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Letter

Prediction of nonlinear evolution character of energetic-particle-driven instabilities

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Abstract

A general criterion is proposed and found to successfully predict the emergence of chirping oscillations of unstable Alfvénic eigenmodes in tokamak plasma experiments. The model includes realistic eigenfunction structure, detailed phase-space dependences of the instability drive, stochastic scattering and the Coulomb drag. The stochastic scattering combines the effects of collisional pitch angle scattering and micro-turbulence spatial diffusion. The latter mechanism is essential to accurately identify the transition between the fixed-frequency mode behavior and rapid chirping in tokamaks and to resolve the disparity with respect to chirping observation in spherical and conventional tokamaks.

Keywords: fast ions, non-linear dynamics, wave chirping, plasma instabilities

(Some figures may appear in colour only in the online journal)

In fusion grade plasmas, there is a population of energetic particles (EPs) with typical energies substantially greater than those of the thermal background. These particles provide an energy-inverted population, that through the kinetic wave-particle interaction with Alfvén waves, can induce instabilities that jeopardize plasma confinement [1, 2]. The nature of these oscillations vary considerably (with the possibility of several bifurcations [3, 4]), with two typical non-linear scenarios being: (a) the excitation of a slowly evolving amplitude with a nearly fixed-frequency oscillation and (b) coherent oscillations that chirp in frequency at timescales much shorter than that of the plasma equilibrium modification. These scenarios lead to dominant diffusive and convective transport of EPs, respectively.

This letter addresses two outstanding and inter-connected issues that, in spite of their major relevance for the transport of EPs in future-generation burning plasmas, are currently not understood. The first issue is what plasma conditions most strongly determine the likelihood of each non-linear saturation scenario in experiments. The second is why the chirping response (observed in all major tokamaks, e.g. DIII-D [5], NSTX [6, 7], JET [8], MAST [9], JT-60U [10, 11], ASDEX-U [12]) is much more common in spherical tokamaks than in conventional tokamaks, being especially rare in DIII-D. This classification is important in anticipating whether EP-induced instabilities in burning plasma experiments will likely lead to steady oscillations, where quasi-linear theory [13–15] would be expected to described EP transport or chirping, which would then require new theoretical tools to assess the consequences of the induced EP transport.

In this letter we show that a previous approach that attempted to simplify the needed input that the theory requires

[16] is insightful but limited for making accurate predictions of experimental scenarios. Here we employ a generalized formulation and show that its predictions are in accordance with observations. This analysis reveals that micro-turbulence, even while producing no observable effect on the beam ion transport, provides the vital mechanism in determining which non-linear regime is more likely for a mode as well as the mode transition from one regime to the other, as parameters of an experiment change in time.

We focus the analysis on the onset of a mode non-linear evolution near marginal stability. The interaction Hamiltonian between a particle (at position $\mathbf{r}$, with velocity $\mathbf{v}$) and a tokamak eigenmode with frequency ω can be written as

$$q_{\mathrm{EP}}\mathbf{A}(\mathbf{r},t)\cdot\mathbf{v}=C(t)\sum_j V_j(\mathcal{E},P_\varphi,\mu)\mathrm{e}^{\mathrm{i}(j\theta_a-n\varphi_a-\omega t)}, \quad (1)$$

where $\mathbf{A}(\mathbf{r},t)$ is the perturbed vector potential in a gauge where the electrostatic potential vanishes, $\mathcal{E}$ is the unperturbed energy, P_φ is the canonical angular momentum, μ is the magnetic moment (all per unit EP mass), the summation is over all integers j, φ_a and θ_a are the action angles of the unperturbed orbit in the toroidal and poloidal directions, q_{EP} is the charge of an EP and n is a fixed quantum number for the toroidal angular response of a perturbed linear wave in an axisymmetric tokamak. V_j accounts for the wave-particle energy exchange. It is essentially the integral of the projection of the resonant particle current onto the eigenmode electric field and can be calculated by taking the inverse transform of equation (1) (as in equation 12 of [17]). Upon a suitable normalization, the complex amplitude $C(t)$ has been shown to be governed by an integro-differential cubic equation that is nonlocal in time [3, 16, 18][6],

$$\frac{\mathrm{d}C(t)}{\mathrm{d}t}-C(t)=-\sum_j\mathrm{d}\Gamma\mathcal{H}\int_0^{t/2}\mathrm{d}\tau\tau^2C(t-\tau)$$
$$\times\int_0^{t-2\tau}\mathrm{d}\tau_1\mathrm{e}^{-\hat{\nu}^3_{\mathrm{stoch}}\tau^2(2\tau/3+\tau_1)+\mathrm{i}\hat{\nu}^2_{\mathrm{drag}}\tau(\tau+\tau_1)}$$
$$\times C(t-\tau-\tau_1)C^*(t-2\tau-\tau_1) \quad (2)$$

where $\mathcal{H}=2\pi\omega\delta(\Omega_j)\left|V_j\right|^4\left(\frac{\partial\Omega_j}{\partial I}\right)^3\frac{\partial f}{\partial\Omega}$, with f being the equilibrium distribution function. We assume a low frequency mode for which μ is conserved. Then $\partial/\partial I\equiv-n\partial/\partial P_\varphi+\omega\partial/\partial\mathcal{E}$ (with the variable I being defined as in appendix 1 of [18]). The resonance condition is given by $\Omega_j=\omega+n\omega_\varphi-j\omega_\theta=0$, where ω_θ and ω_φ are the mean poloidal and toroidal transit frequencies of the equilibrium orbit. The phase-space integration is given by $\int\mathrm{d}\Gamma...=(2\pi)^3\sum_{\sigma_\parallel}\int\mathrm{d}P_\varphi\times\int\mathrm{d}\mathcal{E}/\omega_\theta\int m_{\mathrm{EP}}c\mathrm{d}\mu/q_{\mathrm{EP}}...$, where m_{EP} is the mass of EPs, c is the light speed and $\sigma_\parallel$ accounts for counter- and co-passing particles. The effective collisional operator can be cast in the form $C_{\mathrm{coll}}[f]=\nu^3_{\mathrm{scatt}}\frac{\partial^2 f}{\partial\Omega^2}+\nu^2_{\mathrm{drag}}\frac{\partial f}{\partial\Omega}$, where ν_{scatt} and ν_{drag} are understood to be the effective pitch-angle scattering and drag (slowing down) coefficients, defined in equation 6 of [16]. ν_{stoch} is the effective stochasticity, which includes ν_{scatt}. In equation (2), the circumflex denotes normalization with respect to $\gamma=\gamma_L-\gamma_d$ (growth rate minus damping rate) and t is the time normalized with respect to the same quantity. Vlasov simulation codes have shown [19, 20] that the blow-up solutions of (2) (as described in [3]) are precursors to chirping behavior.

[6] The cubic equation was derived independently for vortex flow in fluids; see [37].

The type of nonlinear evolution of a wave destabilized by a perturbing EP drive is strongly dependent on the kernel of the integrals of equation (2), specifically on the ratio between the effective stochastic relaxation felt by the EPs and the effective drag rate, as well as the linear growth rate. In [16], equation (2) was simplified by using characteristic values for the collisional ν_{scatt} and ν_{drag} and conditions for the existence and stability of solutions of the cubic equation were derived. In figure 1, we test for the first time this prediction against modes measured in different tokamaks. In order to determine mode properties, we employ the kinetic-MHD code NOVA [21] to compute eigenstructures and the frequency continua and gaps. Its kinetic postprocessor NOVA-K [22, 23] is used to calculate perturbative contributions that can stabilize and destabilize MHD eigenmodes. In addition, NOVA-K is also employed to compute resonant surfaces in $(\mathcal{E},P_\varphi,\mu)$ space. In the analysis, we considered modes for which the calculated drive and damping rates are each much smaller than the mode frequency, which enables the use of a perturbative approach for studying the mode properties. In order to characterize the mode being observed in the experiment, NSTX reflectometer measurements are compared to the mode structures computed by NOVA, by employing a similar procedure as the one used in [7, 24]. In DIII-D, similar identification is performed using Electron Cyclotron Emission (ECE) data [25].

We see from figure 1 that about half of the chirping NSTX modes lie in a region where stable steady modes are predicted by [16]. For the DIII-D experimental cases that produced fixed-frequency modes, the predictions of [16] are mostly in agreement although one point is borderline and another one may be unstable enough to be in a chirping regime. Hence we see that using the simplified, although elaborate, modeling akin to that used in [16], might be in satisfactory agreement with DIII-D data but is generally not satisfactory for much of the NSTX and TFTR data. This comparison indicates that the use of a single characteristic value, as being representative of the entire phase space, for ν_{scatt} (considered the only contribution to ν_{stoch}) and ν_{drag}, although insightful, appears insufficient to provide quantitative predictions for practical tokamak cases. This conclusion motivated the pursuit of a theoretical method to take into account important missing elements, such as spatial mode structures and local phase-space contributions on multiple resonant surfaces of the wave-particle interaction terms, all of which are needed in toroidal geometry. The appropriate weightings for the various needed quantities can be expressed in the action-angle formulation. A necessary, although not sufficient, condition for chirping solutions is that the right hand side of (2) be positive. The resonance condition, represented by $\delta\left(\Omega_j\left(P_\varphi,\mathcal{E},\mu\right)\right)$, allows one of the phase-space integrals to be eliminated. Upon integration over τ_1 and redefinition of the integration variable $z=\nu_{\mathrm{drag}}\tau$ one finds the following criterion for the non-existence of steady solutions of (2):

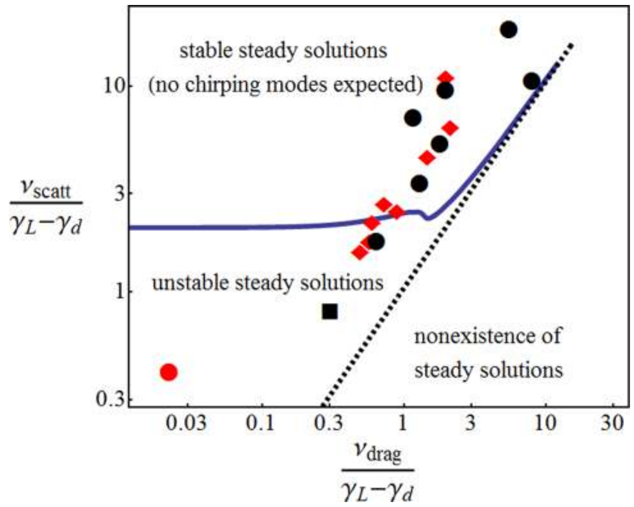

Figure 1. Comparison between analytical predictions with experiment when single characteristic values for phase space parameters are chosen. The dotted line delineates the region of existence of steady amplitude solutions of the cubic equation (2) while the solid line delineates the region of stability, as predicted by [16]. Modes that chirped are represented in red and the ones that were steady are in black, as experimentally observed for TAEs, RSAEs and BAAEs in DIII-D (circular discs), TAEs in NSTX (diamonds) and TAE in TFTR (square).

$$Crt = \frac{1}{N}\sum_{j,\sigma_\parallel}\int \mathrm{d}P_\varphi \int \mathrm{d}\mu \frac{|V_j|^4}{\omega_\theta \nu_{\mathrm{drag}}^4}\left|\frac{\partial \Omega_j}{\partial I}\right|\frac{\partial f}{\partial I} Int < 0 \tag{3}$$

where

$$Int \equiv \mathrm{Re}\int_0^\infty \mathrm{d}z \frac{z}{\frac{\nu_{\mathrm{stoch}}^3}{\nu_{\mathrm{drag}}^3}z - \mathrm{i}} \exp\left[-\frac{2}{3}\frac{\nu_{\mathrm{stoch}}^3}{\nu_{\mathrm{drag}}^3}z^3 + \mathrm{i}z^2\right]. \tag{4}$$

For the resonances to be linearly destabilizing to positive energy waves, *Int* (plotted in figure 2) is the only component of the criterion (3) that can be negative from the phase-space regions which contribute positively to the instability growth. N is a normalization factor consisting of the same sum that appears in equation (3) except for *Int*. Thus, in NOVA-K we use the contributions from each resonance weighted in accord with the appropriate eigenfunction (that fits the measured field structure) and the position in phase space of the resonant interaction. We will see that this procedure produces quite a different conclusion from the less detailed method that uses a single characteristic factor, as is the case in figure 1.

Non-steady oscillations, with the likelihood of chirping, are predicted to occur if $Crt < 0$ while a steady (fixed-frequency) solution exists if $Crt > 0$. However, we see from figure 2 that *Int* could be an order of magnitude larger in phase space regions where $\nu_{\mathrm{stoch}}/\nu_{\mathrm{drag}} \lesssim 1.04$ compared with regions where $\nu_{\mathrm{stoch}}/\nu_{\mathrm{drag}} \gtrsim 1.04$. Hence, because of this disparity, it can turn out that a choice of the use of a single characteristic value for $\nu_{\mathrm{stoch}}/\nu_{\mathrm{drag}}$, would lead to a positive value for *Crt* while the use of the appropriately weighted average leads to a negative value for *Crt*. Such a change is indeed the case for all the TFTR and DIII-D modes and for most of the NSTX

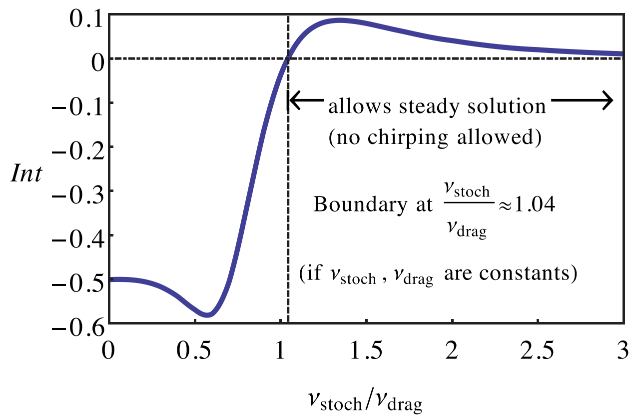

Figure 2. *Int* (equation (4)) plotted in terms of local values of $\nu_{\mathrm{stoch}}/\nu_{\mathrm{drag}}$.

modes shown in figure 1, where ν_{stoch} was considered simply as ν_{scatt}. The reason for this sensitivity is that there will always be a contribution to *Crt* from a phase space region where $\nu_{\mathrm{scatt}}/\nu_{\mathrm{drag}} \ll 1$ because our modeling of the pitch angle scattering coefficient goes to zero as μ vanishes [26]. Hence even when the characteristic value of $\nu_{\mathrm{scatt}}/\nu_{\mathrm{drag}}$ is substantially greater than unity, one still can find that $Crt < 0$.

The above observation indicates that pitch-angle scattering ν_{scatt} may not always be the dominant mechanism in determining ν_{stoch}. Hence, we now introduce the contribution of fast-ion electrostatic micro-turbulence for the determination of ν_{stoch} through the following procedure introduced by Lang and Fu [27]. The TRANSP code [28] is employed to obtain the thermal ion radial thermal conductivity, χ_i (which is essentially the particle diffusivity, D_i [29]) based on power balance. The heat diffusivity due to collisions is subtracted out and the remaining diffusivity is attributed to micro-turbulence interaction with the ions. Then the EPs diffusivity is estimated by using the scalings determined in a gyrokinetic simulation of electrostatic turbulence [30], which for passing particles gives $D_{\mathrm{EP}} \approx 5D_iT_i/E_{\mathrm{EP}}$. In the experiments we analyzed, the drive was mostly from the passing particles and therefore we used this relation as an estimate for D_{EP}. The response of the resonant EPs to perturbing fields is essentially one-dimensional [18] and produces steep gradients in the EP distribution in this perturbing direction. We can then accurately account for the diffusion that is directed in all phase space directions, by projecting the actual diffusion from all these directions onto the steepest gradient path defined by the one-dimensional dynamics, using the specific relation given by equation (2) of [27]. Details of the method are given in [26].

In considering the classical transport processes, we only included the dominant transport process of pitch angle scattering, which is larger than energy scattering by a factor $\sim E_{\mathrm{EP}}/T_i$. Also, the collisional effects from beam-beam interactions have been neglected since they are smaller than the beam-background interactions by a factor $\sim n_b/n_i$, with n_b being the density of the EPs injected by the neutral beam and n_i the background ion density. RF fields have been shown to

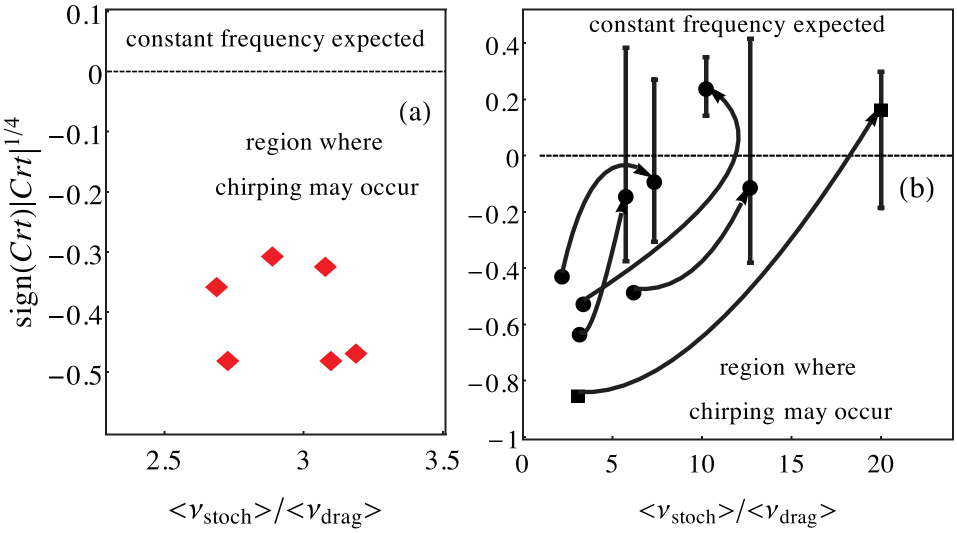

Figure 3. Numerical values for $|Crt|^{1/4}$ multiplyed by the sign of *Crt* as a function of $\langle\nu_{\text{stoch}}\rangle/\langle\nu_{\text{drag}}\rangle$. Modes that chirped are represented in red and the ones that were steady are in black, as experimentally observed in (*a*) NSTX (diamonds) and (*b*) DIII-D (discs) and TFTR (squares). The arrows represent the effect of micro-turbulence. The bars represent by how much the prediction for the modes change if we double the turbulent diffusity (upper bars) or divide it by 2 (lower bars). In the NSTX case, the points hardly move upon the addition of spatial diffusion to collisional scattering.

be a determining component for the suppression of chirping in CTX [31]. Our analysis, however, was only focused on shots where the EPs were created by neutral beam injection, hence there was no need to account for diffusion from RF waves.

Figure 3 shows values of $|Crt|^{1/4}$ multiplyed by the sign of *Crt*, as a function of the ratio of phase-space averaged stochasticity and drag for modes of figure 1. This representation provides better visualization than simply plotting *Crt*, especially close to the steady/chirping boundary and is chosen because of the fourth power dependence $Int \approx 1.022(\nu_{\text{stoch}}/\nu_{\text{drag}})^{-4}$ for $\nu_{\text{stoch}}/\nu_{\text{drag}} \gg 1$. Figure 3(*a*) shows chirping modes in NSTX and figure 3(*b*) shows steady modes in DIII-D and TFTR. The curved arrows represent how the prediction for a mode is affected by micro-turbulence-induced scattering of EPs. It has a strong effect on DIII-D and TFTR (bringing the modes to the steady region, or at least very close to it) while its effect is imperceptible for the chirping modes in NSTX. This is because, unlike in conventional tokamaks, thermal ion transport in spherical tokamaks (STs) is usually close to neoclassical levels [32, 33] even though the electron transport is anomalous. NSTX modes in figure 3(*a*) are only able to transition to the fixed-frequency region when ν_{stoch} is artificially multiplied by a factor from 10 to 50, depending on the specific mode, which indicates the robustness of the chirping prediction. For the analyzed DIII-D discharges, the background turbulence is believed to be mostly in the ITG range. In figure 3(*b*), we artificially multiply the predicted turbulent stochasticity for the resonant energetic ions by factors of 2 and 1/2 (shown by the error bars) in order to understand how sensitive the evaluated criterion is with respect to uncertainties in the inferred EP turbulence level. It turns out that *Crt* can be rather sensitive near the positive/negative transition due to the dependence of *Int* on the ratio $\nu_{\text{stoch}}/\nu_{\text{drag}}$ but becomes quite insensitive to deviations of χ_i as the point moves away from this borderline.

Guided by the theory, we have then examined chirping modes that rarely appear in DIII-D tokamak. A series of dedicated shots were performed on DIII-D to study the transition to the chirping regime. These shots had high ion temperaturein the core (10–12 keV) and strong toroidal rotation (up to 50 kHz on axis). We find that the chirping onset correlates very closely with conditions where the thermal ion transport had drastically decreased (more specifically, near the L to H transition), as shown in figure 4. This is attributed to the decrease in micro-turbulence-induced transport, which also causes decreased EP transport. Alfvénic modes only started chirping when the thermal ion conductivity dropped to values lower than 0.3 m^2 s^{-1}. An example of the evaluation of the criterion (3) is DIII-D shot 152 828 (figure 4(*c*)). Before chirping starts (at $t = 920$ ms, when $D_{\text{th},i} \approx 0.55$ m^2 s^{-1}) the calculated criterion is $Crt = +0.001$. During the early phase of chirping (at $t = 955$ ms, when $D_{\text{th},i} \approx 0.25$ m^2 s^{-1}) the value is $Crt = -0.013$, i.e. the mode has transitioned from the positive (steady) region to the negative region of *Crt*, therefore allowing chirping, in agreement with the observation. We note that mode stability can be sensitive to ion and electron kinetic effects, not captured by NOVA, and to the time evolving equilibrium as the modes transition to chirping. However, even though the growth rate changes, the most relevant parameter for the chirping criterion in figure 4 is the substantial drop microturbulence levels. This is because microturbulence stochasticity enters directly the chirping-relevant ratio $\nu_{\text{stoch}}/\nu_{\text{drag}}$, while the growth rate affects the phase-space averaging, which has a minor effect.

The micro-turbulence interaction with EPs is a key factor that can determine the nature of mode saturation regime (quasi-steady and chirping) and also the transition between them. It also explains the longstanding question of why chirping Alfvénic modes are ubiquitous in STs and rare in conventional tokamaks. Experimentally, the EP transport due to micro-turbulence is too low compared to Alfvénic-induced transport [34, 35]. Yet, its effect on EP transport can be crucial in determining the character of emerging Alfvénic oscillations

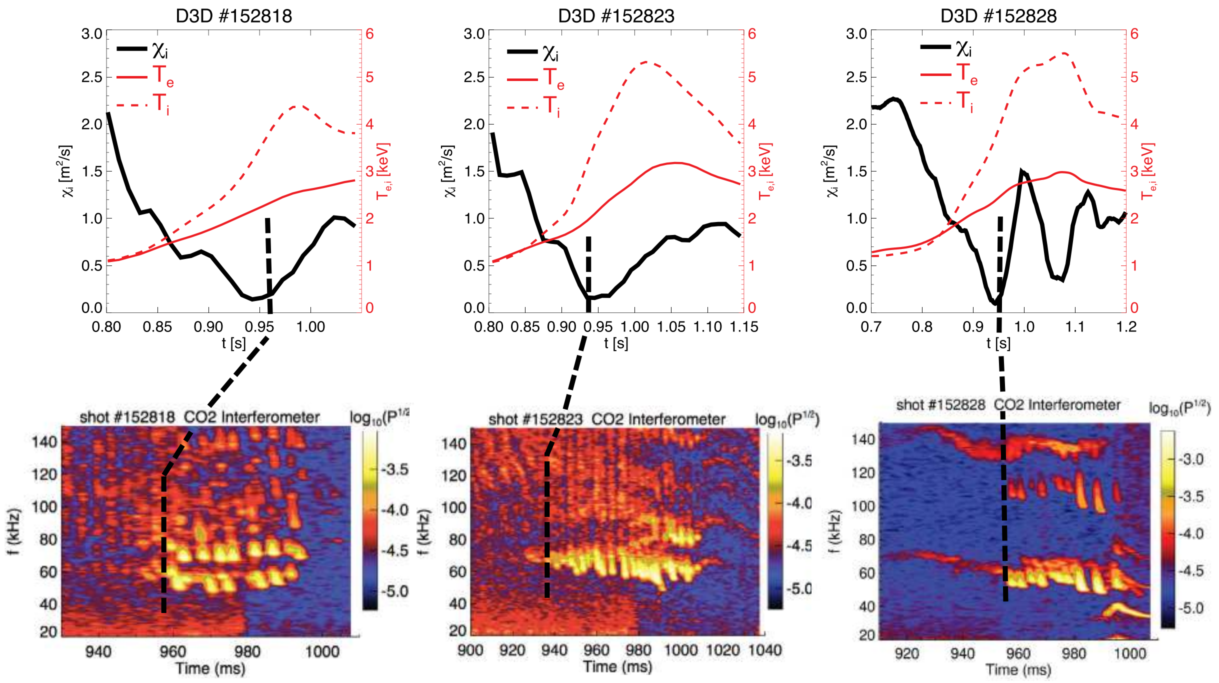

Figure 4. Correlation in DIII-D between the emergence of chirping and the development of low diffusivity, as calculated by TRANSP at the radius where the mode is peaked.

and the type of transport caused by them. This suggests that micro-turbulence simulations employed to predict the thermal plasma transport of future burning plasma devices must also be factored in to considerations of the drive and saturation of modes driven by EPs.

This work provides a means for choosing which of the two extreme scenarios is most likely to be relevant for predicting the character of the energetic particle transport, based on the sign of *Crt*. For a negative *Crt* the physical conditions are established to enable a nonlinear BGK-like mode [36] to form, where the frequency remains locked to a particle resonance frequency as particles trapped by the wave are convected in phase space which, for the Alvénic instabilities, primarily causes resonant energetic particles to flow across field lines. Alternatively, a positive *Crt* represents the lack of chirping and indicates that the details of the nonlinear particle transport might be described by a quasilinear diffusion theory [13–15]. Therefore, the application of this criterion should be important in the planning and modeling of scenarios for future fusion plasma experiments.

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