Predictive Functional Control Based on Fuzzy Model for Heat-Exchanger Pilot Plant

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Abstract—**In this paper, a new method of predictive control is presented. In this approach, a well-known method of predictive functional control is combined with fuzzy model of the process. The prediction is based on fuzzy model given in the form of Takagi–Sugeno (T–S) type. The proposed fuzzy predictive control has been evaluated by implementation on heat-exchanger plant, which exhibits a strong nonlinear behavior. It has been shown that in the case of nonlinear processes, the approach using fuzzy predictive control gives very promising results. The proposed approach is potentially interesting in the case of batch reactors, heat-exchangers, furnaces, and all the processes that are difficult to model.**

Index Terms—**Fuzzy identification, predictive control, real-time control.**

I. Introduction

In recent years, the predictive control has become a very important area of research. It is based on the prediction of the output signal y at each sampling instant. The prediction is obtained implicitly or explicitly according to the model of the controlled process. Using the actual predictive control law, the control signal is calculated which forces the predicted process output signal to follow to the reference signal in way to minimize the difference between the reference and the output signal in the area between certain time horizons. The fundamental methods that are essentially based on the principle of predictive control are Clarke's method, (generalized predictive control [1]), Richalet's method (model algorithmic control and predictive functional control [2]), Cutler's method (dynamic matrix control [3]), De Keyser's method (extended prediction self-adaptive control [4]), and Ydstie's method (extended horizon adaptive control [5]).

In this paper, we are discussing a new method of predictive control. This approach combines a well-known method of predictive functional control together with fuzzy model of the process. The prediction is based on a global linear model, which is obtained by fuzzy model given in the form of Takagi–Sugeno (T–S) type. The predictive control based on a fuzzy model is capable to control also very difficult processes such as strongly nonlinear processes, processes with long time delay and nonminimum phase processes. The controllers based on prediction strategy also exhibit remarkable robustness with respect to model mismatch and unmodeled dynamics. The proposed fuzzy predictive control has been evaluated by implementation on heat-exchanger plant, which exhibits a strong nonlinear behavior. It has been shown that in the case of nonlinear processes the approach is potentially interesting in the case of batch reactors, heat-exchangers, furnaces, and all the processes that are difficult to model.

The paper is organized in the following way. In Section II, the heat-exchanger pilot plant is presented, Section III deals with the fuzzy identification. In Section IV, the concept of predictive control and predictive control based on fuzzy model is given and finally, the implementation of the proposed control algorithm on a real temperature plant is discussed in Section V.

II. Heat-Exchanger Pilot Plant

The problem of heat-exchanger control with sensors and actuators limitation represents a serious problem from the point of optimal energy consumption. The problem lies in the nonlinearity of the system behavior. The objective of our investigation, a real temperature plant, consists of a plate heat-exchanger, a reservoir with heated water, two thermocouples, and a motor driven valve. The plate heat exchanger, through which hot water from an electrically heated reservoir is continuously circulated in the counter-current flow to cold process fluid (cold water). The thermocouples are located in the inlet and outlet flows of the exchanger; both flow rates can be visually monitored. Power to the heater may be controlled by time proportioning control using the external control loop. The flow of the heating fluid can be controlled by the proportional motor driven valve. A schematic diagram of the plant is shown in Fig. 1. The temperature of heated water $T_{sp}(k)$ is measured on the temperature sensor TC4, which is on the outlet of the secondary circuit, the temperature of cold water in the inlet of secondary circuit $T_{ep}(k)$ is measured on the temperature sensor TC3 and $T_{ec}(k)$ represents the temperature of hot water in the inlet of the primary circuit which is measured on the temperature sensor TC1. The primary circuit flow $F_c(k)$ is measured on optical flow sensor F2 and is defined by motor driven valve and the secondary flow $F_p(k)$ is measured on the optical flow sensor F1.

The controlled variable of our problem is the temperature in the secondary circuit T_{sp}, which is manipulated with the flow F_c, which is a function of motor driven valve current V_{mdv}. The current on motor driven valve V_{mdv} is actual manipulated variable of the process. Furthermore, the heat-exchanger is just one part of the plant, so the sensors and the actuators should also be modeled. The predictive functional control requires an internal model of the process. For nonlinear systems with well-understood physical phenomena fundamental modeling is preferable. Although the physical phenomena in the case

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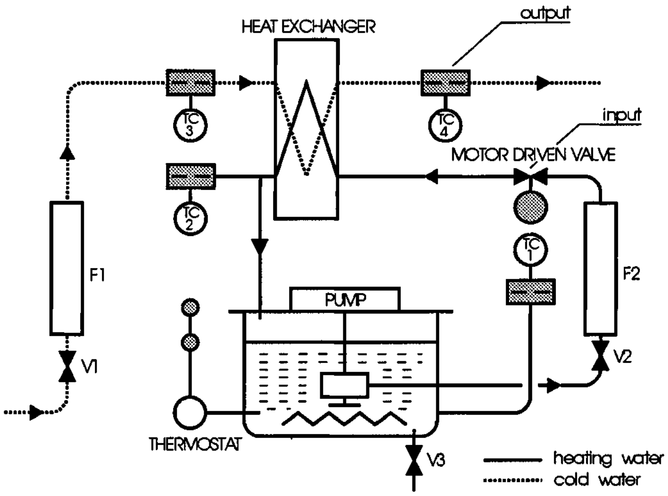

Fig. 1. The heat-exchanger pilot plant.

of heat-exchanger are well investigated, there are still some physical parameters that should be estimated assuming a certain structure of the process dynamics. The simplified first-principle model of heat-exchanger is described by the following differential equations:

$$\tau_2(T_{sp})\dot{T}_{sp} + T_{sp} = \gamma T_{ep} + (1-\gamma)T_{ec} \tag{1}$$

where the generalized formula for γ is given in the literature [6] and can be written as

$$\gamma = \frac{1 + k_c\left(\dfrac{1}{F_c}\right)^m}{1 + k_c\left(\left(\dfrac{1}{F_c}\right)^m + \left(\dfrac{1}{F_p}\right)^m\right)} \tag{2}$$

where k_c and m are unknown constants and τ_2 is an unknown function of operating point. All those parameters should be estimated using classical optimization methods on real-time data.

During the operation of the heat-exchanger, some of the system variables (the flow F_p of the secondary circuit and the temperature T_{ec} at the inlet of the primary circuit of the heat-exchanger) are approximately constant. Our main goal is to control the temperature T_{sp} by changing the position of the motor driven valve. The position of the valve is driven with the current signal $V_{mdv}[mA]$. To fulfill this goal, we should first model the relation between the position of the motor driven valve and the temperature in the secondary circuit T_{sp}. Instead of gray-box model, black-box fuzzy model of the process is obtained by experimental modeling. Although the process is very complex, it could be presented as a model with approximately first order dynamics with small time delay which could be neglected and changeable parameters of the model according to the operating point.

III. Fuzzy Identification

The fuzzy model represents a static nonlinear mapping between input and output variables. Dynamic systems are usually modeled by feeding back delayed input and output signals. The

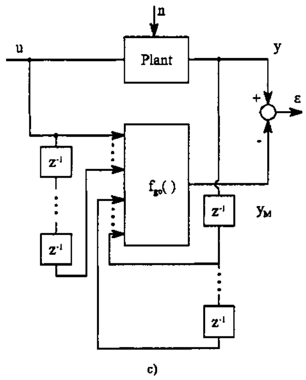

Fig. 2. The generalized output error identification model.

common nonlinear model structure is nonlinear autoregressive with exogenous (NARX) input model, which gives the mapping between the past input–output data and the predicted output of the model

$$\hat{y}_M(k+1) = \mathcal{F}(y(k), y(k-1), \ldots, y(k-n+1), u(k), \ldots, u(k-m+1)) \tag{3}$$

where $(y(k),\ y(k-1), \ldots, y(k-n+1)$ and $u(k), u(k-1) \ldots, u(k-m+1)$ denote the delayed model output and input signals, respectively. The fuzzy model therefore approximates the function $\mathcal{F}$. The model is called *generalized output error model*.

Fuzzy modeling or identification aims at finding a set of fuzzy IF-THEN rules with well defined parameters that can describe the given input–output behavior of the process.

The approach of T–S fuzzy modeling [7]–[9], which was used in our case to model the plant dynamic can be treated as universal approximator (UA) which can approximate continuous functions to an arbitrary precision [10]–[12].

The T–S fuzzy model is based on the type of rule which can be written as

$$\begin{aligned} \mathbf{R}^j\text{:}\quad & \textbf{if } x_1 \textbf{ is } A_1^j \textbf{ and} \ldots \textbf{and } x_N\ is\ A_N^j \\ & \textbf{then } y = f^j(x_1 \ldots x_N) \end{aligned} \tag{4}$$

where $x_1 \ldots x_N$ are the inputs, A_1^j is a subset of the input space, y is the output, and f^j is a function that can be in general nonlinear, but it is usually linear.

The fuzzy logic model which is treated in this paper belongs to the class of NARX variable [13] models and will be denoted by fuzzy logic/neural net autoregresive with exogenous (FNARX) variable. The models for the identification and control of nonlinear dynamic systems are given and detailed described in literature [14], [15]. Fig. 2 represents the fuzzy model structure used in our case.

The identification procedure involves the structure identification of the plant and the estimation of the unknown parameters. In the case of the FNARX models, the structure is usually chosen *ad hoc* and then improved by some optimization procedure. The estimation of the unknown parameters is divided into the estimation of the parameters of antecedent membership function and the estimation of the consequence parameters. In the case of T–S type models with center of singletons defuzzification, respectively, the generalized error models and assuming properly tuned antecedent parameters, the estimation of consequent parameters is especially simple since the problem is linear in unknown parameters and the least squares technique can be used.

A. Global Linear Model Based on T–S Fuzzy Model

In the case of the heat-exchanger pilot plant, the rule of T–S fuzzy model can be written as follows:

$$\mathbf{R}^i: \quad \textbf{if } x_1 \textbf{ is } \mathbf{A}_{\boldsymbol{i}} \textbf{ then } y = a_i x_1 + b_i x_2 + r_i, \quad i = 1, \dots K. \tag{5}$$

The input variables of the fuzzy model are x_1 and x_2, y is the output variable, A_i, $i = 1, \dots K$ are membership functions of variable x_1. The **if** parts or the antecedents of the rules describe fuzzy regions in the space of one or more input variables and the **then** parts or the consequents are linear functions of the inputs.

Defining the output temperature of the heat-exchanger T_{sp} as the output or controlled variable y_p and defining the current signal to the motor driven valve V_{mdv} as the input or manipulated variable u the fuzzy model in (5) is rewritten in the following form:

$$\mathbf{R}^i: \textbf{ if } y_p(k) \textbf{ is } \mathbf{A}_{\boldsymbol{i}} \textbf{ then } y_p(k+1) = a_i y_p(k) + b_i u(k) + r_i, \quad i = 1, \dots, K \tag{6}$$

where $u(k)$ and $y_p(k)$ are input variables of the fuzzy system, $y_p(k+1)$ is an output variable and A_i are membership functions where $i = 1, \dots, n_a$. The number of membership functions of the input variable $y_p(k)$ defines the number of rules $K = n_a$. The membership functions have to cover the whole operating area of the closed-loop system. The output of T–S model is than given by the following equation:

$$y_p(k+1) = \sum_{i=1}^{K} \left(\beta_i(\boldsymbol{\varphi}(k)) \left(a_i y_p(k) + b_i u(k) + r_i\right)\right) \tag{7}$$

where $\boldsymbol{\varphi}(k)$ represents the regressor which consists of input and output signals. The normalized degree of fulfillment $\beta_i(\boldsymbol{\varphi}(k))$ is given in the following equation:

$$\beta_i(\boldsymbol{\varphi}(k)) = \frac{\mu_{A_i}(y_p(k))}{\sum_{i=1}^{K} \mu_{A_i}(y_p(k))}. \tag{8}$$

The normalized degrees of fulfillment for the whole set of rules can are written in vector form as

$$\boldsymbol{\beta}^T = [\beta_1 \quad \beta_2 \quad \dots \quad \beta_K]. \tag{9}$$

Due to (7) and (9) the process can be modeled in fuzzy form as

$$y_p(k+1) = \boldsymbol{\beta}^T \mathbf{a} y_p(k) + \boldsymbol{\beta}^T \mathbf{b} u(k) + \boldsymbol{\beta}^T \mathbf{r} \tag{10}$$

where $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{r}$ stand for fuzzified parameters of the process that have constant elements

$$\begin{aligned} \mathbf{a}^T &= [a_1 \quad a_2 \quad \dots \quad a_K] \\ \mathbf{b}^T &= [b_1 \quad b_2 \quad \dots \quad b_K] \\ \mathbf{r}^T &= [r_1 \quad r_2 \quad \dots \quad r_K]. \end{aligned} \tag{11}$$

The parameters of the fuzzy model are obtained on measured input–output data using least square optimization method. The optimization approach in this case is different from the approaches that are reported in literature [7], [9], [15], [16]. This novel approach results in a fuzzy model, which gives a more accurate fuzzy model in the sense of the parameters. This approach is based on decomposition of the data matrix $\boldsymbol{\Psi}$ into K submatrices $\boldsymbol{\Psi}_1, \boldsymbol{\Psi}_2, \dots, \boldsymbol{\Psi}_K$. This means that the parameters of each rule are calculated separately. This leads to a better estimate of the fuzzy parameters or the variance of the estimated parameters are smaller than in the classical approach. The reason for this fact lies in a better conditioning of submatrices $\boldsymbol{\Psi}_1, \boldsymbol{\Psi}_2, \dots, \boldsymbol{\Psi}_K$.

The algorithm of calculating the fuzzy model parameters a_i, b_i, and r_i for $i = 1, \dots K$ will be given next. The algorithm is based on (10), which describes the fuzzy model of the observed process. Assuming the normalized degrees of fulfillment, which are also time dependent, this leads to

$$\sum_i \beta_i(k) = \boldsymbol{\beta}^T(k) \boldsymbol{I} = 1 \tag{12}$$

where $\boldsymbol{I}$ stands for unity vector. According to the normalized degrees of fulfillment (10) can be written in the following form:

$$\boldsymbol{\beta}^T(k) \boldsymbol{I} y_p(k+1) = \boldsymbol{\beta}^T(k) \mathbf{a} y_p(k) + \boldsymbol{\beta}^T(k) \mathbf{b} u(k) + \boldsymbol{\beta}^T(k) \mathbf{r}. \tag{13}$$

This leads to the form of the fuzzy model described in the following equation:

$$\sum_{i=1}^{K} \beta_i(k) y_p(k+1) = \sum_{i=1}^{K} \left(\beta_i(k) a_i y_p(k) + \beta_i(k) b_i u(k) + \beta_i(k) r_i\right). \tag{14}$$

Equation (14) can be separated into K equations which represent the participation of a certain rule to the whole output variable of the fuzzy model. This results in the following:

$$\mathbf{R}^1: \quad \beta_1(k) y_p(k+1) = \beta_1(k) a_1 y_p(k) + \beta_1(k) b_1 u(k) + \beta_1(k) r_1 \tag{15}$$

$$\mathbf{R}^2: \quad \beta_2(k) y_p(k+1) = \beta_2(k) a_2 y_p(k) + \beta_2(k) b_2 u(k) + \beta_2(k) r_2 \tag{16}$$

$$\vdots \tag{17}$$

$$\mathbf{R}^K: \quad \beta_K(k) y_p(k+1) = \beta_K(k) a_K y_p(k) + \beta_K(k) b_K u(k) + \beta_K(k) r_K. \tag{18}$$

To obtain the fuzzy model parameters a_i, b_i, and r_i for $i = 1, \dots K$ the following form of regressor will be used for each rule:

$$\boldsymbol{\psi}_i(k) = [\beta_i(k)y_p(k) \quad \beta_i(k)u(k) \quad \beta_i(k)1]. \tag{19}$$

Composing the regressors of a certain rule for the whole group of input–output data pairs the regression matrix $\boldsymbol{\Psi}_i$ is obtained

$$\boldsymbol{\Psi}_i = \begin{bmatrix} \beta_i(1)y_p(1) & \beta_i(1)u(1) & \beta_i(1)1 \\ \beta_i(2)y_p(2) & \beta_i(2)u(2) & \beta_i(2)1 \\ \vdots & \vdots & \vdots \\ \beta_i(N)y_p(N) & \beta_i(N)u(N) & \beta_i(N)1 \end{bmatrix} \tag{20}$$

where N stands for the number of data pairs. The regressor is added to the regression matrix when the following criteria is fulfilled:

$$\beta_i(k) > \delta, \qquad k = 1, \dots, N \tag{21}$$

where δ is equal to the estimated variance of the noise. According to the criterion in (21) and assuming a sufficient input excitation of the process the matrix $\boldsymbol{\Psi}$ will be properly conditioned. This is important for matrix inversion which is necessary to get the fuzzy model parameters.

The output variable which corresponds to the rule $\mathbf{R}^i$ is written in the following form:

$$y_p^i(k+1) = \beta_i(k)y_p(k+1) \tag{22}$$

and will combine an output data vector

$$\boldsymbol{Y}_p^i = \begin{bmatrix} \beta_i(1)y_p(1) \\ \beta_i(2)y_p(2) \\ \vdots \\ \beta_i(N)y_p(N) \end{bmatrix}. \tag{23}$$

The fuzzy model for the rule $\mathbf{R}^i$ in the matrix form is written as follows:

$$y_p^i(k+1) = \boldsymbol{\psi}_i(k)\boldsymbol{\theta}_i \tag{24}$$

where vector θ_i contains the fuzzy model parameters for the rule $\mathbf{R}^i$.

The fuzzy model parameters for the rule $\mathbf{R}^i$ are obtained using least-square optimization method

$$\boldsymbol{\theta}_i = (\boldsymbol{\Psi}_i^T\boldsymbol{\Psi}_i)^{-1}\boldsymbol{\Psi}_i^T\boldsymbol{Y}_p^i \tag{25}$$

where $\boldsymbol{\theta}_i$ consists of parameters a_i, b_i and r_i

$$\boldsymbol{\theta}_i = [a_i \quad b_i \quad r_i]. \tag{26}$$

By calculating the fuzzy model parameters for the whole group of rules, the fuzzified model parameters are obtained as it is given in (11).

The model in (10) represents a linear model with changeable parameters that are called the global linear parameters and are given in the following:

$$\begin{aligned} \tilde{a}(\boldsymbol{\varphi}(k)) &= \boldsymbol{\beta}^T(\boldsymbol{\varphi}(k))\mathbf{a} \\ \tilde{b}(\boldsymbol{\varphi}(k)) &= \boldsymbol{\beta}^T(\boldsymbol{\varphi}(k))\mathbf{b} \\ \tilde{r}(\boldsymbol{\varphi}(k)) &= \boldsymbol{\beta}^T(\boldsymbol{\varphi}(k))\mathbf{r}. \end{aligned} \tag{27}$$

This procedure can be viewed as an instantaneous linearization of the process dynamics. The described instantaneous linearization gives the parameters of a global linear model that depend on the input regressor vector. In other words, the model parameters are spanned on input regressor vector that depends on the model structure. The global linear parameters of the process can be used directly in the case of adaptive and predictive control where the controllers adapt to the dynamic changes on line.

IV. Predictive Functional Control Based on Fuzzy Model

Model-based predictive control (MBPC) is a control strategy based on the explicit use of a dynamic model of the process to predict the future behavior of the process output signal over a certain (finite) horizon and to evaluate control actions to minimize a certain cost function. The predictive control law is in general obtained by minimization of the following criterion:

$$J(u,k) = \sum_{j=N_1}^{N_2} (y_m(k+j) - y_r(k+j))^2 + \lambda \sum_{j=1}^{N_u} u^2(k+j) \tag{28}$$

where $y_m(k+j)$, $y_r(k+j)$ and $u(k+j)$ stand for j-step ahead prediction of process output signal, reference trajectory, and control signal, respectively. N_1, N_2 and N_u are minimum, maximum, and control horizon, respectively, and λ weights the relative importance of control and output variables. The basic predictive control strategy and basic parameters are presented in Fig. 3. The predictive control law adopts a receding policy, which means that at each time instant, the optimal control sequence according to the criterion in (28) is obtained, but only the first element in this sequence is applied to the plant. The procedure is repeated in the next time instant.

MBPC stands for a collection of several different techniques all based on the same principles. Originally, the algorithms have been developed for linear systems, but the basic idea of prediction has been extended to nonlinear systems [17], [18]. In our approach, the basic principles of predictive functional control are applied. In this case, the prediction of the process output is given by a fuzzy process model. In this section the basics of predictive functional control based on a fuzzy model (FPFC) for the first-order system are introduced. The fundamental principles of predictive functional control [2], [6] are very strong and easy to understand because they are natural and can be rapidly grasped because the skilled plant operators always observe these principles. The global linear model of the smooth nonlinear process of

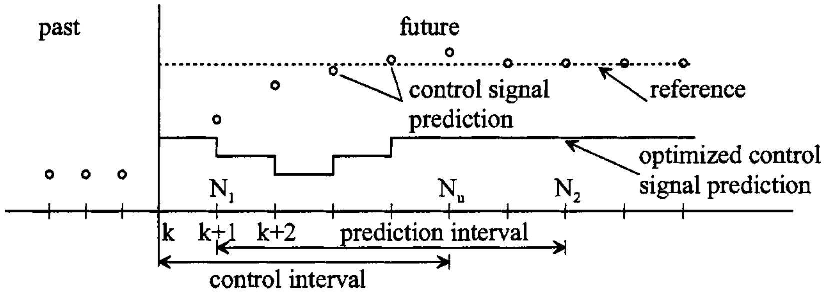

Fig. 3. The basic predictive control strategy.

the first order is described by the following difference equation with global linear parameters:

$$y_m(k+1) = \tilde{a}_m y_m(k) + \tilde{b}_m u(k) + \tilde{r}_m. \tag{29}$$

Closed-loop behavior of the system is defined by reference trajectory which is given in the form of reference model. The control goal is to determine the future control action so that the area between predicted output and reference trajectory over a certain prediction horizon (N_1, N_2) is minimized. The reference model in the case of first-order system is given by the following difference equation:

$$y_r(k+1) = a_r y_r(k) + b_r w(k) \tag{30}$$

where the reference model parameters should be chosen to fulfill the following equation:

$$\frac{b_r}{1-a_r} = 1. \tag{31}$$

This choice ensures that reference-model output tracks a constant reference signal w and it enables the reference trajectory tracking.

According to (31) it follows that:

$$y_r(k+1) = a_r y_r(k) + (1-a_r) w(k). \tag{32}$$

In the case of fuzzy predictive functional control, one single horizon is assumed $N_1 = N_2$, which is called coincidence horizon H. At this horizon the predicted output value coincide with the reference trajectory. To derive the control law also the constant future manipulated variable $u(k) = u(k+1) = \ldots = u(k+H-1)$ and $\lambda = 0$ has to be taken into account. The H-step ahead prediction of the process output based on fuzzy model is calculated assuming constant global process parameters over the whole prediction horizon and is given by

$$y_m(k+H) = \tilde{a}_m^H y_m(k) + \frac{\tilde{b}_m}{1-\tilde{a}_m}(1-\tilde{a}_m^H)u(k) + \frac{1}{1-\tilde{a}_m}(1-\tilde{a}_m^H)\tilde{r}_m. \tag{33}$$

The reference trajectory prediction is given by the following equation:

$$y_r(k+H) = a_r^H y_r(k) + (1-a_r^H) w(k). \tag{34}$$

The main idea of FPFC is the equivalence of the objective increment of the process and the model output increment. The objective increment Δ_p is defined as the difference between predicted reference trajectory $y_r(k+H)$ and actual process output signal $y_p(k)$

$$\Delta_p = y_r(k+H) - y_p(k). \tag{35}$$

Assuming (34) the objective increment Δ_p is defined as follows:

$$\Delta_p = a_r^H y_r(k) + (1-a_r^H) w(k) - y_p(k). \tag{36}$$

The model output increment Δ_m is defined by

$$\Delta_m = y_m(k+H) - y_m(k). \tag{37}$$

According to (33), the model output increment is the following:

$$\Delta_m = \tilde{a}_m^H y_m(k) + \frac{\tilde{b}_m}{1-\tilde{a}_m}(1-\tilde{a}_m^H)u(k) + \frac{1}{1-\tilde{a}_m}(1-\tilde{a}_m^H)\tilde{r}_m - y_m(k). \tag{38}$$

From the above equations and the goal of FPFC, which is described with the following:

$$\Delta_m = \Delta_p \tag{39}$$

the control law of FPFC is obtained in explicit analytical form as

$$u(k) = \frac{a_r^H (y_r(k) - w(k)) + (w(k) - y_p(k))}{\frac{\tilde{b}_m}{1-\tilde{a}_m}(1-\tilde{a}_m^H)} + \frac{y_m(k)}{\frac{\tilde{b}_m}{1-\tilde{a}_m}} - \frac{\tilde{r}_m}{\tilde{b}_m}. \tag{40}$$

The control law given in (40) is usually simplified into the form given in the following:

$$u_s(k) = \frac{(1-a_r^H)(w(k) - y_p(k))}{\frac{\tilde{b}_m}{1-\tilde{a}_m}(1-\tilde{a}_m^H)} + \frac{y_m(k)}{\frac{\tilde{b}_m}{1-\tilde{a}_m}} - \frac{\tilde{r}_m}{\tilde{b}_m}. \tag{41}$$

This simplification is justified in

$$\begin{aligned} \frac{a_r^H (y_r(k) - w(k))}{\frac{\tilde{b}_m}{1-\tilde{a}_m}(1-\tilde{a}_m^H)} &= \frac{a_r^H (y_p(k) + \epsilon(k) - w(k))}{\frac{\tilde{b}_m}{1-\tilde{a}_m}(1-\tilde{a}_m^H)} \\ &= \frac{a_r^H (y_p(k) - w(k))}{\frac{\tilde{b}_m}{1-\tilde{a}_m}(1-\tilde{a}_m^H)} - \frac{a_r^H \epsilon(k)}{\frac{\tilde{b}_m}{1-\tilde{a}_m}(1-\tilde{a}_m^H)} \end{aligned} \tag{42}$$

where $\epsilon(k)$ stands for

$$\epsilon(k) = y_r(k) - y_p(k) \tag{43}$$

and defines the reference-model error. Assuming a stable reference model the parameter a_r should satisfy the following condition:

$$0 < a_r < 1. \tag{44}$$

According to the integral action of predictive functional control, which is due to the second term on the right side of (41) or the internal model of the process, which is in the inner positive feedback loop, the reference-model error $\epsilon(k)$ tends to zero. This means that the last term in (42) tends to zero

$$\frac{a_r^H}{\frac{\tilde{b}_m}{1-\tilde{a}_m}(1-\tilde{a}_m^H)}\epsilon(k) \doteq 0. \tag{45}$$

Equation (42) is according to the previous equation simplified to the following:

$$\frac{a_r^H(y_r(k)-w(k))}{\frac{\tilde{b}_m}{1-\tilde{a}_m}(1-\tilde{a}_m^H)} \doteq \frac{a_r^H(y_p(k)-w(k))}{\frac{\tilde{b}_m}{1-\tilde{a}_m}(1-\tilde{a}_m^H)}. \tag{46}$$

This justifies the simplification of the control law in (40) to the simplified control law in (41).

The predictive functional control scheme introduce the incremental law through the internal model of the plant. This internal model in the positive feedback loop multiplied by inverse process model gain enables the integral action to the control system. This ensures that plant output tracks the reference-model output also in the case of the input and output disturbances [2], [6].

The transformation of the internal model feedback loop from (41) to transfer function results in

$$G_{im}(z) = \frac{(1-\tilde{a}_m z^{-1})}{(1-z^{-1})} \tag{47}$$

which clearly indicate the integral action of predictive functional control law.

V. Predictive Control based on Fuzzy Model for Heat-Exchanger Pilot Plant

The predictive approach discussed in the previous section has been implemented on a real temperature plant, the heat-exchanger pilot plant. The model of the plant can be given as model of first-order dynamics with a small time delay which is neglected. The parameters of the plant vary according to the operating point. The relation which is modeled by fuzzy model is presented in the following:

$$T_{sp}(k+1) = \mathcal{F}(T_{sp}(k), V_{mdv}(k)). \tag{48}$$

The fuzzy model which was developed on real-time data of the plant consists of six rules and is of the first order. The fuzzy parameters of the model depends only on physical variable T_{sp} or with other words, depends on process output y_p. The fuzzy model in form of fuzzy rules is presented in (49) with sampling time $T_s = 4$ s. The sampling time was chosen to fulfill the requirements of sampling in different operating points

$$\begin{aligned}
\mathbf{R}^1:&\quad \textbf{if } y_p(k) \textbf{ is } \mathbf{A}_1 \textbf{ then } y_p(k+1)\\
&\qquad = 0.9797y_p(k) + 0.0645u(k) - 0.0227\\
\mathbf{R}^2:&\quad \textbf{if } y_p(k) \textbf{ is } \mathbf{A}_2 \textbf{ then } y_p(k+1)\\
&\qquad = 0.9764y_p(k) + 0.0907u(k) - 0.1160\\
\mathbf{R}^3:&\quad \textbf{if } y_p(k) \textbf{ is } \mathbf{A}_3 \textbf{ then } y_p(k+1)\\
&\qquad = 0.9705y_p(k) + 0.1724u(k) - 0.6004\\
\mathbf{R}^4:&\quad \textbf{if } y_p(k) \textbf{ is } \mathbf{A}_4 \textbf{ then } y_p(k+1)\\
&\qquad = 0.9685y_p(k) + 0.1283u(k) - 0.1274\\
\mathbf{R}^5:&\quad \textbf{if } y_p(k) \textbf{ is } \mathbf{A}_5 \textbf{ then } y_p(k+1)\\
&\qquad = 0.9804y_p(k) + 0.0304u(k) + 0.4304\\
\mathbf{R}^6:&\quad \textbf{if } y_p(k) \textbf{ is } \mathbf{A}_6 \textbf{ then } y_p(k+1)\\
&\qquad = 0.9781y_p(k) + 0.0135u(k) + 0.8471.
\end{aligned} \tag{49}$$

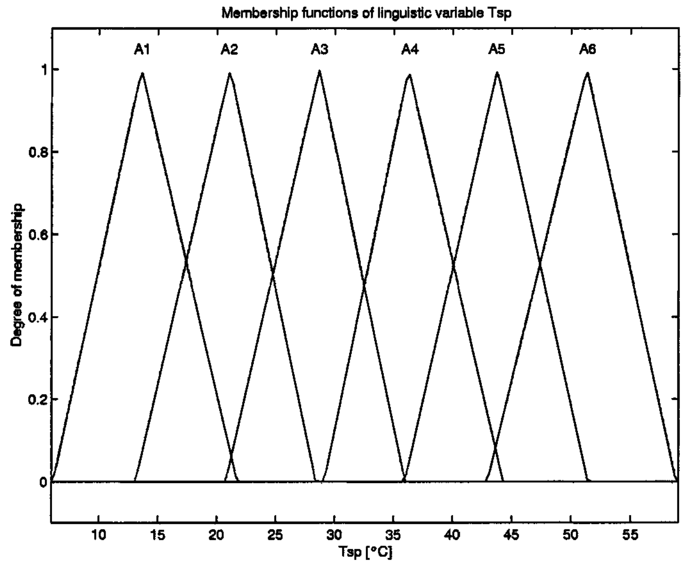

Fig. 4. The linguistic variable T_{sp}.

The linguistic variable of T_{sp} is shown in Fig. 4, where it is shown that operating domain is divided into six membership functions. The characteristics of the plant behavior are shown in Fig. 5, where a statical characteristic measured on the plant data is shown and the gain of the process that is also measured on the plant data is compared with the fuzzy model based estimation of the process gain. It is shown that process gain varies significantly according to the operating point. The fuzzy model in (49) was used as an internal model in the predictive functional algorithm. The global linear parameters are calculated instantly in each sampling instant as shown in (27). With respect to those parameters, the predictive functional control law is calculated due to (41). The prediction horizon is normally chosen as integer number in the interval defined in

$$1 \le H < \frac{T_r}{2T_s} \tag{50}$$

where T_r stands for the reference-model time constant and T_s for sampling time. In our case, it was chosen due to the frequency characteristics study of the input and output disturbance

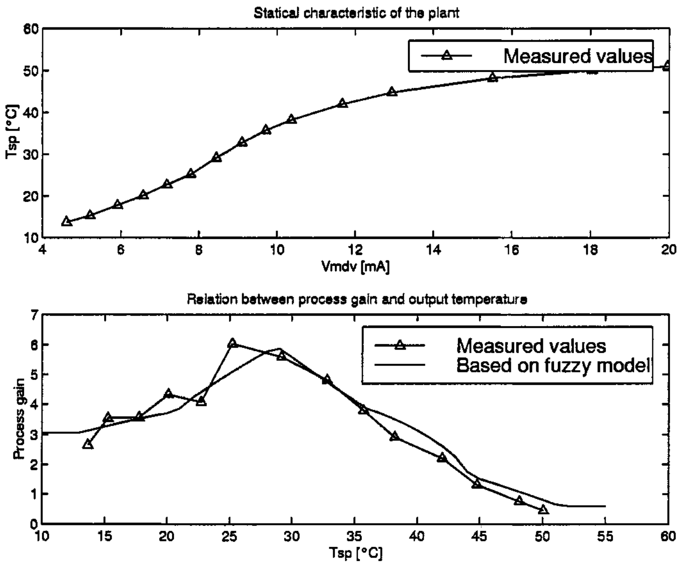

Fig. 5. The plant and fuzzy model characteristics.

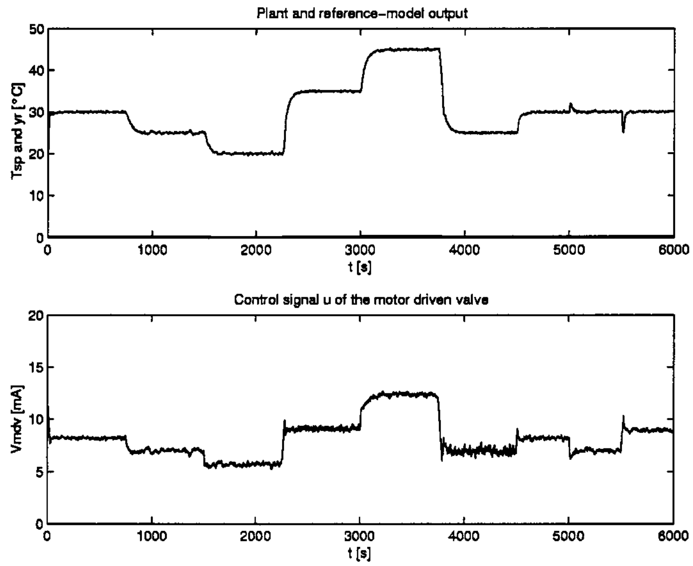

Fig. 6. The plant output temperature, the reference-model temperature, and the control original.

transfer functions and the open-loop transfer function. The coincidence parameter H has a very small influence on the input disturbance frequency response, in the case of the output disturbance frequency response the magnitude is moving to the lower frequencies when the coincidence horizon is getting higher. The higher value of coincidence horizon would also shift the magnitude of the open-loop frequency response to the lower frequencies, which means a slower response to the reference signal changes. In our application a predictive horizon has been chosen as $H = 8$ according to the previous study of frequency characteristics. The reference-model time constant T_r is chosen approximately equal to the time constant of the plant. The choice was made to show only the phenomena of perfect reference-model tracking in the case of nonlinear plant gain using fuzzy predictive functional control. The reference-model time constant was chosen as $T_r = 160$ s, this results in the discrete transfer function

$$G_r(z^{-1}) = \frac{0.0247z^{-1}}{1 - 0.9753z^{-1}} \tag{51}$$

with the sampling time $T_s = 4$ s. The reference-model time constant T_r or in a discrete time domain the parameter a_r, is the most important tuning parameter in the case of FPFC. The parameter a_r has great influence on the input and output disturbance rejection. By decreasing the parameter a_r, the magnitude of frequency response characteristic is shifted to the higher frequencies in both cases. A lower value of the parameter a_r also reflects in a faster response to the reference signal changes.

The output of the process T_{sp} and the reference-model output $y_r(k)$ are shown in Fig. 6. The inlet temperature of the heated water T_{ec} is controlled using a simple ON–OFF controller and varies between 60 °C and 65 °C. Also the outlet flow F_p and the inlet temperature T_{ep} are slightly changing during the operation period. In the last period shown in Fig. 6, a disturbance was made changing the outlet flow F_p from the normal value. This variation which has a great influence to the process dynamics was rejected very fast. Due to the leak of sensors the outlet flow F_p can only be displayed and cannot be sampled. That is also the main reason that it has not been used as an input variable to the process model. The problem of heat-exchanger control with sensor limitation represents a serious problem. To obtain a good control action is difficult because of the nonlinear system behavior. The results which have been obtained using the proposed fuzzy predictive algorithm exhibits a very good performance in both modes, in model following mode, and disturbance rejection mode.

VI. Conclusion

In this paper, fuzzy predictive control scheme is presented. The development of a new fuzzy predictive scheme was motivated by the unsatisfactory results obtained by using conventional techniques. Regarding the real-time experiments realized on the heat-exchanger plant, it can be seen that the novel algorithm introduces a great robustness and satisfactory performance also in the presence of model parameters mismatch which was obtained by change of the outlet flow. The proposed approach offers some advantages in the case of nonlinear systems with simple dynamics. The main advantage in comparison to the other modern techniques is simplicity together with excellent performance.

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Drago Matko, photograph and biography not available at time of publication.