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PRECIPITATION IN TRAP MODELS FOR SOLAR HARD X-RAY BURSTS

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SUMMARY

Precipitation of electrons due to collisions necessarily occurs in trap models for hard X-ray bursts and the thick-target emission from the precipitating electrons produces an X-ray spectrum similar in intensity and in spectral shape to that from the trapped electrons. Such a trap-plus-precipitation model combines attractive features and removes some of the difficulties of thick-target and simple trap models. The evolution of the trap-plus-precipitation model is amenable to an analytic description, which is presented, including inversion, to find the injection electron spectrum from the X-ray spectrum. Streaming instabilities are unlikely to be important and resonant scattering is also probably not important for the electrons ($E \gtrsim 100$ keV) which emit most of the X-rays but may well be important for the higher energy electrons which generate microwave bursts.

1. INTRODUCTION

It is widely accepted that solar hard X-ray emission (photon energies ϵ in the range $10\ \text{keV} \lesssim \epsilon \lesssim 100\ \text{keV}$) is bremsstrahlung from electrons with energy E in the range $10\ \text{keV} \lesssim E \lesssim 100\ \text{keV}$. The treatment of the emission is usually restricted to two limiting cases called the *thick-target* and the *thin-target* limits. These limits correspond to the lifetime of the radiating electrons against Coulomb losses being, respectively, much shorter and much longer than other relevant time scales such as the time resolution of the burst. Models for the bursts fall into three classes (Brown 1975): thick-target models, thin-target models and trap models. In thick-target models electrons are injected from an acceleration region into a dense (chromospheric) source region. An important attraction of such models is that the heating of the denser layers by the energetic electrons can account for the associated soft X-ray, *EUV* and optical emissions (Brown 1973a; Syrovatskii & Shmeleva 1973). In thin-target models (Datlowe & Lin 1973) the electrons propagate away from the photosphere and lose only a small fraction of their energy in traversing the source region in the corona. One of the motivations for thin-target models was the relatively great heights of the X-ray sources inferred for behind-the-limb events (Roy & Datlowe 1975). Because both thick- and thin-target models involve streaming of electrons they can account for the observed linear polarization of hard X-rays (Tindo, Mandel'shtam & Shuryghin 1973). Trap models involve energetic electrons confined by a magnetic bottle. An attraction of such models is

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that, as well as producing behind-the-limb events, they account for the association between hard X-ray and microwave bursts in terms of related distributions of electrons ($E \lesssim 100$ keV for the X-ray emission and $E \gtrsim 100$ keV for the microwave emission) in the same magnetic bottle (Takakura & Kai 1966). More recently trap models have been invoked to explain quasi-periodic variations in the X-ray spectrum in terms of pulsations of the magnetic bottle (Brown 1973b; Brown & Hoyng 1975). In thick- and thin-target models all observed variations must be attributed to variations in the injection from the ' black-box ' acceleration region.

Our purposes in this paper are (a) to emphasize that precipitation of electrons necessarily occurs in trap models, as has already been pointed out by Hudson (1972) and Kane (1974), and (b) to point out that when the X-ray emission from the precipitating electrons is taken into account, the trap-plus-precipitation model incorporates most of the attractions of the other models. Furthermore the model is amenable to a quantitative analytic description which is developed here.

The evolution of the energy spectrum of electrons in a trap under the influence of collisional energy losses and precipitation, and with a source (continuous injection) term is described in Section 2. The corresponding total X-ray spectrum including the contributions from the trap and from the precipitating electrons is derived in Section 3. In Section 4 it is shown to be possible in principle to reconstruct the source term for injection into the trap (as a function of E and t) from a knowledge of the X-ray spectrum (as a function of ϵ and t). In Section 5 it is shown that wave–particle interactions including resonant scattering (by whistlers) are probably not important for electrons with $E \lesssim 100$ keV, which result is anticipated in neglecting the effects of wave–particle interactions in Sections 2–4, and possible implications of resonant scattering for $E \gtrsim 100$ keV are considered briefly.

2. EVOLUTION OF THE ENERGY SPECTRUM

It is reasonable to assume that the evolution of the energy spectrum of energetic electrons confined in a magnetic trap in the solar corona is determined by three effects. The first is injection of electrons into the trap Let $Q(E, t)$ electrons per unit energy be injected into the trap in unit time. (This ' injection ' may be regarded as including the effect on the energy spectrum of pulsations of the trap, and also of any acceleration *in situ* of electrons which previously had insufficient energy to contribute to the X-ray emission, and this would be relevant in, e.g. Brown & Hoyng's (1975) treatment of oscillations of a trap where one would have $Q(E, t) < 0$ during compressions and $Q(E, t) > 0$ during rarefactions (see (45) below).) The second is the loss of energy by individual electrons and only losses due to Coulomb interactions with thermal electrons are considered explicitly here. With Coulomb interactions in mind it is reasonable to neglect diffusion in energy space in comparison with systematic energy losses and to assume that each electron loses energy at the rate $dE/dt = -\nu_E E$ with $\nu_E \propto E^{-3/2}$. (The rate of diffusion in energy space is roughly the thermal energy divided by E times the rate ν_E of systematic energy losses for Coulomb interactions.) The third is loss of electrons from the trap due to precipitation into a denser region where an electron loses its energy in traversing about a scale height (the ' precipitation layer '). Let ν_p be the probability per unit time that an electron be precipitated. Finally let the distribution of electrons in the trap be described by their differential energy spectrum $N(E, t)$, which is the total number of electrons per unit energy range. Then, provided that the spatial variations of all these quantities within the trap region are ignored, the evolution may be described by (*cf.* Lin 1974)

$$\frac{\partial}{\partial t}N(E,t) = \frac{\partial}{\partial E}\{\nu_E E N(E,t)\} - \nu_p N(E,t) + Q(E,t) \tag{1}$$

in general ν_E and ν_p may be functions of time t as well as of E.

General solution of (1)

It is possible to solve (1) in general provided that ν_E and ν_p do not depend on t. (This amounts to neglecting time variations in the ambient density n_0—*cf.* (11) and (12).) Though n_0 will actually vary with time in an oscillating trap, the main effects on the X-ray emission arise from the effect of the varying field on the fast electron spectrum (Brown & Hoyng 1975). Firstly introduce the Laplace transforms of $N(E, t)$ and $Q(E, t)$

$$\begin{Bmatrix} n(E,p) \\ q(E,p) \end{Bmatrix} = \int_0^\infty dt\, e^{-pt} \begin{Bmatrix} N(E,t) \\ Q(E,t) \end{Bmatrix}, \tag{2}$$

and then Laplace transform (1):

$$pn(E,p) - N(E,0) = \frac{\partial}{\partial E}\{\nu_E E n(E,p)\} - \nu_p n(E,p) + q(E,p). \tag{3}$$

Introducing the integrating factor exp $[G(E,p)]$ with $G(E,p) = H(E) + ph(E)$, and with

$$H(E) = \int dE \frac{\nu_p}{\nu_E E}, \qquad h(E) = \int \frac{dE}{\nu_E E}, \tag{4}$$

the solution of (3) becomes

$$n(E,p) = \frac{1}{\nu_E E}\int_E^\infty dE' \exp\,[H(E)-H(E')] \exp\,[p\{h(E)-h(E')\}] \times\{N(E,0)+q(E,p)\}. \tag{5}$$

The inversion involves only elementary Laplace transforms:

$$N(E,t) = \frac{1}{\nu_E E}\int_E^\infty dt' \exp\,[H(E)-H(E')]\,\{N(E,0)\,\delta(t-h(E')+h(E)) + Q(E,t-h(E')+h(E))\,\theta(t-h(E')+h(E))\}, \tag{6}$$

where $\theta(t)$ is the step function ($\theta(t) = 0$ for $t<0$ and $\theta(t) = 1$ for $t>0$). The solution (6) may be rewritten in the form

$$N(E,t) = \frac{\partial E_0}{\partial E} N(E_0,0) \exp\,[H(E)-H(E_0)] + \frac{1}{\nu_E E}\int_E^{E_0} dE' \exp\,[H(E)-H(E')]\, Q(E', t-h(E')+h(E)) \tag{7}$$

where E_0, which is regarded as a function of E and t, is defined as the solution of

$$t - h(E_0) + h(E) = 0. \tag{8}$$

Physically E_0 is the energy which an electron would have to have had at $t = 0$ for its energy to be E at time t when E varies according to $dE/dt = -\nu_E E$.

Precipitation

To proceed we need to estimate the rate of precipitation ν_p. An extensive literature exists on precipitation of electrons from the magnetosphere of the Earth due to wave–particle interactions involving whistlers, e.g. Kennel & Petschek (1966), Roberts (1969), Kennel (1969). The precipitation results from scattering of the electrons into the loss cone and this scattering is described in terms of diffusion in pitch angle. The theory is not sensitive to the specific scattering mechanism and it may be applied to scattering due to Coulomb interactions, in which case pitch angle diffusion occurs at a rate ν_D which is just twice the rate ν_E of collisional energy loss (Trubnikov 1965). It might be remarked that Hudson (1972) estimated $\nu_E/\nu_D \approx 0{\cdot}35$; as opposed to $\nu_E/\nu_D = 0{\cdot}5$ here. A simple physical interpretation is implicit in Trubnikov's (1965) derivation of the latter: fast electrons are scattered at the same rate by both thermal electrons and thermal ions, and energy losses occur at the same rate per thermal particle, but only the interactions with thermal electrons cause significant changes in energy. The result follows for a fully-ionized hydrogen plasma.

The loss cone is the range of pitch angles α which would correspond to mirror points at or below the precipitation layer. Let $\alpha < \alpha_0$ (and $\alpha > \pi - \alpha_0$) be the loss cone at the top of the trap where the magnetic induction has its minimum value B_0. Let B_p be the magnetic induction in the precipitation layer and then α_0 is given by

$$\alpha_0 = \arcsin\,(B_0/B_p)^{1/2}. \tag{9}$$

Electrons of different energy have different precipitation layers and hence B_p and α_0 must be functions of E in general. However, this dependence should be a weak one because the scale height for B is large compared with the scale height for density.

There are simple approximations to the precipitation rate ν_p in two opposite limiting cases which Kennel (1969) called the weak and the strong diffusion limits:

$$\nu_p \approx \begin{cases} \nu_D & \text{for} \quad \nu_D \ll \tfrac{1}{2}\alpha_0^2\nu_b & \text{(10a)} \\ \tfrac{1}{2}\alpha_0^2\nu_b & \text{for} \quad \nu_D \gg \tfrac{1}{2}\alpha_0^2\nu_b & \text{(10b)} \end{cases}$$

where $\alpha_0^2 \ll 1$ has been assumed and where ν_b is the bounce frequency for electrons in the trap. These two limits correspond to the loss cone being approximately empty and approximately full respectively.

Here it is assumed that the weak diffusion limit (10a) applies, and this assumption is discussed in Sections 5 and 6. (Though the real flare situation may represent an intermediate case, these gross limiting cases should encompass the possible range of trap behaviour.) Furthermore both the pitch angle scattering and the energy losses are assumed to be due only to Coulomb interactions with

$$\nu_D = r\nu_E, \qquad \nu_E = \nu_0 E^{-3/2} \tag{11}$$

and

$$\nu_0 \approx 5 \times 10^{-9}\, n_0\ \mathrm{s^{-1}\,(keV)^{3/2}} \tag{12}$$

where n_0 is the number density ($\mathrm{cm^{-3}}$) of thermal electrons. The case $r = 2$ applies to the trap-plus-precipitation model with Trubnikov's (1965) value for the ratio ν_D to ν_E. (Hudson's (1972) value $\nu_E/\nu_D = 0{\cdot}35$ corresponds to $r \approx 3$.) The case $r = 0$ would correspond to a trap with no precipitation.

Specific solutions

With (11) the solution of (8) is

$$E_0 = E(1+\tfrac{3}{2}\nu_0 E^{-3/2}t)^{2/3}, \tag{13}$$

and then with $\partial E_0/\partial E = (E_0/E)^{-1/2}$ the general solution (7) becomes

$$N(E,t) = (E_0/E)^{-r-1/2}\, N(E_0, 0) + \frac{E^{1/2}}{\nu_0}\int_E^{E_0} dE'(E'/E)^{-r}\, Q\left(E', t-\frac{2}{3\nu_0}(E'^{3/2}-E^{3/2})\right). \tag{14}$$

It is instructive to compare the trap-plus-precipitation model ($r = 2$) with a trap model without precipitation ($r = 0$) and also with a trap model with precipitation but no energy losses (limit $r \to \infty$). In this last case it is elementary to solve (1), with $\nu_E = 0$, directly:

$$N(E,t) = \exp[-\nu_p t]\, N(E, 0) + \int_0^t dt' \exp[-\nu_p(t-t')]\, Q(E, t'), \tag{15}$$

with $\nu_p = 2\nu_0 E^{-3/2}$ by assumption here.

(a) *Initial injection.* Suppose firstly that there is an initial injection ($N(E, 0) \neq 0$) and no continuous injection ($Q(E, t) = 0$ for $t > 0$). Then (13)–(15) with

$$N(E, 0) = KE^{-\delta} \tag{16}$$

lead to the following results:

(i) Trap-plus-precipitation model ($r = 2$)

$$N(E,t) = \left(\frac{E_0}{E}\right)^{-5/2} N(E_0, 0) \approx \begin{cases} KE^{-\delta}\{1-(\delta+\tfrac{5}{2})\,\nu_0 E^{-3/2}t\} & \text{for} \quad E \gg E_T \\ \left(\dfrac{E}{E_T}\right)^{5/2} KE_T^{-\delta} & \text{for} \quad E \ll E_T \end{cases} \tag{17}$$

with

$$E_T = (\tfrac{3}{2}\nu_0 t)^{2/3}. \tag{18}$$

The result $N(E, t) \propto E^{5/2}$ for $E \ll E_T$ applies for any $N(E, 0)$ and is not restricted to the power law case (16).

(ii) Trap model without precipitation ($r = 0$)

$$N(E,t) = \left(\frac{E_0}{E}\right)^{-1/2} N(E_0, 0) \approx \begin{cases} KE^{-\delta}\{1-(\delta+\tfrac{1}{2})\,\nu_0 E^{-3/2}t\} & \text{for} \quad E \gg E_T \\ \left(\dfrac{E}{E_T}\right)^{1/2} KE_T^{-\delta} & \text{for} \quad E \ll E_T. \end{cases} \tag{19}$$

(iii) Trap model with precipitation and without energy losses

$$N(E,t) = \exp[-\nu_D t]\, N(E, 0) \approx \begin{cases} KE^{-\delta}\{1-2\nu_0 E^{-3/2}t\} & \text{for } E \gg E_T \\ KE^{-\delta}\exp\left\{-\dfrac{4}{3}\left(\dfrac{E_T}{E}\right)^{3/2}\right\} & \text{for } E \ll E_T. \end{cases} \tag{20}$$

In all cases the energy spectrum has a maximum at an energy $E \approx E_T$ which increases with time as $E_T \propto t^{2/3}$. Below the bend (19) implies that energy losses alone would cause $N(E, t)$ to rise as $E^{1/2}$. This may be interpreted in terms of a flux of particles in energy space, $F(E, t)$ say. Conservation of particles implies $\partial N(E, t)/\partial t + \partial F(E, t)/\partial E = 0$ in the absence of sources and losses, and comparison of this equation with (1), for $\nu_p = 0$ and $Q(E, t) = 0$, implies $F(E, t) = -\nu_0 E^{-1/2} N(E, t)$ (the negative sign corresponds to a flux from high to low energies). For $E \ll E_T$ the flux in energy space must become independent of E because there are neither sources nor sinks of particles, and hence $E^{-1/2}N(E, t)$ must become independent of E, as implied by (19). On the other hand, in the absence of energy losses, precipitation causes an exponential decrease with time in the number of particles with $E \ll E_T$, and hence, as implied by (2), an exponential rise of the energy spectrum with energy for $E \ll E_T$. The case where both precipitation and energy losses occur is intermediate between these two and leads to the $E^{5/2}$ dependence of $N(E, t)$ for $E \ll E_T$ in the particular case $\nu_p = 2\nu_E$.

(b) *Continuous injection.* Now suppose that there is no initial injection ($N(E, 0) = 0$) and that the continuous injection is independent of time. Furthermore suppose the injection spectrum is a power law (δ, A constant)

$$Q(E, t) = AE^{-\delta}\theta(t). \tag{21}$$

Then (13)–(15) lead to the following results:

(i) Trap-plus-precipitation model ($r = 2$)

$$\begin{aligned} N(E, t) &= \frac{AE^{-\delta}}{(\delta+1)\,\nu_0 E^{-3/2}}\{1-(1+\tfrac{3}{2}\nu_0 E^{-3/2}t)^{-2(\delta+1)/3}\} \\ &= \begin{cases} AE^{-\delta}t & \text{for} \quad E \gg E_T \\ \dfrac{A}{(\delta+1)\,\nu_0} E^{-(\delta-3/2)} & \text{for} \quad E \ll E_T, \end{cases} \end{aligned} \tag{22}$$

with E_T given by (18).

(ii) Trap model without precipitation ($r = 0$)

$$\begin{aligned} N(E, t) &= \frac{AE^{-\delta}}{(\delta-1)\,\nu_0 E^{-3/2}}\{1-(1+\tfrac{3}{2}\nu_0 E^{-3/2}t)^{-2(\delta-1)/3}\} \\ &\approx \begin{cases} AE^{-\delta}t & \text{for} \quad E \gg E_T \\ \dfrac{A}{(\delta-1)\,\nu_0} E^{-(\delta-3/2)} & \text{for} \quad E \ll E_T. \end{cases} \end{aligned} \tag{23}$$

(iii) Trap model with precipitation and without energy losses

$$\begin{aligned} N(E, t) &= \frac{AE^{-\delta}}{2\nu_0 E^{-3/2}}\{1-\exp(-2\nu_0 E^{-3/2}t)\} \\ &\approx \begin{cases} AE^{-\delta}t & \text{for} \quad E \gg E_T \\ \dfrac{A}{2\nu_0} E^{-(\delta-3/2)} & \text{for} \quad E \ll E_T. \end{cases} \end{aligned} \tag{24}$$

In all three cases, which differ from each other in only minor ways, the spectrum hardens by 1·5 in spectral index for $E \lesssim E_T$. However, the fact that the evolution of

$N(E, t)$ is similar in all three cases does not mean that the X-ray spectrum would be the same because in cases (i) and (iii) precipitating electrons contribute thick-target X-ray emission.

3. THE X-RAY SPECTRUM

The X-ray emission in the trap-plus-precipitation model may be regarded as consisting of two parts. One part, $I_t(\epsilon, t)$ say, is emission from electrons in the trap. The other part, $I_p(\epsilon, t)$ say, is thick-target emission from the precipitating electrons.

Following Brown (1975) explicit expressions for I_t and I_p may be written down as follows. Let $\sigma(\epsilon, E)$ be the cross-section for the emission of bremsstrahlung and let $\bar{n}$ be the appropriate average value of the number density of thermal protons in the trap. Then one has

$$I_t(\epsilon, t) = \frac{\bar{n}(2/m)^{1/2}}{4\pi R^2} \int_\epsilon^\infty dE E^{1/2} \sigma(\epsilon, E)\, N(E, t), \tag{25}$$

where R (≈ 1 AU) is the distance to the source. In a thick target an electron with initial energy E radiates

$$(2/m)^{1/2} \int^E dE' \frac{E'^{1/2} n \sigma(\epsilon, E')}{(-dE'/dt)}$$

photons per unit energy range ϵ in traversing the source. The dependence on n would be complicated in general (because the electron passes through an inhomogeneous region) but it simplifies greatly for Coulomb interactions for which $-dE/dt$ is proportional to $nE^{-1/2}$ and the dependences on n cancel. Consequently one may replace $n/(-dE'/dt)$ by $\bar{n}/\nu_0 E'^{-1/2}$ where $\bar{n}$ and ν_0 correspond to values in the trap. With $\nu_p N(E, t)$ precipitating electrons per unit energy range and per unit time one has

$$I_p(\epsilon, t) = \frac{\bar{n}(2/m)^{1/2}}{4\pi R^2 \nu_0} \int_\epsilon^\infty dE \nu_p N(E, t) \int_\epsilon^E dE_0 E_0 \sigma(\epsilon, E_0). \tag{26}$$

In accord with the discussion in Section 2 we set $\nu_p = 2\nu_0 E^{-3/2}$ and then the dependences on ν_0 cancel in (26).

After a partial integration in (26) the total X-ray spectrum may be written in the form

$$I(\epsilon, t) = I_t(\epsilon, t) + I_p(\epsilon, t) = \frac{\bar{n}(2/m)^{1/2}}{4\pi R^2} \int_\epsilon^\infty dE E^{1/2} N_{\rm eff}(E, t)\, \sigma(\epsilon, E) \tag{27}$$

with

$$N_{\rm eff}(E, t) = N(E, t) + 2E^{1/2} \int_E^\infty dE_0 E_0^{-3/2} N(E_0, t). \tag{28}$$

The effective energy spectrum $N_{\rm eff}(E, t)$ may be interpreted as the equivalent energy spectrum in a trap model without precipitation, i.e. $N_{\rm eff}(E, t)$ in a trap model without precipitation would produce the same X-ray spectrum as $N(E, t)$ would in a trap-plus-precipitation model.

It is of interest to estimate the relative contributions to the total X-ray emission from electrons in the trap and from precipitating electrons. If we assumed a power law energy spectrum $N(E, t) \propto E^{-\delta}$ then (28) gives

$$N_{\rm eff}(E, t) = \{1 + 2/(\delta + \tfrac{1}{2})\}\, N(E, t),$$

i.e. $I_p(\epsilon, t)/I_t(\epsilon, t) = 2/(\delta+\frac{1}{2})$. However, even if a power law energy spectrum is injected the energy spectrum hardens at lower energies due to the effects of Coulomb interactions. Hence the estimate of $2/(\delta+\frac{1}{2})$ with δ characteristic of the injection spectrum would be an underestimate of the ratio of $I_p(\epsilon, t)$ to $I_t(\epsilon, t)$. Alternatively if the explicit expression for the Bethe–Heitler cross-section, i.e.

$$\sigma(\epsilon, E) = \frac{K_{BH}}{\epsilon E} \ln \frac{1+\sqrt{1-\epsilon/E}}{1-\sqrt{1-\epsilon/E}} \tag{29}$$

where K_{BH} is a constant, is inserted in (26) the E_0-integration may be performed and (26) may be rewritten as

$$I_p(\epsilon, t) = \frac{2\bar{n}(2/m)^{1/2}}{4\pi R^2} \int_\epsilon^\infty dE E^{1/2} N(E, t)\, \tilde{\sigma}(\epsilon, E) \tag{30}$$

with

$$\tilde{\sigma}(\epsilon, E) = \frac{1}{E^2}\int_\epsilon^E dE_0 E_0 \sigma(\epsilon, E_0) = \frac{K_{BH}}{\epsilon E} \times \left\{\left(1-\frac{\epsilon}{2E}\right) \ln\frac{1+\sqrt{1-\epsilon/E}}{1-\sqrt{1-\epsilon/E}} - \sqrt{1-\epsilon/E}\right\}. \tag{31}$$

For a sufficiently hard energy spectrum the X-ray emission occurs predominantly at $\epsilon \ll E$ and then $\tilde{\sigma}(\epsilon, E)$ is nearly equal to $\sigma(\epsilon, E)$. In this limit the ratio of $I_p(\epsilon, t)$ to $I_t(\epsilon, t)$ would be two.

Thus we conclude that $I_p(\epsilon, t)$ is proportional to $I_t(\epsilon, t)$ (for each ϵ), contrary to a statement by Hudson (1972) that the former spectrum would be the harder, and that the constant of proportionality is in the range

$$\frac{2}{\delta+\frac{1}{2}} < \frac{I_p(\epsilon, t)}{I_t(\epsilon, t)} < 2 \tag{32}$$

where δ is the (negative) spectral index of the injection energy spectrum into the trap. The physical interpretation of this identity of the spectra from trap and thick target contributions here is that although thick target X-ray spectra are harder than trap spectra for a given electron injection spectrum (due to collisional hardening in the thick target—Brown 1971) the electron spectrum injected by collisional precipitation from a trap is steeper than in the trap itself. In the present situation the two collisional effects thus cancel.

4. INVERSION TO FIND $Q(E, t)$

It is interesting that it is possible to derive the injection spectrum $Q(E, t)$ from the X-ray spectrum $I(\epsilon, t)$ for the trap-plus-precipitation model.

The first step is to invert (27) to find $N_{eff}(E, t)$ from $I(\epsilon, t)$. Brown (1971) performed this inversion explicitly for the case where $\sigma(\epsilon, E)$ is given by the Bethe–Heitler formula (29). Brown's (1975) version of the inversion formula gives

$$N_{eff}(E, t) = \frac{4R^2}{K_{BH}\bar{n}} \left(\frac{m}{2}\right)^{1/2} \int_E^\infty d\epsilon \frac{G(\epsilon, t)}{\sqrt{\epsilon - E}}, \tag{33}$$

with

$$G(E, t) = \left(1+3\epsilon\frac{\partial}{\partial\epsilon}+\epsilon^2\frac{\partial^2}{\partial\epsilon^2}\right) I(\epsilon, t). \tag{34}$$

The next step is to invert (28) to find $N(E, t)$ given $N_{\text{eff}}(E, t)$:

$$N(E, t) = N_{\text{eff}}(E, t) - 2E^{5/2} \int_E^\infty dE_0 E_0^{-7/2} N_{\text{eff}}(E_0, t). \tag{35}$$

The final step is to use (1), with $\nu_p = \nu_D = 2\nu_E$ and $\nu_E = \nu_0 E^{-3/2}$, to find $Q(E, t)$ given $N(E, t)$:

$$Q(E, t) = 2\nu_0 E^{-3/2} N(E, t) - \frac{\partial}{\partial E}\{\nu_0 E^{-1/2} N(E, t)\} + \frac{\partial}{\partial t} N(E, t). \tag{36}$$

With (33) and (35), (36) reduces to

$$Q(E, t) = \frac{4R^2}{K_{\text{BH}}\bar{n}} \left(\frac{m}{2}\right)^{1/2} \left[\nu_0 E^{-3/2} \int_E^\infty \frac{d\epsilon}{\sqrt{\epsilon - E}} \left(\tfrac{1}{2} - E\frac{\partial}{\partial \epsilon}\right) G(\epsilon, t) \right.$$
$$\left. + E^{5/2} \int_E^\infty dE_0 E_0^{-7/2} \int_{E_0}^\infty \frac{d\epsilon}{\sqrt{\epsilon - E_0}} \left(\tfrac{1}{2} - E_0 \frac{\partial}{\partial \epsilon}\right) \frac{\partial}{\partial t} G(\epsilon, t)\right], \tag{37}$$

where we omit several intermediate steps involving the E-derivative in (36) and including partial integrations.

Let us consider the particular case of a power law X-ray spectrum

$$I(\epsilon, t) = a(t)\, \epsilon^{-\gamma(t)}. \tag{38}$$

Brown & Hoyng (1975) represented X-ray data in terms of a spectrum of the form (38) in studying the evolution of X-ray events, and consequently the injection spectrum $Q(E, t)$ corresponding to the X-ray spectrum (38) is of practical interest. Substitution of (38) in (34) and thence in (37) gives

$$Q(E, t) = \frac{4R^2}{K_{\text{BH}}\bar{n}} \left(\frac{m}{2}\right)^{1/2} (\gamma - 1)^2 \gamma B(\gamma - \tfrac{1}{2}, \tfrac{1}{2})\, E^{-\gamma+1/2}$$
$$\times \left[a\nu_0 E^{-3/2} + \frac{\dot{a}}{\gamma + 2} - \frac{a\dot{\gamma}}{\gamma + 2} (\ln E + f(\gamma))\right]. \tag{39}$$

Here the time dependences of $a(t)$ and $\gamma(t)$ are left understood and $\dot{a}$ and $\dot{\gamma}$ denote $da(t)/dt$ and $d\gamma(t)/dt$ respectively. $B(x, y)$ is the beta function and $f(\gamma)$ is the function

$$f(\gamma) = \psi(\gamma) - \psi(\gamma - \tfrac{1}{2}) + \frac{1}{\gamma + 2} - \frac{1}{\gamma}, \tag{40}$$

with $\psi(x) = d \ln \Gamma(x)/dx$. To within 10 per cent over the range $3 \lesssim \gamma \lesssim 10$ $f(\gamma)$ may be approximated by $f/(3\gamma)$ and within 2 per cent for $4 \lesssim \gamma \lesssim 8$ by

$$f(\gamma) \approx 1/(3{\cdot}8 + 2{\cdot}4\gamma). \tag{41}$$

The term involving $\ln E$ is unpleasant in that because of it (39) is not obviously independent of the choice of units. However, the offending term can be combined with the preceding term in the form

$$E^{-\gamma}(\dot{a} + a\dot{\gamma} \ln E) = \frac{\partial}{\partial t}(aE^{-\gamma}) \tag{42}$$

whose dimensions are obviously acceptable being those of $\partial I(\epsilon, t)/\partial t$.

The first term on the right-hand side of (39) is proportional to the rate of collisional losses. This term appears because collisional losses tend to harden the energy spectrum at lower energies and this would cause the X-ray spectrum to harden at lower energies. Consequently, in order to obtain a simple power law X-ray spectrum one must offset the collisional losses by injecting an additional soft component at lower energies. The first term may be neglected if the X-ray spectrum changes at a rate much greater than the rate of collisional losses. The next two terms may be combined in the form (42), while the final term is roughly $(\dot{\gamma}/3\gamma)/(\dot{a}/a)$ times these terms and can be neglected for semi-quantitative purposes. Thus for rapid changes ($\dot{a}/a \gg \nu_0 E^{-3/2}$) in the X-ray spectrum one may approximate (39) by

$$Q(E, t) \approx \frac{4R^2}{K_{BH}\bar{n}} F(\gamma) \left(\frac{mE}{2}\right)^{1/2} \frac{\partial}{\partial t} I(E, t) \tag{43}$$

with

$$F(\gamma) = \frac{(\gamma-1)^2 \gamma}{\gamma+2} B(\gamma-\tfrac{1}{2}, \tfrac{1}{2}). \tag{44}$$

Thus the injection spectrum is related directly (although only approximately) to the time rate of change of the X-ray spectrum.

It should be emphasized that 'injection' does not necessarily require the addition of new particles. A relevant example of $Q(E, t) \neq 0$ without the total number of energetic electrons changing would be due to oscillations of the trap. Suppose the energy of individual particles changes at a rate $(dE/dt)_{osc}$ due to oscillations of the trap, e.g. $(dE/dt)_{osc} = (\dot{B}/B)\,E$. Then there would be an effective injection spectrum

$$Q_{osc}(E, t) = -\frac{\partial}{\partial E}\{(dE/dt)_{osc}\, N(E, t)\}. \tag{45}$$

Brown (1973b) and Brown & Hoyng (1975) have interpreted quasi-periodic variations in X-ray spectra in terms of oscillations of the flux tube and we are re-examining their interpretation in terms of the foregoing theory. Thus although rapid variations in the X-ray spectrum in trap-plus-precipitation models is attributed to changes in the injection spectrum, as in a thick-target model, the 'injection' may be due to oscillations and other changes in the magnetic flux tube to which the electrons are confined.

At the same time we hope to apply (39) to the derivation of Q, for particular flares, from ESRO TD1A data (Hoyng *et al.* 1976). This will give some insight into the electron acceleration process and at the same time reveal the limits imposed on our inversion procedure by the finite resolution of the data. In doing so it will be necessary in the first instance to neglect a number of effects on the spectrum (*cf.* Brown 1975). Our use of the isotropic bremsstrahlung cross-section should be a good approximation since the electron distribution in a trap as a whole is very nearly isotropic (McClymont 1976). The effect of photospherically back-scattered photons may not be negligible but (*cf.* Henoux 1975) with an isotropic primary source the main effect would appear to be a constant scaling factor between I and Q, with little effect on the spectral distribution since Compton back-scattering is nearly energy independent at energies $\geqslant$ 20 keV.

5. WAVE–PARTICLE INTERACTIONS

It has been assumed above that wave–particle interactions are unimportant. Wave–particle interactions which should be considered are, firstly, resonant scattering of the trapped electrons (including the loss cone instability) and, secondly, streaming instabilities associated with the precipitating electrons. (We discuss these processes in connection with other-than-trap models elsewhere (Brown & Melrose 1975).)

Two streaming instabilities need to be discussed. The first is the growth of Langmuir waves (the ' two-stream ' instability) which develops when the distribution of electrons has a positive gradient in velocity space along the streaming direction. A positive gradient could arise due to the faster electrons outpacing the slower electrons, but this is not relevant to situations involving steady precipitation. It could also arise due to the hardening of the energy spectrum by 1·5 in spectral index due to the effect of Coulomb interactions, *cf.* (22)–(24), but this effect would produce a positive gradient in velocity space only for $\delta < 1$, which is not the case in practice, and furthermore it is offset due to the precipitation spectrum ($\propto \nu_0 N(E, t)$) being softer by 1·5 in spectral index than the spectrum in the trap. Hence growth of Langmuir waves should not occur.

The other streaming instability is a current instability due to the thermal electrons flowing relative to the ions at faster than the ion sound speed and causing ion sound waves to grow. The flow of the electrons is in a return current which neutralizes the current associated with the precipitating electrons. For the flow speed induced by $\dot{\mathcal{N}}$ electrons per second precipitating over an area A(cm^2) in a plasma with ambient electron number density n_e(cm^{-3}) and temperature T_e(K) to exceed the ion sound speed requires

$$\dot{\mathcal{N}}/A \gtrsim 0{\cdot}9 \times 10^4\, n_e T_e^{1/2}. \tag{46}$$

(It is also necessary for the electrons to be hotter than the ions and this is likely to be the case due to the preferential transfer of energy to thermal electrons rather than ions by the energetic electrons.) It may well be that (46) is marginally satisfied and that the current instability limits the rate of precipitation (i.e. limits $\dot{\mathcal{N}}$) to the value determined by the approximate equality in (46) (Brown & Melrose 1976). The implications of the current instability are not entirely clear and we ignore it here.

Resonant scattering would have a large effect on the model if it were important because any enhanced scattering, as compared with Coulomb interactions, would enhance the rate of precipitation and cause ν_p to have a different energy dependence from that ($\nu_D \propto E^{-3/2}$) assumed in Section 2. Enhanced scattering would probably cause the precipitation to be in the strong diffusion limit (10b). However, resonant scattering of non-relativistic electrons by whistlers involves a threshold condition $v \gtrsim 43 v_A$ (e.g. Kennel & Petschek 1966; Melrose 1974) which is probably not satisfied for the electrons which emit hard X-rays in a trap model.

The condition $v > 43 v_A$ corresponds to

$$E \gtrsim E_{\min} \approx 5{\cdot}2(v_A/10^8 \text{ cm s}^{-1})^2 \text{ keV}. \tag{47}$$

In fact the electron energy needs to be considerably in excess of this threshold in order for whistlers to grow due to a loss cone instability (e.g. Melrose 1974). The value of the Alfvén speed

$$v_A = 2{\cdot}18 \times 10^{11}\, B n_e^{-1/2} \text{ cm s}^{-1} \tag{48}$$

is limited rather severely in a trap model. On the one hand the magnetic pressure $B^2/8\pi$ must be sufficient to confine the energetic electrons. On the other hand n_e

must be low enough for the collisional lifetime of individual electrons to be comparable with the observed duration of a burst. Together these imply a lower limit to v_A. Furthermore quasiperiodic oscillations in X-ray emission over $\approx 10^2$ s (Hoyng *et al.* 1976) imply that the typical size, L, of the source divided by v_A must be of order 10^2 s. It is unlikely that v_A in any trap model could differ by more than a factor of 2 or 3 from the value $v_A = 4{\cdot}2 \times 10^8$ cm s^{-1} deduced for a specific model by Brown & Hoyng (1975). Thus we conclude that resonant scattering is unlikely to be important for the electrons with $E \lesssim 100$ keV involved in the hard X-rays emission.

However, it is interesting that the electrons with $E \gtrsim 100$ keV are important in generating microwave bursts and that resonant scattering is likely to be important for such electrons. In particular, Takakura (1973) suggested that the electron spectrum has a steeper slope above 100 keV and that this can account for part of the existing inconsistency when one attempts to attribute hard X-rays and microwaves to a single distribution of electrons in a trap model. It is tempting to speculate that such a steepening might be associated with the threshold (47) for resonant scattering.

'*Knees*' *in X-ray spectra*

The suggested steepening of the energy spectrum arises from evidence that X-ray spectra tend to steepen at higher energies (Cline, Holt & Hones 1969; Frost 1969; Kane & Anderson 1970; Frost & Dennis 1971). Hoyng *et al.* (1976) found that the steepening occurs typically in the range 60–100 keV and that the spectral index γ increases by from 0·5 to 1·5. There are two ways in which one might hope to account for such 'knees' in terms of the threshold for resonant scattering.

The first is that as a result of the enhanced scattering one expects the electron energy spectrum to steepen at higher energies. Let $E_R(>E_{min})$ be the energy at which resonant scattering becomes important and consider constant injection in the form (21), namely, $Q(E, t) = AE^{-\delta}\theta(t)$, into a trap in which energy losses can be neglected in comparison with precipitation losses so that the evolution of the energy spectrum is described by (15). For strong diffusion (10b) the precipitation rate is proportional to the bounce frequency ν_b which is proportional to the speed of the electron. Hence, if the E-dependence of $\alpha_0{}^2$ is ignored, one has

$$\nu_p = \nu_{p0} E^{1/2} \tag{49}$$

with ν_{p0} roughly independent of E. Then (15) gives

$$\begin{aligned} N(E, t) &= \frac{AE^{-\delta}}{\nu_{p0}E^{1/2}}(1 - \exp[-\nu_{p0}E^{1/2}t]) \\ &\approx \begin{cases} AE^{-\delta}t & \text{for} \quad E \ll (\nu_{p0}t)^{-2} \\ \dfrac{A}{\nu_{p0}} E^{-(\delta+1/2)} & \text{for} \quad E \gg (\nu_{p0}t)^{-2} \end{cases} \end{aligned} \tag{50}$$

where we assumed $(\nu_{p0}t)^{-2} > E_R$. Thus the energy spectrum steepens by 0·5 in spectral index at higher energies.

However, the precipitating electrons dominate in the X-ray emission in this case and it does not follow that the X-ray spectrum also steepens. In fact the total

X-ray spectrum can be written in the form (27) with the effective energy spectrum

$$N_{\rm eff}(E, t) = N(E, t) + \frac{\nu_{\rm p0}}{\nu_0} E^{1/2} \int_E^\infty dE_0 E_0^{1/2} N(E_0, t), \qquad (51)$$

which is dominated by (the final term) the precipitating electrons. In fact $N_{\rm eff}$ is harder than $N(E, t)$ by two in spectral index and hence the X-ray spectrum should not steepen by 0·5 but should rather flatten by $2-0{\cdot}5 = 1{\cdot}5$. Thus the steepening of the electron energy spectrum due to the enhanced scattering cannot explain the observations in a model with continuous injection.

The other idea is that the enhanced scattering at higher energies causes the electron spectrum to be steeper at higher energies from the outset, i.e. the injection spectrum has a knee. (It is not acceptable to assume that resonant scattering preferentially precipitates out the higher energy electrons initially because these electrons would then produce an initially very hard X-ray component for which there is no evidence.) Resonant scattering could impede the escape from the acceleration region into the trap. However, this idea is not amenable to a quantitative treatment because of our lack of knowledge of the acceleration mechanism and of the injection process.

It may be concluded that resonant scattering of trapped electrons is unlikely to be important for electrons with energy $\lesssim 100$ keV, and although it appears reasonable that the energy at which resonant scattering becomes important might correspond to the observed knee in many X-ray spectra, there is no obvious mechanism which would allow enhanced scattering to cause such a knee.

6. DISCUSSION

The foregoing discussion may be summarized as follows:

(1) Precipitation is necessarily important in any trap model. This is because the rate of precipitation due to (Coulomb) scattering into the loss cone is just twice the rate of collisional energy losses, which is also proportional to the rate of emission of (X-ray) photons through bremsstrahlung. The X-ray emission from the precipitating electrons is similar in intensity, spectral shape and time profile to that from the trapped electrons, provided that the scattering is due to Coulomb interactions.

(2) Resonant scattering probably is not important for the electrons ($E \lesssim 100$ keV) which emit most of the observed X-rays, and probably is important for the higher energy electrons involved in the emission of microwaves. The energy at which resonant scattering is likely to become important plausibly corresponds to the energy at which X-ray spectra have a ‘knee’ (the spectra tend to be convex, steepening at energies between 60 and 100 keV). However, the mechanism by which resonant scattering could account for the observed spectra has not been identified. (Resonant scattering should cause a preferential precipitation of higher energy electrons which would produce an X-ray spectrum which flattens at higher energies, i.e. a concave spectrum.)

(3) The evolution of the electron energy spectrum and of the X-ray spectrum may be treated analytically in the trap-plus-precipitation model, and it is possible in principle to determine the electron injection spectrum (as a function of energy and time) from a knowledge of the energy and time dependences of the X-ray spectrum.

(4) Rapid variations in the X-ray spectrum (at faster than the collisional energy loss rate) imply an ' injection ' energy spectrum (into the trap) which is proportional to the time rate of change of the X-ray spectrum (*cf.* (43)). This situation is similar to thick-target (and thin-target) models where time variations must be attributed to variations in the injection spectrum. However, the X-ray spectrum for a given injection spectrum is different in the trap-plus-precipitation and thick-target models, and more importantly, the ' injection ' in the trap-plus-precipitation model may involve the effects of pulsations in the trap (*cf.* (45)), rather than injection of new particles.

One point which has not been discussed is the assumption that scattering due to Coulomb interactions in the trap leads to weak diffusion (10a) rather than strong diffusion (10b). By way of illustration, for the parameters in the detailed trap model of Brown & Hoyng (1975) we find

$$\nu_D \approx 2\times 10^{-3}\,(E/25\text{ keV})^{-3/2}\,\text{s}^{-1},\quad \tfrac{1}{2}\alpha_0{}^2\nu_b \approx 0{\cdot}1\,\alpha_0{}^2\,(E/25\text{ keV})^{1/2}\,\text{s}^{-1}.$$

Weak diffusion requires $\nu_D \ll \frac{1}{2}\alpha_0{}^2\nu_b$ and with a reasonable choice of $\alpha_0{}^2$ (in the range $10^{-2} \gtrsim \alpha_0{}^2 \lesssim 10^{-1}$ say, *cf.* (9)) one concludes that for the lower energy electrons the weak diffusion limit could be only marginally satisfied if at all. The qualitative effect of strong diffusion dominating at low energies would be that, in Section 2, we overestimate the number of precipitating electrons at low energy and hence overestimate the X-ray spectrum at low energies. However, our estimate of the X-ray intensity would be too large by a factor of no more than 2 at energies where strong diffusion dominates.

Finally it is worth emphasizing that the necessary inclusion of precipitation (and the associated X-ray emission) in a trap model lead to a trap-plus-precipitation model which incorporates most of the attractions of other models. These include:

(a) As in a thick-target model the precipitating electrons could heat the denser layers (Hudson 1972; Brown 1973a; Syrovatskii & Shmeleva 1973) and so account for flare-associated soft X-ray *EUV* and optical emissions. Roughly two-thirds of the energy of electrons injected in the trap ends up in direct heating of the denser layers (to see this consider the integral

$$\int_0^\infty dt \int dEE$$

of (1) with $\nu_D = 2\nu_E$), and so ample energy is precipitated.

(b) As in thick- and thin-target models the directed motion of the precipitating electrons should produce polarized X-rays (Tindo *et al.* 1973). (We might remark that in the strong diffusion limit there is little directed motion and little polarization would be expected in energy ranges where this occurs.)

(c) Unlike a thick-target model, the trap-plus-precipitation model (and thin-target models) are compatible with the inference from behind-the-limb events that the X-rays source are relatively high ($\geqslant 10^4$ km) in the corona (Datlowe 1975), but at the same time allows chromospheric dumping.

(d) Also unlike a thick-target model, the acceleration energy spectrum required to produce the observed X-ray spectrum in a trap-plus-precipitation model is compatible with the energy spectrum of electrons observed *in situ* at the orbit of the Earth. It should be emphasized that the X-ray spectrum from collisionally precipitating electrons in a trap-plus-precipitation model is not the same as that in a thick-target model with direct injection contrary to the conclusion implied by

Hudson (1972). Hudson pointed out that 'the thick-target process requires an electron spectrum steeper by a factor $\approx E^{-1\cdot 5}$' than for a thin target. However, in the trap-plus-precipitation model the number of precipitating electrons per unit energy and per unit time is $\nu_p N(E, t)$ which is in fact just a factor $\approx E^{-1\cdot 5}$ steeper than $N(E, t)$ for $\nu_p = \nu_D = 2\nu_0 E^{-3/2}$. (Note also that the thick-target spectrum referred to is a flux spectrum while the trap spectrum is a number spectrum. This is unclear in Hudson's conclusion.)

(e) As in any trap model, an explanation for quasi-periodic variations (e.g. Hoyng *et al.* 1976) and other changes in the X-ray spectrum can be sought in terms of pulsations and other changes in the trap, e.g. Brown & Hoyng (1975), and in a trap-plus-precipitation model there is the additional possibility of modulating the X-ray emission by modulating the precipitation. Indeed the initial motivation for this investigation was to seek an explanation for the initial (two-staged) phase in the 1972 August 4 event (Brown & Hoyng 1975) in terms of the onset of precipitation during the injection. However, although the trap-plus-precipitation model offers a variety of possibilities, we have not been able to find a convincing explanation for the 'dog-leg' portion in Brown & Hoyng's $\mathscr{F}$–γ diagram.

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