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PREDICTION OF TWO-PHASE CRITICAL FLOW RATE

By
S. LEVY

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ATOMIC POWER EQUIPMENT DEPARTMENT
GENERAL ELECTRIC
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APPROVED BY: ______________________
E. Janssen, Project Engineer
Two-Phase Pressure Losses

Approved By: ______________________
D. H. Imhoff, Manager
Engineering Development

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SUMMARY

An analytical model to predict two-phase critical flow rate is proposed. The model is based upon thermal equilibrium, a "lumped" treatment of the two-phase velocity (each phase is represented by a single mean velocity), and upon the neglect of frictional and hydrostatic pressure losses. A comparison of the proposed predictions with available test results and previous analyses shows that:

1. The present model agrees very well with the published test data.

2. In contrast to all other analyses, the model requires no assumption about the gas void fraction.

INTRODUCTION

When fluid escapes from a high pressure reservoir into lower pressure surroundings, the escape flow rate first increases as the differential pressure between the reservoir and the surroundings is raised. If the surroundings pressure continues to decrease, the flow rate soon reaches a maximum value which cannot be exceeded by further lowering of the pressure outside the reservoir. This "critical" flow condition has been observed in both single-phase and two-phase systems. Single-phase critical flow has been studied in details and is known to occur when the fluid escape velocity equals the velocity of sound. Its prediction is covered in most hydrodynamic books. [1] [2]

In recent years, considerable attention has been focused on the subject of two-phase critical flow. The main reason is that the critical flow condition determines the maximum possible escape rate of a high pressure fluid from the system in which it is contained. For instance, it specifies the highest rate at which coolant can be lost from the primary or secondary system of a high pressure water cooled nuclear reactor. The escape rate, in turn, fixes the pressure within the containment building or plant enclosure. It also establishes whether the reactor fuel can be uncovered in the course of the accident.

For this reason, several experimental and analytical studies of two-phase critical flow have been published in the last few years. Most of the experimental investigations have been performed with a steam-water mixture and utilize a pipe extending from a high pressure reservoir into a receiver tank. The recent test results of Isbin, Moy, and DaCruz, [3] Faletti, [4] Fauske, [5] and Zaloudek [6] are especially worthy of note because they accurately determined the pressure at the critical point from their pressure profile measurements along the pipes. They utilized a large number of pressure taps which are essential to obtaining good data, particularly at the pipe exit where rapid depressurization of the fluid occurs.

Several analytical models have also been proposed to predict two-phase critical flow. The first model developed was the homogeneous model where the two-phase flowing mixture was treated as a single phase homogeneous fluid of uniform velocity. Under these conditions, the critical flow rate G_H is defined by the same relation as in single phase flow

$$G_H^2 = - g_c \ (dP/dv_H)_S \qquad (1)$$

where P is the fluid pressure, v_H the homogeneous specific volume, and S the entropy. For homogeneous flow,

$$v_H = v_L \ (1 - x) + v_G x \qquad (2)$$

Values of the homogeneous flow rate G_H can be readily computed from Equations (1) and (2) and the thermodynamic properties of the fluids involved. Convenient charts of G_H for steam-water mixtures have, for instance, been published by Tippets.[7] The predicted values are, however, considerably lower than the measured flow rates.

Linning[8] made the first attempt to improve upon the homogeneous model. He utilized a simplified annular model of uniform liquid and gas velocity, and defined the critical flow condition as that where the two-phase energy, continuity, and momentum equation become incompatible. His prediction still falls well below the experimental results.

More recently, Isbin, Moy, and Da Cruz,[3] Massena,[9] and Fauske[5] have developed improved models. All of these analyses are of the "lumped" type where each of the two phases is represented by a single mean velocity. If friction and head losses are neglected, the critical flow rate is given by

$$G_S^2 = - g_c \left(dP/dv_M\right)_S \tag{3}$$

where v_M is given by

$$v_M = \left(v_G x^2/\alpha\right) + v_L (1 - x)^2/(1 - \alpha) \tag{4}$$

Equations (3) and (4) specify the critical flow G_S if the relation between the gas volume fraction α and its weight fraction x is known. Isbin, Moy, and Da Cruz[3] used the Martinelli-Lockhart[10] correlation for void fraction. Massena[9] utilized a modified form of Armand's correlation.[11] These two methods are about equivalent and give fair correspondence with the test results except at low weight fraction where they deviate from the data by more than 50 percent.

Fauske noted that critical flow was attained when the pressure gradient reached a finite, but maximum, slope in the test section. He postulated that this maximization resulted from the variation of slip ratio and found that

$$\alpha = \left[1 + (1 - x)\sqrt{v_L/v_G}/x\right]^{-1} \tag{5}$$

Fauske's model agrees well with the test results and is superior to all other published models especially at low gas weight fraction.

The purpose of this paper is to present an analytical solution of two-phase critical flow. It is based upon a lumped model. The model, however, requires no additional assumption about the gas volume fraction. The analytical model and its basic equations are presented first; next, the predicted values are compared with available test results.

DERIVATION OF EQUATIONS

The present analysis is based upon a lumped model where the liquid and gas are each represented by a single mean velocity, U_L and U_G. If the static pressure drop is the same for the two phases, the following momentum equations can be written[12]

$$dP + \rho_L U_L \, dU_L / g_c = (dP/dy)_{LTP} \, dy - \rho_L \sin\theta \, dy \quad (6)$$

$$dP + (1/g_c A_G) \, d(A_G \rho_G U_G^2) + (U_L / g_c A_G) \, d(A_L \rho_L U_L) = (dP/dy)_{GTP} \, dy - \rho_G \sin\theta \, dy \quad (7)$$

If frictional and head losses are neglected, Equations (6) and (7) can be multiplied respectively by A_L and A_G and added to give

$$dP + (1/g_c A) \, d\left[A_G \rho_G U_G^2 + A_L \rho_L U_L^2\right] = 0 \quad (8)$$

Introduction of the gas weight and void fraction from

$$\left.\begin{aligned} \rho_L A_L U_L &= GA(1-x) \\ \rho_G A_G U_G &= GAx \\ (A_G/A) &= \alpha \, ; \, (A_L/A) = 1-\alpha \end{aligned}\right\} \quad (9)$$

gives

$$dP + (1/g_c) G^2 \, d\left[(V_G x^2/\alpha) + V_L (1-x)^2/(1-\alpha)\right] = dP + (1/g_c) G^2 \, dV_M = 0$$

The above equation can be rewritten to yield

$$G^2 = g_c (dP/dV_M) \quad (10)$$

The total mass flow rate per unit area will be maximum when the process is isentropic and Equation (10) reduces to Equation (3).

If we now subtract Equation (6) from Equation (7) and utilize the same assumptions as above, there results

$$(G^2/g\rho_L) \, d\left[(1-x)^2/(1-\alpha) + x^2 \rho_L/\alpha\rho_G - (1/2)(1-x)^2/(1-\alpha)^2\right] = 0 \quad (11)$$

Equation (11) can be integrated with $\alpha = O$ at $x = O$ so that [12]

$$x = \frac{\alpha\,(1-2\alpha) + \alpha\sqrt{(1-2\alpha)^2 + \alpha\left[(2\rho_L/\rho_G)\,(1-\alpha)^2 + \alpha\,(1-2\alpha)\right]}}{(2\rho_L/\rho_G)\,(1-\alpha)^2 + \alpha\,(1-2\alpha)} \tag{12}$$

In other words, for the special case of no friction and head loss, no further assumptions about α are needed and the relation between α and x is given by Equation (12) and the momentum model of reference (12).

Under these conditions, the critical flow G_S can be calculated readily. The entropy S of the two-phase fluid is first defined as

$$S = S_L\,(1-x) + S_G\,x \tag{13}$$

so that at constant entropy

$$dS = (\partial S/\partial P)_x\,dP + (\partial S/\partial x)_P\,dx = O \tag{14}$$

and

$$dx = -\left[(\partial S/\partial P)_x/(\partial S/\partial x)_P\right]dP = -\left[x(dS_G/dP) + (1-x)(dS_L/dP)\right]/(S_G - S_L) \tag{15}$$

Similarly,

$$dV_M = (\partial V_M/\partial P)_x\,dP + (\partial V_M/\partial x)_P\,dx \tag{16}$$

and substitution of Equation (15) gives

$$(d\,V_M/dP)_S = (\partial V_M/\partial P)_x - (\partial V_M/\partial x)_P\left[x\,(dS_G/dP) + (1-x)(dS_L/dP)\right]/(S_G - S_L) \tag{17}$$

The partial derivatives $(\partial V_M/\partial P)_x$ and $(\partial V_M/\partial x)_P$ are obtained by differentiating Equations (4) and (12)

$$(\partial V_M/\partial x)_P = \frac{\left[V_L(1-x)^2\right]\left[2(1-x)\ V_L/(1-\alpha) - 2x\,V_L/\alpha - (1-x)\,V_L/(1-\alpha)^2\right]}{(1-\alpha)^3\left[V_L\,(1-x)^2/(1-\alpha)^2 - x^2\ V_G/\alpha^2 - V_L(1-x)^2/(1-\alpha)^3\right]} - \frac{V_L\,(1-x)}{(1-\alpha)^2} \quad (18)$$

$$(\partial V_M/\partial P)_x = \frac{\left[V_L\,(1-x)^2\,x^2/\alpha\right]\left[(V_G/V_L)\ (dV_L/dP) - (dV_G/dP)\right]}{(1-\alpha)^3\left[V_L\,(1-x)^2/(1-\alpha)^2 - x^2\,V_G/\alpha^2 - V_L(1-x)^2/(1-\alpha)^3\right]} +$$

$$(1/2)\,(dV_L/dP)\left[1 + (1-x)^2/(1-\alpha)^2\right] \quad (19)$$

Equations (3), (12), (17), (18), and (19) define the desired solution. The calculation method proceeds as follows:

1. For given values of P, V_L, V_G, and corresponding derivatives, a value of α is assumed, and x is calculated from Equation (12).

2. The derivatives $(\partial V_M/\partial x)_P$ and $(\partial V_M/\partial P)_x$ are computed from Equations (18) and (19).

3. The total derivative $(dV_M/dP)_S$ is obtained from Equation (17) and its substitution in Equation (3) gives the critical flow rate G_S.

It should be noted that if it is desired to assume that the flow process is other than isentropic, the same method can be employed. For instance, for an isenthalpic process, all the above equations apply except for replacing the enthropy S_G by the enthalpy h_G, and S_L by h_L in Equation (17). Equation (10) will then give the corresponding isenthalpic critical flow, G_h.

DISCUSSION OF RESULTS

The proposed analysis involves three major assumptions: the neglect of frictional and head losses, thermal equilibrium, and the lumped nature of the solution. The first assumption is not serious; as noted previously, it is commonly made in critical flow models, including single-phase solutions. It is not expected to produce a significant error, since critical flow conditions occur over very short distances and the momentum interchange dominates. The second assumption excludes metastable flow systems*, i. e., critical discharge from very short tubes.[13] [14] The third assumption is more serious; the liquid and gas distributions are expected to vary along the test section and to depend upon the flow pattern at every position. Our understanding of two-phase flow is such that the appropriate velocity distributions and flow patterns cannot be predicted accurately at this time. Furthermore, the flow conditions are changing so rapidly along the test section in a critical flow test that it is difficult to foresee how an accurate prediction of flow patterns and velocities could ever be developed. The approximate lumped nature of the present model must still be recognized, even though more basic models may not yield more accurate answers as they are developed.

Critical flow rates were calculated from the proposed model for steam-water mixtures at various pressures. The results are shown in Figure 1 for isentropic and isenthalpic processes. The critical flow rates are plotted in terms of steam quality. The corresponding total reservoir enthalpy h_o is

$$h_o = h_L\ (1-x) + h_G x + (G^2/2g_c) \left[(x^3\ V_G^{\ 2}/\alpha^2) + (1-x)^3\ V_L^{\ 2}/(1-\alpha)^2 \right] (1/J) \quad (20)$$

The isentropic predictions fall above the isenthalpic ones as expected. An interesting aspect of the solutions is that they exhibit a maximum in terms of steam quality. The maximum shifts to higher steam qualities as the pressure is raised. Within the range of presently available data, the maximum occurs at steam qualities below 1 percent. At these conditions, the accuracy in determining the experimental steam quality is rather poor, and the test results cannot be expected to confirm the occurrence of the predicted maximum. This confirmation must await tests at much higher pressure.

Comparisons of the predictions with experimental results are shown on Figures 2 to 6. The agreement is good over the entire range of test results. The analysis falls slightly below the data at high steam quality and slightly above it for steam qualities below 0. 2. It should be noted that the test steam qualities are calculated on the basis of a homogeneous model so that

$$h_o = h_L\ (1-x) + h_G x + G^2\ V_H^{\ 2}/2g_c J \quad (21)$$

* This is inherent in the assumption that $\alpha = 0$ at $x = 0$.

If Equation (20) instead of Equation (21) had been used to reduce the test data, the experimental points would shift to the right, particularly those in the mid to low steam quality range. This would mean that the analytical model would tend to underpredict the critical flow rates at all steam qualities, but the departure would still vary from only 5 percent to at most 10 percent.

A comparison between the present model and previously published analyses is shown on Figure 7 for steam-water mixtures at pressures of 30 to 50 psia. In addition to the models of references 3, 5, and 9, a vapor choking model which assumes that the steam vapor travels at sonic velocity is illustrated on the same graph. It is seen again that the proposed model gives good agreement with Zaloudek data up to and including the low steam quality region. The present model is superior to the homogeneous and vapor choking model and the models proposed in references 3 and 9. It is about equivalent to the Fauske's model. The present model, however, has the advantage that once the frictional and head losses are neglected, no further assumptions about vapor slip are needed. Such an assumption was required in all previous analyses, including the homogeneous model which inherently postulates equal gas and liquid velocity.

CONCLUSIONS

1. A 'lumped' analytical model to predict two-phase critical flow rate is derived.

2. The model gives very good agreement with available experimental results.

NOMENCLATURE

A	Cross sectional area of flow, ft^2
G	Mass flow rate per unit area, $lb_m/sec\text{-}ft^2$
g_c	Constant in Newton's law of motion, $(lb_m/lb_f)(ft/sec^2)$
h	Enthalpy, Btu/lb_m
J	Mechanical equivalent of heat, $ft\text{-}lb_f/Btu$
P	Pressure, lb_f/ft^2
S	Entropy, $Btu/lb_m\ R$
U	Velocity, ft/sec
x	Vapor or gas weight fraction, nondimensional
y	Distance measured along flow direction, ft
α	Vapor or gas void fraction, nondimensional
θ	Angle of inclination of test section
ρ	Density, lb_m/ft^3

Subscripts

G	Gas or Vapor
GTP	Gas or vapor in two-phase flow
H	Homogeneous
h	Isenthalpic
L	Liquid
LTP	Liquid in two-phase flow
M	Momentum
S	Isentropic

REFERENCES

1. Liepmann, H. W. and Puckett, A. E., "An Introduction to the Aerodynamics of a Compressible Fluid", John Wiley and Sons, New York, 1947.

2. Shapiro, A. H., "The Dynamics and Thermodynamics of Compressible Fluid Flow", The Ronald Press, New York, 1953.

3. Isbin, H. S., Moy, J. E., and DaCruz, A. J. R., "Two-Phase, Steam-Water Critical Flow", A.I.Ch.E. Journal, 3, No. 3, 361, 1957.

4. Faletti, D. W., "Two-Phase Critical Flow of Steam-Water Mixtures", Ph. D. Thesis, University of Washington, 1959.

5. Fauske, H., "Critical-Two-Phase, Steam-Water Flows, Heat Transfer and Fluid Mechanics Institute", Stanford University Press, 1961.

6. Zaloudek, F.R., "The Low-Pressure Critical Flow Behavior of Steam-Water Mixtures in Constant Area Flow Passages", Hanford Atomic Products Operation Report, HW-68934, 1961.

7. Tippets, F. E., "Water Wall Rupture in a High Pressure Reactor - Hydraulic and Heat Transfer Effects", Hanford Atomic Products Operation Report, HW-40388, 1955.

8. Linning, D. L., "The Adiabatic Flow of Evaporating Fluids in Pipes of Uniform Bore", Proc. Inst. Mech. Engrs., IB, 64, 1952.

9. Massena, W. A., "Steam-Water Critical Flow Using the Separated Flow Model", Hanford Atomic Products Operation Report, HW-65739, 1960.

10. Lockhart, R. W., and Martinelli, R. C., "Proposed Correlation of Data for Isothermal Two-Phase, Two-Component Flow in Pipes", Chemical Engineering Progress, Vol. 45, 1944.

11. Armand, A. A., "The Resistance during the Movement of a Two-Phase System in Horizontal Pipes", AERE-Trans-828, March 1959.

12. Levy, S., "Steam-Slip-Theoretical Prediction from Momentum Model", Journal of Heat Transfer, May 1960.

13. Fauske, H. K., and Min, T. C., "A Study of the Flow of Saturated Freon-II Through Apertures and Short Tubes", ANL-6667, 1963.

14. Zaloudek, F. R., "The Critical Flow of Hot Water Through Short Tubes", HW-77594, 1963.

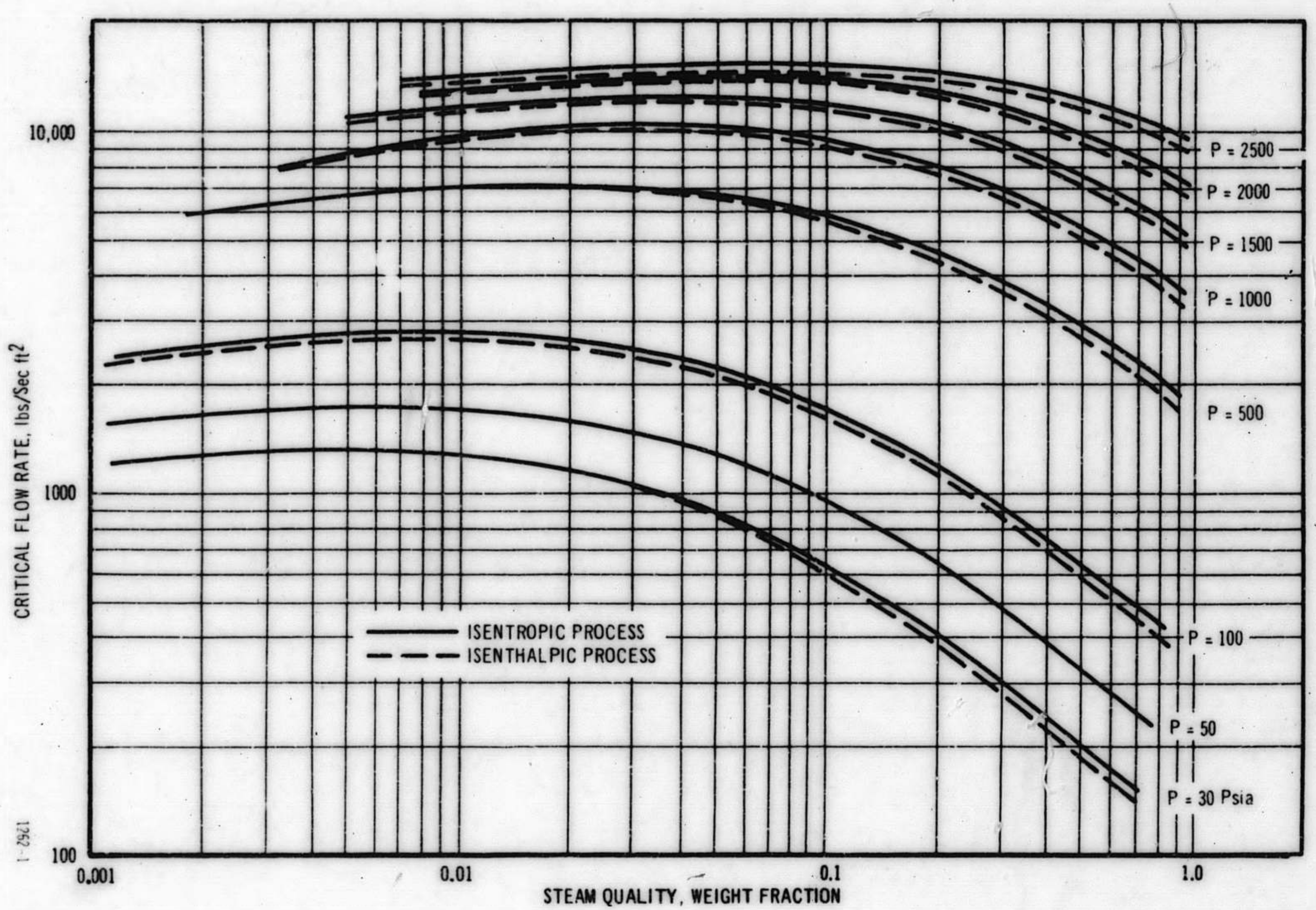

Figure 1. Critical Flow Rate Prediction For Steam-Water Mixture.

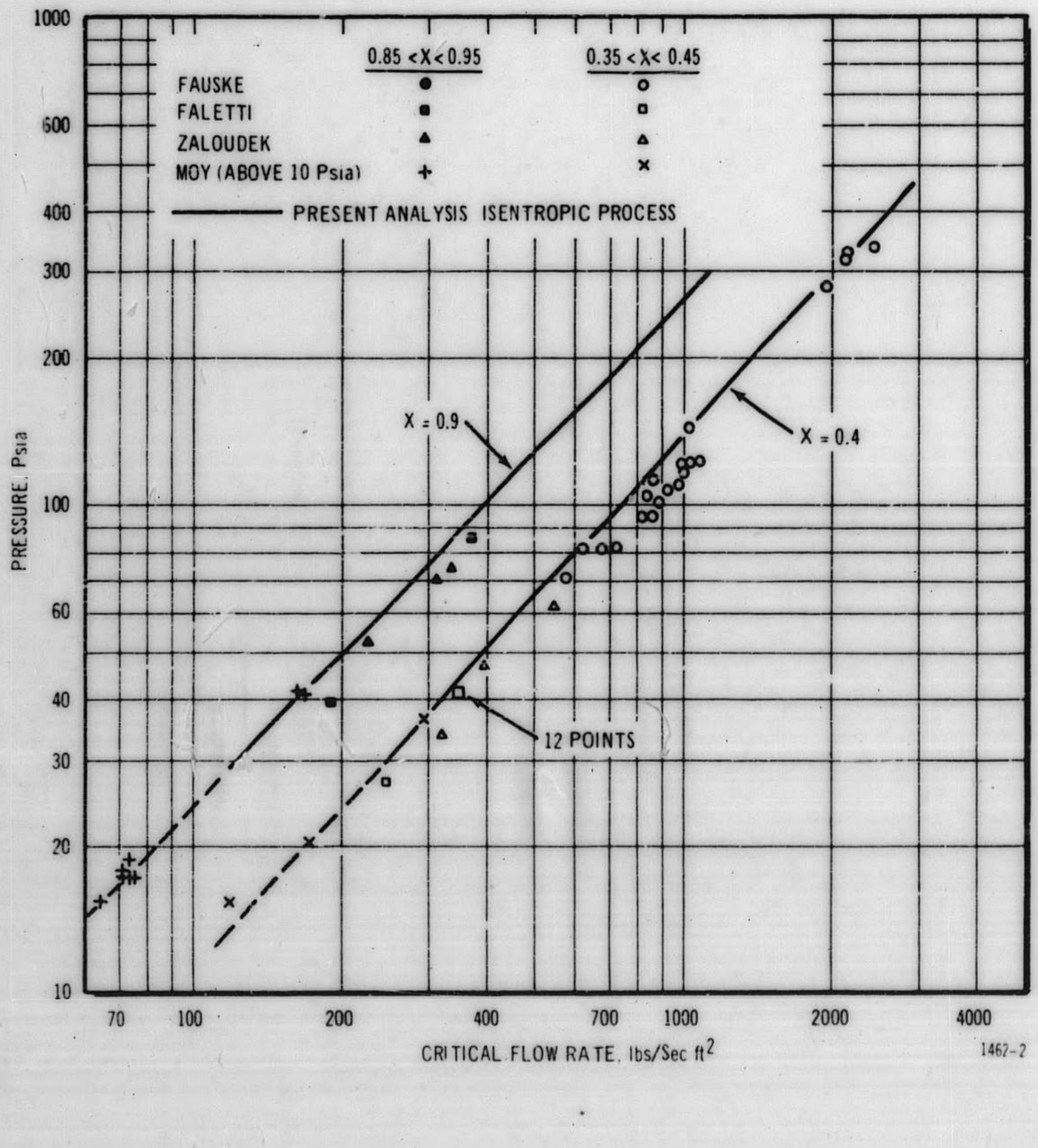

Figure 2. Comparison of Predictions with Test Results of Steam-Water Mixtures. ($0.85 < x < 0.95$ and $0.35 < x < 0.45$)

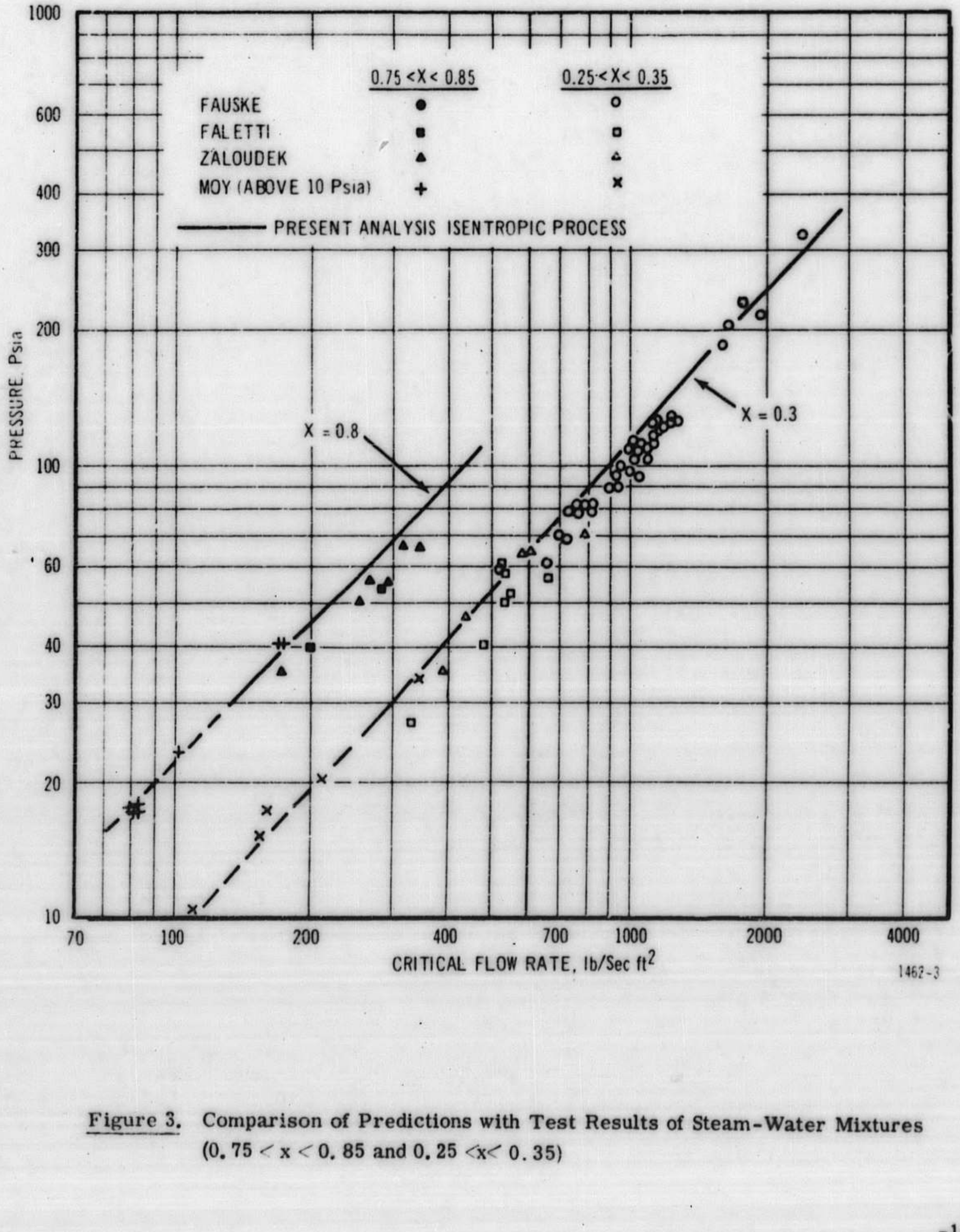

Figure 3. Comparison of Predictions with Test Results of Steam-Water Mixtures ($0.75 < x < 0.85$ and $0.25 < x < 0.35$)

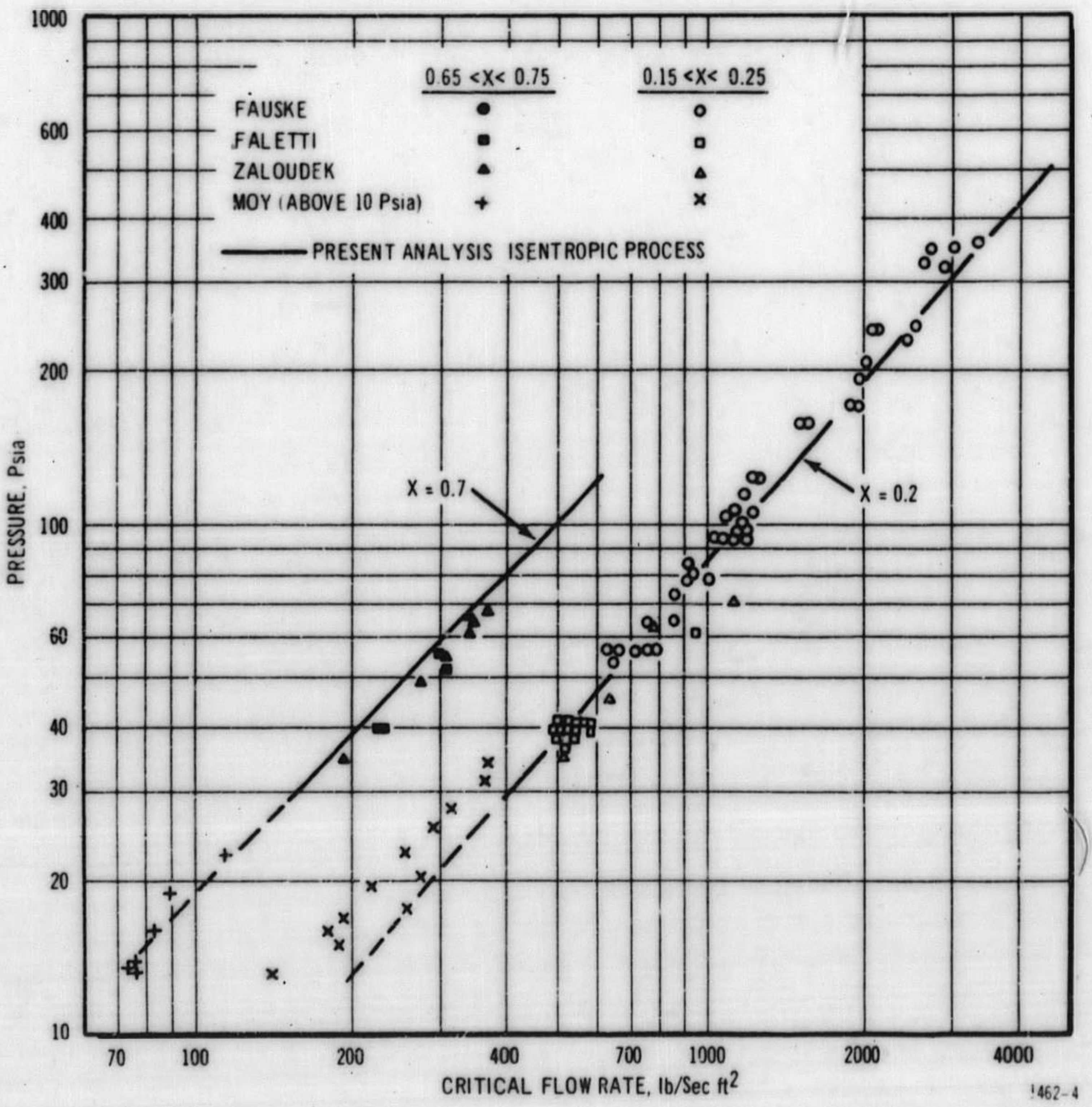

Figure 4. Comparison of Predictions with Test Results of Steam-Water Mixtures. ($0.65 < x < 0.75$ and $0.15 < x < 0.25$)

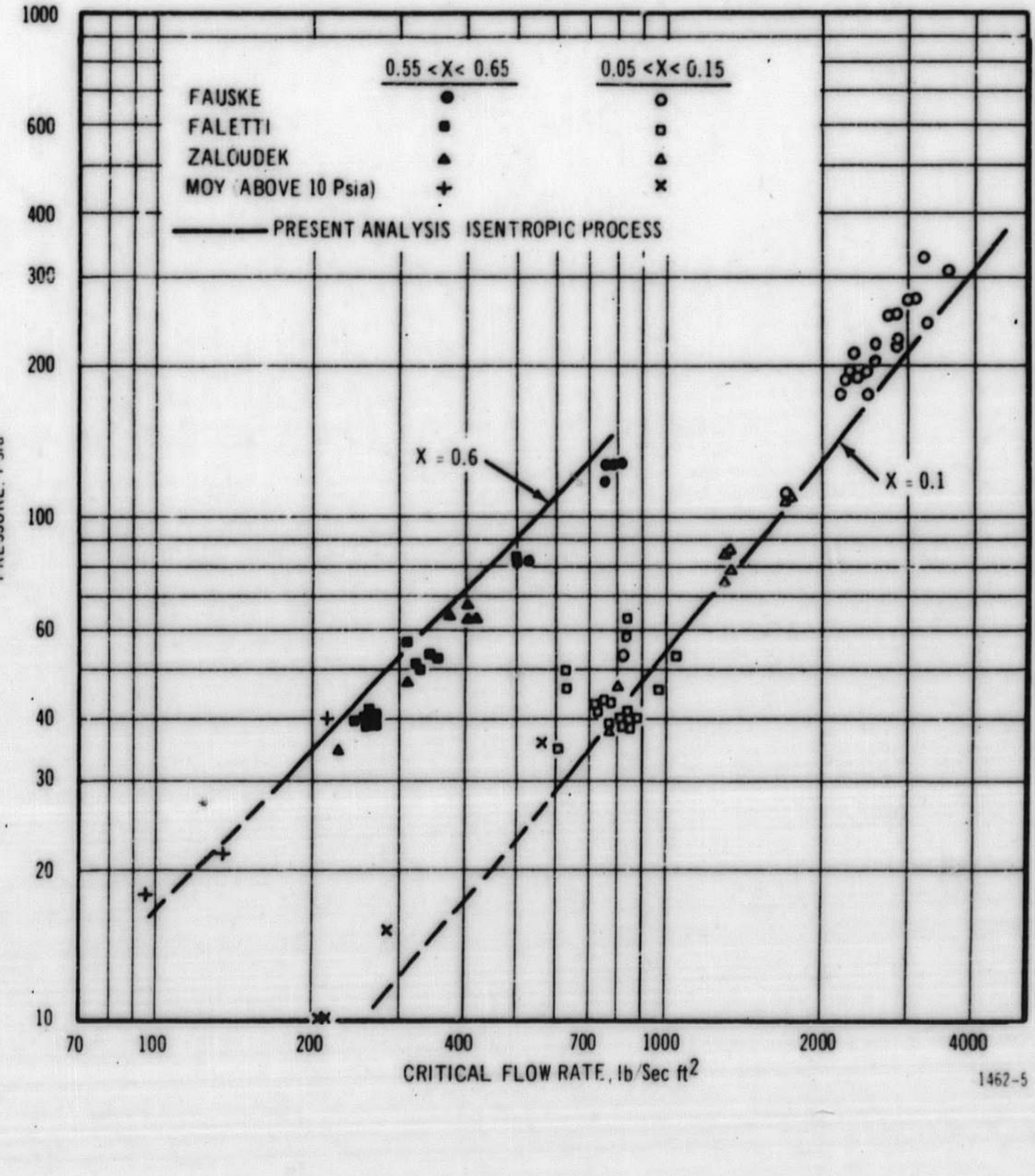

Figure 5. Comparison of Predictions with Test Results of Steam-Water Mixtures ($0.55 < x < 0.65$ and $0.05 < x < 0.15$)

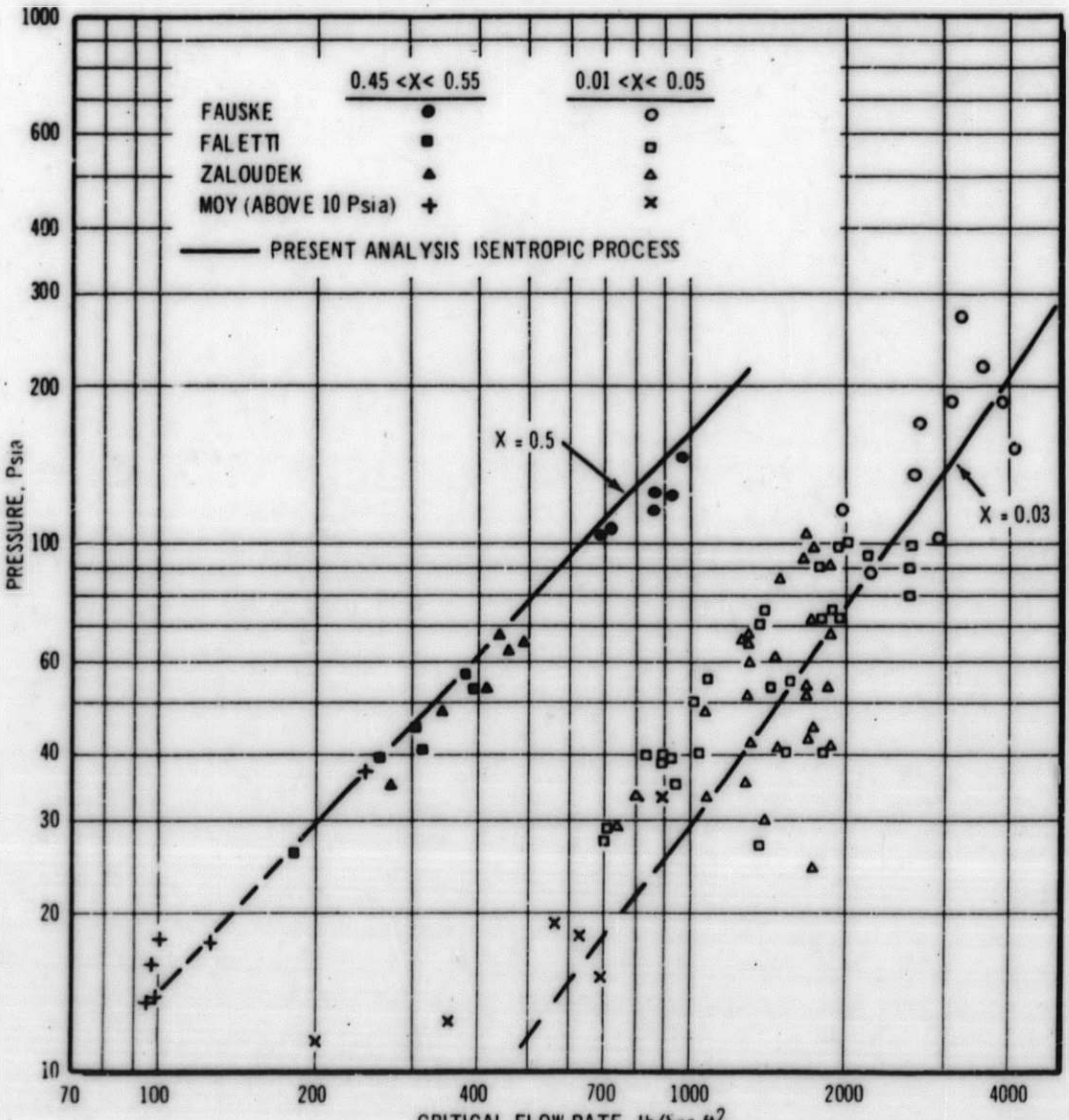

Figure 6. Comparison of Predictions with Test Results of Steam-Water Mixtures (0.45 <x< 0.55 and 0.01 <x< 0.05)

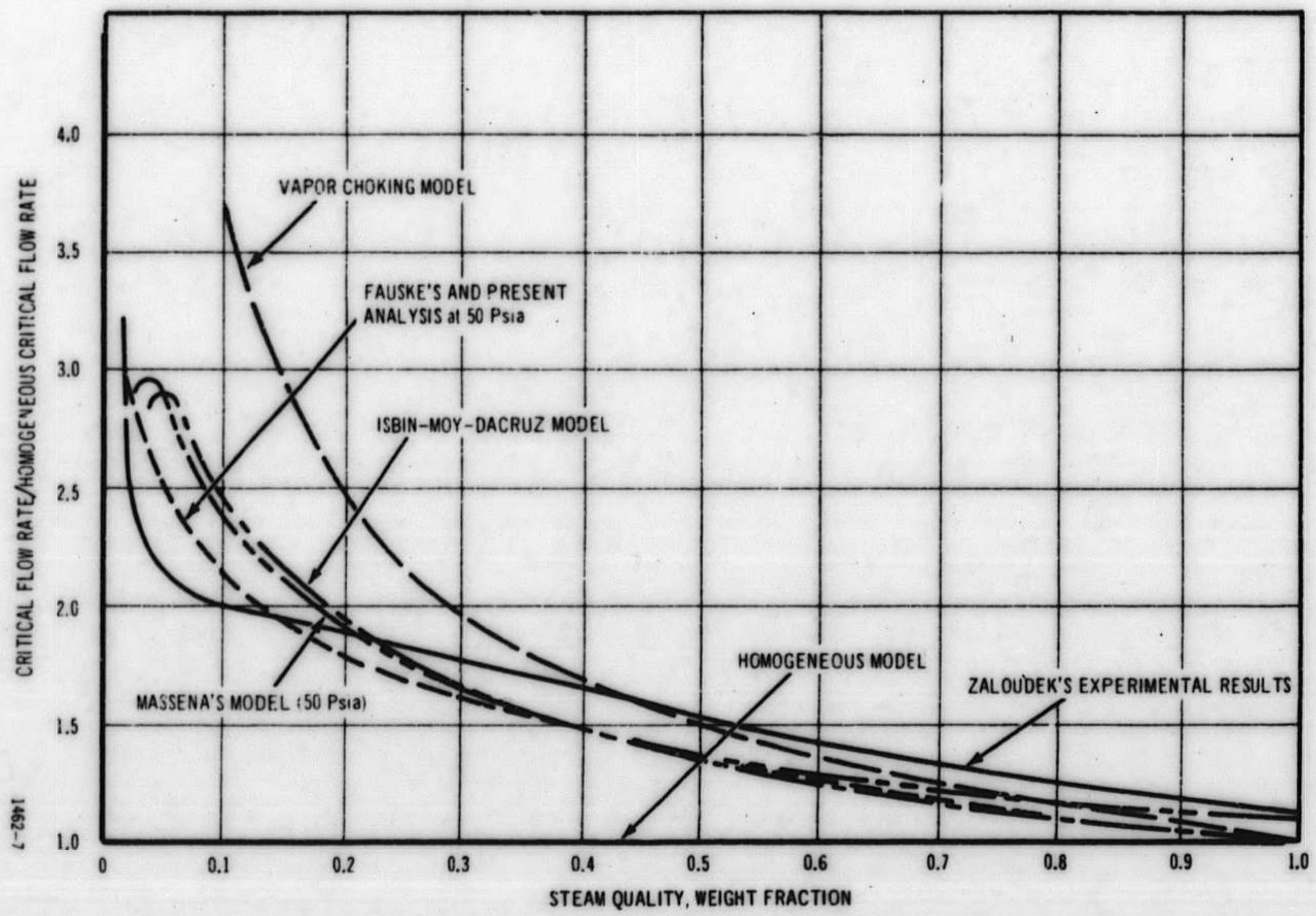

Figure 7. Comparison of Theoretical Predictions for Steam-Water Mixtures.