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Abstract approved : ______________________
Anton Polensek

To apply accurate procedures of structural analysis that are now available, the behavior of nailed joints in light-frame wood buildings under long-term loads needs to be studied. Such a behavior can best be evaluated by testing specimens under constant loads, which requires relatively simple testing arrangements. To provide for a practical use of the constant-load test results, theoretical models were developed that predict the behavior of nailed joints under varying loads that are subjected to wood structures in service.

Existing models and principles were used to develop five new general models, all of which account for the nonlinear viscous-viscoelastic behavior of nailed joints. The models incorporated the modified superposition principle and strain hardening principle. Heaviside function and Fourier series were also incorporated to describe varying loads that can be either discrete or mathematically defined continuous function.

To develop experimental data needed for the formulation and verification of the models developed,

joints made of Douglas-fir lumber, plywood and 6d nails were tested under four constant and four varying loads. The experimental data for constant loads were used to formulate specific theoretical models which were further modified for varying loads. The comparison between the predicted and the corresponding test results shows a very good agreement for all the specific models. The models that include the modified superposition principle are the most accurate and the simplest to apply to nailed joints under discrete load functions. Fourier series representation of varying-load functions shows a great potential for practical applications, because it can represent the service loads more accurately than the discrete approximation.

The specific models presented are limited to the type of joints used in this investigation. Other types of joints need to be tested under constant loads to develop appropriate equations that describe their creep behavior under varying loads.

PREDICTION MODELS FOR CREEP BEHAVIOR OF NAILED JOINTS BETWEEN DOUGLAS-FIR LUMBER AND PLYWOOD

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

PREDICTION MODELS FOR CREEP BEHAVIOR OF NAILED JOINTS BETWEEN DOUGLAS-FIR LUMBER AND PLYWOOD

I. INTRODUCTION

In light-framed wood buildings, nailed joints connect framing and sheathing elements into a composite structural system. Important characteristics of composite behavior are load sharing between elements of various stiffnesses and partial composite action between framing-lumber and sheathing materials. These two characteristics have recently been incorporated into theoretical analysis of walls (32,33,34) and floors (35,37). However, the methods have not been applied practically on a wide scale, partially because the behavior of nailed joints under long-term loading lacks definition. Thus, deformation of nailed joints under long-term loading or creep should be studied. Theoretical models and procedures are needed to predict creep under varying load from the data obtained from constant-load tests. This investigation is aimed at developing and verifying such models. The models investigated are Five-element Model (5-E), Modified Five-element Model (M5-E), Viscous-viscoelastic Model 1 (V-VE1), Viscous-viscoelastic Models 2 (V-VE2), and Viscous-viscoelastic Model 3 (V-VE3).

1.1. Justification

In the actual environment, structural wood systems are under loads that are caused by gravity, winds, earthquakes and humans. The response of structural wood systems to these loads often is nonlinear and very complicated to define. Therefore, current design procedures for wood structures use simplified assumptions. For example, wood-stud wall system is represented by a set of identical, independent beams/columns. Such a representation does not account for load sharing among studs and partial composite action between wall coverings and studs. However, procedures were recently developed that consider load sharing and composite action in wall and floor systems (31,32,33,34,36). In order to apply these procedures, load-duration and creep effects on physical properties of lumber, sheathing materials and joints needed to be defined.

Load-duration behavior of lumber has been extensively studied (7,16,38). Similar studies have been also conducted on various types of composition boards (6,25,28, 29,30,31). On the other hand, studies on load-duration behavior of joints are only few (15,17,23,36), even though nailed joints are the most common type of joints in wood constructions. Therefore, studying creep behavior of nailed joints is essential in improving the existing design of wood structures.

The composite action between wood frames and wall coverings is a substantial factor in overall strength and stiffness of light-framed wood buildings (36). The degree of composite action depends on the stiffness of joints which decreases under long-term loads. Therefore, long-term behavior of nailed joints should be known before the composite action can be included into a design procedure that gets approved by building codes.

Most studies on creep behavior of nailed joints have been limited to the behavior under constant loads, because constant loads are easy to apply in testing. However, the actual service loads on wood buildings are not constant but vary with time. Long-term varying loads are very difficult and expensive to apply in the laboratory, because each load change usually requires time-comsuming manual manipulation by research personnel. Furthermore, complexities are introduced by variation in material properties. A testing program that would include all the major materials and loads is not practical due to a prohibitively large number of specimens.

A practical alternative is offered by testing under constant loads, because they require simple arrangements and procedures. Thus, if creep behavior under varying loads could be predicted from the data of tests under constant loads, much time and money would be saved in evaluating the behavior of wood system under varying loads. One such a model had been developed and demonstrated the possibility of predicting the long-term performance of nailed joints under varying loads (36). The model was developed and verified for increasing load function only. Therefore, it needs to be developed and verified for decreasing load functions. Furthermore, additional models which are more general, easy to use and perhaps more accurate are needed, because the model accuracy may depend on the type of joints and load functions.

1.2. Objectives

The overall objective was to theoretically model creep behavior of nailed joints. The specific objectives were :

1. To evaluate the feasibility of existing theoretical concepts to model creep behavior of nailed joints,
2. To modify the most promising existing concepts and develop theoretical models and computer programs that accurately predict creep of nailed joints under varying load functions, and
3. To assess the accuracy of the models and computer programs by physical testing of typical nailed joints.

II. LITERATURE REVIEW

Wood has long been recognized and studied as a viscoelastic material. Most sheathing materials in U.S. housing construction are wood-based and, thus, can be regarded as viscoelastic. A few viscoelastic models have been developed to study the creep behavior of wood and wood-based materials, but studies on creep of nailed joints are scarce and strictly experimental (17,23). Only one example of theoretical modeling was found in the literature (36); its significance was in demonstrating the feasibility of using viscoelastic model for creep of nailed joints.

Next, the most important existing viscoelastic models are first introduced and then studied for possible applications to nailed joints. The models are four-element, three-integral representation and viscous-viscoelastic models.

2.1. Four-element model

The four-element or Burger's model is one of the most widely used mechanical models for viscoelastic materials. The basic elements of this model are spring and dashpot which represent elasticity and viscosity, respectively. Combinations of springs and dashpots can represent the behavior of many materials quite accurately. The simplest combination that has one spring and one dashpot in series is the Maxwell model (Figure 2.1a) and the Kelvin model with the spring and dashpot in parallel (Figure 2.1b). The Maxwell model simulates the instantaneous elastic displacement and recovery. The Kelvin model shows the delayed elasticity. But neither of these two models can describe the behavior of viscoelastic materials.

The simplest model that does describe the viscoelastic behavior successfully is a four-element model (Figure 2.2) which is a combination of Maxwell and Kelvin model. Its constitutive equation can be derived from the strain response of the model under constant load :

$$\varepsilon(t) = \sigma/E_1 + \sigma/E_2\{1-EXP(-E_2 t/\eta_2)\} + \sigma t/\eta_1 \qquad (2\text{-}1)$$

in which $\varepsilon(t)$ = strain as a function of time, t; σ = stress; E_1 and E_2 = spring stiffnesses; and η_1 and η_2 = dashpot viscosities. In four-element model, the first, second and third term represent instantaneous elastic, delayed elastic and viscous strain, respectively.

There are several procedures for evaluating parameters of the four-element model. For instance, Moslemi (27) applied such a model to describe creep behavior of hardboard under static ramp loading. He divided the total creep into three parts : instantaneous elastic, delayed elastic and viscous strain. The

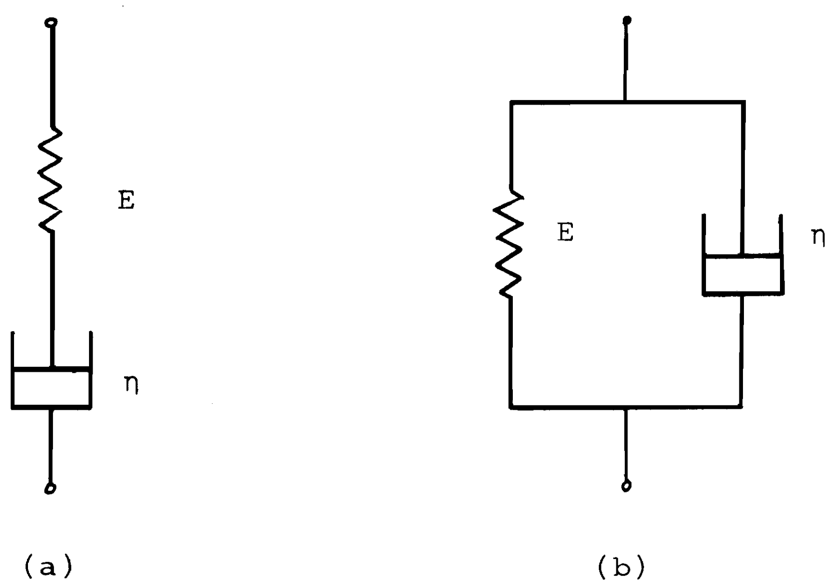


Figure 2.1. Maxwell (a) and Kelvin model (b). (E = spring stiffness, η = dashpot viscosity)

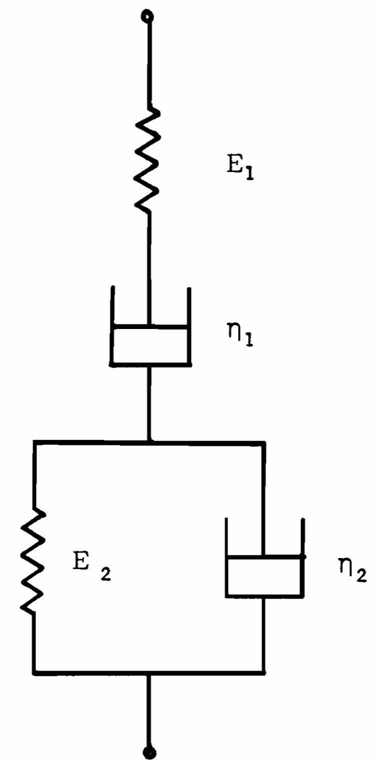


Figure 2.2. Four-element or Burger's model.

constitutive equation developed by him under load function $P(t) = At$ was :

$$\varepsilon(t) = At/E+[At/E+A\eta_1/E^2\{EXP(-Et/\eta_1)-1\}]+ At^2/2\eta_2 \qquad (2\text{-}2)$$

in which all symbols were defined earlier. In this model, the stiffness of both springs should be equal.

Szabo and Ifju (38) applied a four-element model to describe creep of wood beams under stresses due to moisture adsortion and desorption. They obtained the parameters of the constitutive equation by assuming that the slope of strain-time curve of specimens under constant load remained constant after t > 120 hours.

Pierce et al. (6,28,29,30) found the constitutive equation of the four-element model for chipboard by employing constant-load tests :

$$\varepsilon(t) = B_1+B_2\{1-EXP(-B_3t)\}+B_4t \qquad (2\text{-}3)$$

in which B_1, B_2, B_3 and B_4 are the parameters obtained by nonlinear least square curve fitting of experimental data. The model was applied to study the influence of moisture content, stress (6,29,30) and temperature (6) on creep of chipboard.

Pierce et al. (31) modified equation (2-3) to describe the nonlinear viscosity of chipboard. They replaced the linear dashpot in the Maxwell model by a nonlinear dashpot (Figure 2.3) in which the viscosity was expressed as a power function of time. The corresponding constitutive equation has five parameters, among which B and B are nonlinear :

$$\varepsilon\ (t) = B_1 + B_2\{1-EXP(-B_3t)\} + B_4t^{B_5} \qquad (2\text{-}4)$$

Again, nonlinearleast squares curve fitting was employed to get Bi, i = 1,...,5.

Thus, the existing investigations demonstrate that four-element model can successfully predict creep of wood and wood-based materials. Therefore, it was expected to be also successful to predict creep of nailed joints.

2.2. Three-integral representation

Multiple-integral representations have been widely used to predict the behavior of viscoelastic material, such as plastic, under long-term loads (8,11,12,26,27). One form is a three-integral representation which has been recognized as the one that provides adequate exactness and simplicity in representing nonlinear viscoelasticity. In deriving three-integral equations for continuous load functions, Findley et al. (11) applied first, a continuum mechanics approach and then, a modified superposition principle, both of which gave this expression :

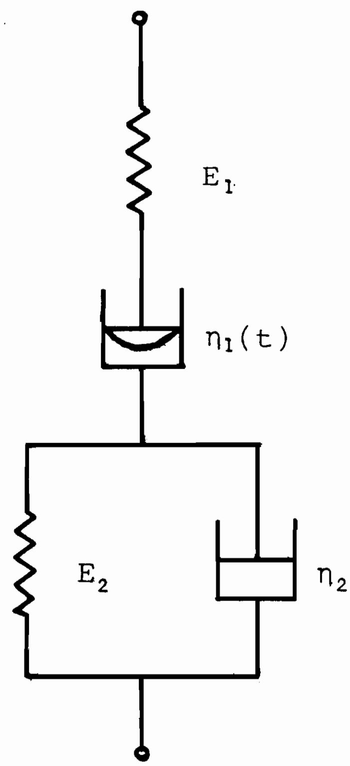


Figure 2.3. Nonlinear four-element model.
(η_1 (t) = viscosity of nonlinear dashpot)

$$\varepsilon(t) = \int_o^t F_1(t-Z_1)\,\dot{\sigma}(Z_1)dZ_1 + \int_o^t\int_o^t F_2(t-Z_1, t-Z_2)\,\dot{\sigma}(Z_1)\,\dot{\sigma}(Z_2)dZ_1dZ_2 + \int_o^t\int_o^t\int_o^t F_3(t-Z_1, t-Z_2, t-Z_3)\,\dot{\sigma}(Z_1)\,\dot{\sigma}(Z_2)\,\dot{\sigma}(Z_3)dZ_1dZ_2dZ_3 \qquad (2\text{-}5)$$

in which Z_1, Z_2 and Z_3 = arbitrary time points between zero and t; $\dot{\sigma}(t)$ = derivative of stress with respect to t; and $F_1(t)$, $F_2(t)$ and $F_3(t)$ are known as kernel functions (8,11,12), which represent the time-dependent properties of material and can be derived by testing specimens under constant stress.

For constant stress, equation (2-5) becomes (11,24, 26,36) :

$$\varepsilon(\sigma, t) = F_1(t)\sigma + F_2(t)\sigma^2 + F_3(t)\sigma^3 \qquad (2\text{-}6)$$

in which $F_1(t)$, $F_2(t)$ and $F_3(t)$ are the same kernel functions as those in equation (2-5).

Several investigations (8,11,12,24,26,27) have shown that kernel functions can be expressed as power functions:

$$Fi(t) = Fi^o + Fi^+(t) \qquad (2\text{-}7)$$

where Fi^o = constant; $Fi^+(t)$ = power function of time obtained from constant load tests; and i = 1,2,3.

In equation (2-5), F_1, F_2 and F_3 are functions of one, two and three variables, respectively. Numerous tests are required to evaluate these functions and no one has determined all the functions that are required for any material under all possible stress conditions (11). Findley et al. (11,12) developed three simplified forms that can be evaluated from tests under constant loads. First is the product form :

$$F_2(t-Z_1,t-Z_2) = [F_2(t-Z_1)F_2(t-Z_2)] \quad (2\text{-}8)$$

$$F_3(t-Z_1,t-Z_2,t-Z_3) = [F_3(t-Z_1)F_3(t-Z_2)F_3(t-Z_3)] \quad (2\text{-}9)$$

Second is the first additive form :

$$F_2(t-Z_1,t-Z_2) = F_2(2t-Z_1-Z_2) \quad (2\text{-}10)$$

$$F_3(t-Z_1,t-Z_2,t-Z_3) = F_3(3t-Z_1-Z_2-Z_3) \quad (2\text{-}11)$$

and third is the second additive form :

$$F_2(t-Z_1,t-Z_2) = 1/2[F_2(t-Z_1)+F_2(t-Z_2)] \quad (2\text{-}12)$$

$$F_3(t-Z_1,t-Z_2,t-Z_3) = 1/3[F_3(t-Z_1)+F_3(t-Z_2)+ F_3(t-Z_3)] \quad (2\text{-}13)$$

These forms of kernel functions were applied successfully to several man-made materials such as plastic (11,12) and nailed joint (36).

An alternative to these forms is the modified superposition principle which was developed on the basis of the Boltzmann superposition principle for linear materials (11). Because of its importance, it is discussed in detail next.

2.2.1. Modified superposition principle (MSP)

MSP was developed by Findley et al. (8,11) to describe the nonlinear behavior of viscoelastic materials, such as plastic (8,12,26), aluminum (9,10,19,20) and stainless steel (3,4,5,24).

The development was based on the Boltzmann linear superposition principle which is illustrated in Figure 2.4. The creep displacement in Figure 2.4b can be obtained by resolving the loading stress function (Figure 2.4a) into two constant stresses shown in Figures 2.4c and 2.4e and then, superimposing the corresponding creep displacements shown in Figures 2.4d and 2.4f. However, this principle can be used for linear materials only (11).

MSP is a nonlinear superposition of creep displacements as illustrated in Figure 2.5. Varying stresses in Figure 2.5a are resolved into two stresses shown in Figures 2.5c and 2.5e. Superposition of resulting creep displacements in Figures 2.5d and 2.5f produces the creep displacement in Figure 2.5b under stress function shown in Figure 2.5a.

For stepwise stresses, MSP can be expressed as the following sum(11) :

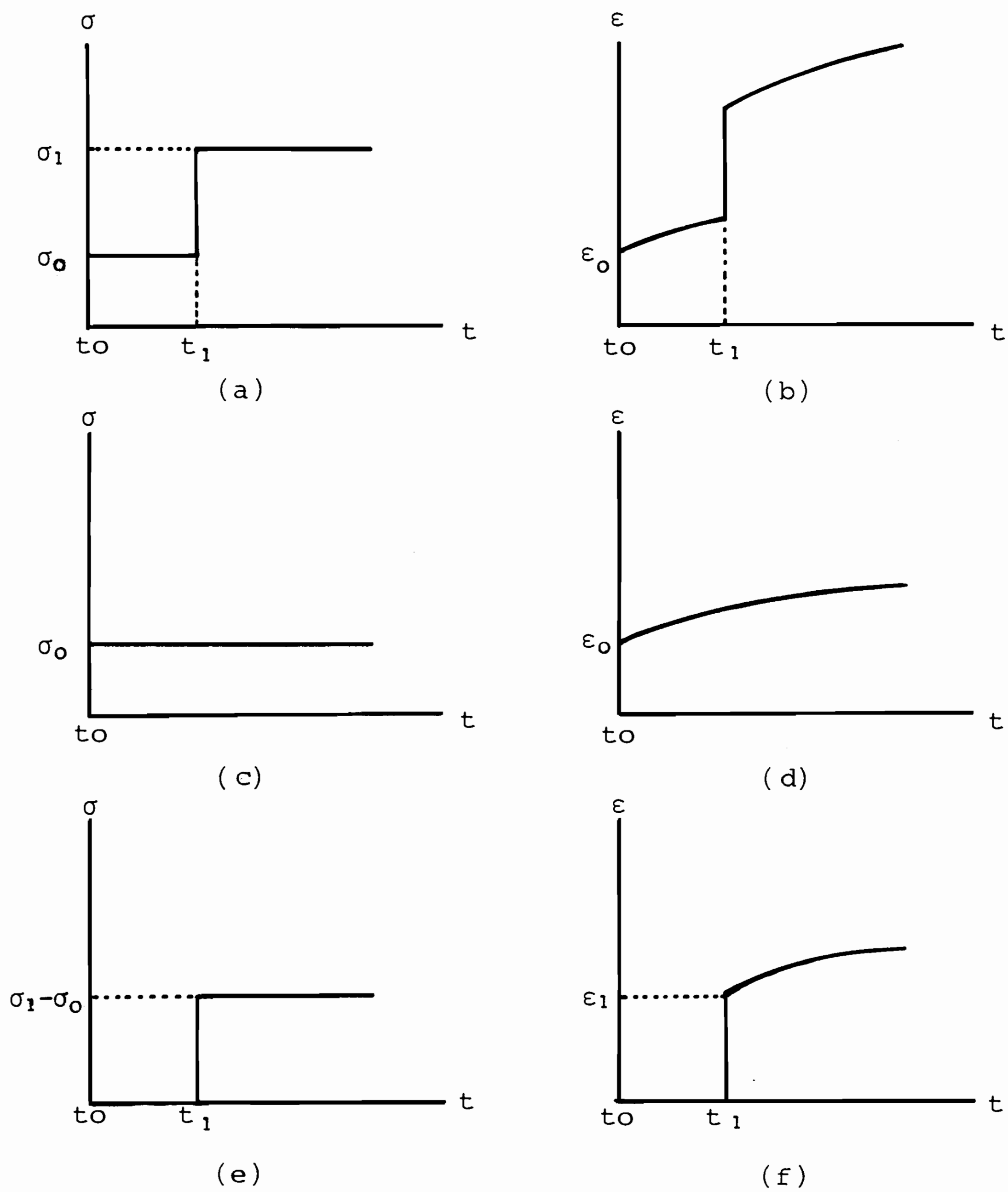


Figure 2.4. Boltzmannn superposition principle;(a) stress function, (b) strain-time curve, (c) first step in stress function, (d) strain under the first step, (e) second step in stress function, and (f) strain under the second step.

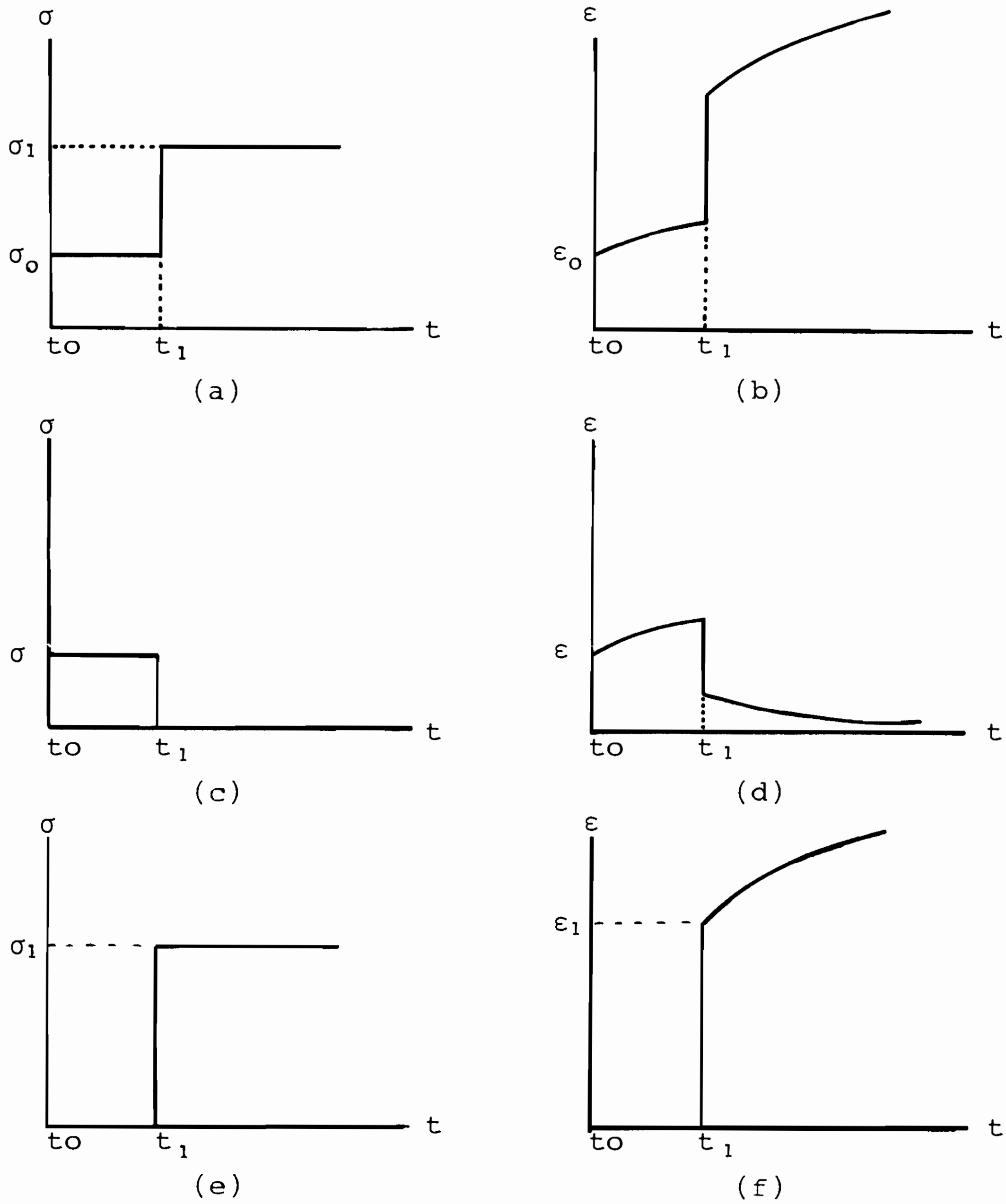


Figure 2.5. Modified superposition principle; (a) stress function, (b) strain-time curve, (c) first step in stress function, (d) strain under the first step, (e) second step in stress function, and (f) strain under the second step.

$$\varepsilon(t) = \sum_{i=o}^{N-1} [f(\sigma_i, t-t_i) - f(\sigma_{i-1}, t-t_i)], \quad t>t_{N-1} \qquad (2\text{-}14)$$

where ε = strain; N = number of stressing steps; σ_i = stress level for i = 1,...,(N-1); and $f(\sigma, t)$ = creep function. For continuous stress functions, the strain can be expressed as an integral form, that is, as an addition of an infinite number of infinitesimal steps of stress (11) :

$$\varepsilon(t) = \int_o^t \frac{\partial f(\sigma(Z), t-Z)}{\partial \sigma(Z)} \frac{d\sigma(Z)}{dZ} dZ \qquad (2\text{-}15)$$

where Z is the integration variable identified in equation (2-5).

For instance, if a creep function is obtained by a three-integral form under constant load (equation(2-6)), then the creep displacement under varying stress function in Figure 2.5a can be obtained from equations (2-6) and (2-14) :

$$\begin{aligned} \varepsilon(t) &= f(\sigma o, t-to) + f(\sigma_1, t-t_1) - f(\sigma o, t-t_1) \\ &= F_1(t-to)\sigma o + F_2(t-to)\sigma o^2 + F_3(t-to)\sigma o^3 + \\ &\quad F_1(t-t_1)\sigma_1 + F_2(t-t_1)\sigma_1^2 + F_3(t-t_1)\sigma_1^3 - \\ &\quad F_1(t-t_1)\sigma_o - F_2(t-t_1)\sigma_o^2 - F_3(t-t_1)\sigma_o^3 \end{aligned} \qquad (2\text{-}16)$$

The equations obtained by MSP can be used to represent creep under varying-stress functions.

Originally, three-integral representation in equation (2-5) was proposed to represent creep under arbitrary stress functions. However, it is practically impossible to get the kernel functions in equation (2-5). Therefore, MSP provides a viable alternative in expressing the creep of nailed joints under varying stress.

2.3. Viscous-viscoelastic model

Most of the existing works have concentrated on creep of materials that have the same instantaneous displacement and recovery. However, there are many materials, including nailed joints, which do not fully recover the instantaneous displacement upon unloading. For such materials, the viscous-viscoelastic model developed by Findley et al. (3,4,5,9,10,19,20) is very useful.

The basis for this model is in subdividing the total creep strain into five components :

$$\varepsilon_t = \varepsilon_e + \varepsilon_p + \varepsilon_{pv} + \varepsilon_{NV} + \varepsilon_{de} \qquad (2\text{-}17)$$

where ε_t = total strain; ε_e = instantaneous elastic strain which is time-independent and recoverable; ε_p = instantaneous plastic strain which is time-independent and nonrecoverable; ε_{pv} and ε_{NV} = positive and negative viscous strains, respectively, which are time-dependent and nonrecoverable; and ε_{de} = delayed elastic strain which is time-dependent and recoverable. This concept was applied to develop theoretical models in this study.

In experiments, Findley et al. used viscoelastic materials, such as stainless steel (3,4,5) and aluminum (9,10,19,20), in which the instantaneous recovery was the same as the instantaneous displacement. Thus, they assumed that the instantaneous plastic strain was zero. For total creep strain, they employed a power function :

$$\varepsilon_t = \varepsilon^{\circ} + \varepsilon^{+} t^n = \varepsilon_e + \varepsilon_{VE} + \varepsilon_v \tag{2-18}$$

in which ε° and ε^{+} are parameters obtained from tests; ε_{pv} and ε_{NV} are expressed as one term that is time-dependent nonrecoverable strain, ε_v; and ε_{de} is expressed as time-dependent recoverable strain, ε_{VE}. Three-integral representation was then applied to express the total creep :

$$\begin{aligned} \varepsilon_t &= F_1\sigma + F_2\sigma^2 + F_3\sigma^3 \\ &= (F_1^{\circ} + F_2^{+}(t))\sigma + (F_2^{\circ} + F_2^{+}(t))\sigma^2 + \\ &\quad (F_3^{\circ} + F_3^{+}(t))\sigma^3 \end{aligned} \tag{2-19}$$

where $F_i = F_i^{\circ} + F_i^{+}(t)$, i=1,2,3, are kernel functions; and F_i° and F_i^{+} are parameters obtained from experiments.

Findley et al. postulated that ε_{VE} and ε_v are functions of the same power of time, n. Therefore, equation (2-18) becomes :

$$\varepsilon_t = \varepsilon_e + Bt^n + Mt^n \qquad (2\text{-}20)$$

where B and M are experimental coefficients of recoverable and nonrecoverable strain terms, respectively. Equation (2-20) is often written as :

$$\varepsilon_t = \varepsilon_e + M(R+1)t^n \qquad (2\text{-}21)$$

where R = B/M. By comparing equations (2-18) and (2-21), expressions for ε_{VE} and ε_v can be written as :

$$\varepsilon_{VE} = \{R/(R+1)\}\,\varepsilon^+ t^n \qquad (2\text{-}22)$$

$$\varepsilon_v = \{1/(R+1)\}\,\varepsilon^+ t^n \qquad (2\text{-}23)$$

Expression for time-dependent strain, $F(\sigma)$, follows from equations (2-18) and (2-19) :

$$\varepsilon^+ t^n = F(\sigma) = F_1^+(t)\,\sigma + F_2^+(t)\sigma^2 + F_3^+(t)\sigma^3 \qquad (2\text{-}24)$$

Findley et al. found that no strain was developed under the stress less than certain limit. To incorporate this concept into the model, they further introduced the creep limit, $\sigma *$, into equation (2-24) :

$$F(\sigma) = F_1^+(t)(\sigma - \sigma *) + F_2^+(t)(\sigma - \sigma *)^2 + F_3^+(t)(\sigma - \sigma *)^3 \qquad (2\text{-}25)$$

Next, Findley et al. applied MSP to recoverable strain of equation (2-22) and strain hardening principle to nonrecoverable strain of equation (2-23). The result was the expression for creep under varying loads. The constitutive equation for ε_{VE} under three-step increasing loads was found to be (19) :

$$\varepsilon_{VE}(t) = \{R/(R+1)\}[F(\sigma_1)\{t^n-(t-t_1)^n\}+ F(\sigma_2)\{(t-t_1)^n-(t-t_2)^n\}+ F(\sigma_3)(t-t_2)^n],\ t>t_2 \qquad (2\text{-}26)$$

where $F(\sigma)$ is the same as in equation (2-25).

2.3.1. Strain Hardening principle (SHP)

Many theories have been proposed to predict the creep rate in terms of other variables than stress (13). Among those, SHP has been the most successful (8,11,13). In SHP, strain rate is expressed as a function of strain and stress.

The basis of SHP is the postulation that the creep under constant stress can be represented by a function of stress and time (13), the simplest form of which is :

$$\varepsilon = A\,\sigma^n t^m \qquad (2\text{-}27)$$

where A, n and m are constants. Differentiating equation

(2-27) gives the strain rate :

$$d\varepsilon/dt = \dot{\varepsilon} = mA\,\sigma^{n}\,t^{m-1} \qquad (2\text{-}28)$$

Expression for time variable, t, can be obtained by imposing 1/m power on both sides of equation (2-27) and solving for t :

$$t = (\varepsilon)^{1/m}/(A\,\sigma^{n})^{1/m} \qquad (2\text{-}29)$$

which is then substituted into equation (2-28) to obtain :

$$d\varepsilon/dt = mA\sigma^{n}\,[\,\varepsilon^{1/m}/(A\,\sigma^{n})^{1/m}]^{m-1} \qquad (2\text{-}30)$$

or

$$[\,\varepsilon^{(1/m-1)}/m]d\varepsilon = (A\,\sigma^{n})^{1/m}dt \qquad (2\text{-}31)$$

Integrating both sides of equation (2-31) yields :

$$\varepsilon^{1/m} = \int (A\sigma^{n})^{1/m}dt \qquad (2\text{-}32)$$

Therefore,

$$\varepsilon = [\int (A\,\sigma^{n})^{1/m}dt]^{m} \qquad (2\text{-}33)$$

Findley et al. (4,5,9,10) applied this principle to the nonrecoverable strain of equation (2-23) to get the expression for the nonrecoverable viscous strain under three-step increasing load :

$$\varepsilon_v = \{1/(R+1)\}[\{F(\sigma_1)\}^{1/n}(t_1)+\{F(\sigma_2)\}^{1/n}(t_2-t_1)+ +\{F(\sigma_3)\}^{1/n}(t-t_2)]^n \quad , \; t>t_2 \qquad (2\text{-}34)$$

The expression for the total creep under three-step increasing load is the sum of equations (2-26), (2-34) and the instantaneous elastic strain at the time of interest. The prediction of this viscous-viscoelastic model agreed closely with the experimental data for aluminum (9,10,19, 20) and stainless steel (3,4,5).

2.5. Creep and stiffness of nailed joints

There have been many studies (1,14,15,17,22,23,36) on the strength properties of nailed joints, but only a few (17,23,36) have dealt with creep of nailed joints.

Jenkins et al. (17) studied creep-related stiffness loss of nailed joints between Douglas-fir stud and plywood. They approximated the stiffness of nailed joints, K_j, by the secant modulus :

$$K_j(P,t) = (P_j - P_{j-1})/[S_t(P,t_j) - S_t(P,t_{j-1})] \; (\text{lb/in}) \qquad (2\text{-}35)$$

where S_t = total creep; P = load; t = time; and j-1 and j = successive intervals defining the secant modulus. The major problem with this study was the use of linear superposition for creep curves under constant loads to get creep under increasing loads by employing the assumption that creep behavior of nailed joints is linear. Another limitation was in restricting the study within the effect of increasing loads only.

Polensek (36) developed a theoretical model which can be used to predict creep of nailed joints under increasing loads from the data of constant load tests. He fitted the data to power functions :

$$Sc = at^n \qquad (2\text{-}36)$$

in which Sc = creep slip; a and n = constants obtained from experiments; and t = time. Mack (25) used the same power function. Polensek employed the three-integral representation, equation (2-6), to represent the creep under constant load :

$$S(t) = F_1(t)P + F_2(t)P^2 + F_3(t)P^3 \qquad (2\text{-}37)$$

where S(t) = creep slip; P = load; and $F_i(t)$ = kernel functions for i = 1,2,3. The corresponding equation for stepwise load is :

$$S(t) = \sum_{i=o}^{n} (\Delta P_i) F_1(t-t_i) + \sum_{i=o}^{n} \sum_{j=o}^{n} (\Delta P_i)(\Delta P_j) F_2(t-t_i, t-t_j) + \sum_{i=o}^{n} \sum_{j=o}^{n} \sum_{k=o}^{n} (\Delta P_i)(\Delta P_j)(\Delta P_k) F_3(t-t_i, t-t_j, t-t_k) \qquad (2\text{-}38)$$

where $\Delta P_{i, j, k}$ = step size in stepwise load.

To approximate the two- and three-variable kernel functions in equation (2-38), Polensek employed the product and two additive forms which were discussed in Section 2.2. In applying these approximations to nailed joints, he found that any of these three forms gives the accurate predictions of creep slip under increasing loads.

III. THEORETICAL PROCEDURE

It was not known at the beginning of this investigation whether the existing viscoelastic models for wood and wood-based materials can be directly applied to nailed joints. Therefore, several models for materials, such as plastic and aluminum, were studied for possible application to nailed joints. Three of the most promising models were the four-element, three-integral and viscous-viscoelastic model, but they did not include all the mechanisms of nailed joint behavior. Therefore, the three models were modified in this investigation, which produced five new models : Five-element, Modified Five-element and Viscous-viscoelastic Model 1, 2 and 3. This chapter describes the development of each of these models.

3.1. Theoretical principles

The theoretical procedure employed in this study is summarized in Figure 3.1. Five models were developed from the existing models and concepts. To modify the models for varying loads, two approaches were chosen, one based on MSP and SHP, and the other on MSP only. The developments included three load functions : discrete function, Heaviside function and Fourier series. Among those three, discrete load function was used in both approaches, while the other two were used in Approach 2

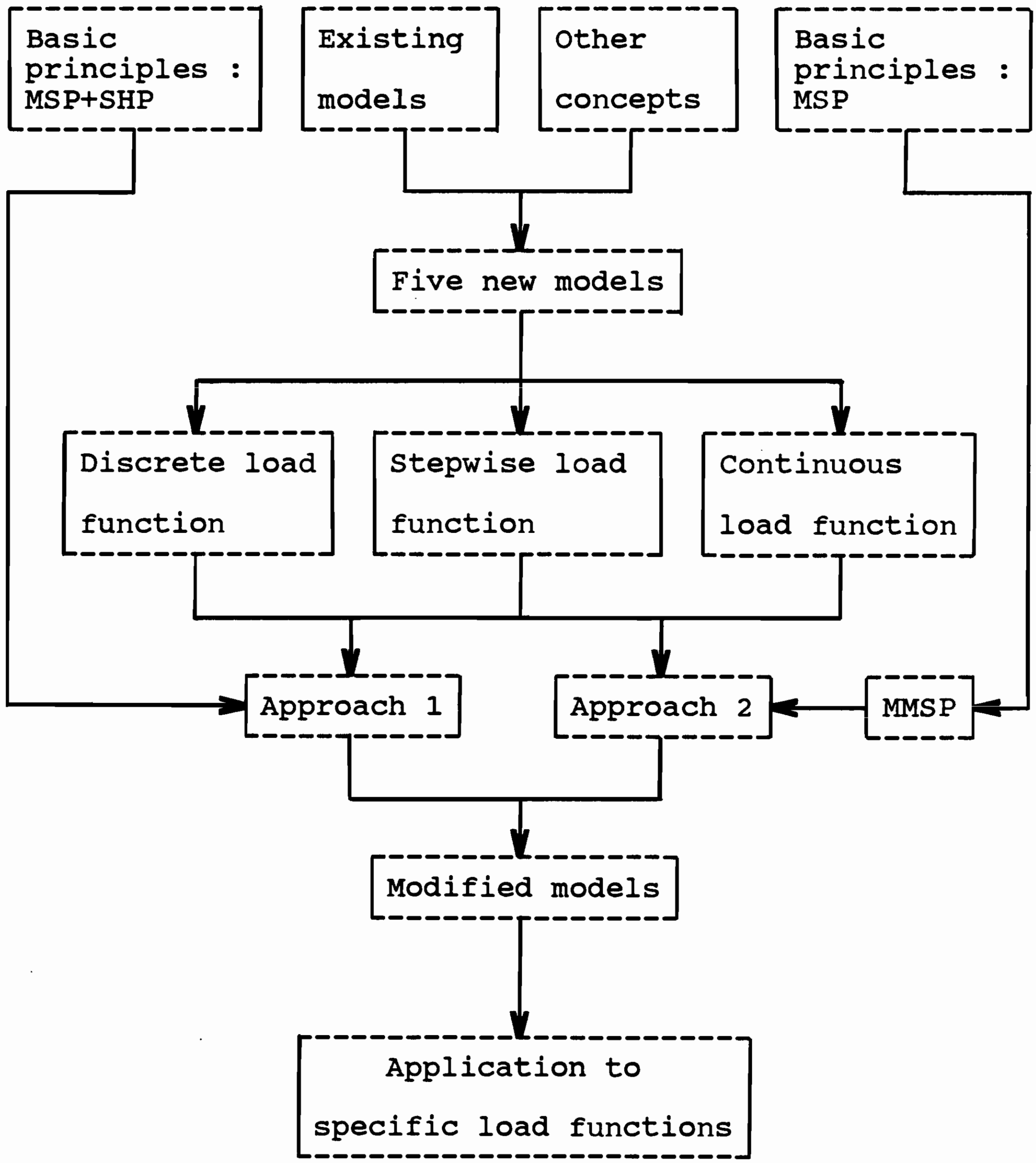


Figure 3.1. Theoretical procedure.

only. The developed models were then applied to specific load functions to get the corresponding expressions for the creep slip. The basic ideas incorporated into modeling are introduced first.

3.1.1. Basic concepts

It is often convenient to divide the time-dependent behavior of viscoelastic materials into instantaneous elastic, delayed elastic and viscous component. The first two components are recoverable but the third is not.

Past investigations have shown that wood and wood-based materials are viscoelastic materials (6,7,16,22,28, 29,30,38). Nails are made of common steel which is also viscoelastic material. Therefore, it may be postulated that nailed joints between lumber and sheathing material also behave viscoelastically. However, crushed wood around embedded nails introduces additional complexity. Preliminary testing in this investigation showed that the time-dependent behavior of nailed joints could not be described accurately by any of the existing viscoelastic models. For instance, Figure 3.2 illustrates a typical behavior of nailed joints under long-term load. The instantaneous recovery upon unloading, DG = BC, is much less than the instantaneous slip, AC, while the instantaneous displacement and recovery are the same for viscoelastic materials.

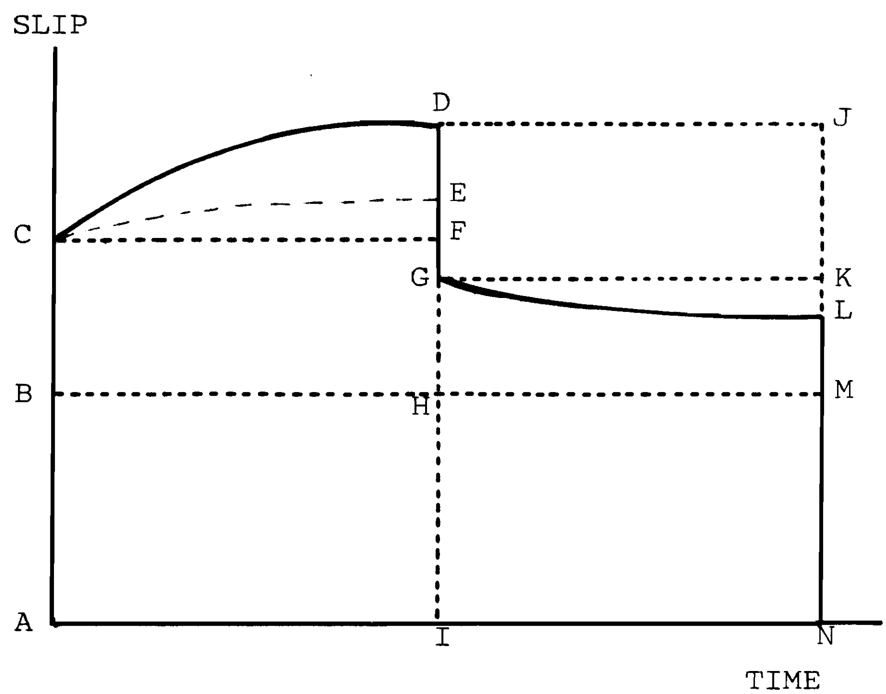


Figure 3.2. Typical creep curve of nailed joints.

In this investigation, three useful assumptions have been adopted. The first is that the instantaneous and delayed elastic displacement are totally recoverable. The second states that the plastic and viscous displacement are not recoverable. The third assumption reasons that because only elastic displacement is totally recoverable, the elastic displacement-versus-time curve should be the same as the recoverable displacement-versus-time curve. Thus, the total slip of nailed joints can be divided into four components, which are similar to the division introduced in the viscous-viscoelastic model discussed in Chapter 2.

In this study, the total slip is visualized as a combination of components that can be explained physically and represented mathematically (Figure 3.2). As mentioned before, the instantaneous elastic slip, BC, is the same as the instantaneous recovery, DG, and time-independent. The delayed elastic slip,CEF, is equal to the delayed recovery, GLK, and time-dependent. The rest in the instantaneous slip, AB, is plastic, time-independent and nonrecoverable. Under constant load, the instantaneous plastic slip is developed immediately upon loading and remains constant during loading and unloading period. The rest of total creep slip, CDE, is viscous, time-dependent and nonrecoverable. Therefore, the total creep slip, ACDI,can be divided into four components which are the instantaneous elastic, BCFH, instantaneous plastic, ABHI, delayed elastic, CEF, and viscous slip, CDE.

The components in Figure 3.2 are similar to those employed by Findley et al. (3,4,5), whose model lacks the instantaneous plastic displacement. However, in nailed joints, the instantaneous plastic slip due to compressed damages of wood fibers around the nail is significant and must be included.

The models developed in this study consist of four components described above, which are connected in series and may be added to give the total creep slip :

$$S_t = Se + Sp + S_{de} + Sv \qquad (3\text{-}1)$$

where S_t = total creep slip; Se = instantaneous elastic slip; Sp = instantaneous plastic slip; S_{de} = delayed elastic slip; and Sv = viscous slip.

3.1.2. Application of MSP to nailed joint (MMSP)

MSP discussed in Section 2.2 was proposed by Findley et al. (8) for materials having the same instantaneous displacement and recovery. In their model, the instantaneous plastic displacement was assumed to be zero, and all the creep displacement was assumed to be recoverable. Thus, the same equation was used to describe both creep displacement and recovery. However, in nailed joints, the instantaneous plastic slip is larger than the instantaneous elastic slip and the viscous slip is not recoverable (Figure 3.2). Therefore, MSP can be applied to elastic slip but not to total creep slip.

In viscous-viscoelastic model, Findley et al.(3,9,19) employed MSP and SHP for recoverable and nonrecoverable displacements, respectively. The same method was employed in this study and is refered to as Approach 1. However, the predictions by Approach 1 were smaller than the experimental data because the viscous slip predicted by SHP was too small. Thus, a further modification was carried out, which resulted in modified MSP or MMSP, and refered to as Approach 2.

Mathematically, MMSP was accomplished as follows. In Section 2.3.1, equation (2-33) defined the viscous displacement predicted by SHP as :

$$\varepsilon v = [\int_o^t (A \sigma^n)^{1/m} dz]^m \tag{3-2}$$

Under stepwise load function, equation (3-2) becomes :

$$\varepsilon v = [(A \sigma_o^n)^{1/m} t_1 + (A\sigma_1^n)^{1/m}(t_2 - t_1) + \ldots + (A \sigma_{N-1}^n)^{1/m}(t - t_{N-1})]^m , \quad t > t_{N-1} \tag{3-3}$$

where N = number of steps. The properties of m power are :

$(A+B+C+\ldots)^m$ $< \alpha$, $= \alpha$ and $> \alpha$ if $m<1$, $m=1$ and $m>1$, respectively in which $\alpha = A^m + B^m + C^m + \ldots$ (3-4)

In equation (3-3), m is always less than one. Therefore, the maximum value of equation (3-3) equals :

$$\varepsilon v = [\int_0^t (A\sigma^n)^{1/m} dz]^m$$
$$< [A\sigma_0^n (t_1)^m + A\sigma_1^n (t_2 - t_1)^m + \ldots +$$
$$A\sigma_{N-1}^n (t - t_{N-1})^m],\ t>t_{N-1} \quad (3\text{-}5)$$

In Approach 2 for nailed joints, the maximum value of equation (3-5) was used to define the viscous slip, Sv, in which load, P, and slip, S, replace stress, σ, and strain, ε. Figure 3.3 graphically illustrates this approach. The load function in Figure 3.3a is composed of two steps. Under each step, the creep slip is divided into two parts, the recoverable and nonrecoverable slip. For the recoverable slip, Sr, MSP can be applied as shown in Figures 3.3c and 3.3d. For the nonrecoverable slip, Sn, the maximum value of equation (3-5) is used, which can be justified as follows. Upon unloading Po at time t_1, the instantaneous plastic slip, Sp, is recovered and the viscous slip, Sv, remains constant (Figure 3.3e). Upon loading P_1 at time t_1, the instantaneous plastic and viscous slip are developed (Figure 3.3f). Finally, combining all four graphs, Figures 3.3c, 3.3d, 3.3e and 3.3f, gives Figure 3.3b which is the total slip, S_t, under

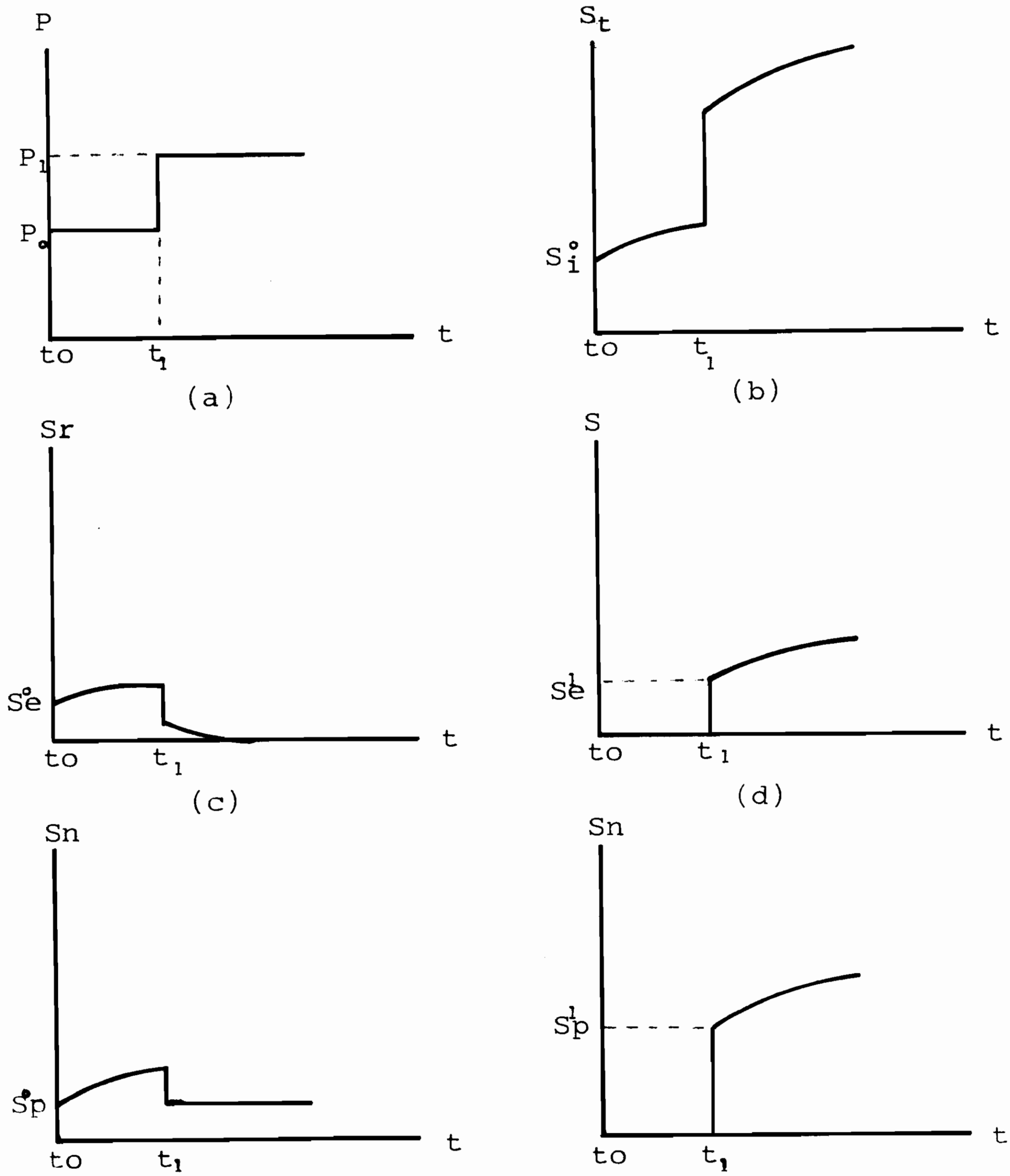


Figure 3.3. MMSP for nailed joints; (a) Load function, (b) slip under load function (a), (c) recoverable slip in the first step, (d) recoverable in the second step, (e) nonrecoverable slip in the first step, and (f) nonrecoverable slip in the second step.

load function in Figure 3.3a. Thus :

$$S_t = Sr(Po,t-to)+Sr(P_1,t-t_1)-Sr(Po,t-t_1)+Sp_1+$$
$$Sv(Po,t_1-to)+Sv(P_1,t-t_1), \quad t>t_1 \qquad (3-6)$$

Approach 1 and Approach 2 involve assuming that the stepwise decreasing load is the same as the stepwise increasing load acting in the opposite direction to the existing load and having magnitude of the maximum load just before unloading minus the actual load, that is, it is assumed as compressive force. It is further assumed that the compressive force induces elastic displacement or recovery only until the compressive force exceeds the existing load. By applying MSP to this compressive force, the recovery upon partial unloading can be calculated.

3.1.3. Stiffness loss due to creep

An additional concept must be included in the modeling of nailed joints. Previous studies (17,36) have shown that the stiffness of nailed joints decreases with creep magnitude. However, any specific definition of this effect has not been reported. The experimental data of this study showed that the creep-related stiffness loss affects the recovery under stepwise decreasing load. Therefore, the effect of stiffness loss should be included in the creep modeling.

In this study, the stiffness of nailed joint will be defined by the secant slip modulus, that is the ratio of total load to the corresponding slip. For instance, if the load function in Figure 3.4a is applied to nailed joints, the creep slip curve becomes that shown in Figure 3.4b and the corresponding load-slip curve becomes Figure 3.4c. If load P_3 is applied instantaneously, the instantaneous slip will be Sa. However, under the load function in Figure 3.4a, the slip is developed as much as S_t (Figure 3.4b). The difference, Sc, between S_t and Sa is the delayed creep slip developed under the stepwise load shown in Figure 3.4a.

The stiffness of nailed joints in Figure 3.4c may be defined by the slope of line OA when P is applied instantaneously :

$$Ka = P_3 / Sa \tag{3-7}$$

where Ka = stiffness or slip modulus of nailed joints under instantaneous loading. When the stepwise increasing load function (Figure 3.4a) is applied, the slope of line OB in Figure 3.4c gives the slip modulus of nailed joints:

$$Ks = P_3 / S_t = P_3 / (Sa + Sc) \tag{3-8}$$

where Ks = slip modulus of nailed jonts under stepwise loading. Therefore, the stiffness reducing factor, Rs, is

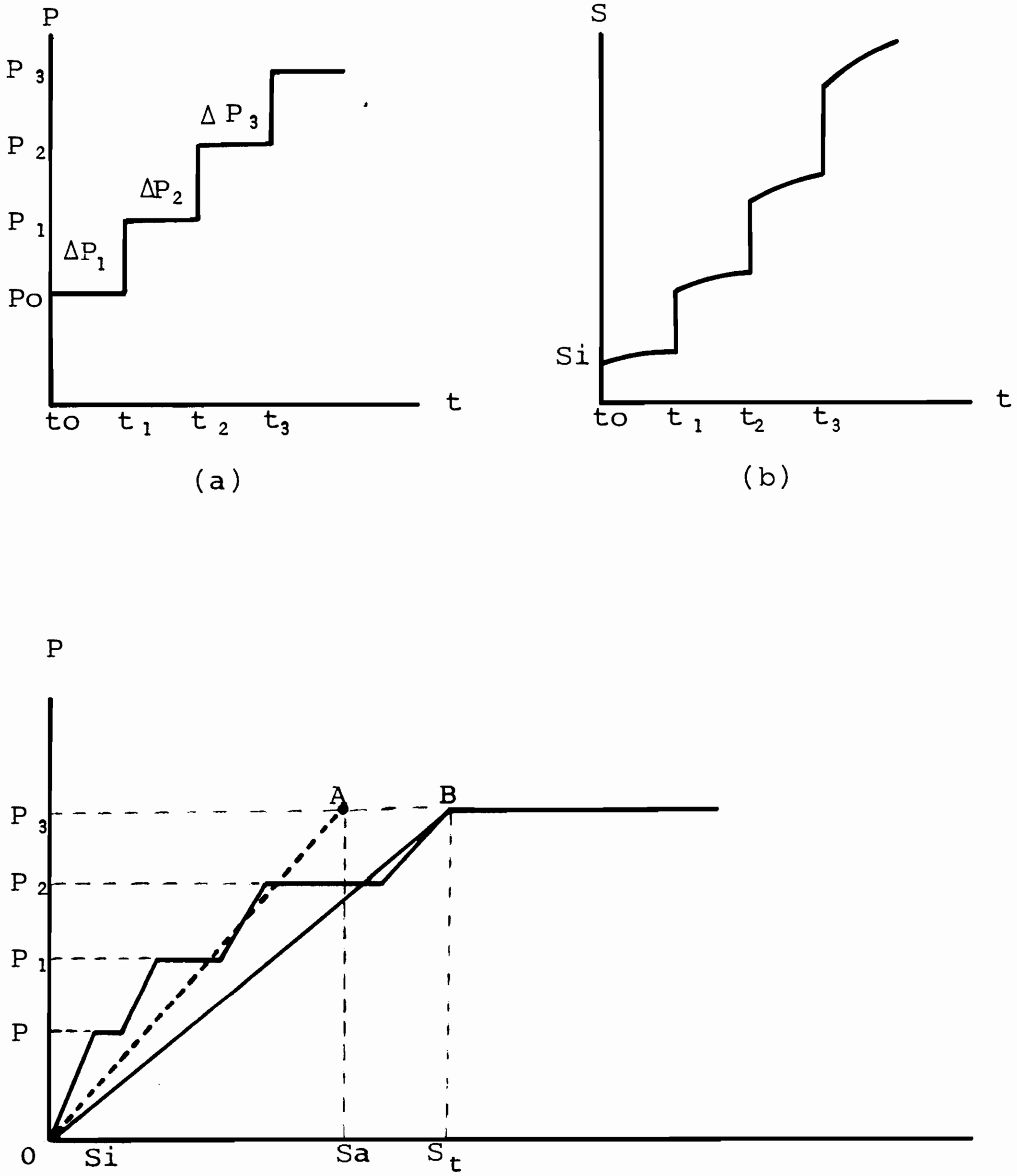


Figure 3.4. Stiffness loss due to creep in nailed joints; (a) load function, (b) slip under load function (a), and (c) load-slip curve of nailed joints under load function (a).

the ratio of Ks to Ka :

$$Rs = Ks / Ka = Sa / S_t = Sa / (Sa + Sc) \tag{3-9}$$

Factor Rs depends on the shape of the load function and the duration of each step.

Under the decreasing stepwise loading, the recovery is affected by creep under each load step. The recovery rate is slowed because of creep acting opposite to recovery. The testing showed that the recovery under stepwise unloading was much less than the recovery under instantaneous unloading. Therefore, it was assumed in the modeling that, upon stepwise unloading, the recovery is reduced as much as the stiffness reducing rate under the reversed stepwise loading. For example, the following procedure would account for the stepwise decreasing load function in figure 3.5a. First, the stiffness loss under the reversed load function in Figure 3.5b and the corresponding stiffness reducing factor would be determined by equations (3-6) through (3-9) (Figure 3.5c). Then, the reduced recovery (dot line in Figure 3.5d) would be calculated by multiplying the stiffness reducing factor to the slip recovery obtained by MSP. In the experimental part of this study, it was shown that, in nailed joints, the recovery under partial unloading was almost negligible. Thus, the stiffness reducing factor was applied only to the complete unloading step at the end of

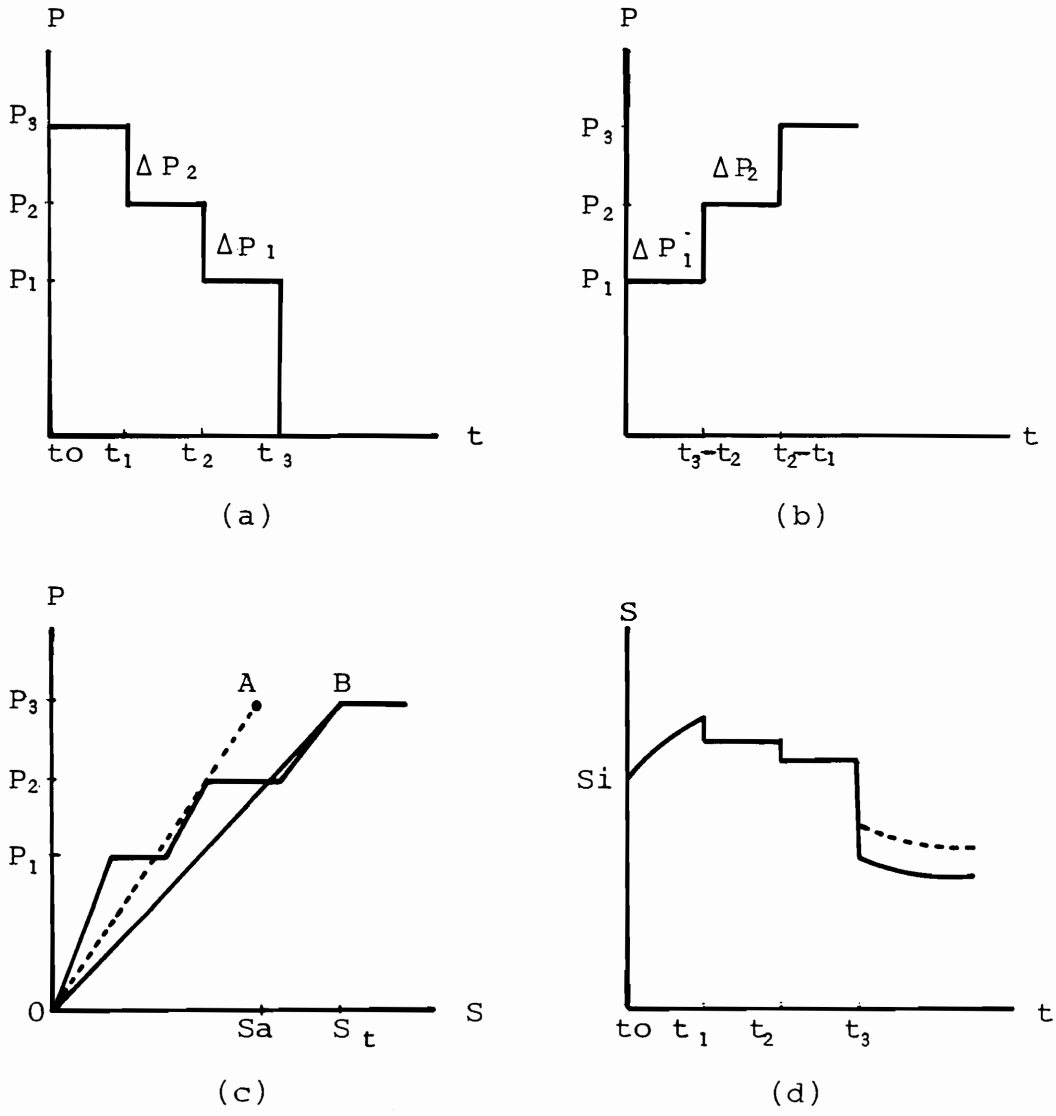


Figure 3.5. Recovery under stepwise decreasing load; (a) decreasing load function, (b) reversed increasing load function, (c) load-slip curve under load function (b), and (d) slip-time curve under load function (a).

the decreasing load function. The recovery predicted by this procedure agreed well with the experimental data.

3.2. Theoretical models

The development of the five new models are discussed next.

3.2.1. Five-element Model (5-E)

As mentioned before, the four-element model is too simple to represent the creep behavior of nailed joints. Therefore, a concept of nonlinear viscosity, originally introduced by Pierce et al. (31), was added to the four-element model. Specifically, a nonlinear dashpot with variable flow rate replaced the single linear dashpot in four-element model (Figure 2.3). This was accomplished by expressing the viscosity of the nonlinear dashpot as a power function of time :

$$\eta_1 \ (t) = at^n \qquad (3\text{-}10)$$

in which $\eta_1 (t)$ = nonlinear viscosity of dashpot as a function of time, t, and a and n = constants obtained from experiments. Expressing the relation between load, P, and viscous slip, Sv, as :

$$P = \eta_1(t)\ (dSv\ /\ dt) \tag{3-11}$$

and integrating gave :

$$Pt = \eta_1\ (t)Sv \tag{3-12}$$

$$Sv = Pt\ /\ \eta_1(t) \tag{3-13}$$

Finally, the substitution of equation (3-10) into (3-13) resulted in :

$$Sv = Pt\ /\ at^n\ = APt^M \tag{3-14}$$

where A = 1/a and M = 1-n.

However, such a model did not have the component describing the instantaneous plastic displacement, so that it still could not describe the behavior of nailed joints. Thus, a new element for the instantaneous plastic slip was added to the nonlinear four-element model. The result was the Five-element Model (5-E) which is shown in Figure 3.6. The new element, refered to as a nonlinear time-hardening element, describes the nonlinear instantaneous plastic slip upon loading and becomes a rigid element immediately after the instantaneous slip takes place and remains constant until the load increases.

The plasticity of the nonlinear time-hardening element, $\mu(P)$, was assumed to be a power function of load. Therefore, its load-slip relation was found to be :

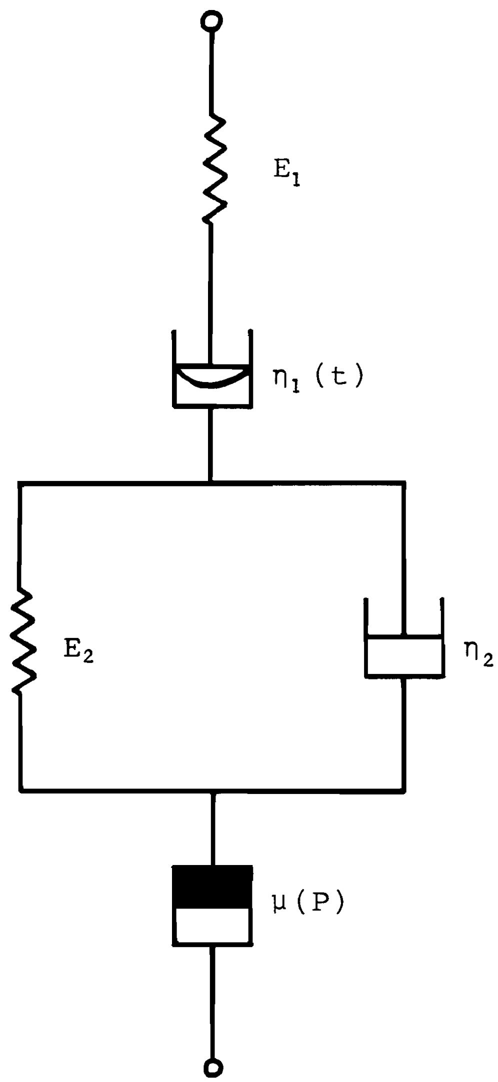


Figure 3.6. Five-element Model.

$$Sp = BP^{m} \tag{3-15}$$

where B and m are constants obtained from experiemnts. Now, the constitutive equation for Model 5-E could be written :

$$S_t = P/E_1 + P/E_2 [1-EXP\{-(E_2/\eta_2)t\}] + APt^{M} + BP^{m} \tag{3-16}$$

where E , E , η_2 , A, B, M and m are parameters determined from experimental data.

Under constant load, equation (3-16) can be further simplified to give :

$$S = A_1 + A_2\{1-EXP(-A_3t)\} + A_4 t^{M} + A_5 \tag{3-17}$$

where A_1, A_2, A_3, A_4, A_5 and M are parameters obtained from the constant load tests. To apply this method, a set of different parameters is needed for each load level, which make Model 5-E difficult to apply to practical problems.

3.2.2. Modified Five-element Model (M5-E)

In applying Model 5-E, it is not practical to evaluate and use several sets of coefficients for various load levels when predicting the creep slip under varying load. Thus, Model 5-E was modified to allow its application with only one set of coefficients for all the load levels. The new model is refered to as the Modified Five-element Model (M5-E).

Tests have shown that the instantaneous elastic and viscous slip of nailed joints are not linear with respect to load. To include the nonlinearity of the instantaneous elastic slip, the linear single spring in Model 5-E was replaced by a nonlinear single spring (Figure 3.7). The stiffness of the nonlinear spring was assumed to be a power function of the load. Thus, the load-slip relation in this spring could be expressed as follows :

$$Se = B_r P^{N_1} \tag{3-18}$$

where B_1 and N_1 are constants obtained from experiments.

The viscosity of nonlinear dashpot in Model M5-E, η_1, was assumed to be the power function of both load and time to include the nonlinearity :

$$\eta_1 (t) = bP^{\ell} t^m \tag{3-19}$$

where b, ℓ and m again are constants obtained from experiments. Then, the load-slip relation was derived similarly to the procedure described in Section 3.2.1 :

$$Sv = B_4 P^{N_2} t^{N_3} \tag{3-20}$$

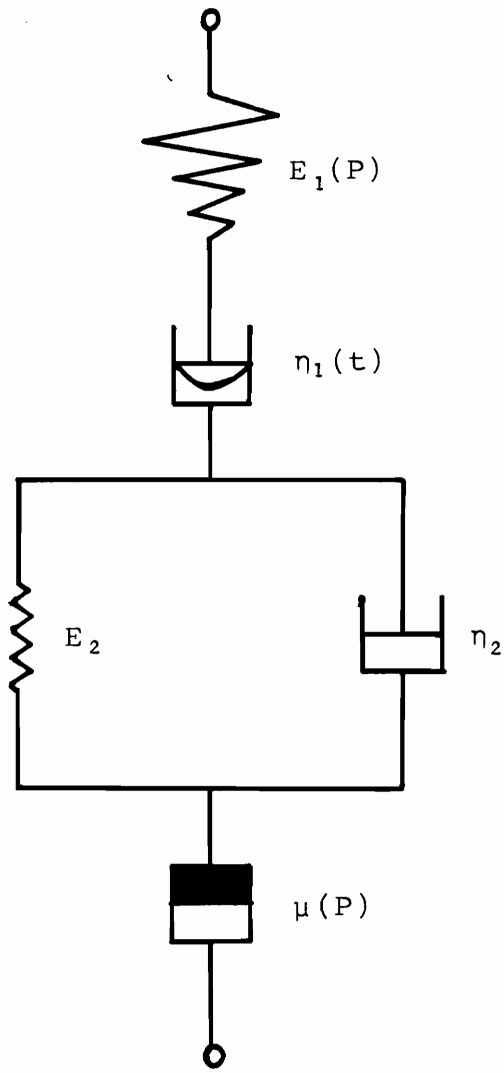


Figure 3.7. Modified Five-element Model.

where B_4, N_2 and N_3 are constants obtained from experiments.

The constitutive equation of Model M5-E was obtained by incorporating these modifications into equations (3-16) and (3-17), which resulted in :

$$S_t = B_1 P^{N_1} + B_2 P\{1-EXP(-B_3 t)\} + B_4 P^{N_2} t^{N_3} + B_5 P^{N_4} \tag{3-21}$$

in which B_1, B_2, B_3, B_4, B_5, N_1, N_2, N_3 and N_4 are parameters obtained from the results of constant load tests. In Model M5-E, nonlinearity of the instantaneous elastic and viscous slip is included. Therefore, equation (3-21) can be employed for all load levels with only one set of parameters.

3.2.3. Viscous-viscoelastic Model 1 (V-VE1)

The three integral representation describing nonlinear viscoelastic behavior of materials cannot be directly applied to represent the time-dependent behavior of the material showing the instantaneous plastic displacement. Therefore, another approach was selected for nailed joints. It was based on equation (3-1) that defines the slip as a sum of four components :

$$S_t = Se + Sp + S_{de} + Sv \tag{3-22}$$

which can also be expressed as a sum of two components, recoverable, Sr, and nonrecoverable slip, Sn, where :

$$Sr = Se + S_{de} \tag{3-23}$$

$$Sn = Sp + Sv \tag{3-24}$$

In nailed joints, the shapes of the curves of equations (3-23) and (3-24) are similar to the shape of the curve representing viscoelastic creep behavior. Therefore, it was possible to analogize that the three-integral representation could be used for both Sr and Sn as follows :

$$Sr = F_1(t)P + F_2(t)P^2 + F_3(t)P^3 \tag{3-25}$$

$$Sn = F_4(t)P + F_5(t)P^2 + F_6(t)P^3 \tag{3-26}$$

where $F_i(t)$, i=1-6, are kernel functions obtained from the results of constant load tests. Thus, the total slip equaled :

$$S_t = Sr + Sn = F_1(t)P + F_2(t)P^2 + F_3(t)P^3 + F_4(t)P + F_5(t)P^2 + F_6(t)P^3 \tag{3-27}$$

Equation (3-27) will be called the Viscous-viscoelastic Model 1 (V-VE1).

3.2.4. Viscous-viscoelastic Model 2 (V-VE2)

Model V-VE2 consists of three-integral representation for recoverable slip and power functions for nonrecoverable slip. Thus, equation (3-25) was first applied to define Sr and then, equations (3-15) and (3-20) were used to define Sp and Sv, respectively, to give :

$$S_t = F_1(t)P+F_2(t)P^2+ F_3(t)P^3+B_4P^{N_2}t^{N_3}+B_5P^{N_4} \qquad (3\text{-}28)$$

Equation (3-28) is a combination of Models M5-E and V-VE1 and refered to as the Viscous-viscoelastic Model 2 (V-VE2).

3.2.5. Viscous-viscoelastic Model 3 (V-VE3)

This is another combination of Models M5-E and V-VE1. In Model V-VE3, power functions are used to express the recoverable slip and the nonrecoverable slip is defined by the three-integral representation. Thus, using the first two terms in equation (3-21) to define Sr and equation (3-26) to define Sn gave :

$$S_t = B_1P^{N_1}+B_2P\{1-EXP(-B_3t)\}+$$
$$F_4(t)P+F_5(t)P^2+F_6(t)P^3 \qquad (3\text{-}29)$$

This equation is refered to as the Viscous-viscoelastic

Model 3 (V-VE3).

3.2.6. Model applications to varying load

To predict the creep of nailed joints under varying load, further modifications of the models developed were needed. In this investigation, two modification concepts, identified as Approach 1 and Approach 2, which were previously introduced in Section 3.1, were used to further modify the models.

3.2.6.1. Application of Approach 1

Table 3.1 summarizes the models modified according to Approach 1. Each model is divided into two parts, recoverable and nonrecoverable slip, to apply MSP and SHP, respectively, as given in Table 3.1 in which equations (3-30) through (3-33) are defined as follows :

$$Sr = B_1 P^{N_1} + B_2 P\{1-EXP(-B_3 t)\} \tag{3-30}$$

$$Sr = F_1(t)P + F_2(t)P^2 + F_3(t)P^3 \tag{3-31}$$

$$Sn = B_4 P^{N_2} t^{N_3} + B_5 P^{N_4} \tag{3-32}$$

$$Sn = F_4(t)P + F_5(t)P^2 + F_6(t)P^3 \tag{3-33}$$

Approach 1 could not be applied to Model 5-E which is discrete, because Approach 1 requires the integration of Heaviside function that is not possible for Model 5-E.

Table 3.1. Numbers identifying equations whose sum defines the models developed by Approach 1.

Model	Equation No. for					
	Basic part		Modification part		Loading part	
	Sr	Sn	Ss^1	Ss^2	S_1[a]	S_2[b]
M5-E	3-30	3-32	3-34	3-37 or 3-38	3-49	3-51
V-VE1	3-31	3-33	3-34	3-45 or 3-46	3-50	3-51
V-VE2	3-31	3-32	3-34	3-37 or 3-38	3-49	3-51
V-VE3	3-30	3-33	3-34	3-45 or 3-46	3-50	3-51

(a) Slip under the four-step increasing load function in figure 3.4a.

(b) Slip under the three-step decreasing load function in figure 3.5a.

By applying equation (2-18) of MSP to the recoverable slip in equations (3-30) and (3-31), the recoverable slip under stepwise load, S_s^1, was obtained :

$$S_s^1 = \sum_{i=0}^{N-1}[Sr(P_i, t-t_i)-Sr(P_{i-1}, t-t_i)], \quad t>t_{N-1} \qquad (3\text{-}34)$$

where Sr is given by equation (3-30) for Models M5-E and V-VE3 and by equation (3-31) for Models V-VE1 and V-VE2, and N = number of steps.

The nonrecoverable slip under varying load, S_s^2, for Models M5-E and V-VE2 was obtained by applying SHP to equation (3-32) as follows. Equation (3-32) was differentiated first :

$$dSn/dt = B_4 N_3 P^{N_2} t^{(N_3-1)} \qquad (3\text{-}35)$$

and solved for time :

$$t = [(Sn-B_5 P^{N_4})/(B_4 P^{N_2})]^{1/N_3} \qquad (3\text{-}36)$$

Substituting equation (3-36) into equation (3-35) and integrating gave :

$$S_s^2 = B_5 P^{N_4}+[\int_0^t (B_4 P^{N_2})^{1/N_3} dt]^{N_3} \qquad (3\text{-}37)$$

which, under stepwise load function, became :

$$Ss^2 = B_5P^{N_4} + [(B_4 Po^{N_2})^{\frac{1}{N_3}} t_1 + (B_4 P^{N_2})^{\frac{1}{N_3}} (t_2 - t_1) + \ldots + (B_4 P^{N_2}_{N-1})^{\frac{1}{N_3}} (t - t_{N-1})]^{N_3}, \quad t > t_{N-1} \qquad (3\text{-}38)$$

SHP was applied to equation (3-33) to obtain the nonrecoverable slip under varying load, $S\acute{s}^2$, for Models V-VE1 and V-VE3. This was made possible by having the same power of time in all the kernel functions in equation (3-33) as follows :

$$F_4(t) = F_4^\circ + F_4^+ t^m \qquad (3\text{-}39)$$

$$F_5(t) = F_5^\circ + F_5^+ t^m \qquad (3\text{-}40)$$

$$F_6(t) = F_6^\circ + F_6^+ t^m \qquad (3\text{-}41)$$

where F_4°, F_5°, F_6°, F_4^+, F_5^+, F_6^+ and m are parameters obtained from constant load tests. Finally, equation (3-33) became :

$$Sn = (F_4^\circ P + F_5^\circ P^2 + F_6^\circ P^3) + (F_4^+ P + F_5^+ P^2 + F_6^+ P^3) t^m$$
$$= G_1(P) + G_2(P) t^m \qquad (3\text{-}42)$$

which was again solved for time :

$$t = [\{Sn - G_1(P)\}/G_2(P)]^{1/m} \qquad (3\text{-}43)$$

and differentiated to give :

$$dSn/dt = mG_2(P) t^{(m-1)} \qquad (3\text{-}44)$$

Substituting equation (3-43) into equation (3-44) and integrating the result yielded :

$$Ss^2 = G_1(P)+[\int_0^t \{G_2(P)\}^{1/m} dt]^m \qquad (3-45)$$

which under stepwise load function equaled :

$$Ss^2 = G_1(P_{N-1})+[\{G_2(Po)\}^{1/m}t_1+\{G_2(P_1)\}^{1/m}(t_2-t_1)+ \\ \ldots+\{G_2(P_{N-1})\}^{1/m}(t-t_{N-1})]^m \quad , \quad t>t_{N-1} \qquad (3-46)$$

The total creep slip under varying load, Ss, is the sum of Ss^1 and Ss^2 :

$$Ss = Ss^1 + Ss^2 \qquad (3-47)$$

As discussed in Section 3.1.2, the decreasing load was considered as the compressive load acting in the opposite direction to the existing load, so that MSP could be applied in the same way as for increasing load. Thus, the total slip under decreasing load function was found to be:

$$Ss = Smax - Ss^1 \qquad (3-48)$$

where Smax = maximum slip before unloading and Ss^1 is given by equation (3-34). The stiffness reducing factor, Rs, was applied to the last step of the load function as discussed in Section 3.1.3.

Next, the total creep slip was evaluated for the four-step increasing load function defined in Figure 3.4a and the three-step decreasing load function defined in Figure 3.5a. The results are shown in Table 3.1 in which equations (3-49), (3-50) and (3-51) are given as follows :

$$Ss = Sr(Po,t-to)+Sr(P_1,t-t_1)-Sr(Po,t-t_1)+ Sr(P_2,t-t_2)-Sr(P_1,t-t_2)+Sr(P_3,t-t_3)- Sr(P_2,t-t_3)+B_5 P_3^{N_4}+[(B_4 Po^{N_2})^{\frac{1}{N_3}}(t_1-to)+ (B_4 P_1^{N_2})^{\frac{1}{N_3}}(t_2-t_1)+(B_4 P_2^{N_2})^{\frac{1}{N_3}}(t_3-t_2)+ (B_4 P_3^{N_2})^{\frac{1}{N_3}}(t-t_3)]^{N_3}, \quad t>t_3 \qquad (3\text{-}49)$$

$$Ss = Sr(Po,t-to)+Sr(P_1,t-t_1)-Sr(Po,t-t_1)+ Sr(P_2,t-t_2)-Sr(P_1,t-t_2)+Sr(P_3,t-t_3)- Sr(P_2,t-t_3)+G_1(P_3)+[\{G_2(Po)\}^{1/m}(t_1-to)+ \{G_2(P_1)\}^{1/m}(t_2-t_1)+\{G_2(P_2)\}^{1/m}(t_3-t_2)+ \{G_2(P_3)\}^{1/m}(t-t_3)]^m, \quad t>t_3 \qquad (3\text{-}50)$$

$$Ss = Smax-Sr(\Delta P_2,t-t_1)-Sr(\Delta P_2+\Delta P_1,t-t_2)+ Sr(\Delta P_2,t-t_2)-Rs[Sr(P_3,t-t_3)- Sr(\Delta P_2+\Delta P_1,t-t_3)], \quad t>t_3 \qquad (3\text{-}51)$$

3.2.6.2. Application of Approach 2

In Approach 2, MMSP is applied to the total slip. Thus, MSP was applied to the recoverable slip and for the nonrecoverable slip, the procedure described in Section

3.1.2 was used. The general expression for Approach 2 under stepwise load function was derived from equation (3-6) :

$$Ss = \sum_{i=o}^{N-1}[Sr(P_i, t-t_i) - Sr(P_{i-1}, t-t_i) + Sp(P_{N-1})] + \sum_{i=o}^{N-1} Sv(P_i, t_{i+1}-t_i) + Sv(P_{N-1}, t-t_{N-1}), \quad t>t_{N-1} \tag{3-52}$$

To use this equation in the models, the model constitutive equations should be divided into three parts, the recoverable, Sr, instantaneous plastic, Sp, and viscous slip, Sv. The definition for each part of the models is given in table 3.2 in which equations (3-53) through (3-61) are defined as follows :

$$Sr = A_1 + A_2\{1-EXP(-A_3 t)\} \tag{3-53}$$

$$Sr = B_1 P^{N_1} + B_2 P\{1-EXP(-B_3 t)\} \tag{3-54}$$

$$Sr = F_1(t)P + F_2(t)P^2 + F_3(t)P^3 \tag{3-55}$$

$$Sp = A_5 \tag{3-56}$$

$$Sp = B_5 P^{N_4} \tag{3-57}$$

$$Sp = F_4^{\circ}P + F_5^{\circ} P^2 + F_6^{\circ} P^3 \tag{3-58}$$

$$Sv = A_4 t^M \tag{3-59}$$

$$Sv = B_4 P^{N_2} t^{N_3} \tag{3-60}$$

$$Sv = (F_4^{+}P + F_5^{+}P^2 + F_6^{+}P^3)t^m \tag{3-61}$$

Equation (3-52) of Approach 2 was applied to all five models where Sr, Sp and Sv are defined by equation numbers given in Table 3.2. For decreasing load, MSP was applied

Table 3.2. Numbers identifying equations whose sum defines the models developed by Approach 2.

Model	Equation No. for					
	Basic part			Modification part	Loading part	
	Sr	Sp	Sv	$Ss^1 + Ss^2$	S_1 [a]	S_2 [b]
5-E	3-53	3-56	3-59	3-52	3-62	3-51
M5-E	3-54	3-57	3-60	3-52	3-62	3-51
V-VE1	3-55	3-58	3-61	3-52	3-62	3-51
V-VE2	3-55	3-57	3-60	3-52	3-62	3-51
V-VE3	3-54	3-58	3-61	3-52	3-62	3-51

(a) Slip under the four-step increasing load function in figure 3.4a.

(b) Slip under the three-step decreasing load function in figure 3.5a.

to the recoverable slip only. Consequently, equation (3-48) also can be used for deceasing load in which Sr is given in Table 3.2.

For instance, the total creep slip under load functions in Figures 3.4a and 3.5a were evaluated as described above and is presented in Table 3.2 in which equation (3-62) is defined by :

$$\begin{aligned} Ss = {} & Sr(Po,t-to)+Sr(P_1,t-t_1)-Sr(Po,t-t_1)+ \\ & Sr(P_2,t-t_2)-Sr(P_1,t-t_2)+Sr(P_3,t-t_3)- \\ & Sr(P_2,t-t_3)+Sv(Po,t_1-to)+Sv(P_1,t_2-t_1)+ \\ & Sv(P_2,t_3-t_2)+Sv(P_3,t-t_3)+Sp(P_3), \quad t>t_3 \end{aligned} \tag{3-62}$$

3.3. Representation of load functions

In Section 3.2.6, the models were modified under discrete loads. These discrete loads can be represented by a equation using Heaviside step function or Fourier approximation. In this Section, two modification ideas for the models are discussed. Two ideas are the applications of Approach 2 with the load functions represented by Heaviside function and Fourier approximation, respectively.

3.3.1. Stepwise load function represented by Heaviside function

The procedure described in Section 3.1 is a discrete analysis in which the load function is not defined as a continuous equation. Stepwise varying load is not continuous, but it can be expressed as a continuous equation in terms of Heaviside unit step function (11,18). The application of Heaviside and Dirac delta function to the models developed is presented next.

3.3.1.1. Difinition

Heaviside unit step function (Figure 3.8a) is defined by (11) :

$$H(t-a) = \begin{cases} 1 & \text{if } t > a \\ 1/2 & \text{if } t = a \\ 0 & \text{if } t < a \end{cases} \qquad (3-63)$$

where t = time and a = constant.

Dirac delta function (Figure 3.8b) is defined as follows (11,18) :

$$\delta(t-a) = \begin{cases} \infty & \text{if } t = a \\ 0 & \text{if } t \neq a \end{cases} \qquad (3-64)$$

and

$$\int_{-\infty}^{+\infty} (t-a)dt = \int_{a-}^{a+} (t-a)dt = 1 \qquad (3-65)$$

Dirac delta function actually is the derivative of Heaviside unitstep function :

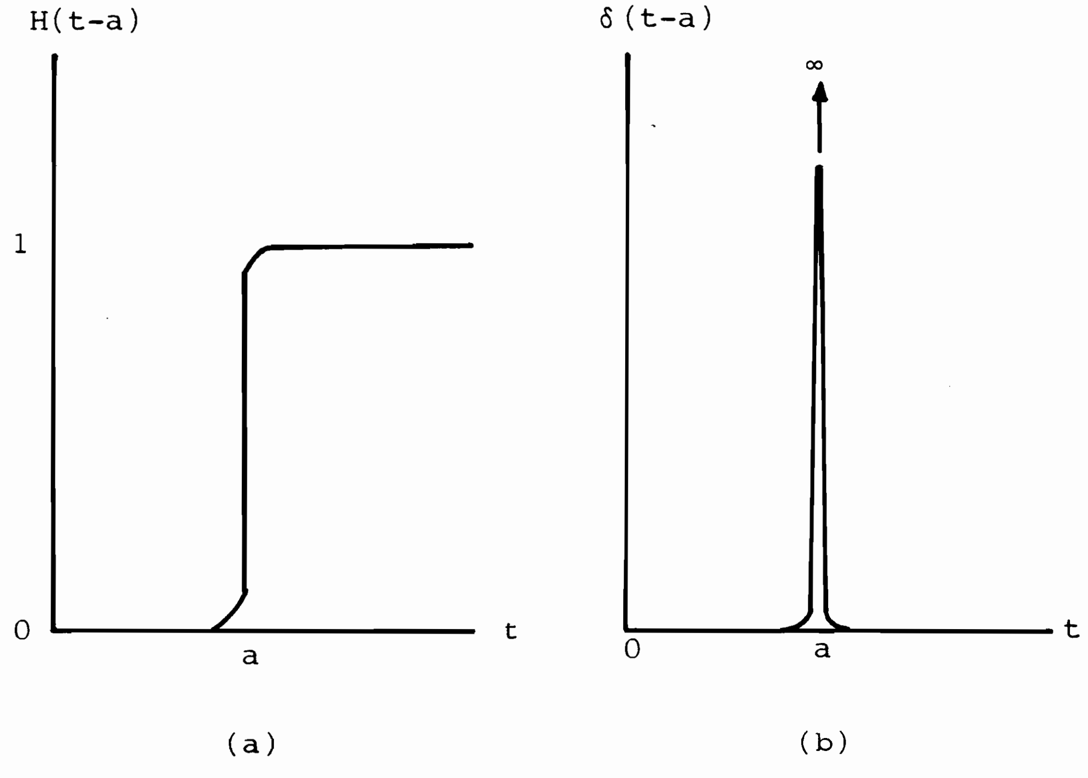


Figure 3.8. Heaviside unit step function (a) and Dirac delta function (b).

$$d\{H(t-a)\} / dt = \dot{H}(t-a) = \delta(t-a) \tag{3-66}$$

The integral of Heaviside and Dirac delta function has following properties :

$$\int_0^\infty H(t-a)\ \delta(t-b)dt = \begin{cases} 0 & \text{if } a>b \\ 1/2 & \text{if } a=b,\ a\geq 0 \\ 0 & \text{if } a=b,\ a<0 \\ 1 & \text{if } a<b,\ b\geq 0 \\ 0 & \text{if } a<b,\ b<0 \end{cases} \tag{3-67}$$

$$\int_0^\infty \{H(t-a)\}^N\ \delta(t-a)dt = \begin{cases} 1/(N+1) & \text{if } a>0 \\ 0 & \text{if } a<0 \end{cases} \tag{3-68}$$

where a, b and N are constants.

3.3.1.2. Application

Because it was expected that the results of this procedure were similar to the results of discrete analysis described in Section 3.1, the application was performed for load function 7 only. The resulting models were then modified by Approach 2.

For period A, the load function of Figure 3.9 is :

$$P(t)=PoH(t-to) = \begin{cases} 0 & \text{if } t<to \\ Po & \text{if } t>to \end{cases} \tag{3-69}$$

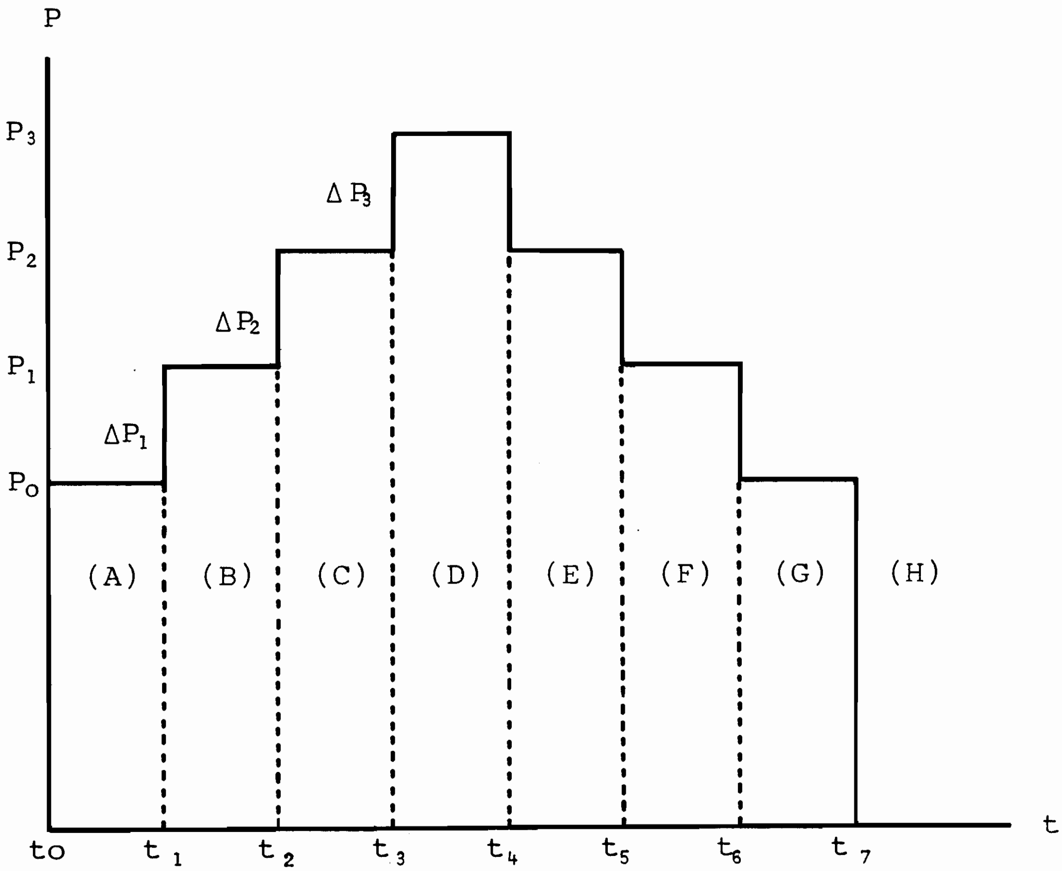


Figure 3.9. Load function 7.

where Po = load in the first step and to = initial time of the first step. The load function in period A and B of Figure 3.9 equals :

$$P(t) = PoH(t-to) + \Delta P_1 H(t-t_1) \qquad (3\text{-}70)$$

where $P_1 = Po + \Delta P_1$ = load level during the second step. By repeating this procedure for all steps, the overall load function equals :

$$P(t) = PoH(t-to) + \Delta P_1 H(t-t_1) + \Delta P_2 H(t-t_2) + \Delta P_3 H(t-t_3) - \Delta P_3 H(t-t_4) - \Delta P_2 H(t-t_5) - \Delta P_1 H(t-t_6) - PoH(t-t_7),\ t>t_7 \qquad (3\text{-}71)$$

where $P_2 = Po + \Delta P_1 + \Delta P_2$ = load level during the third step; $P_3 = Po + \Delta P_1 + \Delta P_2 + \Delta P_3$ = load level during the fourth step; and t_i = initial time of each step for i = 0,1,...,7.

Equation (3-52) of MMSP could be rewritten for stepwise increasing-load function to give :

$$Ss = \sum_{i=o}^{N-1} [Sr(P_i, t-t_i) - Sr(P_{i-1}, t-t_i) + Sp(P_i) - Sp(P_{i-1})] + \sum_{i=o}^{N-2} Sv(P_i, t_{i+1} - t_i) + Sv(P_{N-1}, t-t_{N-1}),\ t>t_{N-1} \qquad (3\text{-}72)$$

The first four terms in this equation are the expression for MSP and can be represented by an integral formfor a continuous load function. Thus, equation (3-72) became :

$$Ss = \int_o^t [\, \partial\{Sr(P(z),t-z)\}/\partial P(z) + d\{Sp(P(z))\}/dP(z)\}]\, \dot{P}(z)dz + \sum_{i=o}^{N-2} Sv(P_i, t_{i+1}-t_i) + Sv(P_{N-1}, t-t_{N-1}),\quad t>t_{N-1} \qquad (3\text{-}73)$$

The integrals in equation (3-73) were solved by direct integration. The remaining terms in equation (3-73) were solved in the same way as discussed in Section 3.2.6. Model 5-E was not employed because it could not be integrated. Therefore, only the remaining models were modified by this procedure.

3.3.1.2.1. Modified five-element Model (M5-E)

Model M5-E was divided into three components represented by equations (3-53), (3-56) and (3-59). By employing equations (3-53) and (3-56), the integrals in equation (3-73) were expressed as :

$$\begin{aligned} Ss^1 &= \int_o^t [B_1 N_1 \{P(z)\}^{N_1-1} + B_2\{1-EXP(-B_3(t-z))\} + B_5 N_4 \{P(z)\}^{N_4-1}]\, \dot{P}(z)dz \\ &= B_1 N_1 \int_o^t \{P(z)\}^{N_1-1}\dot{P}(z)dz + B_2 \int_o^t \dot{P}(z)dz - B_2 \int_o^t EXP(-B_3(t-z))\dot{P}(z)dz + B_5 N_4 \int_o^t \{P(z)\}^{N_4-1}\dot{P}(z)dz \end{aligned} \qquad (3\text{-}74)$$

in which load function, P(t), was given by equation (3-69) and its derivative, $\dot{P}(t)$, equalled :

$$\dot{P}(z) = Po\,\delta(t) + \Delta P_1\,\delta(t-t_1) + \Delta P_2\,\delta(t-t_2) + \Delta P_3\,\delta(t-t_3) - \Delta P_3\,\delta(t-t_4) - \Delta P_2\,\delta(t-t_5) - \Delta P_1\,\delta(t-t_6) - Po\,\delta(t-t_7) \qquad (3\text{-}75)$$

Next, equation (3-74) was solved for each step of load function 7 by substituting equations (3-71) and (3-75) into equation (3-74), and by applying equations (3-65), (3-66) and (3-67).

The second and third terms in equation (3-73) were expressed by applying equation (3-59) :

$$Ss^2 = \sum_{i=o}^{N-2} B_4\, P_i^{N_2}\,(t_{i+1}-t_i)^{N_3} + B_4 P_{N-1}^{N_2}\,(t-t_{N-1})^{N_3} \qquad (3\text{-}76)$$

Creep slip during the loading part of load function 7 is a combination of equations (3-74) and (3-76). For example, the creep slip in period D of Figure 3.9 was found to be :

$$\begin{aligned} Ss = {} & B_1\,[Po^{N_1} + N_1\{(P_1)^{N_1-1}\Delta P_1 + (P_2)^{N_1-1}\Delta P_2 + (P_3)^{N_1-1}\Delta P_3\}] + \\ & B_2\,[P_3 - \{PoEXP(-B_3(t-to)) + \Delta P_1 EXP(-B_3(t-t_1)) + \\ & \Delta P_2 EXP(-B_3(t-t_2)) + \Delta P_3 EXP(-B_3(t-t_3))]+ \\ & B_5\,[Po^{N_4} + N_4\{(P_1)^{N_4-1}\Delta P_1 + (P_2)^{N_4-1}\Delta P_2 + (P_3)^{N_4-1}\Delta P_3\}] + \\ & 4[Po^{N_2}(t_1-to)^{N_3} + P_1^{N_2}(t_2-t_1)^{N_3} + P_2^{N_2}(t_3-t_2)^{N_3} + \\ & P_3^{N_2}(t-t_3)^{N_3}],\quad t_3 < t < t_4 \end{aligned} \qquad (3\text{-}77)$$

The solution for the unloading portion was based on the basic assumption that the decreasing load could be considered as a compressive force applied in the opposite direction to the existing load. The compressive-force function, Pc(t), and its derivative, $\dot{P}c(t)$, were derived as :

$$Pc(t) = \Delta P_3 H(t-t_4) + \Delta P_2 H(t-t_5) + \Delta P_1 H(t-t_6) + PoH(t-t_7) \qquad (3-78)$$

$$\dot{P}c(t) = \Delta P_3 \; \delta(t-t_4) + \Delta P_2 \; \delta(t-t_5) + \Delta P_1 \; \delta(t-t_6) + Po \; \delta(t-t_7) \qquad (3-79)$$

For unloading, MSP can be applied, because only recoverable slip is related to unloading steps. Thus :

$$Ss = Smax - Rs[\int_{t_4}^{t} \frac{\partial \{Sr(P_C(z), t-z)\}}{\partial P_C(z)} \dot{P}_C(z)dz], \quad t>t_4 \qquad (3-80)$$

in which Smax = maximum slip at time t4 before unloading; Rs = stiffness reducing factor; and Sr = recoverable slip defined by equation (3-53). As suggested in Section 3.1.3, stiffness reducing factor, Rs, was applied only at the end of the last step. The integration in equation (3-80) can be solved either directly or numerically. For example, the creep slip for period H of Figure 3.9 was found directly to be :

$$Ss = Smax - Rs[B_1 [\Delta P_3^{N_1} + N_1 \{ (\Delta P_3 + \Delta P_2)^{N_1-1} \Delta P_2 +$$
$$(\Delta P_3 + \Delta P_2 + \Delta P_1)^{N_1-1} \Delta P_1 + (P_3)^{N_1-1} Po] + B_2 [P_3 -$$

$$\{ \Delta P_3 EXP(-B_3 (t-t_4))+ \Delta P_2 EXP(-B_3 (t-t_5))+$$

$$\Delta P_1 EXP(-B_3 (t-t_6))+PoEXP(-B_3 (t-t_7)) \}]] \quad (3\text{-}81)$$

3.3.1.2.2. Viscous-viscoelastic Model 1 (V-VE1)

Model V-VE1 is composed of three parts which are represented by equations (3-54), (3-57) and (3-60). MMSP and the corresponding equation (3-73) were applied to Model V-VE1. The integrals in equation (3-73) were written as :

$$Ss^1 = \int_o^t F_1 (t-z)\dot{P}(z)dz+2 \int_o^t F_2 (t-z)P(z)\dot{P}(z)dz+$$
$$3 \int_o^t F_3 (t-z)\{P(z)\}^2 \dot{P}(z)dz+$$
$$\int_o^t \{F_4^\circ +2F_5^\circ P(z)+3F_6^\circ (P(z))^2 \}\dot{P}(z)dz \quad (3\text{-}82)$$

The load function, P(t), and its derivative, $\dot{P}(t)$, were defined by equation (3-71) and (3-75), respectively. Equation (3-82) can be integrated either directly or numerically. The rest terms in equation (3-73) were determined as :

$$Ss^2 = \sum_{i=o}^{N-2} (F_4^+ P_i + F_5^+ P_i^2 + F_6^+ P_i^3) (t_{i+1} - t_i)^m +$$
$$(F_4^+ P_{N-1} + F_5^+ P_{N-1}^2 + F_6^+ P_{N-1}^3) (t-t_{N-1})^m, \; t>t_{N-1} \quad (3\text{-}83)$$

Adding equation (3-82) to equation (3-83) gave creep slip under load function 7 for Model V-VE1.

Next, Model V-VE1 and equations (3-82) and (3-83) were used to evaluate the slip for period D of Figure 3.9. The resulting equation was found to be :

$$\begin{aligned}
Ss = {} & F_1(t)Po + F_1(t-t_1)\Delta P_1 + F_1(t-t_2)\Delta P_2 + F_1(t-t_3)\Delta P_3 + \\
& F_2(t)Po^2 + 2F_2(t-t_1)P_1\Delta P_1 + 2F_2(t-t_2)P_2\Delta P_2 + \\
& 2F_2(t-t_3)P_3\Delta P_3 + F_3(t)Po^3 + 3F_3(t-t_2)P_1^2\Delta P_1 + \\
& 3F_1(t-t_2)P_2^2\Delta P_2 + 3F_3(t-t_3)P_3^2\Delta P_3 + F_4^{\circ}P_3 + \\
& 2F_5^{\circ}(Po^2 + P_1\Delta P_1 + P_2\Delta P_2 + P_3\Delta P_3) + 3F_6^{\circ}(Po^3 + \\
& P_1^2\Delta P_1 + P_2^2\Delta P_2 + P_3^2\Delta P_3) + (F_4^{+}Po + F_5^{+}Po^2 + \\
& F_6^{+}Po^3)t^m + (F_4^{+}P_1 + F_5^{+}P_1^2 + F_6^{+}P_1^3)(t_2-t_1)^m + \\
& (F_4^{+}P_2 + F_5^{+}P_2^2 + F_6^{+}P_2^3)(t_3-t_2)^m + \\
& (F_4^{+}P_3 + F_5^{+}P_3^2 + F_6^{+}P_3^3)(t-t_3)^m, \quad t>t_3 \qquad (3\text{-}84)
\end{aligned}$$

For unloading, decreasing loads were again regarded as a compressive forces. The bases for determining the slip under unloading function were the force function and its derivative given in equations (3-78) and (3-79). For MSP, the resulting equation was identical to equation (3-80) in which Sr was given by equation (3-54).

The same development was also applied to period H in Figure 3.9. The total slip was found to be :

$$\begin{aligned}
Ss = {} & Smax - Rs[F_1(t-t_4)\Delta P_3 + F_1(t-t_5)\Delta P_2 + F_1(t-t_6)\Delta P_1 + \\
& F_1(t-t_7)Po + F_2(t-t_4)\Delta P_3^2 + 2F_2(t-t_5)(\Delta P_3 + \Delta P_2)\Delta P_2 + \\
& 2F_2(t-t_6)(\Delta P_3 + \Delta P_2 + \Delta P_1)\Delta P_1 + 2F_2(t-t_7)P_3\ Po + \\
& F_3(t-t_4)\Delta P_3^3 + 3F_3(t-t_5)(\Delta P_3 + \Delta P_2)^2\Delta P_2 + \\
& 3F_3(t-t_6)(\Delta P_3 + \Delta P_2 + \Delta P_1)^2\Delta P_1 +
\end{aligned}$$

$$3F_3(t-t_7)P_3^2\ Po],\ t>t_7 \tag{3-85}$$

Models V-VE2 and V-VE3 are combinations of Models M5-E and V-VE1. The corresponding total slip could be easily obtained from equations (3-74), (3-76), (3-82) and (3-83) following the procedure of Section 3.2.6.2, but the development was not carried out in this dissertation.

3.3.2. Continuous load function represented by Fourier series

Next, a procedure was developed for loading represented by continuous functions. Such a procedure offers advantages when the load is difficult to be represented by a simple equation that is easy to integrate. The development was based on a Fourier series approximation, because such an approximation can easily be used for varying-load functions.

3.3.2.1. Definition of Fourier approximation

Fourier series approximation of load function P(t) is a trigonometric series defined by (18) :

$$\begin{aligned} Fp(t) &= (1/Tp)\int_0^{Tp} P(t)dt + \\ &\sum_{i=1}^{\infty} [(2/Tp)\{\int_0^{Tp} P(t)\cos(i\omega t)dt\}\cos(i\omega t) + \\ &(2/Tp)\{\int_0^{Tp} P(t)\sin(i\omega t)dt\}\sin(i\omega t)] \\ &= Ao + \sum_{i=1}^{\infty} [Ai\ \cos(i\omega t) + Bi\ \sin(i\omega t)] \end{aligned} \tag{3-86}$$

where P(t) = load function; Tp = period; and $\omega = 2\pi/Tp$.

To illustrate such a procedure, Fourier series approximation was applied to load function 7 of figure 3.9. First, load function 7 was divided into two parts (Figure 3.10), part a consisting of a constant load of 60 lb and part b consisting of stepwise varying load which was load function 7 minus 60 lb.

Part b which shows odd periodic extension by dot line in Figure 3.10 was approximated by a Fourier sine series (18). In this case, equation(3-86) was valid, in which :

$$Ao = 60$$

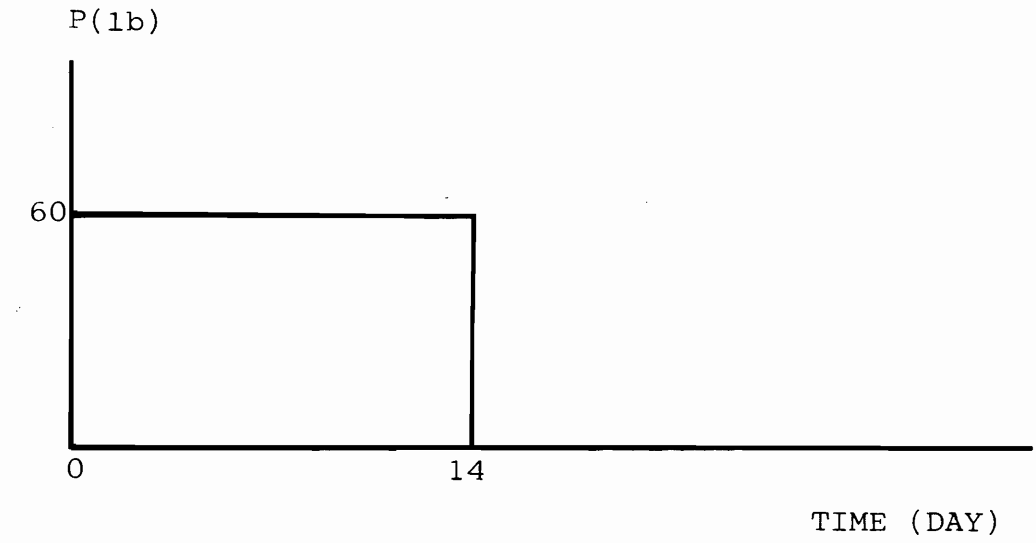
$$Ai = 0$$

$$Bi = (2/Tp)\int_0^{Tp} P(t)\ \sin(i\omega t)dt$$

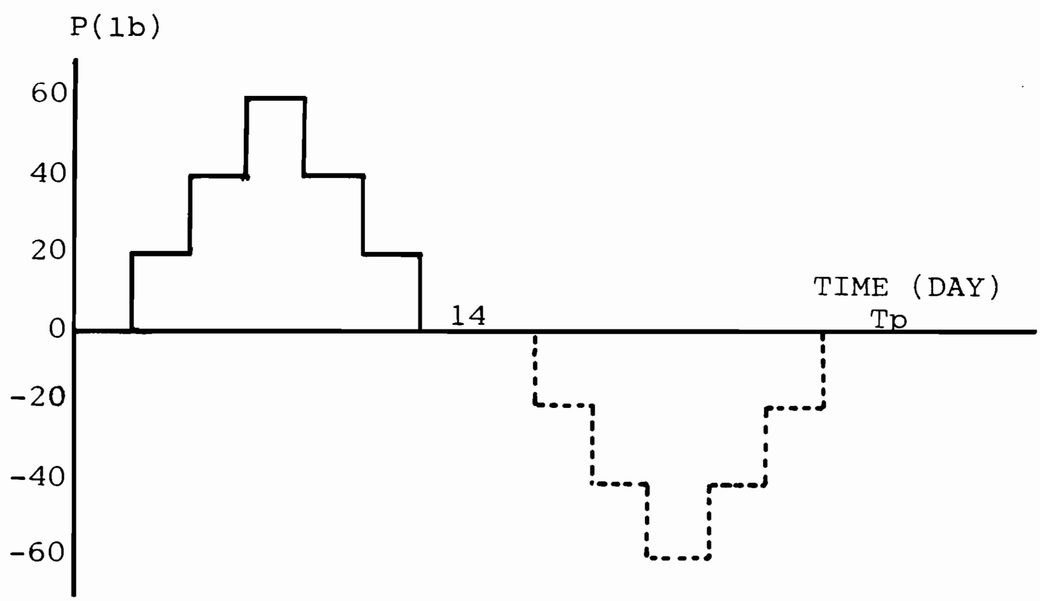
$$= (2/Tp)\int_0^{Tp} P(t)\ \sin\ (2i\pi/Tp)t\ dt \qquad (3\text{-}87)$$

For load function 7, t7 is 14 days or 20,160 minutes and Tp is 28 days or 40,320 minutes, for which equation (3-87) was found to be :

$$\begin{aligned} Bi = -(1/i\pi)[&P_1\{\cos(2i\pi/7)+\cos(6i\pi/7)+ \\ &\cos(8i\pi/7)+\cos(12i\pi/7)-\cos(i\pi/7)- \\ &\cos(5i\pi/7)-\cos(9i\pi/7)-\cos(13i\pi/7)\}+ \\ &P_2\{\cos(3i\pi/7)+\cos(5i\pi/7)+\cos(9i\pi/7)+ \\ &\cos(11i\pi/7)-\cos(2i\pi/7)-\cos(4i\pi/7)- \\ &\cos(10i\pi/7)-\cos(12i\pi/7)\}+P_3\{\cos(4i\pi/7)+ \\ &\cos(10i\pi/7)-\cos(3i\pi/7)-\cos(11i\pi/7)\}] \end{aligned} \qquad (3\text{-}88)$$



(a)



(b)

Figure 3.10. Division and odd periodic extension of load function 7 for Fourier approximation.

After having defined all the coefficients, load function 7 was fully defined :

$$Fp(t) = Ao + \sum_{i=1}^{\infty} Bi \sin(i\omega t) \qquad (3\text{-}89)$$

In programming, "i" was selected to be 20, because preliminary calculations showed that 20 assures convergence at sufficient accuracy.

3.3.2.2. Application of Fourier approximation

Next, Fourier sine series was applied to the models developed and the models were modified by Approach 2. For varying load, MMSP was represented by equation (3-73) in which the derivative of the load function equalled :

$$\dot{F}p(t) = \sum_{i=1}^{\infty} i\omega B_i \cos(i\omega t) \qquad (3\text{-}90)$$

Model 5-E could not be used in this procedure because it is discrete.

3.3.2.2.1. Modified five-element Model (M5-E)

Model M5-E can be divided into three parts that are associated with equations (3-53), (3-56) and (3-59). The general expression for MMSP under continuous load function was found to be equation (3-73), in which the integrals equalled :

$$Ss^1 = B_1 N_1 \int_0^t \{Fp(z)\}^{N_1-1} \dot{F}p(z)dz + B_2[\int_0^t \{1-EXP(-B_3(t-z))\}\dot{F}p(z)dz] + B_5 N_4 \int_0^t \{Fp(z)\}^{N_4-1} \dot{F}p(z)dz \quad (3\text{-}91)$$

The remaining terms in equation (3-73) were expressed as :

$$Ss^2 = \sum_{i=1}^{N-2} B_4 Fp^{N_2} (t_{i+1}-t_i)^{N_3} + B_4 Fp_{N-1}^{N_2} (t-t_{N-1})^{N_3}, \; t>t_{N-1} \quad (3\text{-}92)$$

For continuous load function, the viscous slip was obtained by equation (3-92). The total slip under load function 7 was expressed by a Fourier series as a combination of equations (3-91) and (3-92). When a numerical method is used to solve equation (3-91), equations (3-91) and (3-92) should both have the same step size.

Next, the unloading part of the function was expressed by :

$$Fc(t) = Fmax - Fp(t), \; t>tmax \quad (3\text{-}93)$$

where Fc(t) = unloading part of the equation; Fmax = maximum load at time tmax; and Fp(t) is defined by equation (3-89). The derivative of equation (3-93) equalled :

$$\dot{F}c(t) = -\dot{F}p(t), \ t>tmax \tag{3-94}$$

where $\dot{F}p(t)$ was defined by equation (3-90).

Next, MSP was applied to the unloading part of the function in which the recoverable slip was defined as :

$$\begin{aligned} Ss &= Smax - Rs[\int_{tmax}^{t}\{\partial Sr(Fc(z),t-z)/\partial Fc(z)\}\dot{F}c(z)dz] \\ &= Smax - Rs[B_1N_1 \int_{tmax}^{t}\{Fc(z)\}^{N_1-1}\dot{F}c(z)dz+ \\ &\quad B[\int_{tmax}^{t}\{1-EXP(-B_3(t-z))\}\dot{F}c(z)dz]], \ t>tmax \end{aligned} \tag{3-95}$$

where Smax = maximum slip at time tmax and the other symbols were previously defined. The integration of equation (3-95) can be carried out either by direct or numerical method. As before, the stiffness reducing factor, Rs, was applied to the last step only.

3.3.2.2.2. Viscous-viscoelastic Model 1 (V-VE1)

Model V-VE1 is divided into three parts that were defined by equations (3-54), (3-57) and (3-60). Again, the slip under varying load was defined by equation (3-73) of MMSP in which the integrals equalled :

$$\begin{aligned} Ss^1 &= \int_0^t F_1(t-z)\dot{F}p(z)dz+2\int_0^t F_2(t-z)Fp(z)\dot{F}p(z)dz+ \\ &\quad 3\int_0^t F_3(t-z)\{Fp(z)\}^2\dot{F}p(z)dz+\int_0^t\{F_4^\circ+2F_5^\circ\ Fp(z)+ \\ &\quad 3F_6^\circ\ \{Fp(z)\}^2\}\dot{F}p(z)dz \end{aligned} \tag{3-96}$$

where Fp(t) and $\dot{F}p(t)$ were given by equations(3-89) and (3-90), respectively. The remaining terms in equation (3-73) were found to be :

$$Ss^2 = \sum_{i=o}^{N-2}(F_4^+ Fp_i + F_5^+ Fp_i^2 + F_6^+ Fp_i^3)(t_{i+1}-t_i)^m + (F_4^+ Fp_{N-1} + F_5 Fp_{N-1}^2 + F_6 Fp_{N-1}^3)(t-t_{N-1})^m,$$

$$t>t_{N-1} \qquad (3-97)$$

Equation (3-97) is a discrete approximation of a continuous load function. The sum of equations (3-96) and (3-97) gives the total slip under load function 7 approximated by Fourier series. When equation (3-95) is evaluated by a numerical integration, it is convenient to use the same step size for both, equations (3-96) and (3-97).

For unloading, the force function and its derivative were expressed by equations (3-93) and (3-94), respectively. When MSP was applied, the total slip became :

$$\begin{aligned} Ss &= Smax - Rs[\int_{tmax}^{t} \{\partial Sr(Fc(z),t-z)/\partial Fc(z)\}\dot{F}c(z)dz \\ &= Smax - Rs[\int_{tmax}^{t} F_1(t-z)\dot{F}c(z)dz + \\ &\quad 2\int_{tmax}^{t} F_2(t-z)Fc(z)\dot{F}c(z)dz + \\ &\quad 3\int_{tmax}^{t} F_3(t-z)\{Fc(z)\}^2\dot{F}c(z)dz] \qquad (3-98) \end{aligned}$$

Again, equation (3-98) can be evaluated by direct or numerical integration. The stiffness reducing factor, Rs, was applied to the last step only.

Models V-VE2 and V-VE3 can be easily obtained from equations (3-91), (3-92) and (3-95) for Model M5-E and from equations (3-96), (3-97) and (3-98) for Model V-VE1. However, these derivations were not presented in this dissertation.

VI. EXPERIMENTAL PROCEDURE

To formulate and verify the models developed, creep experimental values are required for nailed joints under constant and varying loads. Therefore, a testing study was carried out, which included the most common joint type in building construction. The joint type selected was nailed joint between framing member and plywood sheathing.

4.1. Joint specimens

Material selection and joint specimen construction are introduced in this section.

4.1.1. Material selection

Douglas-fir lumber of nominal size 2-by 4-inches was selected as representative framing member. Initially, 350 12-inch long pieces of lumber were cut from studs left over from another project at the Forest Products Laboratory, Oregon State University. The pieces containing too much pith, checks, splits or knots were excluded because these defects could produce joints that were not typical of the overall statistical population.

The sheathing material consisted of 3/8-inch thick Douglas-fir plywood of sheathing grade, because it represented commonly used sheathing materials in construction stock. It was selected from 4-by 8-foot panels bought in a local lumber yard. In the laboratory, sections of 4-by 15-inches were cut from the panels. The sections having weak adhesion, checks, splits and knots were excluded, because they would not produce typical joints.

Next, the materials were stored in a conditioning room at 12-percent equillibrium moisture content (EMC) with forced air circulation until they reached EMC. Their moisture content were checked periodically by an electric moisture meter to monitor their equilibration.

Six penny galvanized box nails were selected to make joints, because this type is commonly used in wall construction. The length and diameter of the nail are 2.0 in. and 0.98 in., respectively. The nails having a crushed head, shank or extremely rough surface were excluded.

4.1.2. Evaluation of specific gravity

Specific gravity is often visualized as an easily measured property of wood, which is closely related to other mechanical properties. Therefore, it was evaluated for all 350 pieces of lumber selected.

In this investigation, specific gravity was based on the volume at 12 % moisture content and evaluated by :

$$SG = Ww / (1.12\ V) \qquad (4\text{-}1)$$

where Ww = weight of wood and V = volume of wood at 12-percent moisture content. In equation (4-2), moisture contents of all pieces of lumber were assumed to be 12-percent and the volumes were obtained by measuring the thickness, width and length of each piece.

4.1.3. Evaluation of elastic bearing constant

Elastic bearing constant is related to the stiffness of nailed joints. It is defined as the elastic spring modulus for the wood under the nail and expressed in psi per inch of wood deformation. In nailed joint, the wood can be visualized as a foundation and the nail as a beam. Under small deflection, the nail and the wood behave elastically. Therefore, the elastic spring constant of wood is important in indicating the stiffness of nailed joints.

The elastic bearing constant depends on the testing method and the equation of calculating it. In this investigation, elastic bearing constants were evaluated by the modulus of subgrade reaction of soil foundation (2), because it was suitable for nailed joints and relatively easy to test. In this method, the elastic bearing constant is defined by :

$$Ko = q / y \quad (lb/in.^3) \qquad (4-2)$$

where Ko = elastic bearing constant; q = stress in wood ($lb/in.^2$); and y = deflection under nail (in.). For nailed joints, q equals :

$$q = P / A \tag{4-3}$$

where :

$$A = D L \tag{4-4}$$

in which P = load (lb); A = contact area between nail and wood (in.); D = diameter of nail (in.); and L = contact length between nail and wood (in.). The contact length, L, was obtained by :

$$L = Ln - Lp \tag{4-5}$$

where Ln = length of nail (in.) and Lp = thickness of plywood (in.). Substituting equations (4-3), (4-4) and (4-5)into equation (4-2) resulted in :

$$Ko = P / [yD(Ln-Lp)] \ (lb/in.^3) \tag{4-6}$$

in which P, y, D, Ln and Lp are variables measured in experiments.

The testing arrangement for the elastic bearing constant is shown in Figure 4.1. The nails were placed on the cross section of the lumber pieces and compression load parallel to the grain was applied. A steel loading

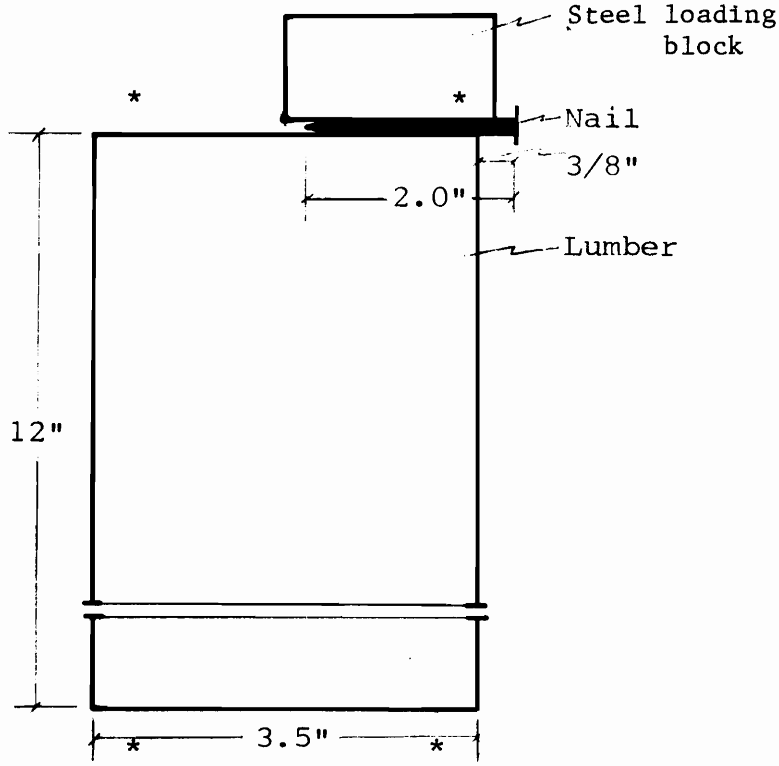


Figure 4.1. Testing arrangement for evaluating elastic bearing constant.

block was used to apply load to the nail. The contact length of the nail equalled the actual penetrating length, that is the length of nail minus the thickness of plywood. In Figure 4.1, the tests were performed on four corners (* marks) of each section of lumber and the average of four was taken as the elastic bearing constant of that section.

4.1.4. Specimen construction

The specific gravity and elastic bearing constant were the basis for selecting 25 lumber sections used for nailed joints specimens. The specific gravities and elastic bearing constants of 350 lumber sections are shown in Figure 4.2. These histograms were assumed to represent the Douglas-fir lumber. The average was 0.45 for specific gravity and 183,929 $lb/in.^3$ for elastic bearing constant. The 25-specimen lumber sample was selected from 350 sections, so that the histograms of this sample matched those in Figure 4.2. The average for the 25-specimen sample was 0.45 for specific gravity and 185,693 $lb/in.^3$ for elastic bearing constant. Furthermore, the selected lumber specimens were checked visually to assure that they were free of defects.

Nailed joints were constructed by hammer-driving until the stud and plywood made firm contact and the nail head was flat with the surface of the plywood.

Each of 25 lumber and plywood specimens was used

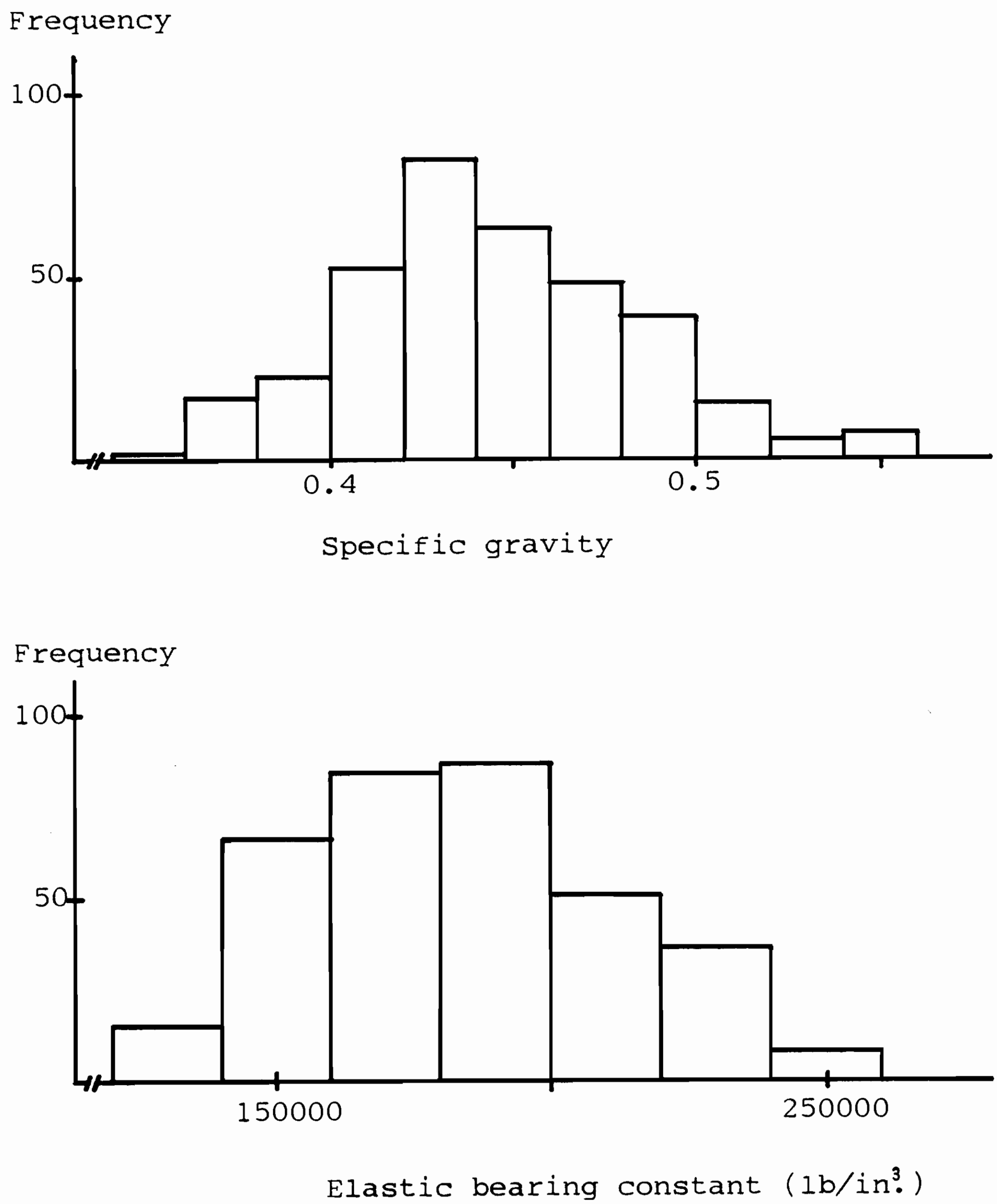


Figure 4.2. Distribution of specific gravities and elastic bearing constants of Douglas-fir lumber specimens.

eight times, once for each of eight load functions, to reduce the variation in material properties. After testing each nailed joint, the nail was carefully pulled out, and a new joint was assembled by hammer-driving a new nail into the plywood and lumber at the point one inch apart from the previous nailing point. The framing members were used four times on each narrow edge (Figure 4.3) and the plywood sections were used eight times (Figure 4.3). The reuse of lumber and plywood to make new specimen was expected to minimize the variation in creep readings for the eight load functions due to material variability.

4.2. Testing arrangement

The nailed-joint specimens (Figure 4.4) were the same as those developed in an earlier investigation (17,22). The type selected introduces almost pure shear and has considerably less moment than the joint in the ASTM testing arrangement (17,22). Steel straps were used to connect specimen to the frame supporting at the top and to the weight at the bottom in Figure 4.4.

In constant-load tests, dial gauges were used to monitor the creep slip instead of linear variable differential transformers (LVDTs). To compensate the difference in measuring instruments between constant-and varying-load tests, the measurements of the dial gauges

Figure 4.3. Douglas-fir lumber and plywood specimens used in testing.

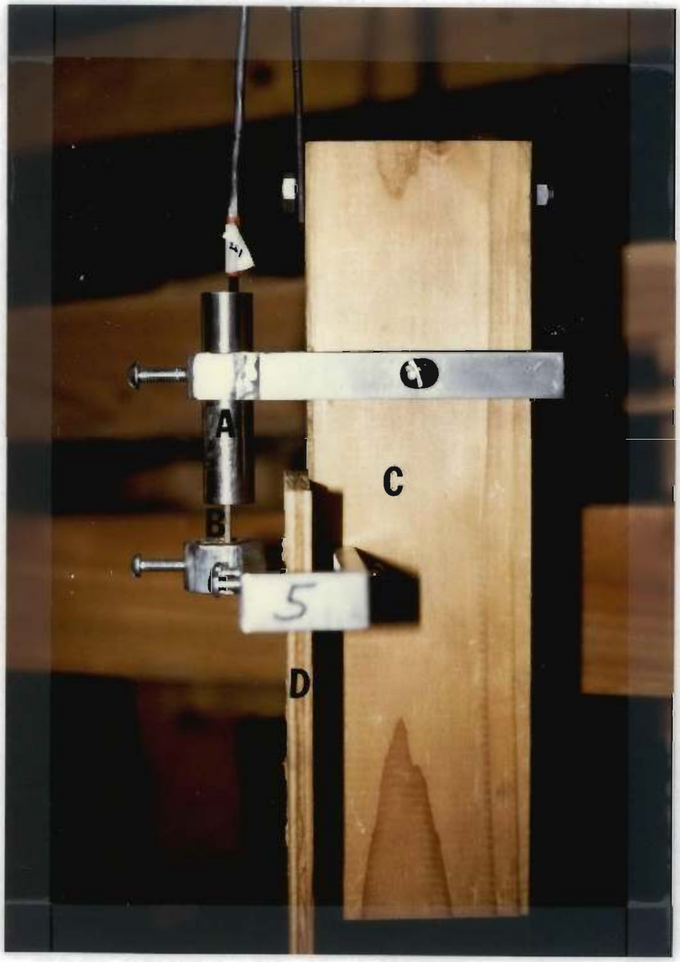


Figure 4.4. Testing arrangement for varying-load tests; A = LVDT, B = string to LVDT core, C = stud and D = plywood.

and LVDTs were calibrated by one vernier caliper. The readings were taken visually at intervals of 1, 2.5, 5, 10, 20, 30, 60, 120, 240, 480, 1440 minutes and daily afterwards.

In varying-load tests, LVDTs were used. They were connected to 14-channel analogue input box (A in Figure 4.5) which had eight AC-LVDT and six DC-LVDT channels. The signals were then sent to a 21X-micrologger (B in Figure 4.5) which was used to collect data at a preprogrammed scanning rate of one second for the first two hours and 30 minutes afterwards. Finally, the data were stored on cassette tapes by a tape recorder (C in Figure 4.5) which was connected to the 21X-micrologger. The data on the cassette tapes were recovered by an IBM microcomputer for the analysis.

The loads were applied and removed by a small lifting device that was made specifically for this testing (Figure 4.6). The device minimized the jerk and vibration which could develop during the loading and unloading processes.

4.3. Testing procedure

All the tests were performed in the conditioning room at 12-percent EMC, because the behavior of nailed joints under load is affected by the change in EMC. During the tests, checking the dry and wet bulb temperatures showed that EMC varied between 11.5 % and 12.5 %, which is within

Figure 4.5. The arrangement for data acquisition system; A = 16 channel analogue input box, B = 21 X-micrologger, C = cassette tape recorder.

Figure 4.6. Loading and unloading device.

the acceptable limits.

4.3.1. Load functions

The eight load functions employed in the tests are defined in Figures 4.7 (constant-load functions) and 4.8 (varying-load functions). The load range between 60 and 120 lb was selected, because this range represents the load which can be applied to the wall in the actual environment. The varying-load functions were selected to include loading, unloading, increasing, decreasing and cyclic loading, because these are basic patterns of the actual loads developed by several natural occurances.

4.3.2. Description of constant-load tests

In constant-load tests, sets of two or three specimens were loaded in series to reduce the time and space required for the testing. Therefore, the specimens at the top had six and those at the middle had three more pounds of weight than those at the bottom. The test results were corrected for this over-loading by using piecewise-linear interpolation to bring the slip to the target load. Piecewise-linear interpolation was sufficient because of the small corrections involved.

The loading schedule in this tests is given in Table 4.1. The specimens were tested in 10 series sets, among

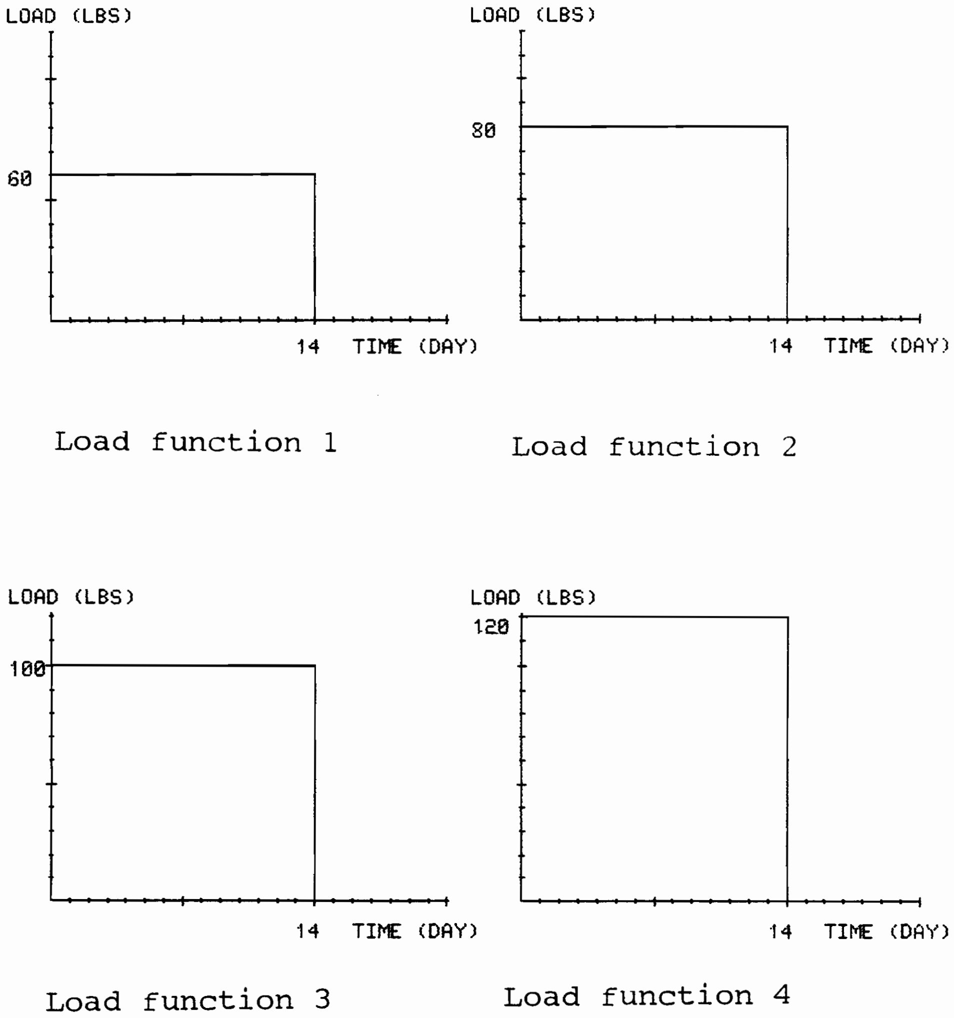


Figure 4.7. Constant-load functions used in testing to develop data needed to form the models.

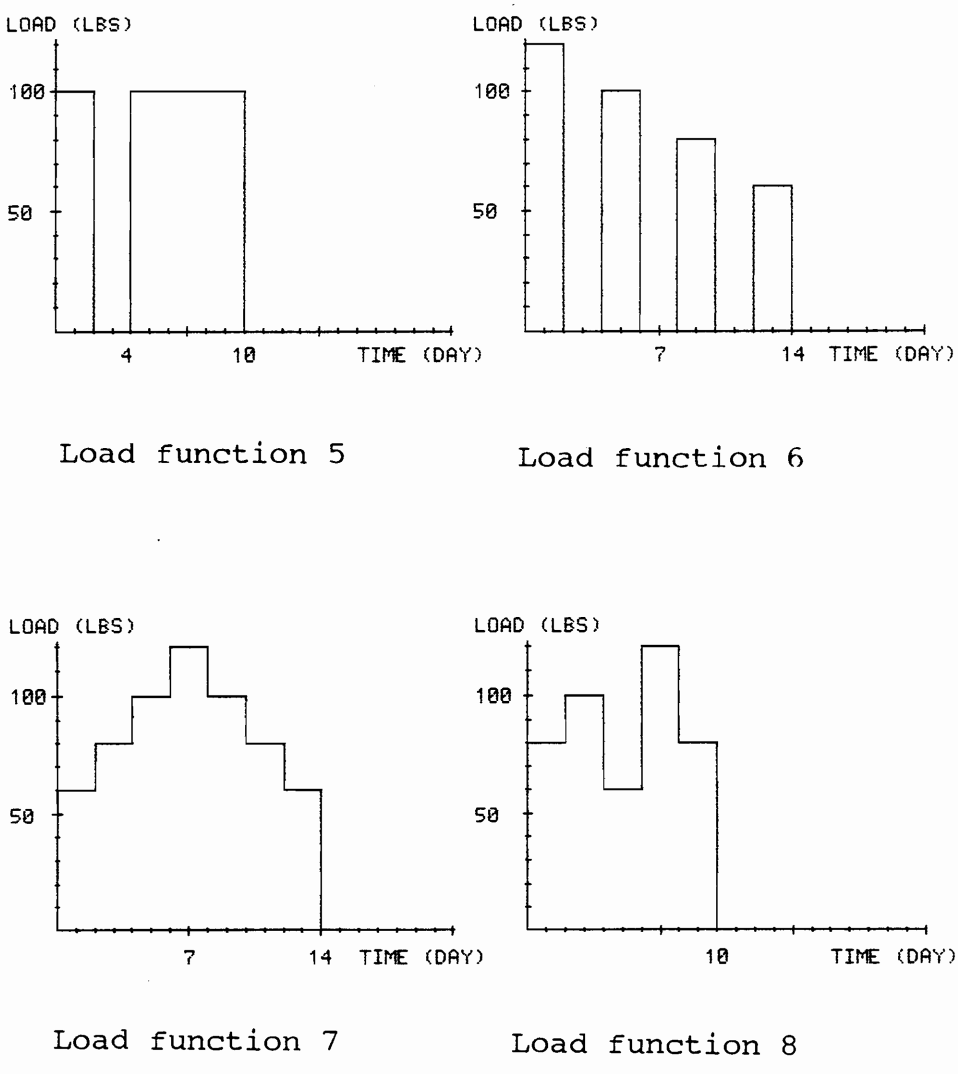


Figure 4.8. Varying-load functions used in testing to develop data for model verification.

which the first five sets had three specimens and the remaining five had two specimens. The specimens in two or three sets were simultaneously subjected to the same load function. The tests were run with all four load functions simultaneously to reduce the potential variation in the room conditioning.

After the loads were removed, the recovery was measured for one week. Among 25 specimens, five were subjected to more dynamic load than others due to the unexpected mistakes during the loading processes and showed much more instantaneous slip than the specimens which had only static loads. Therefore, the results of those five were excluded from the analysis, and varying-load tests were also conducted with the other 20 specimens.

4.3.3. Description of varying-load tests

Ten specimens were tested with only one specimen in each set. In Table 4.1, test No. 5, 6, 7 and 8 were scheduled for the first ten specimens and were the same as test No. 9, 10, 11 and 12 which were scheduled for the remaining ten specimens. Load functions 5 and 8 took 10 days and load functions 6 and 7 took 14 days to complete and the recovery was measured for one week afterwards.

The loading or unloading was accomplished with the help of a lifting device to assist with the gradual

Table 4.1. The assignment of load-function numbers (defined in Figures 4.7 and 4.8) in the testing schedule.

Test type	Test No.	The No. of load function used in the specimen groups 1 - 10									
		1	2	3	4	5	6	7	8	9	10
Constant load test	1	1	1	1	2	2	3	3	3	4	4
	2	3	3	3	4	4	1	1	1	2	2
	3	2	2	2	1	1	4	4	4	3	3
	4	4	4	4	3	3	2	2	2	1	1
Varying load test	5	5	5	5	6	6	7	7	7	8	8
	6	7	7	7	8	8	5	5	5	6	6
	7	6	6	6	5	5	8	8	8	7	7
	8	8	8	8	7	7	6	6	6	5	5
	9	5	5	5	6	6	7	7	7	8	8
	10	7	7	7	8	8	5	5	5	6	6
	11	6	6	6	5	5	8	8	8	7	7
	12	8	8	8	7	7	6	6	6	5	5

application and minimize the jerk. The partial loading and partial unloading were performed manually. Preliminary tests showed that the device reduced the loading and unloading jerk.

4.3.4. Procedure of comparing experimental and theoretical data

To compare the theoretical predictions with the experimental data, the square of the correlation coefficient, R^2, and the sum of the squares of errors, SSE, were calculated, which are defined as follows :

$$R^2 = \frac{\left[\sum_{i=1}^{N} (S-\bar{S})(S^*-\bar{S}^*)\right]^2}{\left[\sum_{i=1}^{N}(S-\bar{S})^2 \sum_{i=1}^{N}(S^*-\bar{S}^*)^2\right]} \qquad (4\text{-}7)$$

$$SSE = \sum_{i=1}^{N}(S-S^*)^2 \qquad (4\text{-}8)$$

where S = experimental data; $\bar{S}$ = mean of experimental data; S^* = predicted data; $\bar{S}^*$ = mean of predicted data; and N = number of observations.

V. RESULTS AND DISCUSSION

To develop and verify the workable models of the theoretical concepts introduced in Chapter 3, experimental observations for creep of nailed joints under constant and varying loads were needed. This chapter describes the formulation and verification of the workable models for nailed joints tested in Chapter 4. The models can predict the behavior of nailed plywood-to-wood joints under any type of loading function.

5.1. Experimental results

This section contains the results of testing described in Chapter 4.

5.1.1. Creep under constant load

The raw data obtained from the tests under constant loads of Chapter 4, which are the mean values of 20 specimens, are given in Appendix B.1. The graphs that are based on the data in Appendix B and presented in this chapter were drawn by an IBM PC/XT microcomputer and TEKTRONICS graphic plotter using the software developed in another FRL study in the Forest Products Department.

The mean time-slip traces from these data are shown in Figure 5.1. Under 60-lb load, the delayed slip and

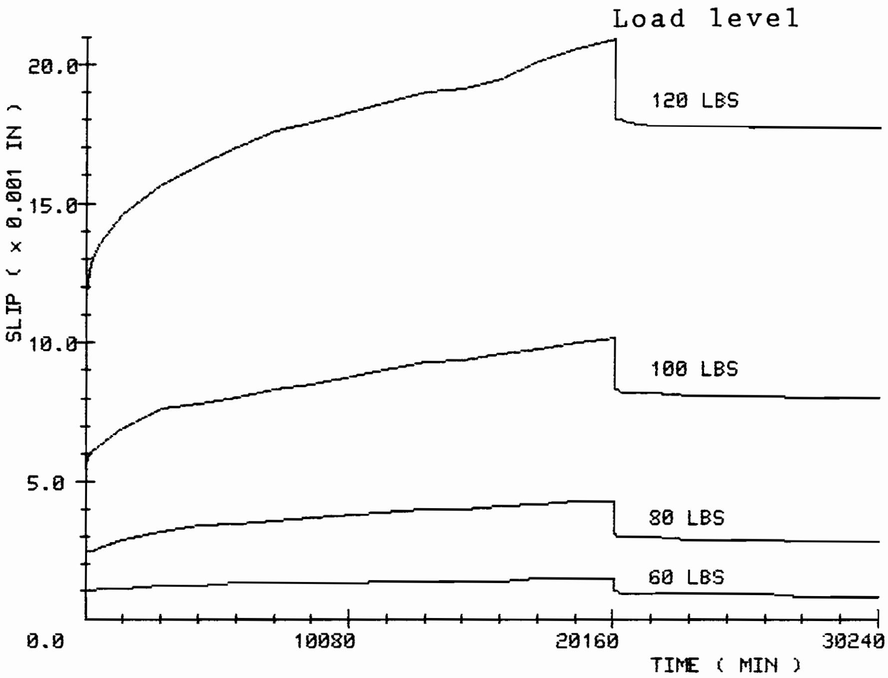


Figure 5.1. Creep of nailed joints under constant-load functions.

Table 5.1. Recoverable slip of nailed joints under constant-load functions.

Time(min.)	Creep slip (0.001 in.) under constant load (lb)			
	60	80	100	120
0.0	0.5	1.1	1.7	2.7
1.0	0.5	1.2	1.7	2.7
2.5	0.5	1.2	1.8	2.8
5.0	0.5	1.2	1.8	2.8
10.0	0.5	1.2	1.9	2.9
20.0	0.5	1.2	1.9	2.9
30.0	0.5	1.2	1.9	2.9
60.0	0.5	1.2	1.9	2.9
120.0	0.5	1.3	1.9	2.9
240.0	0.6	1.3	2.0	2.9
480.0	0.6	1.3	2.0	3.0
1440.0	0.6	1.3	2.0	3.1
2880.0	0.6	1.4	2.1	3.1
4320.0	0.6	1.4	2.1	3.1
5760.0	0.6	1.4	2.1	3.2
7200.0	0.7	1.5	2.2	3.2
10080.0	0.7	1.5	2.2	3.2
12960.0	0.7	1.5	2.2	3.2
15840.0	0.7	1.6	2.2	3.3
20160.0	0.7	1.6	2.2	3.3

Table 5.2. Nonrecoverable slip of nailed joints under constant-load functions.

Time(Min.)	Creep slip (0.001 in.) under constant load (lb)			
	60	80	100	120
0.0	0.4	1.0	2.9	6.4
1.0	0.5	1.0	3.2	7.4
2.5	0.5	1.1	3.3	7.6
5.0	0.5	1.1	3.4	8.0
10.0	0.5	1.1	3.4	8.2
20.0	0.5	1.2	3.6	8.6
30.0	0.5	1.2	3.7	8.8
60.0	0.5	1.3	3.9	9.2
120.0	0.5	1.3	4.0	9.6
240.0	0.5	1.3	4.1	10.1
480.0	0.5	1.3	4.3	10.5
1440.0	0.5	1.6	4.9	11.5
2880.0	0.6	1.8	5.5	12.6
4320.0	0.6	2.0	5.7	13.3
5760.0	0.6	2.1	5.9	13.8
7200.0	0.6	2.1	6.1	14.4
10080.0	0.6	2.3	6.6	15.1
12960.0	0.7	2.5	7.1	15.8
15840.0	0.7	2.6	7.4	16.2
20160.0	0.8	2.7	8.0	17.6

even the total slip were negligible. However, it increased as the load level increased. Under 120-lb load, the delayed slip was almost twice the magnitude of the instantaneous slip.

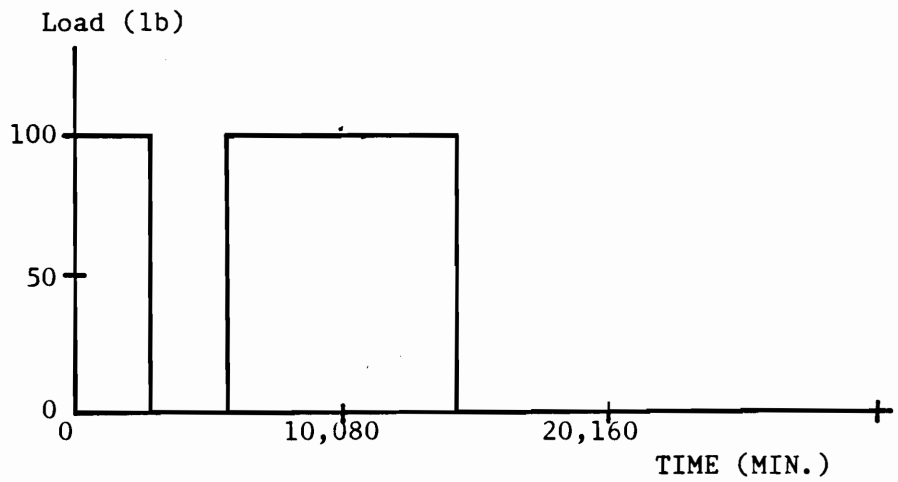
The slip recovery was relatively small compared to the slip under loading. The slip recovered immediately after unloading was about 1/2 to 1/3 of the instantaneous slip. At 60 and 80 lb, the immediate slip recovery was close to 1/2 of the instantaneous slip. The delayed recovery was negligible for all four load levels.

The total slip under each load was divided into four components : Se, S_{de} , Sp and Sv which are defined in Chapter 3. The four components are given in Table 5.1 for recoverable slip and in Table 5.2 for nonrecoverable slip. Three components : Se, Sp and Sv display nonlinear relationship with the load. The experimental slip in Tables 5.1 and 5.2 were used to evaluate the parameters for the workable creep models.

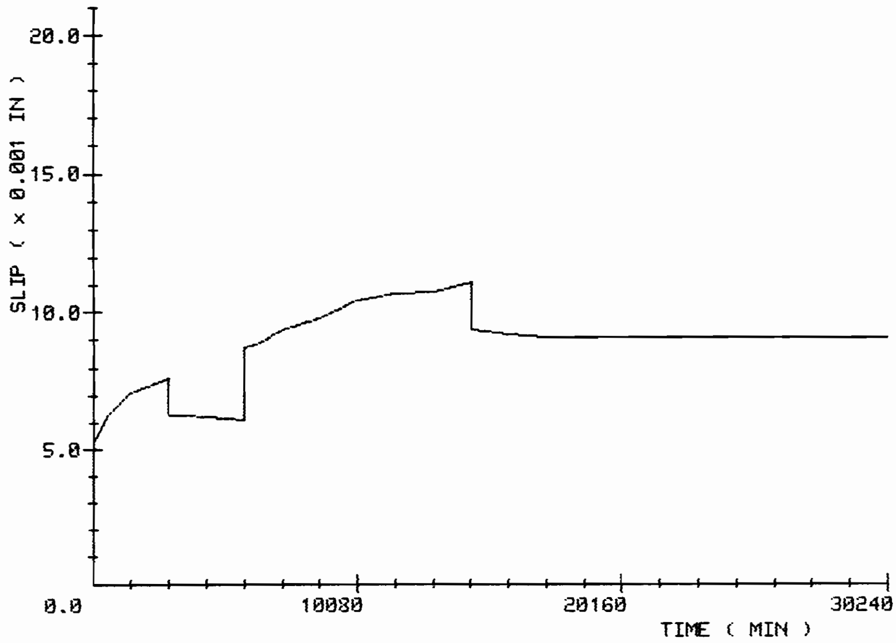
5.1.2. Creep under varying load

The means of the raw data for 20 nailed joints specimens tested under varying-load functions are presented in Appendix B.2. The mean time-slip traces for load functions 5, 6, 7 and 8 are presented in Figures 5.2, 5.3, 5.4 and 5.5, respectively.

When removing the load at t=2880 minutes (2 days),

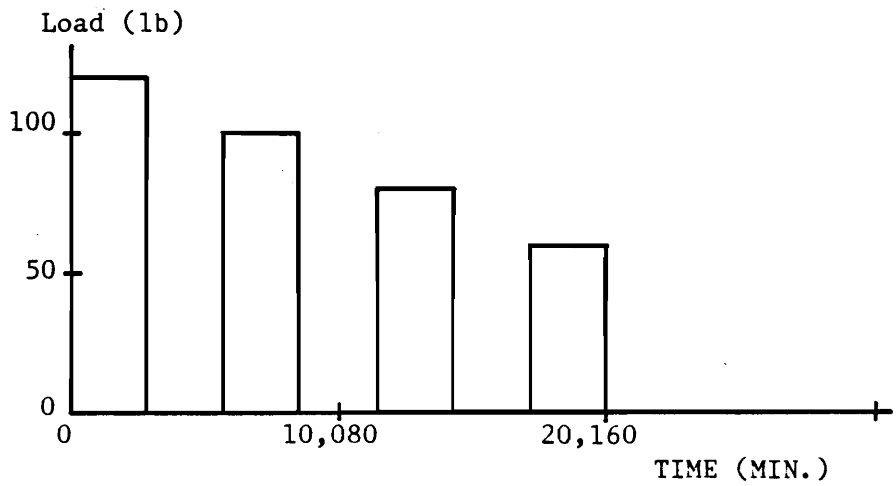


(a) Load function

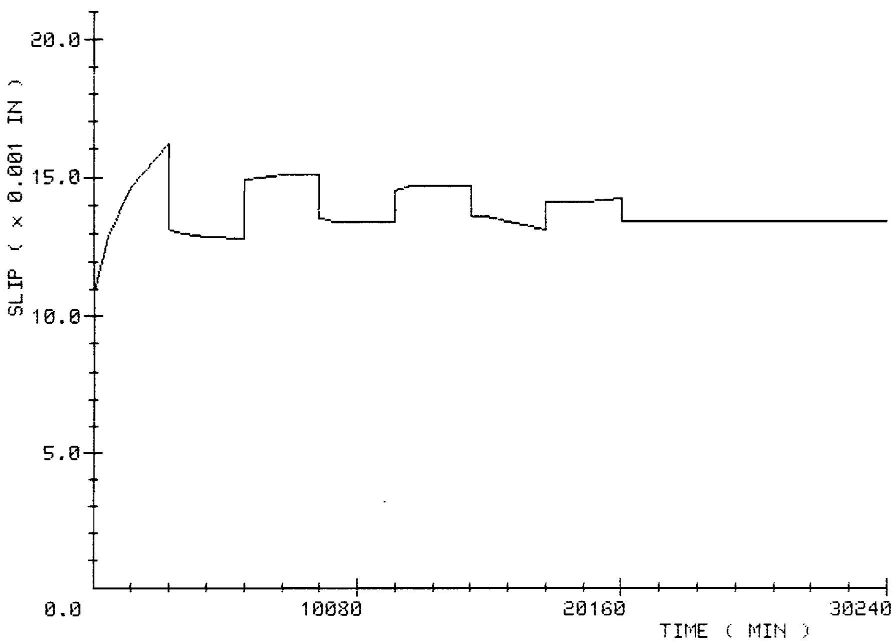


(b) Slip response

Figure 5.2. Creep of nailed joints under load function 5.

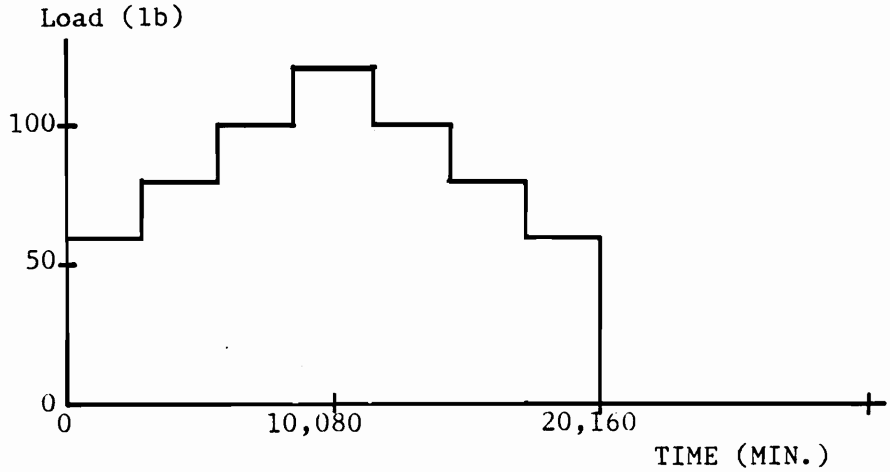


(a) Load function

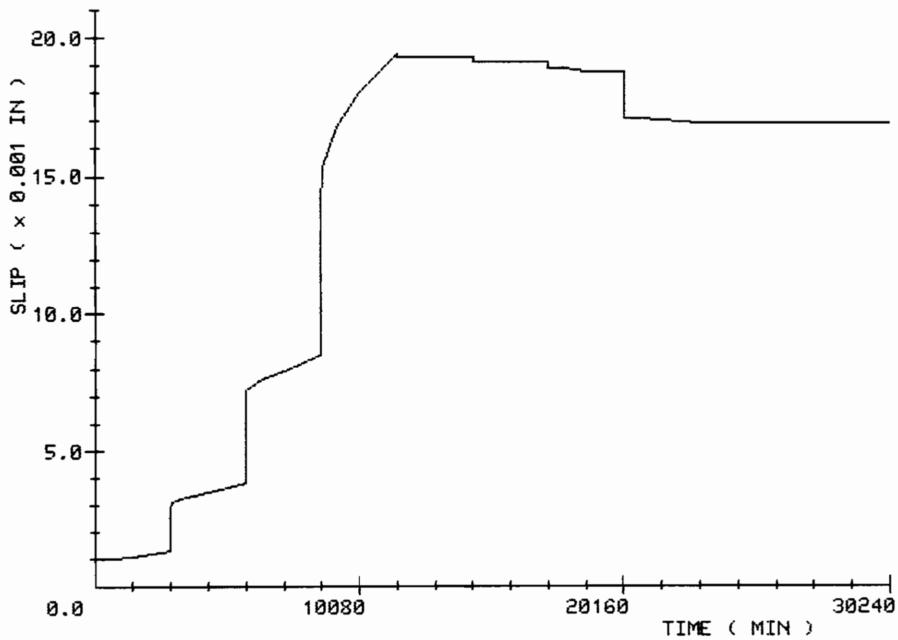


(b) Slip response

Figure 5.3. Creep of nailed joints under load function 6.

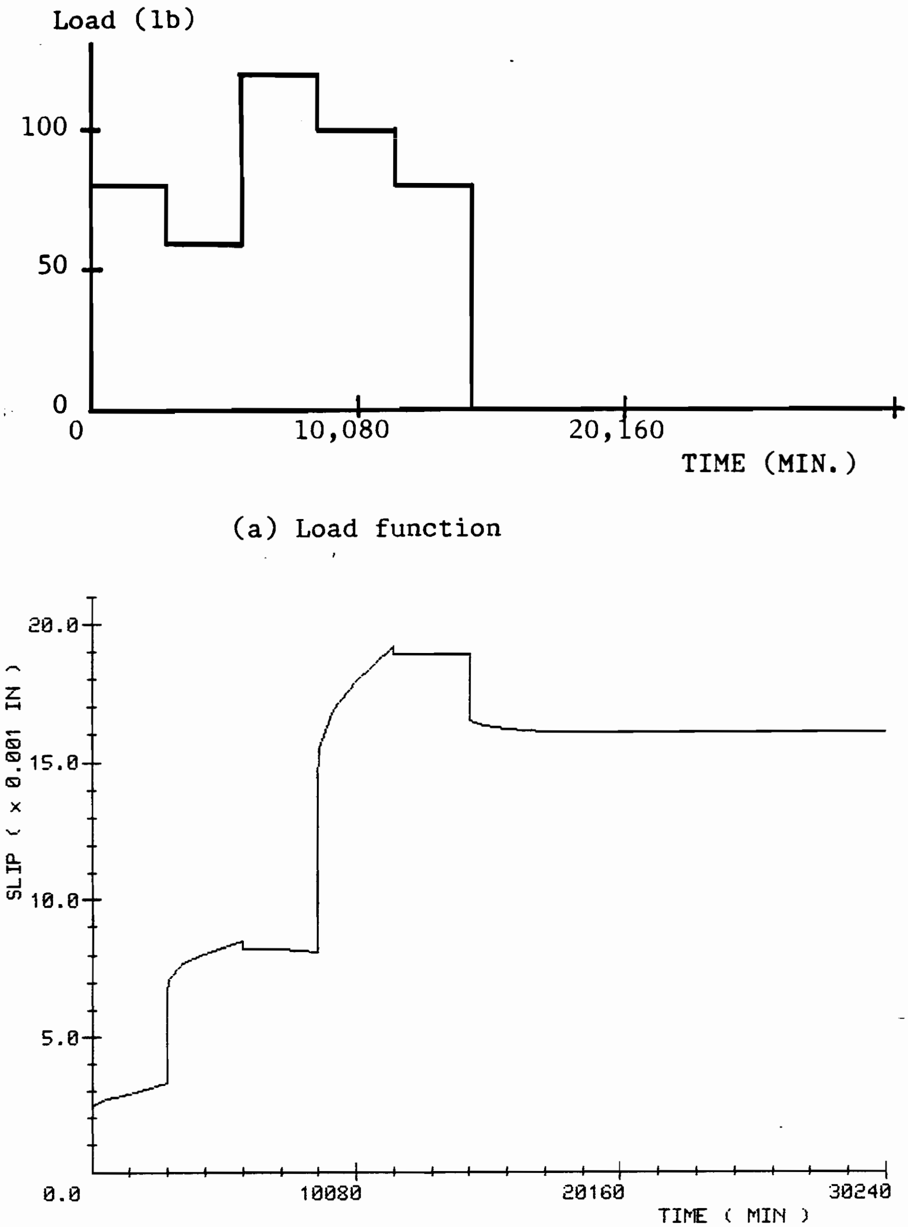


(a) Load function



(b) Slip response

Figure 5.4. Creep of nailed joints under load function 7.



(a) Load function

(b) Slip response

Figure 5.5. Creep of nailed joints under load function 8.

the trace in Figure 5.2 displays a recovery of about 1/3 of the instantaneous slip, which remained almost constant until reloading. Upon reloading at t=5760 minutes (4 days), the slip quickly reached the magnitude observed just before unloading. After unloading at the end of the load function, about 1/3 of the instantaneous slip observed in the previous step was recovered again and the subsequent delayed recovery was negligible. From the observations discussed, it was postulated that the recovery was not affected by the loading history.

Figure 5.3 depicts the slip under uniform loads of 120, 100, 80 and 60 lb. After the initial loading of 120 lb, the slip increased stiffly. Upon unloading, about 1/3 of the instantaneous slip was recovered, which was followed by a small amount of delayed recovery. In the 100-lb step, both the slip and recovery were smaller than that in the first step. The same trend was observed in the subsequent loading and unloading steps of 80 and 60 lb. The instantaneous slip and recovery were almost the same in each step except the first step, and their magnitudes were similar to the instantaneous elastic slip under constant-load functions. These observations underlined the general understanding that the prior loading history affects the creep. In this study, the following specific observation was made : if the present load is smaller than the previous maximum load, only elastic slip is developed upon loading and is recovered upon unloading.

Figure 5.4 depicts the creep slip under load function 7. During stepwise load increase, the slip increased nonlinearly. Even though the load increment was the same, the slip increase got progressively larger with higher loads. Therefore, it was deduced that the present load level and the previous loading history affected the slip in next step. The recovery during the stepwise load decrease was considerably smaller than that in the complete unloading step at the end of decreasing load.

Figure 5.5 shows the slip response to load function 8. Partial unloading of 40 lb at t=5,760 minutes (4 days) gave a negligible but almost the same recovery as that of a 40 pound partial unloading at t=11,520 minutes (8 days). This indicates that the recovery under decreasing load is not affected by the present load level and the previous loading history. By observing the creep slip under load functions 7 and 8, it can be postulated that the decreasing load can be treated as the compressive force, which acts in the opposite direction to the existing load and causes the elastic displacement only.

The observations and postulations presented in Figures 5.2 through 5.5 were included in theoretical modeling discussed in Chapter 3.

5.2. Formulation of theoretical models

The parameters of five models were obtained from the results of constant load tests. The data were fitted by a nonlinear least squares procedure (21,28,31) for Models 5-E and M5-E and by the procedure reported earlier (36) for Model V-VE1. The curve fitting procedures are discussed next.

5.2.1. Five-element Model (5-E)

The constitutive equation for Model 5-E is equation (3-17) which is composed of two parts, Sr and Sn. Each part includes two linear and one nonlinear parameters. The nonlinear parameters are A for Sr and m for Sn. The experimental data for each component are given in Tables 5.1 and 5.2.

The curve fitting was performed as follows. The recoverable part in equation (3-17) was rewritten as :

$$Sr = A_1 + A_2\{1-EXP(-A_3 t)\}+e \qquad (5\text{-}1)$$

where e = error in curve fitting. Inversion of equation (5-1) gives :

$$e = Sr-[A_1+A_2\{1-EXP(-A_3 t)\}] \qquad (5\text{-}2)$$

The sum of the squares of the errors equalled :

$$SSE = \sum_{i=1}^{N} e_i^2 = \sum_{i=1}^{N} [S_{ri} - A_1 - A_2\{1-EXP(-A_3\ t\ )\}]^2 \qquad (5\text{-}3)$$

where N is the number of observations. To minimize equation (5-3), the partial derivatives should be zeros :

$$\partial\ SSE/\ \partial A = -2\sum_{i=1}^{N} [S_{ri} - A_1 - A_2\{1-EXP(-A_3\ t_i)\}] = 0 \qquad (5\text{-}4)$$

$$\partial\ SSE/\ \partial A = -2\sum_{i=1}^{N} \{1-EXP(-A_3\ t_i)\}\ [S_{ri} - A_1 - A_2\{1-EXP(-A_3\ t_i)\}] = 0 \qquad (5\text{-}5)$$

$$\partial\ SSE/\ \partial A = -2A_2\sum_{i=1}^{N} t_i\ EXP(-A_3\ t_i)\ [S_{ri} - A_1 - A_2\{1-EXP(-A_3\ t_i)\}] = 0 \qquad (5\text{-}6)$$

Substituting in equations (5-4) and (5-5) as follows :

$$E_i = EXP(-A_3\ t_i) \qquad (5\text{-}7)$$

results in :

$$NA_1 + A_2(N - \sum_{i=1}^{N} E_i) = \sum_{i=1}^{N} S_{ri} \qquad (5\text{-}8)$$

$$A_1(N - \sum_{i=1}^{N} E_i) + A_2\ (N - 2\sum_{i=1}^{N} E_i + \sum_{i=1}^{N} E_i^2) = \sum_{i=1}^{N} S_{ri} - \sum_{i=1}^{N} S_{ri} E_i \qquad (5\text{-}9)$$

Equations (5-8) and (5-9) are linear with respect to parameters A1 and A2, and they were rewritten in a matrix form :

$$\begin{bmatrix} N & N - \sum_{i=1}^{N} E_i \\ N - \sum_{i=1}^{N} E_i & N - 2\sum_{i=1}^{N} E_i + \sum_{i=1}^{N} E_i^2 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{N} S_{ri} \\ \sum_{i=1}^{N} S_{ri} - \sum_{i=1}^{N} S_{ri} E_i \end{Bmatrix} \qquad (5\text{-}10)$$

which was solved by Gauss elimination procedure to give :

$$\begin{bmatrix} 1 & 1-\sum_{i=1}^{N}E_i/N \\ 0 & \sum_{i=1}^{N}E_i^2-(\sum_{i=1}^{N}E_i)^2/N \end{bmatrix}\begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix}=\begin{Bmatrix} \sum_{i=1}^{N}S_{ri}/N \\ \sum_{i=1}^{N}S_{ri}\sum_{i=1}^{N}E_i/N-\sum_{i=1}^{N}S_{ri}E_i \end{Bmatrix} \quad (5\text{-}11)$$

from which :

$$A_2 = [1/\{\sum_{i=1}^{N}E_i^2-(\sum_{i=1}^{N}E_i)^2/N\}](\sum_{i=1}^{N}S_{ri}\sum_{i=1}^{N}E_i/N-\sum_{i=1}^{N}S_{ri}E_i) \quad (5\text{-}12)$$

$$A_1 = \sum_{i=1}^{N}S_{ri}/N-A_2(1-\sum_{i=1}^{N}E_i/N) \quad (5\text{-}13)$$

Equations (5-12) and (5-13) were solved by first assuming a reasonable estimate for A_3 in equation (5-7). Based on assumed A_3 , A_1 and A_2 were calculated and then the estimated A_3 should be checked by applying equation (5-6). If equation (5-6) was not equal to zero, the next estimate of A_3 could be obtained by Newton's formula :

$$A_3'' = A_3' -[(\partial SSE/\partial A_3)/(\partial^2 SSE/\partial A_3^2)] \quad (5\text{-}14)$$

where A_3' = current estimate, and A_3'' = estimate in the next step. The second derivative of SSE with respect to A_3 in equation (5-14) was obtained by differentiating equation (5-6) :

$$\partial^2 SSE/\partial A^2 = 2A_2[\sum_{i=1}^{N}S_{ri}t_i^2E_i-(A_1+A_2)\sum_{i=1}^{N}t_i^2E_i+ 2A_2\sum_{i=1}^{N}t_i^2E_i^2] \quad (5\text{-}15)$$

Table 5.3. Parameters in equation (3-17) defining Model 5-E.

LOAD	A1	A2	A3	A4	A5	m
60	0.5118	0.19014	0.0002981	0.0004536	0.2941	0.57
80	1.2130	0.40501	0.0001523	0.0288380	1.0509	0.41
100	1.8438	0.34536	0.0005148	0.1451320	3.0947	0.35
120	2.8434	0.42717	0.0003385	0.5136500	7.1030	0.30

Substituting the current estimate and equation (5-15) into equation (5-14) calculated the estimate of A_3 in the next step, which became again the current estimate in the next step. This process was repeated until equation (5-6) approached zero.

The nonrecoverable part of equation (3-17) was obtained by the same method as the recoverable part. A BASIC program for an IBM microcomputer was developed for above nonlinear least square procedure (Appendix C.1). The parameters of the models for nailed joints between Douglas-fir lumber and plywood under constant load were evaluated. They are given in Table 5.3.

5.2.2. Modified Five-element Model (M5-E)

The constitutive equation for Model 5-E is equation (3-21) in which B_1, B_2, B_4 and B_5 are linear and B_3, N_1, N_2, N_3 and N_4 are nonlinear. Nonlinear least square fitting of the experimental data could not be used, because the number of nonlinear parameters is too high. Therefore, the parameters of Model M5-E were obtained from the parameters of Model 5-E because Models 5-E and M5-E are obtained from the same data and have the same components except the nonlinear terms for the instantaneous elastic and viscous slip of Model M5-E.

Equation (3-17) of Model 5-E and equation (3-21) of Model M5-E consist of four components, each of which has a

Table 5.4. Parameters in equation (3-21) defining Model M5-E.

Parameter	Value
B_1	5.7464 X 10
B_2	0.003812
B_3	3.2590 X 10
B_4	3.1916 X 10
B_5	4.8121 X 10
N_1	2.2838
N_2	4.3212
N_3	0.3500
N_4	4.9026

similar form for both models. Therefore, by comparing each part of equation (3-21) with the corresponding part of equation (3-17), the parameters of Model M5-E were obtained in terms of the parameters of Model 5-E. They were found to be :

$$B_1 P^{N_1} = A_1 \tag{5-16}$$

$$B_2 P = A_2 \tag{5-17}$$

$$B_3 = A_3 \tag{5-18}$$

$$B_4 P^{N_2} = A_4 \tag{5-19}$$

$$N_3 = m \tag{5-20}$$

$$B_5 P^{N_4} = A_5 \tag{5-21}$$

Equations (5-17), (5-18) and (5-20) show linear relation between the parameters of Model M5-E, B_2, B_3 and N_3, and the parameters of Model 5-E, A_2, A_3 and m. Therefore, B_2, B_3 and N_3 were obtained as the averages of the values for A_2, A_3 and m given in Table 5.3, because the table shows values for each of the four load levels. In equations (5-16), (5-19) and (5-21), parameters B_1, B_4 B_5, N_1, N_2 and N_4 have nonlinear relation with A_1, A_4 and A_5, which are the parameters of Model 5-E. Thus, they were evaluated by nonlinear least square fitting of the values for A_1, A_4 and A_5 given in Table 5.3. The analysis resulted in Table 5.4.

5.2.3. Viscous-viscoelastic Model 1 (V-VE1)

Model V-VE1 can be represented by equation (3-27), in which the first three terms represent recoverable slip and the remaining represent nonrecoverable slip. For the nailed joints investigated, the experimental data needed to formulate equation (3-27) are given in Tables 5.1 and 5.2.

To obtain the kernel functions for the three integral representation, the procedure proposed by Polensek (36) was employed. For instance, the recoverable slip was represented by :

$$Sr = F_1(t)P+F_2(t)P^2+F_3(t)P^3 \tag{5-22}$$

and rewritten in a matrix form :

$$\{Sr\} = [P]\ \{F(t)\} \tag{5-23}$$

where :

$$\{Sr\} = \begin{Bmatrix} S_{r1} \\ S_{r2} \\ S_{r3} \end{Bmatrix} \tag{5-24}$$

$$[P] = \begin{bmatrix} P_1 & P_1^2 & P_1^3 \\ P_2 & P_2^2 & P_2^3 \\ P_3 & P_3^2 & P_3^3 \end{bmatrix} \tag{5-25}$$

$$\{F(t)\} = \begin{Bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \end{Bmatrix} \tag{5-26}$$

and kernel functions were obtained by inversion of equation (5-23) :

$$\{F(t)\} = [P]^{-1} \{Sr\} \tag{5-27}$$

To get kernel functions, the data under load levels P1 = 60 lbs, P2 = 100 lbs and P3 = 120 lbs were used. Therefore, the inverse of load matrix, $[P]^{-1}$, gave :

$$[P]^{-1} = \begin{bmatrix} 1/12 & -9/100 & 1/24 \\ -11/7200 & 9/4000 & -1/900 \\ 1/14400 & -1/80000 & 1/144000 \end{bmatrix} \tag{5-28}$$

in which the unit of matrix elements is (1/lb). In equation (5-23), Sr was expressed by power functions :

$$\{Sr\} = \{Sr^{\circ} + Sr^{+}(t)\} \tag{5-29}$$

and kernel functions, $\{F(t)\}$, were represented by equations (3-39), (3-40) and (3-41). Thus, equation (5-27) equaled :

$$\{F^{\circ} + F^{+}(t)\} = [P]^{-1} \{Sr^{\circ} + Sr^{+}(t)\} \tag{5-30}$$

in which $\{F^{\circ}+F^{+}(t)\} = \{F^{\circ}\}+\{F^{+}(t)\}$, and $\{F^{\circ}\}$ and $\{F^{+}(t)\}$ were obtained as follows :

$$\{F^{\circ}\} = [P]^{-1} \{Sr^{\circ}\} \tag{5-31}$$

$$\{F^{+}(t)\} = [P]^{-1} \{Sr^{+}(t)\} \tag{5-32}$$

Next, these procedures were applied to the nailed joints investigated in this study. The experimental data given in Table 5.1 were fitted to the power functions by applying nonlinear least squares analysis to give :

$$S_{r1} = 0.477+0.013222\ t^{0.30}, \text{ for } P=60 \text{ lb} \quad (5\text{-}33)$$

$$S_{r2} = 1.701+0.104614\ t^{0.17}, \text{ for } P=100 \text{ lb} \quad (5\text{-}34)$$

$$S_{r3} = 2.544+0.217091\ t^{0.12}, \text{ for } P=120 \text{ lb} \quad (5\text{-}35)$$

and kernel functions were obtained in the same way as before. They are :

$$F_1(t) = [-7.340+1.1020\ t^{0.30}-9.4153\ t^{0.17}+ 9.0455\ t^{0.12}]\ 0.001 \quad (5\text{-}36)$$

$$F_2(t) = [0.272-0.0202\ t^{0.30}+0.2354\ t^{0.17}- 0.2412\ t^{0.12}]\ 0.001 \quad (5\text{-}37)$$

$$F_3(t) = [-0.00028+0.00009\ t^{0.30}-0.0013\ t^{0.17}+ 0.0015\ t^{0.12}]\ 0.001 \quad (5\text{-}38)$$

For nonrecoverable slip, the method was the same. The experimental data for nonrecoverable slip (Table 5.2) were fitted to the power functions that had the same power of time:

$$S_{n1} = 0.2825+0.003878\ t^{0.35}, \text{ for } P=60 \text{ lb} \quad (5\text{-}39)$$

$$S_{n2} = 3.0947+0.145132\ t^{0.35}, \text{ for } P=100 \text{ lb} \quad (5\text{-}40)$$

$$S_{n3} = 7.5054+0.306340\ t^{0.35}, \text{ for } P=120 \text{ lb} \quad (5\text{-}41)$$

from which the following kernel functions were obtained :

$$F_4(t) = [57.74367+0.025453\ t^{0.35}]\ 0.001 \quad (5\text{-}42)$$

$$F_5(t) = [-1.80786-0.019755\ t^{0.35}]\ 0.001 \quad (5\text{-}43)$$

$$F_6(t) = [0.01540+0.00034\ t^{0.35}]\ 0.001 \quad (5\text{-}44)$$

Next, kernel functions evaluated in equations (5-36), (5-37) and (5-38) for Sr and (5-42), (5-43) and (5-44) for Sn were substituted into equation (3-27) to give the creep slip of Model V-VE1 for nailed joints under constant load.

5.2.4. Viscous-viscoelastic Model 2 (V-VE2) and Model3 (V-VE3)

Models V-VE2 and V-VE3 are combinations of Models M5-E and V-VE1. The constitutive equations of Models V-VE2 and V-VE3 are given in equations (3-28) and (3-29), respectively, whose parameters can be obtained from Models M5-E and V-VE1. The procedure is easy to carry out and is not presented in this dissertation.

5.2.5. Fourier series approximation of load function 7

Load function 7 was applied as an example of continuous-load function. The solution involved the approximation of load function 7 by equation (3-89) of Fourier sine series in Section 3.3. The coefficients were obtained by equation (3-88) and are given in Table 5.5,

Table 5.5. Fourier sine-series coefficients in equation (3-88) defining load function 7

Coefficient	Value
Ao	60.0
B_1	44.48648
B_3	-11.05122
B_5	0.27989
B_7	-3.63783
B_9	0.15550
B_{11}	-3.01394
B_{13}	3.42207
B_{15}	2.96574
B_{17}	-1.95024
B_{19}	0.07365

$B_i \simeq 0.0$ when $i = 2,4,6,8,10,12,14,16,18,20$

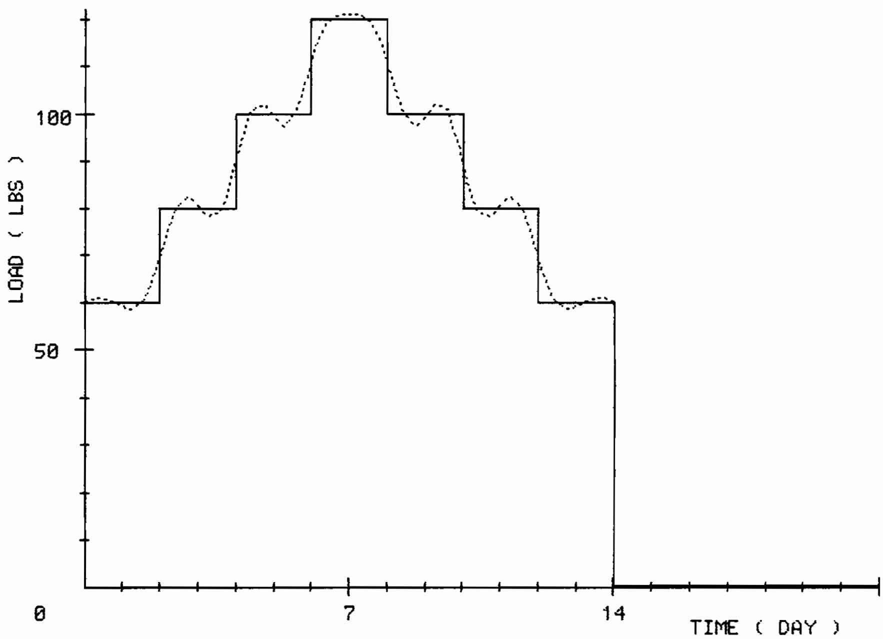


Figure 5.6. Load function 7 and its Fourier sine series approximation.

in which the number of terms in the series was 20. The approximated curve is shown in Figure 5.6.

5.3. Verification of theoretical models

The theoretical models for varying loads were obtained according to the procedure described in Chapter 3, and the specific parameters for nailed joints investigated were developed in Section 5.2. In this Section, the predictions of the theoretical models developed were compared to each other and to the experimental data.

5.3.1. Response to constant-load functions

The predictions of the models under constant-load functions are presented in Figure 5.7. Although the data under 80-lb loading were not used to construct Models V-VE1, V-VE2 and V-VE3, their predictions were also valid for 80-lb loading as shown in Figure 5.7. Tables 5.6 and 5.7 show the statistical indicators, R^2 and SSE, for each of the five models under constant-load functions, respectively.

5.3.2. Accurary of Solutions developed

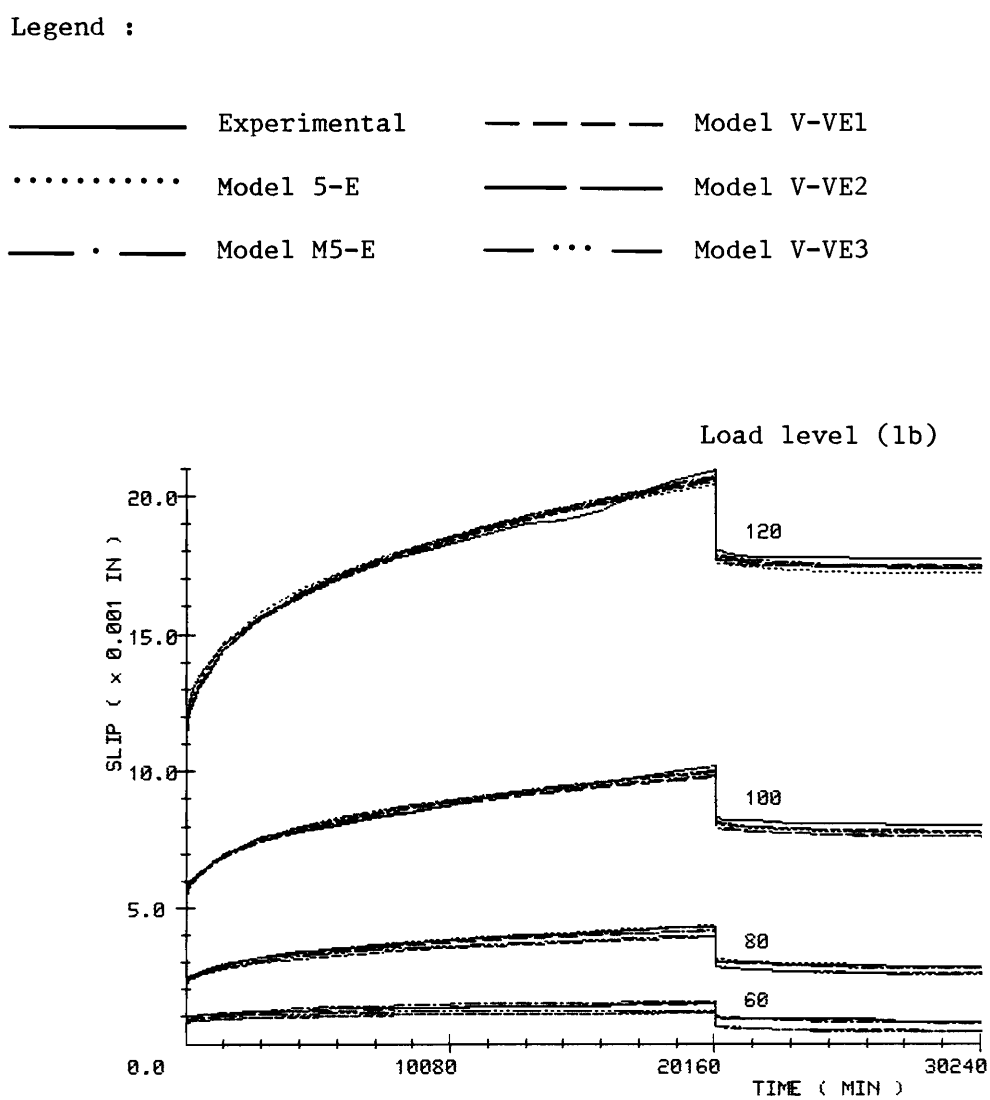


Figure 5.7. Experimental and theoretical data obtained under constant-load functions.

Table 5.6. The values of R^2 for the models under constant-load functions.

MODEL	60 lbs	80 lbs	100 lbs	120 lbs
5-E	0.7421	0.9065	0.9545	0.9734
M5-E	0.8219	0.9139	0.9500	0.9724
V-VE1	0.7601	0.9102	0.9598	0.9757
V-VE2	0.7507	0.9100	0.9560	0.9759
V-VE3	0.7275	0.9069	0.9546	0.9722

Table 5.7. The values of SSE for the models under constant-load functions.

MODEL	60 lbs	80 lbs	100 lbs	120 lbs
5-E	3.7566	1.3866	4.0542	11.3995
M5-E	0.5280	1.1841	5.4727	10.4951
V-VE1	3.5773	3.0573	3.5035	10.0897
V-VE2	0.6271	1.2769	5.2385	9.5246
V-VE3	2.8768	2.7416	3.8514	10.8314

For modifications of the models, four Solutions were proposed in chapter 3. They are :

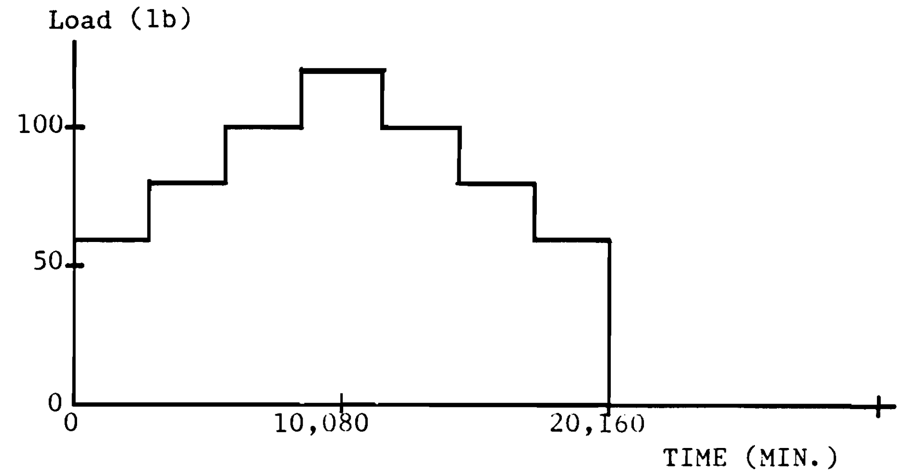
Solution 1) Application of Approach 1 with discrete load function;

Solution 2) Application of Approach 2 with discrete load function;

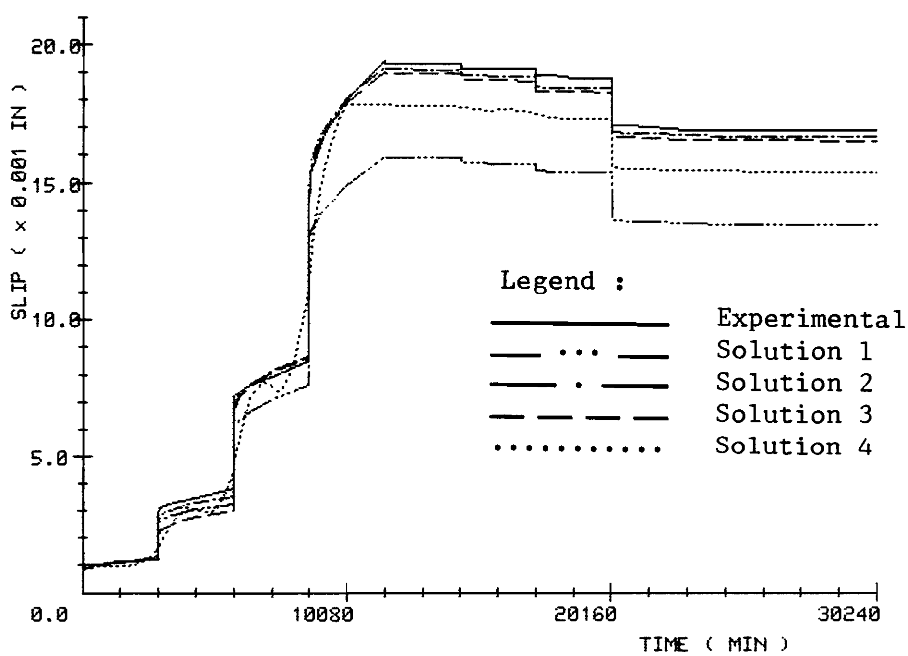
Solution 3) Application of Approach 2 with load function represented by Heaviside function; and

Solution 4) Application of Approach 2 with load function represented by Fourier series.

These Solutions were applied to the models except Model 5-E for load function 7 and then compared to one another and to the experimental data. The creep values predicted by each Solution and each model were computed by BASIC programs for an IBM microcomputer which are recorded in Appendices C.6 (Solution 1), C.4 (Solution 2), C.7 (Solution 3) and C.8 (Solution 4). The predicted results are given in Appendices B.7 (Solution 1), B.5 (Solution 2), B.8 (Solution 3) and B.9 (Solution 4). The results are also presented graphically in Figures 5.8, 5.9, 5.10 and 5.11 for Models M5-E, V-VE1, V-VE2 and V-VE3, respectively. Model 5-E was expressed by Solution 2 because all Solutions except Solution 2 require integration, while Model 5-E is discrete and cannot be

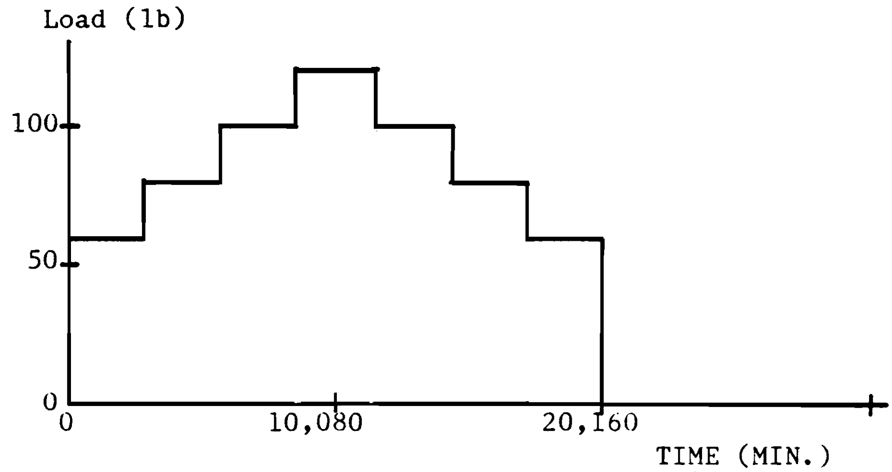


(a) Load function

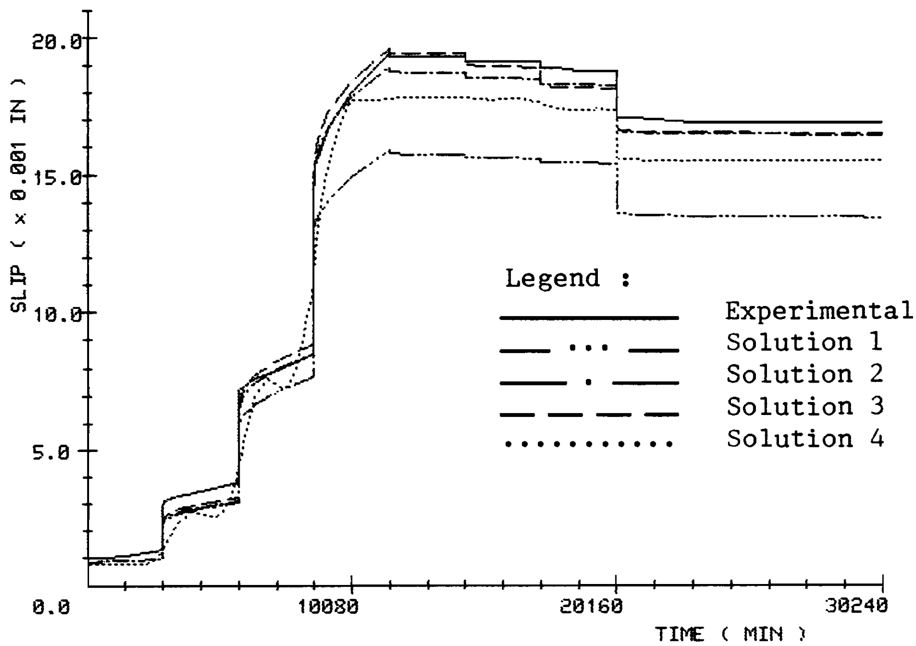


(b) Slip response

Figure 5.8. Effect of Solutions used on the creep of Model M5-E under load function 7.

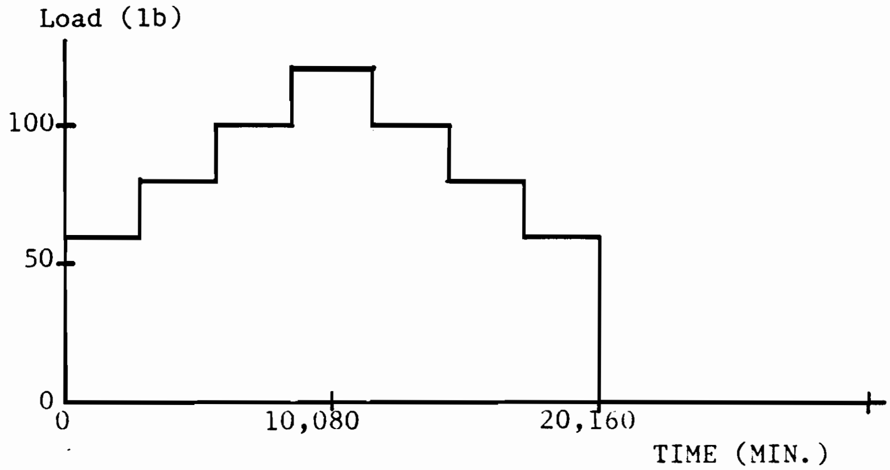


(a) Load function

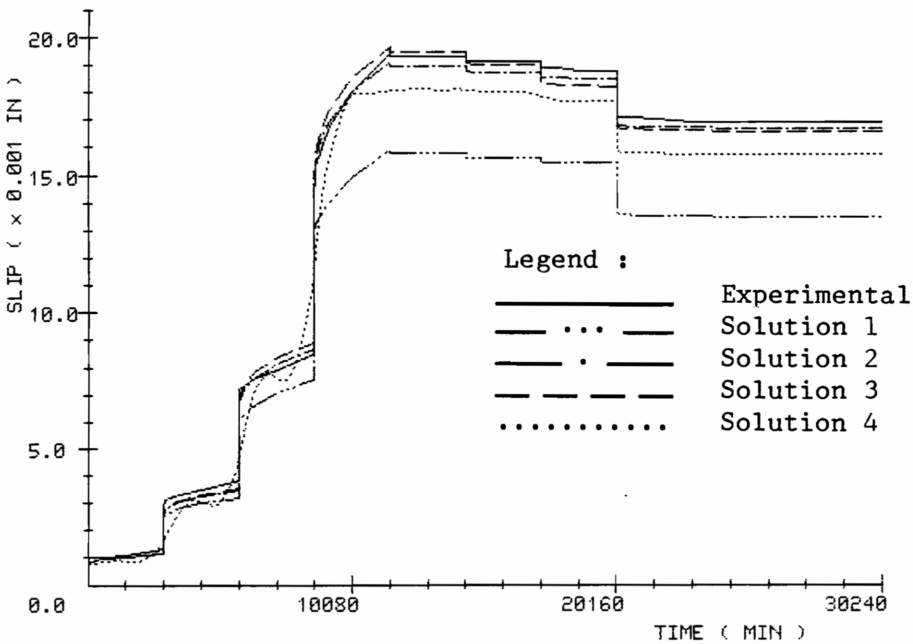


(b) Slip response

Figure 5.9. Effect of Solutions used on the creep of Model V-VE1 under load function 7.

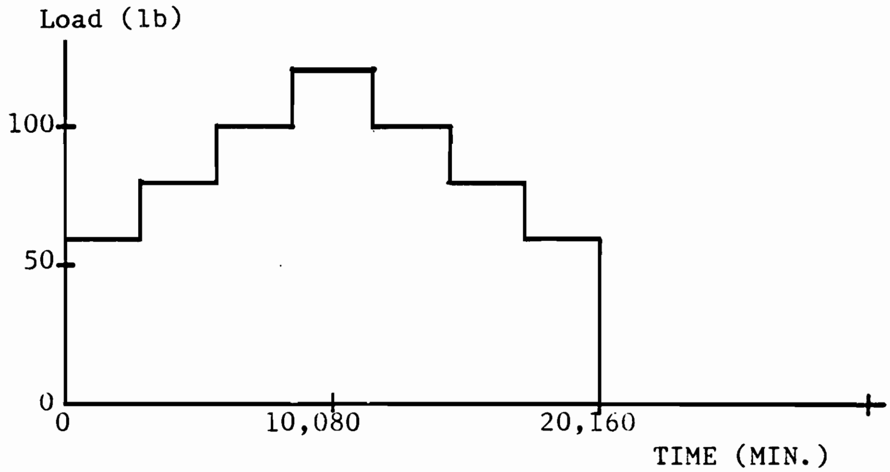


(a) Load function

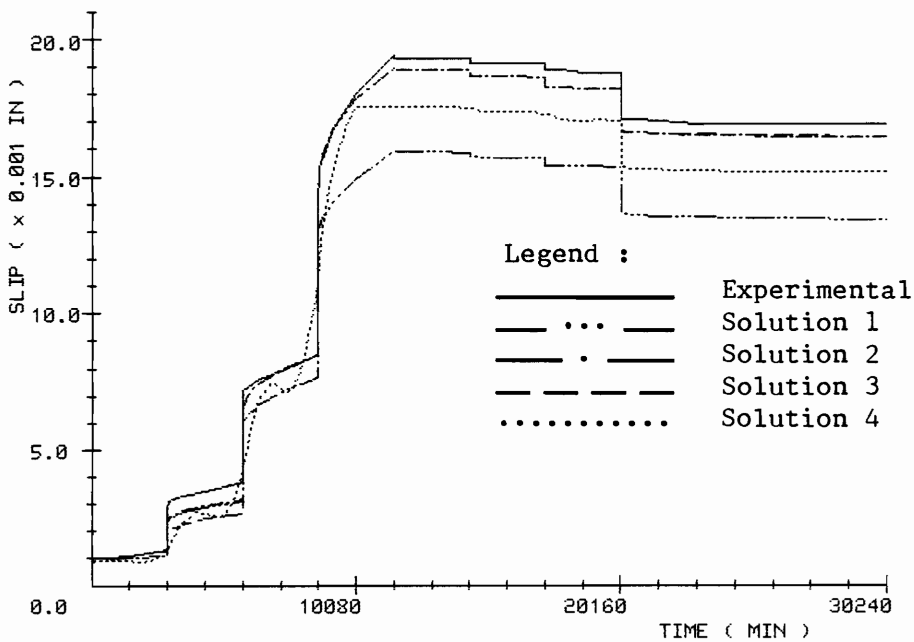


(b) Slip response

Figure 5.10. Effect of Solutions used on the creep of Model V-VE2 under load function 7.



(a) Load function



(b) Slip response

Figure 5.11. Effect of Solutions used on the creep of Model V-VE3 under load function 7.

integrated.

For all four models, Solution 1 predicted the lowest slip values, which were the worst among the four Solutions as illustrated in Figures 5.8 through 5.11, especially at high load levels. Solution 4 predicted intermediate values, but the predicted curves were wavy. The predictions by Solutions 2 and 3 were superior to the other two and very close to the experimental data. For Model M5-E (Figure 5.8), Solution 2 yielded slightly better agreement than that of Solution 3. For Models V-VE1 (Figure 5.9) and V-VE2 (Figure 5.10), Solution 3 gave a slightly closer agreement than Solution 2, but both Solutions gave results that were very close to each other for Model V-VE3 (Figure 5.11). Thus, results by Solutions 2 and 3 appear to give about the same accuracy. However, Solution 2 is much easier to apply than Solution 3, because the latter involves complicated integration of Heaviside function. For these reasons, Solution 2 is the preferred one among the four Solutions.

Solution 4 gave a relatively accurate prediction although it was wavy and underpredicted the slip during unloading steps. The underprediction was due to the shape of the approximated load function by Fourier sine series (Figure 5.6), which had its maximum at t=10,080 minutes (7 days) and decreased from that point on. However, the actual load function had its maximum between t=8,640 minutes (6 days) and t=11,520 minutes (8 days).

Therefore, the approximated load function had shorter loading period under maximum load than the actual load function. For this reason, the predictions of the models by Solution 4 were below the experimental data at the maximum load and during unloading. Therefore, if the load function could be approximated more exactly by Fourier series, the results would have agreed better. Random load functions, such as those caused by winds or earthquakes, cannot be applied to the models directly, because they cannot be integrated. However, for such functions, Solution 4 is relatively easy to apply and expected to produce acceptable aaccuracy.

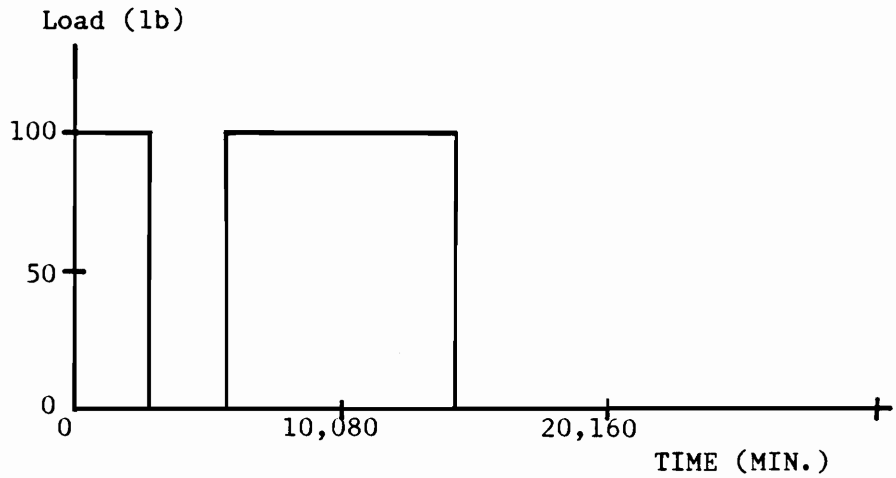
The comparisons in Figures 5.8 through 5.11 show that Solution 2 is not only the most accurate but also the easiest to apply for stepwise loads. Thus, only Solution 2 was extended to the remaining experimental load functions.

5.3.3. Response to varying-load functions

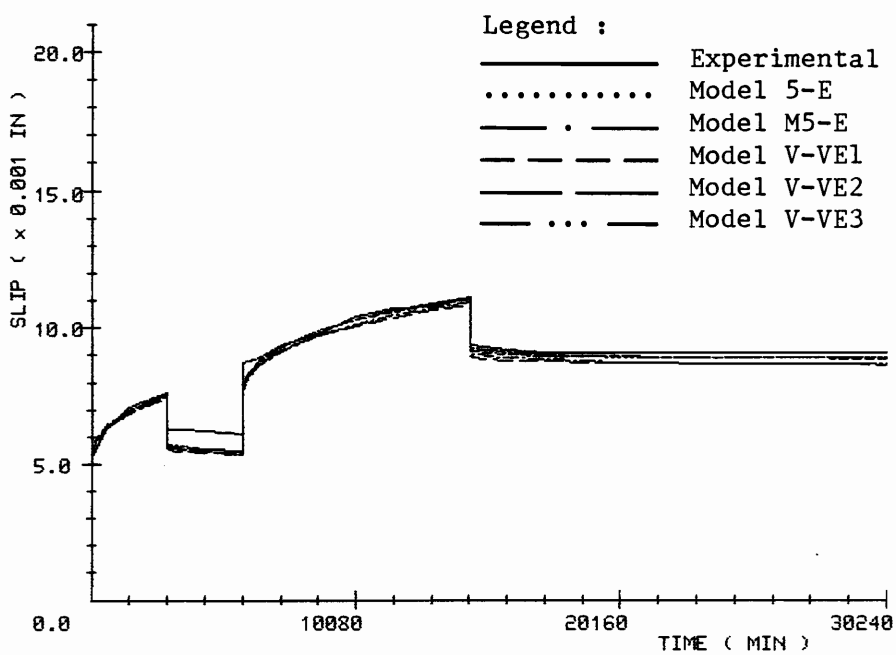
In this section, the predictions of the developed models that were modified by Solution 2 are compared with the experimental observations for the creep under varying-load functions.

5.3.3.1. Creep under load function 5

The five models developed in this study were solved



(a) Load function



(b) Slip response

Figure 5.12. Comparison between the experimental and theoretical data for load function 5.

Table 5.8. The values of R^2 and SSE for the models developed for load function 5

Model	R^2	SSE (0.000001 in.)
5-E	0.9612	5.80
M5-E	0.9562	8.39
V-VE1	0.9610	6.15
V-VE2	0.9583	8.84
V-VE3	0.9583	5.92

Table 5.9. The values of R^2 and SSE for the models developed for load function 6.

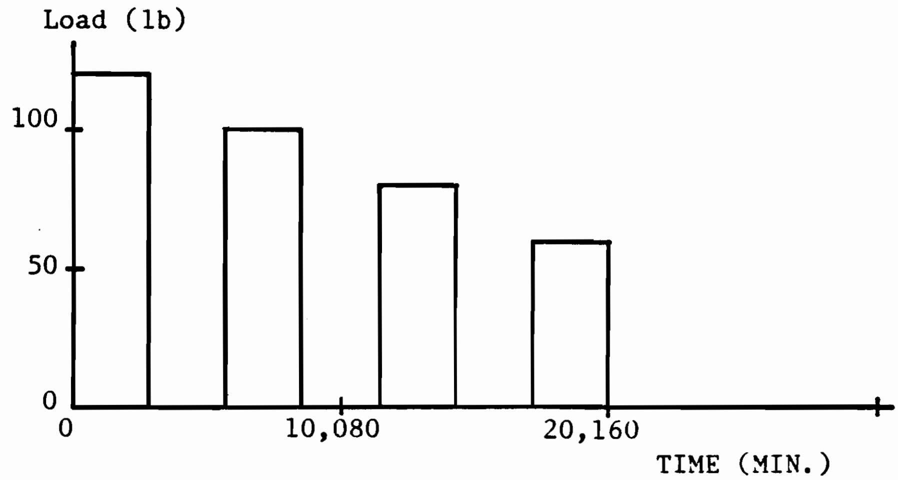
Model	R^2	SSE (0.000001 in.)
5-E	0.8462	25.64
M5-E	0.7982	39.17
V-VE1	0.8204	40.39
V-VE2	0.8222	38.32
V-VE3	0.7916	41.18

for load function 5 (Figure 4.8). Their predictions were computed by an IBM microcomputer and the program is recorded in Appendix C.2. The predicted data are given numerically in Appendix B.3 and graphically in Figure 5.12. The statistical indicators, R^2 and SSE, measuring the closeness of theoretical and experimental data are presented in Table 5.8.

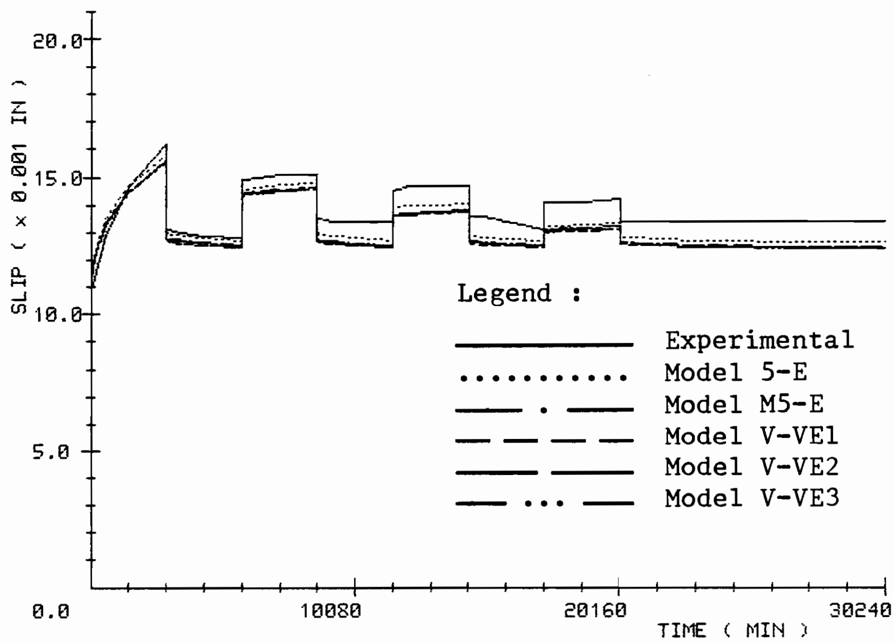
As shown in Figure 5.12, the predictions are very close to each other and to the experimental data for all the models. Although the predictions are somewhat lower than the experimental data during the period from t=2,880 minutes (2 days) to t=5,760 minutes (4 days), the overall predictions agree quite well with the experimental data. Their R^2-values are all greater than 0.95 (Table 5.8). The values of R^2 indicates that Model 5-E is slightly better than others, and the predictions by Model V-VE1 are similar to Model V-VE3 and slightly better than Models M5-E and V-VE2. However, these differences are almost negligible and, for all practical purposes, the correlation is excellent and about the same for all the models.

5.3.3.2. Creep under load function 6

Again, an IBM microcomputer was used to compute the creep for the models developed under the load function 6 (Figure 4.8). The numerical results are given in Appendix



(a) Load function



(b) Slip response

Figure 5.13. Comparison between the experimental and theoretical data for load function 6.

B.4 and graphical ones are shown in Figure 5.13. The computer program is presented in Appendix C.3. The corresponding R^2 and SSE are summarized in Table 5.9.

The correlation for the load function 6 is not as good as those for other load functions, although all the R^2-values of the models are about 0.8 which still indicates a good fit. Again, the prediction by Model 5-E is superior, while Models M5-E, V-VE1 and V-VE2 are similar to each other but slightly better than Model V-VE3.

The inferior correlation for load function 6 in comparison to other functions was caused by the tedious loading and unloading procedure used in testing. Although the loading process was performed carefully with a special device, the application of large load in each cycle of the load function 6 induced dynamic jerk, which in turn caused additional instantaneous slip. Thus, the slip upon loading was greater than the recovery upon unloading when only the action of the static load was removed. This difference was accumulated with the number of steps. In the theoretical models, however, only static-loading effect was considered. This observation is evidenced by the difference between the experimental and theoretical data, which is getting larger with the number of steps.

5.3.3.3. Creep under load function 7

Creep under the load function 7 was already discussed

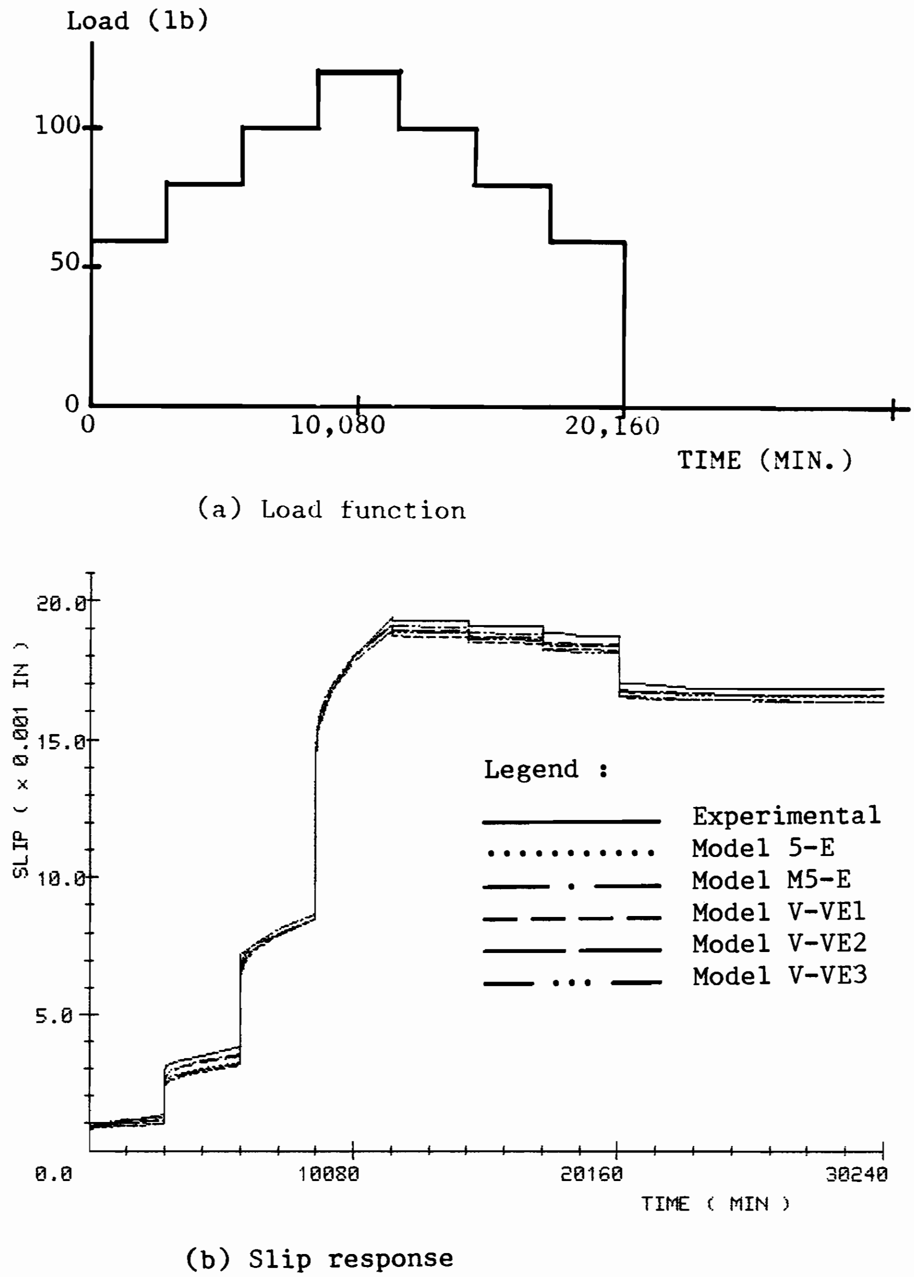


Figure 5.14. Comparison between the experimental and theoretical data for load function 7.

Table 5.10. The values of R^2 and SSE for the models developed for load function 7.

Model	R^2	SSE (0.000001 in.)
5-E	0.9986	7.63
M5-E	0.9979	6.45
V-VE1	0.9977	13.24
V-VE2	0.9981	6.60
V-VE3	0.9977	11.40

Figure 5.11. The values of R^2 and SSE for the models developed for load function 8.

Model	R^2	SSE (0.000001 in.)
5-E	0.9967	22.36
M5-E	0.9966	18.70
V-VE1	0.9972	14.83
V-VE2	0.9969	15.07
V-VE3	0.9968	18.56

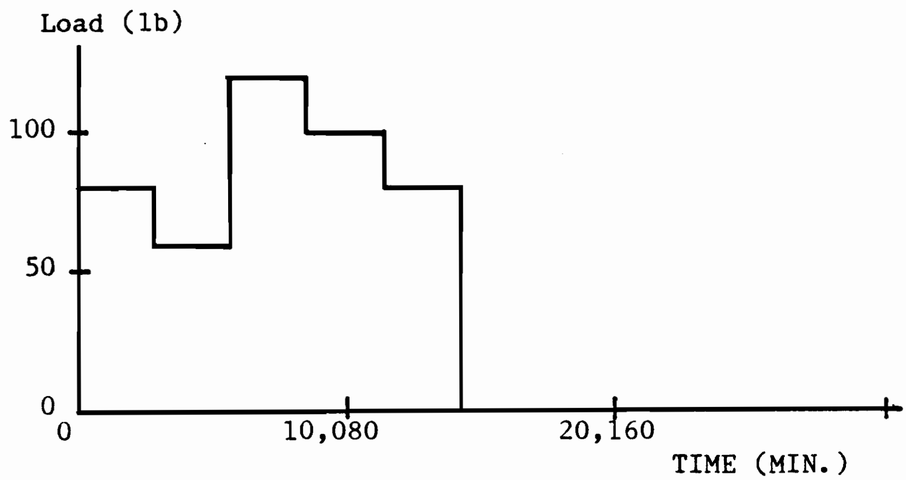
in Section 5.3.2. In this section, only the predictions of the models by Solution 2 are presented. The results are given numerically in Appendix B.5 and graphically in Figure 5.14. The statistical indicators, R^2 and SSE, are shown in Table 5.10.

The predictions of the models developed agree closely with the experimental data, although they are slightly below the experimental data in the second loading step and in the unloading steps. All the R^2- values are greater than 0.99. It appears that Models 5-E, M5-E and V-VE2 are similar to each other and slightly more accurate than Models V-VE1 and V-VE3.

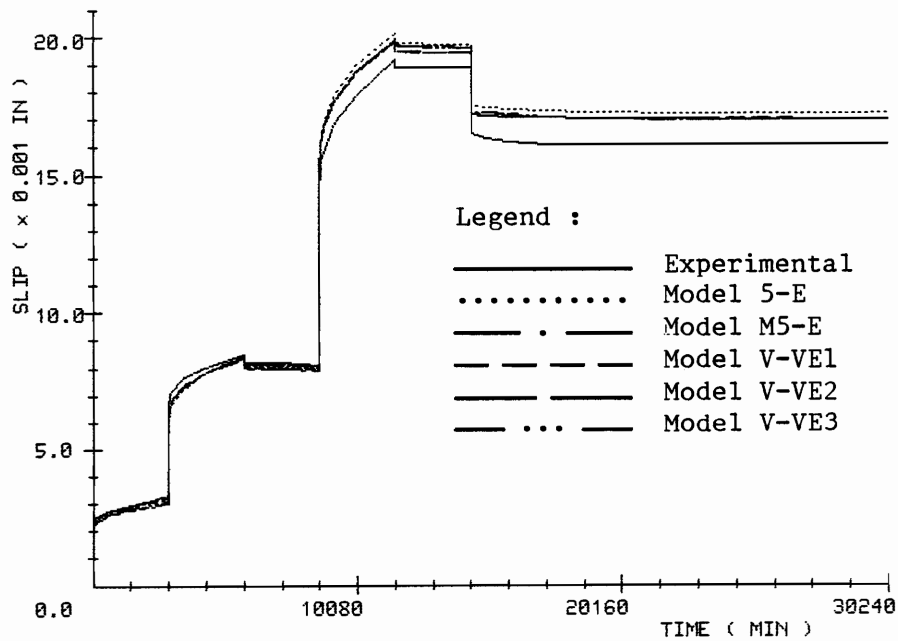
5.3.3.4. Creep under load function 8

As for the other functions, a program (Appendix C.5) and an IBM microcomputer were used to predict the creep under load function 8 (Figure 4.8). The results are presented in Appendix B.6 and Figure 5.15. The corresponding R^2 and SSE are given in Table 5.11.

The predictions of the models are somewhat larger than but quite close to the experimental data under the 120-pound load and in the unloading steps. All the R^2- values are greater than 0.99. Models V-VE1 and V-VE2 are slightly better than the other models.



(a) Load function



(b) Slip response

Figure 5.15. Comparison between the experimental and theoretical data for load function 8.

IV. CONCLUSION AND RECOMMENDATION

6.1. Conclusions

The conclusions reached in this investigation pertain to the experimental and theoretical findings. The conclusions derived on the basis of the experimental investigation are :

1. The delayed slip under low levels of constant loads is negligible.
2. The recovery that occurs immediately after unloading is about 1/2 to 1/3 of the instantaneous slip developed upon loading.
3. In constant-load tests, the instantaneous and delayed recovery can be regarded as the instantaneous and delayed elastic slip due to loading. The rest in the instantaneous and delayed slip due to loading can be regarded as the instantaneous plastic and viscous slip, respectively. The assumption that only elastic slip is recovered produces accurate theoretical predictions, but potential errors due to possible recovery of plastic or viscous slip cannot be detected because of the small overall recovery.
4. If the same weight is applied again after unloading, the slip quickly reaches the magnitude

obtained just before unloading.

5. If the subsequent load in the repetitive uniform loading is less than the maximum load in the previous loading steps, only elastic slip develops and recovers in both loading and unloading.
6. Under stepwise increasing load, the instantaneous elastic, plastic and viscous slip increase nonlinearly with respect to load.
7. Loading history has an effect on the slip under increasing load, but it has no effect on the recovery under decreasing load.

The following conclusions were derived on the basis of the theoretical investigation :

1. Generally, the creep predictions of the five models developed in this study agree closely with the experimental data.
2. In the theoretical modeling, stepwise decreasing load can be regarded as the compressive force acting opposite to the existing load.
3. Stiffness of nailed joints is reduced due to the creep under long-term load. The recoveryduring decreasing loads is reduced as much as stiffness reducing factor obtained under the reversed increasing-load function.

4. Solution 2, which is the modification of the models by modified superposition principle applied to nailed joints (MMSP), gives the most accurate predictions of the creep slip under varying-load functions investigated and is the easiest to use for stepwise-load functions.
5. Solution 4 in which the models are modified by MMSP and the load function is represented by Fourier series gives the results which are quite accurate but somewhat wavy.

6.2. Recommendations

To make the theoretical models developed practically useful, the following recommendations are made :

1. The behavior of nailed joints under actual service load functions, such as those caused by winds and earthquakes, should be studied and the models should be verified for these functions.
2. The representation of the service load functions by simple equations that can be integrated should be studied, so that they can be incorporated into the models.
3. The procedure of applying the models to the theoretical analysis of wood building should be developed.
4. The effect of creep in nailed joints on the overall behavior of wood components or structures under long-termload should be studied.
5. The models developed are accurate for the nailed joints between Douglas-fir lumber and plywood. It is anticipated that they will work equally well for other joint types and materials, but additional investigations are needed to evaluate material constants and demonstrate the accuracy.

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APPENDICES

Appendix A. List of symbols used in this study.

EMC	=	Equillibrium moisture content
EXP	=	Exponential function
5-E	=	Five-element Model
FRL	=	Forest Research Laboratory
M5-E	=	Modified Five-element Model
MSP	=	Modified superposition principle
MMSP	=	Modified MSP for nailed joints
OSU	=	Oregon State University
SHP	=	Strain hardening principle
V-VE1	=	Viscous-viscoelastic Model 1
V-VE2	=	Viscous-viscoelastic Model 2
V-VE3	=	Viscous-viscoelastic Model 3
A, Ao, A_1, A_2,...	=	Constants
A'_3, A''_3	=	First and second estimate for A_3
a	=	Constant
B, B_1, B_2,..., b	=	Constants
C	=	Constant
D	=	Diameter of nail
E, E_1, E_2	=	Stiffnesses of springs
Ei	=	$EXP(-A_3 ti)$
e, ei	=	Errors in curve fitting
$F_1(t)$, $F_2(t)$,...	=	Kernel functions
F°_i, F^{+}_2	=	Constants
$F^{+}_i(t)$	=	Power function of time

Fp(t), $\dot{F}p(t)$	=	Load function and its derivative approximated by Fourier series
Fc(t), $\dot{F}c(t)$	=	Reversed compressive force function and its derivative for decreasing load functions approximated by Fourier series
$G_1(P)$, $G_2(P)$	=	Function of load
H(t-a), $\dot{H}(t-a)$	=	Heaviside unit step function and its derivative
i, j, k	=	Constants
Ka	=	Stiffness modulus of nailed joints for instantaneous loading
Kj	=	Stiffness modulus of nailed joints
Ko	=	Elastic bearing constant
Ks	=	Stiffness modulus of nailed joints for stepwise loading
L	=	Length of nail penetration into framing members
Ln	=	Length of nail
Lp	=	Thickness of plywood
M, m	=	Constants
N	=	Number of steps or observations
N_1, N_2,...	=	Constants
P	=	Load
Po, P_1,...	=	Load level in stepwise load function
P(t), $\dot{P}(t)$	=	Load function and its derivative
Pc(t), $\dot{P}c(t)$	=	Reversed compressive load function and its derivative for decreasing load function.
q	=	Stress applied to wood from nail
R	=	Constant
R^2	=	Square of the correlation coefficient

Rs	=	Stiffness reducing rate
S, $\bar{S}$	=	Experimental data and their mean
S*, $\bar{S}$*	=	Theoretical data and their mean
SG	=	Specific gravity
SSE	=	Sum of the squares of errors
Sa	=	Instantaneous slip
Sc	=	Delayed or creep slip
S_{de}	=	Delayed elastic slip
Se	=	Instantaneous elastic slip
Smax	=	Maximum slip just before unloading
Sn	=	Nonrecoverable slip under constant load
Sp	=	Instantaneous plastic slip
Sr	=	Recoverable slip under constant load
Ss^1	=	Recoverable slip under stepwise load
Ss^2	=	Nonrecoverable slip under stepwise load
St	=	Total slip
Sv	=	Viscous slip
S_1 , S_2	=	Slip under example load functions
t	=	Time
to, t_1,...	=	Initial time for each step in stepwise-load function
tmax	=	Time when the maximum slip occurs
Tp	=	Period used in Fourier approximation of load function 7
V	=	Volume of wood at 12-percent moisture content
Vs	=	Volume of wood at specified moisture content

Wo	=	Ovendry weight of wood
Ww	=	Weight of wood at 12-percent moisture content
ω	=	Angular velocity = $2\pi/Tp$
y	=	Deflection under the nail
$z, z_1, z_2, \ldots$	=	Integration variables
α	=	Constant = $A^m + B^m + C^m + \ldots$
ε, ε	=	Strain and strain rate
$\varepsilon^{\circ}, \varepsilon^{+}$	=	constants
$\varepsilon_{de}, \varepsilon_{VE}$	=	Delayed elastic strain
ε_e	=	Instantaneous elastic strain
ε_{NV}	=	Negative viscous strain
ε_p	=	Instantaneous plastic strain
ε_{pv}	=	Positive viscous strain
ε_t	=	Total strain
ε_v	=	Viscous strain
$\varepsilon_o, \varepsilon_1, \ldots$	=	Strain under stepwise stressing
η_1, η_2	=	Viscosity of linear dashpot
$\eta_1(t)$	=	Viscosity of nonlinear dashpot
$\mu(P)$	=	Plasticity of nonlinear time-hardening element
σ, σ	=	Stress and stress rate
$\sigma_o, \sigma_1, \ldots$	=	Stress levels in stepwise stress function
σ^*	=	Creep limit stress
$\Delta P_{i,j,k}$	=	Step sizes in stepwise load functions
$\delta(t-a)$	=	Dirac delta function

Appendix B.1. Experimental data for creep of nailed joints under constant-load functions.

TIME	60 LBS	80 LBS	100 LBS	120 LBS
0.0	0.9	2.1	4.6	9.1
1.0	1.0	2.2	4.7	10.1
2.5	1.0	2.3	5.0	10.4
5.0	1.0	2.3	5.2	10.8
10.0	1.0	2.3	5.3	11.1
20.0	1.0	2.4	5.5	11.5
30.0	1.0	2.4	5.6	11.7
60.0	1.0	2.5	5.8	12.1
120.0	1.0	2.5	5.9	12.5
240.0	1.0	2.5	6.1	13.0
480.0	1.1	2.6	6.3	13.5
1440.0	1.1	2.9	6.7	14.6
2880.0	1.2	3.2	7.6	15.7
4320.0	1.2	3.4	7.8	16.4
5760.0	1.3	3.5	8.0	17.0
7200.0	1.3	3.6	8.3	17.6
8640.0	1.3	3.7	8.5	17.9
12960.0	1.4	4.0	9.3	19.0
14400.0	1.4	4.0	9.4	19.1
15840.0	1.4	4.1	9.6	19.5
17280.0	1.5	4.2	9.8	20.1
18720.0	1.5	4.3	10.0	20.6
20160.0	1.5	4.3	10.2	20.9
20160.0	1.0	3.2	8.5	18.2
20161.0	1.0	3.1	8.5	18.2
20162.5	1.0	3.1	8.4	18.1
20165.0	1.0	3.1	8.4	18.1
20170.0	1.0	3.1	8.3	18.0
20180.0	1.0	3.1	8.3	18.0
20190.0	1.0	3.1	8.3	18.0
20220.0	1.0	3.1	8.3	18.0
20280.0	1.0	3.0	8.3	18.0
20400.0	0.9	3.0	8.2	18.0
20640.0	0.9	3.0	8.2	17.9
21600.0	0.9	3.0	8.2	17.8
23040.0	0.9	2.9	8.1	17.8
24480.0	0.9	2.9	8.1	17.8
25920.0	0.9	2.9	8.1	17.7
27360.0	0.8	2.8	8.0	17.7
28800.0	0.8	2.8	8.0	17.7
30240.0	0.8	2.8	8.0	17.7

Appendix B.2. Experimental data for creep of nailed joints under varying-load functions.

TIME	5	6	7	8
0.0	4.2	7.5	0.9	2.0
1.0	4.8	9.1	1.0	2.3
30.0	5.3	10.9	1.0	2.5
600.0	6.3	12.9	1.0	2.7
1440.0	7.1	14.6	1.1	2.9
2880.0	7.6	16.2	1.3	3.3
2880.0	6.4	13.4	2.5	5.8
2881.0	6.4	13.2	2.8	6.5
2910.0	6.3	13.1	3.1	7.1
3480.0	6.3	13.0	3.3	7.7
4320.0	6.2	12.9	3.5	8.0
5760.0	6.1	12.8	3.8	8.5
5760.0	8.1	14.5	6.0	8.2
5761.0	8.4	14.7	6.6	8.2
5790.0	8.7	14.9	7.2	8.2
6360.0	8.9	15.0	7.6	8.2
7200.0	9.4	15.1	7.9	8.2
8640.0	9.8	15.1	8.5	8.1
8640.0	-	13.6	12.4	12.7
8641.0	-	13.5	13.8	14.1
8670.0	-	13.5	15.3	15.5
9240.0	-	13.4	16.8	16.9
10080.0	10.4	13.4	18.0	17.9
11520.0	10.7	13.4	19.4	19.2
11520.0	-	14.4	19.3	18.9
11521.0	-	14.5	19.3	18.9
11550.0	-	14.5	19.3	18.9
12120.0	-	14.7	19.3	18.9
12960.0	10.8	14.7	19.3	18.9
14400.0	11.1	14.7	19.3	18.9
14400.0	9.5	13.7	19.1	16.7
14401.0	9.5	13.7	19.1	16.6
14430.0	9.4	13.6	19.1	16.5
15000.0	9.3	13.6	19.1	16.3
15840.0	9.2	13.4	19.1	16.2
17280.0	9.1	13.1	19.1	16.1
17280.0	-	14.0	19.1	-
17281.0	-	14.1	18.9	-
17310.0	-	14.1	18.9	-
17880.0	-	14.1	18.9	-
18720.0	9.1	14.1	18.8	16.1
20160.0	9.1	14.2	18.8	16.1
20160.0	9.1	13.5	17.2	16.1
20161.0	-	13.5	17.1	-
20190.0	-	13.4	17.1	-
20760.0	-	13.4	17.1	-
21600.0	9.1	13.4	17.0	16.1
23040.0	9.1	13.4	16.9	16.1
24480.0	9.1	13.4	16.9	16.1
27360.0	-	13.4	16.9	-
30240.0	-	13.4	16.9	-

Appendix B.3. Theoretical predictions by Solution 2 for the creep of nailed joints under load function 5.

TIME	5-E	M5-E	V-VE1	V-VE2	V-VE3
0.00	4.94	4.95	4.78	4.76	4.97
1.00	5.08	5.09	5.03	5.00	5.11
5.00	5.19	5.19	5.17	5.14	5.22
10.00	5.27	5.26	5.26	5.23	5.32
30.00	5.42	5.41	5.44	5.40	5.45
60.00	5.56	5.54	5.60	5.55	5.58
120.00	5.73	5.71	5.79	5.74	5.76
240.00	5.97	5.93	6.03	5.97	5.99
480.00	6.27	6.22	6.33	6.27	6.28
600.00	6.39	6.33	6.45	6.38	6.40
1440.00	6.97	6.88	6.98	6.90	6.96
2880.00	7.56	7.46	7.53	7.43	7.56
2880.00	5.72	5.58	5.85	5.75	5.68
2881.00	5.72	5.58	5.74	5.64	5.68
2885.00	5.72	5.58	5.71	5.61	5.68
2890.00	5.72	5.58	5.69	5.57	5.68
2910.00	5.71	5.58	5.66	5.56	5.68
2940.00	5.71	5.57	5.64	5.54	5.68
3000.00	5.70	5.57	5.61	5.51	5.67
3120.00	5.68	5.55	5.57	5.48	5.65
3360.00	5.64	5.53	5.55	5.45	5.63
3480.00	5.63	5.51	5.54	5.44	5.61
4320.00	5.54	5.44	5.49	5.39	5.54
5760.00	5.45	5.35	5.43	5.35	5.45
5760.00	7.30	7.22	7.14	7.03	7.33
5761.00	7.44	7.36	7.38	7.28	7.47
5765.00	7.55	7.47	7.53	7.42	7.58
5770.00	7.62	7.54	7.61	7.50	7.65
5790.00	7.78	7.69	7.80	7.68	7.81
5820.00	7.92	7.82	7.95	7.83	7.94
5880.00	8.09	7.99	8.14	8.02	8.11
6000.00	8.33	8.21	8.37	8.25	8.34
6240.00	8.63	8.49	8.69	8.54	8.64
6360.00	8.75	8.61	8.80	8.65	8.75
7200.00	9.33	9.15	9.34	9.17	9.32
8640.00	9.92	9.73	9.89	9.71	9.91
10080.00	10.32	10.13	10.28	10.08	10.33
11520.00	10.63	10.45	10.58	10.38	10.65
12960.00	10.88	10.70	10.84	10.63	10.92
14400.00	11.10	10.93	11.07	10.85	11.14
14400.00	9.26	9.05	9.39	9.17	9.27
14401.00	9.26	9.05	9.28	9.06	9.27
14405.00	9.26	9.05	9.25	9.03	9.27
14410.00	9.26	9.05	9.23	9.02	9.27
14430.00	9.25	9.05	9.20	8.98	9.27
14460.00	9.25	9.04	9.18	8.96	9.26
14520.00	9.24	9.04	9.15	8.93	9.25
14640.00	9.22	9.02	9.12	8.91	9.24
14880.00	9.18	9.00	9.09	8.87	9.21
15000.00	9.17	8.98	9.08	8.86	9.20
15840.00	9.08	8.91	9.03	8.81	9.13
17280.00	8.99	8.82	8.99	8.77	9.04
18720.00	8.95	8.76	8.96	8.74	8.98
20160.00	8.93	8.73	8.94	8.72	8.95
21600.00	8.92	8.71	8.93	8.71	8.92
23040.00	8.92	8.69	8.91	8.69	8.91
24480.00	8.91	8.68	8.90	8.68	8.90
25920.00	8.91	8.68	8.89	8.67	8.90
27360.00	8.91	8.68	8.88	8.66	8.89
28800.00	8.91	8.67	8.87	8.65	8.89
30240.00	8.91	8.67	8.86	8.64	8.89

Appendix B.4. Theoretical predictions by Solution 2 for the creep of nailed joints under load function 6.

TIEM	5-E	M5-E	V-VE1	V-VE2	V-VE3
0.00	9.75	10.35	10.02	10.03	10.35
1.00	10.46	10.66	10.54	10.55	10.65
5.00	10.78	10.90	10.82	10.83	10.89
10.00	10.97	11.05	10.99	11.00	11.04
30.00	11.38	11.37	11.35	11.36	11.36
60.00	11.71	11.65	11.66	11.67	11.64
120.00	12.12	12.02	12.04	12.05	12.00
240.00	12.64	12.49	12.52	12.54	12.47
480.00	13.28	13.09	13.13	13.15	13.07
600.00	13.52	13.33	13.35	13.38	13.30
1440.00	14.66	14.45	14.43	14.46	14.42
2880.00	15.82	15.64	15.55	15.58	15.60
2880.00	12.97	12.79	13.03	13.07	12.76
2881.00	12.97	12.79	12.81	12.85	12.76
2885.00	12.97	12.79	12.77	12.81	12.76
2890.00	12.97	12.79	12.75	12.78	12.76
2910.00	12.97	12.79	12.71	12.74	12.75
2940.00	12.96	12.79	12.68	12.72	12.75
3000.00	12.96	12.78	12.65	12.69	12.74
3120.00	12.94	12.76	12.62	12.65	12.72
3360.00	12.91	12.73	12.58	12.62	12.69
3480.00	12.89	12.71	12.57	12.61	12.68
4320.00	12.81	12.62	12.52	12.56	12.59
5760.00	12.71	12.52	12.48	12.52	12.48
5760.00	14.55	14.39	14.16	14.20	14.35
5761.00	14.55	14.39	14.27	14.31	14.35
5765.00	14.55	14.39	14.30	14.34	14.35
5770.00	14.55	14.39	14.32	14.36	14.35
5790.00	14.56	14.39	14.35	14.39	14.36
5820.00	14.56	14.40	14.37	14.41	14.36
5880.00	14.57	14.41	14.40	14.44	14.37
6000.00	14.59	14.42	14.43	14.46	14.38
6240.00	14.63	14.45	14.46	14.50	14.41
6360.00	14.64	14.46	14.47	14.51	14.42
7200.00	14.73	14.53	14.52	14.56	14.50
8640.00	14.82	14.62	14.56	14.60	14.59
8640.00	12.97	12.75	12.88	12.91	12.71
8641.00	12.97	12.75	12.77	12.81	12.71
8645.00	12.97	12.75	12.74	12.78	12.71
8650.00	12.97	12.75	12.72	12.76	12.71
8670.00	12.97	12.74	12.69	12.73	12.71
8700.00	12.96	12.74	12.67	12.71	12.70
8760.00	12.95	12.73	12.64	12.68	12.70
8880.00	12.93	12.72	12.61	12.65	12.68
9120.00	12.90	12.69	12.58	12.62	12.66
9240.00	12.88	12.68	12.57	12.61	12.64
10080.00	12.79	12.61	12.52	12.56	12.57
11520.00	12.71	12.52	12.48	12.52	12.48
11520.00	13.92	13.64	13.48	13.52	13.61
11521.00	13.92	13.64	13.52	13.56	13.61
11525.00	13.92	13.64	13.54	13.58	13.61
11530.00	13.92	13.64	13.55	13.59	13.61
11550.00	13.92	13.65	13.57	13.61	13.61
11580.00	13.92	13.65	13.59	13.62	13.61
11640.00	13.93	13.65	13.61	13.64	13.62
11760.00	13.93	13.67	13.63	13.66	13.63

12000.00	13.75	13.69	13.65	13.69	13.65
12120.00	13.96	13.70	13.66	13.70	13.66
12960.00	14.00	13.76	13.70	13.74	13.72
14400.00	14.06	13.83	13.74	13.77	13.79
14400.00	12.85	12.70	12.74	12.77	12.66
14401.00	12.85	12.70	12.70	12.73	12.66
14405.00	12.85	12.70	12.68	12.71	12.66
14410.00	12.85	12.70	12.67	12.70	12.66
14430.00	12.85	12.70	12.65	12.68	12.66
14460.00	12.85	12.70	12.63	12.67	12.66
14520.00	12.84	12.69	12.61	12.65	12.65
14640.00	12.84	12.68	12.59	12.63	12.64
14880.00	12.82	12.66	12.56	12.60	12.62
15000.00	12.82	12.65	12.56	12.59	12.61
15840.00	12.77	12.59	12.52	12.55	12.55
17280.00	12.71	12.52	12.48	12.52	12.48
17280.00	13.22	13.10	12.95	12.99	13.06
17281.00	13.22	13.10	12.97	13.00	13.06
17285.00	13.22	13.10	12.97	13.01	13.06
17290.00	13.22	13.10	12.98	13.02	13.06
17310.00	13.22	13.10	12.99	13.03	13.06
17340.00	13.22	13.10	13.00	13.03	13.07
17400.00	13.23	13.11	13.01	13.05	13.07
17520.00	13.23	13.12	13.02	13.06	13.08
17760.00	13.24	13.13	13.04	13.07	13.10
17880.00	13.25	13.14	13.04	13.08	13.10
18720.00	13.29	13.19	13.07	13.11	13.15
20160.00	13.33	13.24	13.10	13.13	13.20
20160.00	12.82	12.66	12.62	12.66	12.62
20161.00	12.82	12.66	12.61	12.65	12.62
20165.00	12.82	12.65	12.60	12.64	12.62
20170.00	12.82	12.65	12.60	12.63	12.62
20190.00	12.81	12.65	12.59	12.62	12.62
20220.00	12.81	12.65	12.58	12.61	12.61
20280.00	12.81	12.65	12.57	12.60	12.61
20400.00	12.80	12.64	12.55	12.59	12.60
20640.00	12.79	12.62	12.54	12.58	12.59
20760.00	12.79	12.61	12.53	12.57	12.58
21600.00	12.75	12.57	12.51	12.54	12.53
23040.00	12.71	12.52	12.48	12.52	12.48
24480.00	12.68	12.48	12.46	12.50	12.45
25920.00	12.66	12.46	12.45	12.48	12.42
27360.00	12.65	12.45	12.43	12.47	12.41
28800.00	12.64	12.44	12.42	12.46	12.40
30240.00	12.64	12.44	12.41	12.45	12.40

Appendix B.5. Theoretical predictions by Solution 2 for the creep of naild joints under load function 7.

TIME	5-E	M5-E	V-VE1	V-VE2	V-VE3
0.00	0.81	1.09	0.77	0.98	0.88
1.00	0.81	1.10	0.79	1.01	0.88
5.00	0.81	1.11	0.80	1.03	0.88
10.00	0.81	1.12	0.81	1.04	0.88
30.00	0.81	1.14	0.82	1.07	0.89
60.00	0.81	1.16	0.83	1.09	0.89
120.00	0.82	1.18	0.84	1.12	0.90
240.00	0.83	1.21	0.86	1.15	0.92
480.00	0.85	1.25	0.88	1.20	0.94
600.00	0.85	1.27	0.89	1.22	0.95
1440.00	0.90	1.37	0.94	1.29	1.01
2880.00	0.96	1.48	0.98	1.38	1.08
2880.00	2.42	2.54	2.25	2.43	2.36
2881.00	2.44	2.58	2.28	2.50	2.36
2885.00	2.47	2.61	2.31	2.54	2.38
2890.00	2.49	2.63	2.33	2.57	2.39
2910.00	2.53	2.67	2.38	2.62	2.43
2940.00	2.57	2.71	2.42	2.66	2.46
3000.00	2.62	2.76	2.47	2.72	2.51
3120.00	2.69	2.82	2.54	2.79	2.58
3360.00	2.78	2.91	2.64	2.88	2.67
3480.00	2.82	2.95	2.68	2.91	2.71
4320.00	3.01	3.13	2.86	3.08	2.91
5760.00	3.23	3.33	3.07	3.27	3.13
5760.00	5.91	6.12	5.83	6.01	5.95
5761.00	6.02	6.21	6.04	6.16	6.09
5765.00	6.11	6.28	6.15	6.24	6.19
5770.00	6.16	6.32	6.22	6.29	6.25
5790.00	6.27	6.41	6.35	6.40	6.37
5820.00	6.37	6.50	6.47	6.48	6.48
5880.00	6.50	6.60	6.61	6.60	6.61
6000.00	6.68	6.75	6.78	6.75	6.79
6240.00	6.91	6.94	7.01	6.94	7.01
6360.00	7.00	7.02	7.10	7.01	7.10
7200.00	7.46	7.40	7.53	7.39	7.54
8640.00	7.94	7.84	8.00	7.82	8.02
8640.00	12.95	13.24	12.86	13.10	13.00
8641.00	13.32	13.41	13.34	13.38	13.37
8645.00	13.53	13.54	13.57	13.52	13.59
8650.00	13.65	13.62	13.70	13.61	13.71
8670.00	13.90	13.80	13.97	13.80	13.97
8700.00	14.11	13.96	14.18	13.97	14.18
8760.00	14.36	14.17	14.45	14.18	14.44
8880.00	14.68	14.45	14.77	14.46	14.77
9120.00	15.07	14.83	15.19	14.83	15.19
9240.00	15.22	14.98	15.35	14.98	15.35
10080.00	15.97	15.73	16.12	15.72	16.13
11520.00	16.79	16.60	16.96	16.57	16.99
11520.00	16.62	16.55	16.80	16.41	16.94
11521.00	16.62	16.55	16.79	16.40	16.94
11525.00	16.62	16.55	16.79	16.40	16.94
11530.00	16.62	16.55	16.79	16.40	16.94
11550.00	16.62	16.55	16.78	16.40	16.94
11580.00	16.62	16.55	16.78	16.39	16.94
11640.00	16.61	16.55	16.78	16.39	16.94
11760.00	16.61	16.54	16.77	16.39	16.93
12000.00	16.61	16.54	16.77	16.38	16.93

12120.00	16.61	16.54	16.77	16.38	16.92
12960.00	16.59	16.52	16.76	16.37	16.91
14400.00	16.58	16.50	16.75	16.36	16.89
14400.00	16.41	16.32	16.59	16.20	16.71
14401.00	16.41	16.32	16.58	16.20	16.71
14405.00	16.41	16.32	16.58	16.19	16.71
14410.00	16.41	16.32	16.58	16.19	16.71
14430.00	16.41	16.32	16.58	16.19	16.71
14460.00	16.41	16.32	16.57	16.19	16.71
14520.00	16.41	16.32	16.57	16.18	16.70
14640.00	16.40	16.31	16.57	16.18	16.70
14880.00	16.40	16.30	16.56	16.17	16.69
15000.00	16.40	16.30	16.56	16.17	16.69
15840.00	16.38	16.28	16.54	16.16	16.67
17280.00	16.36	16.25	16.53	16.14	16.64
17280.00	15.89	15.90	16.37	15.98	16.29
17281.00	15.89	15.90	16.37	15.98	16.29
17285.00	15.89	15.90	16.36	15.97	16.29
17290.00	15.89	15.90	16.36	15.97	16.29
17310.00	15.89	15.90	16.36	15.97	16.29
17340.00	15.89	15.90	16.35	15.97	16.29
17400.00	15.88	15.90	16.35	15.96	16.29
17520.00	15.88	15.89	16.35	15.96	16.28
17760.00	15.86	15.89	16.34	15.95	16.27
17880.00	15.86	15.88	16.34	15.95	16.27
18720.00	15.83	15.86	16.32	15.93	16.25
20160.00	15.79	15.83	16.30	15.91	16.22
20160.00	9.03	14.28	14.87	14.48	14.67
20161.00	8.64	14.28	14.71	14.33	14.67
20165.00	8.40	14.28	14.68	14.30	14.67
20170.00	8.25	14.28	14.67	14.28	14.67
20190.00	7.94	14.28	14.65	14.26	14.67
20220.00	7.69	14.28	14.63	14.24	14.67
20280.00	7.38	14.27	14.62	14.23	14.66
20400.00	6.99	14.27	14.60	14.21	14.66
20640.00	6.50	14.25	14.58	14.19	14.64
20760.00	6.32	14.25	14.58	14.19	14.63
21600.00	5.48	14.20	14.55	14.16	14.59
23040.00	4.63	14.15	14.53	14.14	14.54
24480.00	4.04	14.12	14.51	14.12	14.51
25920.00	3.59	14.10	14.50	14.11	14.49
27360.00	3.21	14.09	14.49	14.10	14.48
28800.00	2.89	14.09	14.48	14.09	14.47
30240.00	2.60	14.08	14.47	14.09	14.47

Appendix B.6. Theoretical predictions by Solution 2 for the creep of nailed joints under load function 8.

TIME	5-E	M5-E	V-VE1	V-VE2	V-VE3
0.00	2.26	2.16	1.93	2.03	2.06
1.00	2.29	2.21	2.02	2.12	2.11
5.00	2.32	2.25	2.08	2.18	2.15
10.00	2.34	2.28	2.12	2.22	2.17
30.00	2.38	2.33	2.19	2.30	2.23
60.00	2.42	2.38	2.25	2.36	2.27
120.00	2.48	2.45	2.33	2.44	2.34
240.00	2.55	2.54	2.42	2.54	2.42
480.00	2.66	2.66	2.54	2.67	2.53
600.00	2.70	2.71	2.58	2.71	2.58
1440.00	2.91	2.95	2.79	2.93	2.81
2880.00	3.16	3.21	3.00	3.16	3.05
2880.00	5.84	6.00	5.85	5.89	5.96
2881.00	5.98	6.14	6.05	6.09	6.11
2885.00	6.09	6.25	6.18	6.21	6.22
2890.00	6.16	6.32	6.25	6.28	6.29
2910.00	6.32	6.46	6.42	6.44	6.44
2940.00	6.46	6.59	6.56	6.57	6.57
3000.00	6.63	6.76	6.73	6.74	6.74
3120.00	6.86	6.97	6.95	6.96	6.96
3360.00	7.16	7.24	7.24	7.23	7.25
3480.00	7.28	7.35	7.34	7.33	7.36
4320.00	7.84	7.86	7.85	7.83	7.88
5760.00	8.41	8.40	8.38	8.34	8.44
5760.00	8.07	8.16	8.07	8.03	8.21
5761.00	8.07	8.16	8.06	8.02	8.21
5765.00	8.07	8.16	8.05	8.01	8.21
5770.00	8.07	8.16	8.05	8.01	8.21
5790.00	8.07	8.16	8.04	8.00	8.20
5820.00	8.07	8.16	8.04	8.00	8.20
5880.00	8.07	8.16	8.03	7.99	8.20
6000.00	8.06	8.15	8.02	7.98	8.19
6240.00	8.05	8.14	8.01	7.97	8.18
6360.00	8.05	8.14	8.01	7.97	8.18
7200.00	8.03	8.11	7.99	7.95	8.15
8640.00	8.00	8.07	7.97	7.93	8.11
8640.00	14.34	14.77	14.43	14.41	14.78
8641.00	14.85	15.08	14.94	14.92	15.09
8645.00	15.17	15.31	15.20	15.19	15.32
8650.00	15.36	15.46	15.37	15.36	15.47
8670.00	15.76	15.78	15.72	15.71	15.79
8700.00	16.10	16.07	16.02	16.01	16.07
8760.00	16.51	16.42	16.39	16.38	16.43
8880.00	17.02	16.88	16.86	16.86	16.88
9120.00	17.65	17.48	17.45	17.45	17.47
9240.00	17.89	17.70	17.67	17.67	17.70
10080.00	18.99	18.78	18.72	18.73	18.77
11520.00	20.10	19.91	19.81	19.83	19.90
11520.00	19.76	19.68	19.49	19.51	19.66
11521.00	19.76	19.68	19.48	19.50	19.66
11525.00	19.76	19.68	19.48	19.50	19.66
11530.00	19.76	19.68	19.47	19.49	19.66
11550.00	19.76	19.68	19.47	19.49	19.66
11580.00	19.75	19.68	19.46	19.48	19.66
11640.00	19.75	19.68	19.46	19.47	19.66
11760.00	19.75	19.67	19.45	19.46	19.65
12000.00	19.74	19.66	19.44	19.45	19.64

12120.00	19.74	19.65	19.43	19.45	19.64
12960.00	19.71	19.62	19.41	19.43	19.61
14400.00	19.68	19.59	19.40	19.41	19.57
14400.00	17.50	17.30	17.48	17.50	17.28
14401.00	17.50	17.30	17.30	17.31	17.28
14405.00	17.50	17.30	17.26	17.28	17.28
14410.00	17.50	17.30	17.24	17.26	17.28
14430.00	17.50	17.29	17.21	17.23	17.28
14460.00	17.49	17.29	17.19	17.21	17.27
14520.00	17.49	17.29	17.17	17.19	17.27
14640.00	17.47	17.27	17.15	17.17	17.26
14880.00	17.45	17.25	17.13	17.14	17.23
15000.00	17.44	17.24	17.12	17.14	17.22
15840.00	17.38	17.18	17.09	17.10	17.16
17280.00	17.30	17.10	17.06	17.07	17.08
18720.00	17.25	17.05	17.04	17.05	17.03
20160.00	17.23	17.02	17.02	17.04	17.00
21600.00	17.21	17.00	17.01	17.03	16.99
23040.00	17.20	16.99	17.00	17.02	16.97
24480.00	17.19	16.98	16.99	17.01	16.97
25920.00	17.19	16.98	16.98	17.00	16.96
27360.00	17.19	16.98	16.98	16.99	16.96
28800.00	17.19	16.98	16.97	16.99	16.96
30240.00	17.19	16.97	16.96	16.98	16.96

Appendix B.7. Theoretical predictions by Solution 1 under load function 7.

TIME	M5-E	V-VE1	V-VE2	V-VE3
0.0	0.83	0.76	0.73	0.87
1.00	0.85	0.78	.76	0.87
5.00	0.86	0.79	0.78	0.87
10.00	0.87	0.80	0.77	0.88
30.00	0.89	0.81	0.82	0.88
60.00	0.90	0.82	0.84	0.89
120.00	0.93	0.84	0.87	0.90
240.00	0.96	0.86	0.90	0.91
480.00	1.00	0.88	0.95	0.93
600.00	1.02	0.89	0.96	0.94
1440.00	1.12	0.93	1.04	1.00
2080.00	1.22	0.97	1.12	1.07
2880.00	2.54	2.15	2.43	2.26
2881.00	2.55	2.19	2.46	2.27
2885.00	2.55	2.23	2.48	2.30
2870.00	2.56	2.25	2.42	2.32
2910.00	2.57	2.31	2.52	2.37
2740.00	2.60	2.36	2.55	2.41
3000.00	2.64	2.43	2.60	2.47
3120.00	2.71	2.51	2.67	2.55
3360.00	2.81	2.62	2.77	2.65
3480.00	2.85	2.66	2.81	2.67
4320.00	3.05	2.85	3.01	2.89
5760.00	3.27	3.05	3.21	3.11
5760.00	6.06	5.91	5.95	6.02
5761.00	6.07	5.97	6.02	6.02
5765.00	6.07	5.99	6.04	6.03
5770.00	6.08	6.01	6.05	6.04
5790.00	6.11	6.06	6.09	6.08
5820.00	6.16	6.12	6.14	6.13
5880.00	6.23	6.22	6.23	6.22
6000.00	6.37	6.37	6.36	6.37
6240.00	6.57	6.60	6.57	6.60
6360.00	6.66	6.69	6.66	6.70
7200.00	7.10	7.16	7.10	7.17
8640.00	7.60	7.67	7.50	7.69
8640.00	13.01	12.93	12.87	13.07
8641.00	13.01	13.04	12.98	13.07
8645.00	13.02	13.06	13.00	13.08
8650.00	13.03	13.08	13.02	13.09
8670.00	13.08	13.14	13.08	13.14
8700.00	13.15	13.21	13.16	13.20
8760.00	13.28	13.33	13.29	13.33
8880.00	13.52	13.56	13.52	13.55
9120.00	13.90	13.92	13.90	13.92
9240.00	14.06	14.08	14.07	14.08
10080.00	14.94	14.93	14.93	14.94
11520.00	15.75	15.91	15.93	15.73
11520.00	15.92	15.78	15.80	15.90
11521.00	15.92	15.78	15.80	15.90
11525.00	15.92	15.78	15.80	15.90

11530.00	15.92	15.78	15.80	15.90
11550.00	15.92	15.77	15.79	15.90
11580.00	15.92	15.77	15.77	15.70
11640.00	15.92	15.77	15.79	15.90
11760.00	15.91	15.77	15.79	15.89
12000.00	15.91	15.76	15.78	15.89
12120.00	15.71	15.76	15.78	15.87
12960.00	15.90	15.75	15.77	15.88
14400.00	15.88	15.75	15.77	15.86
14400.00	15.74	15.62	15.64	15.72
14401.00	15.74	15.62	15.64	15.72
14405.00	15.74	15.62	15.64	15.72
14410.00	15.74	15.62	15.64	15.72
14430.00	15.74	15.62	15.63	15.72
14460.00	15.74	15.61	15.63	15.72
14520.00	15.74	15.61	15.63	15.72
14640.00	15.74	15.61	15.63	15.72
14880.00	15.73	15.60	15.62	15.71
15000.00	15.73	15.60	15.62	15.71
15840.00	15.71	15.59	15.61	15.69
17280.00	15.69	15.58	15.60	15.37
17280.00	15.42	15.46	15.48	15.40
17281.00	15.42	15.45	15.47	15.40
17285.00	15.42	15.45	15.47	15.40
17290.00	15.42	15.45	15.47	15.40
17310.00	15.42	15.45	15.47	15.40
17340.00	15.42	15.45	15.46	15.40
17400.00	15.42	15.44	15.46	15.40
17520.00	15.42	15.44	15.46	15.40
17760.00	15.41	15.43	15.45	15.39
17880.00	15.41	15.43	15.45	15.39
18720.00	15.39	15.42	15.44	15.37
20160.00	15.37	15.41	15.43	15.35
20160.00	13.64	13.82	13.84	13.62
20161.00	13.64	13.66	13.68	13.62
20165.00	13.64	13.63	13.65	13.62
20170.00	13.64	13.62	13.64	13.62
20190.00	13.64	13.60	13.62	13.62
20220.00	13.64	13.58	13.60	13.62
20280.00	13.63	13.57	13.58	13.61
20400.00	13.62	13.55	13.57	13.60
20640.00	13.61	13.53	13.55	13.59
20760.00	13.60	13.52	13.54	13.58
21600.00	13.56	13.50	13.52	13.54
23040.00	13.51	13.48	13.50	13.49
24480.00	13.48	13.46	13.48	13.46
25920.00	13.46	13.45	13.47	13.44
27360.00	13.45	13.44	13.46	13.43
28800.00	13.44	13.43	13.45	13.42
30240.00	13.44	13.42	13.44	13.42

Appendix B.8. Theoretical predictions by Solution 3 for load function 7.

TIME	M5-E	V-VE1	V-VE2	V-VE3
0.00	0.83	0.76	0.73	0.87
1.00	0.85	0.78	0.76	0.87
5.00	0.86	0.79	0.78	0.87
10.00	0.87	0.80	0.79	0.88
30.00	0.89	0.81	0.82	0.88
60.00	0.90	0.82	0.84	0.89
120.00	0.93	0.84	0.87	0.90
240.00	0.96	0.86	0.90	0.91
480.00	1.00	0.88	0.95	0.93
600.00	1.02	0.89	0.96	0.94
1440.00	1.12	0.93	1.04	1.00
2880.00	1.22	0.97	1.12	1.07
2880.00	2.02	2.23	2.48	1.76
2881.00	2.07	[illegible]	2.48	1.81
2885.00	2.11	2.28	2.55	1.85
2890.00	2.14	2.32	2.58	1.88
2910.00	2.20	2.40	2.66	1.93
2940.00	2.24	2.46	2.73	1.97
3000.00	2.31	2.54	2.81	2.04
3120.00	2.39	2.64	2.91	2.11
3360.00	2.51	2.76	3.04	2.22
3480.00	2.55	2.80	3.09	2.26
4320.00	2.76	3.01	3.31	2.46
5760.00	2.99	3.23	3.55	2.67
5760.00	6.23	6.22	6.37	6.08
5761.00	6.37	6.29	6.44	6.22
5765.00	6.48	6.43	6.57	6.33
5770.00	6.54	6.52	6.66	6.40
5790.00	6.69	6.70	6.84	6.56
5820.00	6.82	6.86	6.99	6.69
5880.00	6.98	7.05	7.18	6.86
6000.00	7.19	7.30	7.42	7.08
6240.00	7.46	7.61	7.71	7.36
6360.00	7.57	7.72	7.83	7.46
7200.00	8.07	8.27	8.36	7.98
8640.00	8.59	8.84	8.91	8.52
8640.00	13.93	14.29	14.32	13.90
8641.00	14.24	14.50	14.54	14.20
8645.00	14.47	14.77	14.81	14.44
8650.00	14.62	14.94	14.98	14.58
8670.00	14.94	15.31	15.34	14.91
8700.00	15.22	15.61	15.65	15.18
8760.00	15.58	16.00	16.04	15.54
8880.00	16.04	16.49	16.54	15.99
9120.00	16.62	17.11	17.16	16.57
9240.00	16.84	17.35	17.39	16.79
10080.00	17.90	18.45	18.51	17.85
11520.00	19.01	19.60	19.66	18.95
11520.00	18.97	19.44	19.50	18.90
11521.00	18.97	19.43	19.50	18.90
11525.00	18.97	19.43	19.49	18.90
11530.00	18.97	19.43	19.49	18.90
11550.00	18.97	19.43	19.49	18.90
11580.00	18.96	19.42	19.49	18.90
11640.00	18.96	19.42	19.48	18.90
11760.00	18.96	19.42	19.48	18.90
12000.00	18.95	19.41	19.47	18.89

12120.00	18.95	19.41	19.47	18.89
12960.00	18.94	19.40	19.46	18.88
14400.00	18.92	19.39	19.45	18.86
14400.00	18.74	19.01	19.08	18.68
14401.00	18.74	19.03	19.09	18.68
14405.00	18.74	19.02	19.08	18.67
14410.00	18.74	19.01	19.07	18.67
14430.00	18.74	19.00	19.06	18.67
14460.00	18.73	18.99	19.05	18.67
14520.00	18.73	18.98	19.04	18.67
14640.00	18.73	18.97	19.03	18.67
14880.00	18.72	18.95	19.02	18.66
15000.00	18.72	18.95	19.01	18.66
15840.00	18.70	18.93	18.99	18.64
17280.00	18.67	18.90	18.97	18.61
17280.00	18.32	18.31	10.37	18.26
17281.00	18.32	18.35	18.41	18.26
17285.00	18.32	18.33	18.39	18.26
17290.00	18.32	18.32	18.38	18.26
17310.00	18.32	18.30	18.36	18.26
17340.00	18.32	18.28	18.34	18.26
17400.00	18.32	18.26	18.32	18.25
17520.00	18.31	18.24	18.30	18.25
17760.00	18.30	18.22	18.28	18.24
17880.00	18.30	18.21	18.27	18.24
18720.00	18.28	18.17	18.23	18.21
20160.00	18.25	18.13	18.20	18.19
20160.00	16.71	16.89	16.95	16.65
20161.00	16.71	16.74	16.80	16.65
20165.00	16.71	16.71	16.77	16.65
20170.00	16.71	16.69	16.75	16.65
20190.00	16.71	16.67	16.73	16.65
20220.00	16.71	16.65	16.71	16.65
20280.00	16.70	16.64	16.70	16.64
20400.00	16.70	16.62	16.68	16.63
20640.00	16.68	16.60	16.66	16.62
20760.00	16.67	16.59	16.65	16.61
21600.00	16.63	16.56	16.63	16.57
23040.00	16.58	16.54	16.60	16.52
24480.00	16.55	16.52	16.58	16.49
25920.00	16.53	16.50	16.56	16.47
27360.00	16.52	16.49	16.55	16.46
28800.00	16.51	16.48	16.54	16.45
30240.00	16.51	16.47	16.53	16.45

Appendix B.9. Theoretical predictions by Solution 4 under load function 7.

TIME	M5-E	V-VE1	V-VE2	V-VE3
1.00	0.84	0.76	0.73	0.87
5.00	0.84	0.76	0.73	0.87
10.00	0.85	0.76	0.74	0.87
30.00	0.87	0.77	0.76	0.88
60.00	0.88	0.77	0.77	0.88
120.00	0.89	0.78	0.78	0.89
240.00	0.93	0.79	0.82	0.90
480.00	0.96	0.81	0.86	0.91
600.00	0.98	0.81	0.87	0.92
1080.00	0.99	0.81	0.87	0.91
1440.00	0.98	0.78	0.87	0.89
2880.00	1.66	1.29	1.57	1.37
2881.00	1.66	1.29	1.58	1.37
2885.00	1.67	1.30	1.59	1.38
2890.00	1.69	1.31	1.61	1.39
2910.00	1.75	1.36	1.67	1.44
2940.00	1.80	1.41	1.72	1.49
3000.00	1.91	1.50	1.83	1.58
3120.00	2.13	1.71	2.06	1.78
3360.00	2.53	2.11	2.48	2.17
3480.00	2.72	2.30	2.67	2.35
3960.00	3.15	2.74	3.11	2.70
4320.00	3.10	2.67	3.05	2.72
5760.00	4.65	4.24	4.61	4.28
5761.00	4.65	4.26	4.62	4.29
5765.00	4.69	4.29	4.66	4.32
5770.00	4.73	4.34	4.70	4.37
5790.00	4.91	4.52	4.88	4.55
5820.00	5.04	4.66	5.01	4.69
5880.00	5.33	4.96	5.30	4.78
6000.00	5.92	5.58	5.89	5.60
6240.00	6.85	6.56	6.84	6.58
6360.00	7.25	6.97	7.23	6.90
6840.00	7.88	7.63	7.87	7.64
7200.00	7.65	7.38	7.63	7.40
8640.00	11.52	11.33	11.49	11.36
8641.00	11.54	11.35	11.51	11.38
8645.00	11.61	11.43	11.59	11.45
8650.00	11.71	11.52	11.68	11.55
8670.00	12.08	11.91	12.06	11.73
8700.00	12.33	12.16	12.31	12.18
8760.00	12.84	12.66	12.81	12.69
8880.00	13.85	13.66	13.82	13.69
9120.00	15.23	15.01	15.20	15.04
9240.00	15.80	15.57	15.77	15.60
9720.00	17.32	17.03	17.27	17.07
10080.00	18.01	17.71	17.96	17.77
11520.00	18.01	17.79	18.04	17.76
11521.00	18.01	17.79	18.04	17.76
11525.00	18.01	17.79	18.04	17.76
11530.00	18.01	17.79	18.04	17.76

11550.00	18.01	17.80	18.05	17.76
11580.00	18.01	17.80	18.05	17.76
11640.00	18.01	17.81	18.06	17.76
11760.00	18.01	17.82	18.07	17.76
12000.00	17.79	17.84	18.09	17.75
12120.00	17.99	17.85	18.10	17.74
12600.00	17.97	17.85	18.10	17.72
12960.00	17.97	17.85	18.10	17.72
14400.00	17.91	17.83	18.08	17.66
14401.00	17.91	17.83	18.08	17.66
14405.00	17.71	17.83	18.08	17.66
14410.00	17.91	17.83	18.08	17.66
14430.00	17.91	17.83	18.08	17.66
14460.00	17.90	17.83	18.08	17.65
14520.00	17.89	17.82	18.07	17.64
14640.00	17.87	17.80	18.05	17.62
14880.00	17.83	17.77	18.01	17.58
15000.00	17.81	17.75	18.00	17.56
15480.00	17.79	17.73	17.98	17.54
15840.00	17.80	17.75	18.00	17.55
17280.00	17.65	17.61	17.85	17.40
17281.00	17.65	17.61	17.85	17.40
17285.00	17.65	17.60	17.85	17.40
17290.00	17.65	17.60	17.85	17.40
17310.00	17.64	17.60	17.85	17.39
17340.00	17.63	17.59	17.84	17.38
17400.00	17.61	17.57	17.82	17.36
17520.00	17.58	17.53	17.78	17.33
17760.00	17.52	17.46	17.71	17.27
17880.00	17.49	17.43	17.68	17.24
18360.00	17.44	17.38	17.63	17.17
18720.00	17.44	17.38	17.63	17.19
20160.00	17.43	17.38	17.63	17.10
20161.00	15.70	15.66	15.91	15.45
20165.00	15.70	15.64	15.89	15.45
20170.00	15.70	15.62	15.87	15.45
20170.00	15.70	15.60	15.85	15.45
20220.00	15.70	15.59	15.83	15.45
20280.00	15.69	15.57	15.82	15.44
20400.00	15.69	15.56	15.80	15.44
20640.00	15.67	15.54	15.79	15.43
20760.00	15.67	15.54	15.79	15.42
21240.00	15.65	15.52	15.77	15.40
21600.00	15.63	15.52	15.77	15.39
23040.00	15.59	15.51	15.76	15.34
23041.00	15.59	15.51	15.76	15.34
23045.00	15.59	15.51	15.76	15.34
23050.00	15.59	15.51	15.76	15.34
23070.00	15.59	15.51	15.76	15.34
23100.00	15.59	15.51	15.76	15.34
23160.00	15.59	15.51	15.76	15.34
23280.00	15.59	15.51	15.76	15.34
23520.00	15.58	15.51	15.76	15.33
23640.00	15.58	15.51	15.75	15.33
24120.00	15.57	15.50	15.75	15.32

24480.00	15.57	15.50	15.75	15.32
25920.00	15.55	15.50	15.75	15.30
25921.00	15.55	15.50	15.75	15.30
25925.00	15.55	15.50	15.75	15.30
25930.00	15.55	15.50	15.75	15.30
25950.00	15.55	15.50	15.75	15.30
25980.00	15.55	15.50	15.75	15.30
26040.00	15.55	15.50	15.75	15.30
26160.00	15.55	15.50	15.75	15.30
26400.00	15.55	15.50	15.75	15.30
26520.00	15.55	15.50	15.75	15.30
27000.00	15.54	15.50	15.75	15.29
27360.00	15.54	15.50	15.75	15.29
28800.00	15.54	15.50	15.74	15.29
28801.00	15.54	15.50	15.74	15.29
28805.00	15.54	15.50	15.74	15.29
28810.00	15.54	15.50	15.74	15.29
28830.00	15.54	15.50	15.74	15.29
28860.00	15.54	15.50	15.74	15.29
28920.00	15.54	15.50	15.74	15.29
29040.00	15.53	15.49	15.74	15.29
29280.00	15.53	15.49	15.74	15.28
29400.00	15.53	15.49	15.74	15.28
29880.00	15.53	15.49	15.74	15.28
30240.00	15.53	15.49	15.74	15.28

Appendix C.1. Program for nonlinear least squares curve fitting.

```
100 DEFINT I-N
110 OPEN "A:INDATA.001" FOR INPUT AS #1
120 INPUT "No. of points and initial value ?";N,B
130 DIM X(N),Y(N),Z(N),V(N)
140 FOR I=1 TO N
150 INPUT #1,X(I),Y(I)
160 NEXT I
170 LPRINT X(N),Y(N)
180 L=0
190 REM "Make summations."
200 SUM1=0!
210 SUM2=0!
220 SUM3=0!
230 SUM4=0!
240 SUM5=0!
250 SUM6=0!
260 FOR I=1 TO N
270 Z(I)=X(I)^B
280 SUM1=SUM1+Z(I)
290 SUM2=SUM2+(Z(I)^2)
300 SUM3=SUM3+Y(I)
310 SUM4=SUM4+Y(I)*Z(I)
320 NEXT I
330 REM "Get coefficient A and C."
340 C=(SUM3-(SUM4*SUM1)/SUM2)/(N-(SUM1^2)/SUM2)
350 A=SUM4/SUM2-(SUM1/SUM2)*C
360 REM "Check if dY/dB=0.0."
370 FOR I=1 TO N
380 V(I)=LOG(X(I))
390 SUM5=SUM5+(Y(I)*Z(I)-A*(Z(I)^2)-C*Z(I))*V(I)
400 NEXT I
410 REM "If dY/dB=0.0, print answers."
420 DY1=-2!*(A)*SUM5
430 IF ABS(DY1)<1E-10 THEN GOTO 550
440 REM "if dY/dB is not equal to 0.0, repeat ."
450 REM "Newton's method."
460 FOR I=1 TO N
470 SUM6=SUM6+(Y(I)*Z(I)-2!*A*(Z(I)^2)-C*Z(I))*(V(I)^2)
480 NEXT I
490 DY2=-2!*(A)*SUM6
500 M=L/10
510 IF L=M*10 THEN LPRINT A,B,C,DY1,L
520 L=L+1
530 B=B-(DY1/DY2)
540 GOTO 190
550 LPRINT A,B,C
560 END
```

Appendix C.2. Program for the creep predicted by Solution 2 under load function 5.

```
100 REM "Creep slip by Solution 2 under loading function 5."
110 OPEN "A:CRVARY.505" FOR OUTPUT AS #1
120 DEFINT I-N
130 DIM T(12)
140 N=12
150 FOR I=1 TO N
160 INPUT "Time interval";T(I)
170 NEXT I
180 REM "Input some constants."
190 DT=2880!
200 HT=1440!
210 DP=20!
220 PT=0!
230 PLOD=100!
240 A=.35
250 REM "Define some functions for 5-E."
260 DEF FNFP(PL)=4.8121E-10*(PL^4.9026)
270 DEF FNFF100(TM)=1.844+.34536*(1!-EXP((-.0005148)*TM))
280 DEF FNFT100(TM)=.14513*(TM^.35)
290 FF=3.095
300 REM "Define some functions for M5-E."
310 EN=-.0003259
320 AN=5.07464E-05
330 BN=3.1916E-10
340 CN=4.3212
350 DEF FNDE(PL,TM)=AN*(PL^2.28375)+(PL*.003812)*(1!-EXP(EN*TM))
360 DEF FNDV(PL,TM)=BN*(PL^CN)*(TM^A)
370 REM "Define some functions for V-VE1."
380 DEF FNF1(TM)=(-7.34+1.102*(TM^.3)-9.415601*(TM^.17)+9.0455*(TM^.12))*.001
390 DEF FNF2(TM)=(.272-.0202*(TM^.3)+.2354*(TM^.17)-.2412*(TM^.12))*.001
400 DEF FNF3(TM)=(-.0003+.00009*(TM^.3)-.0013*(TM^.17)+.0015*(TM^.12))*.001
410 DEF FNF4(TM)=(57.74367+.025433*(TM^A))*.001
420 DEF FNF5(TM)=(-1.80786-.019755*(TM^A))*.001
430 DEF FNF6(TM)=(.015399+.00034*(TM^A))*.001
440 DEF FNVE(PL,TM)=FNF1(TM)*PL+FNF2(TM)*(PL^2)+FNF3(TM)*(PL^3)
450 DEF FNVV(PL,TM)=FNF4(TM)*PL+FNF5(TM)*(PL^2)+FNF6(TM)*(PL^3)
460 DEF FNVG(PL)=(57.74367*PL-1.80786*(PL^2)+.015399*(PL^3))*.001
470 REM "Compute the creep slip under the first step of loads."
480 FOR I=1 TO N
490 TIME=PT+T(I)
500 S5F=FNFF100(TIME)+FNFT100(TIME)+FF
510 VM=FNDV(PLOD,TIME)+FNFP(PLOD)
520 VE=FNDE(PLOD,TIME)
530 EV=FNVE(PLOD,TIME)
540 VV=FNVV(PLOD,TIME)
550 SME=VE+VM
560 SV1=EV+VV
570 SV2=EV+VM
580 SV3=VE+VV
590 PRINT #1, USING "######.##";TIME;S5F;SME;SV1;SV2;SV3
600 NEXT I
610 S1=S5F
620 S2=SME
630 S3=SV1
640 S4=SV2
650 S5=SV3
```

```
660 REM "Compute the creep slip under the second step of loads."
670 PT=TIME
680 FOR I=1 TO N
690 TIME=PT+T(I)
700 TIM1=TIME-DT
710 S5F=S1-FNFF100(TIM1)
720 VE=FNDE(PLOD,TIM1)
730 EV=FNVE(PLOD,TIM1)
740 SME=S2-VE
750 SV1=S3-EV
760 SV2=S4-EV
770 SV3=S5-VE
780 PRINT #1, USING "######.##";TIME;S5F;SME;SV1;SV2;SV3
790 NEXT I
800 S1=S5F
810 S2=SME
820 S3=SV1
830 S4=SV2
840 S5=SV3
850 REM "Compute the creep slip under the third step of loads."
860 PT=TIME
870 FOR I=1 TO N
880 TIME=PT+T(I)
890 TIM2=TIME-2*DT
900 S5F=S1+FNFF100(TIM2)+FNFT100(TIM2)
910 VM=FNDV(PLOD,TIM2)
920 VE=FNDE(PLOD,TIM2)
930 EV=FNVE(PLOD,TIM2)
940 VV=FNVV(PLOD,TIM2)-FNVG(PLOD)
950 SME=S2+VE+VM
960 SV1=S3+EV+VV
970 SV2=S4+EV+VM
980 SV3=S5+VE+VV
990 PRINT #1, USING "######.##";TIME;S5F;SME;SV1;SV2;SV3
1000 IF TIME=>8640! THEN GOTO 1020
1010 NEXT I
1020 IF TIME=>14400! THEN GOTO 1050
1030 TIME=TIME+HT
1040 GOTO 890
1050 S1=S5F
1060 S2=SME
1070 S3=SV1
1080 S4=SV2
1090 S5=SV3
1100 REM "Compute the creep slip under the forth step of loads."
1110 PT=TIME
1120 FOR I=1 TO N
1130 TIME=PT+T(I)
1140 TIM3=TIME-3*DT
1150 S5F=S1-FNFF100(TIM3)
1160 VE=FNDE(PLOD,TIM3)
1170 EV=FNVE(PLOD,TIM3)
1180 SME=S2-VE
1190 SV1=S3-EV
1200 SV2=S4-EV
1210 SV3=S5-VE
1220 PRINT #1, USING "######.##";TIME;S5F;SME;SV1;SV2;SV3
1230 IF TIME=>17280! THEN GOTO 1250
1240 NEXT I
1250 IF TIME=>30240! THEN GOTO 1280
1260 TIME=TIME+HT
1270 GOTO 1140
1280 CLOSE #1
1290 END
```

Appendix C.3. Program for the creep predicted by Solution 2 under load function 6.

```
100 REM "Creep slip by Solution 2 under loading function 6."
110 OPEN "A:CRVARY.606" FOR OUTPUT AS #1
120 DEFINT I-N
130 DIM T(12)
140 N=12
150 FOR I=1 TO N
160 INPUT "Time interval";T(I)
170 NEXT I
180 REM "Input some constants."
190 DT=2880!
200 HT=1440!
210 DP=20!
220 PT=0!
230 PLOD=120!
240 A=.35
250 ID1=1
260 ID2=1
270 ID3=1
280 REM "Define some functions for 5-E."
290 DEF FNFP(PL)=4.8121E-10*(PL^4.9026)
300 DEF FNFF60(TM)=.5120001+.19014*(1!-EXP((-.0002981)*TM))
310 DEF FNFF80(TM)=1.213+.40501*(1!-EXP((-.0001523)*TM))
320 DEF FNFF100(TM)=1.344+.34536*(1!-EXP((-.0005148)*TM))
330 DEF FNFF120(TM)=2.843+.42717*(1!-EXP((-.0003385)*TM))
340 DEF FNFT120(TM)=.51365*(TM^.3)+7.103
350 REM "Define some functions for M5-E."
360 EN=-.0003259
370 AN=5.07464E-05
380 BN=3.1916E-10
390 CN=4.3212
400 DEF FNDE(PL,TM)=AN*(PL^2.28375)+(PL*.003812)*(1!-EXP(EN*TM))
410 DEF FNDV(PL,TM)=BN*(PL^CN)*(TM^A)
420 REM "Define some functions for V-VE."
430 DEF FNF1(TM)=(-7.34+1.102*(TM^.3)-9.415301*(TM^.17)+7.0455*(TM^.12))*.001
440 DEF FNF2(TM)=(.272-.0202*(TM^.3)+.23541*(TM^.17)-.2412*(TM^.12))*.001
450 DEF FNF3(TM)=(-.0003+.00009*(TM^.3)-.0016*(TM^.17)+.0015*(TM^.12))*.001
460 DEF FNF4(TM)=(57.74367+.025453*(TM^A))*.001
470 DEF FNF5(TM)=(-1.80786-.019755*(TM^A))*.001
480 DEF FNF6(TM)=(.015399+.00034*(TM^A))*.001
490 DEF FNVE(PL,TM)=FNF1(TM)*PL+FNF2(TM)*(PL^2)+FNF3(TM)*(PL^3)
500 DEF FNVV(PL,TM)=FNF4(TM)*PL+FNF5(TM)*(PL^2)+FNF6(TM)*(PL^3)
510 REM "Compute the creep slip under the first step of loads."
520 FOR I=1 TO N
530 TIME=PT+T(I)
540 S5F=FNFF120(TIME)+FNFT120(TIME)
550 VM=FNDV(PLOD,TIME)+FNFP(PLOD)
560 VE=FNDE(PLOD,TIME)
570 EV=FNVE(PLOD,TIME)
580 VV=FNVV(PLOD,TIME)
590 SME=VE+VM
600 SV1=EV+VV
610 SV2=EV+VM
620 SV3=VE+VV
630 PRINT #1, USING "######.##";TIME;S5F;SME;SV1;SV2;SV3
640 NEXT I
650 S1=S5F
660 S2=SME
670 S3=SV1
680 S4=SV2
690 S5=SV3
```

```
700 REM "Compute the creep slip under the second step of loads."
710 PT=TIME
720 FOR I=1 TO N
730 TIME=PT+T(I)
740 TIM1=TIME-DT
750 S5F=S1-FNFF120(TIM1)
760 VE=FNDE(PLOD,TIM1)
770 EV=FNVE(PLOD,TIM1)
780 SME=S2-VE
790 SV1=S3-EV
800 SV2=S4-EV
810 SV3=S5-VE
820 PRINT #1, USING "######.##";TIME;S5F;SME;SV1;SV2;SV3
830 NEXT I
840 IF ID1=0 THEN GOTO 870 ELSE GOTO 850
850 PLOD=PLOD-DP
860 ID2=ID2+1
870 S1=S5F
880 S2=SME
890 S3=SV1
900 S4=SV2
910 S5=SV3
920 ID3=ID3+1
930 REM "Compute the creep slip under the next step of loads."
940 PT=TIME
950 FOR I=1 TO N
960 TIME=PT+T(I)
970 TIM2=TIME-ID3*DT
980 IF ID2=2 THEN SF=FNFF100(TIM2)
990 IF ID2=3 THEN SF=FNFF80(TIM2)
1000 IF ID2=4 THEN SF=FNFF60(TIM2)
1010 VE=FNDE(PLOD,TIM2)
1020 EV=FNVE(PLOD,TIM2)
1030 IF ID1=1 THEN SN=1! ELSE SN=-1!
1040 S5F=S1+SN*SF
1050 SME=S2+SN*VE
1060 SV1=S3+SN*EV
1070 SV2=S4+SN*EV
1080 SV3=S5+SN*VE
1090 PRINT #1, USING "######.##";TIME;S5F;SME;SV1;SV2;SV3
1100 IF TIME=>23040 THEN GOTU 1140
1110 NEXT I
1120 IF ID1=1 THEN ID1=0 ELSE ID1=1
1130 GOTO 840
1140 TIME=TIME+HT
1150 IF TIME>30240 THEN GOTO 1170
1160 GOTO 970
1170 CLOSE #1
1180 END
```

Appendix C.4. Program for the creep predicted by Solution 2 under load function 7.

```
100 REM "Creep slip by Solution 2 under loading function 7."
110 DEFINT I-N
120 DIM T(12)
130 N=12
140 FOR I=1 TO N
150 INPUT "Time interval";T(I)
160 NEXT I
170 REM "Input some constants."
180 DI=2880!
190 HT=1440!
200 DP=20!
210 PT=0!
220 PLOD=60!
230 A=.35
240 CF=.765
250 REM "Define some functions for 5-E."
260 DEF FNFP(PL)=4.8121E-10*(PL^4.9026)
270 DEF FNFB(TM)=.171+.06338*(1!-EXP((-.0002981)*TM))
280 DEF FNFC(TM)=.341+.12676*(1!-EXP((-.0002781)*TM))
290 DEF FNFF60(TM)=.5120001+.19014*(1!-EXP((-.0002981)*TM))
300 DEF FNFT60(TM)=.0004536*(TM^.57)
310 DEF FNFF80(TM)=1.213+.40501*(1!-EXP((-.0001523)*TM))
320 DEF FNFT80(TM)=.028838*(TM^.41)
330 DEF FNFF100(TM)=1.844+.34536*(1!-EXP((-.0005148)*TM))
340 DEF FNFT100(TM)=.14513*(TM^.35)
350 DEF FNFF120(TM)=2.843+.42717*(1!-EXP((-.0003385)*TM))
360 DEF FNFT120(TM)=.51365*(TM^.3)
370 FP1=.294
380 FP2=1.051
390 FP3=3.095
400 FP4=7.103
410 REM "Define some functions for M5-E."
420 EN=-.0003259
430 AN=5.07464E-05
440 BN=3.1916E-10
450 CN=4.3212
460 DEF FNDE(PL,TM)=AN*(PL^2.28375)+(PL*.003812)*(1!-EXP(EN*TM))
470 DEF FNDV(PL,TM)=BN*(PL^CN)*(TM^A)
480 REM "Define some functions for V-VE1."
490 DEF FNF1(TM)=(-7.34+1.102*(TM^.3)-9.415301*(TM^.17)+9.0455*(TM^.12))*.001
500 DEF FNF2(TM)=(.272-.0202*(TM^.3)+.23541*(TM^.17)-.2412*(TM^.12))*.001
510 DEF FNF3(TM)=(-.00028+.000092*(TM^.3)-.001308*(TM^.17)+.001508*(TM^.12))*.00
1
520 DEF FNF4(TM)=(57.74367+.025453*(TM^A))*.001
530 DEF FNF5(TM)=(-1.80786-.019755*(TM^A))*.001
540 DEF FNF6(TM)=(.015399+.00034*(TM^A))*.001
550 DEF FNVE(PL,TM)=FNF1(TM)*PL+FNF2(TM)*(PL^2)+FNF3(TM)*(PL^3)
560 DEF FNVV(PL,TM)=FNF4(TM)*PL+FNF5(TM)*(PL^2)+FNF6(TM)*(PL^3)
570 DEF FNVG(PL)=(57.74367*PL-1.80786*(PL^2)+.015399*(PL^3))*.001
580 REM "Compute the creep slip under the first step of loads."
590 LPRINT "      TIME        5-E       M5-E       V-VE1       V-VE2      V-VE3"
600 FOR I=1 TO N
610 TIME=PT+T(I)
620 S5F=FNFF60(TIME)+FNFT60(TIME)+FP1
630 VM=FNDV(PLOD,TIME)+FNFP(PLOD)
640 VE=FNDE(PLOD,TIME)
650 EV=FNVE(PLOD,TIME)
660 VV=FNVV(PLOD,TIME)
670 SME=VE+VM
680 SV1=EV+VV
690 SV2=EV+VM
700 SV3=VE+VV
710 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
720 NEXT I
```

```
730 REM "Compute the creep slip under the second step of loads."
740 PT=TIME
750 PL1=PLOD+DP
760 FOR I=1 TO N
770 TIME=PT+T(I)
780 TIM1=TIME-DT
790 S5F=FNFF60(TIME)+FNFF80(TIM1)-FNFF60(TIM1)+FNFT60(DT)+FNFT80(TIM1)+FP2
800 VM=FNFP(PL1)+FNDV(PLOD,DT)+FNDV(PL1,TIM1)
810 VE=FNDE(PLOD,TIME)+FNDE(PL1,TIM1)-FNDE(PLOD,TIM1)
820 EV=FNVE(PLOD,TIME)+FNVE(PL1,TIM1)-FNVE(PLOD,TIM1)
830 VV=FNVV(PLOD,DT)+FNVV(PL1,TIM1)-FNVG(PLOD)
840 SME=VE+VM
850 SV1=EV+VV
860 SV2=EV+VM
870 SV3=VE+VV
880 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
890 NEXT I
900 REM "Compute the creep slip under the third step of loads."
910 PT=TIME
920 PL2=PL1+DP
930 FOR I=1 TO N
940 TIME=PT+T(I)
950 TIM1=TIME-DT
960 TIM2=TIME-2*DT
970 S5F=FNFF60(TIME)+FNFF80(TIM1)-FNFF60(TIM1)+FNFF100(TIM2)-FNFF80(TIM2)+FNFT60
(DT)+FNFT80(DT)+FNFT100(TIM2)+FP3
980 VM=FNFP(PL2)+FNDV(PLOD,DT)+FNDV(PL1,DT)+FNDV(PL2,TIM2)
990 VE=FNDE(PLOD,TIME)+FNDE(PL1,TIM1)-FNDE(PLOD,TIM1)+FNDE(PL2,TIM2)-FNDE(PL1,TI
M2)
1000 EV=FNVE(PLOD,TIME)+FNVE(PL1,TIM1)-FNVE(PLOD,TIM1)+FNVE(PL2,TIM2)-FNVE(PL1,T
IM2)
1010 VV=FNVV(PLOD,DT)+FNVV(PL1,DT)+FNVV(PL2,TIM2)-FNVG(PLOD)-FNVG(PL1)
1020 SME=VE+VM
1030 SV1=EV+VV
1040 SV2=EV+VM
1050 SV3=VE+VV
1060 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
1070 NEXT I
1080 REM "Compute the creep slip under the forth step of loads."
1090 PT=TIME
1100 PL3=PL2+DP
1110 FOR I=1 TO N
1120 TIME=PT+T(I)
1130 TIM1=TIME-DT
1140 TIM2=TIME-2*DT
1150 TIM3=TIME-3*DT
1160 A5F=FNFF60(TIME)+FNFF80(TIM1)-FNFF60(TIM1)+FNFF100(TIM2)-FNFF80(TIM2)+FNFF1
20(TIM3)-FNFF100(TIM3)
1170 S5F=A5F+FNFT60(DT)+FNFT80(DT)+FNFT100(DT)+FNFT120(TIM3)+FP4
1180 VM=FNFP(PL3)+FNDV(PLOD,DT)+FNDV(PL1,DT)+FNDV(PL2,DT)+FNDV(PL3,TIM3)
1190 VE=FNDE(PLOD,TIME)+FNDE(PL1,TIM1)-FNDE(PLOD,TIM1)+FNDE(PL2,TIM2)-FNDE(PL1,T
IM2)+FNDE(PL3,TIM3)-FNDE(PL2,TIM3)
1200 EV=FNVE(PLOD,TIME)+FNVE(PL1,TIM1)-FNVE(PLOD,TIM1)+FNVE(PL2,TIM2)-FNVE(PL1,T
IM2)+FNVE(PL3,TIM3)-FNVE(PL2,TIM3)
1210 VV=FNVV(PLOD,DT)+FNVV(PL1,DT)+FNVV(PL2,DT)+FNVV(PL3,TIM3)-FNVG(PLOD)-FNVG(P
L1)-FNVG(PL2)
1220 SME=VE+VM
1230 SV1=EV+VV
1240 SV2=EV+VM
1250 SV3=VE+VV
1260 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
1270 NEXT I
1280 REM "Compute the creep slip under the fifth step of loads."
1290 PT=TIME
1300 S1=S5F
1310 S2=SME
1320 S3=SV1
1330 S4=SV2
1340 S5=SV3
```

```
1350 FOR I=1 TO N
1360 TIME=PT+T(I)
1370 TIM4=TIME-4*DT
1380 S5F=S1-FNFB(TIM4)
1390 VE=FNDE(DP,TIM4)
1400 VED=FNVE(PLOD,TIM4)/3!
1410 SME=S2-VE
1420 SV1=S3-VED
1430 SV2=S4-VED
1440 SV3=S5-VE
1450 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
1460 NEXT I
1470 REM "Compute the creep slip under the sixth step of loads."
1480 PT=TIME
1490 DDP=2!*DP
1500 FOR I=1 TO N
1510 TIME=PT+T(I)
1520 TIM4=TIME-4*DT
1530 TIM5=TIME-5*DT
1540 S5F=S1-(FNFB(TIM4)+FNFC(TIM5)-FNFB(TIM5))
1550 VE=(FNDE(DP,TIM4)+FNDE(DDP,TIM5)-FNDE(DP,TIM5))
1560 VED=(FNVE(PLOD,TIM4)+FNVE(PLOD,TIM5)*2!-FNVE(PLOD,TIM5))/3!
1570 SME=S2-VE
1580 SV1=S3-VED
1590 SV2=S4-VED
1600 SV3=S5-VE
1610 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
1620 NEXT I
1630 REM "Compute the creep slip under the seventh step of loads."
1640 PT=TIME
1650 FOR I=1 TO N
1660 TIME=PT+T(I)
1670 TIM4=TIME-4*DT
1680 TIM5=TIME-5*DT
1690 TIM6=TIME-6*DT
1700 S5F=S1-(FNFB(TIM4)+FNFC(TIM5)-FNFB(TIM5)+FNFF60(TIM6)-FNFC(TIM6))
1710 VE=(FNDE(DP [illegible] IM4)+FNDE(DDP,TIM5)-FNDE(DP,TIM5)+FNDE(PLOD,TIM6)-FNDE(DDP,TIM
6))
1720 VED=(FNVE(PLOD,TIM4)+FNVE(PLOD,TIM5)*2!-FNVE(PLOD,TIM5 [illegible] VE(PLOD,TIM6)*3!-
FNVE(PLOD,TIM6)*2!)/3!
1730 SME=S2-VE
1740 SV1=S3-VED
1750 SV2=S4-VED
1760 SV3=S5-VE
1770 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
1780 NEXT I
1790 REM "Compute the recovery after complete unloading."
1800 PT=TIME
1810 FOR I=1 TO N
1820 TIME=PT+T(I)
1830 TIM4=TIME-4*DT
1840 TIM5=TIME-5*DT
1850 TIM6=TIME-6*DT
1860 TIM7=TIME-7*DT
1870 S5F=S1-(FNFB(TIM4)+FNFC(TIM5)-FNFB(TIM5)+FNFF60(TIM6)-FNFC(TIM6)+FNFF120(TI
M7)-FNFF60(TIM7))*CF
1880 VE=(FNDE(DP,TIM4)+FNDE(DDP,TIM5)-FNDE(DP,TIM5)+FNDE(PLOD,TIM6)-FNDE(DDP,TIM
6)+FNDE(PL3,TIM7)-FNDE(PLOD,TIM7))*CF
1890 VED=((FNVE(PLOD,TIM4)+FNVE(PLOD,TIM5)*2!-FNVE(PLOD,TIM5)+FNVE(PLOD,TIM6)*3!
-FNVE(PLOD,TIM6)*2!)/3!+FNVE(PL3,TIM7)-FNVE(PLOD,TIM7))*CF
1900 SME=S2-VE
1910 SV1=S3-VED
1920 SV2=S4-VED
1930 SV3=S5-VE
1940 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
1950 IF TIME>23040 THEN GOTO 1970
1960 NEXT I
1970 TIME=TIME+HT
1980 IF TIME>30240 THEN GOTO 2000
1990 GOTO 1830
2000 END
```

Appendix C.5. Program for the creep predicted by Solution 2 under load function 8.

```
100 REM "Creep slip by Solution 2 under loading function 8."
110 OPEN "A:CRVARY.808" FOR OUTPUT AS #1
120 DEFINT I-N
130 DIM T(12)
140 N=12
150 FOR I=1 TO N
160 INPUT "Time interval";T(I)
170 NEXT I
180 REM "Input some constants."
190 DT=2880!
200 HT=1440!
210 DP=20!
220 DDP=40!
230 PT=0!
240 PLOD=80!
250 A=.35
260 CF=.891
270 REM "Define some functions for 5-E."
280 DEF FNFP(PL)=4.8121E-10*(PL^4.9026)
290 DEF FNFB(TM)=.171+.06338*(1!-EXP((-.0002981)*TM))
300 DEF FNFC(TM)=.341+.12676*(1!-EXP((-.0002981)*TM))
310 DEF FNFF60(TM)=.5120001+.19014*(1!-EXP((-.0002981)*TM))
320 DEF FNFT60(TM)=.0004536*(TM^.57)
330 DEF FNFF80(TM)=1.213+.40501*(1!-EXP((-.0001523)*TM))
340 DEF FNFT80(TM)=.028838*(TM^.41)
350 DEF FNFF100(TM)=1.844+.34536*(1!-EXP((-.0005148)*TM))
360 DEF FNFT100(TM)=.14513*(TM^.35)
370 DEF FNFF120(TM)=2.843+.42717*(1!-EXP((-.0003385)*TM))
380 DEF FNFT120(TM)=.51365*(TM^.3)
390 FP1=.294
400 FP2=1.051
410 FP3=3.095
420 FP4=7.103
430 REM "Define some functions for M5-E."
440 EN=-.0003259
450 AN=5.07464E-05
460 BN=3.1916E-10
470 CN=4.3212
480 DEF FNDE(PL,TM)=AN*(PL^2.28375)+(PL*.003812)*(1!-EXP(EN*TM))
490 DEF FNDV(PL,TM)=BN*(PL^CN)*(TM^A)
500 REM "Define some functions for V-VE1."
510 DEF FNF1(TM)=(-7.34+1.102*(TM^.3)-9.415301*(TM^.17)+9.0455*(TM^.12))*.001
520 DEF FNF2(TM)=(.272-.0202*(TM^.3)+.23541*(TM^.17)-.2412*(TM^.12))*.001
530 DEF FNF3(TM)=(-.0003+.00009*(TM^.3)-.0013*(TM^.17)+.0015*(TM^.12))*.001
540 DEF FNF4(TM)=(57.74367+.025453*(TM^A))*.001
550 DEF FNF5(TM)=(-1.80786-.019755*(TM^A))*.001
560 DEF FNF6(TM)=(.015399+.00034*(TM^A))*.001
570 DEF FNVE(PL,TM)=FNF1(TM)*PL+FNF2(TM)*(PL^2)+FNF3(TM)*(PL^3)
580 DEF FNVV(PL,TM)=FNF4(TM)*PL+FNF5(TM)*(PL^2)+FNF6(TM)*(PL^3)
590 DEF FNVG(PL)=(57.74367*PL-1.80786*(PL^2)+.015399*(PL^3))*.001
600 REM "Compute the creep slip under the first step of loads."
610 FOR I=1 TO N
620 TIME=PT+T(I)
630 S5F=FNFF80(TIME)+FNFT80(TIME)+FP2
640 VM=FNDV(PLOD,TIME)+FNFP(PLOD)
650 VE=FNDE(PLOD,TIME)
660 EV=FNVE(PLOD,TIME)
670 VV=FNVV(PLOD,TIME)
680 SME=VE+VM
690 SV1=EV+VV
700 SV2=EV+VM
710 SV3=VE+VV
720 PRINT #1, USING "######.##";TIME;S5F;SME;SV1;SV2;SV3
730 NEXT I
```

```
740 REM "Compute the creep slip under the second step of loads."
750 PT=TIME
760 PL1=PLOD+DP
770 FOR I=1 TO N
780 TIME=PT+T(I)
790 TIM1=TIME-DT
800 S5F=FNFF80(TIME)+FNFF100(TIM1)-FNFF80(TIM1)+FNFT80(DT)+FNFT100(TIM1)+FP3
810 VM=FNFP(PL1)+FNDV(PLOD,DT)+FNDV(PL1,TIM1)
820 VE=FNDE(PLOD,TIME)+FNDE(PL1,TIM1)-FNDE(PLOD,TIM1)
830 EV=FNVE(PLOD,TIME)+FNVE(PL1,TIM1)-FNVE(PLOD,TIM1)
840 VV=FNVV(PLOD,DT)+FNVV(PL1,TIM1)-FNVG(PLOD)
850 SME=VE+VM
860 SV1=EV+VV
870 SV2=EV+VM
880 SV3=VE+VV
890 PRINT #1, USING "######.##";TIME;S5F;SME;SV1;SV2;SV3
900 NEXT I
910 REM "Compute the creep slip under the third step of loads."
920 PT=TIME
930 PL2=PLOD-DP
940 S1=S5F
950 S2=SME
960 S3=SV1
970 S4=SV2
980 S5=SV3
990 FOR I=1 TO N
1000 TIME=PT+T(I)
1010 TIM2=TIME-2*DT
1020 S5F=S1-FNFC(TIM2)
1030 VE=FNDE(DDP,TIM2)
1040 EV=FNVE(PL2,TIM2)*2!/3!
1050 SME=S2-VE
1060 SV1=S3-EV
1070 SV2=S4-EV
1080 SV3=S5-VE
1090 PRINT #1, USING "######.##";TIME;S5F;SME;SV1;SV2;SV3
1100 NEXT I
1110 REM "Compute the creep slip under the forth step of loads."
1120 PT=TIME
1130 PL3=PL1+DP
1140 S1=S5F
1150 S2=SME
1160 S3=SV1
1170 S4=SV2
1180 S5=SV3
1190 FOR I=1 TO N
1200 TIME=PT+T(I)
1210 TIM3=TIME-3*DT
1220 S5F=S1+FNFF120(TIM3)-FNFF60(TIM3)+FNFT120(TIM3)+FP4-FP3
1230 VE=FNDE(PL3,TIM3)-FNDE(PL2,TIM3)
1240 VM=FNDV(PL3,TIM3)+FNFP(PL3)-FNFP(PL1)
1250 EV=FNVE(PL3,TIM3)-FNVE(PL2,TIM3)
1260 VV=FNVV(PL3,TIM3)-FNVG(PL1)
1270 SME=S2+VE+VM
1280 SV1=S3+EV+VV
1290 SV2=S4+EV+VM
1300 SV3=S5+VE+VV
1310 PRINT #1, USING "######.##";TIME;S5F;SME;SV1;SV2;SV3
1320 NEXT I
```

```
1330 REM "Compute the creep slip under the fifth step of loads."
1340 PT=TIME
1350 S1=S5F
1360 S2=SME
1370 S3=SV1
1380 S4=SV2
1390 S5=SV3
1400 FOR I=1 TO N
1410 TIME=PT+T(I)
1420 TIM4=TIME-4*DT
1430 S5F=S1-FNFC(TIM4)
1440 VE=FNDE(DDP,TIM4)
1450 EV=FNVE(PL2,TIM4)*2!/3!
1460 SME=S2-VE
1470 SV1=S3-EV
1480 SV2=S4-EV
1490 SV3=S5-VE
1500 PRINT #1, USING "######.##";TIME;S5F;SME;SV1;SV2;SV3
1510 NEXT I
1520 REM "Compute the creep slip after complete unloading."
1530 PT=TIME
1540 FOR I=1 TO N
1550 TIME=PT+T(I)
1560 TIM4=TIME-4*DT
1570 TIM5=TIME-5*DT
1580 S5F=S1-(FNFC(TIM4)+FNFF120(TIM5)-FNFC(TIM5))*CF
1590 VE=(FNDE(DDP,TIM4)+FNDE(PL3,TIM5)-FNDE(DDP,TIM5))*CF
1600 EV=((FNVE(PL2,TIM4)-FNVE(PL2,TIM5))*2!/3!+FNVE(PL3,TIM5))*CF
1610 SME=S2-VE
1620 SV1=S3-EV
1630 SV2=S4-EV
1640 SV3=S5-VE
1650 PRINT #1, USING "######.##";TIME;S5F;SME;SV1;SV2;SV3
1660 IF TIME>17280 THEN GOTO 1680 ELSE 1670
1670 NEXT I
1680 TIME=TIME+HT
1690 IF TIME>30240 THEN GOTO 1710 ELSE GOTO 1700
1700 GOTO 1560
1710 CLOSE #1
1720 END
```

Appendix C.6. Program for the creep predicted by Solution 1 under load function 7.

```
100 REM "Creep slip by Solution 2 under loading function 7."
110 DEFINT I-N
120 DIM T(12)
130 N=12
140 FOR I=1 TO N
150 INPUT "Time interval";T(I)
160 NEXT I
170 REM "Input some constants."
180 DT=2080!
190 HT=1440!
200 DP=20!
210 PT=0!
220 PLOD=60!
230 A=.35
240 REM "Define some functions for 5-E."
250 DEF FNFP(PL)=4.8121E-10*(PL^4.9026)
260 DEF FNFB(TM)=.171+.06338*(1!-EXP((-.0002981)*TM))
270 DEF FNFF60(TM)=.5120001+.19014*(1!-EXP((-.0002981)*TM))
280 DEF FNFT60(TM)=FNFF60(TM)+.0004536*(TM^.57)+.294
290 DEF FNFF80(TM)=1.213+.40501*(1!-EXP((-.0001523)*TM))
300 DEF FNFT80(TM)=FNFF80(TM)+.028838*(TM^.41)+1.051
310 DEF FNFF100(TM)=1.844+.3454*(1!-EXP((-.0005148)*TM))
320 DEF FNFT100(TM)=FNFF100(TM)+.14513*(TM^.35)+3.095
330 DEF FNFF120(TM)=2.843+.42717*(1!-EXP((-.0003385)*TM))
340 DEF FNFT120(TM)=FNFF120(TM)+.51365*(TM^.3)+7.103
350 REM "Define some functions for M5-E."
360 EN=-.0003254
370 AN=5.07464E-05
380 BN=3.1916E-10
390 CN=4.3212
400 DEF FNDE(PL,TM)=AN*(PL^2.28375)+(PL*.003812)*(1!-EXP(EN*TM))
410 DEF FNDV(PL,TM)=BN*(PL^CN)*(TM^A)+FNFP(PL)
420 DEF FNDS(PL,TM)=TM*((BN*PL^CN)^(1!/A))
430 REM "Define some functions for V-VE1."
440 DEF FNF1(TM)=(-7.34+1.102*(TM^.3)-9.415301*(TM^.17)+9.0455*(TM^.12))*.001
450 DEF FNF2(TM)=(.272-.0202*(TM^.3)+.23538*(TM^.17)-.2412*(TM^.12))*.001
460 DEF FNF3(TM)=(-.00028+.000092*(TM^.3)-.001308*(TM^.17)+.001508*(TM^.12))*.00
1
470 DEF FNF4(TM)=(57.74367+.025453*(TM^A))*.001
480 DEF FNF5(TM)=(-1.80786-.019755*(TM^A))*.001
490 DEF FNF6(TM)=(.015399+.00034*(TM^A))*.001
500 DEF FNVE(PL,TM)=FNF1(TM)*PL+FNF2(TM)*(PL^2)+FNF3(TM)*(PL^3)
510 DEF FNVV(PL,TM)=FNF4(TM)*PL+FNF5(TM)*(PL^2)+FNF6(TM)*(PL^3)
520 DEF FNVG(PL)=(57.74367*PL-1.80786*(PL^2)+.015399*(PL^3))*.001
530 DEF FNVH(PL,TM)=(((.025453*PL-.019755*(PL^2)+.00034*(PL^3))*.001)^(1!/A))*T
540 REM "Compute the creep slip under the first step of loads."
550 LPRINT "     TIME       5-E      M5-E       V-VE1      V-VE2      V-VE3"
560 FOR I=1 TO N
570 TIME=PT+T(I)
580 S5F=FNFT60(TIME)
590 VM=FNDV(PLOD,TIME)
600 VE=FNDE(PLOD,TIME)
610 EV=FNVE(PLOD,TIME)
620 VV=FNVV(PLOD,TIME)
630 SME=VE+VM
640 SV1=EV+VV
650 SV2=EV+VM
660 SV3=VE+VV
670 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
680 NEXT I
```

```
690 REM "Compute the creep slip under the second step of loads."
700 PT=TIME
710 PL1=PLOD+DP
720 FOR I=1 TO N
730 TIME=PT+T(I)
740 TIM1=TIME-DT
750 S5F=FNFT60(TIME)+FNFT80(TIM1)-FNFT60(TIM1)
760 VM=FNFP(PL1)+((FNDS(PLOD,DT)+FNDS(PL1,TIM1))^A)
770 VE=FNDE(PLOD,TIME)+FNDE(PL1,TIM1)-FNDE(PLOD,TIM1)
780 EV=FNVE(PLOD,TIME)+FNVE(PL1,TIM1)-FNVE(PLOD,TIM1)
790 VV=FNVG(PL1)+((FNVH(PLOD,DT)+FNVH(PL1,TIM1))^A)
800 SME=VE+VM
810 SV1=EV+VV
820 SV2=EV+VM
830 SV3=VE+VV
840 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
850 NEXT I
860 REM "Compute the creep slip under the third step of loads."
870 PT=TIME
880 PL2=PL1+DP
890 FOR I=1 TO N
900 TIME=PT+T(I)
910 TIM1=TIME-DT
920 TIM2=TIME-2*DT
930 S5F=FNFT60(TIME)+FNFT80(TIM1)-FNFT60(TIM1)+FNFT100(TIM2)-FNFT80(TIM2)
940 VM=FNFP(PL2)+((FNDS(PLOD),DT)+FNDS(PL1,DT)+FNDS(PL2,TIM2))^A)
950 VE=FNDE(PLOD,TIME)+FNDE(PL1,TIM1)-FNDE(PLOD,TIM1)+FNDE(PL2,TIM2)-FNDE(PL1,TI
M2)
960 EV=FNVE(PLOD,TIME)+FNVE(PL1,TIM1)-FNVE(PLOD,TIM1)+FNVE(PL2,TIM2)-FNVE(PL1,TI
M2)
970 VV=FNVG(PL2)+((FNVH(PLOD,DT)+FNVH(PL1,DT)+FNVH(PL2,TIM2))^A)
980 SME=VE+VM
990 SV1=EV+VV
1000 SV2=EV+VM
1010 SV3=VE+VV
1020 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
1030 NEXT I
1040 REM "Compute the creep slip under the forth step of loads."
1050 PT=TIME
1060 PL3=PL2+DP
1070 FOR I=1 TO N
1080 TIME=PT+T(I)
1090 TIM1=TIME-DT
1100 TIM2=TIME-2*DT
1110 TIM3=TIME-3*DT
1120 A5F=FNFT60(TIME)+FNFT80(TIM1)-FNFT60(TIM1)+FNFT100(TIM2)-FNFT80(TIM2)
1130 S5F=A5F+FNFT120(TIM3)-FNFT100(TIM3)
1140 VM=FNFP(PL3)+((FNDS(PLOD,DT)+FNDS(PL1,DT)+FNDS(PL2,DT)+FNDS(PL3,TIM3))^A)
1150 VE=FNDE(PLOD,TIME)+FNDE(PL1,TIM1)-FNDE(PLOD,TIM1)+FNDE(PL2,TIM2)-FNDE(PL1,T
IM2)+FNDE(PL3,TIM3)-FNDE(PL2,TIM3)
1160 AEV=FNVE(PLOD,TIME)+FNVE(PL1,TIM1)-FNVE(PLOD,TIM1)+FNVE(PL2,TIM2)-FNVE(PL1,
TIM2)
1170 EV=AEV+FNVE(PL3,TIM3)-FNVE(PL2,TIM3)
1180 VV=FNVG(PL3)+((FNVH(PLOD,DT)+FNVH(PL1,DT)+FNVH(PL2,DT)+FNVH(PL3,TIM3))^A)
1190 SME=VE+VM
1200 SV1=EV+VV
1210 SV2=EV+VM
1220 SV3=VE+VV
1230 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
1240 NEXT I
1250 REM "Compute the creep slip under the [illegible] step of loads."
1260 PT=TIME
1270 S1=S5F
1280 S2=SME
1290 S3=SV1
1300 S4=SV2
1310 S5=SV3
```

```
1320 FOR I=1 TO N
1330 TIME=PT+T(I)
1340 TIM4=TIME-4*DT
1350 S5F=S1-FNFB(TIM4)*.765
1360 VE=FNDE(DP,TIM4)*.765
1370 VED=FNVE(PLOD,TIM4)*.765/3!
1380 SME=S2-VE
1390 SV1=S3-VED
1400 SV2=S4-VED
1410 SV3=S5-VE
1420 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
1430 NEXT I
1440 REM "Compute the creep slip under the sixth step of loads."
1450 PT=TIME
1460 DDP=2!*DP
1470 FOR I=1 TO N
1480 TIME=PT+T(I)
1490 TIM4=TIME-4*DT
1500 TIM5=TIME-5*DT
1510 S5F=S1-(FNFB(TIM4)+FNFB(TIM5))*.765
1520 VE=(FNDE(DP,TIM4)+FNDE(DDP,TIM5)-FNDE(DP,TIM5))*.765
1530 VED=(FNVE(PLOD,TIM4)+FNVE(PLOD,TIM5))*.765/3!
1540 SME=S2-VE
1550 SV1=S3-VED
1560 SV2=S4-VED
1570 SV3=S5-VE
1580 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
1590 NEXT I
1600 REM "Compute the creep slip under the seventh step of loads."
1610 PT=TIME
1620 FOR I=1 TO N
1630 TIME=PT+T(I)
1640 TIM4=TIME-4*DT
1650 TIM5=TIME-5*DT
1660 TIM6=TIME-6*DT
1670 S5F=S1-(FNFB(TIM4)+FNFB(TIM5)+FNFB(TIM6))*.765
1680 VE=(FNDE(DP,TIM4)+FNDE(DDP,TIM5)-FNDE(DP,TIM5)+FNDE(PLOD,TIM6)-FNDE(DDP,TIM
6))*.765
1690 VED=(FNVE(PLOD,TIM4)+FNVE(PLOD,TIM5)+FNVE(PLOD,TIM6))*.765/3!
1700 SME=S2-VE
1710 SV1=S3-VED
1720 SV2=S4-VED
1730 SV3=S5-VE
1740 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
1750 NEXT I
1760 REM "Compute the recovery after complete unloading."
1770 PT=TIME
1780 FOR I=1 TO N
1790 TIME=PT+T(I)
1800 TIM4=TIME-4*DT
1810 TIM5=TIME-5*DT
1820 TIM6=TIME-6*DT
1830 TIM7=TIME-7*DT
1840 S5F=S1-(FNFB(TIM4)+FNFB(TIM5)+FNFB(TIM6)+FNFF120(TIM7)-FNFF60(TIM7))*.765
1850 VE=(FNDE(DP,TIM4)+FNDE(DDP,TIM5)-FNDE(DP,TIM5)+FNDE(PLOD,TIM6)-FNDE(DDP,TIM
6)+FNDE(PL3,TIM7)-FNDE(PLOD,TIM7))*.765
1860 VED=((FNVE(PLOD,TIM4)+FNVE(PLOD,TIM5)+FNVE(PLOD,TIM6))/3!+FNVE(PL3,TIM7)-FN
VE(PLOD,TIM7))*.765
1870 SME=S2-VE
1880 SV1=S3-VED
1890 SV2=S4-VED
1900 SV3=S5-VE
1910 LPRINT USING "+#####.##";TIME,S5F,SME,SV1,SV2,SV3
1920 IF TIME>23040 THEN GOTO 1940
1930 NEXT I
1940 TIME=TIME+HT
1950 IF TIME>30240 THEN GOTO 1970
1960 GOTO 1800
1970 END
```

Appendix C.7. Program for the creep predicted by Solution 3 under load function 7.

```
100 REM "Creep slip by Solution 2 under loading function 7."
110 REM "Load function is expressed as a continuous function which is sum of
        Heaviside unit step functions."
120 OPEN "A:CRVARY.D07" FOR OUTPUT AS #1
130 DEFINT I-N
140 DIM T(12)
150 N=12
160 FOR I=1 TO N
170 INPUT "Time interval";T(I)
180 NEXT I
190 REM "Input some constants."
200 DI=2880!
210 HT=1440!
220 DP=20!
230 DDP=40!
240 PT=0!
250 PLOD=60!
260 A=.35
270 CF=.765
280 REM "Define some functions for M5-E."
290 EN=-.0003254
300 E2=.003812
310 AN=5.07464E-05
320 BN=3.1916E-10
330 CN=4.8121E-10
340 AM=2.28375
350 BM=4.3212
360 CM=4.9026
370 DEF FNDI(PL)=AN*(PL^AM)
380 DEF FNDE(PL)=AN*AM*((PL+.5*DP)^(AM-1!))*DP
390 DEF FNDD(PL,TM)=PL*EXP(EN*TM)
400 DEF FNDV(PL,TM)=BN*(PL^BM)*(TM^A)
410 DEF FNDP(PL)=CN*CM*((PL+.5*DP)^(CM-1!))*DP
420 PI=CN*(PLOD^CM)
430 REM "Define some functions for V-VE1."
440 DEF FNF1(TM)=(-7.34+1.102*(TM^.3)-9.415301*(TM^.17)+9.0455*(TM^.12))*.001
450 DEF FNF2(TM)=(.272-.0202*(TM^.3)+.23538*(TM^.17)-.2412*(TM^.12))*.001
460 DEF FNF3(TM)=(-.00028+.000092*(TM^.3)-.001308*(TM^.17)+.001508*(TM^.12))*.00
1
470 DEF FNF4(TM)=(57.74367+.025453*(TM^A))*.001
480 DEF FNF5(TM)=(-1.80786-.019755*(TM^A))*.001
490 DEF FNF6(TM)=(.015399+.00034*(TM^A))*.001
500 DEF FNVE(PL,TM)=FNF1(TM)*PL+FNF2(TM)*(PL^2)+FNF3(TM)*(PL^3)
510 DEF FNVD(PL,TM)=2!*FNF2(TM)*PL*DP+3!*FNF3(TM)*(PL+DP)*DP
520 DEF FNVV(PL,TM)=(.025453*PL-.019755*(PL^2)+.00034*(PL^3))*(TM^A)*.001
530 DEF FNVG(PL)=(57.74367*PL-1.80786*(PL^2)+.015399*(PL^3))*.001
540 DEF FNVP(PL)=(-2!*1.80786*PL*DP+3!*.015399*PL*(PL+DP)*DP)*.001
550 REM "Compute the creep slip under the first step of loads."
560 FOR I=1 TO N
570 TIME=PT+T(I)
580 VM=FNDV(PLOD,TIME)+PI
590 VE=FNDI(PLOD)+PLOD*E2-FNDD(PLOD,TIME)*E2
600 EV=FNVE(PLOD,TIME)
610 VV=FNVV(PLOD,TIME)+FNVG(PLOD)
620 SME=VE+VM
630 SV1=EV+VV
640 SV2=EV+VM
650 SV3=VE+VV
660 PRINT #1, USING "######.##";TIME;SME;SV1;SV2;SV3
670 NEXT I
```

```
680 REM "Compute the creep slip under the second step of loads."
690 PT=TIME
700 PL1=PLOD+DP
710 FOR I=1 TO N
720 TIME=PT+T(I)
730 TIM1=TIME-DT
740 VM=PI+FNDP(PLOD)+FNDV(PLOD,DT)+FNDV(PL1,TIM1)
750 VE=FNDI(PLOD)+FNDE(PLOD)+PL1*E2-(FNDD(PLOD,TIME)+FNDD(DP,TIM1))*E2
760 EV=FNVE(PLOD,TIME)+FNVE(DP,TIM1)+FNVD(PLOD,TIM1)
770 VV=FNVV(PLOD,DT)+FNVV(PL1,TIM1)+FNVG(PLOD)+FNVG(DP)+FNVP(PLOD)
780 SME=VE+VM
790 SV1=EV+VV
800 SV2=EV+VM
810 SV3=VE+VV
820 PRINT #1, USING "######.##";TIME;SME;SV1;SV2;SV3
830 NEXT I
840 REM "Compute the creep slip under the third step of loads."
850 PT=TIME
860 PL2=PL1+DP
870 FOR I=1 TO N
880 TIME=PT+T(I)
890 TIM1=TIME-DT
900 TIM2=TIME-2*DT
910 VM=PI+FNDP(PLOD)+FNDP(PL1)+FNDV(PLOD,DT)+FNDV(PL1,DT)+FNDV(PL2,TIM2)
920 VE=FNDI(PLOD)+FNDE(PLOD)+FNDE(PL1)+PL2*E2-(FNDD(PLOD,TIME)+FNDD(DP,TIM1)+FND
D(DP,TIM2))*E2
930 EV=FNVE(PLOD,TIME)+FNVE(DP,TIM1)+FNVE(DP,TIM2)+FNVD(PLOD,TIM1)+FNVD(PL1,TIM2
)
940 VV=FNVV(PLOD,DT)+FNVV(PL1,DT)+FNVV(PL2,TIM2)+FNVG(PLOD)+FNVG(DP)*2!+FNVP(PLO
D)+FNVP(PL1)
950 SME=VE+VM
960 SV1=EV+VV
970 SV2=EV+VM
980 SV3=VE+VV
990 PRINT #1, USING "######.##";TIME;SME;SV1;SV2;SV3
1000 NEXT I
1010 REM "Compute the creep slip under the forth step of loads."
1020 PT=TIME
1030 PL3=PL2+DP
1040 FOR I=1 TO N
1050 TIME=PT+T(I)
1060 TIM1=TIME-DT
1070 TIM2=TIME-2*DT
1080 TIM3=TIME-3*DT
1090 VM=PI+FNDP(PLOD)+FNDP(PL1)+FNDP(PL2)+FNDV(PLOD,DT)+FNDV(PL1,DT)+FNDV(PL2,DT
)+FNDV(PL3,TIM3)
1100 VE=FNDI(PLOD)+FNDE(PLOD)+FNDE(PL1)+FNDE(PL2)+PL3*E2-(FNDD(PLOD,TIME)+FNDD(D
P,TIM1)+FNDD(DP,TIM2)+FNDD(DP,TIM3))*E2
1110 EV=FNVE(PLOD,TIME)+FNVE(DP,TIM1)+FNVE(DP,TIM2)+FNVE(DP,TIM3)+FNVD(PLOD,TIM1
)+FNVD(PL1,TIM2)+FNVD(PL2,TIM3)
1120 VV=FNVV(PLOD,DT)+FNVV(PL1,DT)+FNVV(PL2,DT)+FNVV(PL3,TIM3)+FNVG(PLOD)+FNVG(D
P)*3!+FNVP(PLOD)+FNVP(PL1)+FNVP(PL2)
1130 SME=VE+VM
1140 SV1=EV+VV
1150 SV2=EV+VM
1160 SV3=VE+VV
1170 PRINT #1, USING "######.##";TIME;SME;SV1;SV2;SV3
1180 NEXT I
1190 REM "Compute the creep slip under the fifth step of loads."
1200 PT=TIME
1210 S2=SME
1220 S3=SV1
1230 S4=SV2
1240 S5=SV3
1250 FOR I=1 TO N
1260 TIME=PT+T(I)
1270 TIM4=TIME-4*DT
```

```
1280 VE=FNDI(DP)+DP*E2-FNDD(DP,TIM4)*E2
1290 EV=FNVE(PLOD,TIM4)/3!
1300 SME=S2-VE
1310 SV1=S3-EV
1320 SV2=S4-EV
1330 SV3=S5-VE
1340 PRINT #1, USING "######.##";TIME;SME;SV1;SV2;SV3
1350 NEXT I
1360 REM "Compute the creep slip under the sixth step of loads."
1370 PT=TIME
1380 DDP=2!*DP
1390 FOR I=1 TO N
1400 TIME=PT+T(I)
1410 TIM4=TIME-4*DT
1420 TIM5=TIME-5*DT
1430 VE=FNDI(DP)+FNDE(DP)+DDP*E2-(FNDD(DP,TIM4)+FNDD(DP,TIM5))*E2
1440 EV=(FNVE(PLOD,TIM4)+FNVE(PLOD,TIM5)+FNVD(PLOD,TIM5))/3!
1450 SME=S2-VE
1460 SV1=S3-EV
1470 SV2=S4-EV
1480 SV3=S5-VE
1490 PRINT #1, USING "######.##";TIME;SME;SV1;SV2;SV3
1500 NEXT I
1510 REM "Compute the creep slip under the seventh step of loads."
1520 PT=TIME
1530 FOR I=1 TO N
1540 TIME=PT+T(I)
1550 TIM4=TIME-4*DT
1560 TIM5=TIME-5*DT
1570 TIM6=TIME-6*DT
1580 VE=FNDI(DP)+FNDE(DP)+FNDE(DDP)+PLOD*E2-(FNDD(DP,TIM4)+FNDD(DP,TIM5)+FNDD(DP
,TIM6))*E2
1590 EV=(FNVE(PLOD,TIM4)+FNVE(PLOD,TIM5)+FNVE(PLOD,TIM6)+FNVD(PLOD,TIM5)+FNVD(PL
OD,TIM6)*2!)/3!
1600 SME=S2-VE
1610 SV1=S3-EV
1620 SV2=S4-EV
1630 SV3=S5-VE
1640 PRINT #1, USING "######.##";TIME;SME;SV1;SV2;SV3
1650 NEXT I
1660 REM "Compute the recovery after complete unloading."
1670 PT=TIME
1680 DTP=PLOD+DP
1690 FOR I=1 TO N
1700 TIME=PT+T(I)
1710 TIM4=TIME-4*DT
1720 TIM5=TIME-5*DT
1730 TIM6=TIME-6*DT
1740 TIM7=TIME-7*DT
1750 VE=(FNDI(DP)+FNDE(DP)+FNDE(DDP)+FNDE(PL1)*3!+PL3*E2-(FNDD(DP,TIM4)+FNDD(DP,
TIM5)+FNDD(DP,TIM6)+FNDD(PLOD,TIM7))*E2)*CF
1760 EV=((FNVE(PLOD,TIM4)+FNVE(PLOD,TIM5)+FNVE(PLOD,TIM6)+FNVE(PLOD,TIM7)*3!+FNV
D(PLOD,TIM5)+FNVD(PLOD,TIM6)*2!)/3!+2!*FNF2(TIM7)*PLOD*PLOD+3!*FNF3(TIM7)*PLOD*P
L3*PLOD)*CF
1770 SME=S2-VE
1780 SV1=S3-EV
1790 SV2=S4-EV
1800 SV3=S5-VE
1810 PRINT #1, USING "######.##";TIME;SME;SV1;SV2;SV3
1820 IF TIME>23040 THEN GOTO 1840
1830 NEXT I
1840 TIME=TIME+HT
1850 IF TIME>30240 THEN GOTO 1870
1860 GOTO 1710
1870 CLOSE #1
1880 END
```

Appendix C.8. Program for the creep predicted by Solution 4 under load function 7.

```
100 REM "Creep slip by Solution 2 under load function 7."
110 REM "Load function 7 is approximated by a Fourier series."
120 DEFINT I-N
130 OPEN "CRVARY.F07" FOR OUTPUT AS #1
140 INPUT "No. of terms in the Fourier series";N
150 DIM B(100),T(20)
160 FOR I=1 TO 12
170 INPUT "Time interval";T(I)
180 NEXT I
190 DT=2880!
200 TP=40320!
210 PHI=3.14159
220 P0=0!
230 P1=20!
240 P2=40!
250 P3=60!
260 ST1=0!
270 ST2=0!
280 ST4=0!
290 ST8=0!
300 SS1=0!
310 SS2=0!
320 SS4=0!
330 SS8=0!
340 ID=0
350 UME=0!
360 UVE=0!
370 A0=60!
380 PL=60!
390 REM "Get coefficients of the Fourier series."
400 GOSUB 1360
410 FOR I=1 TO N
420 LPRINT "    A" TAB(6) I TAB(10) "=" TAB(12) B(I)
430 NEXT I
440 REM "Get the creep slip under the Fourier series load function."
450 REM "Give some parameters for theoretical models."
460 TIME=0!
470 TIM2=0!
480 B1=2.28375
490 B11=1.28375
500 B2=4.3212
510 B3=.35
520 B4=4.9026
530 AN=5.07464E-05
540 BN=3.191604E-10
550 CN=4.812104E-10
560 ER=.003812
570 EN=-.0003259
580 A1=.3
590 A2=.17
600 A3=.12
610 A4=1!/.35
620 F=.001
630 CF1=.765
640 REM "Define some functions.
650 DEF FNF1(X)=(-7.34+1.102*(X^A1)-9.415301*(X^A2)+9.0455*(X^A3))*F
660 DEF FNF2(X)=(.272-.0202*(X^A1)+.2354*(X^A2)-.2412*(X^A3))*F
670 DEF FNF3(X)=(-.0003+.00009*(X^A1)-.0013*(X^A2)+.0015*(X^A3))*F
680 DEF FNFG(X)=(57.74367*X-1.80786*(X^2)+.0154*(X^3))*F
690 DEF FNFH(X,Y)=(.02545*X-.01976*(X^2)+.00034*(X^3))*((Y^B3)*.062)*F
700 SI=AN*(A0^B1)
710 U = -7.34*A0+.272*(A0^2)-.0003*(A0^3))*F
```

```
720 FOR L=1 TO 12
730 TIN=TIME
740 TIME=TIM2+T(L)
750 IF ID=2 THEN GOTO 950
760 IF T(L)<=30! THEN DH=1! ELSE DH=10!
770 IF T(L)=>480! THEN DH=40!
780 REM "Evaluate the integrals."
790 IF ID1=1 THEN GOTO 800 ELSE GOTO 820
800 TIM1=TIM2
810 TIM2=10080!
820 GOSUB 1590
830 IF ID1=1 THEN TIM2=TIM1
840 REM "Compute the creep slip."
850 IF ID=1 THEN GOTO 1030
860 IF TIME=0! THEN AFV=CN*(PLL^B4) ELSE AFV=ST4+CN*(PLL^B4)
870 SE=AN*B1*ST1+ER*(ST2-ST3)+SI
880 VE=ST5+2!*ST6+3!*ST7+VI
890 VV=FNFG(PLL)+ST8
900 SME=SE+AFV
910 SV1=VE+VV
920 SV2=VE+AFV
930 SV3=SE+VV
940 GOTO 1090
950 TIM3=TIME-20160!
960 UVE=FNF1(TIM3)*A0+FNF2(TIM3)*((2!*A0)^2-A0^2)+FNF3(TIM3)*((2!*A0)^3-A0^3)
970 UME=AN*((2!*A0)^B1-A0^B1)+ER*A0*(1!-EXP(EN*TIM3))
980 SME=S1-UME*CF1
990 SV1=S2-UVE*CF1
1000 SV2=S3-UVE*CF1
1010 SV3=S4-UME*CF1
1020 GOTO 1090
1030 SE=AN*B1*ST1+ER*(ST2-ST3)
1040 VE=ST5+2!*ST6+3!*ST7
1050 SME=S1-SE
1060 SV1=S2-VE
1070 SV2=S3-VE
1080 SV3=S4-SE
1090 PRINT #1, USING "######.##";TIME;SME;SV1;SV2;SV3
1100 IF TIME=10080! THEN GOTO 1110 ELSE GOTO 1210
1110 S1=SME
1120 S2=SV1
1130 S3=SV2
1140 S4=SV3
1150 SS1=0!
1160 SS2=0!
1170 SS3=0!
1180 SS4=0!
1190 ID1=1
1200 PM=PL
1210 IF TIME=>10080! THEN ID=1
1220 IF TIME=>20160! THEN ID=2
1230 NEXT L
1240 IF TIME =>30240 THEN GOTO 1340
1250 TIM2=TIME
1260 IF ID1=1 THEN ID1=2
1270 IF TIME=20160! THEN GOTO 1290
1280 GOTO 720
1290 S1=SME
1300 S2=SV1
1310 S3=SV2
1320 S4=SV3
1330 GOTO 720
1340 CLOSE #1
1350 END
```

```
1360 REM "This subroutine computes coefficients of the Fourier series."
1370 FOR I=1 TO N
1380 C1=COS(I*PHI/7!)
1390 C2=COS(2!*I*PHI/7!)
1400 C3=COS(3!*I*PHI/7!)
1410 C4=COS(4!*I*PHI/7!)
1420 C5=COS(5!*I*PHI/7!)
1430 C6=COS(6!*I*PHI/7!)
1440 B(I)=(P1*(C2-C5+C6-C1)+P2*(C3-C4+C5-C2)+P3*(C4-C3))*(-2!)/(I*PHI)
1450 NEXT I
1460 RETURN
1470 REM "This subroutine computes load at given time from the Fourier series."
1480 PL=0!
1490 PD=0!
1500 FOR I=1 TO N
1510 CPT=2!*I*PHI/TP
1520 SC=SIN(CPT*TN)
1530 CC=COS(CPT*TN)
1540 PL=PL+B(I)*SC
1550 PD=PD+B(I)*CC*CPT
1560 NEXT I
1570 PL=PL+A0
1580 RETURN
1590 REM "This subroutine evaluates integral by the trapezoidal rule."
1600 TN=TIM2
1610 ST1=0!
1620 ST2=0!
1630 ST3=0!
1640 ST5=0!
1650 ST6=0!
1660 ST7=0!
1670 TT=TIME-TN
1680 REM "Compute load at given time from the Fourier series."
1690 PK=PL
1700 GOSUB 1470
1710 PA=(PK+PL)/2!
1720 IF ID=0 THEN GOTO 1760
1730 PL=PM-PL
1740 IF PL<0! THEN PL=0!
1750 PD=-PD
1760 IF TN=TIM2 THEN GOTO 1770 ELSE GOTO 1790
1770 FT0=PL
1780 FTI=PD
1790 IF TN=>TIN THEN GOTO 1860
1800 ST3=ST3+EXP(EN*TT)*PD
1810 ST5=ST5+FNF1(TT)*PD
1820 ST6=ST6+FNF2(TT)*PL*PD
1830 ST7=ST7+FNF3(TT)*(PL^2)*PD
1840 TN=TN+DH
1850 GOTO 1670
1860 IF TN=TIN THEN GOTO 1870 ELSE GOTO 1890
1870 FP0=PL
1880 FPI=PD
1890 ST1=ST1+(PL^B11)*PD
1900 ST2=ST2+PD
1910 ST3=ST3+EXP(EN*TT)*PD
1920 ST5=ST5+FNF1(TT)*PD
1930 ST6=ST6+FNF2(TT)*PL*PD
1940 ST7=ST7+FNF3(TT)*(PL^2)*PD
1950 IF TN=TIN THEN GOTO 1990 ELSE GOTO 1960
```

```
1960 IF ID=1 THEN GOTO 1990 ELSE GOTO 1970
1970 ST4=ST4+BN*(PA^B2)*((DH^B3)*.062)
1980 ST8=ST8+FNFH(PA,DH)
1990 TN=TN+DH
2000 IF TN>TIME THEN GOTO 2010 ELSE GOTO 1670
2010 TS=TIME-TIN
2020 ST1=DH*(ST1-((FPO^B11)*FPI+(PL^B11)*PD)/2!)+SS1
2030 ST2=DH*(ST2-(FPI+PD)/2!)+SS2
2040 ST3=DH*(ST3-(EXP(EN*TS)*FTI+EXP(EN*TT)*PD)/2!)
2050 ST5=DH*(ST5-(FNF1(TS)*FTI+FNF1(TT)*PD)/2!)
2060 ST6=DH*(ST6-(FNF2(TS)*FTO*FTI+FNF2(TT)*PL*PD)/2!)
2070 ST7=DH*(ST7-(FNF3(TS)*(FTO^2)*FTI+FNF3(TT)*(PL^2)*PD)/2!)
2080 SS1=ST1
2090 SS2=ST2
2100 SS3=ST3
2110 SS4=ST4
2120 SS5=ST5
2130 SS6=ST6
2140 SS7=ST7
2150 SS8=ST8
2160 PLL=PL
2170 IF ID=1 THEN PLL=PM-PL
2180 PDD=PD
2190 IF ID=1 THEN PDD=-PD
2200 IF TIM2=0! THEN GOTO 2550
2210 IF TIM2=10080! THEN GOTO 2550
2220 TN=0!
2230 IF ID=1 THEN TN=10080!
2240 DH=40!
2250 ST3=0!
2260 ST5=0!
2270 ST6=0!
2280 ST7=0!
2290 TT=TIME-TN
2300 GOSUB 1470
2310 IF ID=0 THEN GOTO 2350
2320 PL=PM-PL
2330 IF PL<0! THEN PL=0!
2340 PD=-PD
2350 IF TN=0! THEN GOTO 2370
2360 IF TN=10080! THEN GOTO 2370 ELSE GOTO 2390
2370 FTO=PL
2380 FTI=PD
2390 ST3=ST3+EXP(EN*TT)*PD
2400 ST5=ST5+FNF1(TT)*PD
2410 ST6=ST6+FNF2(TT)*PL*PD
2420 ST7=ST7+FNF3(TT)*(PL^2)*PD
2430 TN=TN+DH
2440 IF TN>TIM2 THEN GOTO 2460
2450 GOTO 2290
2460 IF ID=0 THEN TS=TIME ELSE TS=TIME-10080!
2470 ST3=DH*(ST3-(EXP(EN*TS)*FTI+EXP(EN*TT)*PD)/2!)
2480 ST5=DH*(ST5-(FNF1(TS)*FTI+FNF1(TT)*PD)/2!)
2490 ST6=DH*(ST6-(FNF2(TS)*FTO*FTI+FNF2(TT)*PL*PD)/2!)
2500 ST7=DH*(ST7-(FNF3(TS)*(FTO^2)*FTI+FNF3(TT)*(PL^2)*PD)/2!)
2510 ST3=ST3+SS3
2520 ST5=ST5+SS5
2530 ST6=ST6+SS6
2540 ST7=ST7+SS7
2550 RETURN
```