Precision Machine Design

Lecture 2 : Error Analysis

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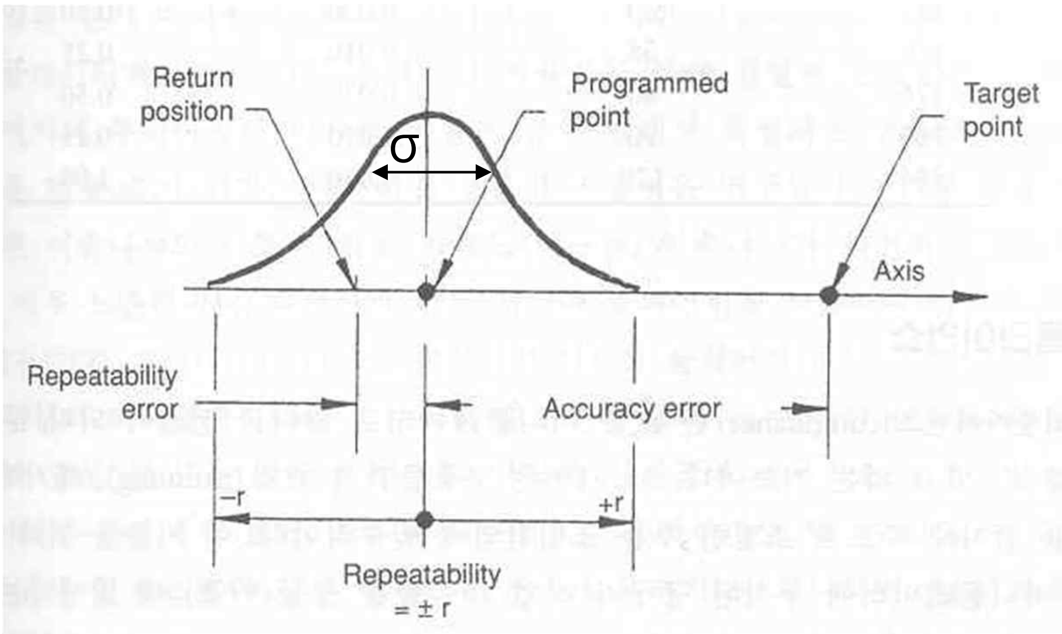
References : 1. Dae Gab Gweon, Precision Machine Design, KAIST
2. Alexander H. Slocum, Precision Machine Design, MIT

1. Definition of Error

- Systematic error (=) : repeatable, compensable
- Random error (=) : corrected by feedback control

σ : 표준 편차(Standard deviation)



2. Error Types

1) Geometric errors
2) Kinematic errors
3) External load induced errors
4) Thermal errors
5) Dynamic errors
6) Modeling errors
7) Instrumentation errors
8) Computational errors
9) Additional source of errors
10) Variations in supply systems

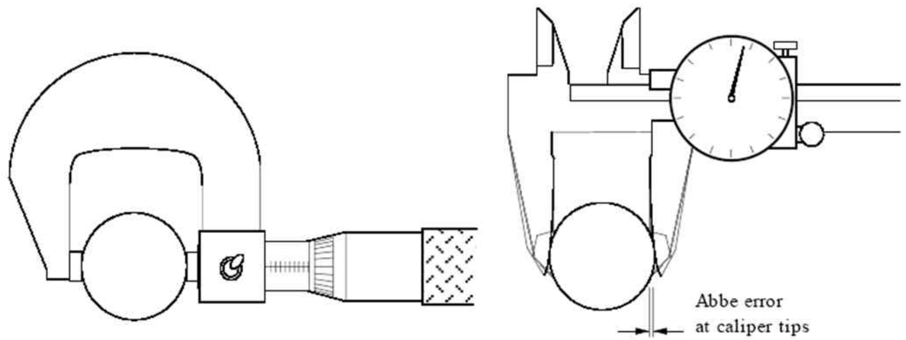
(1) Geometric Errors

A. Errors in the form of

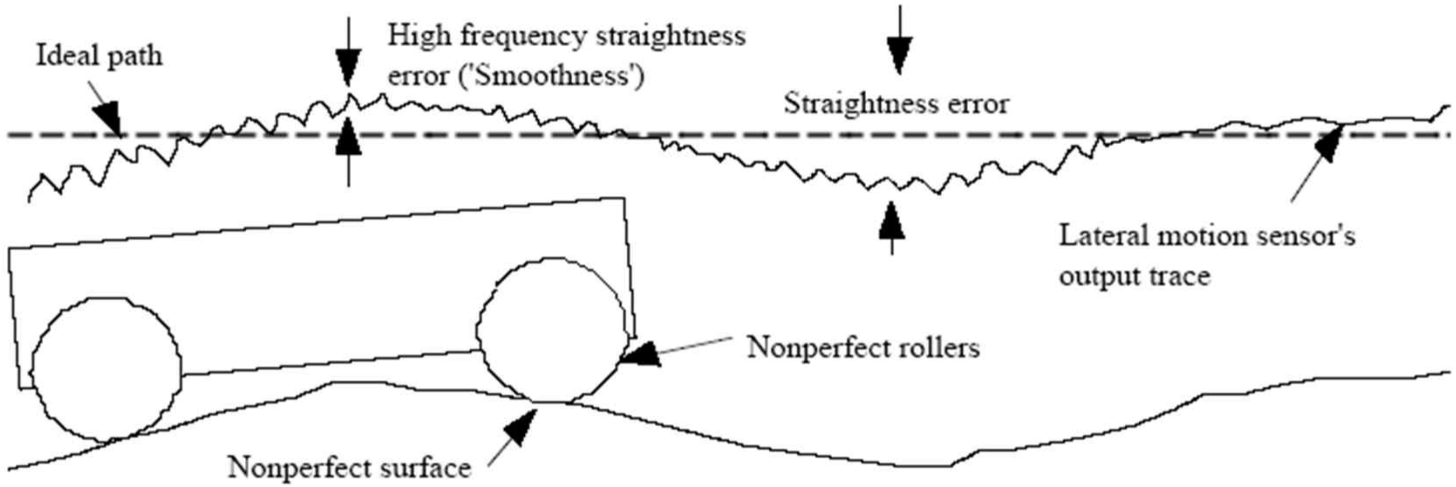
- ✓ Component straightness error due to machining errors.
- ✓ Component straightness error due to gravity loading.

Ex.)

Abbe error

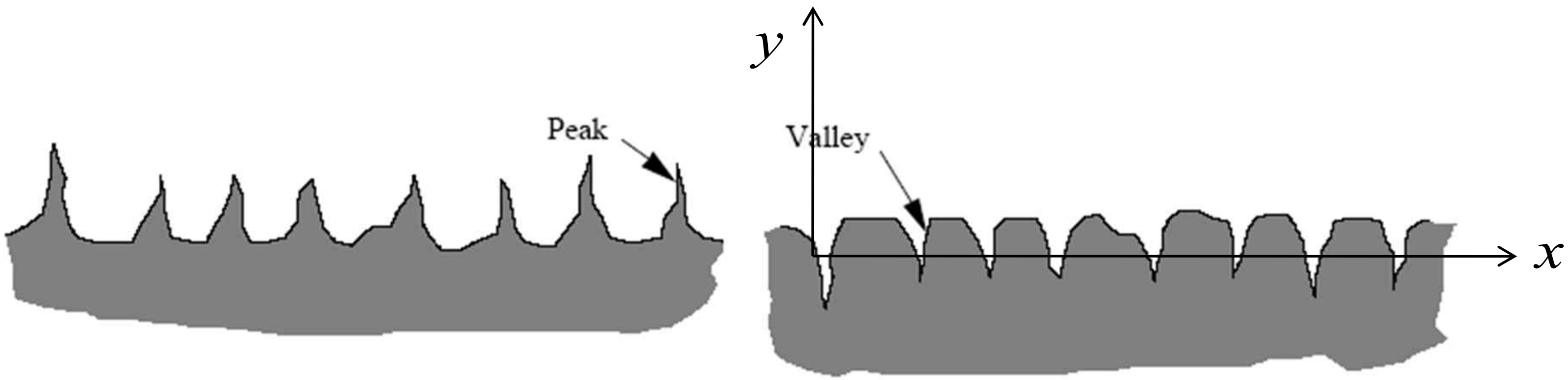


Surface straightness error



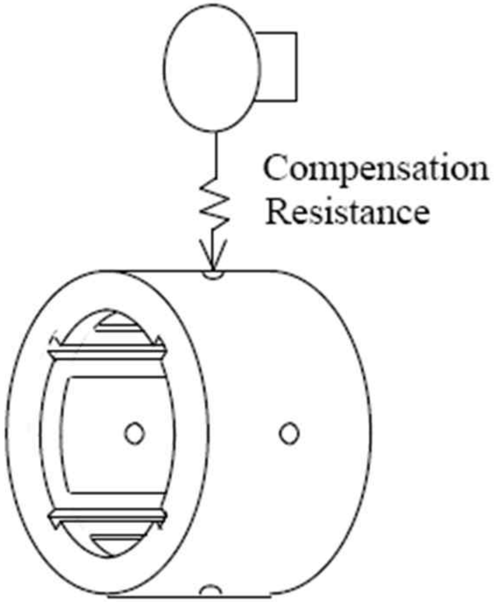
B. Surface finish effects:

- ✓ The surface finish of a bearing not only affects life, it also can affect accuracy.
- ✓ Surfaces with positive (left) and negative (right) skewness.
- ✓ Roughness parameter, R_a (average roughness):

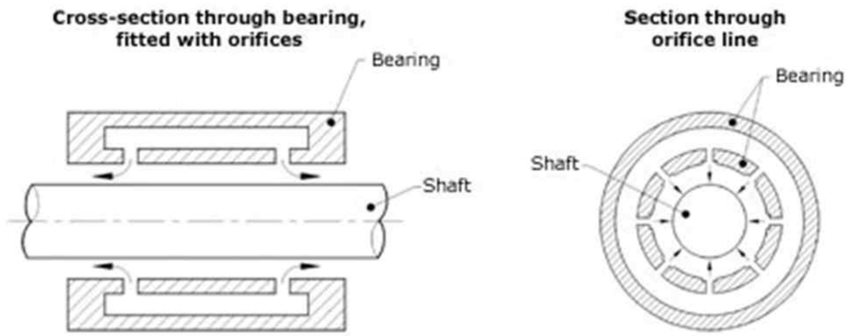


Positive skewness Negative skewness

- ✓ Hydrostatic and aerostatic(gas) bearings are insensitive to surface finish effects.
- ✓ Surface finish should be at least [   ] smaller (e.g., 1 μm) than the bearing clearance (e.g., 10 μm).



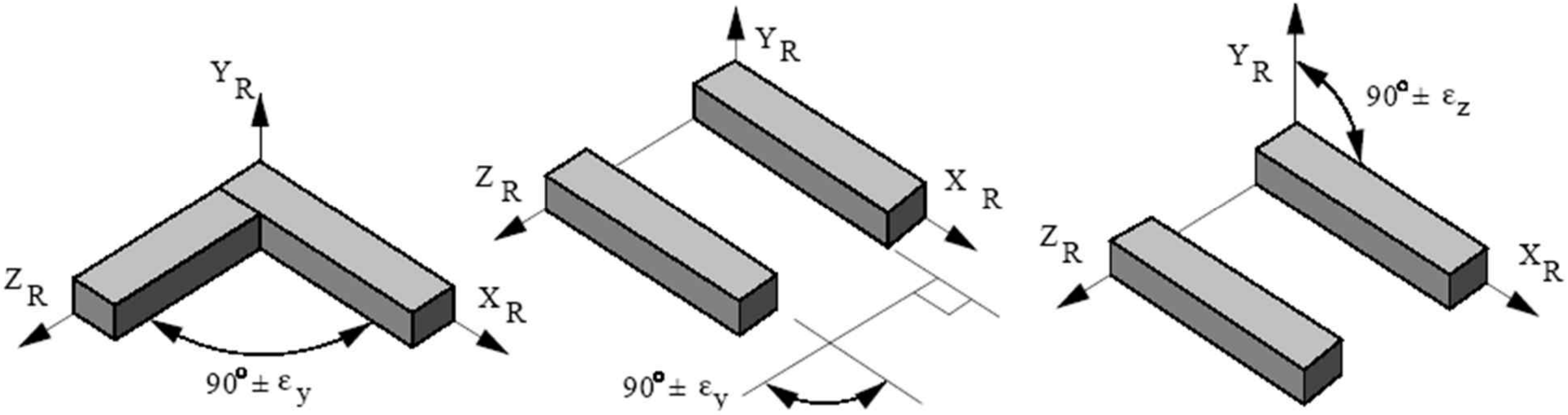
Conventional hydrostatic bearing
(P = about 40 atm)



Aerostatic bearing
(P $<$ 8 atm)

(2) Kinematic Errors

A. Errors in motion due to



B. **Improper offsets(translational) between components.**

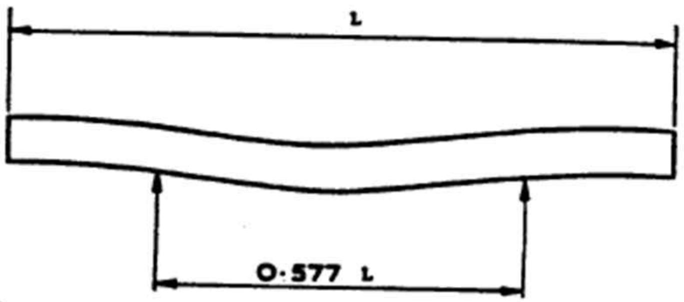
- ✓ Spindle axis set too high above tailstock axis on a lathe

C. **Improper component dimension.**

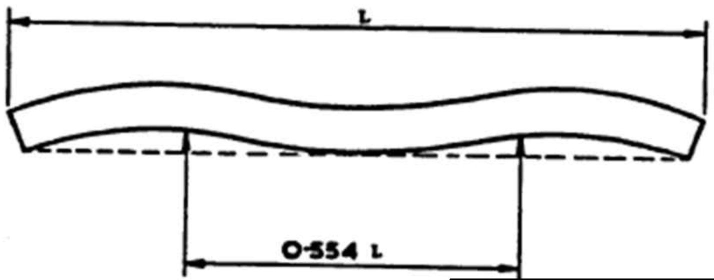
- ✓ Link length
- ✓ Bearing location on a kinematic vee and flat system

(3) External Load Induced Errors

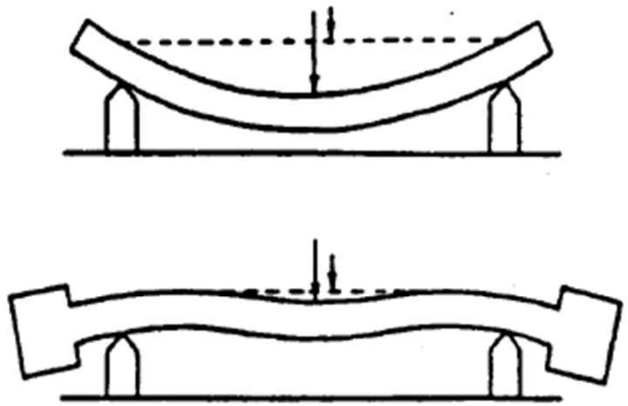
A. Deflection due to it's own weight.



Position of support to bring



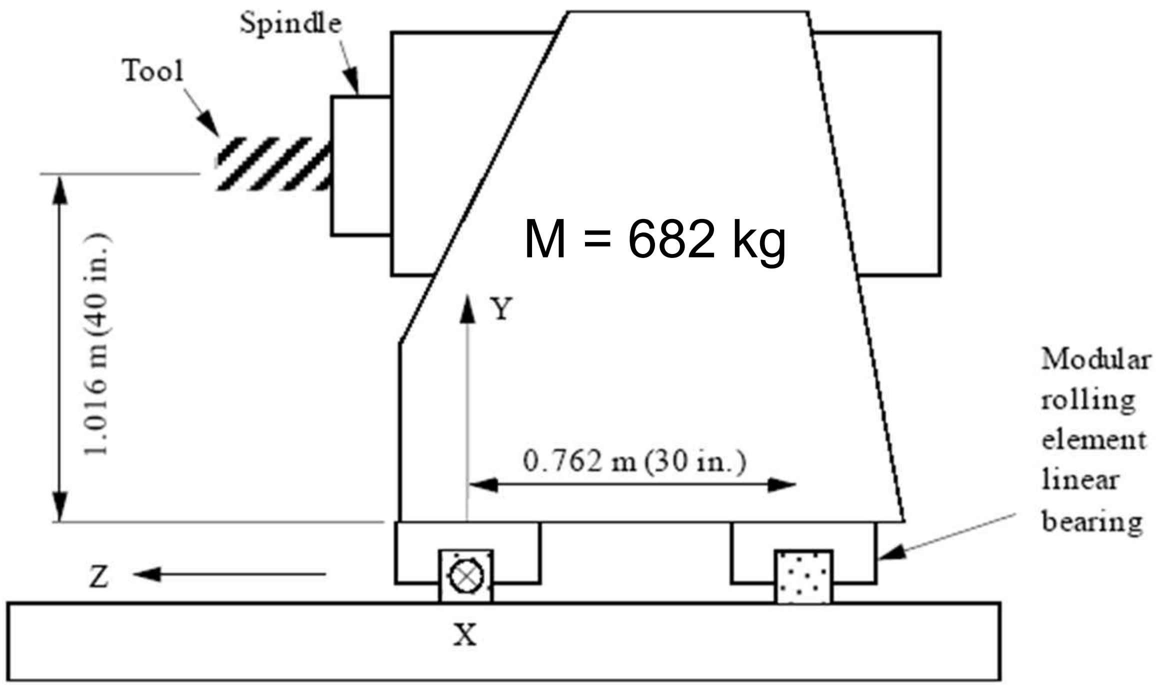
Position of support for



Use of counter weight

B. Errors induced by inertia force

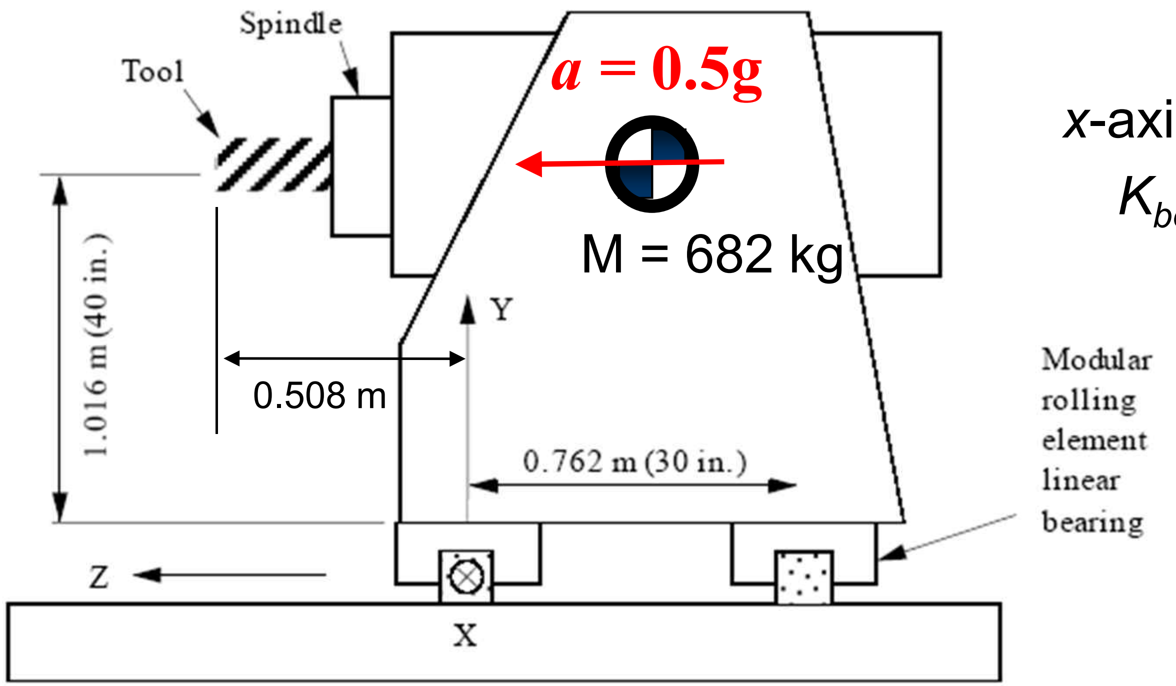
- ✓ A high-speed machine has an X axis carriage that holds two other axes and the spindle
- ✓ High productivity – high speed – high acceleration
- ✓ The dominant compliance is due to the X axis bearing whose Y deflection result in angular motion about the X axis.



✓ With a Z acceleration "a", the Y axis deflection of the rear X axis bearing and the roll of the structure about the X axis are:

$$\delta_{y-rear\ bearing} =$$

$$\theta_x =$$



x-axis bearing stiffness :

$K_{bearing}$ = 350 MN/m

- ✓ Homogeneous Transformation matrix :

- ✓ Tool position after deformation :

- ✓ Tool tip position error :

(4) Thermal Errors

A. Mean temperature other than 68 °F (20 °C).

B. Gradients in environment's temperature.

C. Errors caused by

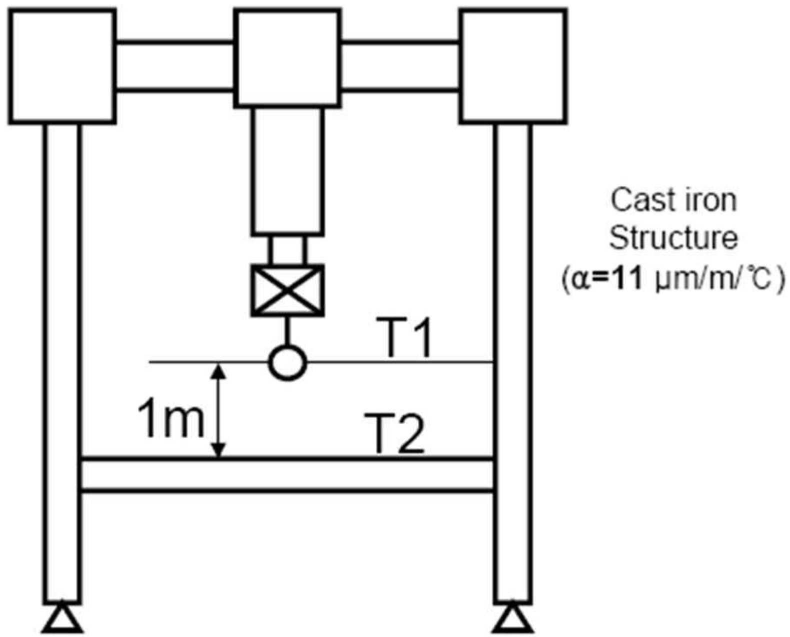
❖ External heat sources:

- ✓ Mean temperature of the room.
- ✓ Sun shining through the window onto the machine.
- ✓ Nearby machine's hot air vent.
- ✓ Overhead lights.
- ✓ Operator's body heat.

❖ Internal heat sources:

- ✓ Motors.
- ✓ Bearings.
- ✓ Machining process.
- ✓ Pumps.
- ✓ Expansion of compressed fluids.

Ex) Precision coordinate measuring machine

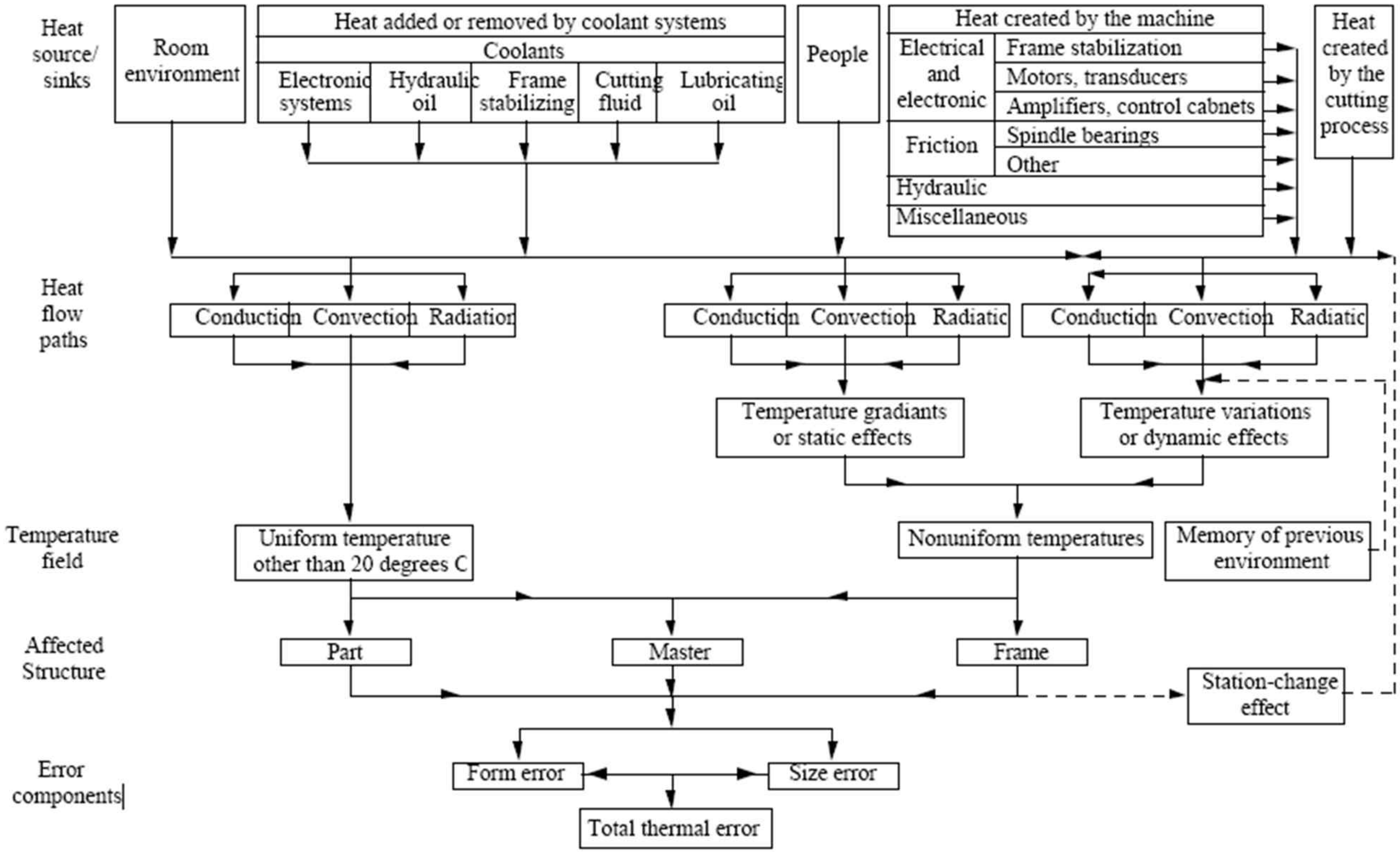


Temp. gradient in inspection room

1℃/m.height (linear gradient)

Thermal expansion (measuring error)

- Thermal effects in manufacturing and metrology

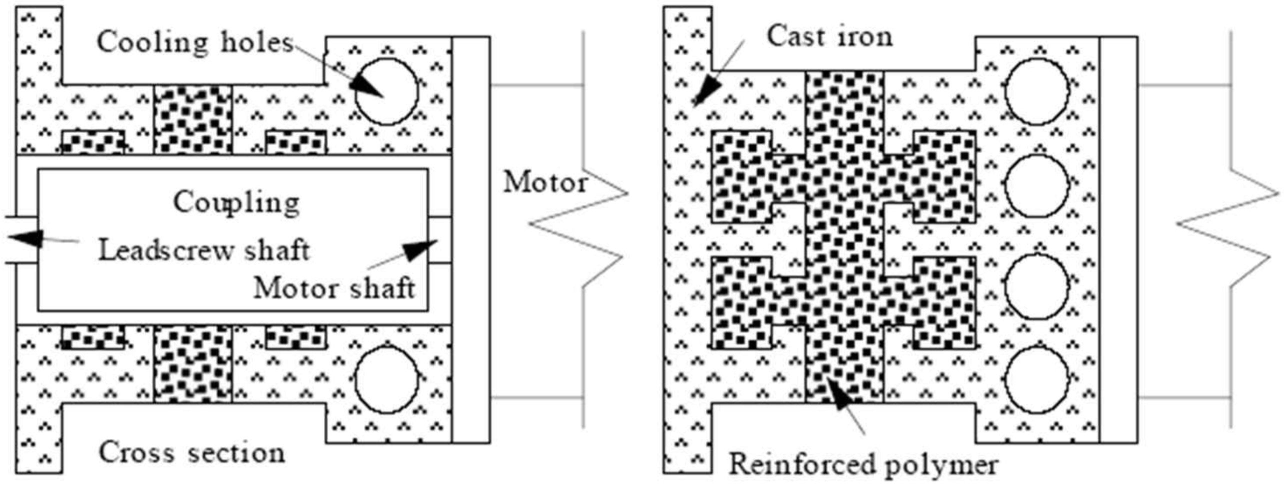


D. Design strategies:

a.

b. Maximize conductivity, OR insulate

- ✓ Conduct insulation : use thermal breaks (insulators) or coolant
  Keep the temperature the same in the building all year!
- ✓ Convection insulation : use sheet metal or plastic cowlings.
- ✓ Radiation insulation : Plastic PVC curtains are very effective at blocking infrared radiation.



Thermal break : Motor to shaft

Ex) Common error in machine tools :

- The bed may be subjected to a flood of temperature ______

- The base will be ______, which may vary wildly in old buildings.

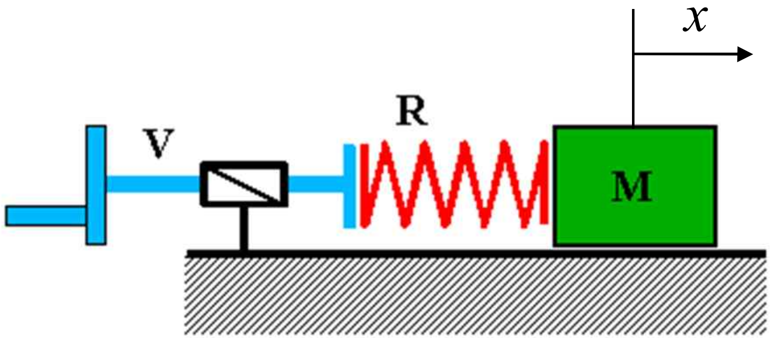
- A gradient will exist from the base to the machine's working volume

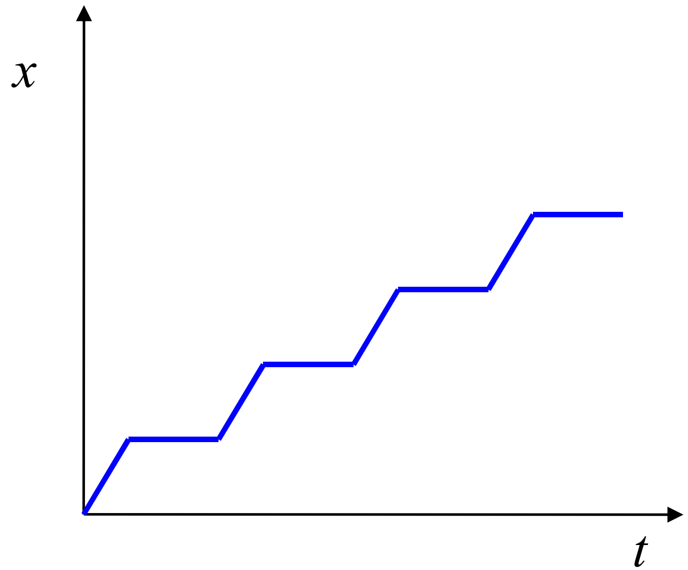
(5) Dynamic Errors

A. **Static friction causes start-up forces which deform the structure and make control more difficult.**

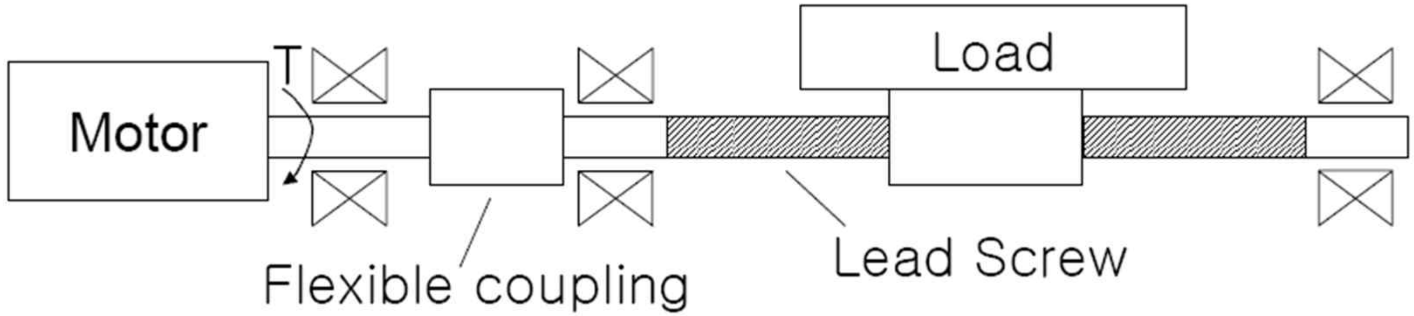
B. **Dynamic friction causes forces which deform the structure, but makes control more easy (however, dynamic friction causes heat to be generated).**

Ex)





Ex) Motion Transmission System

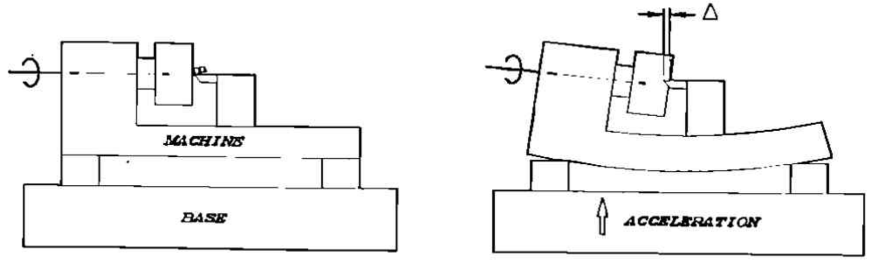


$T = T_{coup.deform} + T_{friction} + T_{inertia}$

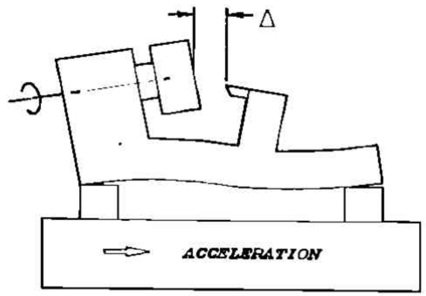
Self oscillating motion(limit cycling) due to the stored energy($T_{coup.deform}$ and $T_{inertia}$) usually

C. causes errors throughout the structure.

- ✓ Vibration transmission through the floor.
- ✓ Rotating mechanical components (e.g., motors, transmissions).
- ✓ Rolling bearings (e.g., microstructure making noise as asperities grind together).
- ✓ Turbulence in fluid supply lines.
- ✓ Sound pressure.
- ✓ Aerostatic instability in air bearings (pneumatic hammer).

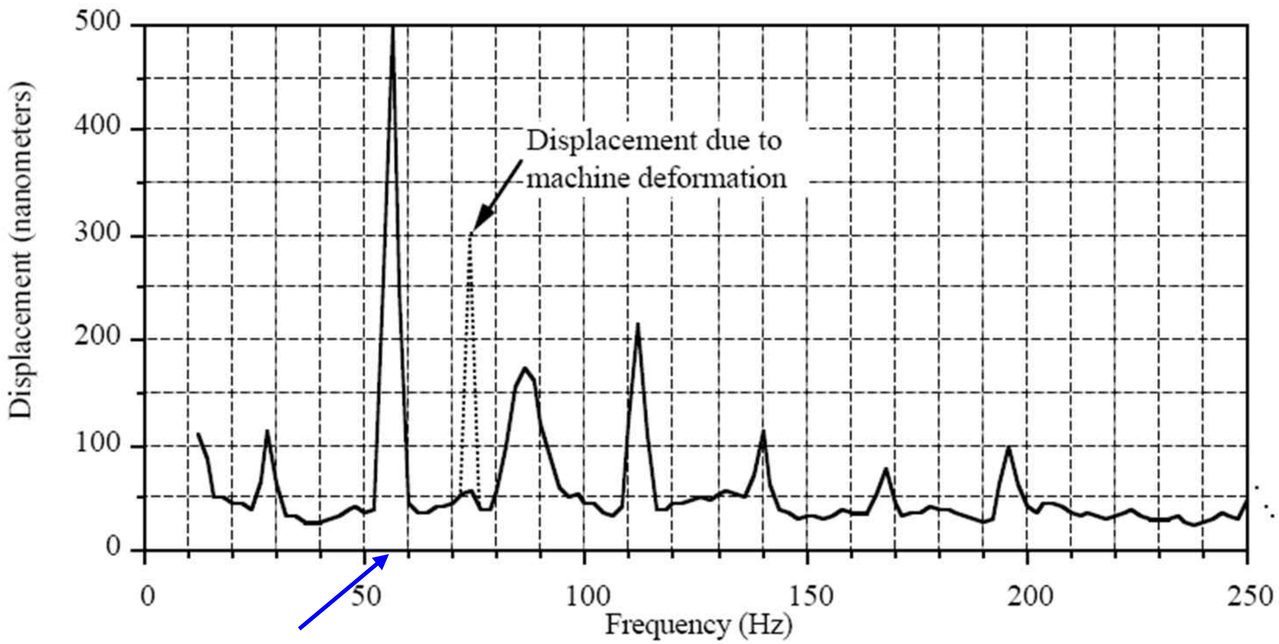


- Structural deformations



From D. B. DeBra
CIRP Keynote 1992

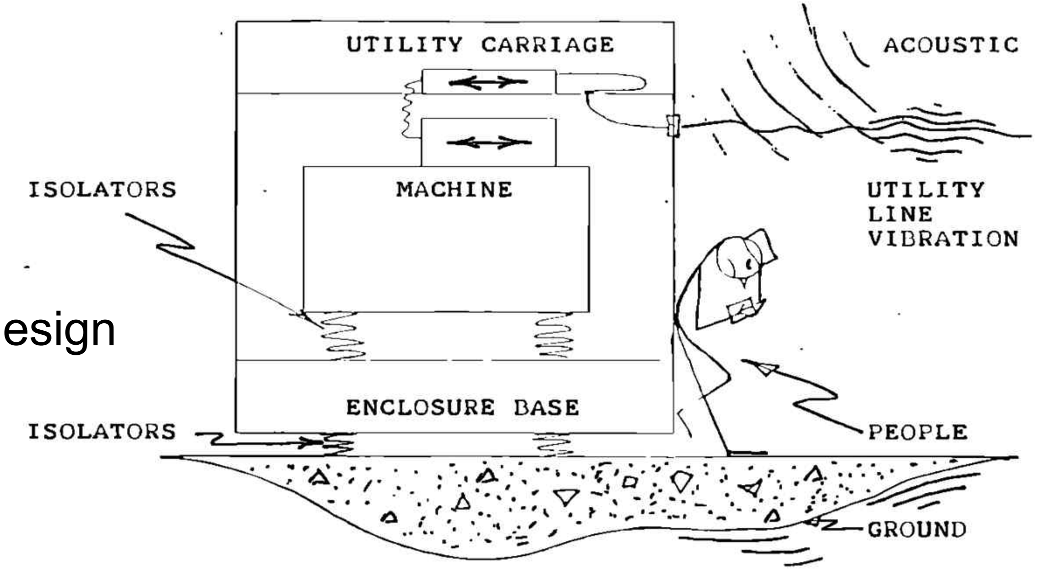
● Frequency spectrum of a rolling element bearing spindle's radial error motion.



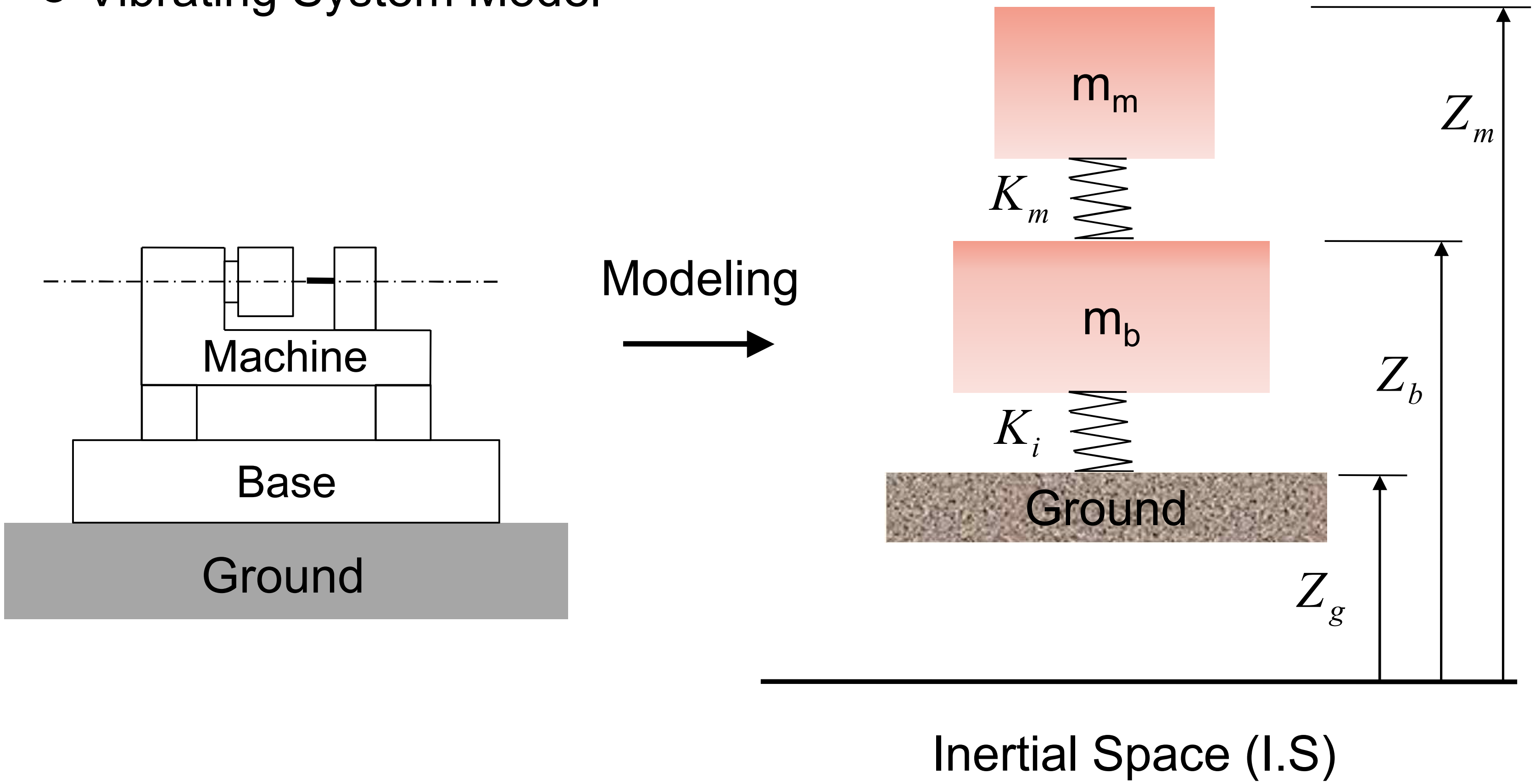
D. Dynamic errors are part of a detective game:

- ✓ An accelerometer and a signal analyzer are critical tools for gathering clues.
- ✓ Modal analysis software and the Fourier Transform are critical tools for tracking the villains!
- ✓ The best way to avoid dynamic problems is to build in as much damping as possible into the system.
- ✓ 62.5 grams of prevention is worth a kilogram of cure.

● Basic concepts for precision instrument design

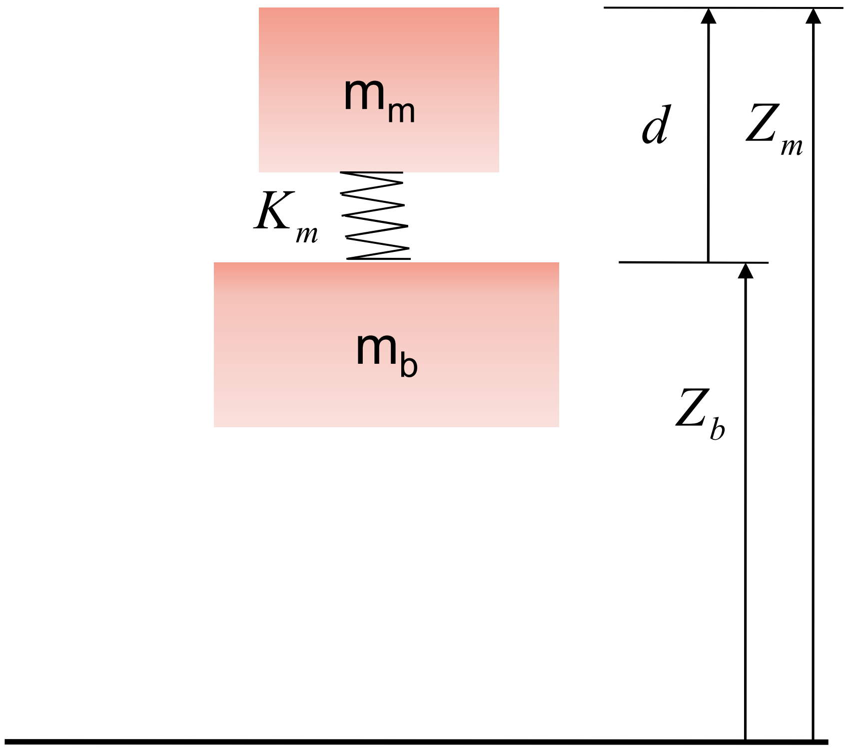


- Vibrating System Model



- K_m permits distortion of machine when m_m is accelerated.
- K_i is added to isolate the machine from ground motion.

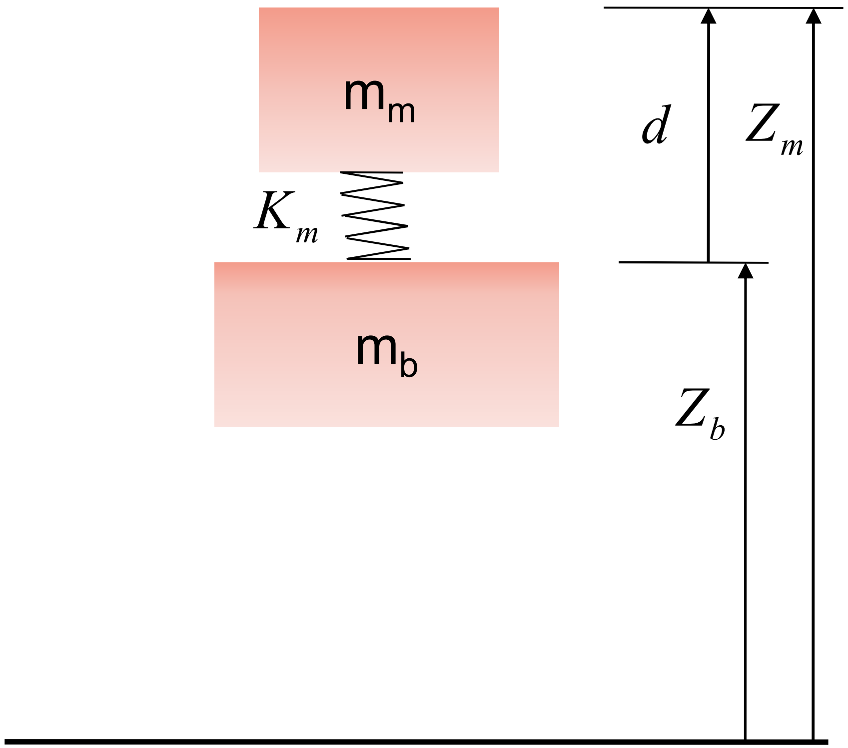
- Machine Distortion



Inertial Space (I.S)

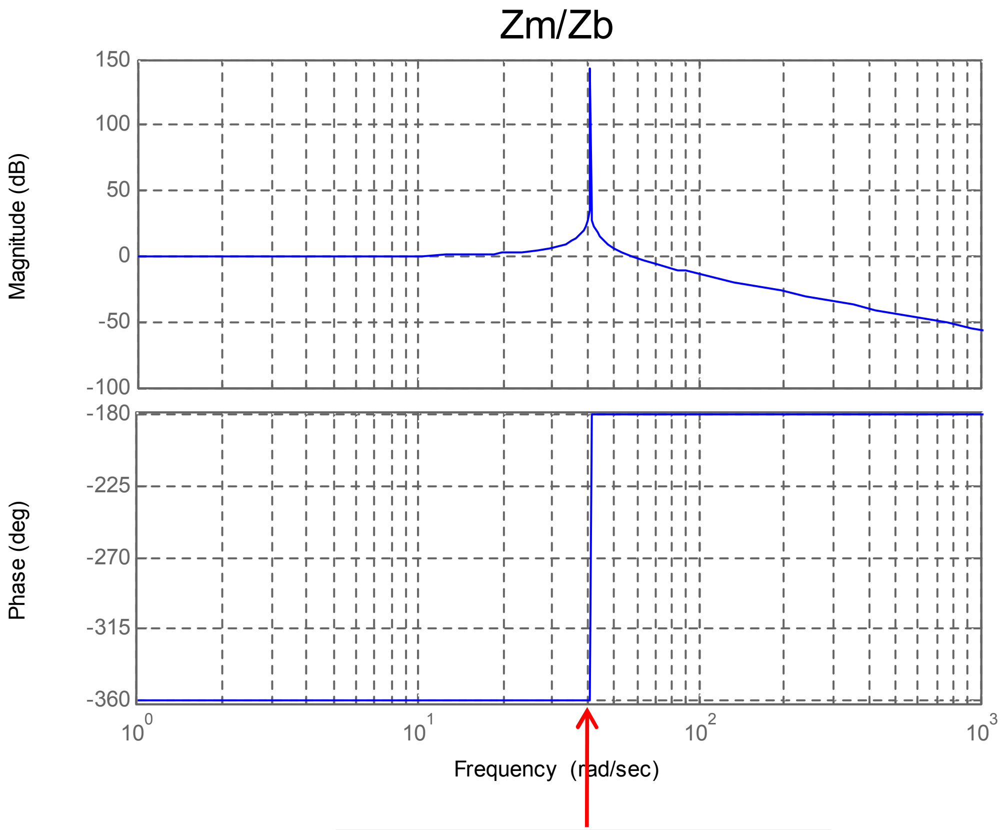
- Dynamic equation

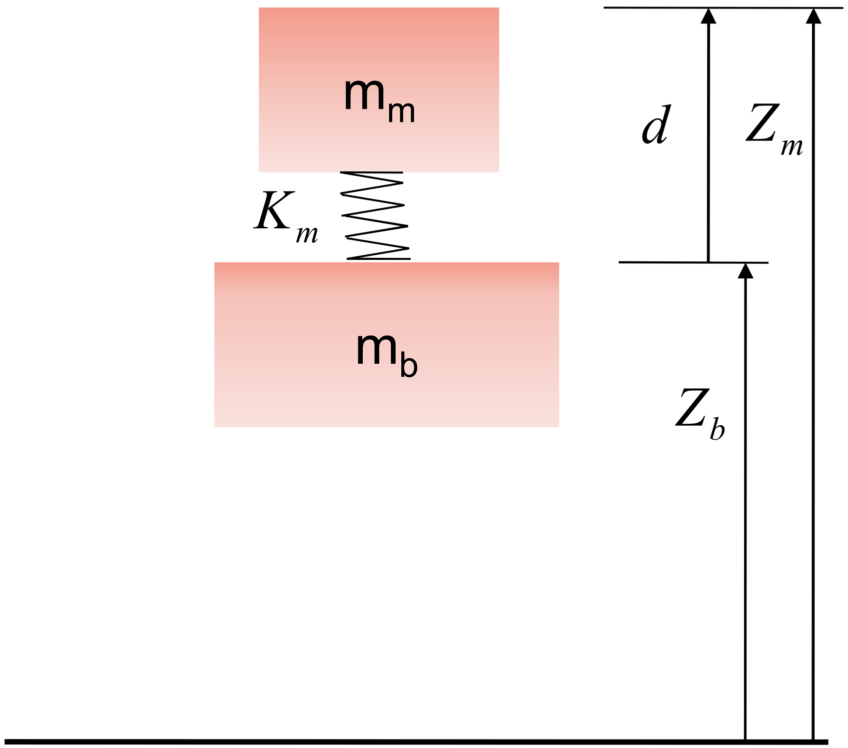
- Transfer Functions



Inertial Space (I.S)

- Bode Plot





Inertial Space (I.S)

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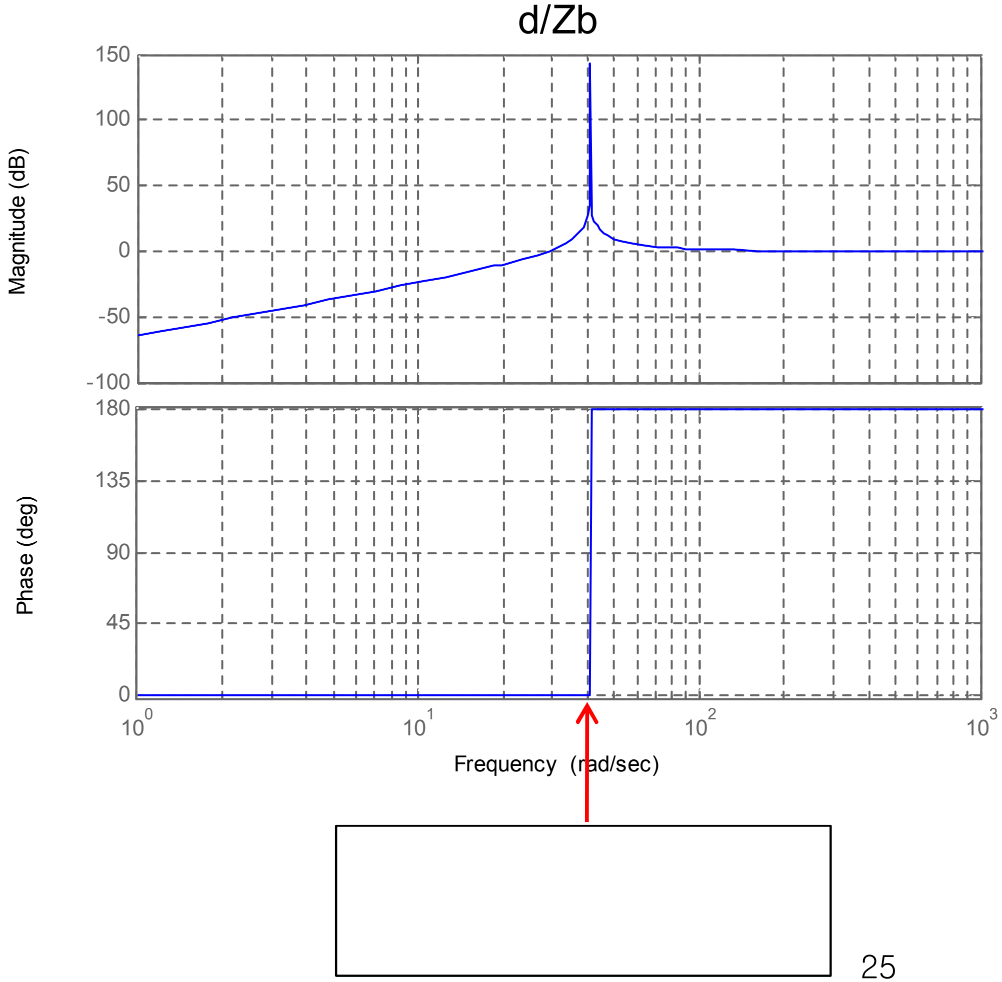
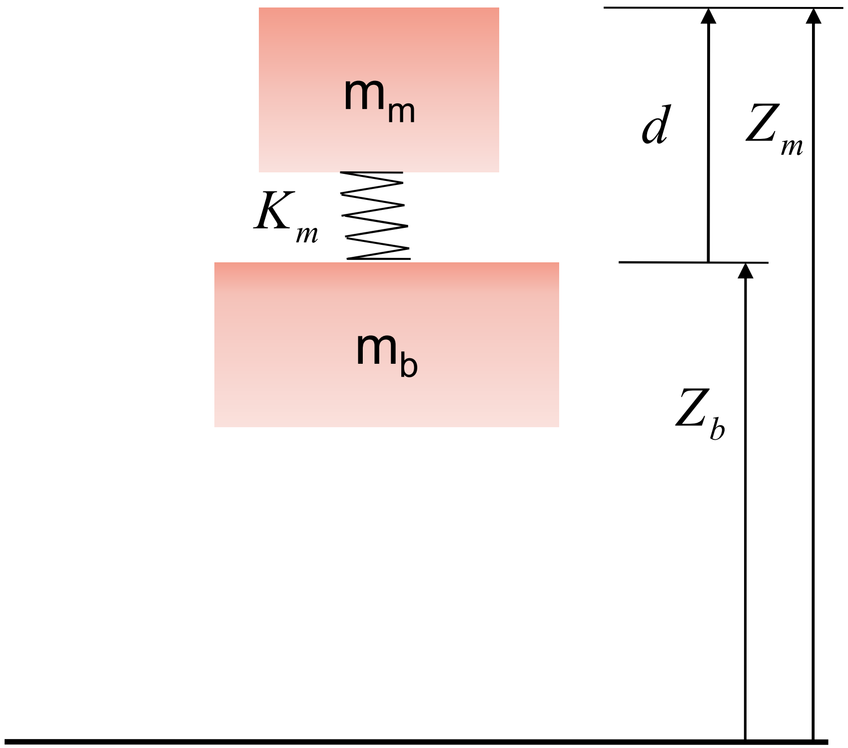




Inertial Space (I.S)

- Dynamic equation

At low freq. ($\omega << \omega_n$) : $\ddot{Z}_b \approx \ddot{Z}_m$

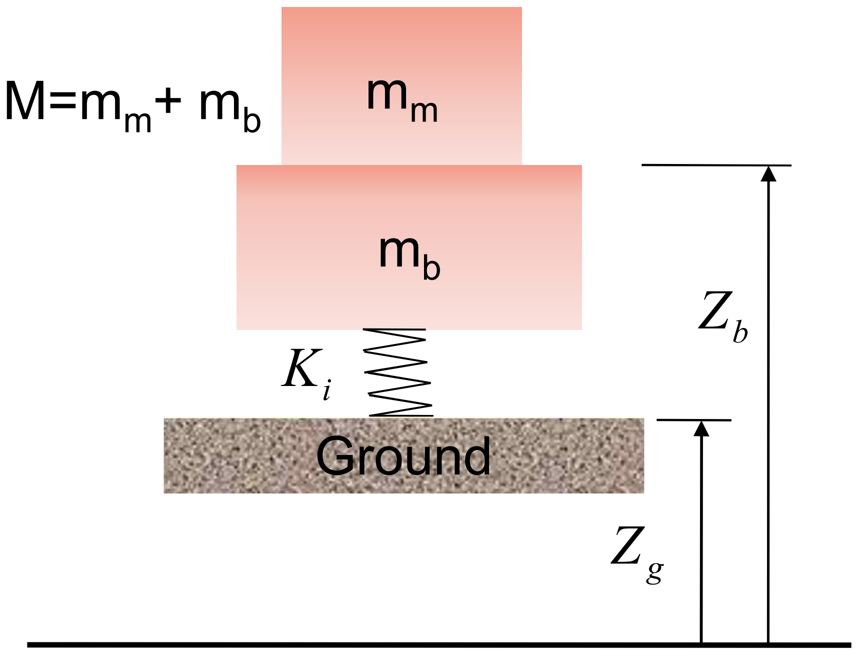
For sine wave : $Z_b = A \sin \omega t$

Thus, $A = \dfrac{K_m |d|}{m_m \omega^2}$

- Ex) for d < 10 nm, $\omega \approx$ 10 Hz,

$$\left| \frac{K_m}{m_m} \right| \approx 10^5 \text{ sec}^{-2} \quad \therefore A < 0.3 \mu m$$

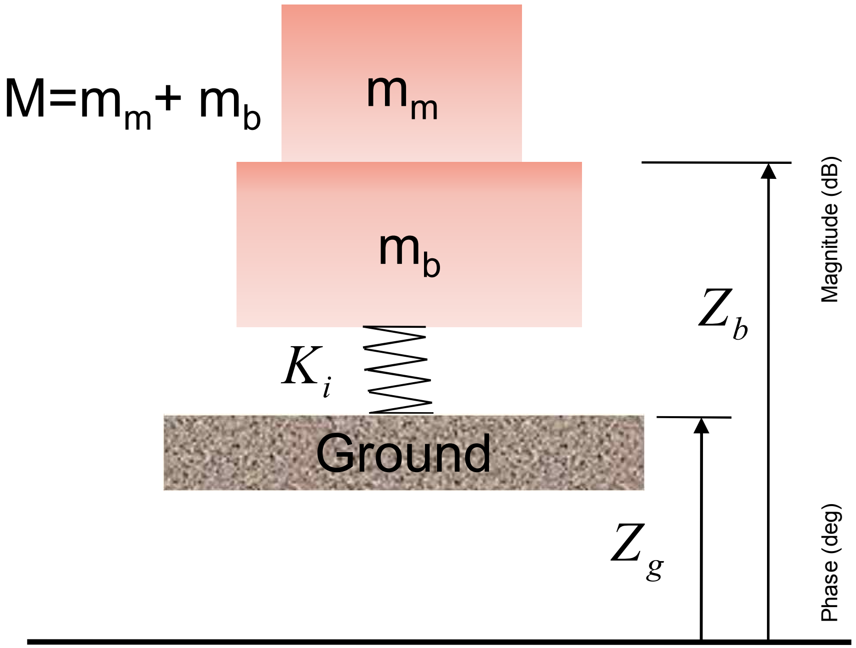
● Ground Vibration



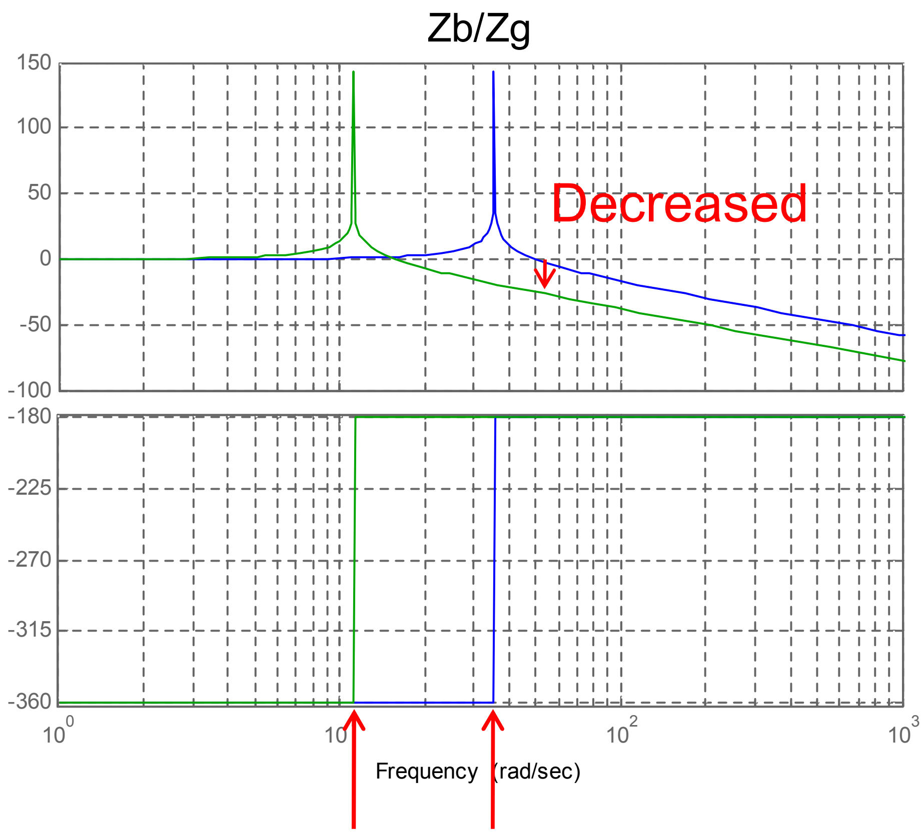
Inertial Space (I.S)

- Dynamic equation

- Transfer Functions



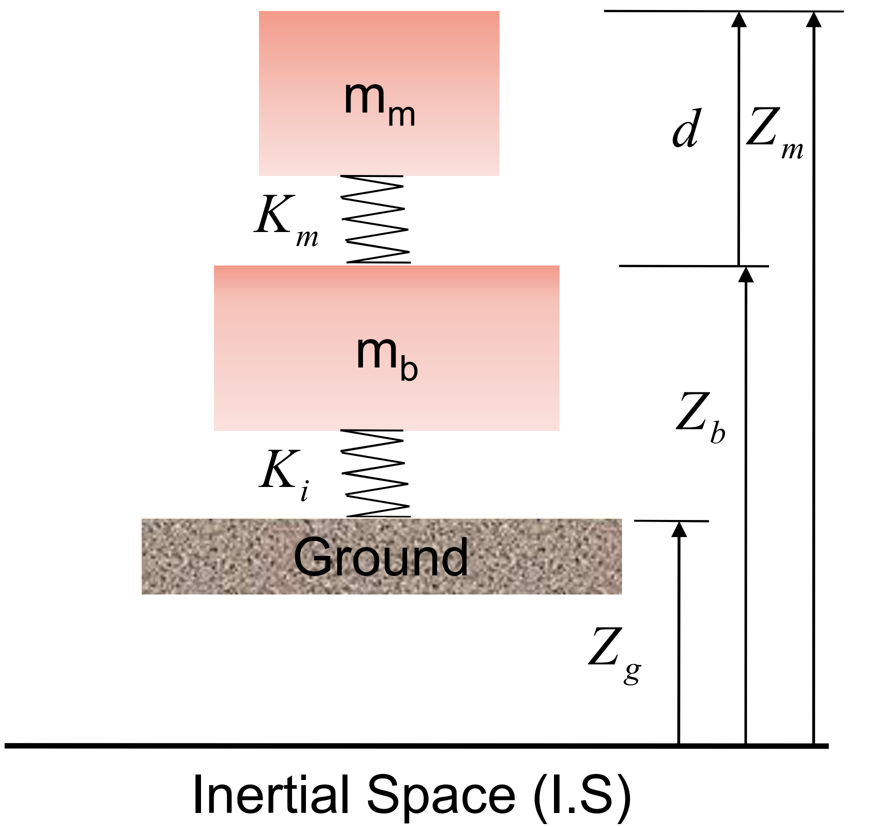
Inertial Space (I.S)

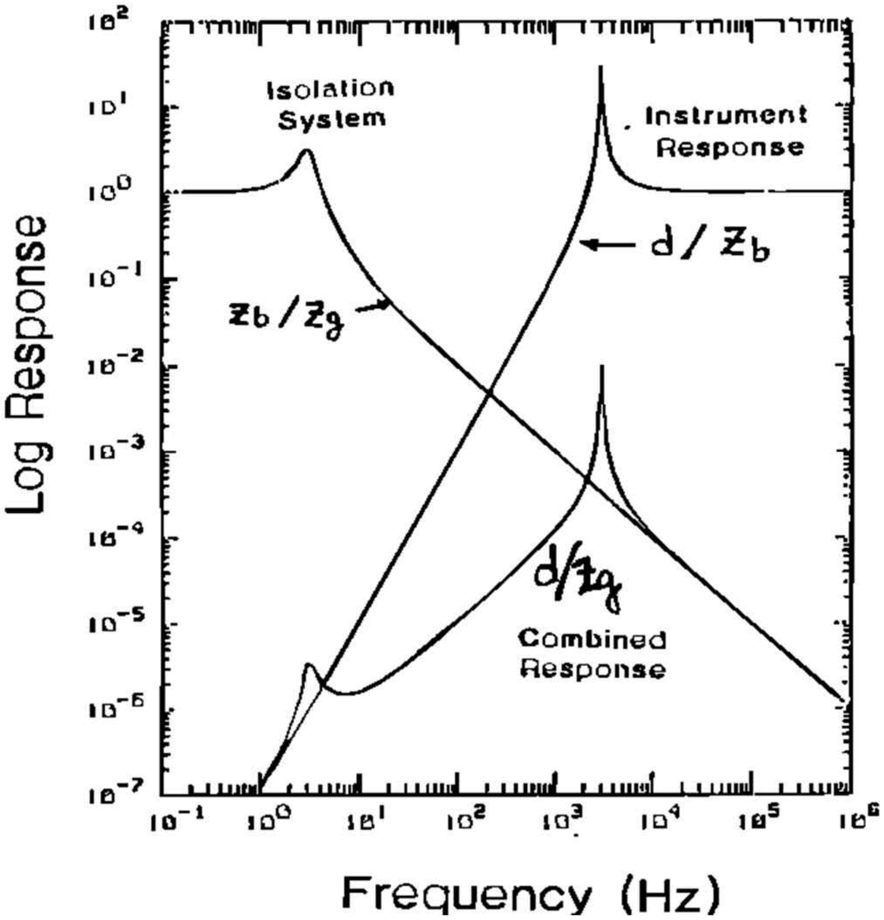


$$\omega_{n,2} = \sqrt{\frac{0.1K_i}{M}} \quad \omega_{n,1} = \sqrt{\frac{K_i}{M}}$$

- No isolation at << ω_n
- low ω_n is desirable for isolation
- Limit : $Mg/Ki = g/\omega_n^2 < \delta max$ →

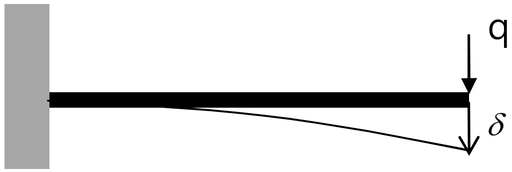
- Key Points


m_m
K_m
m_b
K_i
Ground
d
Z_m
Z_b
Z_g
Inertial Space (I.S)


Isolation System
Instrument Response
d / Z_b
Z_b / Z_g
d/Z_g
Combined Response
Log Response
Frequency (Hz)

(6) Modeling Errors

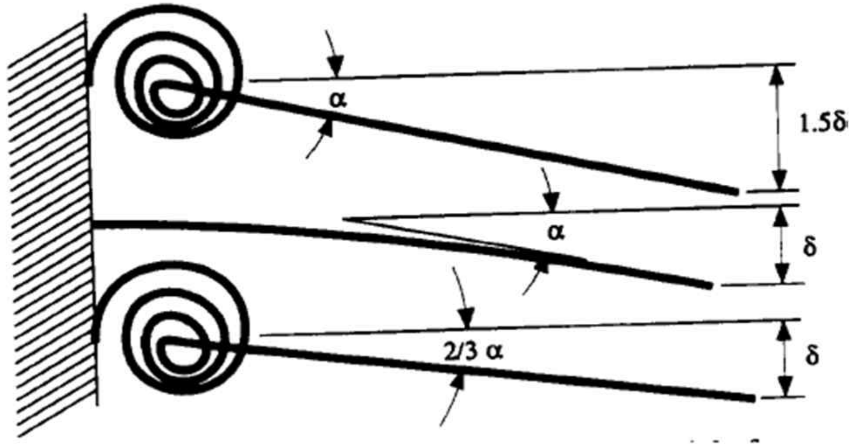
Cantilever beam



$$\delta = \frac{qL^3}{3EI}$$

$$\theta = \frac{qL^2}{2EI}$$

Modeling →



- Lateral deflection matched

	Cantilever beam	Angular spring	Deflection error
$K_{lateral}$			
$K_{angular}$			

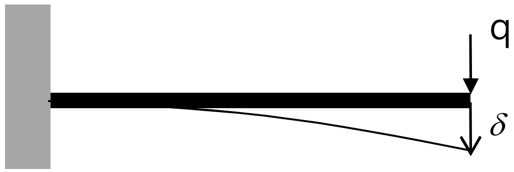
- Slope at ends matched

	Cantilever beam	Angular spring	Deflection error
$K_{lateral}$			
$K_{angular}$			

(6) Modeling Errors

- Calculation of stiffness of cantilever beam

Cantilever beam



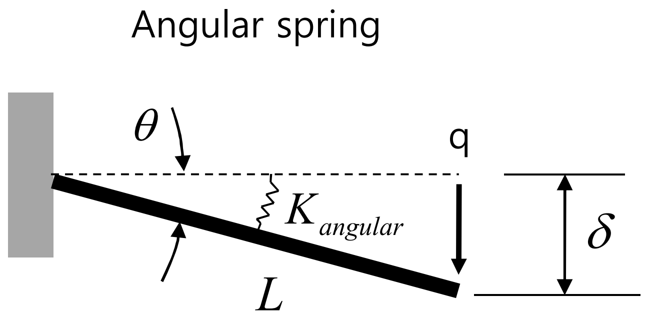
$$\delta = \frac{qL^3}{3EI}$$

$$\theta = \frac{qL^2}{2EI}$$

- Force =
- Torque =

(6) Modeling Errors

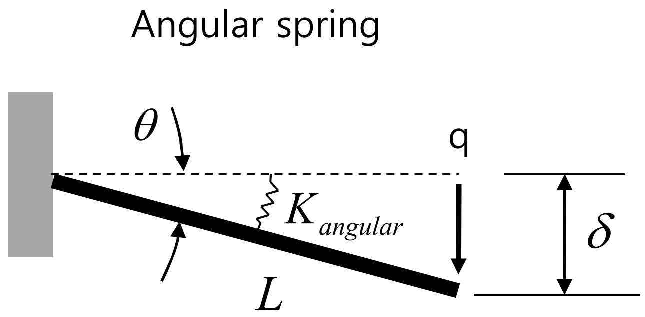
- Calculation of stiffness of angular spring



- deflection 을 일치시켜서 stiffness 를 계산하면, 즉 $\delta = \dfrac{qL^3}{3EI}$ 라면,

(6) Modeling Errors

- Calculation of stiffness of angular spring



- 기울기각을 일치시켜서 stiffness 를 계산하면, 즉 $\theta = \dfrac{qL^2}{2EI}$ 라면,

(7) Instrumentation Errors

A. Errors associated with sensors:

- Intrinsic accuracy.
- Interpolation.
- Mounting errors:
  - ✓ Position.
  - ✓ Mounting stress.

B. Calibration (error associated with the mastering process).

(8) ETC.

A. Computational errors:

- Error introduced in the analysis algorithms.
- Rounding off errors due to hardware.

B. Additional sources of error (often very difficult to model):

- Humidity.
- Loose Joints.
- Dirt.

C. Variations in supply systems:

- Electricity.
- Fluid pressure.
- Operator inattention.

D. Assembly errors

E. Operators