

PREDICTIONS OF MIXED NON-GAUSSIAN COSMOLOGICAL DENSITY FIELDS FOR THE COSMIC MICROWAVE BACKGROUND RADIATION

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ABSTRACT

We present simulations of the cosmic microwave background radiation (CMBR) power spectrum for a class of mixed, non-Gaussian, primordial random fields. We assume a skew-positive mixed model with adiabatic inflation perturbations plus additional isocurvature perturbations possibly produced by topological defects. The joint probability distribution used in this context is a weighted combination of Gaussian and non-Gaussian random fields, such as $P(\delta) = (1 - \alpha)f_1(\delta) + \alpha f_2(\delta)$, where $f_1(\delta)$ is a Gaussian distribution, $f_2(\delta)$ is a non-Gaussian general distribution, and α is a scale-dependent mixture parameter. Results from simulations of CMBR temperature and polarization power spectra show a distinct signature for very small deviations ($\lesssim 0.1\%$) from a pure Gaussian field. We discuss the main properties of such mixed models, as well as their predictions, and suggestions on how to apply them to small-scale CMBR observations. A reduced χ^2 test shows that the contribution of an isocurvature fluctuation field is not ruled out in actual CMBR observations, even in the *Wilkinson Microwave Anisotropy Probe* first-year sky map.

Subject headings: cosmic microwave background — cosmology: theory — methods: numerical

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1. INTRODUCTION

One of the main goals of cosmology today is to determine the origin of primordial density fluctuations. Since cosmic microwave background radiation (CMBR) carries the intrinsic statistical properties of cosmological perturbations, it is considered the most powerful tool for investigating the nature of cosmic structure. Tests for the Gaussianity of CMBR anisotropy can discriminate between various cosmological models for structure formation.

The most accepted model for structure formation assumes initial quantum fluctuations created during inflation and amplified by gravitational effects. The standard inflation model predicts an adiabatic, uncorrelated random field with a nearly flat, scale-invariant spectrum on scales larger than $\sim 1^\circ - 2^\circ$ (Guth 1981; Salopek, Bond, & Bardeen 1989; Bardeen, Steinhardt, & Turner 1983). Simple inflationary models also predict that the random field follows a nearly Gaussian distribution, where just small deviations from Gaussianity are allowed (e.g., Gangui et al. 1994). However, larger deviations are also possible in a wide class of alternative models, such as nonstandard inflation models with a massless axion field (Allen, Grinstein, & Wise 1987), with multiple scalar fields (Salopek et al. 1989), with a massive scalar field (Koyama, Soda, & Taruya 1999), and with variations in the Hubble parameter (Barrow & Coles 1990). Cosmic defect models (Kibble 1976; Magueijo & Brandenberger 2000) also predict the creation of non-Gaussian random fields. In hybrid inflation models (Battye & Weller 1998; Battye, Magueijo, & Weller 1999), structure is formed by a combination of (inflation-produced) adiabatic and (topological

defect-induced) isocurvature density fluctuations. The topological defects are assumed to appear during the phase transition that marks the end of the inflationary epoch. In this scenario the fields are uncorrelated, are combined by a weighted average, and obey non-Gaussian statistics.

The interest in non-Gaussian structure formation models is not unjustified. Indeed, the increasing number of galaxies observed at high redshifts clearly disfavors standard inflationary models with Gaussian initial conditions, which predict that these objects should be very rare in the universe (e.g., Weymann et al. 1998). In addition, statistical evidence for a small level of non-Gaussianity in the anisotropy of the CMBR has been found in the *COBE* Differential Microwave Radiometer (DMR) 4 year sky maps (Ferreira, Magueijo, & Górski 1998) and in the *Wilkinson Microwave Anisotropy Probe* (*WMAP*) first-year observations (Chiang et al. 2003). On the other hand, recent CMBR anisotropy observations on large (Smoot et al. 1992; Bennett et al. 1996), intermediate (de Bernardis et al. 2000; Hanany et al. 2000), and small angular scales (Halverson et al. 2002; Stompor et al. 2001; Mason et al. 2003; Pearson et al. 2003; Peiris et al. 2003; Kogut et al. 2003) seem to be in reasonable agreement with inflation predictions. Hence, it is difficult to either accept or rule out the non-Gaussian contribution to structure formation.

Actually, if one looks at all the available data, one possible interpretation suggests an intermediate situation in which realistic initial conditions for structure formation have small but significant departures from Gaussianity. A scenario in which the details of such departures are understood and calculated may offer a new alternative for the evolution of cosmological perturbations (for a present overview, see, e.g., Gordon 2001;

Gordon et al. 2001) and the formation of structures in the universe. Hopefully, with the improved quality of CMBR observations both on the ground and on board stratospheric balloons, and with data coming from the *WMAP* and *Planck* satellite missions (Tauber 2000; Wright 2000) in the very near future, we will be able to unveil the statistical properties of the density field with good precision. This will finally allow us to choose, among the large number of available candidates, the cosmological models that adequately fit the observational data.

In this work we explore the hypothesis that the initial conditions for structure formation do not necessarily build a single, one-component random field but a weighted combination of two or more fields. In particular, we are interested in simple mixtures of two fields, one of them being a dominant Gaussian process. In a previous work, Ribeiro, Wuensche, & Letelier (2000, hereafter RWL00) used such a model to probe the galaxy cluster abundance evolution in the universe and found that even a very small level of non-Gaussianity in the mixed field may introduce significant changes in the cluster abundance rate. Now we investigate the effects of mixed models on the CMBR power spectrum, considering a general class of finite mixtures and always combining a Gaussian with a second field to produce a positive-skewness density fluctuation field. For this combination we adopt a scale-dependent mixture parameter and a power-law initial spectrum, $P(k) = Ak^n$. CMBR temperature and polarization power spectra are simulated for a flat, Λ -dominated cold dark matter (Λ CDM) model, while varying some cosmological parameters, and the temperature fluctuations are estimated. We show how the shape and amplitude of the fluctuations in CMBR are dependent on such mixed fields and how we can distinguish a standard adiabatic Gaussian field from a mixed non-Gaussian field.

This paper is organized as follows: in § 2 we discuss the main properties of the mixed models. In § 3 we present the simulation results for a standard cosmological Λ CDM model, mixing Gaussian and lognormal, exponential, Maxwellian, and χ^2 distributions. Simulations for different combinations of cosmological parameters are presented in § 4. In § 5 we finally summarize and discuss the possibilities of using the proposed mixed-composition model for parameter estimation in future small-scale CMBR observations.

2. GENERAL DESCRIPTION

2.1. Non-Gaussian Random Fields

The use of statistical methods to describe the structure formation process in the universe is due to the lack of complete knowledge about the density fluctuation field $\delta(\mathbf{x})$ at any time t . This led us to treat $\delta(\mathbf{x})$ as a random field in three-dimensional space and to assume the universe as a random realization from a statistical ensemble of possible universes. In general, it is possible to assure Gaussianity of this field simply by invoking the central limit theorem. However, in order to better understand the process of structure formation, it is necessary to investigate the existence and (in case they exist) contribution of non-Gaussian effects to the primordial density field.

Non-Gaussianity implies an infinite range of possible statistical models. Hence, the usual approach to this subject is to examine specific classes of non-Gaussian fields. The general procedure for creating a wide class of non-Gaussian models is to admit the existence of an operator that transforms Gaussianity into non-Gaussianity according to a specific rule. For a small level of non-Gaussianity, the perturbation theory

works well for the density field. For instance, we can define a zero-mean random field ψ that follows a local transformation F on an underlying Gaussian field:

$$\psi(\mathbf{x}) = F(\phi) \equiv \alpha\phi(\mathbf{x}) + \epsilon[\phi^2(\mathbf{x}) - \langle\phi^2(\mathbf{x})\rangle], \quad (1)$$

where α and ϵ are free parameters of the model. In the limit $\alpha \rightarrow 0$, ψ is χ^2 distributed, while for $\epsilon \rightarrow 0$, one recovers a Gaussian field. The field described by ψ is physically motivated in the context of nonstandard inflation models (e.g., Falk et al. 1993; Gangui et al. 1994). Besides, the transformation in equation (1) can be considered as a Taylor expansion of more general non-Gaussian fields (e.g., Coles & Barrow 1987; Verde et al. 2000). We should also note that $P(\phi)$ is a Gaussian probability density function (PDF), while $P(\psi) = \int W(\psi|\phi)P(\phi) d\phi$, where $W(\psi|\phi)$ is the transition probability from ϕ to ψ (e.g., Taylor & Watts 2000; Matarrese, Verde, & Jimenez 2000).

An alternative approach to studying non-Gaussian fields is that proposed by RWL00, in which the PDF itself is modified as a mixture: $P(\psi) = \alpha f_1(\phi) + (1 - \alpha)f_2(\phi)$, where $f_1(\phi)$ is a (dominant) Gaussian PDF and $f_2(\phi)$ is a second distribution, with α being a parameter between 0 and 1. This parameter gives the absolute level of Gaussian deviation, while $f_2(\phi)$ modulates the shape of the resulting non-Gaussian distribution. The particular choice of RWL00 was to define f_2 as a lognormal distribution. Instead of supporting this model with a specific inflation picture, the authors take the simple argument that the real PDF of the density field cannot be strongly non-Gaussian (from *COBE* data constraints [Stompor et al. 2001] and from *WMAP* [Komatsu et al. 2003]), and, at the same time, it should be approximately lognormal in the nonlinear regime (from Abell 1958 and Abell, Corwin, & Olowin 1989 cluster data; see Plionis & Valdarnini 1995). Indeed, Coles & Jones (1991) argued that the lognormal distribution provides a natural description for the density fluctuation field resulting from Gaussian initial conditions in the weakly nonlinear regime. Hence, RWL00 envisage α as a function of time that turns a nearly Gaussian PDF at recombination ($\alpha \approx 1$) into a nearly lognormal distribution ($\alpha \approx 0$) over the nonlinear regime. The problem of formulating a direct relationship between the PDF at two different times is not considered by RWL00, but it could be done in the context of extended perturbation theory (Colombi et al. 1997). It is important to note that a mixture of distributions including a lognormal component implies the existence of a nonperturbative contribution of type e^ϕ in the primordial density field, such that the transformation F becomes

$$\psi(\mathbf{x}) = F(\phi) = \alpha\phi(\mathbf{x}) + (1 - \alpha)e^{\phi(\mathbf{x})}. \quad (2)$$

The physics of the field described by ψ is studied elsewhere (A. L. B. Ribeiro et al. 2004, in preparation). Here, in continuity with the work of RWL00, we investigate the implications of mixed models for the CMBR power spectrum. We take the attitude that, in the face of the difficulties in completely describing the primordial density field and its evolution, it is valid to take the predictions of a tentative model such as that of RWL00 and compare them with observations. Our primary aim is just to find a successful and simple idealization for observed phenomena. In § 2.2 we describe the technical details of the mixed models and how to use them to make predictions for the CMBR.

2.2. The Mixed Models

A Gaussian random field is one in which the Fourier components δ_k have independent, random, and uniformly distributed phases. In this case, the PDF in Fourier space is

$$P(\delta_k) \propto \exp\left(-\frac{1}{2} \sum_k \frac{|\delta_k|^2}{\sigma_k^2}\right). \quad (3)$$

Such a condition means that the phases are noncorrelated in space and assures that the statistical properties of the Gaussian fields are completely specified by the two-point correlation function or, equivalently, by its power spectrum $P(k) = |\delta_k|^2$, which contains information about the density fluctuation amplitude of each scale k . In an isotropic and homogeneous universe, k represents only the wavevector amplitude.

In the mixed scenario, we suppose that the field has a PDF of the form

$$P(\delta_k) \propto (1 - \alpha)f_1(\delta_k) + \alpha f_2(\delta_k). \quad (4)$$

In general, the PDF of the Fourier components $P(\delta_k)$ is not equal to the PDF of the field $P(\delta)$. However, we can always consider a non-Gaussian distribution field, such as a combination of a Gaussian and a non-Gaussian distribution. This means that a mixed distribution can be applied even to real and Fourier space in a non-Gaussian context, but this does not mean that the real and Fourier spaces have the same kind of combination. In the mixed context, the Fourier components δ_k have only a small fraction of correlated phase in space represented by the second distribution. The first field will always be the Gaussian component, and a possible effect of the second component is to modify the Gaussian field to have a positive tail. The parameter α in equation (4) allows us to modulate the contribution of each component to the resulting field. The Gaussian component represents the adiabatic (or isentropic) inflation field, and the second component may represent the effect of adding an isocurvature field produced by some primordial mechanism acting on the energy distribution, such as topological defects. The two-component random field can be generated by taking $\delta_k = P(k)\nu^2$, where ν is a random number with a distribution given by equation (4). Then, the mean fluctuation $\langle \delta^2(x) \rangle$ is proportional to

$$\int_k P(k) \left\{ \int_\nu [(1 - \alpha)f_1(\nu) + \alpha f_2(\nu)] \nu^2 d\nu \right\} d^3k. \quad (5)$$

The primordial power spectrum of the mixed field has the form

$$P(k)^{\text{mix}} \equiv M^{\text{mix}}(\alpha)P(k), \quad (6)$$

where $P(k)$ represents a power-law spectrum and $M^{\text{mix}}(\alpha)$ is the mixture term, which accounts for the statistics effect of a new component, a functional of f_1 and f_2 :

$$M^{\text{mix}}(\alpha) \equiv \int_\nu [(1 - \alpha)f_1(\nu) + \alpha f_2(\nu)] \nu^2 d\nu. \quad (7)$$

Resolving this integral assuming f_1 to be the Gaussian distribution, the mixture term is

$$M^{\text{mix}}(\alpha) = 1 - \alpha + \alpha \int_\nu f_2(\nu) \nu^2 d\nu. \quad (8)$$

In this work we explore the case of a positive-skewness model, in which the second field adds to the Gaussian component a positive tail representing a number of rare peaks in the density fluctuation field. To represent the effect of adding an isocurvature field, we have chosen the well-known log-normal, exponential, Rayleigh, Maxwellian, and χ^2 distributions as the second component. These random fields have already been used to calculate the size and number of positive and negative peaks in the CMBR distribution, under the assumption that it possesses a single, non-Gaussian component (Coles & Barrow 1987), but, to our present knowledge, they have never been used in this context of mixed fields.

Like the hybrid inflation models (Battye & Weller 1998; Battye et al. 1999), mixture models consider the scenario in which structure is formed by both adiabatic density fluctuations produced during inflation and active isocurvature perturbations created by cosmic defects during the phase transition that marks the end of the inflationary epoch. Nevertheless, the mixed scenario considers a possible correlation between the adiabatic and the isocurvature fields only in the postinflation universe. Therefore, the fluctuations in superhorizon scales are strictly uncorrelated. While the hybrid inflation scenario considers the super- and subdegree scales of CMBR anisotropy due to uncorrelated strings and inflation fields, the mixed model considers an effective mixed, correlated field acting inside the Hubble horizon, on subdegree scales. To allow for this condition and keep a continuous mixed field, the mixture parameter is defined as a scale-dependent parameter, $\alpha \equiv \alpha(k)$. The simplest choice for $\alpha(k)$ is a linear function of k :

$$\alpha(k) \equiv \alpha_0 k. \quad (9)$$

In this case, the mixture term is a function of k , $M(\alpha_0, k)$, and the mixed primordial power spectrum is

$$\begin{aligned} P(k)^{\text{mix}} &\equiv M^{\text{mix}}(\alpha_0, k)P(k) \\ &= k^n + M(\alpha_0)k^{n+1}, \end{aligned} \quad (10)$$

where $M(\alpha_0)$ represents only the coefficient dependence, α_0 . In the case of a pure Gaussian field, $\alpha_0 \approx 0$, and the mixed power spectrum is a simple power-law spectrum. In the case of a mixed field, the phase correlations between both fields are estimated by the integral in equation (8), on mixture scales defined by equation (9).

3. MIXED NON-GAUSSIAN FIELDS IN THE COSMIC MICROWAVE BACKGROUND RADIATION

Since radiation and matter were tightly coupled up to 3×10^5 yr, understanding the statistical properties of the CMBR can be an extremely powerful tool for investigating the Gaussian nature of the cosmological density fluctuation field. Comparing theoretical predictions and observational data, it is possible to select the models that account for the best description of the temperature field. The statistical nature of the initial conditions is a basic assumption for an algorithm that generates CMBR power spectra. In this work, instead of assuming the usual Gaussian initial conditions, we analyze the consequences of using mixed (non-Gaussian) initial conditions to generate CMBR power spectra.

To estimate a CMB temperature power spectrum, we need to evaluate the evolution of fluctuations generated in the early universe through the radiation-dominated era and recombination. In a mixed model with a small deviation from Gaussianity

(i.e., a possible small fraction of correlated phases in space), we can, for simplicity, ignore the coupled phase evolution through the radiation-dominated era and consider only the evolution of the amplitude of the Fourier modes within the framework of linearized perturbation theory. Following this approach, the evolution of both adiabatic and isocurvature components of the mixed density field are considered as independent processes. Only their effective amplitude correlation is considered at the last scattering surface of the photons from CMBR. To compute the independent evolution, we have used the Linger function of the COSMICS code package¹ for an adiabatic Λ CDM mode and an isocurvature Λ CDM mode. The Linger function integrates the coupled, linearized, Einstein, Boltzmann, and fluid equations governing the evolution of scalar metric perturbations and density perturbations for photons, baryons, and CDM in a perturbed, flat, Robertson-Walker universe, in a synchronous gauge. The Linger function does generate the photon density field for computing the CMBR anisotropy.

To allow for a possible coupling of both fields in the last scattering surface, the CMBR temperature and polarization power spectra were estimated by a mixed photon density function incorporated into the original COSMICS package in estimation of the multipole moments, C_l :

$$C_l = 4\pi \int_0^{k_{\max}} d^3k P^{\text{mix}}(k) (\Delta_l^{\text{mix}})^2(k, \tau). \quad (11)$$

The function Δ_l^{mix} represents the mixed photon density field in the last scattering surface, defined by

$$\Delta_l^{\text{mix}} \equiv (1 - \alpha_0 k) \Delta_l^{\text{adi}} + \alpha_0 k \Delta_l^{\text{iso}}, \quad (12)$$

where Δ_l^{adi} and Δ_l^{iso} are the photon density function estimated by COSMICS for adiabatic and isocurvature seed initial conditions, respectively.

Since we are not considering the correlation between modes on different scales (while we are considering just linear evolution), the amplitude of the mixed field is obtained by the moment

$$\begin{aligned} \langle \Delta_l^{\text{mix}}(k) \Delta_l^{\text{mix}}(k)^* \rangle &= \langle |\Delta_l^{\text{mix}}|^2 \rangle \\ &= \left\langle |\Delta_l^{\text{adi}}|^2 + \alpha_0^2 k^2 \left(|\Delta_l^{\text{iso}}|^2 \right. \right. \\ &\quad \left. \left. + |\Delta_l^{\text{adi}}|^2 - 2|\Delta_l^{\text{iso}}| |\Delta_l^{\text{adi}}| \right) \right. \\ &\quad \left. + 2\alpha_0 k |\Delta_l^{\text{adi}}| |\Delta_l^{\text{iso}}| - 2\alpha_0 k |\Delta_l^{\text{adi}}|^2 \right\rangle, \end{aligned} \quad (13)$$

which contains correlated terms between the amplitudes of the adiabatic and the isocurvature fields, and not only the independent contribution of each field. The coefficients of the amplitudes appearing on the third and fourth lines of equation (13) represent the mixing effect between the fields,

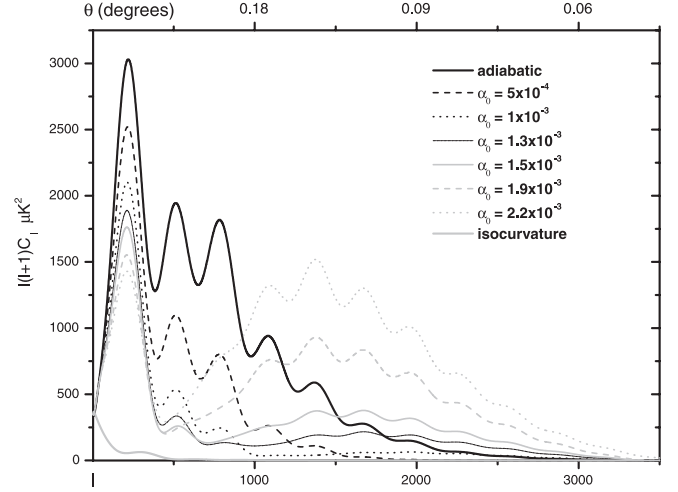


FIG. 1.—CMBR temperature mixed angular power spectrum estimated for a standard Λ CDM model in different mixing degrees. [See the electronic edition of the Journal for a color version of this figure.]

and the mixing parameter α_0 controls the contribution of the isocurvature field relative to that of the adiabatic field. Since we are not considering independent field amplitudes, and we also consider a possible small fraction of phase correlation, we can describe this mixed field by non-Gaussian statistics such as in equation (4).

Inserting equation (13) into equation (11), we obtain a mixed term in the C_l estimation. This condition suggests that the amplitudes of both fields are cross-correlated at the last scattering surface, with a mixing ratio defined by α_0 and in a characteristic scale defined by $\alpha_0 k$. The power spectra estimated by equation (11) consider a flat Λ CDM universe distorted by non-Gaussian statistics, with a constant spectral index in the range of $0.8 \leq n \leq 1.2$. In the case of a pure Gaussian field, $\alpha_0 = 0$, we obtain only the first term of the power-law spectrum in equation (10), and for a pure and noncorrelated adiabatic field, described only by a Gaussian distribution, we obtain the first term in equation (13).

In this section we present the simulations of the CMBR temperature and polarization power spectra for the present

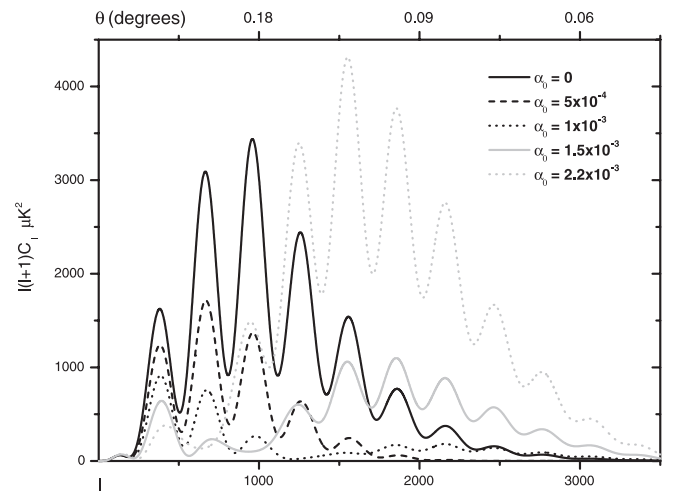


FIG. 2.—CMBR polarization mixed angular power spectrum estimated for a standard Λ CDM model in different mixing degrees. [See the electronic edition of the Journal for a color version of this figure.]

¹ COSMICS: Cosmological Initial Conditions and Microwave Anisotropy Codes, Massachusetts Institute of Technology (<http://arcturus.mit.edu/cosmics>).

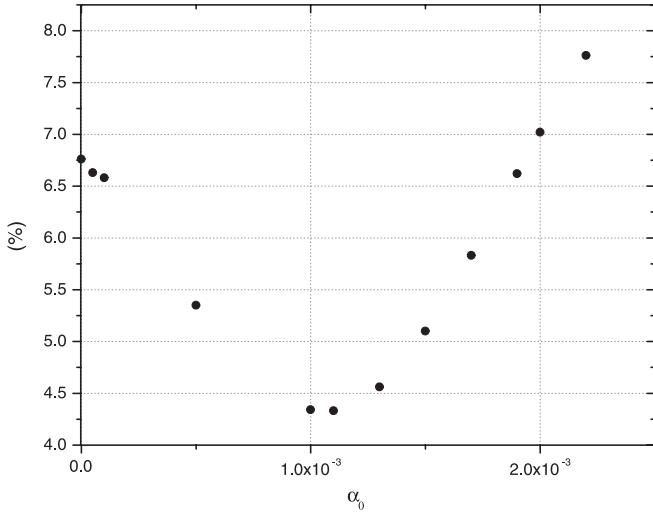


FIG. 3.—CMBR polarized component simulated for a mixed Gaussian-lognormal model in a standard combination of cosmological parameters.

standard Λ CDM model ($H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_0 = 1$, $\Omega_b h^2 = 0.03$, $\Omega_{\text{CDM}} h^2 = 0.27$, $\Omega_\Lambda h^2 = 0.7$, and $n = 1.1$). Another set of simulations, using different values for the above parameters, is presented in § 4. Figure 1 contains the mixed (Gaussian-lognormal) CMBR temperature power spectrum estimated for different values of α_0 . The temperature fluctuations are normalized by the rms quadrupole amplitude estimated by the *COBE* DMR experiment, $Q_{\text{rms}} = 13 \pm 4 \mu\text{K}$ (Smoot et al. 1992). Figure 2 shows the mixed (Gaussian-lognormal) polarization power spectrum.

In Figures 1 and 2 we see how the shape and amplitude of the spectrum change even for small values of α_0 ($\sim 10^{-4}$ – 10^{-3}). The peak intensities are clearly susceptible to the existence of mixed fields, although distinguishing peaks of higher order (second, third, etc.) in the mixed context is not a straightforward task, since their intensities, compared to the first peak, are very low. On the other hand, it is very clear how the peak intensities change while the mixture increases. The effect of increasing α_0 is a power transfer to smaller scales ($l > 1000$), while the superdegree scales are not affected. According to our definition, the effect of $\alpha(k)$ on the fields can be seen only inside the Hubble horizon. In Figure 3 we see that the fraction of the CMBR polarized component depends on the mixture parameter and ranges from 4.3% to 7.7% of the total intensity for $10^{-4} \leq \alpha_0 \leq 10^{-3}$.

We consider the relation between the peak intensities and α_0 to be a key result of this work, since it is a prediction of the mixed model that can be easily tested and, moreover, offers a good and straightforward tool to be applied to the forthcoming data sets from present CMBR experiments and the *WMAP* and *Planck* satellites. Both of them will map the CMBR power spectrum in the l -interval studied in this paper, and the data from the above-mentioned experiments will surely offer a good bench test for this model.

To illustrate the mean properties of the CMBR mixed fluctuations field, we have estimated the mean temperature fluctuations $(\Delta T/T)_{\text{rms}}$:

$$\left(\frac{\Delta T}{T}\right)_{\text{rms}}^2 = \frac{1}{4\pi} \sum_{l=2} (2l+1) C_l. \quad (14)$$

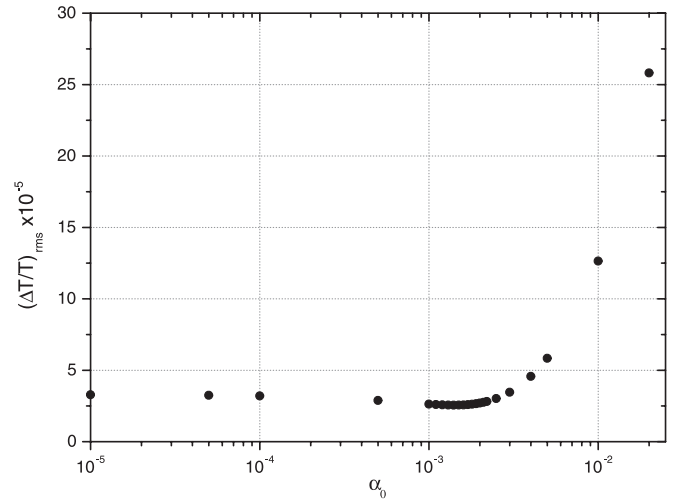


FIG. 4.—CMBR mean fluctuation temperature estimated for a mixed Gaussian-lognormal model in a standard combination of cosmological parameters.

The behavior of $(\Delta T/T)_{\text{rms}}$ for the Gaussian-lognormal mixture is shown in Figure 4. For large values of α_0 ($\alpha_0 > 3 \times 10^{-3}$), we see a fast increase in the temperature fluctuations, probably caused by correlation excess between the mixed fields, resulting in more power in small scales. From these results, we can set an acceptable range for α_0 to be $\alpha_0 \lesssim 3 \times 10^{-3}$.

In Table 1 we present the mean temperature fluctuations $(\Delta T/T)_{\text{rms}}$ estimated for different values of α_0 for mixtures between Gaussian and exponential, lognormal, Maxwellian, and χ^2 , with 1 degree of freedom distributions. For a Rayleigh distribution, the integral in equation (8) has the same value as the integral for an exponential distribution. Therefore, the mixed Gaussian-exponential and the Gaussian-Rayleigh power spectrum are exactly the same. As we can see in Table 1, the difference between various mixture components is quite small for the mixed models, because of a very small change in P^{mix} for different components. For $\alpha_0 \sim 10^{-3}$, the mixed term $M(\alpha_0)$ is about 10^{-3} . To obtain C_l we have to

TABLE 1
MEAN TEMPERATURE FLUCTUATIONS, $(\Delta T/T)_{\text{rms}} \times 10^{-5}$, ESTIMATED FOR SOME MIXTURE COMPONENTS NORMALIZED BY *COBE*

α_0	G + Exp	G + ln	G + Max	G + χ^2
0.....	3.2941	3.2941	3.2942	3.2942
1.0E-05.....	3.2848	3.2847	3.2848	3.2848
5.0E-05.....	3.2478	3.2478	3.2480	3.2480
1.0E-04.....	3.2028	3.2027	3.2028	3.2028
5.0E-04.....	2.8850	2.8850	2.8850	2.8850
1.0E-03.....	2.6250	2.6247	2.6251	2.6251
1.3E-03.....	2.5603	2.5595	2.5605	2.5605
1.5E-03.....	2.5591	2.5579	2.5594	2.5593
1.7E-03.....	2.5915	2.5899	2.5919	2.5919
1.9E-03.....	2.6564	2.6543	2.6569	2.6568
2.1E-03.....	2.7517	2.7487	2.7519	2.7518
2.5E-03.....	3.0184	3.0195	3.0195	3.0193
5.0E-03.....	5.8409	5.8323	5.8433	5.8422
1.0E-02.....	12.6310	12.6413	12.6417	12.6371
6.0E-02.....	62.4921	62.7690	62.7797	62.6557
1.0E-01.....	79.8232	80.3811	80.4025	80.1531

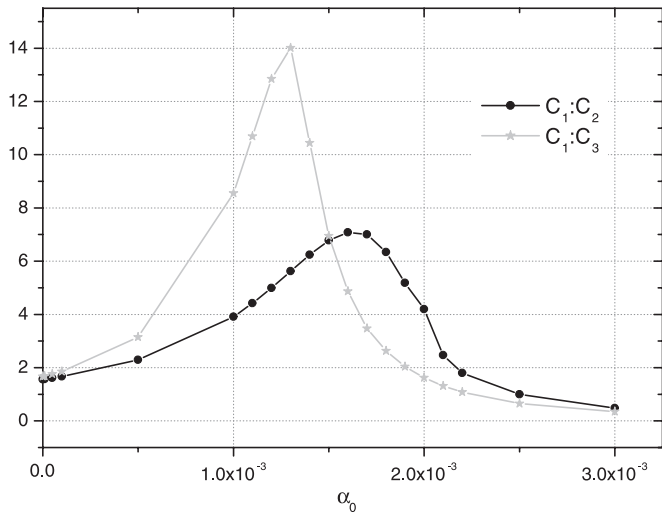


FIG. 5.—Relative amplitude of the first three peaks for a Gaussian-lognormal mixed-model temperature power spectrum, where $C_i : C_j$ means the i th peak amplitude divided by the j th peak amplitude.

multiply $M(\alpha_0)$ by k^{n+1} [$\sim (10^{-1})^{n+1}$] and integrate over dk . Therefore, the difference in C_l due to different mixture components is about $\sim 10^{-4}$ to 10^{-5} , too small compared to the simulation errors ($\sim 10^{-3}$) for the C_l estimation in μK^2 . However, the difference between a mixed non-Gaussian and a pure Gaussian field is evident in both temperature and polarization CMBR power spectra. It is clearly seen that the amplitude fluctuations change with the mixture parameter α_0 , but the main ingredient of the model seems to be the mixed (correlated) photon density field considered in the last scattering surface, and not only the statistical treatment of the phase correlation introduced in P^{mix} . A practical way to discriminate between a pure Gaussian (adiabatic) field and a mixed non-Gaussian one is to compare the polarized component, the mean temperature fluctuations, and the peak amplitudes in the power spectrum.

Multicomponent models resulting in an excess of power in small scales have already been investigated with the aid of CMBR anisotropy simulations. For instance, Bucher, Moodley, & Turok (2000) have investigated two CDM isocurvature modes (with neutrino isocurvature and isocurvature velocity perturbations), evaluated by linearized perturbation theory in

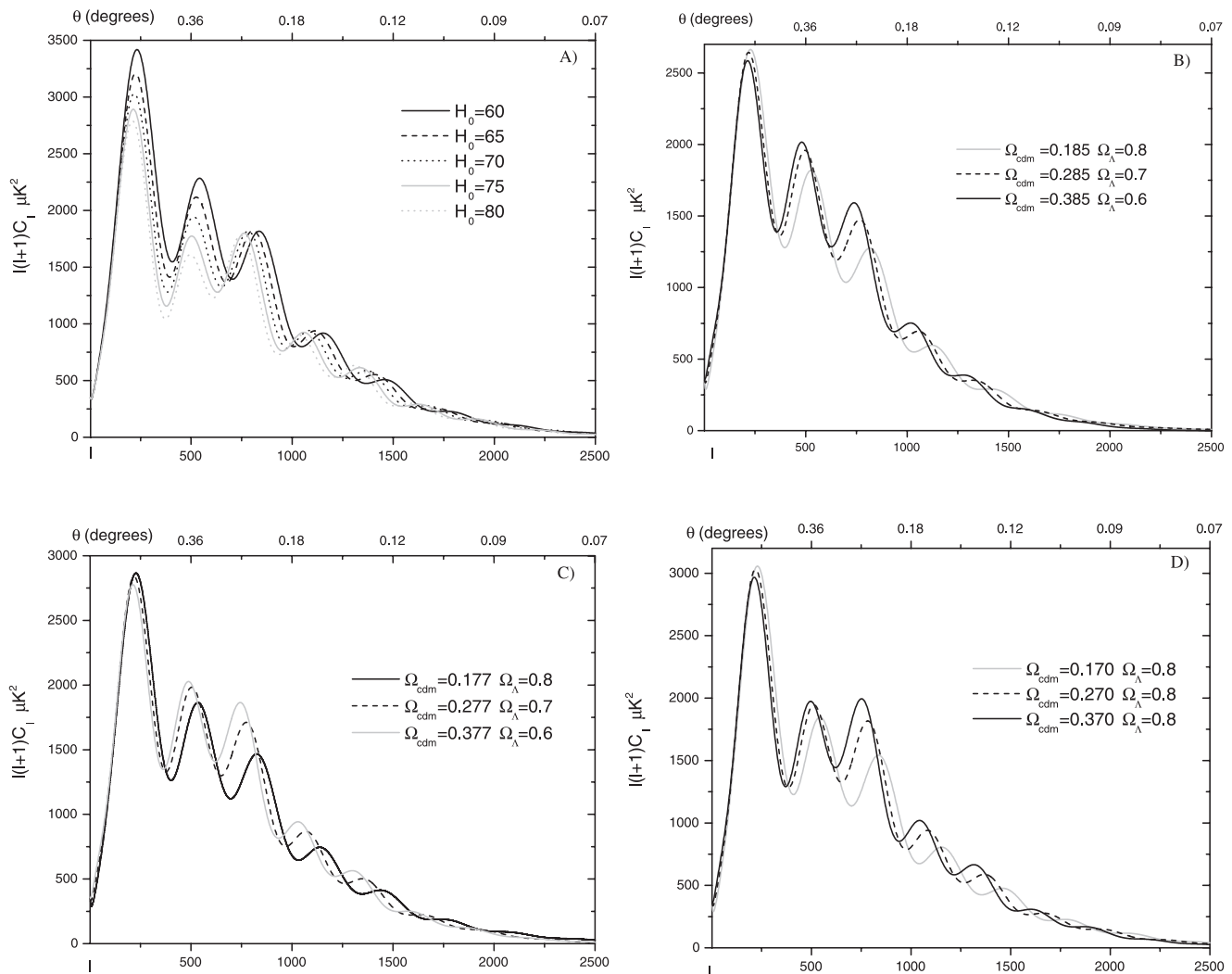


FIG. 6.—Temperature power spectra simulated for a pure Gaussian field combining a wide class of cosmological parameters. In (a) the curves show variations in the Hubble constant for a model with $\Omega_b = 0.03$, $\Omega_{\text{CDM}} = 0.27$, $\Omega_\Lambda = 0.7$, and $n = 1.1$. In (b), (c), and (d) Ω_b is set as 0.015, 0.023, and 0.03, respectively; Ω_{CDM} and Ω_Λ vary while the spectral index is fixed at 1.1, and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

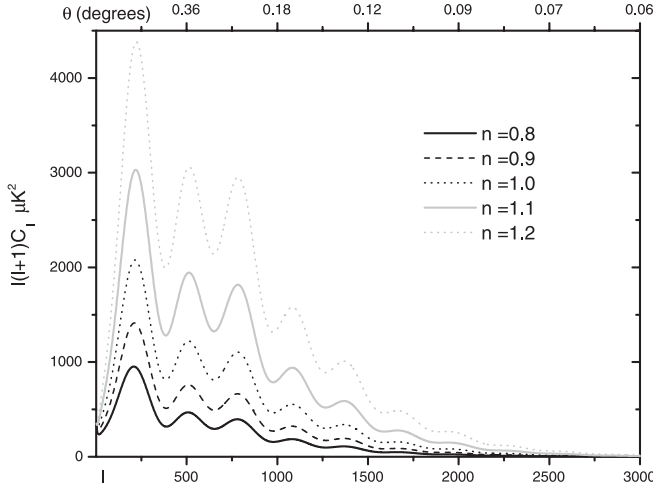


FIG. 7.—Temperature power spectra simulated for a pure Gaussian field for a model with $\Omega_b = 0.03$, $\Omega_{\text{CDM}} = 0.27$, $\Omega_\Lambda = 0.7$, $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and spectral index varying from 0.8 to 1.2.

distinct regular and singular modes (rather than growing and decaying modes), and have found a great variation of peak intensities for different initial power spectra. Another approach with multicomponent models is that of Gordon (2001) and Amendola et al. (2002). They consider a correlated field of adiabatic and isocurvature perturbations produced during a period of cosmological inflation, described in a generic power-law spectrum, and show that correlations can cause the acoustic peak height to increase relative to the plateau of CMBR.

Our point is quite different from those of the above-mentioned authors. We show that it is possible to directly assess and quantify the mixture of a correlated adiabatic and isocurvature non-Gaussian field. Figure 5 shows the relative amplitude of the most distinguished peaks (the first three peaks) for a Gaussian-lognormal mixed-model temperature power spectrum. This plot clearly shows the difference in the relative amplitudes of the peaks for $\alpha_0 < 3 \times 10^{-3}$. This behavior points to another possibility for extracting information from a CMBR power spectrum: the possibility of

detecting weakly mixed density fields, even if we cannot exactly identify the mixture components' distribution.

4. MIXED MODELS AND COSMOLOGICAL PARAMETERS

A key question that will possibly be asked when trying to apply this model to CMBR measurements is whether we can distinguish the effects of a mixed field from those of variations in the cosmological parameters. In order to answer this question and quantify how a given cosmological parameter variation modifies the properties of an a priori unknown mixed field, we ran a number of realizations for a wide range of cosmological parameter values, for both pure Gaussian and mixed fields.

CMBR observational data are in good agreement with the basic preferences of standard inflation scenarios: flat geometry ($\Omega_{\text{tot}} \sim 1$) and a nearly scale invariant primeval spectrum ($n \sim 1$). Nevertheless, degeneracies in parameter space prevent independent determination of various cosmological parameters, such as Ω_b (the baryon density energy), Ω_{CDM} (CDM energy density), Ω_Λ (vacuum energy density), and H_0 (the Hubble constant) (Turner 1999; Efstathiou et al. 2002). Recent CMBR and large-scale structure observations suggest a Hubble constant H_0 in the range $60 < H_0 < 70$ ($65 \pm 5 \text{ km s}^{-1} \text{ Mpc}^{-1}$; Turner 1999) and a positive cosmological constant in the range $0.065 < \Omega_\Lambda < 0.85$ (Efstathiou et al. 2002; Spergel et al. 2003). The mass density of baryons determined by big bang nucleosynthesis is $\Omega_b = 0.019 \pm 0.01 h^{-2}$, which is in good agreement with CMBR observations (Stompor et al. 2001). To be consistent with these estimations, we ran another set of realizations for a pure adiabatic Gaussian field, considering a flat universe and a range of possibilities for four cosmological parameters: $0.8 < n < 1.2$, $0.015 < \Omega_b < 0.03$, $0.6 < \Omega_\Lambda < 0.8$, and $60 < H_0 < 80$. The CDM density was set to $1 - (\Omega_b + \Omega_\Lambda)$, ranging over $0.170 < \Omega_{\text{CDM}} < 0.385$. The temperature power spectra simulated for a pure Gaussian field generated with the above range of parameters are plotted in Figures 6 and 7.

We can see the effect of variations in the Hubble parameter in Figure 6a. When H_0 is increased, the positions of all peaks deviate toward smaller l , while their amplitudes decrease. For fixed Ω_b , the effect of increasing Ω_{CDM} while reducing Ω_Λ is to

TABLE 2
RELATIVE AMPLITUDE OF THE PEAKS AND MEAN TEMPERATURE FLUCTUATION ESTIMATED FOR A WIDE CLASS OF MODELS FOR A PURE GAUSSIAN FIELD

Model	$C_1 : C_2$	$C_1 : C_3$	$(\Delta T/T)_{\text{rms}} \times 10^{-5}$
$H_0 = 70, \Omega_b = 0.015, \Omega_\Lambda = 0.8, n = 1.1$	1.46	2.09	3.057
$H_0 = 70, \Omega_b = 0.015, \Omega_\Lambda = 0.7, n = 1.1$	1.35	1.79	3.121
$H_0 = 70, \Omega_b = 0.015, \Omega_\Lambda = 0.6, n = 1.1$	1.28	1.62	3.148
$H_0 = 70, \Omega_b = 0.023, \Omega_\Lambda = 0.8, n = 1.1$	1.54	1.95	3.157
$H_0 = 70, \Omega_b = 0.023, \Omega_\Lambda = 0.7, n = 1.1$	1.43	1.66	3.221
$H_0 = 70, \Omega_b = 0.023, \Omega_\Lambda = 0.6, n = 1.1$	1.37	1.49	3.255
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.8, n = 1.1$	1.65	1.98	3.226
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.1$	1.56	1.67	3.295
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.6, n = 1.1$	1.50	1.49	3.326
$H_0 = 60, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.1$	1.50	1.88	3.454
$H_0 = 65, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.1$	1.51	1.76	3.368
$H_0 = 75, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.1$	1.63	1.61	3.226
$H_0 = 80, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.1$	1.73	1.57	3.170
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 0.8$	2.03	2.40	2.004
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 0.9$	1.86	2.12	2.349
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.0$	1.70	1.88	2.773
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.2$	1.43	1.48	3.929

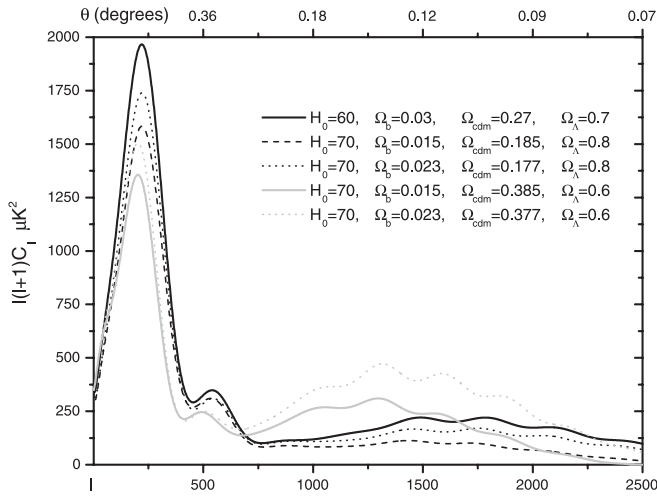


FIG. 8.—Temperature power spectra simulated for a mixed Gaussian-lognormal field for some classes of Ω_b , Ω_{CDM} , Ω_Λ , and H_0 . The spectral index is set as 1.1, and the mixture degree is 1.5×10^{-3} .

shift the second and third acoustic peak positions to smaller l and to higher amplitudes, while the primary peak features are barely disturbed. The first peak has lower intensity for high Ω_{CDM} (and low Ω_Λ), while higher order peaks have higher intensity, as can be seen in Figures 6*b–d*. Figure 7 shows the effect of varying the spectral index n in a pure adiabatic Gaussian field. As n increases, all the acoustic peaks become intensified and slightly shifted to smaller l .

For all the combined cosmological parameters simulated, we have estimated the relative amplitude of the first three acoustic peaks and the mean temperature fluctuation. The estimated values are presented in Table 2. Comparing the values presented in Table 2 with the curves shown in Figure 6, we observe that the relative amplitude of the peaks is lower than 2.1 for a wide combination of cosmological parameters, while the relative amplitude is greater than 2 for a mixed degree in the range $5 \times 10^{-4} < \alpha_0 < 2.1 \times 10^{-3}$. Therefore, when comparing the peak amplitudes, a mixed (adiabatic and isocurvature) spectrum is clearly distinct from a pure

adiabatic one with a different combination of cosmological parameters.

Figure 8 shows the behavior of the temperature power spectrum for mixed models according to the variations in the cosmological parameter values, for five different cosmological models with $\alpha_0 = 1.5 \times 10^{-3}$. The behavior of the acoustic peaks is similar to that seen in Figure 1, with power being transferred to higher l , while the superdegree scales are not affected. Table 3 contains the relative amplitude of the peaks and the mean temperature fluctuation for the 17 simulated mixed models, using $\alpha_0 = 5 \times 10^{-4}$. Comparing the values in Tables 2 and 3, we can verify the difference between the relative amplitude for mixed and pure models for a quite large combination of cosmological parameters. It seems clear that the effect of mixed models on the acoustic peak amplitudes is more pronounced than that achieved through variations of the cosmological parameters in a pure model. Besides that, once the above cosmological parameters are determined with better precision, power spectrum examination can be used to identify a mixed density field and estimate the mixture degree by comparing the peak intensities.

In order to quantify the possible non-Gaussian contribution to the fluctuation field, we have taken a maximum likelihood approach to modeling the power spectrum estimated by several classes of CMB experiments. Our best estimate of the angular power spectrum for the CMB is shown in Figure 9 for a combination of various CMB data prior to *WMAP*, and in Figure 10 are shown the best standard Gaussian and the best estimated mixed-model fits for the first-year *WMAP* data. The χ^2 test shows that the contribution of an isocurvature fluctuation field is not ruled out in actual CMB observations.

5. DISCUSSION

Assuming that the initial fluctuation field is the result of weakly correlated adiabatic and isocurvature mixed fields, we made a number of realizations of CMBR temperature and polarization power spectra and estimated the mean temperature fluctuations combining Gaussian and exponential, lognormal, Rayleigh, Maxwellian, and χ^2 distributions. The contribution of a second field was estimated by the distortion of the power spectrum, and the correlated amplitude for the mixed field was

TABLE 3

RELATIVE AMPLITUDE OF THE PEAKS AND MEAN TEMPERATURE FLUCTUATION ESTIMATED FOR A WIDE CLASS OF MODELS FOR A MIXED GAUSSIAN-LOGNORMAL FIELD WITH A MIXING DEGREE OF 5×10^{-4}

Model	$C_1 : C_2$	$C_1 : C_3$	$(\Delta T/T)_{\text{rms}} \times 10^{-5}$
$H_0 = 70, \Omega_b = 0.015, \Omega_\Lambda = 0.8, n = 1.1$	2.00	3.64	2.366
$H_0 = 70, \Omega_b = 0.015, \Omega_\Lambda = 0.7, n = 1.1$	1.91	3.33	2.386
$H_0 = 70, \Omega_b = 0.015, \Omega_\Lambda = 0.6, n = 1.1$	1.87	3.19	2.391
$H_0 = 70, \Omega_b = 0.023, \Omega_\Lambda = 0.8, n = 1.1$	2.14	3.44	2.449
$H_0 = 70, \Omega_b = 0.023, \Omega_\Lambda = 0.7, n = 1.1$	2.07	3.10	2.472
$H_0 = 70, \Omega_b = 0.023, \Omega_\Lambda = 0.6, n = 1.1$	2.05	2.95	2.503
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.8, n = 1.1$	2.33	3.51	2.513
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.1$	2.29	3.15	2.557
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.6, n = 1.1$	2.30	2.97	2.587
$H_0 = 60, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.1$	2.07	3.32	2.604
$H_0 = 65, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.1$	2.16	3.20	2.565
$H_0 = 75, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.1$	2.49	3.14	2.547
$H_0 = 80, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.1$	2.76	3.18	2.565
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 0.8$	3.00	4.55	1.695
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 0.9$	2.74	4.02	1.930
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.0$	2.50	3.55	4.664
$H_0 = 70, \Omega_b = 0.030, \Omega_\Lambda = 0.7, n = 1.2$	2.11	2.79	2.976

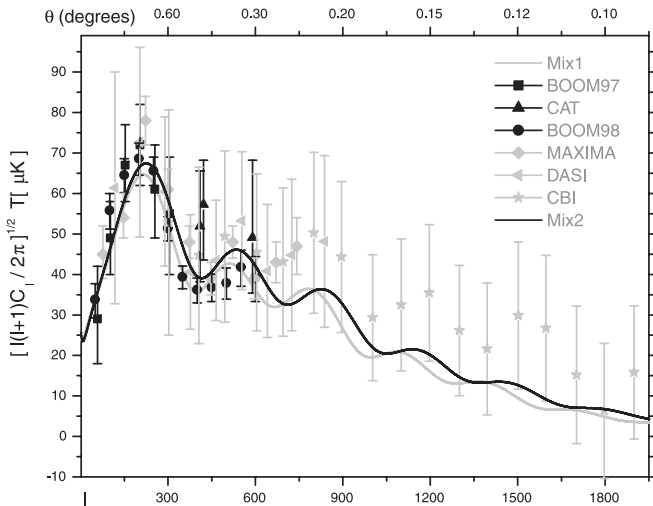


FIG. 9.—Temperature fluctuation estimated for some classes of CMBR experiments and the mixed-model estimate for $\alpha_0 = 5 \times 10^{-4}$ and two combinations of cosmological parameters. For Mix1, $\Omega_b = 0.023$, $\Omega_{\text{CDM}} = 0.177$, $\Omega_\Lambda = 0.8$, $n = 1.1$, and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. For Mix2, $\Omega_b = 0.03$, $\Omega_{\text{CDM}} = 0.27$, $\Omega_\Lambda = 0.7$, $n = 1.1$, and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The reduced χ^2 estimates are 1.010 and 1.000, respectively. [See the electronic edition of the Journal for a color version of this figure.]

also considered. Some important results were obtained. We show that it is possible to directly assess and quantify the mixture of correlated adiabatic and isocurvature non-Gaussian fields. The simulations clearly show the difference in the relative amplitude of the acoustic peaks for a mixed, correlated model. This behavior points to another possibility of extracting information from a CMBR power spectrum: the possibility of detecting weakly mixed density fields, even if we cannot exactly identify the mixture components' distributions. The simulations show that the influence of the specific statistics of the second component in the mixed field is not as important as the cross-correlation between the amplitudes of both fields. This seems to be very important in the CMBR power spectrum estimation. We claim that a physical mechanism responsible for the generation of both fields could result in a distinctive signature in the CMBR. We also show that the results are not strongly affected by the choice of the cosmological parameters, and hence the characteristic behavior of the acoustic peak amplitudes and the polarized component can be used as a cosmological test for the nature of the primordial density field. Indeed, the predictions of mixture models are very distinct from those for pure density fields, especially for small angular scales.

In addition, recent temperature fluctuation CMBR observational data do not rule out a mixture component with a small level of non-Gaussianity, with a mixture coefficient of $\sim 10^{-4}$,

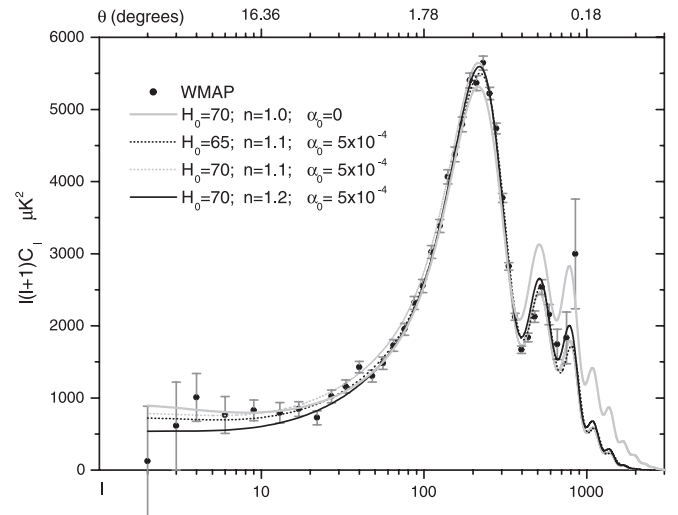


FIG. 10.—Angular power spectrum estimated from the *WMAP* mission, the prediction of the standard Λ CDM model, and the three best-fit Λ CDM mixed models for the CMBR. In all models illustrated, the best cosmological parameter combination found was $\Omega_b = 0.03$, $\Omega_{\text{CDM}} = 0.27$, and $\Omega_\Lambda = 0.7$. The reduced χ^2 estimates are 13.399 for $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $n = 1.0$, and $\alpha_0 = 0$; 1.723 for $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $n = 1.1$, and $\alpha_0 = 5 \times 10^{-4}$; 2.260 for $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $n = 1.2$, and $\alpha_0 = 5 \times 10^{-4}$; and 2.546 for $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $n = 1.1$, and $\alpha_0 = 5 \times 10^{-4}$. [See the electronic edition of the Journal for a color version of this figure.]

as can be seen in Figures 9 and 10. With the new generation of CMBR experiments, especially the expected satellite mission *Planck* (Tauber 2000), we expect to be able to compare more observations on small scales, with better signal-to-noise ratio and larger sky coverage, to the predictions of our class of mixed models and estimate the physical mechanisms responsible for structure formation. Despite the degeneration of the power spectrum for a mixed PDF, we expect to better estimate the statistical description of fluctuations in a mixed scenario by carrying out the investigation, in a non-Gaussian context, of the average correlation function and the correlation function for high-amplitude peaks (A. P. A. Andrade, A. L. B. Ribeiro, & C. A. Wuensche 2004, in preparation).

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