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Prediction of Forces during Drilling of Composite Laminates Using Artificial Neural Network: A New Approach

Drilling of fiber-reinforced plastics (FRP's) is an inevitable machining operation, because it facilitates assembly of several components by means of mechanical fastening. But, drilling of FRP leads to delamination which results in reduced life and efficiency of the FRP part. The delamination that induced during drilling is directly affected by the thrust force and torque. In the present research endeavour, four different types of drill point geometries have been used for making of holes in two different types of composite laminates. The drilling of composite laminate has been conducted at three different levels of spindle speed and feed rate. A new artificial neural network (ANN) approach has been proposed to predict the drilling-induced thrust force and torque. The values of thrust force and torque predicted by the proposed ANN models are in close agreement with the experimental values.

Keywords: *Composites, laminates, drilling, thrust force, torque, artificial neural network.*

1. INTRODUCTION

The use of FRP's has increased to a great extent over the last few years due to their exceptional physical and mechanical properties such as, high strength to weight ratio, high impact resistance, excellent corrosion resistance and ease of manufacturing [1, 2]. The use of these materials has grown in the field of aerospace, aircrafts, automobiles etc. where they have replaced many conventional materials. Generally, primary production of FRP leads to near-net shape product. But, sometimes the product has to be made in parts due to the intricacy in the product design. Thus drilling of FRP's is inevitable as it facilitates the assembly of FRP parts by means of mechanical fastening [3-5]. However, the damage-induced during drilling is one of the major challenges that cannot be completely avoided. It has been found that the thrust force and torque produced during drilling directly affect the drilling-induced damage [6, 7]. It has been stated that high cutting speed and feed rate results in higher values of thrust force and delamination [8, 9]. El-Sonbaty et al. [10] found that both the thrust force and torque increases with drill diameter and feed rate. The effect of tool geometry on the thrust force and torque has also been investigated [11]. The thrust force generated during drilling was identified as the root cause for the occurrence of delamination. Hence, the models for critical thrust force were developed using linear elastic fracture mechanics [12, 13]. A near-linear relationship was found between the delamination factor and average thrust force during drilling of carbon fiber-reinforced plastic (CFRP) composite [14]. On the contrary, it has been reported that it is not only the thrust force that influences the drilling-induced damage; torque also plays significant role [15].

A number of predictive models were developed to predict the critical thrust force during drilling of FRP's [16-18]. Hocheng and Tsao [19, 20] proposed an analytical approach to predict the critical thrust force for different drill point geometries. It was found that the core drill offers the highest critical feed rate followed by candle stick drill, saw drill, step drill and the traditional twist drill. The mathematical models to predict the thrust force, torque and delamination during drilling of composite laminates have also been developed [21, 22]. Various predictive models have been developed, but the need of the hour is to develop more generic models is an area of paramount importance. Hence, ANN predictive models are used to solve complex and non-linear problems, thus saving time and cost of conducting the experiments. Mishra et al. [23] developed an ANN model to predict the drilling-induced damage during drilling of glass fiber-reinforced plastic (GFRP) composites. Latha and Senthilkumar [24] developed a neural network model based on back propagation algorithm to predict the delamination factor during the drilling of GFRP laminates. Athijayamani et al. [25] developed an ANN and a regression model to predict the thrust force and torque during drilling of natural fiber-reinforced hybrid composites. The results revealed that the ANN models are better than the regression models in predicting thrust force and torque. In order to develop the ANN models, the number of neurons in hidden layer and the values of learning rate and momentum factor are found using hit and trial method. Therefore, in the present research initiative, a new ANN approach in context of drilling of composite laminates has been suggested to find the number of neurons in the hidden layer and the values of momentum factor and learning rate.

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2. EXPERIMENTAL DETAILS

2.1 Preparation of composite specimen

Composite laminates of 4 mm thickness were prepared using hand lay-up technique. Araldite LY556 along with hardener HY 951was used as resin. Two different types of GFRP laminates were fabricated, namely, unidirectional GFRP and [(0/90)/0]s GFRP laminates.

2.2 Drilling setup

Drilling of the developed GFRP laminates was conducted on a radial drilling machine using solid carbide drills. Four different types of drill point geometries (4-facet, 8-facet, Jo and parabolic drill) of 4 and 8mm diameter was used for making of holes in the developed laminates. Three different levels of spindle speed (750, 1500 and 2250 rpm) and feed rate (10, 15 and 20 mm/min) were used to conduct the drilling experiments.

2.3 Measurements

Four-component piezoelectric drill dynamometer (Make: Kistler, Type: 9272) was used to record the thrust force and torque signals. The dynamometer is attached to the charge meters (Make: Kistler, Type: 5015) and the output of the charge meters is supplied as an input to the personal computer using an analogue/ digital card.

3. ARTIFICIAL NEURAL NETWORKS: AN INTRODUCTION

An ANN is a computational technique which is inspired by biological neural network system. An ANN is a fast, efficient, accurate and cost effective process-modeling tool. The working principle of an ANN is similar to the human brain. The neurons transmit the information from one to another through synaptic weights of the links [26]. A general symbol of neuron consisting of processing nodes and synaptic connections is shown in Figure 1.

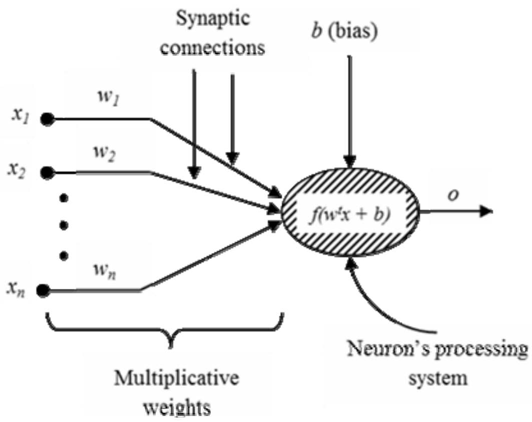

Figure 1. General symbol of an artificial neuron

The weight (w_{ij}) associated with the link is multiplied with the input signal (x_i) transmitted through that link to calculate the weighted function. The net input function is calculated by adding the weighted function with a bias b. The bias is much like a weight, except that it has a constant input of 1. The net activation input for j^{th} neuron is given by:

$$net_j = \sum_{i=1}^{n} w_{ij} x_i + b \tag{1}$$

Where, w_{ij} is the weight of link connecting neuron j to i, and x_i is the input of i^{th} neuron.

Finally, the net input is passed through the transfer function f, which produces the scalar output (o). The neuron output is given by;

$$o = f\left(\sum_{i=1}^{n} w_{ij} x_i + b\right) \tag{2}$$

The function f = (net_j) is referred to as activation function. There are different activation functions like, uni-polar sigmoid function, bi-polar sigmoid function, pure linear etc. The ANN can be trained from a set of training data to find the solution of complex non-linear problems in which the output parameter is depends on one or more input parameters. The predictive models developed on the basis of ANNs are more generic in nature. Moreover, any number of input parameters can be considered without knowing the interrelationship among them. The network is built directly from the experimental data by its self-organizing capabilities. ANN can be easily applied to solve the problems difficult to solve or cannot be solved using mathematical techniques [27].

The working of a neural network is shown in Figure 2. Conceptually, the working principle of a neural network is similar to the mechanical feedback servo control system. A neural network can be trained to perform a particular function by adjusting the values of the connections (weights) between the elements. Generally, neural networks are adjusted, or trained, so that a particular input leads to a specific target output. The difference between the output and the target is called error. The training of the network is stopped when the network output matches the target, or reached the maximum number of iterations, or reached the minimum acceptable error value.

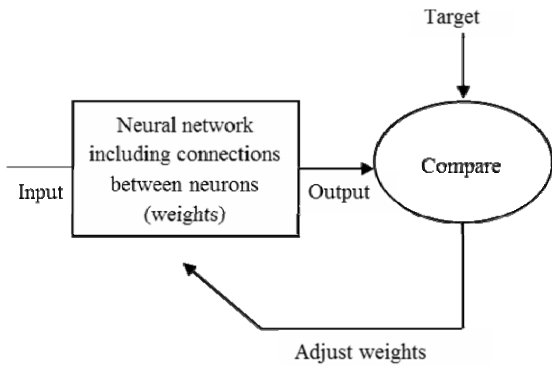

Figure 2. Working principle of a neural network

4. APPLICATION OF ARTIFICIAL NEURAL NETWORKS: A NEW APPROACH

Most of the engineering problems are modeled using neural networks with one hidden layer. Generally, hit and trial method is used to find the number of neurons in the hidden layer and the values of momentum factor and learning rate. Moreover, the ANN is trained for minimum error and the resulting ANN is tested using the test data. In the present research initiative, it is proposed that the ANN should be trained for three different approaches to fit the requirements.

4.1 First approach

The training error should be reduced and then the testing should be done. The training error is given by:

$$E = \frac{1}{2}\sum_{p=1}^{p}\left(\text{target}_p - \text{output}_p\right)^2 \tag{3}$$

where, p is the total number of training pairs.

4.2 Second approach

The overall mean percentage error during the testing of test data should be minimized. The mean percentage error (MPE) is given by;

$$MPE = \sum_{q=1}^{Q}\frac{\left(\text{target}_q - \text{output}_q\right)^2}{\text{target}_q}\times 100 \tag{4}$$

where, Q is the total number of test data pairs.

4.3 Third approach

The maximum percentage error *Max_PE* should be reduced during the testing of test data pairs.

$$Max\_PE = maximum\ \{error\_percent\} \tag{5}$$

where, *{error_percent}* is a set consisting of percentage error for each test data input.
Here, *{error_percent}* = $\{p_1, p_2 \ldots p_q\}$ and

$$P_q = \frac{\left(\text{target}_q - \text{output}_q\right)}{\text{target}_q}\times 100\text{, for } q = 1, 2, 3 \ldots Q$$

When a set of new data is presented to the trained neural network the second and third approach is very important in order to reduce the error. It is sometimes difficult to train and design the ANN for a particular problem as it takes much time to decide the number of neurons in hidden layer and the values of learning rate and momentum factor. Hence, a new approach has been proposed to design the ANN with one hidden layer. The training phase is based on error back propagation algorithm. The proposed procedure helps to find the number of neurons in a hidden layer and the values of momentum factor and learning rate for three different approaches. The principal steps involved in this new ANN approach are as follows:

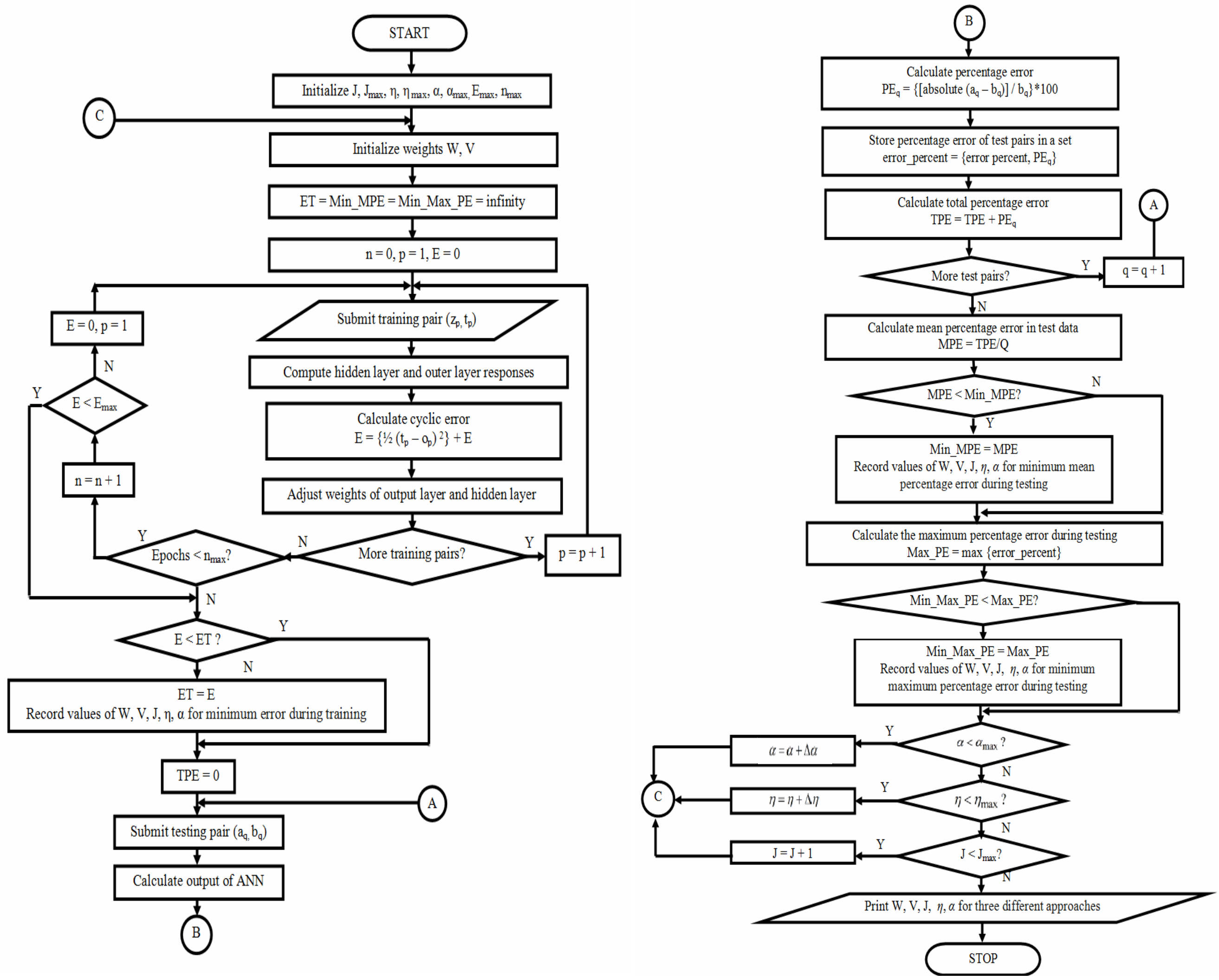

Figure 3. New ANN approach

Step 1: Initialize the number of neurons in hidden layer (J), learning rate (η), momentum factor (α), maximum number of epochs $\eta_{\max}$ and the minimum acceptable error $E_{\max}$.
Step 2: Specify the maximum number of neurons in hidden layer (J_{max}), the maximum learning rate ($\eta_{\max}$) and the maximum momentum factor $\alpha_{\max}$.
Step 3: Initially, set minimum error during training (ET), minimum mean percentage error during testing (Min_MPE) and minimu-maximum percentage error during testing (Min_Max_PE) is equal to infinity.
Step 4: Train the ANN for J neurons in hidden layer, learning rate (η) and momentum factor (α) till the network reached the maximum number of epochs ($\eta_{\max}$) or the minimum acceptable error value ($E_{\max}$). Compute the training error.
Step 5: If, training error is less than ET than set ET as training error and record J, η and α along with weights on connection to the hidden layer V and weights on connections to the hidden layer of the output layer W.
Step 6: Calculate the outputs for the test data with the trained ANN.
Step 7: Calculate the MPE and (Max_MPE) for the test data.
Step 8: If, $MPE < Min\_MPE$ then set $Min\_MPE = MPE$ and record J, η, α, V and W.
Step 9: If, $Max\_PE < Min\_Max\_PE$ then set $Min\_Max\_PE = Max\_PE$ and record J, η, α, V and W.
Step 10: If, $\alpha < \alpha_{\max}$ then $\alpha = \alpha + \Delta\alpha$ and go to Step 4 else go to Step 11.
Note: $\Delta\alpha$ is a small increment in the momentum factor.
Step 11: If, $\eta < \eta_{\max}$ then $\eta = \eta + \Delta\eta$ and go to Step 4 else go to Step 12.
Note: $\Delta\eta$ is a small increment in the learning rate.
Step 12: If, $J < J_{max}$ then $J = J+1$ and go to Step 4 else go to Step 13.
Step 13: Display the values recorded in Step 5, 8 and 9.

The new ANN approach is presented with the help of a flow chart as shown in Figure 3.

5. PREDICTIVE MODELLING OF THRUST FORCE AND TORQUE USING NEW ANN APPROACH

A code based on proposed algorithm was written in MATLAB. Three layer network architectures were used to predict the thrust force and torque as shown in Figure 4. The numbers of neurons in input layer is six. Five for the number of input variables (material, drill point geometry, drill diameter, feed rate and spindle speed) and one for bias. The modelling of thrust force and torque was carried out one at a time. Hence, the output layer consists of one neuron corresponding to one output variable. It is worthy to note that the training of the ANN for one output leads to less complexities and better results.

Assuming that the activation function used in the hidden and the outer layer is sigmoidal, the outputs of the hidden and outer layer were calculated using the following equation:

$$f(net) = \frac{1}{1+e^{-net}} \qquad (6)$$

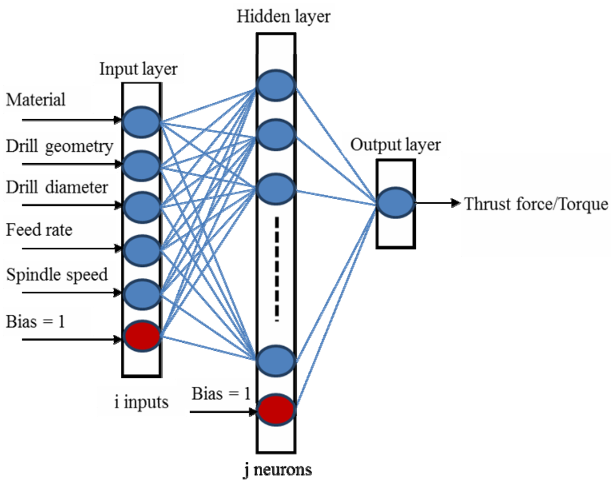

Figure 4. Neural architecture

Initially, a pilot running of the program was done by varying the neurons of hidden layer from four to fifty using 0.1 as value for the learning rate and 0.9 for the momentum factor. This was done in order to obtain the idea of the neurons in the hidden layer. The number of neurons in the hidden layer was finally varied from 10 to 45. The learning rate was kept low and was varied from 0.05 to 3 at intervals of 0.025. The momentum factor was varied between 0.7 and 0.9 at intervals of 0.05.

The data sets were first normalized and a sigmoidal function was used as an activation function. The number of iterations used was 15000. The weights were randomly chosen and were kept below 1, both in the input and the hidden layers. First, 130 data sets were chosen randomly for the purpose of training the algorithm, whereas the remaining 14 data sets were used to test the program.

6. PREDICTION OF THRUST FORCE AND TORQUE

Using the new ANN approach, the modelling was done for both the thrust force and torque. The number of hidden layers and the values of momentum factor and the learning rate were calculated for all the three approaches (first approach: minimum error in training, second approach: minimum mean percentage error during testing, and third approach: minimum-maximum percentage error during testing). The details of the developed ANN models for thrust force and torque are shown in Table 1.

The results obtained from the developed thrust force model based on three different approaches of ANN application, as shown in Figure 5. The model fails during testing when the non-linearity in the data is more and the error during training is minimum.

The results obtained from the thrust force model, which was based on the second approach, have shown exceptional compliance with the experimental values, as can be seen in Figure 5. The results of the thrust force model presented in Table 2 clearly indicate that the second approach is better in copmarison with the other two approaches in terms of predicting drilling-induced thrust force.

Table 1. Details of the ANN models

Model	Characteristics values	Approach		
		First	Second	Third
Thrust force	Number of neurons in hidden layer	35	36	42
	Learning rate	0.25	0.27	0.12
	Momentum factor	0.90	0.80	0.85
Torque	Number of neurons in hidden layer	36	35	20
	Learning rate	0.30	0.15	0.12
	Momentum factor	0.90	0.85	0.90

Table 2. Results of the thrust force model

Characteristics values	Approach		
	First	Second	Third
Mean percentage error in predicted values of training data inputs	2.5103	4.5081	4.7241
Mean percentage error in predicted values of test data inputs	8.2366	3.7436	5.4568
Coefficient of correlation among predicted values and training data outputs	0.9972	0.9888	0.9886
Coefficient of correlation among predicted values and test data outputs	0.9646	0.9851	0.9880

Table 3. Results of the torque model

Characteristics values	Approach		
	First	Second	Third
Mean percentage error in predicted values of training data inputs	2.3569	5.2011	5.3088
Mean percentage error in predicted values of test data inputs	20.5329	7.0732	9.2711
Coefficient of correlation among predicted values and training data outputs	0.9983	0.9889	0.9883
Coefficient of correlation among predicted values and test data outputs	0.8349	0.9749	0.9633

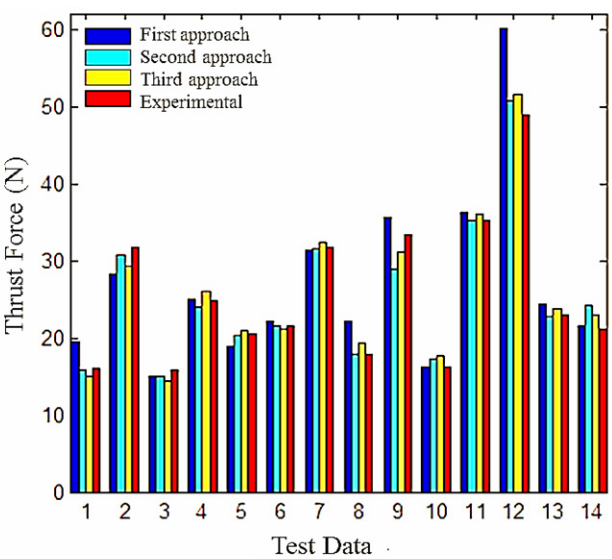

Figure 6. Experimental and predicted values of thrust force for test data sets

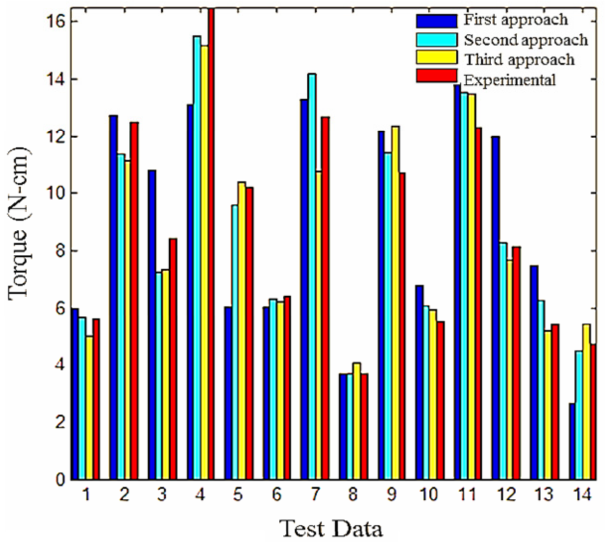

Figure 8. Experimental and predicted values of torque for test data sets

The results obtained from the developed torque model based on three different ANN approaches is shown in Figure 6. It is quite clear from the figure that the torque model based on the second approach gives better results. The results presented in Table 3 also indicate that the second approach is better than the other two approaches in predicting drilling-induced torque.

7. CONCLUSION

In the present research endeavour, the drilling of unidirectional GFRP and [(0/90)/0]s GFRP laminates were conducted using four different carbide drills of 4 and 8mm diameter at three different levels of spindle speed and feed rate. A new ANN approach (first approach: minimum error in training, second approach: minimum mean percentage error during testing, and third approach: minimum-maximum percentage error during testing) has been proposed to develop the predictive thrust force and torque models. The suggested new ANN approach can be easily used to find the number of neurons in a hidden layer and the values of momentum factor and learning rate. The cumbersome hit and trial method used to find the parametric values of the neural network can be avoided if the proposed ANN approach is used to solve the problem. The testing of the developed models using test data reveals that the thrust force and torque model based on the second ANN approach, i.e. minimum mean percentage error is better than the other two approaches. The three approaches suggested are realistic in nature and can be used to develop ANN models for different engineering problems. Furthermore, the predictive force models can be utilized commercially to save time and cost. Further increase in data sets may lead to a better predictive model.

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NOMENCLATURE

FRP's	fiber reinforced plastics
GFRP	glass fiber reinforced plastic
CFRP	carbon fiber reinforced plastic
ANN	artificial neural network
p	number of training data pair
q	number of test data pair
(z_p, t_p)	p^{th} training pair
(a_q, b_q)	q_{th} testing pair
J	number of neurons in hidden layer
η	learning rate
α	momentum factor
n	number of epochs
n_{max}	maximum number of epochs
E_{max}	minimum acceptable error
J_{max}	maximum number of neurons in hidden layer
η_{max}	maximum learning rate
α_{max}	maximum momentum factor
ET	minimum error during training
MPE	mean percentage error
Max_PE	maximum percentage error
Min_MPE	minimum mean percentage error during testing
Min_Max_P	minimum-maximum percentage error during
E	testing
TPE	total percentage error
V	weights on connections of input layer with hidden layer
W	weights on connections of the hidden layer with output layer

ПРЕДВИЂАЊЕ СИЛА ТОКОМ ОБРАДЕ БУШЕЊЕМ КОМПОЗИТНИХ ЛАМИНАТА ПРИМЕНОМ ВЕШТАЧКЕ НЕУРОНСКЕ МРЕЖЕ: НОВИ ПРИСТУП

Vikas Dhawan, Kishore Debnath, Inderdeep Singh, Sehijpal Singh

У раду се проучава осносиметрично, изотермско, стишљиво струјање гаса са клизањем, при малим вредностима Рејнолдсовог броја. Проблем је решен применом једначине континуитета и Навије-Стоксових једначина, заједно са Максвеловим граничним условом првог реда. Аналитички резултати су добијени применом пертурбационе методе. Добијена решења се добро слажу са познатим експеримен–талним резултатима других аутора.