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Precise determination of the f(0)(600) and f(0)(980) pole parameters from a dispersive data analysis — Source link

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Enhanced non-quark-antiquark and non-glueball N_c behavior of light scalar mesons

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We show that the latest and very precise dispersive data analyses require a large and very unnatural fine-tuning of the $1/N_c$ expansion at $N_c = 3$ if the $f_0(600)$ and $K(800)$ light scalar mesons are to be considered predominantly $\bar{q}q$ states, which is not needed for light vector mesons. For this, we use scattering observables whose $1/N_c$ corrections are suppressed further than one power of $1/N_c$ for $\bar{q}q$ or glueball states, thus enhancing contributions of other nature. This is achieved without using unitarized ChPT, but if it is used we can also show that it is not just that the coefficients of the $1/N_c$ expansion are unnatural, but that the expansion itself does not even follow the expected $1/N_c$ scaling of a glueball or a $\bar{q}q$ meson.

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Light scalar resonances play a relevant role for several fields of Physics: For the nucleon-nucleon interaction, because they are largely responsible for the attractive part [1] (with cosmological and anthropic implications). For the QCD nonabelian nature, because some of these resonances have the quantum numbers of the lightest glueball, also common to the vacuum and hence of relevance for the spontaneous chiral symmetry breaking. Moreover, they are also of interest for the saturation [2] of the low energy constants of Chiral Perturbation Theory (ChPT) [3]. However, the precise properties of these mesons, their nature, spectroscopic classification, and even their existence—as for the $K(800)$ or κ—are still the object of an intense debate. In particular, different models [4] suggest that they may not be ordinary quark-antiquark mesons, but tetraquarks, meson molecules, glueballs, or a complicated mixture of all these. The problem, of course, is that we do not know how to solve QCD at low energies.

However, since the QCD $1/N_c$ expansion is applicable at all energies, and the mass and width N_c dependence of $\bar{q}q$ mesons and glueballs is well known [5], the N_c scaling of resonances becomes a powerful tool to classify them and understand their nature. In [6,7], some of us studied the mass and width behavior of light resonances using ChPT—which is the QCD low-energy effective Lagrangian—and unitarization with a dispersion relation. It was found that the poles of the $\rho(770)$ and $K^*(982)$ vectors behave predominantly as expected for $\bar{q}q$ states, whereas those of the $f_0(600)$, also called σ, and $K(800)$ scalars, do not [6]. Still, a possible subdominant $\bar{q}q$ component for the $f_0(600)$ may arise naturally at two loops [7] within ChPT (less so at one loop), but with a mass around 1 GeV or more.

Of course, all these conclusions rely on unitarized ChPT and the assumption that corrections, suppressed just by $1/N_c$, are of natural size. Since $N_c = 3$ in real life, this may not seem as a large suppression, even more when the meaning of ‘‘natural size’’ may not be clear for dimensional parameters. For that reason, unitarized ChPT was useful to change N_c, and reveal the $1/N_c$ scaling, no matter how unnatural the coefficients may appear.

Here, we will provide adimensional observables with corrections suppressed further than $1/N_c$, that can also be applied directly to real data at $N_c = 3$, without the need to extrapolate to larger N_c using unitarized ChPT.

In particular, resonances appearing in elastic two-body scattering are commonly identified by three criteria. The N_c behavior of one of these criteria—the associated pole in the unphysical sheet—was already studied in [6,7]. A second possibility is to define the mass as the energy where the phase shift reaches $\pi/2$, which both for $\pi\pi$ or πK scattering occurs relatively far from the $f_0(600)$ and $K(800)$ pole positions. This criterion was studied in [8] for the $f_0(600)$ with a relatively inconclusive result about its assumed $\bar{q}q$ behavior. A more reliable parametrization and better data were called for and we will provide them here together with more conclusive results. Third, the phase increases very fast in the resonance region and the mass can be identified with the maximum of the phase derivative. All three criteria roughly coincide for narrow resonances, but the most physical definition is the latest, since it identifies the resonance as a metastable state whose lifetime is the inverse of the width. Note that this is the less evident feature both for the $f_0(600)$ and $K(800)$ and thus the phase derivative will become our preferred observable to test their N_c dependence.

Let us then recall that partial waves generically scale as $1/N_c$, except at the resonance mass m_R. Actually, it has been found [8] that if a resonance pole at $s_R = m_R^2 - i m_R \Gamma_R$ behaves as a $\bar{q}q$ [5], i.e. $m_R \sim O(1)$ and $\Gamma_R \sim O(1/N_c)$, then the phase shift satisfies [9]:

$$\delta(m_R^2) = \frac{\pi}{2} - \underbrace{\frac{\mathrm{Re}\,t^{-1}}{\sigma}\bigg|_{m_R^2}}_{O(N_c^{-1})} + O(N_c^{-3}), \qquad (1)$$

$$\delta'(m_R^2) = -\underbrace{\frac{(\mathrm{Re}\,t^{-1})'}{\sigma}\bigg|_{m_R^2}}_{O(N_c)} + O(N_c^{-1}), \qquad (2)$$

where $t(s)$ is the partial wave, $\sigma = 2k/\sqrt{s}$, and k is the meson center of mass momentum. Derivatives are taken with respect to s. The $1/N_c$ counting of the different terms in the equations above comes from the following expansions at $s = m_R^2$ [10]:

$$\mathrm{Re}\, t^{-1} = m_R\Gamma_R\left[\frac{m_R\Gamma_R}{2}(\mathrm{Re}t^{-1})'' - \sigma'\right] + O(N_c^{-3}), \quad (3)$$

$$m_R\Gamma_R = \frac{\sigma}{(\mathrm{Re}t^{-1})'} + O(N_c^{-3}). \quad (4)$$

In brief, the corrections in Eqs. (1)–(4) are suppressed by a further $1/N_c^2$ power due to an expansion on the imaginary part of the pole, which scales like $\Gamma \sim 1/N_c$. As nicely shown in [8], by expanding separately the real and imaginary parts of t^{-1}, only the $1/N_c^{2n+1}$ powers are kept on each expansion, leading to Eqs. (3) and (4).

Since we are interested in adimensional observables whose corrections are suppressed further than just $1/N_c$, we can recast Eqs. (1) and (2) as follows:

$$\left.\frac{\frac{\pi}{2} - \mathrm{Re}t^{-1}/\sigma}{\delta}\right|_{m_R^2} \equiv \Delta_1 = 1 + \frac{a}{N_c^3}, \quad (5)$$

$$\left.-\frac{[\mathrm{Re}t^{-1}]'}{\delta'\sigma}\right|_{m_R^2} \equiv \Delta_2 = 1 + \frac{b}{N_c^2}. \quad (6)$$

Note that we have normalized each equation and extracted the leading $1/N_c$ dependence so that the coefficients a and b should naturally be $O(1)$ or less. It is relatively simple to make a and b much smaller than 1 with cancellations with natural higher order $1/N_c$ contributions, but very unnatural to make them much larger.

Now, in Table I, we show the resulting a and b for the lightest resonances found in $\pi\pi$ and πK elastic scattering. Before describing in detail the calculations, let us observe that *for the* $\rho(770)$ *and* $K^*(892)$ *vector resonances all parameters are of order one or less, as expected for* $\bar{q}q$ *states*. In contrast, *for the* $f_0(600)$ *and* $K(800)$ *scalar resonances we find that all parameters are larger, by two orders of magnitude, than expected for* $\bar{q}q$ *states*. This is one of the main results of this work, and makes the $\bar{q}q$ interpretation of both scalars extremely unnatural.

Let us now describe in detail our calculations and their different degree of precision and reliability. As commented above, the $f_0(600)$ "Breit-Wigner" mass was already studied [8] using Eq. (1), but no conclusion was reached

TABLE I. Normalized coefficients of the $1/N_c$ expansion for different resonances. For $\bar{q}q$ resonances, all them are expected to be of order one or less.

	$\rho(770)$	$K^*(892)$	$f_0(600)$	$K(800)$
a	-0.06 ± 0.01	0.02	-252^{+119}_{-156}	-2527
b	$0.37^{+0.04}_{-0.05}$	0.16	77^{+28}_{-24}	162

on whether the deviations were consistent with the $1/N_c$ suppression or not. This was partly attributed to the limited reliability of the conformal parametrization or unitarized ChPT, whose phase never reaches $\pi/2$, used in [8]. To overcome this caveat, we are now using the recent, very precise and reliable *output* of the data analysis in [11], constrained to satisfy once-subtracted coupled dispersion relations—or GKPY equations—as well as Roy equations, which is therefore model-independent and specially suited to obtain the $f_0(600)$ pole [12]. Note that this analysis incorporates the very recent and reliable data on K_{l4} decays from NA48/2 [13], which is a key factor in attaining high levels of precision. The analysis in [11,12] is also in good agreement with previous dispersive results based on standard Roy equations [24]. We have followed the same rigorous approach for the $\rho(770)$, although, being so narrow, the conformal unconstrained data analysis and the Inverse Amplitude Method (IAM) yield very similar results. The uncertainties we quote for both the $f_0(600)$ and $\rho(770)$ cover the uncertainties in the output of the dispersive representation.

In this work, we also deal with strange resonances in πK scattering. For the scalar $K(800)$, we have also used a rigorous dispersive calculation, namely, that in [14], which uses Roy-Steiner equations to determine the isospin 1/2 scalar channel of πK scattering, although this time, we can only provide a central value. Note, however, that the value of m_R^2 obtained in that analysis is located below threshold, so that the phase shift is ill defined at m_R^2. Nevertheless, we have been using the m_R mass definition to allow for an easier comparison with [8], but the definition $\sqrt{s_R} = m - i\Gamma/2$ is equally valid and is actually the standard choice used in the context of scalar mesons. Moreover, the N_c scaling of Eqs. (1) and (2) does not change if we evaluate the quantities at $s = m^2$, instead of m_R^2, since m^2 differs from m_R^2 in $\Gamma^2/4$, which is $O(N_c^{-2})$. Thus, the values for the $K(800)$ in Table I correspond to this choice. For the vector $K^*(892)$, there are no very precise purely dispersive descriptions of the existing data and we therefore rely on a single partial wave dispersion relation and $SU(3)$ ChPT to one-loop to determine its subtraction constants (this is known as ChPT unitarized with the single-channel IAM [15]), which we will briefly explain in the next section. We have applied the same method to the $\rho(770)$ and the results lie within 50% of their central value when using the GKPY dispersive representation. Since the $K^*(892)$ is narrower than the $\rho(770)$, the IAM is likely to provide a better approximation than in the $\rho(770)$ case, but even with that 50% uncertainty, it is enough to check that the a and b parameters are smaller than 1.

There is, of course, another way of interpreting our results, which is that due to the large $1/N_c$ coefficients of the $f_0(600)$ the series simply does not converge. In particular, Eq. (1), which was thoroughly considered in [8], is obtained as an expansion of $\arctan(x) = x - x^3/3\ldots$. In this way, we could explain why the $a = -0.06 \pm 0.01$ coefficient is so small for the $\rho(770)$: it is simply the effect of calculating $a = \tilde{a}^3/3$ with $\tilde{a} = 0.56^{+0.03}_{-0.04}$, which is now naturally of $O(1)$. We could try the same procedure for the $f_0(600)$, assuming its series expansion is that of a $\bar{q}q$, to find $\tilde{a} = 9.1$, still rather unnatural, but of course, this value makes no sense since the whole series would not be converging and terms higher than $1/N_c^3$ would become dominant.

This is one of the reasons why despite being only suppressed by $1/N_c^2$ instead of $1/N_c^3$, we also provide the expansion in Eq. (6) obtained from the derivative of the amplitude. In this case, the b/N_c^2 term is not the square of a natural $1/N_c$ quantity, i.e.,

$$\frac{b}{N_c^2} = \frac{\mathrm{Re}t^{-1}}{\sigma}\left[\frac{\sigma'}{(\mathrm{Re}t^{-1})'} - \frac{\mathrm{Re}t^{-1}}{\sigma}\right] + O(N_c^{-4}). \qquad (7)$$

Despite containing a cancellation between two $1/N_c$ terms, its value for the $\rho(770)$ is rather natural. However, once again, the value for the scalars is almost two orders of magnitude larger than expected.

In the previous analysis, it is very relevant that the width of the resonance is suppressed with additional $1/N_c$ powers with respect to the mass. Actually, it is rather straightforward to extend the formalism to study the assumption that the $f_0(600)$ could be predominantly a glueball, since then $m_R \sim O(1)$ and $\Gamma_R \sim O(1/N_c^2)$ [5,16]. As a consequence, for the glueball case, the scaling of Eqs. (3) and (4) changes and so does that of $\delta(m_R^2)$ and $\delta'(m_R^2)$:

$$\delta(m_R^2) = \frac{\pi}{2} - \underbrace{\frac{\mathrm{Re}t^{-1}}{\sigma}\bigg|_{m_R^2}}_{O(N_c^{-2})} + O(N_c^{-6}), \qquad (8)$$

$$\delta'(m_R^2) = \underbrace{\frac{(\mathrm{Re}t^{-1})'}{\sigma}\bigg|_{m_R^2}}_{O(N_c^{2})} + O(N_c^{-2}). \qquad (9)$$

Much as it was done in Eqs. (5) and (6), in order to make explicit this further N_c suppression, we can define some new parameters a' and b' that should be of $O(1)$ if the resonance was a glueball:

$$\Delta_1 = 1 + \frac{a'}{N_c^6}, \qquad \Delta_2 = 1 + \frac{b'}{N_c^4}. \qquad (10)$$

Following the same procedure as before, we obtain for the $f_0(600)$, $a' = -6800^{+3200}_{-4200}$ and $b' = 2080^{+760}_{-650}$. In other words, a very dominant or pure glueball nature for the $f_0(600)$ is very disfavored by the $1/N_c$ expansion, even more than the $\bar{q}q$ interpretation. This is because it would require even more unnatural coefficients, this time too large by three to four orders of magnitude.

Of course, as we did for the $\bar{q}q$ case, we could worry about the fact that, due to the $\arctan(x) = x - x^3/3 + \ldots$ expansion, the a' should have been interpreted as $a' = \tilde{a}'^3/3$. But even with that interpretation, we would still find $\tilde{a}' = 27^{+5}_{-7}$, again rather unnatural. Once more, and as happened in the $\bar{q}q$ case, the b' parameter does not correspond to the fourth power of any natural quantity, so that its value is genuinely unnatural, disfavoring the glueball interpretation.

Let us remark that in the case of tetraquarks or molecules, the width is not expected to be suppressed with additional $1/N_c$ powers with respect to the mass of the resonance [16,17]. Thus, our previous formalism does not apply. Furthermore, it is most likely that scalars are a mixture of different components. Therefore, our results, while showing that neither the $\bar{q}q$ or a glueball are favored as dominant components of light scalars, do not exclude that these structures could be mixed with other components that would dominate the $1/N_c$ expansion with a different N_c behavior [18].

In summary, we have shown that if, for the light scalar mesons, we study $\bar{q}q$ or glueball $1/N_c$ expansions as those in Eqs. (5), (6), and (10), their coefficients come out very unnatural, suggesting that these resonances cannot be described as *predominantly* made of a quark and an antiquark or a glueball. Note that, contrary to our previous works [6,7], this conclusion has been reached *from dispersive analyses of data*, without extrapolating to $N_c \neq 3$ using unitarized ChPT.

However, unitarized ChPT will be used next to calculate the $\Delta_i - 1$ observables, in order to show that, for scalars, what really happens is that they do not even follow the $1/N_c$ expansion of $\bar{q}q$ or glueball states given in Eqs. (5), (6), and (10), thus explaining the need for unnatural coefficients if a $\bar{q}q$ or glueball-like expansion is assumed.

A. **The Inverse Amplitude Method:** The elastic IAM [15] uses ChPT to evaluate the subtraction constants and the left cut of a dispersion relation for the inverse of the partial wave. The elastic right cut is exact, since the elastic unitarity condition, $\mathrm{Im}t = \sigma|t|^2$, fixes $\mathrm{Im}t^{-1} = -\sigma$. Note that the IAM is derived only from elastic unitarity, analyticity in the form of a dispersion relation, and ChPT, which is only used at low energies. It satisfies exact elastic unitarity and reproduces meson-meson scattering data up to energies $\sim$1 GeV. It can be analytically continued into the second Riemann sheet where poles associated to resonances are found. In particular, we find the $\rho(770)$ and $f_0(600)$, as well as the $K^*(892)$ and the $K(800)$ resonances as poles in $\pi\pi$ and πK scattering amplitudes, respectively.

The dependence on the QCD number of colors, N_c, is implemented [6,7] through the leading N_c scaling of the ChPT low energy constants (LECs), which is model-independent [3,7,19]. Fortunately, for Eqs. (5) and (6) to hold, only the leading $1/N_c$ behavior is needed. Note also that the IAM does not have any other parameters where uncontrolled N_c-dependence could hide—as it happens in other unitarization methods—so that the IAM allows us to check the scaling of the $\Delta_i - 1$ in Eqs. (5) and (6).

The SU(2) IAM: Only the nonstrange $f_0(600)$ and $\rho(770)$ resonances can be checked, but we can do it by unitarizing with the IAM the corresponding partial waves either to one or two loops. We simply scale $f_{N_c} \to f\sqrt{N_c/3}$, the one-loop constants, as $l^r_{i,N_c} \to l^r_i N_c/3$ and the two-loop ones as $r_{i,N_c} \to r_i(N_c/3)^2$.

Thus, in the two first columns of Fig. 1, we show, for the $\rho(770)$ and $f_0(600)$ resonances, the scaling of the $\Delta_i - 1$ both to one loop (upper panels) and two loops (lower panels). Note that we have normalized them to their $N_c = 3$ value, in order to cancel the leading part of the a and b coefficients and thus extract the leading $1/N_c^k$ behavior of Eqs. (5) and (6). For the one-loop calculations, we use the set of LECs in [20], whereas for the two-loop calculation, we use the fit D from [20,21]. We have checked that similar results are obtained when using other sets of LECs in these references or the estimates from resonance saturation [2].

We can observe that the scaling for the $\rho(770)$ observables overlaps with the expectation for the leading behavior of $\bar{q}q$ states. However, in the case of the $f_0(600)$, the scaling is completely different. To one loop, the $f_0(600)$ observables grow instead of decreasing. Let us note, however, that for N_c larger than $\sim$10, the $f_0(600)$ pole lies on the third quadrant of the complex plane. Before that happens, the value of m_R^2 becomes less than $4m_\pi^2$ and the phase shift has no physical meaning so that Eqs. (5) and (6) do not hold. This behavior does not occur to two loops. Actually, we find again the $f_0(600)$ behavior already observed in [7], where, for N_c close to 3, the width grows as in the one-loop case (and so do the observables here), but for larger N_c, the $f_0(600)$ starts behaving more as a $\bar{q}q$. Note that this $\bar{q}q$ behavior appears at a mass somewhat bigger than 1 GeV. This was a hint of the $f_0(600)$ being a mixture of a predominantly non-$\bar{q}q$ component and, at least, a subdominant $\bar{q}q$ component with a mass much heavier that the physical one, which is the one that survives at large N_c. In terms of the $\Delta_i - 1$ observables defined here, this translates into a growth close to $N_c = 3$ and a decrease at larger N_c. Therefore, it is not only that the a and b coefficients of the $f_0(600)$ are too large as shown in the previous section, but that the scaling itself does not correspond to a $\bar{q}q$ state (and even less so to a glueball). To two-loops, the $\rho(770)$ does not follow exactly the leading behavior of $\bar{q}q$ states but decreases slightly faster, which can be naturally explained due to subleading effects or to a possible small pion cloud contribution.

The SU(3) IAM: Now we can study the scaling of $\Delta_i - 1$ not only for the $\rho(770)$ and $f_0(600)$, but also for the $K^*(892)$ and $K(800)$ resonances, although in this case, the elastic unitarized amplitudes are available only to one loop [22,23]. We have now eight LECs, called $L_i(\mu)$, that scale [3,19] as $L_{i,N_c} \to L_i(N_c/3)$ for $i = 2, 3, 5, 8$, while $2L_1 - L_2$, L_4, L_6 and L_7 do not change with N_c.

In the third and fourth columns of Fig. 1, we show the results found using the set of LECs called Fit II in [22]. Similar results are obtained with Fit I or the estimates from resonance saturation in [2]. In the upper panels, we simply

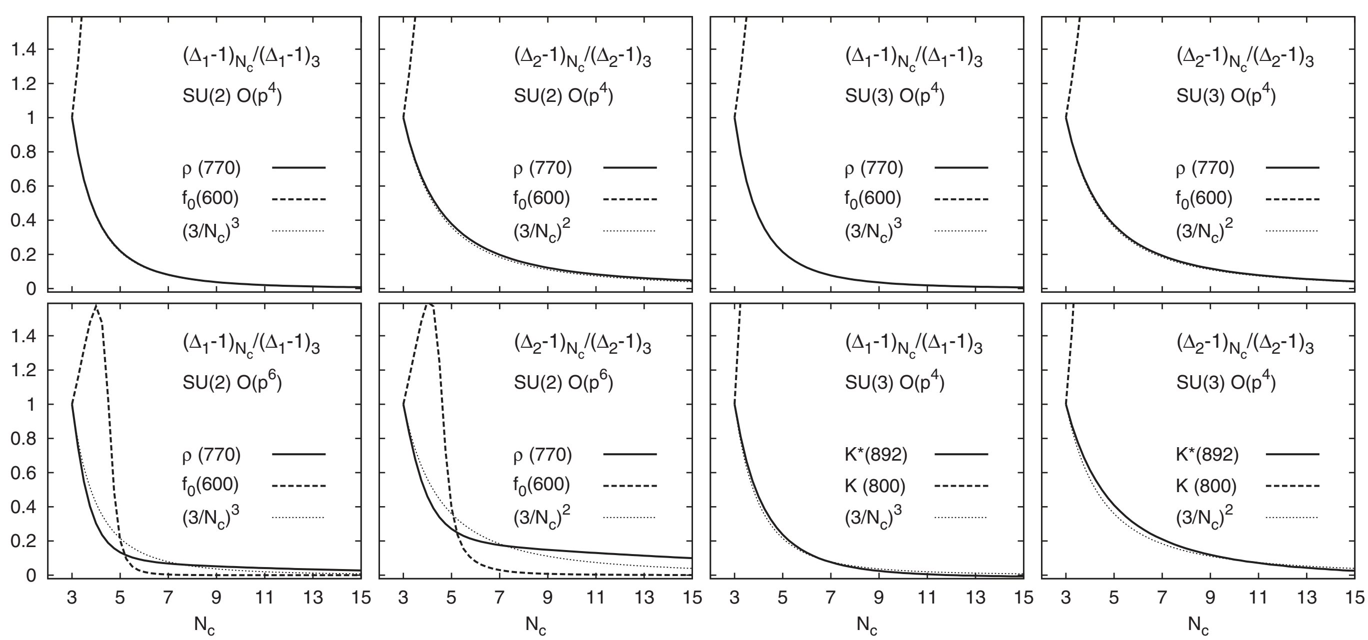

FIG. 1. $1/N_c$ scaling of the $\Delta_i - 1$ observables normalized to their $N_c = 3$ value for light scalar and vector mesons, using unitarized ChPT within $SU(2)$ or $SU(3)$ and to one or two loops, $O(p^4)$ and $O(p^6)$, respectively.

reobtain within the $SU(3)$ formalism the same results we obtained for the $\rho(770)$ and $f_0(600)$ within the $SU(2)$ formalism to one loop. In the lower panels, we show the results for the light vector $K^*(892)$, following nicely the $\bar{q}q$ expectations, as well as the results for the scalar $K(800)$, which has a very similar behavior to the $f_0(600)$, at odds with a dominant $\bar{q}q$ or glueball nature.

Summary: In this work, we have studied the $1/N_c$ expansion of the meson-meson scattering phase-shifts around the pole mass of a $\bar{q}q$ or glueball resonance. In particular, we have defined observables whose corrections are suppressed further than just one power of N_c, paying particular attention to the derivative of the phase, which provides a physical and intuitive definition of a resonance. By using recent and very precise dispersive data analyses, we have shown that if we assume a $\bar{q}q$ or glueball behavior for the $f_0(600)$ and $K(800)$, the coefficients of the expansion of such observables turn out unnaturally large. This is shown without using ChPT or extrapolating beyond $N_c = 3$. Moreover, when using unitarized ChPT, we have shown that it is the very $1/N_c$ scaling of the observables which does not follow the pattern of the $1/N_c$ expansion expected for $\bar{q}q$ or glueball states.

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