zeros, turns out to perform at least as well as CP-OFDM. Simulations results are presented that verify the analysis.**

***Index Terms*—Channel estimation, cyclic prefix (CP), discrete Fourier transform (DFT), least squares (LS), mean squared error (MSE), offset QAM (OQAM), orthogonal frequency division multiplexing (OFDM), pilots, preamble, quadrature amplitude modulation (QAM).**

I. Introduction

Orthogonal-frequency-division multiplexing (OFDM) is currently enjoying popularity in both wired and wireless communication systems [2], mainly because of its immunity to multipath fading, which allows for a significant increase in the transmission rate [18]. Using the cyclic prefix (CP) as a guard interval, OFDM can "reform" a frequency selective channel into a set of parallel flat channels with independent noise disturbances. This greatly simplifies both the estimation of the channel as well as the recovery of the transmitted data at the receiver. However, these advantages come at the cost of an increased sensitivity to frequency offset and Doppler spread. This is due to the fact that, although the subcarrier functions are perfectly localized in time, they suffer from spectral leakage in the frequency domain. Moreover, the inclusion of the CP entails a loss in spectral efficiency, which, in practical systems, can become as high as 25% [2].

An alternative to CP-OFDM, that can mitigate these drawbacks, is provided by a filter bank-based variant employing offset quadrature amplitude modulation (OQAM), known as OFDM/OQAM [8]. In this scheme, pulse shaping is included via an IFFT/FFT-based efficient filter bank, and staggered OQAM symbols, i.e., real symbols at twice the symbol rate of OFDM/QAM, are loaded on the subcarriers [16]. This allows for the pulses to be well localized in both the time and the frequency domains. As a consequence, the system's robustness to frequency offsets and Doppler effects is increased [7] and at the

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same time an enhanced spectral containment, for bandwidth sensitive applications, is offered [1], [17]. Furthermore, although the two OFDM schemes can be seen to exhibit similar peak-to-average power ratio (PAPR) performances, the presence of spectral leakage in OFDM/QAM may, ultimately, generate higher peak power values [19]. Moreover, the use of a CP is not required in the OFDM/OQAM transmission, which may lead to even higher transmission rates [16].[1]

Since nothing is free in this world, the previously mentioned advantages of OFDM/OQAM come at the cost of subcarrier functions being now orthogonal only in the real field, which means that there is always an *intrinsic* imaginary interference among (adjacent) subcarriers [6]. This makes the channel estimation task for OFDM/OQAM systems more challenging, compared to OFDM/QAM. OFDM/OQAM channel estimation has been recently studied for both preamble-based [9], [11] and scattered pilots-based [6], [10] training schemes.

The focus of this correspondence is on the channel estimation task based on a preamble consisting of pilot tones. The question of selecting these tones so as to minimize the channel estimation mean squared error (MSE), subject to a given training energy, is addressed. The case of a *sparse* preamble, built with isolated pilot tones embedded in nulls, is treated.[2] It is shown that an optimal sparse preamble for OFDM/OQAM can be constructed with L_h equispaced and equipowered pilot tones, where L_h denotes the channel length.

Related results have previously been derived for the case of CP-based OFDM/QAM (CP-OFDM) channel estimation. In [15], it was shown that uniform spacing is the best choice given that the pilot tones are equipowered.[3] Equispaced and equipowered pilot tones were shown in [3] to be the optimal CP-OFDM preamble for a given training energy that accounts only for the useful signal, excluding the CP. This correspondence also revisits the problem of optimally selecting the pilot tones in CP-OFDM, when the training energy constraint also includes the CP part. It is shown that, in this case, the pilots should also be *equal*. The effects of such a choice on the resulting PAPR are also discussed. The comparison of optimal sparse preambles for CP-OFDM and OFDM/OQAM turns out to be generally in favor of the latter. We present simulations results that confirm the theoretical analysis.

The rest of the correspondence is organized as follows: In Section II, we describe the discrete-time baseband equivalent model for the OFDM/OQAM and CP-OFDM systems. The optimal sparse preambles for the two OFDM systems are derived in Section III. In Section IV, the MSE performances of the optimal sparse preambles associated with the two systems are compared. Simulations results are reported in Section V. Section VI concludes the correspondence. A more detailed version of this correspondence, including more results and proofs, can be found in http://arxiv.org/abs/0910.3928.

Notation: Vectors and matrices are denoted by bold lowercase and uppercase letters, respectively. Superscripts ^{T} and ^{H} stand for transposition and conjugate transposition. The complex conjugate of a complex number z is denoted by z^*. Also, $\jmath = \sqrt{-1}$. $\|\cdot\|$ is the Euclidean norm. For a matrix $\boldsymbol{A}$, $(\boldsymbol{A})_{i,j}$ denotes its (i,j) entry. The expectation and matrix trace operators are denoted by $E(\cdot)$ and $\mathrm{tr}(\cdot)$, respectively. $\boldsymbol{I}_m$ denotes the mth-order identity matrix, while $\mathbf{0}_{m\times n}$ is the all zeros $m \times n$ matrix.

[1]Nevertheless, this advantage was partly given up in [12] and a CP-based OFDM/OQAM system was proposed for the sake of facilitating the data reception process.

[2]This pilot arrangement is also referred to as *comb-type* [5].

[3]This is no longer valid if there are suppressed (*virtual*) subcarriers. In such a case, the optimal placement is non-uniform [14].

II. System Models

In this section, the CP-OFDM and OFDM/OQAM system models are described, along with some basic concepts that will be used in the sequel.

A. CP-OFDM

Given M subcarriers, the result of the OFDM modulation of a (complex) $M \times 1$ vector $\boldsymbol{x}$ is

$$\boldsymbol{s} = \frac{1}{\sqrt{M}}\boldsymbol{\mathcal{F}}^H\boldsymbol{x}$$

where $\boldsymbol{\mathcal{F}}$ is the $M\times M$ DFT matrix, with entries $(\boldsymbol{\mathcal{F}})_{i,j} = e^{-\jmath 2\pi ij/M}$, $i,j = 0,1,\ldots,M-1$. Prior to transmission, a CP of length ν is prepended to the previous vector, to yield

$$\boldsymbol{s}_{\mathrm{QAM}} = \begin{bmatrix} \mathbf{0}_{\nu\times(M-\nu)} & \boldsymbol{I}_\nu \\ \cdots\cdots & \cdots\cdots \\ \multicolumn{2}{c}{\boldsymbol{I}_M} \end{bmatrix}\boldsymbol{s}. \tag{1}$$

Assume that the CP length is chosen to be the smallest possible one, namely equal to the channel order: $\nu = L_h - 1$ [15]. Moreover, perfect timing and frequency synchronization is assumed. The channel impulse response (CIR), $\boldsymbol{h} = [h_0\ h_1\ \cdots\ h_{L_h-1}]^T$, is assumed to be constant over the duration of an OFDM symbol. The input to the OFDM demodulator, after the CP removal, can then be expressed as

$$\boldsymbol{r} = \boldsymbol{\mathcal{H}}\boldsymbol{s} + \boldsymbol{w}$$

where $\boldsymbol{\mathcal{H}}$ is the Toeplitz circulant matrix with its first row given by $[\,h_0\quad \mathbf{0}_{1\times(M-L_h)}\quad h_{L_h-1}\quad \cdots\quad h_2\quad h_1\,]$ and $\boldsymbol{w}$ is the noise at the receiver front end, assumed to be white Gaussian with zero-mean and variance σ^2. The action of the DFT (FFT) then results in

$$\boldsymbol{y} = \frac{1}{\sqrt{M}}\boldsymbol{\mathcal{F}}\boldsymbol{r} = \mathrm{diag}\,(H_0, H_1, \ldots, H_{M-1})\,\boldsymbol{x} + \boldsymbol{\eta} \tag{2}$$

where $H_m = \sum_{l=0}^{L_h-1} h_l e^{-\jmath 2\pi ml/M}$, $m = 0,1,\ldots,M-1$ is the M-point channel frequency response (CFR) and $\boldsymbol{\eta} = (1/\sqrt{M})\boldsymbol{\mathcal{F}}\boldsymbol{w}$ is the frequency domain noise, with the same statistics as $\boldsymbol{w}$. The CFR estimates, in the least squares (LS) sense, can then be computed as

$$\hat{H}_m = \frac{y_m}{x_m} = H_m + \frac{\eta_m}{x_m}. \tag{3}$$

B. OFDM/OQAM

The baseband discrete-time signal at time instant l, at the output of an OFDM/OQAM synthesis filter bank (SFB) is given by [16]

$$s_{\mathrm{OQAM}}(l) = \sum_{m=0}^{M-1}\sum_n a_{m,n}g_{m,n}(l) \tag{4}$$

where $a_{m,n}$ are real OQAM symbols, and

$$g_{m,n}(l) = g\left(l - n\frac{M}{2}\right)e^{\jmath\frac{2\pi}{M}m\left(l-\frac{L_g-1}{2}\right)}e^{\jmath\varphi_{m,n}},$$

with g being the *real symmetric* prototype filter impulse response (assumed here of unit energy) of length L_g, M being the *even* number of subcarriers, and $\varphi_{m,n} = \varphi_0 + (\pi/2)(m+n) \bmod \pi$, where φ_0 can be arbitrarily chosen[4] [16]. The filter g is usually designed to have length $L_g = KM$, where K, the overlapping factor, takes on values in $1 \le K \le 5$ in practice. The double subscript $(\cdot)_{m,n}$ denotes the

[4]For example, in [16], $\varphi_{m,n}$ is defined as $(m+n)(\pi/2) - mn\pi$.

(m,n)th frequency-time (FT) point. Thus, m is the subcarrier index and n the OQAM symbol time index.[5]

The pulse g is designed so that the associated subcarrier functions $g_{m,n}$ are orthogonal in the real field, that is

$$\Re\left\{\sum_l g_{m,n}(l) g_{p,q}^*(l)\right\} = \delta_{m,p}\delta_{n,q} \tag{5}$$

where $\delta_{i,j}$ is the Kronecker delta (i.e., $\delta_{i,j} = 1$ if $i = j$ and 0 otherwise). This implies that even in the absence of channel distortion and noise, and with perfect time and frequency synchronization, there will be some intercarrier (and/or intersymbol) interference at the output of the analysis filter bank (AFB), which is purely imaginary, i.e.,

$$\sum_l g_{m,n}(l) g_{p,q}^*(l) = \jmath u_{m,n}^{p,q} \tag{6}$$

and is known as *intrinsic* interference [6]. Adopting the commonly used assumption that the channel is (approximately) frequency flat at each subcarrier and constant over the duration of the prototype filter [9], which is true for practical values of L_h and L_g and for well time-localized g's, one can express the AFB output at the pth subcarrier and qth OFDM/OQAM symbol as

$$y_{p,q} = H_{p,q} a_{p,q} + \underbrace{\jmath \underbrace{\sum_{m=0}^{M-1}\sum_n}_{(m,n)\neq(p,q)} H_{m,n} a_{m,n} u_{m,n}^{p,q}}_{I_{p,q}} + \eta_{p,q} \tag{7}$$

where $H_{p,q}$ is the CFR at that FT point, and $I_{p,q}$ and $\eta_{p,q}$ are the associated interference and noise components, respectively. One can easily see that $\eta_{p,q}$ is also Gaussian with zero mean and variance σ^2.

For pulses g that are well localized in both time and frequency, the interference from FT points outside a neighborhood $\Omega_{p,q}$ around (p,q) is negligible. If, moreover, the CFR is almost constant over this neighborhood, one can write (7) as

$$y_{p,q} \approx H_{p,q} c_{p,q} + \eta_{p,q} \tag{8}$$

where

$$c_{p,q} = a_{p,q} + \jmath \sum_{(m,n)\in\Omega_{p,q}} a_{m,n} u_{m,n}^{p,q}. \tag{9}$$

When pilots are transmitted at (p,q) and at points inside its neighborhood $\Omega_{p,q}$, the quantity in (9) can be approximately computed. This can then serve as a *pseudo-pilot* [9] to compute an estimate of the CFR at the corresponding FT point, as

$$\hat{H}_{p,q} = \frac{y_{p,q}}{c_{p,q}} \approx H_{p,q} + \frac{\eta_{p,q}}{c_{p,q}}. \tag{10}$$

III. Optimal Sparse Preambles

For the CP-OFDM system, the preamble structure will consist of one complex vector symbol, as it is common in the literature [15]. Note that each complex CP-OFDM symbol is equivalent to two real vector symbols in the OFDM/OQAM system. We consider an equivalent preamble structure for the OFDM/OQAM system, which consists of one nonzero training vector followed by a zero vector symbol. The latter aims at protecting the nonzero part of the preamble from the intrinsic interference due to the data section of the frame [9].[6]

Definition 1: By *sparse* preamble vector we will mean an $M \times 1$ training vector containing L_h isolated pilots and zeros at the rest of its entries.

In this correspondence, we make the assumption that M/L_h is an integer number, with M being (as usually in practice) a power of two. The channel length, L_h, will thus also be assumed to be a power of two.[7] The following result provides the optimal sparse preamble for CP-OFDM:

Theorem 1: For CP-OFDM, the sparse preamble that minimizes the MSE of the CFR estimates (3), subject to a constraint on the energy of *both* the useful part of the transmitted signal and the CP, consists of *equispaced* and *equal* pilot tones.

Proof: See Appendix I. ■

Remark: Recall that the MSE-optimal sparse preamble for CP-OFDM is built with equispaced and *equipowered*, not necessarily equal pilots, if for the training energy the CP part is not included [15]. However, at least in theory this is not fair since the total amount of energy actually spent for training includes the transmission of the CP as well. According to Theorem 1, if the preamble optimization is to be performed in a fair way, then the pilots should necessarily be all equal. Of course, such a preamble would suffer from a high PAPR. In practice, this can be overcome by transmitting equipowered instead of equal tones, at the expense of a small performance loss.

For the OFDM/OQAM system, the corresponding result is as follows.

Theorem 2: For OFDM/OQAM, the sparse preamble that minimizes the MSE of the CFR estimates (10), subject to an energy constraint, is built with *equispaced* and *equipowered* pilot tones.

Proof: See Appendix II. ■

In view of the above results, preamble vectors containing equal symbols are the only or one of the optimal solutions. Hence, and for the sake of analytical convenience, optimal preambles will henceforth be assumed to consist of all equal pilots.

IV. OFDM/OQAM Sparse Preamble Versus CP-OFDM Sparse Preamble

Let us now compare the estimation performances of the two systems, when using optimal sparse preambles, and with the *same transmitted power for training*. Clearly, in both cases the same model for the received signal, (2), will hold. Moreover, if the spacing of the pilots, M/L_h, is large enough (theoretically equal to or larger than 2), the noise components at the corresponding outputs of the AFB for the OFDM/OQAM system will be uncorrelated. If we do not equalize the powers at the SFB outputs, the two MSEs will obviously be related as $\mathrm{MSE}_{\mathrm{OQAM}} = \mathcal{E}_x^{\mathrm{QAM}}/\mathcal{E}_x^{\mathrm{OQAM}}\,\mathrm{MSE}_{\mathrm{QAM}}$, with $\mathcal{E}_x^{\mathrm{QAM}}, \mathcal{E}_x^{\mathrm{OQAM}}$ being the energies of the QAM and OQAM symbols, respectively. Defining the power ratio for the two systems, $\mathrm{TPR}_{\mathrm{QAM/OQAM}}$, as the ratio of the training power in CP-OFDM over that in OFDM/OQAM, and scaling the output of the OFDM/OQAM SFB by $\sqrt{\mathrm{TPR}_{\mathrm{QAM/OQAM}}}$, we end up with

$$\mathrm{MSE}_{\mathrm{OQAM}} = \frac{\mathcal{E}_x^{\mathrm{QAM}}}{\mathcal{E}_x^{\mathrm{OQAM}}}\frac{\mathrm{MSE}_{\mathrm{QAM}}}{\mathrm{TPR}_{\mathrm{QAM/OQAM}}}.$$

[5]The latter should not be confused with the sample time index l. In fact, the temporal distance between two successive symbol instants $n, n+1$ equals $M/2$ sample time instants.

[6]Note that in wireless standards (e.g., WiMAX [2]), there are sufficiently long guard periods between the uplink and downlink subframes and between frames. Thus, there is no need to worry about intrinsic interference on the preamble vector from previous frames.

[7]It this is not the case, one may zero pad the CIR to $2^{\lceil \log_2 L_h \rceil}$ taps.

It can be shown that the transmit energies in the two systems are given by

$$\mathcal{E}_{\mathrm{OQAM}} = L_h \mathcal{E}_x^{\mathrm{OQAM}}$$

and

$$\mathcal{E}_{\mathrm{QAM}} = L_h \mathcal{E}_x^{\mathrm{QAM}}.$$

The OFDM/OQAM sparse preamble generates L_g nonzero samples at the output of the SFB, while the CP-OFDM sparse preamble yields $M+\nu = M+L_h-1$ samples. The sampling rate at the output of the SFB's is the same for both systems. Hence, to equalize the energies per time unit for the two schemes, we have to form the power ratio as follows:

$$\mathrm{TPR}_{\mathrm{QAM/OQAM}} = \frac{\frac{1}{M+L_h-1} L_h \mathcal{E}_x^{\mathrm{QAM}}}{\frac{1}{L_g} L_h \mathcal{E}_x^{\mathrm{OQAM}}} = \frac{\frac{\mathcal{E}_x^{\mathrm{QAM}}}{\mathcal{E}_x^{\mathrm{OQAM}}} L_g}{M+L_h-1} \quad (11)$$

and finally

$$\mathrm{MSE}_{\mathrm{OQAM}} = \frac{M+L_h-1}{L_g} \mathrm{MSE}_{\mathrm{QAM}}. \quad (12)$$

For example, let $L_h = 32$. Then, for $L_g = M$, the CP-OFDM sparse preamble turns out to be superior to the corresponding OFDM/OQAM sparse preamble, while for $L_g = KM$, with $2 \leq K \leq 5$, the OFDM/OQAM sparse preamble is approximately 3–9 dB better.

Remarks:

1) Note that $(M+L_h-1)/L_g$ is the *ratio of the time durations* of the transmit pulses employed by the two systems.
2) The performance difference can be even greater if we want to achieve a lower PAPR in the CP-OFDM system. We will then have to use *unequal* equipowered pilots, which leads to a slightly worse performance of the CP-OFDM sparse preamble.
3) Nevertheless, *at the cost of increasing the bandwidth in the OFDM/OQAM system*, the OFDM/OQAM and CP-OFDM sparse preambles can become MSE equivalent, in the following way. Note that, due to the good time localization of the OFDM/OQAM pulse, there is always a subinterval of the total pulse duration in the OFDM/OQAM system with the same length as the CP-OFDM modulator output, that carries almost all of the energy of the pulse. In view of the even symmetry of g, we can consider the subinterval $[-\lceil (M+L_h-1)/2 \rceil, \lceil (M+L_h-1)/2 \rceil]$ around its center, where $\lceil a \rceil$ denotes the smallest integer that is not smaller than a. Then, for practical values of M, L_h, it can be easily verified than

$$\sum_{l=-\lceil (M+L_h-1)/2 \rceil}^{\lceil (M+L_h-1)/2 \rceil} g^2(\lceil L_g/2 \rceil + l) \approx 0.99.$$

Therefore, we only need to observe this interval to approximately reconstruct the preamble vector at the receiver. Then the transmit pulses in the two systems have approximately the same duration (albeit with OFDM/OQAM bandwidth increased), thus leading to almost the same MSE performance for the two sparse preambles. Again, the OFDM/OQAM sparse preamble can be slightly better if we use unequal equipowered pilots in the CP-OFDM sparse preamble for a lower PAPR.
4) Clearly, the two systems would exhibit the same MSE performance if they were compared on the basis of equal transmit *energy* instead of equal transmit power.

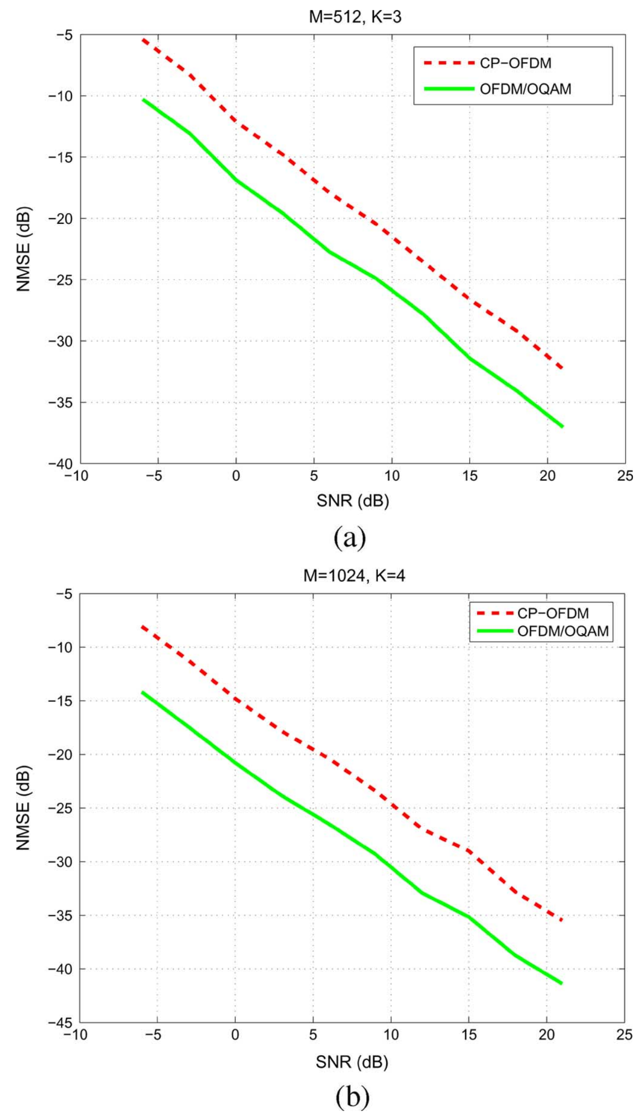

Fig. 1. NMSE performance of the CP-OFDM and OFDM/OQAM sparse preambles: (a) $M = 512, K = 3$; (b) $M = 1024, K = 4$.

V. SIMULATIONS

In this section, we present simulation results to verify our analysis. The channel follows the veh-A model [2]. The CIR is initially generated with 29 taps and then zero padded to the closest power of two, that is, $L_h = 32$ taps. We plot the normalized MSE (NMSE), i.e., $E(\|\boldsymbol{H} - \hat{\boldsymbol{H}}\|^2 / \|\boldsymbol{H}\|^2)$, versus the ratio of the transmit power to the power of the noise. The curves are the result of averaging 200 channel realizations. For each channel realization, 300 different noise realizations are considered. QPSK modulation is employed.

The optimal sparse preambles for the CP-OFDM and OFDM/OQAM systems are compared in Fig. 1. For the OFDM/OQAM system, we have used filter banks as given in [13], [4]. The superiority of the OFDM/OQAM sparse preamble, when the entire transmit pulse duration is considered, is evident. The analytical results can be seen to be approximately verified. Thus, for Fig. 1(a), the theoretical difference is $10\log_{10}[KM/(M+L_h-1)] \approx 4.5$ dB, while for Fig. 1(b), it is approximately 5.9 dB. These values agree with the difference of the experimental curves.

VI. Conclusion

Optimal preamble design for LS channel estimation in CP-OFDM and OFDM/OQAM systems was addressed in this correspondence, for sparse preambles. In contrast to earlier related work on CP-OFDM, the energy spent for the CP transmission was also taken into account when assessing the energy budget for training. This turned out to lead to the requirement of *equal* instead of simply equipowered pilot tones for the CP-OFDM sparse preamble. Equipowered and equispaced pilot tones were shown to comprise the optimal sparse preamble for OFDM/OQAM. The OFDM/OQAM optimal sparse preamble was compared with that of CP-OFDM and shown to allow for a significantly better performance, provided the whole pulse is transmitted when training. Nevertheless, it will perform similarly to CP-OFDM, at the cost of bandwidth expansion, if the tails of the (well time-localized) pulse are left out in the transmission of the preamble. Our analytical results were confirmed via simulations.

Appendix I
Proof of Theorem 1

Let $\mathcal{I}_{L_h} = \{i_0, i_1, \ldots, i_{L_h-1}\}$ be the set of indexes of the nonzero pilot tones in the sparse preamble. Stacking the L_h CFR estimates in the vector $\hat{\boldsymbol{H}}_{L_h}$, we can find the CIR as $\hat{\boldsymbol{h}} = \boldsymbol{F}^{-1}_{L_h\times L_h}\hat{\boldsymbol{H}}_{L_h} = \boldsymbol{h} + \boldsymbol{F}^{-1}_{L_h\times L_h}\varepsilon$, where ε is the $L_h\times 1$ vector with entries $\eta_m/x_m, m\in\mathcal{I}_{L_h}$, and $\boldsymbol{F}_{L_h\times L_h}$ is the $L_h \times L_h$ submatrix of the $M\times M$ DFT matrix $\mathcal{F}$ consisting of its L_h first columns and its rows corresponding to the indexes in $\mathcal{I}_{L_h}$. Thus, the MSE of the above estimate is given by

$$\mathrm{MSE}_{L_h} = \mathrm{tr}\left[\boldsymbol{\mathcal{C}}_{L_h}\left(\boldsymbol{F}_{L_h\times L_h}\boldsymbol{F}^H_{L_h\times L_h}\right)^{-1}\right] \tag{13}$$

where $\boldsymbol{\mathcal{C}}_{L_h} = \sigma^2\mathrm{diag}(1/|x_{i_0}|^2, 1/|x_{i_1}|^2, \ldots, 1/|x_{i_{L_h-1}}|^2)$ is the covariance of ε.

We want to minimize this, subject to the constraint $\sum_{m=0}^{L_h-1}|x_{i_m}|^2 + \frac{1}{M}\boldsymbol{x}^H_{L_h}\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu}\boldsymbol{x}_{L_h} \le \mathcal{E}$, where $\boldsymbol{F}_{M\times\nu}$ is the $M\times\nu$ matrix consisting of the last ν columns of $\mathcal{F}$ and $\boldsymbol{x}_{L_h}$ is the $M\times 1$ sparse preamble vector, with $x_i = 0, i\notin\mathcal{I}_{L_h}$. One can easily verify that the class of equal and equispaced training symbols for the CP-OFDM sparse preamble yields an MSE equal to $L_h\sigma^2/\mathcal{E}$, while it zeroes the CP-energy $(1/M)\boldsymbol{x}^H_{L_h}\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu}\boldsymbol{x}_{L_h}$. It is obvious that, since the CP is a wasted part of energy, the MSE would be minimized if somehow we were able to collect all the energy of the CP and put it in the useful part, namely the first sum of the constraint. To obtain the minimum MSE we can possibly imagine, we consider the relaxed problem[8]:

$$\min_{x_{i_m},\boldsymbol{F}_{L_h\times L_h}} \quad \mathrm{MSE}_{L_h} \tag{14}$$

$$\text{s.t.} \sum_{m=0}^{L_h-1}|x_{i_m}|^2 \le \mathcal{E}. \tag{15}$$

However, it is known [15], [3] that the optimal solution for this problem is the sparse preamble of L_h equispaced and equipowered pilot tones. The minimum achievable MSE is $L_h\sigma^2/\mathcal{E}$, which is also achieved by a sparse preamble of equispaced and *equal* pilot tones. Therefore, we only need to verify that this is the unique class of sparse preamble vectors that achieve this minimum MSE. This is equivalent to proving that the class of sparse preambles with equispaced and equal pilot tones is the only one that zeroes the CP energy.

The submatrix of $\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu}$ involved in the evaluation of $\boldsymbol{x}^H_{L_h}\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu}\boldsymbol{x}_{L_h}$ is given by $[(\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu})_{i,j}]_{i,j\in\mathcal{I}_{L_h}}$ where $\mathcal{I}_{L_h}$ is now any set of the form $\{i_0 + kM/L_h | k = 0, 1, \ldots, L_h - 1\}$ with $i_0 = 0, 1, \ldots, M/L_h - 1$. This submatrix has a very special form:

Lemma 1: All diagonal entries of the submatrix $[(\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu})_{i,j}]_{i,j\in\mathcal{I}_{L_h}}$ are equal to $\nu = L_h - 1$, while all its off-diagonal entries equal -1.

Proof: The general entry of the above submatrix is given by:

$$\left(\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu}\right)_{i,j} = \sum_{l=1}^{\nu} e^{\jmath\frac{2\pi}{L_h}(k_j-k_i)(M-1-\nu+l)} = \sum_{l=1}^{L_h-1} e^{\jmath\frac{2\pi}{L_h}(k_j-k_i)(M-L_h+l)}$$

where $i = i_0 + k_iM/L_h$ and similarly for j. Obviously, for $i = j, (\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu})_{i,i} = L_h - 1, i = 0, 1, \ldots, L_h - 1$. If $i\neq j$, and setting $k = k_j - k_i$,

$$\left(\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu}\right)_{i,j} = \sum_{l=1}^{L_h-1} e^{\jmath\frac{2\pi}{L_h}k(M-L_h+l)} = \sum_{l=1}^{L_h-1} e^{\jmath\frac{2\pi}{L_h}kl}$$

with the assumptions made previously for M, L_h. But

$$0 = \sum_{l=0}^{L_h-1} e^{\jmath\frac{2\pi}{L_h}kl} = 1 + \left(\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu}\right)_{i,j},$$

hence

$$\left(\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu}\right)_{i,j} = -1,\ i\neq j.$$

■

The question now concerns the type of vectors $\boldsymbol{x}_{L_h}$ that make the term $\boldsymbol{x}^H_{L_h}\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu}\boldsymbol{x}_{L_h} = \|\boldsymbol{F}^H_{M\times\nu}\boldsymbol{x}_{L_h}\|^2$ vanish. Suppose that there is such a sparse vector with equispaced and equipowered symbols $x_{i_m} = |x|e^{\jmath\theta_{i_m}}$. Then, there should hold $\boldsymbol{F}^H_{M\times\nu}\boldsymbol{x}_{L_h} = \boldsymbol{0}$, hence $\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu}\boldsymbol{x}_{L_h} = \boldsymbol{0}$. Consider, for example, the inner product of the first row of $\boldsymbol{F}_{M\times\nu}\boldsymbol{F}^H_{M\times\nu}$ with $\boldsymbol{x}_{L_h}$. Then, according to the previous lemma, there should hold $|x|(\nu e^{\jmath\theta_{i_0}} - \sum_{m=1}^{\nu} e^{\jmath\theta_{i_m}}) = 0$ or $\nu e^{\jmath\theta_{i_0}} = \sum_{m=1}^{\nu} e^{\jmath\theta_{i_m}}$. Taking the modulus in both sides, we should have $\nu = |\sum_{m=1}^{\nu} e^{\jmath\theta_{i_m}}|$. But this can only happen when all the exponentials in the sum are collinear and of the same direction in the complex plane, i.e., when all these exponentials are equal.

Conclusion: Among all sparse preamble vectors, it is those with equispaced and equal pilot symbols that yield the *globally* minimum MSE for CP-OFDM.

Appendix II
Proof of Theorem 2

Define the vector of the nonzero SFB output samples for a sparse preamble input:

$$\boldsymbol{s}^{L_h}_{\mathrm{OQAM}} = \left[\sum_{i\in\mathcal{I}_{L_h}} a_{i,0}g_{i,0}(0) \sum_{i\in\mathcal{I}_{L_h}} a_{i,0}g_{i,0}(1)\cdots \sum_{i\in\mathcal{I}_{L_h}} a_{i,0}g_{i,0}(L_g-1)\right]^T.$$

Clearly, for a sparse preamble, we have $M/L_h \ge 2$. We first show the following.

Proposition 1: If $M/L_h \ge 2$, then $\|\boldsymbol{s}^{L_h}_{\mathrm{OQAM}}\|^2 = \sum_{i\in\mathcal{I}_{L_h}} a^2_{i,0}$, i.e., the energy transmitted for training is equal to the energy of the

[8]This is a "genie-aided" problem, i.e., a problem that is unrealistic in practice and only a genie can help us to obtain, since it would lead to the minimum possible achievable MSE.

training vector at the AFB output of the associated ideal (channel- and noise-free) OFDM/OQAM system.

Proof:

$$\left\|\boldsymbol{s}_{\mathrm{OQAM}}^{L_h}\right\|^2 = \sum_{l=0}^{L_g-1}\left|\sum_{i\in\mathcal{I}_{L_h}} a_{i,0}g_{i,0}(l)\right|^2$$
$$= \sum_{m=0}^{L_h-1} a_{i_m,0}\sum_{k=0}^{L_h-1} a_{i_k,0}\sum_{l=0}^{L_g-1} g_{i_m,0}(l)g_{i_k,0}^*(l)$$
$$= \sum_{m=0}^{L_h-1} a_{i_m,0}^2\sum_{l=0}^{L_g-1} |g_{i_m,0}(l)|^2 + \sum_{m,k=0,m\neq k}^{L_h-1} a_{i_m,0}a_{i_k,0}\sum_{l=0}^{L_g-1} g_{i_m,0}(l)g_{i_k,0}^*(l).$$

Obviously, $\sum_{l=0}^{L_g-1}|g_{i_m,0}(l)|^2 = \sum_{l=0}^{L_g-1} g(l)^2 = 1$ and $\sum_{l=0}^{L_g-1} g_{i_m,0}(l)g_{i_k,0}^*(l) = 0, m,k = 0,1,\ldots,L_h-1, m\neq k$ due to our assumption that $M/L_h \geq 2$ and a good frequency localization of g. Therefore, $\|\boldsymbol{s}_{\mathrm{OQAM}}^{L_h}\|^2 = \sum_{m=0}^{L_h-1} a_{i_m,0}^2, i_m\in\mathcal{I}_{L_h}$. ■

The (time domain) MSE expression for the sparse preamble with L_h nonzero pilot tones in the OFDM/ OQAM system is the same as in the CP-OFDM system, i.e., $\mathrm{MSE}_{L_h} = \mathrm{tr}[\boldsymbol{\mathcal{C}}_{L_h}(\boldsymbol{F}_{L_h\times L_h}\boldsymbol{F}_{L_h\times L_h}^H)^{-1}]$. Our optimization problem can therefore be stated as follows:

$$\min_{a_{i_m,0}, i_m\in\mathcal{I}_{L_h}} \mathrm{MSE}_{L_h} \quad (16)$$
$$\text{s.t.} \sum_{m=0}^{L_h-1} a_{i_m,0}^2 \leq \mathcal{E}. \quad (17)$$

But the solution to this problem is known. It is the class of equipowered and equispaced pilot tones [15], [3].

REFERENCES

[1] P. Amini, R. Kempter, and B. Farhang-Boroujeny, "A comparison of alternative filterbank multicarrier methods for cognitive radio systems," presented at the SDR-2006, Orlando, FL, Nov. 13–17, 2006.
[2] J. G. Andrews, A. Ghosh, and R. Muhamed, *Fundamentals of WiMAX: Understanding Broadband Wireless Networking*. Englewood Cliffs, NJ: Prentice-Hall, 2007.
[3] I. Barhumi, G. Leus, and M. Moonen, "Optimal training design for MIMO OFDM systems in mobile wireless channels," *IEEE Trans. Signal Process.*, vol. 51, no. 6, pp. 1615–1624, Jun. 2003.
[4] M. G. Bellanger, "Specification and design of a prototype filter for filter bank based multicarrier transmission," in *Proc. ICASSP-2001*, Salt Lake City, UT, May 7–11, 2001, vol. 4, pp. 2417–2420.
[5] S. Coleri *et al.*, "Channel estimation techniques based on pilot arrangement in OFDM systems," *IEEE Trans. Broadcast.*, vol. 48, no. 3, pp. 223–229, Sep. 2002.
[6] J.-P. Javaudin, D. Lacroix, and A. Rouxel, "Pilot-aided channel estimation for OFDM/OQAM," presented at the VTC-2003 (Spring), Jeju Island, Korea, Apr. 26–29, 2003.
[7] K. P. Kongara, P. J. Smith, and S. Mann, "A comparison of CP-OFDM with IOTA-OFDM under typical system imperfections," in *Proc. 2008 IET Seminar on Wideband and Ultrawideband Systems and Technologies*, London, UK, Nov. 2008, pp. 1–5.
[8] B. Le Floch, M. Alard, and C. Berrou, "Coded orthogonal frequency division multiplex," in *Proc. IEEE*, Jun. 1995, vol. 83, no. 6, pp. 982–996.
[9] C. Lélé *et al.*, "Channel estimation methods for preamble-based OFDM/OQAM modulations," presented at the EW 2007, Paris, France, Apr. 1–4, 2007.
[10] C. Lélé, R. Legouable, and P. Siohan, "Channel estimation with scattered pilots in OFDM/OQAM," in *Proc. SPAWC 2008*, Recife, Brazil, Jul. 6–9, 2008, pp. 286–290.
[11] C. Lélé, P. Siohan, and R. Legouable, "2 dB better than CP-OFDM with OFDM/OQAM for preamble-based channel estimation," in *Proc. ICC 2008*, Beijing, China, May 19–23, 2008, pp. 1302–1306.
[12] H. Lin and P. Siohan, "A new transceiver system for the OFDM/ OQAM modulation with cyclic prefix," in *Proc. PIMRC 2008*, Cannes, France, Sep. 15–18, 2008, pp. 1–5.
[13] K. W. Martin, "Small side-lobe filter design for multitone data-communication applications," *IEEE Trans. Circuits Syst. II, Analog Digit. Signal Process.*, vol. 45, no. 8, pp. 1155–1161, Aug. 1998.
[14] M. Morelli and U. Mengali, "A comparison of pilot-aided channel estimation methods for OFDM systems," *IEEE Trans. Signal Process.*, vol. 49, no. 12, pp. 3065–3073, Dec. 2001.
[15] R. Negi and J. Cioffi, "Pilot tone selection for channel estimation in a mobile OFDM system," *IEEE Trans. Consum. Electron.*, vol. 44, no. 3, pp. 1122–1128, Aug. 1998.
[16] P. Siohan, C. Siclet, and N. Lacaille, "Analysis and design of OFDM/OQAM systems based on filterbank theory," *IEEE Trans. Signal Process.*, vol. 50, no. 5, pp. 1170–1183, May 2002.
[17] A. Skrzypczak *et al.*, "OFDM/OQAM: An appropriate modulation scheme for an optimal use of the spectrum," in *Proc. ISCCSP 2008*, Malta, Mar. 12–14, 2008, pp. 405–410.
[18] R. van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*. Norwood, MA: Artech House, 2000.
[19] D. S. Waldhauser, L. G. Baltar, and J. A. Nossek, "Comparison of filter bank based multicarrier systems with OFDM," in *Proc. APCCAS 2006*, Singapore, Dec. 4–7, 2006, pp. 976–979.

Novel Low-Complexity SLM Schemes for PAPR Reduction in OFDM Systems

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***Abstract*—Selected mapping (SLM) schemes are commonly employed to reduce the peak-to-average power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) systems. It has been shown that the computational complexity of the traditional SLM scheme can be substantially reduced by adopting conversion vectors obtained by using the inverse fast Fourier transform (IFFT) of the phase rotation vectors in place of the conventional IFFT operations [C.-L. Wang and Y. Ouyang, "Low-Complexity Selected Mapping Schemes for Peak-to-Average Power Ratio Reduction in OFDM Systems," IEEE Trans. Signal Process., vol. 53, no. 12, pp. 4652–4660, Dec. 2005]. To ensure that the elements of these phase rotation vectors have an equal magnitude, conversion vectors should have the form of a perfect sequence. This paper presents three novel classes of perfect sequence, each of which comprises certain base vectors and their cyclically shifted versions. Three novel low-complexity SLM schemes are then proposed based upon the unique structures of these perfect sequences. It is shown that while the PAPR reduction performances of the proposed schemes are marginally poorer than that of the traditional SLM scheme, the three schemes achieve a substantially lower computational complexity.**

***Index Terms*—Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR).**

I. INTRODUCTION

Orthogonal-frequency-division multiplexing (OFDM) is a promising technique for high data rate transmission due to its high spectral efficiency and robustness against the interference inherent in multi-path channels. However, OFDM systems suffer the drawback of a high peak-to-average power ratio (PAPR) of the transmitted signal. The literature contains various methods for PAPR reduction in

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