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PREDICTION OF PERIODIC RESPONSE OF BLADES HAVING 3D NONLINEAR SHROUD CONSTRAINTS

J. J. Chen and C. H. Menq*

Coordinate Metrology and Measurement Laboratory
Department of Mechanical Engineering
The Ohio State University
Columbus, OH 43210
U.S.A.

* Corresponding Author (614)292-4232

ABSTRACT

In this paper, a 3D shroud contact model is employed to predict the periodic response of blades having 3D nonlinear shroud constraint. When subjected to periodic excitation, the resulting relative motion at the shroud contact is assumed to be periodic in three-dimensional space. Based on the 3D shroud contact model, analytical criteria are used to determine the transitions between stick, slip, and separation of the contact interface and are used to simulate hysteresis loops of the induced constrained force, when experiencing periodic relative motion. The constrained force can be considered as a feedback force that influences the response of the shrouded blade. By using the Multi-Harmonic Balance Method along with Fast Fourier Transform, the constrained force can be approximated by a series of harmonic functions so as to predict the periodic response of a shrouded blade. This approach results in a set of nonlinear algebraic equations, which can be solved iteratively to yield the periodic response of blades having 3D nonlinear shroud constraint. In order to validate the proposed approach, the predicted results are compared with those of the direct time integration method. The resonant frequency shift, the damping effect, and the jump phenomenon due to nonlinear shroud constraint are examined. The implications of the developed solution procedure to the design of shroud contact are also discussed.

Introduction

In a shrouded blade system, the protruding shrouds constrain the blade motions not only along the contact plane but also along the normal direction of the plane, resulting in very complex contact kinematics. The in-plane tangential relative motion is often two-dimensional, and it can induce stick-slip friction (Menq and Yang, 1998; Yang and Menq, 1997; Griffin and Menq, 1991; Menq *et al*, 1991). On the other hand, the normal relative motion can cause variation of the contact normal load and, in extreme circumstances, separation of contact interface (Menq *et al*, 1986; Yang and Menq, 1998). In previous studies of the shrouded blade systems, Yang and Menq (1997) proposed a two-dimensional model for the contact kinematics. This model retains the normal component of the relative motion that causes normal load variation while degenerating the in-plane tangential component of the relative motion into linear motion. In their study, the assumed blade motion has only two components, namely axial and tangential components. In order to take the radial component into account, Yang *et al* (1998) proposed a simplified three-dimensional shroud contact kinematics, in which the two-dimensional in-plane tangential relative motion is assumed to be elliptical and is decomposed into two linear motions along the principal major and minor axes of the ellipse. A variable normal load friction force model (Yang and Menq, 1997) was then applied separately to each individual linear motion to estimate the equivalent stiffness and damping of the shroud contact.

Yang and Menq (1998) further proposed a three-dimensional shroud contact model, in which the joined effect of the 2D tangential relative motion and the normal relative motion is examined. They developed a set of analytical criteria to determine the transitions among stick, slip, and separation, when experiencing variable normal load. With these transition criteria, the constrained force can be predicted for any given 3D cyclic relative motion across the contact interface. In this paper, the 3D shroud contact model (Yang and Menq, 1998) is employed to obtain the constrained force at the shroud contact of a shrouded blade system. The bladed system is assumed tuned and the assumed blade motion has three components, namely axial, tangential, and radial components. In the shroud contact model, a contact plane is defined and its orientation is assumed invariant. When subjected to periodic excitation, the resulting three-dimensional relative motion at a shroud contact as well as the constrained force are assumed to be periodic. The constrained force can be considered as a feedback force that influences the response of the shrouded blade. By using the Multi-Harmonic Balance Method along with Fast Fourier Transform, the constrained force can be approximated by a series of

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harmonic functions. In the calculation of the nonlinear forced response of a shrouded blade, all the linear degrees of freedom can be condensed to receptance and the modeling of shroud contact can be separated from the complex structure model. This approach results in a set of nonlinear algebraic equations, which can be solved iteratively to yield the periodic forced response of blades having 3D nonlinear shroud constraint.

3D Shroud Contact

Figure 1 shows a shrouded blade system with two neighboring blades contacting each other through the protruding shrouds. When subjected to cyclic excitation, the vibratory motion of the shrouded blade can be assumed to be periodic, and the resulting relative motion across the shroud contact is also periodic in the 3D space. In modeling the shroud contact, a "substructure" can be used to represent the friction interface that contains the contact plane and small portions of the two neighboring shrouds, and the substructure can be modeled as two massless elastic elements that are held together by a preload n_0. The points A and B are the outermost points of these two elastic elements; and the difference of their respective motions can describe the 3D periodic relative motion of the two neighboring shrouds. The periodic relative motion is often not parallel to the contact plane. In order to analyze the induced friction, the periodic relative motion in the 3D space can be decomposed into an in-plane periodic motion on the contact plane and a periodically varying component normal to the contact plane.

Shroud Contact Geometry

Two shrouded blades contact each other through their protruding shrouds are as shown in Figure 2, and the xyz coordinate system (called blade coordinate system) is defined in accordance with the tangential (x), axial (y), and radial (z) directions. The contact plane of the 3D shroud contact is defined by two angles ψ (called shroud angle) and ϕ (called inclination angle). A vuw coordinate system (called shroud coordinate system) can be defined, where v axis is along the normal direction of the contact plane, and u and w axes are on the contact plane. In this paper, the blade coordinate system is defined by using three unit vectors, namely $\begin{bmatrix}\hat{x} & \hat{y} & \hat{z}\end{bmatrix}$, and the shroud coordinate system $\begin{bmatrix}\hat{v} & \hat{u} & \hat{w}\end{bmatrix}$. These two coordinate systems can be related to each other as follows:

$$\begin{bmatrix}\hat{v} & \hat{u} & \hat{w}\end{bmatrix} = \begin{bmatrix}\hat{x} & \hat{y} & \hat{z}\end{bmatrix}\mathbf{T}_0 \tag{1}$$

where $\mathbf{T}_0$ is the coordinate transformation matrix (Yang *et al*, 1998).

$$\mathbf{T}_0 = \begin{bmatrix}\cos\psi\cos\phi & -\sin\psi & \cos\psi\sin\phi \\ \sin\psi\cos\phi & \cos\psi & \sin\psi\sin\phi \\ -\sin\phi & 0 & \cos\phi\end{bmatrix} \tag{2}$$

In current design practice, the inclination angle ϕ is often set to be zero.

The decomposition of the 3D relative motion is shown in Figure 3 schematically. The in-plane periodic motion can induce stick-slip

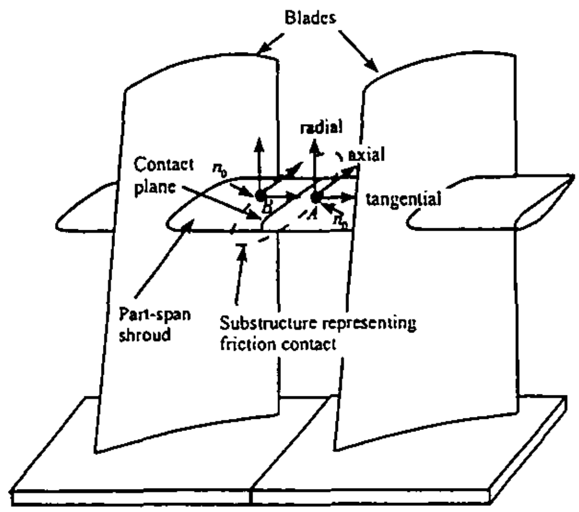

Figure 1 Shroud contact of two neighboring shrouded blades.

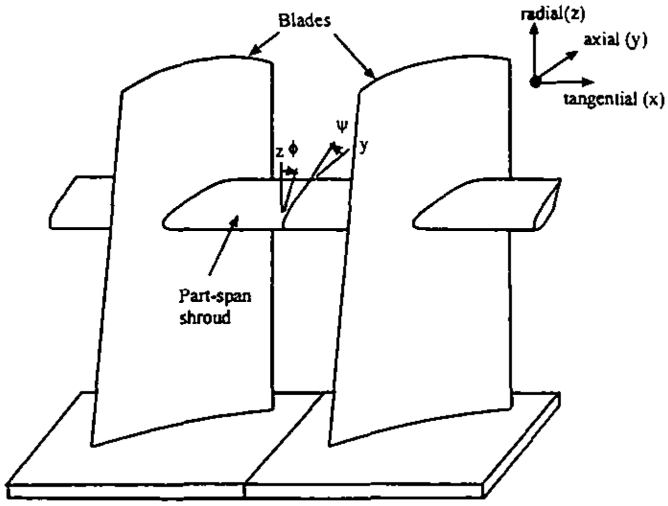

Figure 2 Contact geometry of a shrouded blade system.

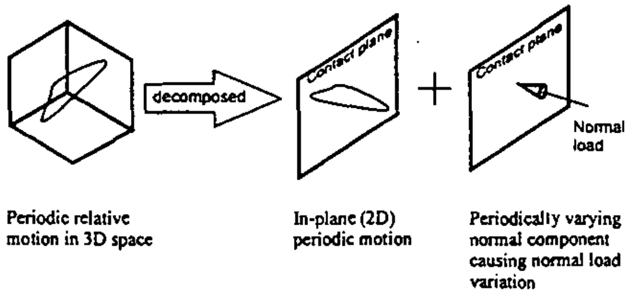

Figure 3 Decomposition of 3D periodic relative motion.

friction, and thus can attenuate the periodic response of the shrouded blades. On the other hand, the normal component tends to alter the normal load across the interface; and this effect, in extreme circumstances, may lead to separation of the interface. It should be noted that the variable normal load is taken as the sum of the initial contact pressure at equilibrium plus a term that is proportional to the periodically varying normal component of the relative motion. Since this decomposition is to transfer the relative motion from the blade coordinate system to the shroud coordinate system, it can be carried out by performing a coordinate transformation on the 3D relative motion by using equation (1).

3D Shroud Contact Model

In the 3D shroud contact model proposed by Yang and Menq (1998), the contact interface between two vibrating shrouds can be modeled as a substructure that contains a massless elastic element and a friction contact point, as depicted in Figure 4. In this model, the elastic element accounts for the shear and normal stiffness of the substructure, and it is characterized by a 2×2 stiffness matrix $\mathbf{K}_u$ for the shear stiffness and a spring constant k_v for the normal stiffness. The friction contact point, that is assumed to obey the Coulomb friction law with the friction coefficient μ when in contact with *Body 2*, can undergo tangential stick-slip motion, and may experience intermittent separation from *Body 2* when the normal relative motion (v) becomes large. The motion of the contact point is denoted as $\mathbf{w}$ in this model. The contact interface is assumed to have either a preload or an initial gap (as designated by n_0). This model allows a negative preload to represent the situation when the interface has an initial gap; the equivalent preload across the interface with a gap e is calculated as $-k_v e$. In this model, $\mathbf{u}$ and v are the input tangential relative motion ($[u\ \ w]^T$) and normal relative motion (v) of the contact interface respectively, and they can be evaluated as the motion of *Body 1* with respect to *Body 2*, where v, u, and w are the relative motions in the shroud coordinate system.

Constrained Force

The constrained force consists of two components: the induced friction force on the contact plane and the variable normal force. Since the friction force is completely characterized by the relative motion, it will not lose generality to assume one of the contacting surfaces is the ground. With this assumption, the input tangential relative motion $\mathbf{u}$, the slip motion of the contact point $\mathbf{w}$, and the induced friction force $\mathbf{f}$ are vectors parallel to the ground; the normal relative motion v and the normal load n are scalars. The friction force, acting on the ground, can be expressed as:

$$\mathbf{f} = \mathbf{K}_u(\mathbf{u} - \mathbf{w}) \tag{3}$$

The normal load is taken as the sum of the preload n_0 plus the variation caused by v, and it can be expressed as:

$$n = \begin{cases} n_0 + k_v v & \text{when } v \geq -n_0/k_v \\ 0 & \text{when } v < -n_0/k_v \end{cases} \tag{4}$$

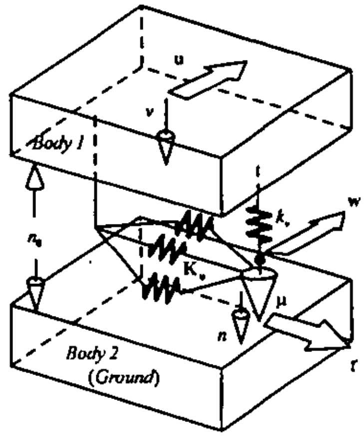

Figure 4 A 3D shroud contact model.

In this paper, the orientation of the contact plane is assumed to be invariant. This assumption is reasonable if the amplitude of shroud relative motion is relatively small when compared to the overall dimension of the shroud interface.

Stick, Slip, and Separation

Depending on the amplitude and phase of the in-plane relative motion and the normal load component of the vibratory motion, the friction contact will either stick, slip, or separate during a cycle of oscillation. The stick and slip conditions can be expressed as follows (Yang and Menq, 1998):

stick condition: $$\left|\mathbf{f} = \mathbf{K}_u\left(\mathbf{u} - \mathbf{u}_0\right) + \mathbf{f}_0\right| < \mu n \qquad \dot{\mathbf{w}} = 0 \tag{5}$$

slip condition: $$\mathbf{f} = \mu n \frac{\dot{\mathbf{w}}}{|\dot{\mathbf{w}}|} \qquad \dot{\mathbf{w}} \neq 0 \tag{6}$$

where $\mathbf{u}_0$ and $\mathbf{f}_0$ are the initial values of $\mathbf{u}$ and $\mathbf{f}$ at the beginning of the stick state. During the cycle of motion, the applied variable normal load may vanish to cause the interface to separate; consequently, the friction force is not present.

Stick/Slip/Separation Transition Criteria

In order to evaluate the resulting periodic constrained force at the shroud contact, analytical criteria are employed to determine the transitions between stick, slip, and separation when experiencing variable normal load. The analytical criteria can be summarized as follows.

(i) Stick-to-slip transition
This transition occurs when the friction force on the tangential plane reaches the varying slip load μn. That is,

$$\left|\mathbf{f} = \mathbf{K}_u(\mathbf{u} - \mathbf{u}_0) + \mathbf{f}_0\right| = \mu n \tag{7}$$

To ensure that the magnitude of the friction force has a tendency to exceed the slip load, the following constraint is imposed:

$$|\dot{\mathbf{f}}| > \mu \dot{n} \tag{8}$$

(ii) Slip-to-stick transition

During the slipping state, according to the Coulomb friction law, the friction force can be solved from an initial value problem:

$$\dot{\mathbf{f}} = \mathbf{K}_u\left(\dot{\mathbf{u}} - \frac{\mathbf{f}^T\mathbf{K}_u\dot{\mathbf{u}} - \mu^2 n\dot{n}}{\mathbf{f}^T\mathbf{K}_u\mathbf{f}}\mathbf{f}\right) \tag{9}$$

The slip-to-stick transition occurs when the velocity of the relative motion $\dot{w}$ equals 0, which implies:

$$\mathbf{f}^T\mathbf{K}_u\dot{\mathbf{u}} - \mu^2 n\dot{n} = 0 \tag{10}$$

Since the initial friction force at the beginning of the slip condition is known, the initial value problem of equation (9) can be solved by using a numerical integration scheme such as the 4th order Runge-Kutta method to obtain the friction force $\mathbf{f}$. Once the friction force is obtained, the criterion of equation (10) can be used to check the occurrence of the slip-to-stick transition.

(iii) Stick/slip-to-separation transition

The transition from stick/slip to separation occurs when the normal load vanishes. In addition, the normal load should be decreasing at this moment to ensure the occurrence of the separation. Hence the transition criteria can be formulated as:

$$n = 0, \qquad \dot{n} < 0 \tag{11}$$

(iv) Separation-to-stick/slip transition

Similarly to the above criterion, the separation ends when the normal load is about to develop on the contact plane. Therefore, the moment of this transition can be determined by the criterion:

$$n = 0, \qquad \dot{n} \geq 0 \tag{12}$$

When the normal load and the friction force begin to develop on the contact plane at the end of the separation, their rate of change at the moment determine whether the following state is either stick or slip.

$$\dot{\mathbf{u}}^T\mathbf{K}_u^T\mathbf{K}_u\dot{\mathbf{u}} < \mu^2\dot{n}^2 \Rightarrow \text{Stick begins} \tag{13}$$

$$\dot{\mathbf{u}}^T\mathbf{K}_u^T\mathbf{K}_u\dot{\mathbf{u}} \geq \mu^2\dot{n}^2 \Rightarrow \text{Slip begins} \tag{14}$$

It should be pointed out that the incipient slipping condition can be regarded as an one-dimensional case, because the friction force is not present at this moment and the slip action will be developed along $\dot{\mathbf{u}}$. Thus, according to the Coulomb friction law, the rate of change of the developing friction force can be expressed as:

$$\dot{\mathbf{f}} = \mu\dot{n}\frac{\dot{\mathbf{u}}}{|\dot{\mathbf{u}}|} \tag{15}$$

Once the friction force develops, it can be further determined by solving the initial value problem of equation (9).

In this paper, these criteria are used to simulate hysteresis loops of the friction contact, when experiencing periodic relative motion. With these hysteresis loops, the resulting constrained force can be characterized by the relative motion between two neighboring shrouds. By using Fast Fourier Transform, the constrained force can be approximated by a series of harmonic functions and employed in Multi-Harmonic Balance Method to solve for the periodic response.

Periodic Response of A Shrouded blade System

In the analysis of a shrouded blade system, a great simplification can be obtained by assuming that the bladed system is tuned, namely each blade of the system has exactly the same dynamic characteristics. In addition, the excitation of interest is that induced by the blades rotating through circumferential variations in the flow field. It can be shown that in effect each blade is exposed to a periodic excitation having the same amplitude but differing in phase by an amount which is proportional to the blade's angular location on the disk. It is assumed that the forced response of the bladed system is also periodic and has the same fundamental period as the excitation[1]. Thus the motion of the blade as well as the nonlinear constrained force can be represented by infinite Fourier series. By truncating these series after the m^{th} terms, an approximate solution assuming that the forced response is periodic can be derived. In this approach, each blade vibrates in the same manner but with a proportional interblade phase difference ($k\varphi$) for k^{th} harmonic component from its adjacent blades. The interblade phase angle is defined as follows:

$$\varphi = \frac{2\pi E}{N} \tag{16}$$

where N is the total number of the blades in the system and E is the engine order of the excitation on the system.

The equation of motion of a shrouded blade subjected to a periodic excitation can be expressed as follows:

$$\mathbf{m}\ddot{\mathbf{x}} + \mathbf{c}\dot{\mathbf{x}} + \mathbf{k}\mathbf{x} = \mathbf{f}_e - \mathbf{f}_N \tag{17}$$

where $\mathbf{x}$ is the nodal displacement vector, $\mathbf{m}$ is the mass matrix, $\mathbf{c}$ is the damping matrix, $\mathbf{k}$ is the stiffness matrix, $\mathbf{f}_e$ is the external periodic excitation, and $\mathbf{f}_N$ is the nonlinear constrained force which is a nonlinear function of the relative motion at the shroud contact. The

[1] Sub-harmonic components can also be included in the approach presented in this paper. However, they are ignored in the current manuscript.

finite element model is three-dimensional and if the model contains n nodes, all the matrices will be $3n \times 3n$ matrices, and all the vectors will be $3n$-element vectors.

The external periodic excitation can be expressed as follows.

$$\mathbf{f}_e(t) = \sum_{k=0}^{m} \mathbf{f}_{e,k} e^{jk\omega t} \qquad (18)$$

where ω is the fundamental excitation frequency and $\mathbf{f}_{e,k}$ is a complex vector representing the amplitude and phase for the k^{th} harmonic component of the excitation. Here the external periodic excitation is assumed to include up to m^{th} harmonic component. It should be pointed out that except for the elements associated with those shroud contact points the other elements of $\mathbf{f}_N$ are zeros. It is clear that the nonlinear aspect of the dynamic problem is embedded in the nonlinear friction force $\mathbf{f}_N$. By using the Modal Analysis Method, the mode shape matrix can be obtained and is denoted as Φ. Using the mode shape matrix and the associated modal information, the receptance of the blade can be derived as:

$$\mathbf{r} = \left[r_{p\ell,k} \right] = \sum_{i=1}^{3n} \left(\Phi_i \Lambda_{i,k} \Phi_i^{\mathrm{T}} \right) \qquad (19)$$

and

$$\Lambda_{i,k} = \left[\left(\mathbf{k}_i - k^2 \omega^2 \mathbf{m}_i \right) + j \left(k\omega \mathbf{c}_i \right) \right]^{-1} \qquad (20)$$

where $r_{p\ell,k}$ is defined as the steady state response of the p^{th} node due to unit k^{th} harmonic excitation force at the ℓ^{th} node, Φ_i is the i^{th} mode shape, $\mathbf{m}_i$ is the i^{th} modal mass, $\mathbf{k}_i$ is the i^{th} modal stiffness, and $\mathbf{c}_i$ is the i^{th} modal damping.

In this paper, the periodic three-dimensional motion in the blade coordinate system can be represented as follows:

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \approx \begin{bmatrix} \sum_{k=0}^{m} \mathbf{A}_{x,k} e^{jk\omega t} \\ \sum_{k=0}^{m} \mathbf{A}_{y,k} e^{jk\omega t} \\ \sum_{k=0}^{m} \mathbf{A}_{z,k} e^{jk\omega t} \end{bmatrix} \qquad (21)$$

where ω is the fundamental oscillating frequency, and $\mathbf{A}_{i,k}$, $i = x, y, z$, are the complex vectors representing the amplitude and phase angle of the k^{th} harmonic component along tangential, axial and radial axes. For a shrouded blade, several pairs of shroud contact points can be defined. For each pair of shroud contact points, one is on the right and the other left and their motions are denoted as $\left[\mathbf{x}_r \quad \mathbf{x}_\ell \right]^{\mathrm{T}}$. For convenience, this vector can be arranged as:

$$\left[\mathbf{x}_r \quad \mathbf{x}_\ell \right]^{\mathrm{T}} = \left[\left(\mathbf{x}_r \quad \mathbf{x}_\ell \right)_k \right]^{\mathrm{T}} = \left[\mathbf{x}_{r,0} \; \mathbf{x}_{\ell,0} \; \mathbf{x}_{r,1} \; \mathbf{x}_{\ell,1} \cdots \mathbf{x}_{r,m} \; \mathbf{x}_{\ell,m} \right]^{\mathrm{T}} \qquad (22)$$

Relative Motion at Shroud Contact

Since the shrouded blade system is assumed tuned, the condition of cyclic symmetry can be applied when deriving the relative motion of a shroud contact. Take the relative motion between the point B of the right shroud and the point A of the left shroud in Figure 1 as an example. First, the motions of the two contact points of the first shrouded blade (left one) are defined and they are $\left[\mathbf{x}_r \quad \mathbf{x}_\ell \right]^{\mathrm{T}}$. Therefore, the motion of point B is now $\mathbf{x}_r$ and the motion of point A differs from $\mathbf{x}_\ell$ with the proportional interblade phase angle ($k\varphi$) for the k^{th} harmonic component. As a result, the relative motion of the two neighboring shrouds, $\mathbf{w}_r$, can be derived as

$$\mathbf{w}_r = \mathbf{T}_1 \left[\left(\mathbf{x}_r \quad \mathbf{x}_\ell \right)_k \right]^{\mathrm{T}}, \quad k = 0, 1, \cdots, m \qquad (23)$$

where $\mathbf{T}_1$ is the interblade relative displacement transformation matrix.

$$\mathbf{T}_1 = \left[diag\left(\mathbf{T}_{1,0}, \mathbf{T}_{1,1}, \cdots, \mathbf{T}_{1,m} \right) \right] \qquad (24)$$

$$\mathbf{T}_{1,k} = \left[\mathbf{I}_{3\times3} \quad -e^{-jk\varphi} \mathbf{I}_{3\times3} \right], \quad k = 0, 1, \cdots, m \qquad (25)$$

It is worthy noting that only the relative motion at the right shroud contact point is derived. Since the shrouded blade system is assumed tuned, the relative motion and the resulting constrained force of the left shroud contact point can be related to those of the right shroud contact point by using the condition of cyclic symmetry. Since both $\mathbf{x}_r$ and $\mathbf{x}_\ell$ are periodic motions, the resulting relative motion $\mathbf{w}_r$ also has periodic trajectory in the 3D space.

Since this relative motion $\mathbf{w}_r$ is in the blade coordinate system, it can be transformed to the shroud coordinate system by using the transformation defined in equation (1):

$$\mathbf{u}_r = \mathbf{T}_{BS}^{\mathrm{T}} \mathbf{w}_r \qquad (26)$$

$$\mathbf{T}_{BS} = \left[diag\left(\mathbf{T}_{0,0}, \mathbf{T}_{0,1}, \cdots, \mathbf{T}_{0,m} \right) \right], \mathbf{T}_{0,k} = \mathbf{T}_0, \quad k = 0, 1, \cdots, m \qquad (27)$$

where $\mathbf{T}_{BS}$ is the coordinate transformation matrix for 3D periodic relative motion. The 3D relative motion in the shroud coordinate system can be expressed as follows:

$$\mathbf{u}_r = \begin{bmatrix} v \\ u \\ w \end{bmatrix} \approx \begin{bmatrix} \sum_{k=0}^{m} \mathbf{A}_{v,k} e^{jk\omega t} \\ \sum_{k=0}^{m} \mathbf{A}_{u,k} e^{jk\omega t} \\ \sum_{k=0}^{m} \mathbf{A}_{w,k} e^{jk\omega t} \end{bmatrix} \qquad (28)$$

where $\mathbf{A}_{i,k}$, $i = v, u, w$, are the complex vectors representing the amplitude and phase angle of the k^{th} harmonic component along v, u and w axes.

Constrained Force at Shroud Contact

After the decomposition, the u and w components of the relative motion follow a periodic trajectory on the contact plane and can induce the stick-slip friction, while the v component can cause the normal load across the interface to vary dynamically. It is apparent that the u and w motions are coupled together when inducing stick-slip friction. Using the 3D contact model proposed by Yang and Menq (1998), the constrained force at the right shroud contact point can be determined. By using Fast Fourier Transform, the constrained force can be approximated by a series of harmonic functions and can be expressed as follows:

$$\mathbf{p}_r = \begin{bmatrix} \mathbf{p}_{rv}(v) \\ \mathbf{p}_{ru}(v,u,w) \\ \mathbf{p}_{rw}(v,u,w) \end{bmatrix} \approx \begin{bmatrix} \sum_{k=0}^{m} \mathbf{p}_{rv,k} e^{jk\omega t} \\ \sum_{k=0}^{m} \mathbf{p}_{ru,k} e^{jk\omega t} \\ \sum_{k=0}^{m} \mathbf{p}_{rw,k} e^{jk\omega t} \end{bmatrix} \tag{29}$$

where $p_{ri,k}$, $i = v, u, w$, are the complex Fourier coefficients of the k^{th} harmonic component along v, u and w axes. Then, the constrained force can be transformed back to the blade coordinate system:

$$\mathbf{f}_r = \mathbf{T}_{BS}\mathbf{p}_r \tag{30}$$

Furthermore, the constrained forces at a pair of shroud contact points can be related to the force at the right shroud contact point using the interblade relative displacement transformation matrix.

$$\begin{bmatrix} \mathbf{f}_r \\ \mathbf{f}_\ell \end{bmatrix} = \mathbf{T}_1^H \mathbf{f}_r \tag{31}$$

where $\mathbf{T}_1^H$ is the complex conjugate transpose of $\mathbf{T}_1$.

Nonlinear Algebraic Equations

When the blade is constrained by its neighboring blades through shroud contacts, the resulting constrained forces are characterized by the displacements of a pair of contact points, $[\mathbf{x}_r \quad \mathbf{x}_\ell]^T$, and they can be considered as feedback forces that influence the response of the blade. This feedback effect along with the contact kinematics is shown in Figure 5. From the nonlinear feedback loop shown in Figure 5, it is evident that in the calculation of nonlinear forced response of a shrouded blade, all the linear degrees of freedom can be condensed to receptance and the modeling of friction contact can be separated from the complex structure model. This approach results in a set of nonlinear algebraic equations, which can be formulated as follows:

$$\mathbf{u}_r = \mathbf{T}_{BS}^T \mathbf{T}_1 \left\{ \mathbf{r}_{ce}\mathbf{f}_e - \mathbf{r}_{cc}\mathbf{T}_1^H \mathbf{T}_{BS}\mathbf{p}_r(\mathbf{u}_r) \right\} \tag{32}$$

where $\mathbf{r}_{ce}$ is the receptance at the shroud contact points due to unit harmonic excitation force, and $\mathbf{r}_{cc}$ is the receptance at the shroud contact points due to unit constrained force. For simplicity of demonstration, each shrouded blade is assumed to contain a pair of shroud contact points. By using the Fast Fourier Transform, the constrained force can be approximated by harmonic functions having the same fundamental frequency as the external periodic excitation, and its amplitude and phase are nonlinear functions of the relative motions of the pair of contact points. By using the Multi-Harmonic Balance Method, the nonlinear algebraic equations become

$$\mathbf{u}_{r,k} = {}^T\mathbf{r}_{ce,k}\mathbf{f}_{e,k} - {}^T\mathbf{r}_{cc,k}\mathbf{p}_{r,k}(\mathbf{u}_r), \quad k = 0,1,2,\cdots m \tag{33}$$

where

$${}^T\mathbf{r}_{ce,k} = \mathbf{T}_{0,k}^T \mathbf{T}_{1,k}\mathbf{r}_{ce,k} \tag{34}$$

$${}^T\mathbf{r}_{cc,k} = \mathbf{T}_{0,k}^T \mathbf{T}_{1,k}\mathbf{r}_{cc,k}\mathbf{T}_{1,k}^H \mathbf{T}_{0,k} \tag{35}$$

This set of nonlinear algebraic equations has the unknown $\{\mathbf{u}_{r,k}\}$, and can be solved iteratively by using Newton-Raphson algorithm. With the solution $\{\mathbf{u}_{r,k}\}$, the constrained force can be obtained by using equation (29). The periodic response of the shrouded blade system can be calculated by using the resulting constrained force together with the receptance.

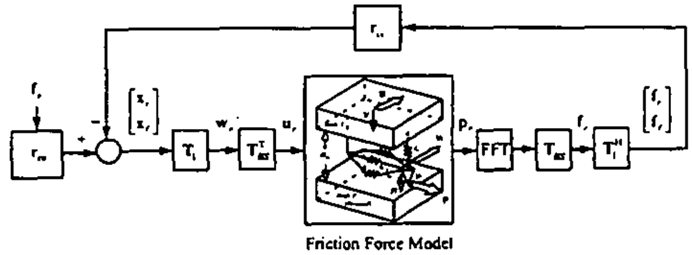

Figure 5 Nonlinear feedback loop of a shrouded blade.

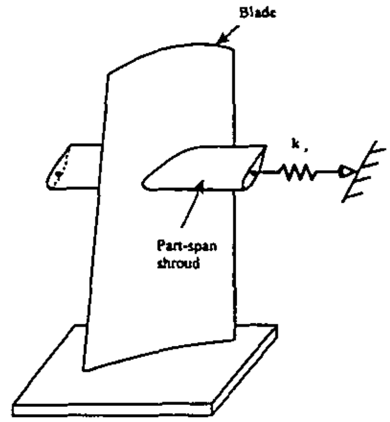

Figure 6 A blade with shroud-to-ground friction damper.

Comparison with Direct Time Integration Method

In order to verify the solution procedure presented in this paper, the predicted results are compared to those of direct time integration method. Since it is very time consuming when using direct time integration method for a bladed system, a simplified system is considered. A blade with shroud-to-ground friction damper, as shown in Figure 6, is tested to validate the proposed solution procedure. In this test, the 3D shroud contact model is employed to predict the periodic response of a shrouded blade, which is constrained by the ground. In this study, the first five vibration modes of the shrouded blade are employed to calculate the receptances and three harmonic terms are included in the calculation of nonlinear forced response. Various levels of preload, ranging from fully separate to fully stuck case, are applied and the predicted results are shown as solid curves in Figure 7, along with discrete data points, which are the results of the direct time integration method. In the figure, the frequency is normalized with respect to the first mode natural frequency, and the amplitude is normalized with respect to the peak value of the fully separation case. It is seen that the Multi-Harmonic Balance Method can accurately predict the periodic response of the shrouded blade when the shroud contact is either preloaded or having an initial gap. In the same figure, the predicted resonance response for the case $n_0 = -5000$ using single-term Harmonic Balance Method is also compared. It is apparent that in this case the nonlinear spring force of the shroud constraint exhibits a hardening effect that causes a jump phenomenon and the single-term Harmonic Balance Method can not accurately predict the resonance response.

Periodic Response of A Shrouded Blade System

The proposed method is applied to predict the periodic response of a shrouded blade system. In this shrouded blade system, the first twenty vibration modes are employed in the analysis.

Shift of Resonant Frequency

It is known that when changing the preload there exist two limit cases, namely the fully separate case and fully stuck case (Yang and Menq, 1998). Since the nonlinear contact force does not appear in the analysis, both cases are linear problems. The fully separate case occurs when the interface has such a large initial gap that the vibrating neighboring shrouds can not make contact with each other. Since the contact force is not present, the resonant response corresponding to the natural frequencies of the system can be clearly seen. On the other hand, when the preload of the interface exceeds a level depending on the external excitation, the shroud contact interface remains fully stuck. In this case, the shroud contact does not dissipate energy. However, it provides additional stiffness, which arises from the shroud constraint, to the system to cause higher resonant frequencies.

The frequency shifts of the first three vibration modes of the shrouded blade system subjected to nodal force external excitation are shown in Figure 8. In the figure the frequency is normalized with respect to the first mode natural frequency, and the amplitude is normalized with respect to the peak of the first mode for the fully separation case. Since the resulting responses along the three axes are similar, only the normalized amplitude of the response along the axial direction is presented in the figure. In the first vibration mode, the resonant peak shifts from fully separation at normalized frequency 1.0 to fully stuck at normalized frequency 2.08, and the amplitude of the peak resonance is reduced by 54.5%. For the second vibration mode, the resonant peak shifts from fully separation at normalized frequency 2.14 to fully stuck at normalized frequency 4.79, and the amplitude of the peak resonance is reduced by 93.3%. For the third vibration mode, the resonant peak shifts from fully separation at normalized frequency 2.33 to fully stuck at normalized frequency 6.85, and the amplitude of the peak resonance is reduced by 89.1%.

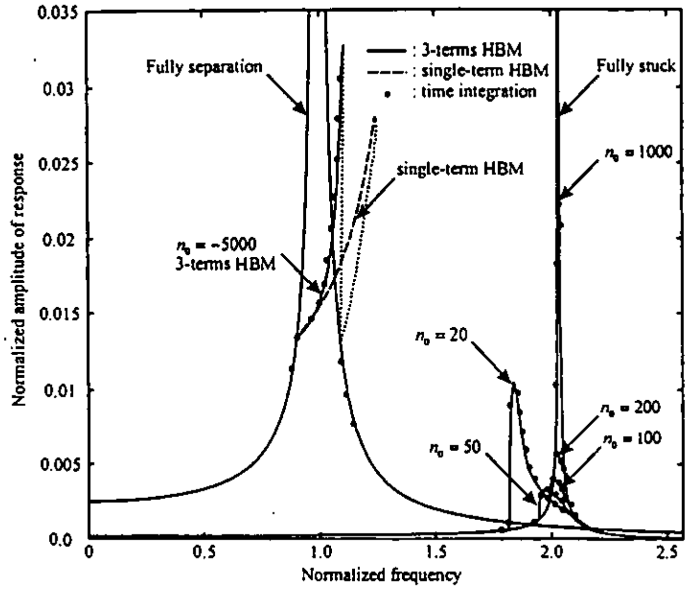

Figure 7 Periodic response of a blade with shroud-to-ground friction damper.

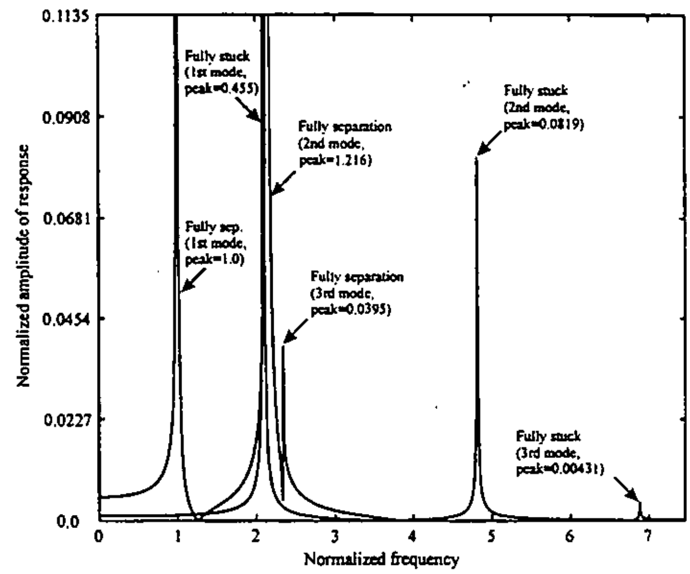

Figure 8 Periodic response of a shrouded blade system: resonant shift of resonant frequency.

Reduction of Resonant Peak

In between the two linear cases, the constraint force consists of nonlinear friction force and the variable normal load. The significance of the variation of the contact normal load depends on the direction of the resulting relative motion at the shroud contact and the orientation of the contact plane. If the variation of the contact normal load is not significant, the effect of a shroud constraint is not very different from that of a platform damper. This is demonstrated by the attenuation effect of the induced friction on the resonant response of the first vibration mode. Figure 9 shows the tracking curves of the first vibration mode of the shrouded blade system when changing the contact preload. Two sets of curves are shown. The solid curves are the predicted results using the 3-terms Harmonic Balance Method and the dashed curves the single-term Harmonic Balance Method. It is seen that as the preload increases, the resonant peak decreases until the minimum response is reached at $n_0 = 2$. Beyond this preload, the damping effect tends to reduce gradually towards the fully stuck case. The preload that gives the minimum response is known as the optimal preload.

Since the induced friction force has super-harmonic components, it is possible that the super-harmonics of the shrouded blade system can be excited and internal resonance can occur. As can be seen in Figure 9, when the preload is 0, the internal resonance can be observed at normalized frequencies 0.75 and 1.45 based on the results using 3-terms Harmonic Balance Method. It appears that the single-term Harmonic Balance Method tends to over-estimate the resonant peak although for the first vibration mode the problem is not obvious.

Jump Phenomenon

In addition to its influence on the friction characteristic, the variable normal load can directly impose nonlinear stiffness on the system. This nonlinear stiffness arises from the intermittent separation of the contact surface during the course of vibration. It has been known that this nonlinearity can result in a multi-valued response that can lead to a jump phenomenon (Thomson, 1988). A multi-valued response can be obtained by using the standard continuation technique (Allgower and Georg, 1990). In this study, a jump phenomenon can be observed at the third vibration mode, as shown in Figure 10, when a moderate preload ($n_0 = 2$) is applied. The increase in the resonant amplitude causes the preloaded interface to separate, and as a result, the interface can not provide stiffness to the system temporarily. The overall effect of the temporary separation is similar to the effect of a "softening spring" that gives rise to the response with a resonance peak bending towards lower frequencies. It should also be noted that one of the multiple solutions from the harmonic balance method shown as the dotted curve is unstable (Thomson, 1988); while separated by the unstable response, the stable response consists of two curves, which are referred to as the upper and lower branches. In this figure, the resonant response predicted by the single-term Harmonic Balance Method is compared to that by the 3-terms Harmonic Balance Method. It is seen that the single-term Harmonic Balance Method well over-estimates the resonant response at the region near jump. Furthermore, the internal resonance can be clearly observed from the predicted results using 3-terms Harmonic Balance Method.

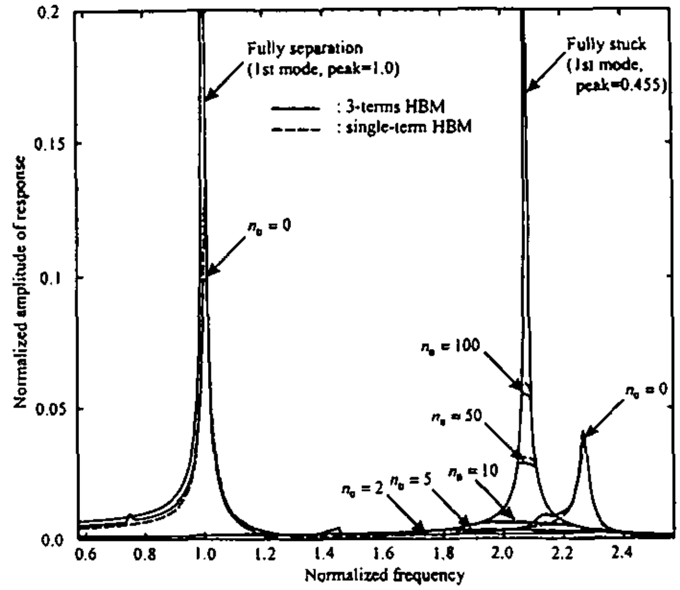

Figure 9 Periodic response of a shrouded blade system: attenuation of resonant amplitude.

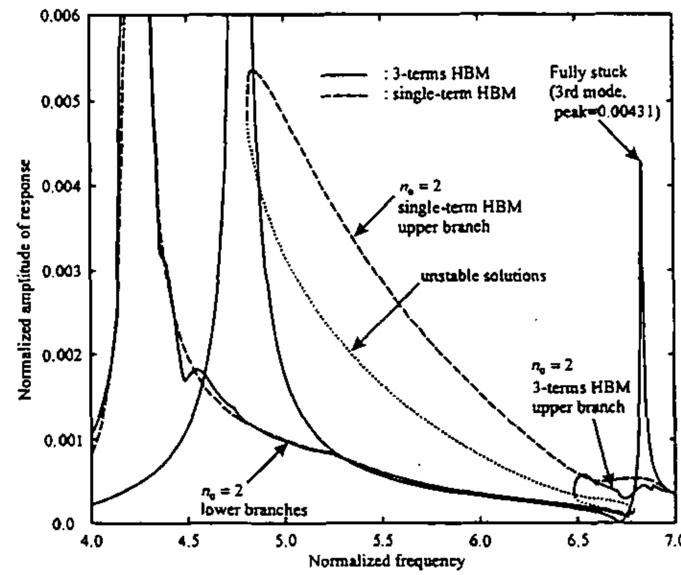

Figure 10 Periodic response of a shrouded blade system: jump phenomenon at third vibration mode.

Conclusion

In this paper, a 3D shroud contact model is employed to predict the periodic response of blades having 3D nonlinear shroud constraint. When subjected to periodic excitation, the resulting relative motion at the shroud contact is assumed to be periodic in three-dimensional space. Based on the 3D shroud contact model, analytical criteria are used to determine the transitions between stick, slip, and separation of the contact interface and are used to simulate hysteresis loops of the induced constrained force, when experiencing periodic relative motion. The constrained force can be considered as a feedback force that influences the response of the shrouded blade. By using the Multi-Harmonic Balance Method along with Fast Fourier Transform, the constrained force can be approximated by a series of harmonic functions so as to predict the periodic response of a shrouded blade. This approach results in a set of nonlinear algebraic equations, which can be solved iteratively to yield the periodic response of blades having 3D nonlinear shroud constraint.

The predicted nonlinear response shows three distinct features: (1) shifted resonant frequency due to the additional spring constant introduced by the shroud constraint, (2) damped resonant response due to the additional friction damping introduced by frictional slip, (3) multi-valued response leading to a jump phenomenon due to intermittent interface separation. The predictive ability of the proposed approach has important implications to the design of the shroud contact. In the design of the shroud contact, the preload is one of the important parameters to control the effectiveness of the shroud contact. Since the attenuation effect of the shroud contact on resonant vibration can be accurately predicted over a wide range of preload using the proposed approach, the designer can achieve the optimal preload to maximize the performance of the shroud contact in dissipating vibratory energy. Moreover, the proposed approach can also facilitate the design of shroud angle, which is another important parameter to be considered in the design of the shroud contact.

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References

Allgower, E. L. and Georg, K., 1990, "Introduction to Numerical Continuation Methods," Springer-Verlag, Berlin, Heidelberg, New York.

Griffin, J. H. and Menq, C. H., 1991, "Friction Damping of Circular Motion and its Implications to Vibration Control," *Transactions of the American Society of Mechanical Engineers, Jaurnal of Vibration and Acoustics* Vol. 113, pp. 225-229.

Menq, C. H., Chidamparam, P. and Griffin, J. H., 1991, "Friction Damping of two-dimensional Motion and Its Application in Vibration Control," *Journal of Sound and Vibration* Vol. 144 No, 3, pp. 427-447.

Menq, C. H., Griffin, J. H. and Bielak, J., 1986, "The Influence of a Variable Normal Load on the Forced Vibration of a Frictionally Damped Structure," *Transactions of the American Society of Mechanical Engineers, Journal of Engineering for Gas Turbines and Power* Vol. 108, pp. 300-305.

Menq, C. H., and Yang, B. D., 1998, "Nonlinear Spring Resistance and Friction Damping of Frictional Constraint Having Two-Dimensional Motion," *Journal af Sound and Vibration*, Vol. 217, No. 1, pp. 127-143.

Thomson, W. T., 1988, "Theory of Vibration with Applications," Prentice Hall, New Jersey.

Yang, B. D. and Menq, C. H., 1997, "Modeling of Friction Contact and Its Application to the Design of Shroud Contact," *ASME Journal of Engineering for Gas Turbines and Power*, Vol. 119, No. 4, pp. 958-963, October 1997.

Yang, B. D., Chen, J. J., and Menq, C. H., 1998, "Prediction of Resonant Response of Shrouded Blades with 3D Shroud Constraint," ASME paper No. 98-GT-485, *The 43rd ASME Gas Turbine & Aeroengine Technical Congress, Exposition and User Symposium*, June 2-5, 1998, Stockholm, Sweden..

Yang, B. D. and Menq, C. H., 1998, "Characterization of 3D Contact Kinematics and Prediction of Resonant Response of Structures Having 3D Frictional Constraint," *Journal of Sound and Vibration*, Vol. 217, No. 5, pp. 909-925.