Predictions for energy distribution and polarization of the positron from the polarized muon decay

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Abstract

The pure lepton decay of the polarized muon is considered, accounting for a new tensor interaction which is outside of the Michel local interactions. This interaction leads to new energy distribution and polarization of the final charged lepton. The presence of such a type of interaction is strongly required for the description of the latest experimental results on the weak radiative pion decay in the full kinematic region. Assuming quark–lepton universality, predictions for a deviation from the Standard Model are made using only one new parameter. They do not contradict the present experimental data and can be verified further in the on-going experiments at PSI and TRIUMF, at least to the level of 3 standard deviations.

1 Introduction

At present only one type of vector particles is known – the gauge one. The quantum field theory of the gauge interactions has been formulated [1] and it has been proved to be unitary and renormalizable [2]. The Higgs mechanism of acquiring mass [3] shows the way of application of this theory to phenomenology [4]. The gauge particles are described by four-vector fields A_α, which are transformed under the Lorentz group as a real representation $(1/2,1/2)$. This *chirally neutral* representation can be constructed as a product of fundamental *chiral* spinor representations $(1/2,0)$ and $(0,1/2)$. The latter correspond to left-handed $\psi_L = \frac{1}{2}(1 - \gamma^5)\psi$ and right-handed $\psi_R = \frac{1}{2}(1 + \gamma^5)\psi$ two-component Weyl spinors, which are related by the discrete P-transformation of the spatial reflection or by the charge conjugation C. So, CP is an underlying symmetry of the Lorentz group. Moreover, the main feature of the Lorentz group as a relativistic generalization of Galileo's invariance is the natural introduction of two non-equivalent *left-handed* and *right-handed* Weyl spinors, which describe different particles and explicitly lead to P violation in nature.

At small transfer momenta, an exchange of the massive gauge vector particles leads effectively to contact four-fermion vector interaction. It is the well known Fermi interaction for the weak processes, which in the case of the muon decay reads

$$\mathcal{L}_V = -\frac{4G_F}{\sqrt{2}} \ (\bar{\mu}_L \gamma_\alpha \nu_\mu) \left(\bar{\nu}_e \gamma_\alpha e_L\right) + \text{h.c.}, \tag{1}$$

with the Fermi coupling constant given by $G_F/\sqrt{2} = g^2/8M_W^2$, where g is the gauge coupling constant and M_W is the mass of the gauge weak boson. This interaction preserves the helicities of the incoming and the outgoing fermions.

The Higgs particles are needed for the symmetry breaking in the Standard Model (SM). They are described by the scalar fields H, which are transformed under a spinless $(0,0)$ representation of the Lorentz group. In an extension of the SM with more than one Higgs doublet, new charged scalar particles can also contribute to the muon decay

$$\mathcal{L}_S = -g_{RR}^S \ \frac{4G_F}{\sqrt{2}} \ (\bar{\mu}_R \nu_\mu) \left(\bar{\nu}_e e_R\right) + \text{h.c.}, \tag{2}$$

with the negative dimensionless coupling constant g_{RR}^S given by $g_{RR}^S G_F/\sqrt{2} = -h^2/8M_H^2$, where h is the Yukawa coupling constant of the lepton currents to the charged Higgs boson with mass M_H. The dimensionless coupling constant g_{RR}^S is introduced in accordance to the notation of ref. [5] and determines the strength of the new interactions relative to the ordinary weak interactions (1), governed by the Fermi coupling constant G_F.

The scalar interaction (2) includes a coupling between right-handed charged leptons and left-handed neutrinos, and therefore it does not preserve the helicities of fermions. In general other scalar interactions with right-handed neutrinos can be written down, but since the neutrino masses are very tiny they do not interfere with the standard weak interaction (1) and their contributions are negligibly small.

It should be noted that the Lorentz group provides the possibility to construct *inequivalent chiral* representations $(1,0)$ and $(0,1)$, which also correspond to particles with spin 1. They can be constructed if one uses only the product either of the left-handed $(1/2,0)$ or of the right-handed $(0,1/2)$ fundamental spinors. Such particles are described by the antisymmetric second-rank tensor fields $T_{\mu\nu}$ and they interact with tensor currents. An example of the presence of the new kind of *chiral particles* in nature is the existence of the axial-vector hadron resonance $b_1(1235)$, which has only anomalous tensor interactions with quarks. The

introduction of such excitations into the Nambu–Jona-Lasinio quark model leads to a successful description of the dynamical properties of spin-1 mesons [6] and to hints that the analogous chiral particles could exist at the electroweak scale as well. An exchange of such spin-1 massive chiral bosons can effectively lead to new four-fermion tensor interactions.

Four-fermion local tensor interactions have been introduced for the muon decay in ref. [7]. However, all of them include the right-handed neutrinos and they do not interfere with the standard weak interaction (1). The local tensor interaction involving both left-handed neutrinos cannot be written down owing to the identity $(\bar{\mu}_R \sigma_{\alpha\beta} \nu_\mu)(\bar{\nu}_e \sigma_{\alpha\beta} e_R) \equiv 0$. However, if we assume that an effective tensor interaction arises from an exchange of the new chiral bosons, then its Lorentz structure should reflect the Lorentz structure of the propagator for such particles. It has been shown [8], that besides the local tensor interactions, the new non-local momentum-transfer-dependent tensor interactions should be introduced. For the muon decay this leads to a new tensor interaction that includes only left-handed neutrinos

$$\mathcal{L}_T = -g_{RR}^T \; \sqrt{2} G_F \; (\bar{\mu}_R \sigma_{\alpha\lambda} \nu_\mu) \frac{4 q_\alpha q_\beta}{q^2} (\bar{\nu}_e \sigma_{\beta\lambda} e_R) + \text{h.c.}, \qquad (3)$$

where g_{RR}^T is the new positive tensor coupling constant. Such type of interaction can interfere with the standard weak interaction (1) and it is more sensitive to experimental detection.

Moreover, the recent results of the PIBETA experiment on the radiative pion decay [9] strongly show evidence for the presence of such type of interaction between lepton and quark tensor currents [10]. If we assume the universality of the new chiral boson coupling to lepton and quark tensor currents the value of the tensor coupling constant can be fixed at $g_{RR}^T \approx 0.013$, in accordance with the explanation of the PIBETA anomaly.

Let us stress again that the most general four-fermion local Lagrangian does not contain a tensor interaction (3) and can provide no model-independent description of the experimental data on the muon decay. It has been shown [11] that the most general four-fermion Lagrangian depending on momentum transfer extends the local one just by two new tensor interactions with coupling constants g_{RR}^T and g_{LL}^T. They lead to a new parametrization even for the case of unpolarized muon decays.

The purpose of the present paper is the calculation of the energy distribution and the polarization of the positron from the decay of polarized muons in the presence of the scalar (2) and the new tensor (3) interactions, which are more sensitive to experimental detection. It will be shown how to discriminate between scalar and tensor contributions. For example, according to Michel analysis, the large transverse component P_{T_1} would indicate a non-zero η parameter and the presence of the scalar interaction (2). On the one hand, the tensor interaction (3) leads to an analogous contribution into P_{T_1}, and therefore the two contributions cannot be distinguished in this case. On the other hand, they lead to different energy distributions for the isotropic part of the positron spectrum, and the anisotropic part is affected only by the interference between the tensor and the $V-A$ interactions. Therefore, the combined analysis of the energy distributions and the polarization of the positron can provide their unambiguous discrimination.

The main goal of this paper is to present the prediction of deviations from the SM in the energy distribution and in the polarization of the positron from the polarized muon decay, obtained on the assumption of the presence of the new tensor interaction (3) with the known coupling constant g_{RR}^T. This analysis is very interesting today, since its predictions could be verified in the near future by on-going experiments at PSI [12] and TRIUMF [13] to a level of at least 3σ.

2

2 The polarized muon decay

Taking into account the dominant contribution from the $V - A$ interaction (1) and allowing only contributions from the scalar (2) and the tensor (3) interactions, we will calculate the differential decay probability of the polarized muon at rest. Neglecting radiative corrections, the neutrino masses as well as the second and higher power of the ratio $x_0 = 2m_e/m_\mu \approx 9.7 \times 10^{-3}$, its general form [14] up to the overall normalization factor A reads

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}x\,\mathrm{d}\cos\vartheta} = \frac{m_\mu^5}{32\pi^3}\frac{A}{16}G_F^2\,x\left[F_{IS}(x) + |\boldsymbol{P}_\mu|\cos\vartheta\,F_{AS}(x)\right]\left[1 + \hat{\boldsymbol{\zeta}}\cdot\boldsymbol{P}_e(x,\vartheta)\right],\qquad(4)$$

where $x = 2E_e/m_\mu$ is the reduced energy of the positron emitted in the direction $\hat{\boldsymbol{x}}_3$ at the angle ϑ with respect to the muon polarization vector $\boldsymbol{P}_\mu$, and with its spin parallel to the arbitrary direction $\hat{\boldsymbol{\zeta}}$. The three components of the electron polarization vector $\boldsymbol{P}_e(x,\vartheta)$ are defined as

$$\boldsymbol{P}_e(x,\vartheta) = P_{T_1}\hat{\boldsymbol{x}}_1 + P_{T_2}\hat{\boldsymbol{x}}_2 + P_L\hat{\boldsymbol{x}}_3,\qquad(5)$$

where $\hat{\boldsymbol{x}}_1$, $\hat{\boldsymbol{x}}_2$ and $\hat{\boldsymbol{x}}_3$ are orthogonal unit vectors defined as follows:

$$\frac{\hat{\boldsymbol{x}}_3 \times \boldsymbol{P}_\mu}{|\hat{\boldsymbol{x}}_3 \times \boldsymbol{P}_\mu|} = \hat{\boldsymbol{x}}_2, \qquad\qquad \hat{\boldsymbol{x}}_2 \times \hat{\boldsymbol{x}}_3 = \hat{\boldsymbol{x}}_1.\qquad(6)$$

The normalization factor

$$A = 16\left(1 + \frac{1}{4}|g_{RR}^S|^2 + 3|g_{RR}^T|^2\right)\qquad(7)$$

is the sum of the relative probabilities for a muon to decay into a positron by the corresponding interactions. It is often forgotten or not stated explicitly that the addition of any new interactions leads effectively to a redefinition of the Fermi coupling constant G_F for the pure $V - A$ interaction.

We will assume that the couplings constants g_{RR}^S and g_{RR}^T are real. It will mean that CP invariance of the interactions holds and the transverse component P_{T_2}, which is perpendicular to the plane spanned by muon spin and positron momentum, is zero.

In the presence of the new tensor interaction (3) the isotropic $F_{IS}(x)$ and the anisotropic $F_{AS}(x)$ parts of the spectrum have almost standard forms:

$$F_{IS}(x) = x(1-x) + \frac{2}{9}\,\rho\,(4x^2 - 3x) + \eta\,x_0(1-x) + \kappa\,x_0,\qquad(8)$$

$$F_{AS}(x) = \frac{1}{3}\xi\,x\left[1 - x + \frac{2}{3}\delta(4x-3)\right] + \kappa\,x_0(2-x),\qquad(9)$$

where the parameters ρ, ξ and δ:

$$\rho = \frac{3}{4}\left\{1 + |g_{RR}^-|^2\right\}\frac{16}{A},\qquad(10)$$

$$\xi = \left\{1 - \frac{7}{4}|g_{RR}^-|^2 + \frac{3}{4}|g_{RR}^+|^2\right\}\frac{16}{A},\qquad(11)$$

$$\xi\delta = \frac{3}{4}\left\{1 - |g_{RR}^-|^2\right\}\frac{16}{A},\qquad(12)$$

can be expressed through only two linear combinations of the coupling constants g_{RR}^S and g_{RR}^T: $g_{RR}^- = \frac{1}{2}g_{RR}^S - g_{RR}^T$ and $g_{RR}^+ = \frac{1}{2}g_{RR}^S + 3g_{RR}^T$.

The so-called "low energy spectral shape parameter"

$$\eta = \frac{1}{2}g_{RR}^S \frac{16}{A} \tag{13}$$

contributes to the isotropic part of the spectrum (8) and does not affect the anisotropic part (9). It appears as a result of the interference of the vector (1) and the scalar (2) interactions.

The main feature of the new tensor interaction (3) is that the analogous parameter

$$\kappa = g_{RR}^T \frac{16}{A} \tag{14}$$

also appears as a result of the interference of the vector (1) and the tensor (3) interactions and contributes to both the isotropic (8) and the anisotropic (9) parts of the spectrum.

The present accuracy in the experimental value of η leads to an uncertainty in the value of the Fermi coupling constant G_F 20 times larger than that of the more precisely known muon lifetime [14]. Moreover, the new parameter κ has never been taken into account. Therefore, its measurement is urgently needed for a model-independent determination of the G_F.

The non-zero components of the positron polarization vector $\boldsymbol{P}_e$ are given by

$$P_{T_1}(x,\vartheta) = \frac{P_\mu \sin\vartheta \ F_{T_1}(x)}{F_{IS}(x) + P_\mu \cos\vartheta \ F_{AS}(x)}, \tag{15}$$

$$P_L(x,\vartheta) = \frac{F_{IP}(x) + P_\mu \cos\vartheta \ F_{AP}(x)}{F_{IS}(x) + P_\mu \cos\vartheta \ F_{AS}(x)}, \tag{16}$$

where we have used $P_\mu = |\boldsymbol{P}_\mu|$, and where

$$F_{IP}(x) = \xi' \, x(1-x) + \frac{2}{9} \, \rho_L \, (4x^2 - 3x) + \kappa \, x_0 x \tag{17}$$

and

$$F_{AP}(x) = \frac{1}{3}\xi'' \, x \left[1 - x + \frac{2}{3}\delta_L(4x-3)\right] - \eta \, \frac{x_0}{3}(1-x) + \kappa \, \frac{x_0}{3}(1+2x) \tag{18}$$

can been written down in a way analogous to $F_{IS}(x)$ and $F_{AS}(x)$ and parametrized by the parameters ξ', ρ_L, ξ'' and δ_L:

$$\xi' = \left\{1 - \frac{3}{4}|g_{RR}^-|^2 - \frac{1}{4}|g_{RR}^+|^2\right\} \frac{16}{A}, \tag{19}$$

$$\rho_L = \frac{3}{4}\left\{1 - |g_{RR}^-|^2\right\}\frac{16}{A}, \tag{20}$$

$$\xi'' = \left\{1 + \frac{7}{4}|g_{RR}^-|^2 - \frac{3}{4}|g_{RR}^+|^2\right\} \frac{16}{A}, \tag{21}$$

$$\xi''\delta_L = \frac{3}{4}\left\{1 + |g_{RR}^-|^2\right\}\frac{16}{A}. \tag{22}$$

The transverse function $F_{T_1}(x)$ has the form

$$F_{T_1}(x) = -\frac{x_0}{6}(1-x)\frac{16}{A} - \eta\frac{x}{3} + \kappa\frac{x}{3}, \tag{23}$$

where the negligibly small $\mathcal{O}(x_0|g_{RR}^\pm|^2)$ terms are omitted.

The first term in (23) is the SM contribution, which is very small, owing to the suppression factor x_0, while the subsequent terms represent scalar and tensor contributions, which are proportional to the new coupling constants g_{RR}^S and g_{RR}^T without suppression. It is interesting to note that the measurement of only the transverse polarization component P_{T_1} cannot allow a discrimination between these two new contributions.

4

3 Tensor interactions and right-handed neutrinos

As far as we have no experimental indications for the presence of the (pseudo)scalar interactions in the weak processes and because of the severe experimental constraint from the ratio $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$ [15], we will not consider them in the following. The previous consideration including the scalar interaction (2) has been done just for the illustrative purpose of comparing its effect with that from the new tensor interaction (3). For example, keeping only the η parameter and omitting the new term with κ could lead to a wrong fit for the isotropic energy distribution. Later we will consider just the opposite case of non-zero value for κ and $\eta = 0$. We will also show that for the anisotropic part of the positron spectrum an effect of the new parameter κ may be important and should be taken into account.

The new tensor interaction between quark and lepton currents [8]

$$
\begin{aligned}
\mathcal{L}_T^{q\ell} &= -\sqrt{2} f_T G_F \ (\bar{d}\sigma_{\alpha\lambda}u) \frac{4q_\alpha q_\beta}{q^2} (\bar{\nu}_e \sigma_{\beta\lambda} e_R) + \text{h.c.} \\
&\equiv -\sqrt{2} f_T G \ (\bar{d}_R \sigma_{\alpha\lambda} u) \frac{4q_\alpha q_\beta}{q^2} (\bar{\nu}_e \sigma_{\beta\lambda} e_R) - \sqrt{2} f_T G \ (\bar{d}_L \sigma_{\beta\lambda} u) (\bar{\nu}_e \sigma_{\beta\lambda} e_R) + \text{h.c.} \quad (24)
\end{aligned}
$$

with the coupling constant $f_T \approx 0.013$ is very suitable [10] for explaining the anomaly in the branching ratio and in the energy distribution for the radiative pion decay [16, 9]. Its destructive interference with the inner bremsstrahlung process in the radiative pion decay leads to an explanation of the lack of events in the high-E_γ/low-E_e kinematic region, where a deficit of events was observed. It is interesting to note that the last term in (24) can be rewritten in this form owing to the identity

$$
(\bar{d}_L \sigma_{\alpha\lambda} u) \frac{4q_\alpha q_\beta}{q^2} (\bar{\nu}_e \sigma_{\beta\lambda} e_R) \equiv (\bar{d}_L \sigma_{\beta\lambda} u) (\bar{\nu}_e \sigma_{\beta\lambda} e_R). \quad (25)
$$

The equality of the coupling constants in the two last terms in (24) is accepted in order to avoid constraints from ordinary pion decay [17].

If one assumes flavour lepton universality, it is possible to apply the same interaction for the τ-decay. For example, in this case the tensor interaction gives a direct contribution to $\tau^- \to \nu_\tau \rho^-$ decay, because the ρ-meson, besides the vector coupling constant f_ρ, has also non-zero tensor coupling constant $f_\rho^T \simeq f_\rho/\sqrt{2}$ [18, 19, 6] with the tensor quark current. It leads to 5% excess in the ρ production in τ-decay with respect to the SM, which explains deviation from CVC prediction [20].

The interaction (24) is the effective interaction and cannot be applied everywhere. It shows good behaviour for any values of the square of momentum transfers besides $q^2 = 0$. This unphysical pole should be cancelled out by some additional fields and interactions in a complete theory for the new chiral bosons, as it happens, for example, in the spontaneously broken gauge symmetry. The interactions (24) and (3) can definitely be applied as the effective interactions for decay processes, where q^2 always has positive values. However, using the interaction (3) it should be impossible to calculate the cross section for the scattering process $\nu_\mu e^- \to \mu^- \nu_e$ ("inverse muon decay"), where the square of the transfer momentum can be equal to zero, without any prescription on how to handle this pole.

Let us note also that the interaction (24) involves both the left-handed and the right-handed quarks. Therefore, in general, we should introduce also the tensor interactions involving the right-handed neutrinos. Inasmuch as the interactions with the right-handed neutrinos do not interfere with the ordinary weak interaction, they cannot lead to the destructive interference observed experimentally. Therefore, introducing the right-handed neutrinos for quark-lepton interactions cannot explain the anomaly in the radiative pion

decay and this introduction seems useless. However, in general we should introduce such type of interactions

$$\mathcal{L}_T^{q\ell R} = -\sqrt{2}f_T G \ (\bar{d}_R \sigma_{\beta\lambda} u)\,(\bar{\nu}_e \sigma_{\beta\lambda} e_L) - \sqrt{2}f_T' G \ (\bar{d}_L \sigma_{\alpha\lambda} u)\,\frac{4q_\alpha q_\beta}{q^2}\,(\bar{\nu}_e \sigma_{\beta\lambda} e_L) + \text{h.c.}, \quad (26)$$

where the coupling constant in the first term is predicted to be the same [8] as in the interaction (24). Assuming quark–lepton universality, the ratio $f_T'/f_T = 2.28 \pm 0.21$ has been fixed from the experimental value of the $K_S - K_L$ mass difference [21]. A similar value $f_T'/f_T = 1 + \sqrt{2} \approx 2.41$ can be predicted from pure theoretical considerations, based on the principle of the least energy exchanged by the new massive chiral bosons [22].

Furthermore, assuming the quark–lepton universality of the tensor interaction we accept that the coupling constant g_{RR}^T for the pure lepton interaction (3) should have the same value as f_T. The corresponding additional interactions for pure lepton interactions can be cast in the form

$$\begin{aligned}
\mathcal{L}_T^R = \ & -g_{RL}^T \ \sqrt{2}G_F \ (\bar{\mu}_L \sigma_{\alpha\beta}\nu_\mu)\,(\bar{\nu}_e \sigma_{\alpha\beta} e_R) - g_{LR}^T \ \sqrt{2}G_F \ (\bar{\mu}_R \sigma_{\alpha\beta}\nu_\mu)\,(\bar{\nu}_e \sigma_{\alpha\beta} e_L) \\
& -g_{LL}^T \ \sqrt{2}G_F \ (\bar{\mu}_L \sigma_{\alpha\lambda}\nu_\mu)\,\frac{4q_\alpha q_\beta}{q^2}\,(\bar{\nu}_e \sigma_{\beta\lambda} e_L) + \text{h.c.} \ .
\end{aligned} \quad (27)$$

If the light right-handed neutrinos exist they can contribute to the muon decay. However, since the neutrinos are practically massless, these interactions do not interfere with the ordinary $V-A$ weak interactions (1) or with the tensor interaction (3), or among themselves. Therefore, they cannot lead to CP violation and their contributions into the muon decay are always proportional to the square of the absolute values of their coupling constants.

The Michel parameters for the general case, when all possible tensor interactions (3), (27) contribute to the muon decay, read

$$A = 16\left\{1 + 3|g_{RR}^T|^2 + 3|g_{RL}^T|^2 + 3|g_{LR}^T|^2 + 3|g_{LL}^T|^2\right\}, \quad (28)$$

$$\rho = \frac{3}{4}\left\{1 + |g_{RR}^T|^2 + |g_{RL}^T|^2 + |g_{LR}^T|^2 + |g_{LL}^T|^2\right\}\frac{16}{A}, \quad (29)$$

$$\xi = \left\{1 + 5|g_{RR}^T|^2 - 5|g_{RL}^T|^2 + 5|g_{LR}^T|^2 - 5|g_{LL}^T|^2\right\}\frac{16}{A}, \quad (30)$$

$$\xi\delta = \frac{3}{4}\left\{1 - |g_{RR}^T|^2 + |g_{RL}^T|^2 - |g_{LR}^T|^2 + |g_{LL}^T|^2\right\}\frac{16}{A}, \quad (31)$$

$$\xi' = \left\{1 - 3|g_{RR}^T|^2 - 3|g_{RL}^T|^2 + 3|g_{LR}^T|^2 + 3|g_{LL}^T|^2\right\}\frac{16}{A}, \quad (32)$$

$$\rho_L = \frac{3}{4}\left\{1 - |g_{RR}^T|^2 - |g_{RL}^T|^2 + |g_{LR}^T|^2 + |g_{LL}^T|^2\right\}\frac{16}{A}, \quad (33)$$

$$\xi'' = \left\{1 - 5|g_{RR}^T|^2 + 5|g_{RL}^T|^2 + 5|g_{LR}^T|^2 - 5|g_{LL}^T|^2\right\}\frac{16}{A}, \quad (34)$$

$$\xi''\delta_L = \frac{3}{4}\left\{1 + |g_{RR}^T|^2 - |g_{RL}^T|^2 - |g_{LR}^T|^2 + |g_{LL}^T|^2\right\}\frac{16}{A}. \quad (35)$$

The first two terms in (27) are the known local Michel tensor interactions, which can be rewritten in the same form as the last term in (27) and as (3), thanks to the identity (25). This form of the interactions allows the introduction of the two new tensor coupling constants g_{RR}^T and g_{LL}^T, which have been set to zero in the Michel approach. It is worth while to note also that the new interactions lead to the same energy distribution for the isotropic spectrum (28), (29) as the ordinary tensor interactions, apart from the interference term with the parameter κ.

However, if we assume the presence of the light right-handed neutrinos, the interactions (26) will contribute to the ordinary pion decay through the electromagnetic radiative corrections [17] due to the different coupling constants f_T and f_T'. This will be in contradiction with the present experimental data for the accepted f_T value. To avoid this problem we will assume that all right-handed neutrinos are very heavy, in accordance to the see-saw mechanism [23] and the latest results from the oscillation experiments. Therefore, they do not contribute either to the pion decay or to the muon decay. Hence, only the coupling constant g_{RR}^T is relevant to low-energy physics.

4   Comparison with experimental data and predictions

Based on the previous considerations, we will compare existing experimental data on the Michel parameters with our hypothesis of small admixture of the new tensor interactions (3) in the muon decay. We will show that it does not contradict the present experimental data; moreover, it tells us where we should be looking for deviations from the SM.

In this section we will also make quantitative predictions for possible deviations from the SM, which may be detected in the on-going experiments at the PSI [12] and TRIUMF [13]. We will assume that all experimental data on the muon decay could be described by only one parameter g_{RR}^T of known value.

Let us begin with a discussion of the well known Michel parameters ρ and η for the isotropic part of the spectrum (8). The first parameter ρ has not been measured since 1969 [24], whilst the most precise result, $\rho = 0.7503 \pm 0.0026$, dates back to 1966 [25]. According to eqs. (29) and (28) the deviation from the SM value of the parameter ρ can be estimated as

$$\rho = \frac{3}{4}\left\{1 + |g_{RR}^T|^2\right\}\frac{16}{A} \approx \frac{3}{4}\left\{1 - 2|g_{RR}^T|^2\right\} \simeq \frac{3}{4}\left\{1 - 3.4 \times 10^{-4}\right\}, \tag{36}$$

which is an order of magnitude less than the present experimental accuracy. Its experimental value has been derived assuming $\eta = 0$, because the parameters ρ and η exhibit a considerable statistical correlation, when determined from fits to a limited part of the spectrum. Therefore, even with a precision of a few parts in 10^{-4} for the Michel parameters, the TWIST experiment could not detect a deviation from the SM in the ρ parameter.

What about the effect of the new parameter κ on ρ? First of all its effect, as well as a possible effect of the η parameter, is suppressed by the small parameter x_0. It can be shown that when the experimental distribution is compared to the relative event density for the theoretical spectrum, the effect of the new term completely diminishes. Therefore, it is practically impossible to detect the new contribution in the isotropic part of the spectrum.

However, unsuppressed contributions of the interference between $V - A$ and the new interactions could be detected from the energy distribution of the transverse polarization component P_{T_1} (15), (23). At the present accuracy the measured P_{T_1} distribution is energy-independent and consistent with zero. The most precise experimental results only put constraints on the average value of P_{T_1}

$$\langle P_{T_1}\rangle = 0.016 \pm 0.023 \text{ [26]}, \qquad \langle P_{T_1}\rangle = 0.005 \pm 0.016 \text{ [12]}. \tag{37}$$

The last one is the preliminary result of the μ_{P_T} experiment at the PSI.

Combining these results we obtain $\langle P_{T_1}\rangle = 0.009 \pm 0.013$. In order to extract an effect of new physics we need to subtract the SM contribution, which is small but not equal to zero. Both experiments were performed in approximately the same kinematical region where the

SM predicts $\langle P_{T_1}^{SM} \rangle \simeq -0.005$. Therefore, the possible effect of new physics is estimated as $\langle P_{T_1}^{NP} \rangle = 0.014 \pm 0.013$. This value should be compared with our prediction $\langle P_{T_1}^{\kappa} \rangle \simeq 0.015$ obtained by taking into account the contribution from the new tensor interaction (3) with the parameter $\kappa \approx g_{RR}^T = 0.013$. This surprising agreement can be checked in the future at the level of 3 standard deviations by the final result of the μ_{P_T} experiment.

If a non-zero effect of new physics will be detected in P_{T_1}, the following question will immediately arise. Is it an effect of the interference with the scalar interaction (2) governed by the parameter η or with the new tensor interaction (3) described by the new parameter κ? To answer this question let us discuss the anisotropic part of the spectrum (9), which is conventionally parametrized by the Michel parameters ξ and δ. In our case we have an additional contribution connected with the interference between the SM and new tensor interactions, while such interference is absent in the case of the scalar interactions. We will show that the parameter δ is very sensitive to the presence of the additional contribution.

To extract the ξ and δ parameters the following asymmetry function

$$\mathcal{A}(x) = P_\mu \frac{F_{AS}(x)}{F_{IS}(x)} \tag{38}$$

is measured in the polarized muon decay. A deviation from the SM in the parameter δ eqs. (31), (30)

$$\delta = \frac{3}{4}\{1 + \Delta\delta\} = \frac{3}{4}\frac{1 - |g_{RR}^T|^2}{1 + 5|g_{RR}^T|^2} \approx \frac{3}{4}\{1 - 6|g_{RR}^T|^2\} \simeq \frac{3}{4}\{1 - 10.1 \times 10^{-4}\} \tag{39}$$

and a non-zero value of the parameter κ can been established from the precise measurement of the zero point $x_z = 1/2 + \Delta x_z$ of the asymmetry, for which $\mathcal{A}(x_z) = 0$, independently of the parameter ξ and the polarization P_μ. It is interesting to note that both effects act in the same direction, leading to a noticeable deviation in x_z from the SM value:

$$\Delta x_z \approx \frac{1}{2}\Delta\delta - 9\kappa x_0 \simeq -16.4 \times 10^{-4}. \tag{40}$$

It was at the edge of the accuracy for the previous experiment at TRIUMF [27] to detect this effect, while it could be pinned down at the 8σ level in the TWIST experiment, thanks to the higher absolute energy calibration 10^{-4} at $x = 1$.

Since the new tensor interaction (24) does not contribute to the pion decay $\pi^+ \to \mu^+\nu_\mu$, the muon polarization P_μ is equal to its SM value. The deviation from the SM in the parameter ξ

$$\xi = \left\{1 + 5|g_{RR}^T|^2\right\}\frac{16}{A} \approx 1 + 2|g_{RR}^T|^2 \simeq 1 + 3.4 \times 10^{-4} \tag{41}$$

has the same value and opposite sign as in the parameter ρ. This value is too small to be detected even in the high-precision TWIST experiment fitting the measured asymmetries to eq. (38). At the same time, the effect from the terms with the parameters δ and κ should be perceptible because of the predicted big deviation in δ (39) and of the large coefficient at the additional κ-term as in eq. (40). For example, the previous TRIUMF experiment [27] had found a 1.6σ effect in δ with the right sign as predicted in this paper; the authors had nevertheless assigned this purely to statistics.

The parameter ξ can be measured also from the integral asymmetry

$$\mathcal{A}' = P_\mu \frac{\int F_{AS}(x)\mathrm{d}x}{\int F_{IS}(x)\mathrm{d}x}. \tag{42}$$

Although the parameters ρ and δ do not contribute to this ratio and the predicted deviation in the parameter ξ is small, the interference κ-terms lead to the dominant contribution

$$\mathcal{A}' \approx \frac{1}{3}P_\mu\xi\frac{1+24\kappa x_0}{1+6\kappa x_0} \approx \frac{1}{3}P_\mu\left\{1 + 18x_0 g_{RR}^T + 2|g_{RR}^T|^2\right\} \simeq \frac{1}{3}P_\mu\left\{1 + 26.0\times 10^{-4}\right\}. \quad (43)$$

The predicted value does not contradict the best present measurements:

$$P_\mu\xi = 1.0027 \pm 0.0079 \pm 0.0030 \text{ [28]}, \qquad P_\mu\xi = 1.0013 \pm 0.0030 \pm 0.0053 \text{ [29]}, \quad (44)$$

and can be verified in the TWIST experiment from a slope determination of the $\cos\vartheta$ distribution.

To conclude our analysis let us consider the effect of the new tensor interaction on the total decay rate Γ and the G_F determination, and how it is connected to the unitarity problem for the first row of the CKM matrix. As has been shown in ref. [11] and as follows also from eq. (4) at $\eta = 0$, the total decay rate is

$$\Gamma = \frac{m_\mu^5 G_F^2}{192\pi^3}\left\{1 + 6x_0 g_{RR}^T + 3|g_{RR}^T|^2\right\}. \quad (45)$$

Then in the presence of the new tensor interaction the experimental value $G_F^{\text{exp}} = (1.16639 \pm 0.00001)\times 10^{-5}$ GeV^{-2} derived from the muon lifetime is related to the Fermi coupling constant G_F as

$$G_F^{\text{exp}} = \sqrt{1 + 6x_0 g_{RR}^T + 3|g_{RR}^T|^2}\ G_F \simeq 1.00063 G_F. \quad (46)$$

Therefore, the real G_F value could be half per mille lower than usually accepted. The matrix element $V_{ud}^{\text{exp}} = 0.9740 \pm 0.0005$ [30] is extracted from the super-allowed $0^+ \to 0^+$ Fermi transitions, using the current experimental value of G_F^{exp}. Therefore, in order to obtain its real value

$$V_{ud} = \frac{G_F^{\text{exp}}}{G_F}V_{ud}^{\text{exp}} \simeq 0.9746 \pm 0.0005, \quad (47)$$

the corrections in G_F have been applied. This value can be accepted as real one, because the new tensor interaction (24) contributes only to the Gamow–Teller transitions and do not affect V_{ud}^{exp} determination.

Using this fact we can calculate the matrix element V_{us} from the unitarity condition

$$V_{us} = \sqrt{1 - |V_{ud}|^2 - |V_{ub}|^2} = 0.2239 \pm 0.0022, \quad (48)$$

where the contribution from V_{ub} can safely be neglected with respect to the accuracy to which V_{ud} can be determined. This value surprisingly coincides with the value 0.2238(30) [31] determined from the ratio of experimental kaon and pion decay widths $\Gamma(K \to \mu\nu)/\Gamma(\pi \to \mu\nu)$ [32] using the lattice calculations of the pseudoscalar decay constant ratio f_K/f_π [33] and assuming unitarity. Here it should be noted that the new tensor interactions do not contribute to the two-body decay of the pseudoscalar mesons. Therefore, the previous determination of V_{us} is valid also in the presence of the new interactions. This fact confirms the recent result of the E865 Collaboration on the K_{e3}^+ branching ratio [34] since the extracted V_{us} value 0.2238(33) [35] is in good agreement with unitarity and the V_{ud} determination from nuclear super-allowed beta decays. Since the value of V_{us} is small, even if the new tensor interactions have an effect on the kaon decays, a deviation in this value will be one order of magnitude smaller than its accuracy.

9

Acknowledgements

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