Cybernetics and Systems Analysis, Vol. 34, No. 4, 1998

PREDICTING THE RELIABILITY OF ALGORITHMIC PROCESSES WITH FUZZY INPUT DATA

A. P. Rotshtein and S. D. Shtovba

UDC 681.3

INTRODUCTION

Many discrete-behavior systems can be analyzed in a unified framework if combined into a class of so-called algorithmic processes (AP). Typical AP include transformation of information in computer systems, performance of scientific research and design projects, technological production processes, instruction processes, and so on. Each of these processes involves a sequence of operations (activities, jobs) unfolding in time whose execution leads to the achievement of a goal, i.e., produces an end product, be it information, documentation, knowledge, etc.

When designing a specific AP, we need to obtain quantitative estimates of the following reliability measures:

p_{AP} – the probability of correct AP execution; this may be interpreted as the reliability of output information, defect-free quality of the output products, reliability of system operation;

t_{AP} – the time or other resources required to execute the AP; this measure is used as an estimate of system productivity or timely goal achievement.

The measures p_{AP} and t_{AP} are interrelated because the AP incorporates checking and support procedures that increase the probability of correct execution of the process but require additional resources.

Models to estimate p_{AP} and t_{AP} are widely used in reliability theory and analysis of man–machine systems [1-3]. In these studies, the modeling is based on the theory of semi-Markov processes [4] whose states correspond to the operators and logical conditions of the given algorithm.

Successful application of AP reliability theory (as also of classical reliability theory [5]) envisages construction of databases with reliability characteristics of the basic elementary operations. However, new operations do not have ex-post statistical estimates of outcomes under real-life conditions. Complex-system designers are therefore often forced to make decisions on the basis of expert judgments, i.e., natural-language statements of the form "the probability of correct execution of the operation is between 0.9-0.99," or "if the equipment is properly maintained and is operated under appropriate conditions, then the reliability is high," or "if the human operator is tired, then the number of errors during the execution of the operation is approximately doubled."

The probabilistic reliability theory [1-3] is incapable of utilizing input data expressed in the form of natural-language expert judgments. It is therefore relevant to try and develop a so-called "fuzzy reliability theory" [6], which in addition to the probabilistic apparatus also uses the tools of fuzzy set theory [7] that can manipulate linguistic expert information.

In this article we propose an approach that generalizes the probabilistic AP reliability models to the case of fuzzy input data and allows for the dependence of data on influential factors through fuzzy logical inference.

MODELING LANGUAGE FOR ALGORITHMIC PROCESSES

For formal description of AP we use the language of Glushkov's algorithmic algebras [8]. In this language, the algorithm operators (statements) are usually denoted by Latin capital letters ($A, B, C, \ldots$) and logical conditions are denoted by Greek lower-case letters ($\alpha, \beta, \gamma, \ldots$), indexed or not. By the regularization theorem [8], every algorithm is representable as a superposition of the following operator structures: the linear structure $B = A_1A_2$; α-disjunction $C = \underset{\alpha}{(A_1 \vee A_2)}$; α-iteration $D = \underset{\alpha}{\{A\}}$.

Translated from Kibernetika i Sistemnyi Analiz, No. 4, pp. 85-93, July-August, 1998. Original article submitted October 2, 1996.

Example 1. The algorithm describing the operation of a ticketing information system can be represented by the formula

$$Y = \underset{\omega_1}{\{A_1\}}\ \{\{A_2 \underset{\omega_2}{(}\ E \vee A_3)\ A_4 \underset{\omega_3}{\}}\ A_5 A_6 \underset{\omega_4}{\}} A_7, \tag{1}$$

where A_1 stands for the submission of a request by a customer and its acceptance by the human operator; ω_1 is validity verification of the incoming request; A_2 is the keyboarding of the request; ω_2 is a visual check of the keyboarded request; E is the recording of the visual check result; A_3 is error correction; A_4 is submission of the request to the central computer; ω_3 is program matching of the request to an established template; A_5 is central computer's decision; A_6 is display of the central computer's decision on the screen; ω_4 is the human operator checking the computer decision; A_7 is printing the ticket and delivering it to the customer.

Each operator A in formula (1) can be represented by a more detailed lower-level algorithm.

PROBABILISTIC ALGORITHM RELIABILITY MODELS

We assume that the following events may occur when an operator A and a logical condition ω are executed:

A^1 (A^0) – correct (incorrect) execution of operator A;

ω^1 (ω^0) – condition ω is *a priori* true (false);

ω^{11} (ω^{10}) – an *a priori* true condition ω is recognized as true (false) during a check;

ω^{00} (ω^{01}) – an *a priori* false condition ω is recognized as false (true) during a check.

The above-listed events are assumed pairwise mutually exclusive.

The probability (Prob) of these events is denoted by

$$p_A^1(p_A^0) = \text{Prob}\, A^1 (\text{Prob}\, A^0);\quad p_\omega(\bar{p}_\omega) = \text{Prob}\, \omega^1 (\text{Prob}\, \omega^0);$$

$$k_\omega^{11}(k_\omega^{10}) = \text{Prob}\, \omega^{11} (\text{Prob}\, \omega^{10});\quad k_\omega^{00}(k_\omega^{01}) = \text{Prob}\, \omega^{00} (\text{Prob}\, \omega^{01}).$$

Mutual exclusivity of the events leads to the relationships

$$p_A^1 + p_A^0 = p_\omega + \bar{p}_\omega = k_\omega^{11} + k_\omega^{10} = k_\omega^{00} + k_\omega^{01} = 1.$$

Note that $k_\omega{}^{10}$ and $k_\omega{}^{01}$ are the probabilities of type I and type II errors when checking condition ω.

The time (or other resources) required to execute the operator A and check the logical condition ω are denoted by t_A and t_ω, respectively.

Given the logic of error-free execution of operator structures, we obtain the following rules for estimating algorithm execution reliability:

$$B = A_1 A_2 \Rightarrow p_B^1 = p_{A_1}^1 \cdot p_{A_2}^1,\quad t_B = t_{A_1} + t_{A_2}, \tag{2}$$

$$C = \underset{\omega}{(}A_1 \vee A_2) \Rightarrow \begin{cases} p_C^1 = p_\omega k_\omega^{11} p_{A_1}^1 + \bar{p}_\omega k_\omega^{00} p_{A_2}^1, \\ t_c = t_\omega + \left(p_\omega k_\omega^{11} + \bar{p}_\omega k_\omega^{01}\right) t_{A_1} + \left(p_\omega k_\omega^{10} + \bar{p}_\omega k_\omega^{00}\right) t_{A_2}, \end{cases} \tag{3}$$

$$D = \underset{\omega}{\{A\}} \Rightarrow \begin{cases} p_D^1 = \dfrac{p_A^1 k_\omega^{11}}{1 - \left(p_A^1 k_\omega^{10} + p_A^0 k_\omega^{00}\right)}, \\ t_D = \dfrac{t_A + t_\omega}{1 - \left(p_A^1 k_\omega^{10} + p_A^0 k_\omega^{00}\right)}. \end{cases} \tag{4}$$

The application of these rules replaces the original algorithm by a single operator with equivalent cost characteristics and equivalent probability of correct execution.

REPRESENTATION OF UNCERTAIN INPUT DATA BY FUZZY SETS

Let q be an uncertain parameter that corresponds to the probability of error-free execution or the cost of executing the operator A or logical condition ω.

The uncertain parameter q is treated as a linguistic variable [7] whose levels are formalized by fuzzy sets with convex membership functions defined on the universal set $U = [\underline{q}, \bar{q}]$, where $\underline{q}$, $\bar{q}$ are the smallest and greatest allowed values of the parameter q. In this case, the uncertain parameter q is identified with the fuzzy number $\tilde{q}$.

Definition 1. The l-form of the uncertain parameter q is the triple

$$\tilde{q} = \langle \underline{q}, \bar{q}, l \rangle, \tag{5}$$

where l is the linguistic estimate of the parameter q in the range $U = [\underline{q}, \bar{q}]$, selected from the term set $L = \{l_1, l_2, \ldots, l_m\}$ such that

$$l_j = \int_U \mu_{l_j}(q) / q,$$

where $\mu_{l_j}(q)$ is the membership function of the value $q \in [\underline{q}, \bar{q}]$ in the term $l_j \in L, j = \overline{1, m}$.

Definition 2. The α-form of the uncertain parameter q is the union of the pairs

$$\tilde{q} = \bigcup_{\alpha \in [0, 1]} \left(\underline{q}_\alpha, \bar{q}_\alpha\right), \tag{6}$$

where $\underline{q}_\alpha (\bar{q}_\alpha)$ is the smallest (greatest) allowed value of q at the α-level of the membership function, i.e.,

$$\mu(\underline{q}_\alpha) = \mu(\bar{q}_\alpha) = \alpha, \quad \mu(\underline{q}) = \mu(\bar{q}) = 0.$$

Assertion 1. If the membership functions $\mu_{l_j}(q)$ of the terms $l_1, l_2, \ldots, l_m$ are given, then the l-form (5) transforms to the α-form (6).

This assertion follows from the definition of the α-level representation of fuzzy sets [7].

Example 2. Assume that the linguistic estimate is selected from the term set L = {low (L), below average (bA), average (A), above average (aA), high (H)} with piecewise-linear membership functions from [6] (Fig. 1). Then we have the following transformation rules:

$$\langle \underline{q}, \bar{q}, \text{low} \rangle = \left(\underline{q}, \bar{q}\right)_0 \cup \left(\underline{q}, \underline{q} + \Delta\right)_{0.5} \cup \left(\underline{q}, \underline{q}\right)_1,$$
$$\langle \underline{q}, \bar{q}, \text{below average} \rangle = \left(\underline{q}, \bar{q}\right)_0 \cup \left(\underline{q}, \underline{q} + 2\Delta\right)_{0.5} \cup \left(\underline{q} + \Delta, \underline{q} + \Delta\right)_1,$$
$$\langle \underline{q}, \bar{q}, \text{average} \rangle = \left(\underline{q}, \bar{q}\right)_0 \cup \left(\underline{q} + \Delta, \bar{q} - \Delta\right)_{0.5} \cup \left(\underline{q} + 2\Delta, \underline{q} + 2\Delta\right)_1,$$
$$\langle \underline{q}, \bar{q}, \text{above average} \rangle = \left(\underline{q}, \bar{q}\right)_0 \cup \left(\underline{q} + 2\Delta, \bar{q}\right)_{0.5} \cup \left(\bar{q} - \Delta, \bar{q} - \Delta\right)_1,$$
$$\langle \underline{q}, \bar{q}, \text{high} \rangle = \left(\underline{q}, \bar{q}\right)_0 \cup \left(\bar{q} - \Delta, \bar{q}\right)_{0.5} \cup \left(\bar{q}, \bar{q}\right)_1,$$

where $\Delta = \dfrac{\bar{q} - \underline{q}}{4}$.

Definition 3. The $l(x)$-form of the uncertain parameter q is the triple

$$\tilde{q} = \langle \underline{q}, \bar{q} l(x) \rangle, \tag{7}$$

where $l(x)$ is the expert knowledge base in the form of systems of fuzzy logical propositions

$$\begin{aligned} &\text{IF } (x_1 = a_1^{j1}) \text{ AND } (x_2 = a_2^{j1}) \ldots \text{ AND } (x_n = a_n^{j1}) \text{ OR} \\ &(x_1 = a_1^{jk_j}) \text{ AND } (x_2 = a_2^{jk_j}) \ldots \text{ AND } (x_n = a_n^{jk_j}), \\ &\text{THEN } l = l_j, \end{aligned} \tag{8}$$

where $a_i^{jp} = \int_{U_i} \mu^{jp}(x_i)/x_i$, $i = \overline{1, n}$, $j = \overline{1, m}$, $p = \overline{1, k_j}$. These propositions link the level l of the parameter $q \in [\underline{q}, \overline{q}]$ with the vector $(x_1, x_2, \ldots, x_n)$ of influential factors, where k_j is the number of disjunctions (OR) in the j-th logical proposition and $\mu^{ij}(x_i)$ is the membership function of the variable $x_i \in U_i$ to the fuzzy term a_i^{jp} estimating the factor x_i in disjunction jp, $i = \overline{1, n}$, $j = \overline{1, m}$, $p = \overline{1, k_j}$.

The $l(x)$-form (6) given above is transformed into l-form (5) by the following assertion.

Assertion 2. To a fixed vector $(x_1^*, x_2^*, \ldots, x_n^*)$ of factors influencing the parameter $q \in [\underline{q}, \overline{q}]$, corresponds the level $l^* \in L$ of this parameter such that

$$\mu_{l^*}(x_1^*, x_2^*, \ldots, x_n^*) = \max_{j=\overline{1,m}} \left[\mu_{l_j}(x_1^*, x_2^*, \ldots, x_n^*)\right], \tag{9}$$

$$\mu_{l_j}(x_1^*, x_2^*, \ldots, x_n^*) = \bigvee_{p=1}^{k_j} \bigwedge_{i=1}^{n} \left\{ \sup_{x_i \in U_i} \left[\mu^{jp}(x_i) \wedge \mu^{jp}(x_i^*)\right]\right\}. \tag{10}$$

This assertion follows from the notion of membership function in fuzzy set theory [7] as generalized in relationships (9) and (10) to the n-dimensional case using fuzzy logical operations $\wedge$ (min) and $\vee$ (max).

Example 3. Consider the operator A_2 in algorithm (1). Assume that the probability $p_{A_2}^1$ of error-free execution of the operator A_2 is in the range [0.95, 0.99] and depends on the following factors: the qualification of the human operator x_1, work stress x_2, fatigue level x_3. The expert knowledge base has the following form:

IF (x_1 = H) AND (x_2 = L) AND (x_3 = L) THEN $p_{A_2}^1$ = H,
IF (x_1 = A) AND (x_2 = aA) AND (x_3 = bA) THEN $p_{A_2}^1$ = A,
IF (x_1 = L) AND (x_2 = H) AND (x_3 = H) THEN $p_{A_2}^1$ = L,

and the fuzzy terms of the membership functions are shown in Fig. 1.

Assume that the following factor levels correspond to the current situation: x_1^* = aA, x_2^* = bA, x_3^* = L. It is required to find the level with probability $p_{A_2}^1$?

By Eq. (10), we find

$$\mu_{\text{H}}(x_1^*, x_2^*, x_3^*) = \sup_{x_1} \left[\mu_{\text{H}}(x_1) \wedge \mu_{\text{aA}}(x_1)\right] \wedge \sup_{x_2} \left[\mu_{\text{L}}(x_2) \wedge \mu_{\text{bA}}(x_2)\right] \wedge$$
$$\wedge \sup_{x_3} \left[\mu_{\text{L}}(x_3) \wedge \mu_{\text{L}}(x_3)\right] = 0.75 \wedge 0.75 \wedge 1 = 0.75,$$

$$\mu_{\text{A}}(x_1^*, x_2^*, x_3^*) = \sup_{x_1} \left[\mu_{\text{A}}(x_1) \wedge \mu_{\text{aA}}(x_1)\right] \wedge \sup_{x_2} \left[\mu_{\text{aA}}(x_2) \wedge \mu_{\text{bA}}(x_2)\right] \wedge$$
$$\wedge \sup_{x_3} \left[\mu_{\text{bA}}(x_3) \wedge \mu_{\text{L}}(x_3)\right] = 0.75 \wedge 0.5 \wedge 0.75 = 0.5,$$

$$\mu_{\text{L}}(x_1^*, x_2^*, x_3^*) = \sup_{x_1} \left[\mu_{\text{L}}(x_1) \wedge \mu_{\text{aA}}(x_1)\right] \wedge \sup_{x_2} \left[\mu_{\text{H}}(x_2) \wedge \mu_{\text{bA}}(x_2)\right] \wedge$$
$$\wedge \sup_{x_3} \left[\mu_{\text{H}}(x_3) \wedge \mu_{\text{L}}(x_3)\right] = 0.43 \wedge 0.43 \wedge 0.33 = 0.33.$$

The term H has the highest degree of membership, and therefore the uncertain probability $p^1_{A_2}$ is described by *l*-form (11) and α-form (12):

$$\tilde{p}^1_{A_2} = \langle 0.95,\ 0.99,\ \text{high}\rangle, \tag{11}$$

$$\tilde{p}^1_{A_2} = (0.95, 0.99)_0 \cup (0.98, 0.99)_{0.5} \cup (0.99, 0.99)_1 . \tag{12}$$

The transition from Eq. (11) to Eq. (12) is governed by the rules of example 2.

GENERALIZING THE RELIABILITY MODELS TO THE FUZZY CASE

Definition 4 (generalization principle [9]). If $y = f(q_1, q_2, \ldots, q_n)$ is a function of n independent variables and the arguments $q_1, q_2, \ldots, q_n$ are specified by fuzzy numbers $\tilde{q}_1, \tilde{q}_2, \ldots, \tilde{q}_n$, then the value of the function $\tilde{y} = f(\tilde{q}_1, \tilde{q}_2, \ldots, \tilde{q}_n)$ is the fuzzy number $\tilde{y}$:

$$\mu_{\tilde{y}}(y^*) = \sup_{\substack{y^* = f(q_1^*, q_2^*, \ldots, q_n^*) \\ q_i^* \in S_{q_i},\ i = \overline{1, n}}} \min_{i = \overline{1, n}} \left(\mu_{\tilde{q}_i}(q_i^*)\right), \tag{13}$$

where S_{q_i} is the support of the fuzzy number $\tilde{q}_i$.

The application of the generalization principle involves a large volume of computational procedures. Therefore for fuzzy numbers in α-form (6) we propose a modified generalization principle.

Definition 5. If the function $y = f(q_1, q_2, \ldots, q_n)$ of n independent variables is given and its arguments q_i are fuzzy numbers $\tilde{q}_i$ defined by the α-form (6) ($i = \overline{1, n}$), then the value of the function $\tilde{y} = f(\tilde{q}_1, \tilde{q}_2, \ldots, \tilde{q}_n)$ is the fuzzy number $\tilde{y}$ represented in α-form:

$$\tilde{y} = \bigcup_{\alpha \in [0, 1]} \left(\underline{y}_\alpha, \overline{y}_\alpha\right),$$

where

$$\underline{y}_\alpha = \inf\left(f\left(q_{1_\alpha}; q_{2_\alpha}, \ldots, q_{n_\alpha}\right)\right);\ \overline{y}_\alpha = \sup\left(f\left(q_{1_\alpha}, q_{2_\alpha}, \ldots, q_{n_\alpha}\right)\right);$$

$$q_{i_\alpha} = \left[\underline{q}_{i_\alpha}, \overline{q}_{i_\alpha}\right],\quad i = \overline{1, n}.$$

Assertion 3. If the fuzzy numbers are defined in α-form, then Definitions 4 and 5 produce the same generalization of the models.

Proof. Application of formula (13) gives

$$\tilde{y} = \sum_{b=1}^{z} \mu_{\tilde{y}}(y_b) \,/\, y_b.$$

Since the membership function is convex, the α-form of the fuzzy number $\tilde{y}$ is written as

$$\tilde{y} = \bigcup_{\alpha \in [0, 1]} \left(\underline{y}_\alpha, \overline{y}_\alpha\right),$$

where $\underline{y}_\alpha = \min\limits_{\forall y_b : \mu_{\tilde{y}}(y_b) = \alpha} (y_b)$; $\overline{y}_\alpha = \max\limits_{\forall y_b : \mu_{\tilde{y}}(y_b) = \alpha} (y_b)$; $b = \overline{1, z}$.

TABLE 1. Characteristics of Operators in Algorithm 1

Operator	$\bar{p}^1_A$	$\bar{t}_A$
A_1	<0.91, 0.95, low>	<10, 26, low>
A_2	<0.95, 0.99, average>	<7, 15, above average>
A_3	<0.95, 0.99, above average>	<1, 5, below average>
A_4	<0.9999, 0.99998, average>	<14, 20, average>
A_5	<0.9999, 1, average>	<1, 2, below average>
A_6	<0.9999, 0.99998, average>	<1, 2, below average>
A_7	0.9999	<10, 18, above average>

TABLE 2. Characteristics of Logical Conditions in Algorithm 1

Logical condition	$\bar{k}^{11}_\omega$	$\bar{k}^{00}_\omega$	$\bar{t}_\omega$
ω_1	<0.93, 0.97, average>	<0.6, 0.7, average>	<10, 26, low>
ω_2	<0.93, 0.97, average>	<0.6, 0.7, below average>	<1, 2, below average>
ω_3	<0.9999, 0.99998, below average>	<0.9999, 0.99996, average>	1
ω_4	<0.96, 0.98, high>	<0.6, 0.7, above average>	<10, 18, above average>

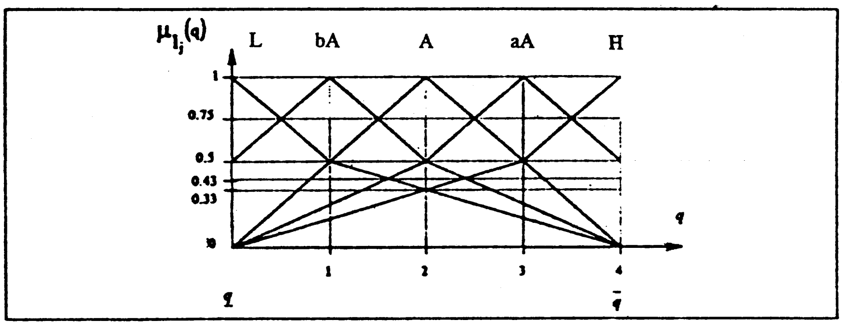

Fig. 1. Piecewise-linear membership functions of fuzzy terms.

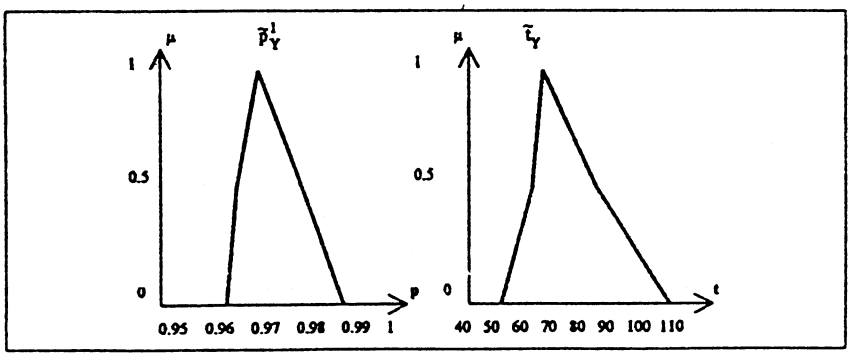

Fig. 2. Membership functions of fuzzy values of execution reliability of algorithm (1).

To prove the assertion, it suffices to show that all y_b such that $\mu_{\tilde{y}}(y_b) = \alpha$ $(b = \overline{1, z})$ are determined by the arguments $q_i \in \left[\underline{q}_{i_\alpha}, \bar{q}_{i_\alpha}\right]$ $(i = \overline{1, n})$. In other words, using arguments at other α-levels does not add new values $y = y^*$ with degree of membership $\mu_{\tilde{y}}(y^*) = \alpha$. Indeed, by convexity of the membership function $\forall \alpha_1, \alpha_2 : \alpha_1 > \alpha_2 \Rightarrow \left[\underline{q}_{i_{\alpha 1}}, \bar{q}_{i_{\alpha 1}}\right] \subset \left[\underline{q}_{i_{\alpha 2}}, \bar{q}_{i_{\alpha 2}}\right]$.

Hence it follows that new values y^* can be obtained by taking argument values q_i^* at lower α-levels $(\mu_{\tilde{q}_i}(q_i^*) < \alpha)$. However, by formula (13), $\min_{i = \overline{1, n}} \left(\mu_{\tilde{q}_i}(q_i^*)\right) < \alpha$. This proves Assertion 3.

The modified generalization principle corresponding to Definition 4 easily produces fuzzy analogues of reliability models of algorithm execution. For instance, for fuzzy input data formula (4) takes the form

$$\tilde{p}_D = \bigcup_{\alpha \in [0, 1]} \left(\underline{p}^1_{D_\alpha}, \bar{p}^1_{D_\alpha}\right), \quad \tilde{t}_D = \bigcup_{\alpha \in [0, 1]} \left(\underline{t}_{D_\alpha}, \bar{t}_{D_\alpha}\right),$$

$$\underline{p}^1_{D_\alpha} = \frac{\underline{p}^1_{A_\alpha} \underline{k}^{11}_{\omega_\alpha}}{1 - \underline{p}^1_{A_\alpha}\left(1 - \underline{k}^{11}_{\omega_\alpha}\right) - \left(1 - \underline{p}^1_{A_\alpha}\right) \underline{k}^{00}_{\omega_\alpha}},$$

$$\bar{p}^1_{D_\alpha} = \frac{\bar{p}^1_{A_\alpha} \bar{k}^{11}_{\omega_\alpha}}{1 - \bar{p}^1_{A_\alpha}\left(1 - \bar{k}^{11}_{\omega_\alpha}\right) - \left(1 - \bar{p}^1_{A_\alpha}\right) \bar{k}^{00}_{\omega_\alpha}}$$

$$\underline{t}_{D_\alpha} = \frac{\underline{t}_{\omega_\alpha} + \underline{t}_{A_\alpha}}{1 - p_1\left(1 - \bar{k}^{11}_{\omega_\alpha}\right) - \left(1 - p_1\right) \underline{k}^{00}_{\omega_\alpha}},$$

$$\bar{t}_{D_\alpha} = \frac{\bar{t}_{\omega_\alpha} + \bar{t}_{A_\alpha}}{1 - p_2\left(1 - \underline{k}^{11}_{\omega_\alpha}\right) - \left(1 - p_2\right) \bar{k}^{00}_{\omega_\alpha}},$$

$$p_1 = \begin{cases} \underline{p}^1_{A_\alpha}, & 1 - \bar{k}^{11}_{\omega_\alpha} - \underline{k}^{00}_{\omega_\alpha} > 0, \\ \bar{p}^1_{A_\alpha}, & 1 - \bar{k}^{11}_{\omega_\alpha} - \underline{k}^{00}_{\omega_\alpha} < 0, \end{cases} \qquad p_2 = \begin{cases} \bar{p}^1_{A_\alpha}, & 1 - \underline{k}^{11}_{\omega_\alpha} - \bar{k}^{00}_{\omega_\alpha} > 0, \\ \underline{p}^1_{A_\alpha}, & 1 - \underline{k}^{11}_{\omega_\alpha} - \bar{k}^{00}_{\omega_\alpha} < 0. \end{cases}$$

Models (2) and (3) can be similarly generalized to the fuzzy case.

Example 4. The l-forms of the input data for estimating the execution reliability of algorithm Y are collected in Tables 1 and 2. Using models (2)-(4) generalized to the fuzzy case, we obtain α-forms of the time requirements $(\tilde{t}_Y)$ and the probability $(\tilde{p}_Y^1)$ of error-free execution of algorithm Y:

$$\tilde{p}^1_Y = (0.959, 0.984)_0 \cup (0.962, 0.974)_{0.5} \cup (0.965, 0.965)_1,$$
$$\tilde{t}_Y = (49.4, 104.2)_0 \cup (59.7, 82.4)_{0.5} \cup (60, 60)_1.$$

These estimates are shown in Fig. 2 and may be interpreted in l-form as

$$\tilde{p}^1_Y = \langle 0.959, 0.984, \text{below average}\rangle, \; \tilde{t}_Y = \langle 49.4, 104.2, \text{below average}\rangle$$

CONCLUSIONS AND POSSIBLE GENERALIZATIONS

The main obstacle to the application of probabilistic reliability models is the absence of input data that reflect real-life conditions describing the operation of the system. The method proposed in this study for estimating the reliability of algorithms is one of the formal approaches to resolving the difficulty with input data by means of linguistic expert information and the principle of fuzzy generalization of analytical models. Contrary to the theory of Markov and semi-Markov models used in reliability theory, the proposed technique is free from complex mathematical procedures that involve convolution of the distribution functions of the system sojourn time in a given state. The proposed system of definitions and assertions is applicable not only in reliability theory: it is also useful in other modeling problems where the input data depend on many factors and expert judgments are the only source of information about these factors.

REFERENCES

1. A. I. Gubinskii, Reliability and Operating Quality of Ergatic Systems [in Russian], Nauka, Leningrad (1982).
2. A. P. Rotshtein, "Probabilistic-algorithmic models of man–machine systems," Avtomatika, No. 5, 81-86 (1987).
3. A. P. Rotshtein, "Algebraic design of fault-free work processes," Usp. Sist. Mash., No. 6, 92-102 (1990).
4. V. S. Korolyuk and A. F. Turbin, Phase Aggregation of Complex Systems [in Russian], Vishcha Shkola, Kiev (1976).
5. B. A. Kozlov and I. A. Ushakov, Handbook of Reliability Calculations in Radio Electronics and Automation [in Russian], Sovetskoe Radio, Moscow (1975).
6. A. Rotshtein, "Fuzzy reliability analysis of man–machine systems," in: Reliability and Safety Analysis Under Fuzziness, Studies in Fuzziness, Physika-Verlag, Springer-Verlag, Berlin (1994), pp. 245-270.
7. L. Zadeh, The Notion of Linguistic Variable and Its Application to Approximate Decision Making [Russian translation], Mir, Moscow (1976).
8. V. M. Glushkov, G. E. Tseitlin, and E. L. Yushchenko, Algebra, Languages, Programming [in Russian], Naukova Dumka, Kiev (1978).
9. A. N. Borisov, O. A. Krumberg, and I. P. Fedorov, Decisions Using Fuzzy Models: Applications [in Russian], Zinatne, Riga (1990).