Predictor-Based Extended-State-Observer Design for Consensus of MASs With Delays and Disturbances

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***Abstract*—In this paper, we study output feedback leader–follower consensus problem for multiagent systems subject to external disturbances and time delays in both input and output. First, we consider the linear case and a novel predictor-based extended state observer is designed for each follower with relative output information of the neighboring agents. Then, leader–follower consensus protocols are proposed which can compensate the delays and disturbances efficiently. In particular, the proposed observer and controller do not contain any integral term of the past control input and hence are easy to implement. Consensus analysis is put in the framework of Lyapunov–Krasovskii functionals and sufficient conditions are derived to guarantee that the consensus errors converge to zero asymptotically. Then, the results are extended to nonlinear multiagent systems with nonlinear disturbances. Finally, the validity of the proposed design is demonstrated through a numerical example of network-connected unmanned aerial vehicles.**

***Index Terms*—Consensus, extended state observer (ESO), input delay and output delay, multiagent systems, nonlinear disturbances, output feedback.**

I. Introduction

IN RECENT years, cooperative control of a group of agents has drawn increased attention due to its high potential in many applications, such as vehicle formation [1], [2], synchronization [3], satellite clusters [4], and distributed sensor network [5]. Consensus control is a fundamental problem in distributed cooperative control of multiagent systems, which deals with the control design to ensure that all the agents achieve the same control objective, such as common output or state values [6]. Laplacian matrix, which plays an important role in consensus control design, is used to describe the network connection. Pioneering results in [7]–[10] are focused on simple agent dynamics, such as single or double integrators. By using local neighbor-to-neighbor interaction or the so called nearest neighbor rule, a group of simple agents can stay together and move in the same direction without centralized coordination. Then, consensus results for higher order linear and nonlinear multiagent systems are addressed in [11]–[19] and the references therein. In term of the number of leader, the researches can be roughly classified into three classes, that is, leaderless consensus (consensus without a leader) [12], leader–follower consensus (or consensus tracking) [16], and containment control where more than one leaders may exist in agent networks [18]. Compared to leaderless consensus, consensus tracking, and containment control have the advantages to determine the final consensus value in advance. Adaptive control-based protocols are developed in [14]–[16] to avoid the use of the *Fiedler* eigenvalue of the Laplacian matrix in control gain design. Most of the consensus protocols are based on the states of neighboring agents. Some results on output feedback consensus can be found in [20]–[22]. Furthermore, reinforcement learning and iterative learning control have also been used for consensus control in [23] and [24]. Recent results on consensus problems are reported in [25]–[29] and the references therein.

With the deepening research on multiagent systems, time delays arising from agents are diverse and cannot be ignored. Communication delay is one source of delay due to the interactions between the agents. Another source of delay is delays in the input and output channel due to the decision-making and signal processing. In applications, for vision-based autonomous robots, camera latency and image processing will cause delays as well. State predictor has been commonly used to deal with input delay. By adding compensation in the controller design, the adverse effect of the input delay could be offset. State-predictor-based controllers are designed in [30] and [31] for single systems and in [32]–[34] for multiagent systems. However, a drawback of the predictor-based methods is that the controllers involve integral terms of the control input, resulting in difficulty for the control implementation. An asymptotic predictor design is first proposed in [35] for linear time invariant systems, as an alternative to methods based on direct integral expressions. A sequential subpredictors approach is introduced in [36] for single linear systems to avoid the use of infinite-dimensional integral term. A truncated prediction feedback (TPF) approach is

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developed to avoid this problem by ignoring the troublesome integral part, and use the prediction based on the exponential of the system matrix. Finite-dimensional TPF controllers have been designed in [37]–[39] for multiagent systems with input and communication delay. Due to the measurement and signal processing, in many scenarios state information cannot be accessed and observer-based output feedback consensus protocols need to be designed. For the output feedback consensus protocols designed in [20]–[22], time delays are not considered. Output feedback TPF consensus controller is given in [37] for some special input-delayed multiagent systems that the eigenvalues of A matrix are on the imaginary axis. There is still a lack of research for general multiagent systems with input/output delay. In addition to the results above, there are some other delays have been studied in consensus control. For example, in [40], distributed time delays is considered for a class of complex networks using sampled data information. Containment control problem with multiple stationary/dynamic leaders are investigated in [41] for second-order multiagent systems with time-varying delays. The exponential leader-following consensus problem is investigated in [42] for a class of nonlinear stochastic networked multiagent systems with partial mixed impulses and unknown time-varying but bounded delays.

One of the fundamental problems in control theory is the disturbance rejection problem, which addresses design of a feedback controller to achieve asymptotic rejection of undesired disturbances in an uncertain system while maintaining closed-loop stability [43]. There are many classical results including H_∞ control, sliding mode control, backstepping and disturbance observer design that address this problem [44]–[48], among many others. Furthermore, consensus output regulation has been a topic for consensus control in recent years which deals with the rejection of deterministic disturbances. Some typical results can be found in [6] and [49] and the references therein.

The main purpose of this paper is to further advance the application of state predictor feedback method in consensus control of multiagent systems with mixed time delays. There are possibly three main contributions. First, unlike [32]–[34] where traditional state predictors are designed for multiagent systems with input delay, in this paper, a new structure of state predictor is designed for each agent without the use of any integral term of the past control input, which greatly reduces the computation burden and improves the practical implementation. Second, the observer-based output feedback consensus problem is considered and the output delay is also taken into account in the observer design. Compared with the consensus state observers designed in [37], the observers here are simpler and the restriction on the structure of A matrix is removed. The result is suitable for general multiagent systems. Third, nonlinear system dynamics and nonlinear disturbances are considered in this paper. An improved predictor-based extended state observer (ESO) design strategy is developed and the nonlinear disturbances can be efficiently estimated. Compared with [25], [46], and [48], the results in this paper are more preferable in practical applications.

The remainder of this paper is organized as follows. Section II presents some notations and the problem formulation. A few preliminary results for consensus analysis are given in Section III. Section IV presents the main results on the consensus control design for general linear multiagent systems. The results are extended to nonlinear multiagent systems in Section V. Simulation results are given in Section VI. Section VII concludes this paper.

II. Problem Statement

Consider a group of $N+1$ agents consisting of N followers and one leader indexed by 0 (here and hereafter, the argument t is omitted excepting delayed arguments)

$$\begin{cases} \dot{x}_i = Ax_i + Bu_i(t-\tau_u) + BF\omega_i \\ y_i = Cx_i\big(t-\tau_y\big) \\ \dot{\omega}_i = S\omega_i \end{cases} \tag{1}$$

where for agent i, $i = 0, 1, \ldots, N$, $x_i \in \mathbb{R}^n$ is the state, $u_i \in \mathbb{R}^q$ is the control input, and $y_i \in \mathbb{R}^p$ is the output. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times q}$, $C \in \mathbb{R}^{p\times n}$, and $F \in \mathbb{R}^{q\times s}$ are constant matrices with (A, B) being controllable and (A, C) being observable, τ_u, $\tau_y > 0$ are the input and the output delay, respectively, $S \in \mathbb{R}^{s\times s}$ is a known constant matrix, $\omega_i \in \mathbb{R}^s$ is external input disturbance with unknown bound. For the leader–follower structure, it is reasonable to assume that the leader has no neighbors, and the leader's control input is zero [15], [50], [51], i.e., $u_0 \equiv 0$ and $\omega_0 \equiv 0$.

The communication connections among agents are described by a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ represents the agents and $\mathcal{E}$ represents the connections between the agents. In the directed graph, $(i, j) \in \mathcal{E}$ represents the communication from the ith agent to the jth agent, but not vice versa. For $N + 1$ agents, the associated adjacency matrix of $\mathcal{G}$ is defined as $\mathcal{A} = [a_{ij}]_{(N+1)\times(N+1)} \in \mathbb{R}^{(N+1)\times(N+1)}$. If there is a connection from agent j to agent i, $a_{ij} = 1$; otherwise $a_{ij} = 0$. The Laplacian matrix $\mathcal{L} = [l_{ij}]_{(N+1)\times(N+1)}$ associated with $\mathcal{A}$ is defined by $l_{ii} = \sum_{j=1}^{N+1} a_{ij}$ and $l_{ij} = -a_{ij}$ when $i \neq j$.

The consensus disturbance rejection problem considered in this paper is to design an observer-based control algorithm for each follower by using its relative output information such that the system (1) can reach state consensus with disturbance rejection. That is, under the proposed algorithm, the following hold for all initial conditions:

$$\lim_{t\to\infty} (x_i - x_0) = 0, \quad \forall i = 1, 2, \ldots, N. \tag{2}$$

Assumption 1: The eigenvalues of S are distinct and on the imaginary axis. Furthermore, the pair (S, BF) is observable.

Assumption 2: The communication topology $\mathcal{G}$ contains a directed spanning tree with the leader as the root.

Remark 1: From a practical point of view, any periodic disturbances can be approximated by sinusoidal functions with different frequencies, and those sinusoidal functions can be formulated as the state variables of the exosystem under Assumption 1 [52]. For the convenience of presentation, in this paper, it is assumed that the disturbance frequency is same for all the agents which determined by S matrix. With some slight adjustments, the results can be extended to $\dot{\omega}_i = S_i\omega_i$ case as shown in [25].

III. PRELIMINARY RESULTS

In this section, we give some preliminary results.

Lemma 1 [14]: Under Assumption 2, the Laplacian matrix $\mathcal{L}$ has the following structure:

$$\mathcal{L} = \begin{bmatrix} 0 & 0_{1\times N} \\ \mathcal{L}_2 & \mathcal{L}_1 \end{bmatrix}$$

where $\mathcal{L}_1 \in \mathbb{R}^{N\times N}$ and $\mathcal{L}_2 \in \mathbb{R}^{N\times 1}$. Furthermore, $\mathcal{L}_1$ is a nonsingular M-matrix and there exists a positive diagonal matrix G such that

$$G\mathcal{L}_1 + \mathcal{L}_1^T G \geq \rho_0 I$$

for some positive constant ρ_0. G can be constructed by setting $G = \text{diag}\{g_1, g_2, \ldots, g_N\}$, where $g = [g_1, g_2, \ldots, g_N]^T = (\mathcal{L}_1^T)^{-1}[1, 1, \ldots, 1]^T$.

Lemma 2 [53]: For a positive definite matrix P, and a function $x : [a, b] \to \mathbb{R}^n$, with $a, b \in \mathbb{R}$ and $b > a$, the following inequality holds:

$$\left(\int_a^b x^T(\tau)\text{d}\tau\right) P \left(\int_a^b x(\tau)\text{d}\tau\right) \leq (b-a)\int_a^b x^T(\tau)Px(\tau)\text{d}\tau.$$

Lemma 3 [54]: For a positive definite matrix P, the following identity holds:

$$e^{A^T t}Pe^{At} - e^{\alpha t}P = -e^{\alpha t}\int_0^t e^{-\alpha\tau}e^{A^T\tau}Re^{A\tau}\text{d}\tau \tag{3}$$

where $\alpha \geq 0$ is a scalar and $R = -A^T P - PA + \alpha P$. Furthermore, if R is positive definite, $\forall t > 0$

$$e^{A^T t}Pe^{At} \leq e^{\alpha t}P. \tag{4}$$

Lemma 4: For any given $a, b \in \mathbb{R}^n$, we have

$$2a^T SQb \leq a^T SPS^T a + b^T Q^T P^{-1}Qb \tag{5}$$

where $P > 0$, S and Q have appropriate dimensions.

IV. PREDICTOR-BASED EXTENDED-STATE-OBSERVER CONSENSUS DESIGN FOR LINEAR CASE

A. Predictor-Based ESO and Controller Design

Let $\eta_i = x_i - x_0$. Then, we have

$$\begin{cases} \dot{\eta}_i = A\eta_i + Bu_i(t-\tau_u) + BF\omega_i \\ \tilde{y}_i = C\eta_i(t-\tau_y) \end{cases} \tag{6}$$

where $\tilde{y}_i = y_i - y_0$.

Define a new state $z_i = [\eta_i^T, \omega_i^T]^T$, which includes the exosystem model. The state-space equation (1) can be rewritten in the augmented form

$$\begin{cases} \dot{z}_i = A_z z_i + B_z u_i(t-\tau_u) \\ \tilde{y}_i = C_z z_i(t-\tau_y) \end{cases} \tag{7}$$

where

$$A_z = \begin{bmatrix} A & BF \\ 0 & S \end{bmatrix}, B_z = \begin{bmatrix} B \\ 0 \end{bmatrix}, C_z = \begin{bmatrix} C & 0 \end{bmatrix}.$$

Define $\bar{z}_i = [\bar{\eta}_i^T, \bar{\omega}_i^T]^T$ as the estimation of the state z_i at time $t+\tau_u$. Then, a predictor-based extended-state-observer is constructed as

$$\dot{\bar{z}}_i = A_z\bar{z}_i + B_z u_i + L\sum_{j=1}^N l_{ij}\big(\tilde{y}_j - C_z\bar{z}_j(t-\tau)\big) \tag{8}$$

where $\tau = \tau_u + \tau_y$, L is the observer gain matrix to be designed later. The estimation error is defined by $\tilde{e}_i = [\tilde{e}_{\eta i}^T, \tilde{e}_{\omega i}^T]^T = z_i - \bar{z}_i(t-\tau_u)$. Then, we have

$$\begin{aligned} \dot{\tilde{e}}_i &= A_z\tilde{e}_i - L\sum_{j=1}^N l_{ij}\big(C_z z_j(t-\tau) - C_z\bar{z}_j(t-\tau-\tau_u)\big) \\ &= A_z\tilde{e}_i - LC_z\sum_{j=1}^N l_{ij}\tilde{e}_j(t-\tau) \\ &= A_z\tilde{e}_i + LC_z v_i(t-\tau) \end{aligned} \tag{9}$$

where $v_i = -\sum_{k=1}^N l_{ik}\tilde{e}_k$. From the error dynamics (9), we have

$$\tilde{e}_i = e^{A_z\tau}\tilde{e}_i(t-\tau) + \int_{t-\tau}^t e^{A_z(t-s)}LC_z v_i(s-\tau)\text{d}s \tag{10}$$

and

$$\begin{aligned} \dot{\tilde{e}}_i &= A_z\tilde{e}_i - LC_z e^{-A_z\tau}\sum_{j=1}^N l_{ij}e^{A_z\tau}\tilde{e}_j(t-\tau) \\ &= A_z\tilde{e}_i - L\bar{C}_z\sum_{j=1}^N l_{ij}\tilde{e}_j \\ &\quad + L\bar{C}_z\sum_{j=1}^N l_{ij}\int_{t-\tau}^t e^{A_z(t-s)}LC_z v_j(s-\tau)\text{d}s \end{aligned}$$

where $\bar{C}_z = C_z e^{-A_z\tau}$. Define $\tilde{e} = [\tilde{e}_1^T, \tilde{e}_2^T, \ldots, \tilde{e}_N^T]^T$ and $v = [v_1^T, v_2^T, \ldots, v_N^T]^T$. The closed-loop system is then described by

$$\dot{\tilde{e}} = \big[(I\otimes A_z) - (\mathcal{L}_1\otimes L\bar{C}_z)\big]\tilde{e} + \lambda \tag{11}$$

where I represents the identity matrix with appropriate dimension, and

$$\lambda = (\mathcal{L}_1\otimes L\bar{C}_z)\int_{t-\tau}^t \Big(I\otimes e^{A_z(t-s)}LC_z\Big)v(s-\tau)\text{d}s.$$

The controller for the ith agent is designed as

$$u_i = -K\bar{\eta}_i - F\bar{\omega}_i = -[K, F]\bar{z}_i \tag{12}$$

where K is the control gain matrix to be designed later. Under control algorithm (12), multiagent systems (6) can be written as

$$\begin{aligned} \dot{\eta}_i &= A\eta_i - BK\bar{\eta}_i(t-\tau_u) - BF\bar{\omega}_i(t-\tau_u) + BF\omega_i \\ &= (A-BK)\eta_i + BB_1\tilde{e}_i \end{aligned}$$

where $B_1 = [K, F]$. Let $\eta = [\eta_1^T, \eta_2^T, \ldots, \eta_N^T]^T$. The closed-loop system is then described by

$$\dot{\eta} = (I\otimes(A-BK))\eta + (I\otimes BB_1)\tilde{e}. \tag{13}$$

Remark 2: Unlike [55], which requires the state of the leader to be accessed by all the followers, in this paper, we assume that only a few followers can access the output information of the leader. Under this assumption, $\tilde{y}_j = y_j - y_0$ in (8) is not available to all the agents. However, based on the relative output information of the neighboring agent, we have

$$\sum_{j=0}^{N} a_{ij}(y_i - y_j) = \sum_{j=1}^{N} a_{ij}(y_i - y_j) + a_{i0}(y_i - y_0) = \sum_{j=1}^{N} a_{ij}(\tilde{y}_i - \tilde{y}_j) + a_{i0}\tilde{y}_i = \sum_{j=1}^{N} l_{ij}\tilde{y}_j.$$

From this equation, it can be verified that observer (8) is implementable even if some agents cannot access the leader's information.

B. Stability Analysis

With the observer and the control law shown in (8) and (12), the control and observer gains are chosen as

$$K = B^T P_1,\ L = cP_2^{-1}\bar{C}_z^T \tag{14}$$

where P_1 and P_2 are positive definite matrices, c is a constant such that $c \geq 2g_{\max}/\rho_0$, $g_{\max} = \max\{g_1, g_2, \ldots, g_N\}$.

For the stability analysis, first we need to establish a bound of the extra integral term λ.

Lemma 5: For the integral term λ, a bound can be established as

$$\|\lambda\|^2 \leq \sigma_0 \int_{t-\tau}^{t} \tilde{e}^T(s-\tau)\tilde{e}(s-\tau)\mathrm{d}s \tag{15}$$

where $\sigma_0 = \tau\|\mathcal{L}_1\|_F^4\rho^4 e^{2\alpha\tau}$, α is a positive number such that $\alpha \geq \lambda_{\max}(A_z^T + A_z)$, ρ is a positive real number such that

$$\rho^2 I \geq c^2\bar{C}_z^T\bar{C}_z P_2^{-1}P_2^{-1}\bar{C}_z^T\bar{C}_z \tag{16}$$

and $\|\cdot\|_F$ denotes the Frobenius norm of a matrix.

Proof: See Appendix A. ■

Based on the above results, the following theorem presents sufficient conditions such that the consensus disturbance rejection problem is solved by using the relative output information.

Theorem 1: For multiagent systems (1) with Assumption 2, the consensus disturbance rejection problem can be solved by the observer (8) and the controller (12) with (14) if there exist positive definite matrices P_1, P_2 and constants $\kappa, \rho > 0$, such that

$$AW + WA^T - BB^T < 0 \tag{17}$$

$$\rho P_2 - c\bar{C}_z^T\bar{C}_z \geq 0 \tag{18}$$

$$\begin{bmatrix} P_2 A_z + A_z^T P_2 - 2\bar{C}_z^T\bar{C}_z + H & P_2 \\ P_2 & -\kappa^{-1} \end{bmatrix} < 0 \tag{19}$$

are satisfied with $W = P_1^{-1}$ and $H = g_{\min}^{-1}B_1^T B_1 + \sigma_1 I$, where $g_{\min} = \min\{g_1, g_2, \ldots, g_N\}$, $\sigma_1 = \kappa^{-1}g_{\max}g_{\min}^{-1}e^{\tau}\sigma_0$, $\sigma_0 = \tau\|\mathcal{L}_1\|_F^4\rho^4 e^{2\alpha\tau}$ is a positive number defined in Lemma 5.

Proof: To start the consensus analysis, we try a Lyapunov function candidate

$$V_0 = \eta^T(I \otimes P_1)\eta + \tilde{e}^T(G \otimes P_2)\tilde{e}. \tag{20}$$

In view of (11) and (13), we have

$$\begin{aligned} \dot{V}_0 &= \eta^T\left[I \otimes \left(A^T P_1 + P_1 A - 2P_1 BB^T P_1\right)\right]\eta \\ &\quad + \tilde{e}^T\left[G \otimes \left(A_z^T P_2 + P_2 A_z\right) - c\left(G\mathcal{L}_1 + \mathcal{L}_1^T G\right) \otimes \bar{C}_z^T\bar{C}_z\right]\tilde{e} \\ &\quad + 2\eta^T(I \otimes P_1 BB_1)\tilde{e} + 2\tilde{e}^T(G \otimes P_2)\lambda \\ &\leq \eta^T\left[I \otimes \left(A^T P_1 + P_1 A - P_1 BB^T P_1\right)\right]\eta \\ &\quad + \tilde{e}^T\left[G \otimes \left(A_z^T P_2 + P_2 A_z - 2\bar{C}_z^T\bar{C}_z + \kappa P_2 P_2\right) + B_1^T B_1\right]\tilde{e} \\ &\quad + \kappa^{-1}g_{\max}\|\lambda\|^2 \end{aligned} \tag{21}$$

where Lemmas 1 and 4 are used for the derivation.

Using (15) and (21), we obtain that

$$\begin{aligned} \dot{V}_0 &\leq \eta^T\left[I \otimes \left(A^T P_1 + P_1 A - P_1 BB^T P_1\right)\right]\eta \\ &\quad + \tilde{e}^T\left[G \otimes \left(A_z^T P_2 + P_2 A_z - 2\bar{C}_z^T\bar{C}_z + \kappa P_2 P_2\right) + B_1^T B_1\right]\tilde{e} \\ &\quad + \kappa^{-1}g_{\max}\sigma_0 \int_{t-\tau}^{t} \tilde{e}^T(s-\tau)\tilde{e}(s-\tau)\mathrm{d}s. \end{aligned} \tag{22}$$

For the delayed term shown in (22), we consider the following Krasovskii functional:

$$V_1 = e^{\tau}\int_{t-\tau}^{t} e^{s-t}\tilde{e}^T(s-\tau)\tilde{e}(s-\tau)\mathrm{d}s + e^{\tau}\int_{t-\tau}^{t} \tilde{e}^T(s)\tilde{e}(s)\mathrm{d}s.$$

A direct evaluation gives that

$$\begin{aligned} \dot{V}_1 &= -e^{\tau}\int_{t-\tau}^{t} e^{s-t}\tilde{e}^T(s-\tau)\tilde{e}(s-\tau)\mathrm{d}s \\ &\quad - \tilde{e}^T(t-2\tau)\tilde{e}(t-2\tau) + e^{\tau}\tilde{e}^T\tilde{e} \\ &\leq -\int_{t-\tau}^{t} \tilde{e}^T(s-\tau)\tilde{e}(s-\tau)\mathrm{d}s + e^{\tau}\tilde{e}^T\tilde{e}. \end{aligned} \tag{23}$$

Let

$$V = V_0 + \kappa^{-1}g_{\max}\sigma_0 V_1. \tag{24}$$

From (22)–(24), we obtain that

$$\dot{V} \leq \eta^T(I \otimes H_1)\eta + \tilde{e}^T(G \otimes H_2)\tilde{e} \tag{25}$$

where

$$H_1 := A^T P_1 + P_1 A - P_1 BB^T P_1 \tag{26}$$

$$H_2 := A_z^T P_2 + P_2 A_z - 2C_z^T C_z + \kappa P_2 P_2 + g_{\min}^{-1}B_1^T B_1 + \sigma_1 I \tag{27}$$

with $\sigma_1 = g_{\min}^{-1}e^{\tau}\kappa^{-1}g_{\max}\sigma_0$ being a positive number defined in Theorem 1.

From the analysis in this section, we know that the control law (12) stabilizes η and $\tilde{e}$ if $H_1 < 0$ and $H_2 < 0$ in (25) are satisfied. Indeed, it is easy to see the conditions $H_1 < 0$ and $H_2 < 0$ are equivalent to the conditions specified in (17) and (19). Furthermore, the condition specified in (18) is equivalent to the condition (16). It implies that η and $\tilde{e}$ converge to zero asymptotically. Hence, the consensus with disturbance rejection in (2) is achieved. ■

Remark 3: The conditions shown in (17)–(19) can be checked by standard LMI routines for a set of fixed values. In particular, we suggest the following step by step algorithm.

1) Solve the LMI equation (17) for W, and then $B_1 = [K, F] = [B^T P_1, F]$ is fixed with $P_1 = W^{-1}$.
2) Fix the value of ρ, κ to some constants $\tilde{\rho}, \tilde{\kappa} > 0$; make an initial guess for the values of $\tilde{\rho}, \tilde{\kappa}$.

3) Solve the LMI equation (19) for P_2 with the fixed values; if a feasible value of P_2 cannot be found, return to step 2) and reset the values of $\tilde{\rho}, \tilde{\kappa}$.
4) Solve the LMI equation (18) for ρ with the feasible value of P_2 obtained in step 3) and make sure that the value of ρ is minimized.
5) If the condition $\tilde{\rho} \geq \rho$ is satisfied, then $(\tilde{\rho}, \tilde{\kappa}, W, P_2)$ is a feasible solution for Theorem 1; otherwise, set $\tilde{\rho} = \rho$ and return to step 3).

Remark 4: As mentioned in [56], unlike low-order time-delayed linear systems, where necessary and sufficient conditions for the stability of such systems have been determined by analyzing the positions of the roots of the characteristic equations, for high-order time-delayed linear systems (including the multiagent case considered in this paper), only sufficient conditions for the stability of such systems have been determined through Lyapunov stability analysis. The delay bound and closed-loop parameters are simultaneously involved in these sufficient conditions, which are typically in the form of LMIs. Thus, to find the upper bound of τ, we may use τ^* to replace τ in (17)–(19) and solve the following optimization problem:

$$\bar{\tau} = \sup_{P_1 \geq 0, P_2 \geq 0, \kappa \geq 0, \rho \geq 0} \{\tau^*\} \text{ s.t. (17)–(19).}$$

V. Predictor-Based Extended-State-Observer Design for Nonlinear Case

In this section, we consider the predictor-based extended-state-observer design for nonlinear case

$$\begin{cases} \dot{x}_i = Ax_i + Bu_i(t-\tau_u) + BF\omega_i + f_1(x_i) \\ y_i = Cx_i(t-\tau_y) \\ \dot{\omega}_i = S\omega_i + f_2(\omega_i) \end{cases} \tag{28}$$

where $f_1(\cdot)$ and $f_2(\cdot)$ are Lipschitz nonlinear functions with Lipschitz constants γ_1 and γ_2, such that

$$\begin{cases} \|f_1(x) - f_1(y)\| \leq \gamma_1 \|x-y\| \\ \|f_2(x) - f_2(y)\| \leq \gamma_2 \|x-y\|. \end{cases} \tag{29}$$

Let $\xi_i = x_i - x_0$. Then, we have

$$\begin{cases} \dot{\xi}_i = A\xi_i + Bu_i(t-\tau_u) + BF\omega_i + \tilde{f}_i(x_i, x_0) \\ \bar{y}_i = C\xi_i(t-\tau_y) \end{cases} \tag{30}$$

where $\bar{y}_i = y_i - y_0$, $\tilde{f}_i(x_i, x_0) = f_1(x_i) - f_1(x_0)$.

Define a new state $\chi_i = [\xi_i^T, \omega_i^T]^T$, which includes the exosystem model. The state-space equation (30) can be rewritten in the augmented form

$$\begin{cases} \dot{\chi}_i = A_\xi \chi_i + B_\xi u_i(t-\tau_u) + \mathcal{F}_i \\ \bar{y}_i = C_\xi \xi_i(t-\tau_y) \end{cases} \tag{31}$$

where

$$A_\xi = \begin{bmatrix} A & BF \\ 0 & S \end{bmatrix}, B_\xi = \begin{bmatrix} B \\ 0 \end{bmatrix}, \mathcal{F}_i = \begin{bmatrix} \tilde{f}_i(x_i, x_0) \\ f_2(\omega_i) \end{bmatrix}$$
$$C_\xi = \begin{bmatrix} C & 0 \end{bmatrix}.$$

Define $\bar{\chi}_i = [\bar{\xi}_i^T, \bar{\omega}_i^T]^T$ as the estimation of the augmented state χ_i at time $t + \tau_u$. Then, a predictor-type observer is constructed as

$$\begin{aligned} \dot{\bar{\chi}}_i &= A_\xi \bar{\chi}_i + B_\xi u_i + \bar{\mathcal{F}}_i \\ &\quad + L \sum_{j=1}^N l_{ij}\big(\bar{y}_j - C_\xi \bar{\chi}_j(t-\tau)\big) \end{aligned} \tag{32}$$

where $\bar{\mathcal{F}}_i = [0, f_2^T(\bar{\omega}_i)]^T$, L is the observer gain matrix. The estimation error is defined by

$$\bar{e}_i = [\bar{e}_{\xi i}^T, \bar{e}_{\omega i}^T]^T = \chi_i - \bar{\chi}_i(t-\tau_u).$$

It follows that:

$$\begin{aligned} \dot{\bar{e}}_i &= A_\xi e_i - L \sum_{j=1}^N l_{ij}\big(C_\xi \chi_j(t-\tau) - C_\xi \bar{\chi}_j(t-\tau-\tau_u)\big) + \tilde{\mathcal{F}}_i \\ &= A_\xi \bar{e}_i - LC_\xi \sum_{j=1}^N l_{ij} \bar{e}_j(t-\tau) + \tilde{\mathcal{F}}_i \end{aligned} \tag{33}$$

where $\tilde{\mathcal{F}}_i = \mathcal{F}_i - \bar{\mathcal{F}}_i(t-\tau_u)$. Similar to (10), we obtain that

$$\begin{aligned} \dot{\bar{e}}_i &= A_\xi \bar{e}_i - L\bar{C}_\xi \sum_{j=1}^N l_{ij} \bar{e}_j + \tilde{\mathcal{F}}_i \\ &\quad + L\bar{C}_\xi \sum_{j=1}^N l_{ij} \int_{t-\tau}^t e^{A_\xi(t-s)} \Big[LC_\xi \bar{v}_j(s-\tau) + \tilde{\mathcal{F}}_j\Big] \mathrm{d}s \end{aligned}$$

where $\bar{v}_j = -\sum_{k=1}^N l_{jk} \bar{e}_k$, $\bar{C}_\xi = C_\xi e^{-A_\xi \tau}$. Define $\bar{e} = [\bar{e}_1^T, \bar{e}_2^T, \ldots, \bar{e}_N^T]^T$, $\bar{v} = [\bar{v}_1^T, \bar{v}_2^T, \ldots, \bar{v}_N^T]^T$, and $\tilde{\mathcal{F}} = [\tilde{\mathcal{F}}_1^T, \tilde{\mathcal{F}}_2^T, \ldots, \tilde{\mathcal{F}}_N^T]^T$. The closed-loop system is then described by

$$\dot{\bar{e}} = \big[\big(I \otimes A_\xi\big) - \big(\mathcal{L}_1 \otimes L\bar{C}_\xi\big)\big]\bar{e} + \bar{\lambda} + \phi \tag{34}$$

where

$$\begin{aligned} \bar{\lambda} &= \big(\mathcal{L}_1 \otimes L\bar{C}_\xi\big) \int_{t-\tau}^t \Big(I \otimes e^{A_\xi(t-s)} LC_z\Big) \bar{v}(s-\tau) \mathrm{d}s \\ \phi &= \big(\mathcal{L}_1 \otimes L\bar{C}_\xi\big) \int_{t-\tau}^t \Big(I \otimes e^{A_\xi(t-s)}\Big) \tilde{\mathcal{F}} \mathrm{d}s. \end{aligned}$$

The controller is designed in the same way as shown in (12)

$$u_i = -K\bar{\xi}_i - F\bar{\omega}_i = -[K, F]\bar{\chi}_i. \tag{35}$$

Under (35), the multiagent systems (30) can be written as

$$\begin{aligned} \dot{\xi}_i &= A\xi_i - BK\bar{\xi}_i(t-\tau_u) - BF\bar{\omega}_i(t-\tau_u) \\ &\quad + BF\omega_i + \tilde{f}_i(x_i, x_0) \\ &= (A - BK)\xi_i + B\bar{B}_1 \bar{e}_i + \tilde{f}_i(x_i, x_0) \end{aligned}$$

where $\bar{B}_1 = [K, F]$. Let $\xi = [\xi_1^T, \xi_2^T, \ldots, \xi_N^T]^T$ and $\tilde{f} = [\tilde{f}_1^T, \tilde{f}_2^T, \ldots, \tilde{f}_N^T]^T$. The closed-loop system is written as

$$\dot{\xi} = (I \otimes (A - BK))\xi + \big(I \otimes B\bar{B}_1\big)\bar{e} + \tilde{f}(x_i, x_0). \tag{36}$$

The control and observer gains are chosen as

$$K = B^T \bar{P}_1, \; L = c\bar{P}_2^{-1} \bar{C}_\xi^T \tag{37}$$

where $\bar{P}_1$ and $\bar{P}_2$ are positive definite matrices, c is the same constant as defined in (14).

For the stability analysis, first we need to establish bounds of the extra terms ϕ and $\bar{\lambda}$.

Lemma 6: For the integral terms ϕ and $\bar{\lambda}$ shown in the error dynamics (34), bounds can be established as

$$\|\phi\|^2 \le \tau\|\mathcal{L}_1\|_F^2\bar{\rho}^2 e^{\bar{\alpha}\tau}\int_{t-\tau}^{t}\left(\gamma_1^2\|\xi\|^2 + \gamma_2^2\|\bar{e}_\omega\|^2\right)\mathrm{d}s$$
$$\|\bar{\lambda}\|^2 \le \tau\|\mathcal{L}_1\|_F^4\bar{\rho}^4 e^{2\bar{\alpha}\tau}\int_{t-\tau}^{t}\bar{e}^T(s-\tau)\bar{e}(s-\tau)\mathrm{d}s$$

where $\bar{\alpha}$ is a positive number such that $\bar{\alpha} \ge \lambda_{\max}(A_\xi^T + A_\xi)$, and $\bar{\rho}$ is a positive number such that

$$\bar{\rho}^2 I \ge c^2\bar{C}_z^T\bar{C}_z\bar{P}_2^{-1}\bar{P}_2^{-1}\bar{C}_z^T\bar{C}_z. \quad (38)$$

Proof: See Appendix B. ■

Theorem 2: For nonlinear multiagent systems (28), the consensus disturbance rejection problem can be solved by the observer (32) and the controller (35) if there exists positive definite matrices $\bar{P}_1, \bar{P}_2$ and constants $\kappa_1, \kappa_2, \kappa_3, \rho > 0$, such that

$$\begin{bmatrix} A\bar{W} + \bar{W}A^T - BB^T + \kappa_1 I & \bar{W} \\ \bar{W} & -\tilde{\gamma}^{-1}\end{bmatrix} < 0 \quad (39)$$
$$\bar{\rho}\bar{P}_2 - c\bar{C}_\xi^T\bar{C}_\xi \ge 0 \quad (40)$$
$$\begin{bmatrix} A_\xi^T\bar{P}_2 + \bar{P}_2A_\xi - 2C_\xi^TC_\xi + \bar{H} & \bar{P}_2 \\ \bar{P}_2 & -\frac{1}{\kappa_2+\kappa_3}\end{bmatrix} < 0 \quad (41)$$

are satisfied with $\bar{W} = \bar{P}_1^{-1}$ and

$$\tilde{\gamma}_1 = \left(\kappa_1^{-1} + \bar{\gamma}_1\right)\gamma_1^2$$
$$\bar{H} = g_{\min}^{-1}\bar{B}_1^T\bar{B}_1 + \bar{\sigma}I + g_{\min}^{-1}\bar{\gamma}_2\bar{D}$$

where $\bar{\gamma}_1 = \kappa_2^{-1}g_{\max}\tau\|\mathcal{L}_1\|_F^2\bar{\rho}^2e^{\bar{\alpha}\tau}\gamma_1^2$, $\bar{\gamma}_2 = \kappa_2^{-1}g_{\max}\tau\|\mathcal{L}_1\|_F^2\bar{\rho}^2e^{\bar{\alpha}\tau}\gamma_2^2$, $\bar{\sigma} = e^\tau\kappa_3^{-1}g_{\max}\tau\|\mathcal{L}_1\|_F^4\bar{\rho}^4e^{2\bar{\alpha}\tau}$, $\bar{D} = [0_{n\times n}\ 0_{n\times s}; 0_{s\times n}\ I_{s\times s}]$.

Proof: To start the consensus analysis, we try a Lyapunov function candidate

$$\bar{V}_0 = \xi^T\left(I\otimes\bar{P}_1\right)\xi + \bar{e}^T\left(G\otimes\bar{P}_2\right)\bar{e}. \quad (42)$$

In view of (11) and (13), we have

$$\begin{aligned}\dot{\bar{V}}_0 =\ & \xi^T\left[I\otimes\left(A^T\bar{P}_1 + \bar{P}_1A - 2\bar{P}_1BB^T\bar{P}_1\right)\right]\xi \\ & + \bar{e}^T\Big[G\otimes\left(A_\xi^T\bar{P}_2 + \bar{P}_2A_\xi\right) - c\left(G\mathcal{L}_1 + \mathcal{L}_1^TG\right) \\ & \times \otimes\bar{C}_\xi^T\bar{C}_\xi\Big]\bar{e} + 2\xi^T\left(I\otimes\bar{P}_1B\bar{B}_1\right)\bar{e} \\ & + 2\bar{e}^T\left(G\otimes\bar{P}_2\right)\bar{\lambda} + 2\xi^T\left(I\otimes\bar{P}_1\right)\tilde{f}(x_i, x_0) \\ & + 2\bar{e}^T\left(G\otimes\bar{P}_2\right)\phi \\ \le\ & \xi^T\Big[I\otimes\Big(A^T\bar{P}_1 - \bar{P}_1BB^T\bar{P}_1 + \kappa_1^{-1}\gamma_1^2I \\ & + \kappa_1\bar{P}_1\bar{P}_1 + \bar{P}_1A\Big)\Big]\xi \\ & + \bar{e}^T\Big[G\otimes\Big(A_\xi^T\bar{P}_2 + \bar{P}_2A_\xi + (\kappa_2+\kappa_3)\bar{P}_2\bar{P}_2 \\ & - 2\bar{C}_\xi^T\bar{C}_\xi\Big) + \bar{B}_1^T\bar{B}_1\Big]\bar{e} \\ & + \kappa_2^{-1}g_{\max}\|\phi\|^2 + \kappa_3^{-1}g_{\max}\|\bar{\lambda}\|^2 \\ \le\ & \xi^T\Big[I\otimes\Big(A^T\bar{P}_1 - \bar{P}_1BB^T\bar{P}_1 + \kappa_1^{-1}\gamma_1^2I \\ & + \kappa_1\bar{P}_1\bar{P}_1 + \bar{P}_1A\Big)\Big]\xi \\ & + \bar{e}^T\Big[G\otimes\Big(A_\xi^T\bar{P}_2 + \bar{P}_2A_\xi + (\kappa_2+\kappa_3)\bar{P}_2\bar{P}_2 \\ & - 2\bar{C}_\xi^T\bar{C}_\xi\Big) + \bar{B}_1^T\bar{B}_1\Big]\bar{e} \\ & + \kappa_2^{-1}g_{\max}\tau\|\mathcal{L}_1\|_F^2\bar{\rho}^2e^{\bar{\alpha}\tau}\int_{t-\tau}^{t}\left(\gamma_1^2\|\xi\|^2 + \gamma_2^2\|\bar{e}_\omega\|^2\right)\mathrm{d}s \\ & + \kappa_3^{-1}g_{\max}\tau\|\mathcal{L}_1\|_F^4\bar{\rho}^4e^{2\bar{\alpha}\tau}\int_{t-\tau}^{t}\bar{e}^T(s-\tau)\bar{e}(s-\tau)\mathrm{d}s\end{aligned}$$

where Lemmas 1, 4, 6, and (29) are used for the derivation.

For the integral terms, we consider the following Krasovskii functionals:

$$\bar{V}_1 = \bar{\gamma}_1\int_{t-\tau}^{t}\xi^T(s)\xi(s)\mathrm{d}s + \bar{\gamma}_2\int_{t-\tau}^{t}\bar{e}_\omega^T(s)\bar{e}_\omega(s)\mathrm{d}s$$
$$\bar{V}_2 = e^\tau\int_{t-\tau}^{t}\bar{e}^{s-t}e^T(s-\tau)\bar{e}(s-\tau)\mathrm{d}s + e^\tau\int_{t-\tau}^{t}\bar{e}^T(s)\bar{e}(s)\mathrm{d}s$$

where $\bar{\gamma}_1 = \kappa_2^{-1}g_{\max}\tau\|\mathcal{L}_1\|_F^2\bar{\rho}^2e^{\bar{\alpha}\tau}\gamma_1^2$ and $\bar{\gamma}_2 = \kappa_2^{-1}g_{\max}\tau\|\mathcal{L}_1\|_F^2\bar{\rho}^2e^{\bar{\alpha}\tau}\gamma_2^2$ are defined in Theorem 2.

Let

$$V = \bar{V}_0 + \bar{V}_1 + \kappa_3^{-1}g_{\max}\tau\|\mathcal{L}_1\|_F^4\bar{\rho}^4e^{2\bar{\alpha}\tau}\bar{V}_2. \quad (43)$$

A direct evaluation gives that

$$\dot{V} \le \xi^T\left(I\otimes\bar{H}_1\right)\xi + \bar{e}^T\left(G\otimes\bar{H}_2\right)\bar{e} \quad (44)$$

where

$$\bar{H}_1 := A^T\bar{P}_1 + \bar{P}_1A - \bar{P}_1BB^T\bar{P}_1 + \kappa_1\bar{P}_1\bar{P}_1 + \tilde{\gamma}_1I \quad (45)$$
$$\begin{aligned}\bar{H}_2 :=\ & A_\xi^T\bar{P}_2 + \bar{P}_2A_\xi + (\kappa_2+\kappa_3)\bar{P}_2\bar{P}_2 - 2C_\xi^TC_\xi \\ & + g_{\min}^{-1}\bar{B}_1^T\bar{B}_1 + \bar{\sigma}I + g_{\min}^{-1}\bar{\gamma}_2\bar{D}.\end{aligned} \quad (46)$$

where $\tilde{\gamma}_1 = (\kappa_1^{-1} + \bar{\gamma}_1)\gamma_1^2$, $\bar{\sigma} = e^\tau\kappa_3^{-1}g_{\max}\tau\|\mathcal{L}_1\|_F^4\bar{\rho}^4e^{2\bar{\alpha}\tau}$, and $\bar{D} = [0_{n\times n}\ 0_{n\times s}; 0_{s\times n}\ I_{s\times s}]$ are defined in Theorem 2.

Following the same analysis in Section IV, the conditions $\bar{H}_1 < 0$ and $\bar{H}_2 < 0$ are equivalent to the conditions specified in (39)–(41) which implies that ξ and $\bar{e}$ converge to zero asymptotically. Hence, the nonlinear consensus disturbance rejection is achieved. ■

Remark 5: Since the values of τ, γ_1, and γ_2 are fixed and they are not the decision variables of the LMIs, a feasible solution may not exist if these values are too large. To avoid this problem, a set of free parameters κ_1, κ_2, and κ_3 are introduced to provide more design degrees of freedom. In addition, if the delay τ is too large to find a feasible solution, an alternative way is to design a series of coupled predictors, each of which is responsible for prediction of one small portion of the delay [36].

Remark 6: In this paper, we consider the known network connection case. The global information Laplacian matrix $\mathcal{L}_1$ can be derived from the information flow among agents. It is adopted for the convenience of presentation of the proposed design. For delay-free multiagent systems, there are some results on fully distributed case, in which the controllers

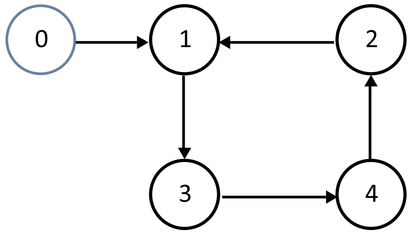

Fig. 1. Communication topology.

are designed without use of any global information (see [14], [16], [25], [46], for instance). How to extend the analysis presented in this paper to the fully distributed case is still an open problem and will be a topic of our future research.

VI. Numerical Example

In this section, an example is used to demonstrate the potential applications of the proposed approach. Suppose a network of five unmanned aerial vehicles (UAVs) are subject to the connection topology specified by the following adjacency matrix:

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Note that the first row all are zeros, as the agent indexed by 0 is chosen as the leader. The communication graph in Fig. 1 shows that only the follower indexed by 1 can get access to the leader and the communication topology contains a directed spanning tree. The dynamics of the ith agent are described by (28), with

$$\dot{x}_i = \begin{bmatrix} x_{i1} \\ x_{i2} \end{bmatrix}, \ A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \ B = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}, f_1(x_i) = \beta_1 \begin{bmatrix} \sin(x_{i1}) & \sin(x_{i2}) \end{bmatrix}.$$

This practical dynamical model of UAV is given in [57]. In this scenario, it is supposed that only the output information is available. Our task in this example is to reject harmonic disturbances in the input channel which generated by a nonlinear external disturbance model (28) with

$$\dot{\omega}_i = \begin{bmatrix} \omega_{i1} \\ \omega_{i2} \end{bmatrix}, \ S = \begin{bmatrix} 0 & -0.1 \\ 0.1 & 0 \end{bmatrix}, F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$f_2 = \beta_2 \begin{bmatrix} (|\omega_{i1} + 1| - |\omega_{i1} - 1|) & 0 \end{bmatrix}$$

which represents an external periodic disturbance with known frequency but without any information of its magnitude and phase. The input delay $\tau_u = 0.05$ s, the output delay $\tau_y = 0.05$ s, and the Lipschitz constants are $\gamma_1 = \beta_1 = 0.03$ and $\gamma_2 = 2\beta_2 = 0.04$. It can be checked that both Assumptions 1 and 2 are satisfied.

The Laplacian matrix $\mathcal{L}_1$ associated with $\mathcal{A}$ is

$$\mathcal{L}_1 = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

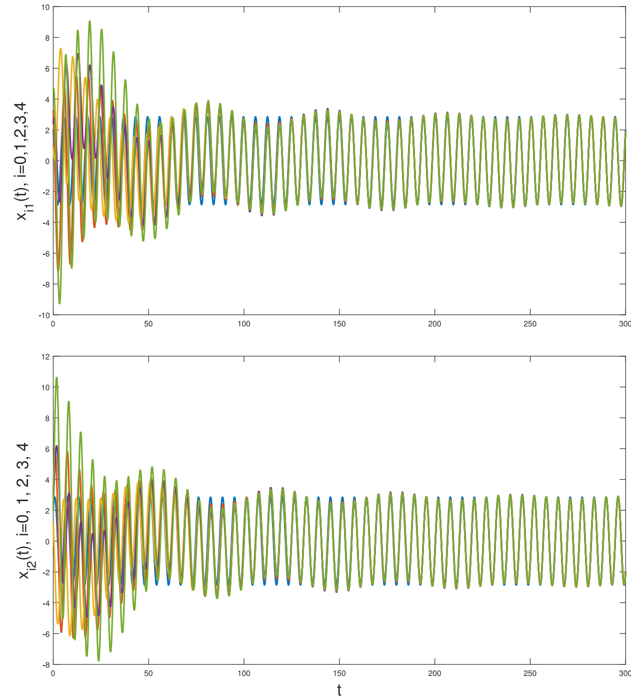

Fig. 2. States trajectories of the five agents.

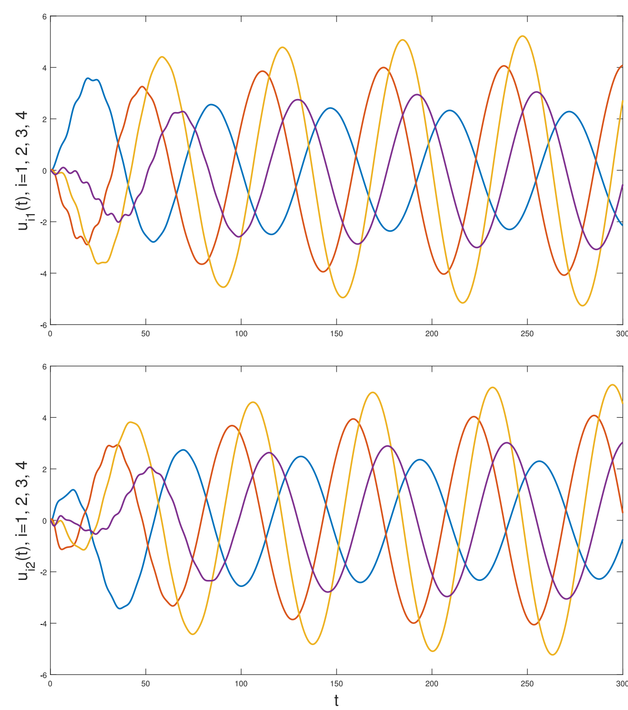

Fig. 3. Control inputs of the four followers.

Following the result shown in Lemma 1, we obtain that $G = \text{diag}\{4, \ 5, \ 7, \ 6\}$ and $\rho_0 = 1.7995$. With $g_{\max} = 7$ and $2g_{\max}/\rho_0 = 7.7799$, we set $c = 8$ in (37).

The initial states of agents are chosen randomly within $[0, \ 5]$, and $u(\theta) = [0, \ 0]^T$, $\forall \theta \in [-\tau_u, 0]$, $\bar{\chi}(\theta) = [0, \ 0]^T$, $\forall \theta \in [-\tau, 0]$. With $\bar{\rho} = 0.2, \kappa_1 = \kappa_2 = \kappa_3 = 0.1$, a feasible solution of the observer gain L is found to be $[\,0.8406 \quad -0.2526 \quad -0.1067 \quad -0.2220\,]$, and a feedback

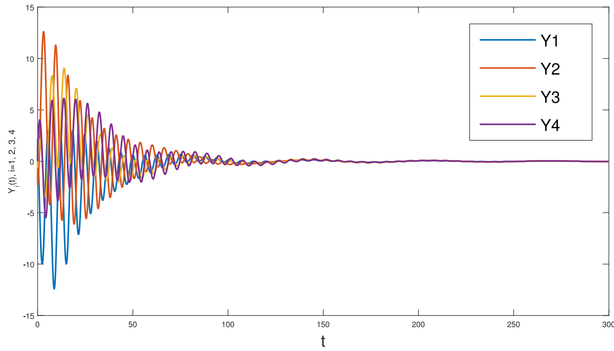

Fig. 4. Relative output error of the four followers.

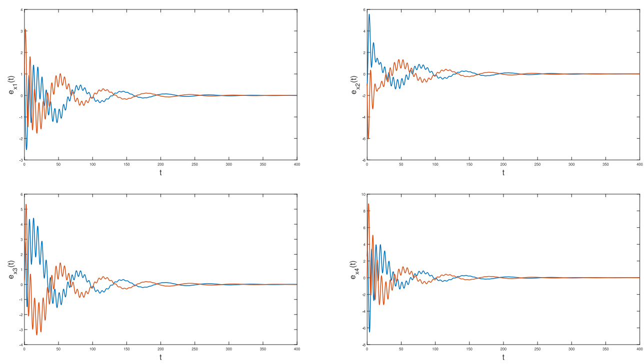

Fig. 5. Estimation errors of the predictor-based state observers.

gain K is found to be

$$K = \begin{bmatrix} 0.0364 & 0.0188 \\ 0.0182 & 0.0376 \end{bmatrix}.$$

Simulation study has been carried out with different disturbances for agents. Fig. 2 shows the simulation results for the trajectories of the states. The control input and the relative output errors, $Y_i = \sum_{j=0}^{N} a_{ij}(y_i - y_j)$, $i = 1, 2, 3, 4$, are shown in Figs. 3 and 4. The state and the disturbance observation errors are shown in Figs. 5 and 6. From the results shown in these figures, it can be seen clearly that all the five agents reach consensus although they are under different disturbances. Therefore, the conditions specified in this paper are sufficient to guarantee the consensus disturbance rejection. Furthermore, compared to our previous results in [32], [33], and [48], the observer-based predictor controllers designed in this paper have some good points. The first point is that the consensus disturbance rejection can be achieved via only relative output information. The second point is that this method not only can deal with input delay but also can deal with output delay and nonlinear disturbances. Finally, the proposed observer-based predictor controllers are easier and safer to implement.

VII. Conclusion

This paper has investigated the observer-based output feedback consensus problem for multiagent systems subject to external disturbance and delays in input and output. Novel predictor-based ESOs are designed, and the delays and disturbances can be compensated efficiently with the proposed

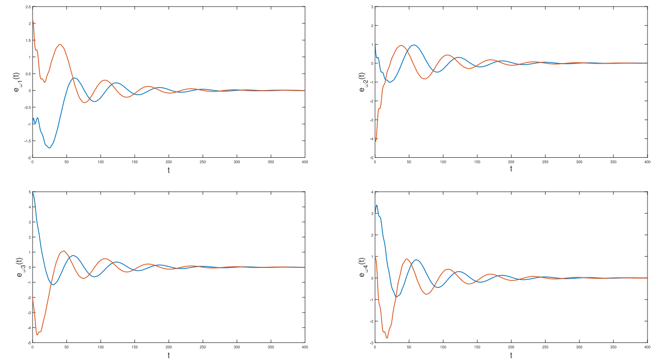

Fig. 6. Estimation errors of the disturbance observers.

controllers. In particular, the observers and the state predictors do not contain any integral term of the past control input, which greatly reduces the computation burden and improves the practical implementation. Consensus analysis is put into the framework of Lyapunov–Krasovskii functionals and sufficient conditions are derived to guarantee that the consensus errors converge to zero asymptotically. The results have been also extended to nonlinear multiagent systems with nonlinear disturbances. Simulation results show the validity of the proposed method and design.

Appendix A
Proof of Lemma 5

Let $\lambda = [\lambda_1^T, \lambda_2^T, \ldots, \lambda_N^T]^T$. From (11), we have

$$\begin{aligned}\lambda_i &= L\bar{C}_z \sum_{j=1}^{N} l_{ij} \int_{t-\tau}^{t} e^{A_z(t-s)} LC_z v_j(s-\tau)\mathrm{d}s \\ &= -L\bar{C}_z \sum_{j=1}^{N} l_{ij} \int_{t-\tau}^{t} e^{A_z(t-s)} LC_z \sum_{k=1}^{N} l_{jk}\tilde{e}_k(s-\tau)\mathrm{d}s.\end{aligned}$$

We define

$$\theta_k = -L\bar{C}_z \int_{t-\tau}^{t} e^{A_z(t-s)} LC_z \tilde{e}_k(s-\tau)\mathrm{d}s.$$

Then, we obtain that

$$\lambda_i = \sum_{j=1}^{N} l_{ij} \sum_{k=1}^{N} l_{jk}\theta_k.$$

Let $\theta = [\theta_1^T, \theta_2^T, \ldots, \theta_N^T]^T$. It then follows that:

$$\|\lambda_i\| \le \sum_{j=1}^{N} |l_{ij}| \sum_{k=1}^{N} |l_{jk}| \|\theta_k\| \le \|l_i\| \|\mathcal{L}_1\|_F \|\theta\|$$

where l_i denotes the ith row of $\mathcal{L}_1$. Therefore, we have

$$\|\lambda\|^2 = \sum_{i=1}^{N} \|\lambda_i\|^2 \le \sum_{i=1}^{N} \|l_i\|^2 \|\mathcal{L}_1\|_F^2 \|\theta\|^2 = \|\mathcal{L}_1\|_F^4 \|\theta\|^2.$$

Next, we need to deal with $\|\theta\|^2$. By Lemma 2, we have

$$\begin{aligned}\|\theta_i\|^2 &= c^2\int_{t-\tau}^{t}\tilde{e}_i^T(s-\tau)C_z^TL^Te^{A_z^T(t-s)}\mathrm{d}s\bar{C}_z^T\bar{C}_zP_2^{-1}\\ &\quad\times P_2^{-1}\bar{C}_z^T\bar{C}_z\int_{t-\tau}^{t}e^{A_z(t-s)}LC_z\tilde{e}_i(s-\tau)\mathrm{d}s\\ &\le \tau\rho^2\int_{t-\tau}^{t}\tilde{e}_i^T(s-\tau)C_z^TL^Te^{A_z^T(t-s)}\\ &\quad\times e^{A_z(t-s)}LC_z\tilde{e}_i(s-\tau)\mathrm{d}s.\end{aligned}$$

In view of Lemma 3 with $P=I$, $\alpha\ge\lambda_{\max}(A_z^T+A_z)$, we have $e^{A_z^Tt}e^{A_zt}\le e^{\alpha t}I$, and

$$\begin{aligned}\|\theta_i\|^2 &\le \tau\rho^2c^2\int_{t-\tau}^{t}e^{\alpha(t-s)}\tilde{e}_i^T(s-\tau)e^{A_z^T\tau}\bar{C}_z^T\bar{C}_zP_2^{-1}\\ &\quad\times P_2^{-1}\bar{C}_z^T\bar{C}_ze^{A_z\tau}\tilde{e}_i(s-\tau)\mathrm{d}s\\ &\le \tau\rho^4e^{2\alpha\tau}\int_{t-\tau}^{t}\tilde{e}_i^T(s-\tau)\tilde{e}_i(s-\tau)\mathrm{d}s.\end{aligned}$$

Then, $\|\theta\|^2$ can be bounded as

$$\|\theta\|^2=\sum_{i=1}^{N}\|\theta_i\|^2\le\tau\rho^4e^{2\alpha\tau}\int_{t-\tau}^{t}\tilde{e}^T(s-\tau)\tilde{e}(s-\tau)\mathrm{d}s.$$

Finally, we have

$$\|\lambda\|^2\le\tau\|\mathcal{L}_1\|_F^4\rho^4e^{2\alpha\tau}\int_{t-\tau}^{t}\tilde{e}^T(s-\tau)\tilde{e}(s-\tau)\mathrm{d}s.$$

This completes the proof.

APPENDIX B
PROOF OF LEMMA 6

Define $\phi=[\phi_1^T,\phi_2^T,\ldots,\phi_N^T]^T$. From (34), we have

$$\phi_i=L\bar{C}_\xi\sum_{j=1}^{N}l_{ij}\int_{t-\tau}^{t}e^{A_\xi(t-s)}\tilde{\mathcal{F}}_j\mathrm{d}s=\sum_{j=1}^{N}l_{ij}\tilde{\phi}_j$$

where $\tilde{\phi}_j=L\bar{C}_\xi\int_{t-\tau}^{t}e^{A_\xi(t-s)}\tilde{\mathcal{F}}_j\mathrm{d}s$. Let $\tilde{\phi}=[\tilde{\phi}_1^T,\tilde{\phi}_2^T,\ldots,\tilde{\phi}_N^T]^T$. Then, we have

$$\|\phi_i\|\le\sum_{j=1}^{N}|l_{ij}|\left\|\tilde{\phi}_j\right\|\le\|l_i\|\left\|\tilde{\phi}\right\|$$

and

$$\|\phi\|^2=\sum_{i=1}^{N}\|\phi_i\|^2\le\sum_{i=1}^{N}\|l_i\|^2\left\|\tilde{\phi}\right\|^2\le\|\mathcal{L}_1\|_F^2\left\|\tilde{\phi}\right\|^2.$$

Next, we need to deal with $\|\tilde{\phi}\|^2$. By Lemmas 2 and 3, we have

$$\begin{aligned}\left\|\tilde{\phi}_i\right\|^2 &= \int_{t-\tau}^{t}\tilde{\mathcal{F}}_i^Te^{A_\xi^T(t-s)}\mathrm{d}s\bar{C}_\xi^TL^TL\bar{C}_\xi\int_{t-\tau}^{t}e^{A_\xi(t-s)}\tilde{\mathcal{F}}_i\mathrm{d}s\\ &\le \tau\bar{\rho}^2\int_{t-\tau}^{t}\tilde{\mathcal{F}}_i^Te^{\left(A_\xi^T+A_\xi\right)(t-s)}\tilde{\mathcal{F}}_i\mathrm{d}s\\ &\le \tau\bar{\rho}^2e^{\bar{\alpha}\tau}\int_{t-\tau}^{t}\left\|\tilde{\mathcal{F}}_i\right\|^2\mathrm{d}s\\ &\le \tau\bar{\rho}^2e^{\bar{\alpha}\tau}\int_{t-\tau}^{t}\left(\gamma_1^2\|\xi_i\|^2+\gamma_2^2\|\bar{e}_{\omega i}\|^2\right)\mathrm{d}s.\end{aligned}$$

Then, $\|\tilde{\phi}\|^2$ can be bounded as

$$\left\|\tilde{\phi}\right\|^2=\sum_{i=1}^{N}\left\|\tilde{\phi}_i\right\|^2\le\tau\bar{\rho}^2e^{\bar{\alpha}\tau}\int_{t-\tau}^{t}\left(\gamma_1^2\|\xi\|^2+\gamma_2^2\|\bar{e}_\omega\|^2\right)\mathrm{d}s.$$

Therefore, we have

$$\|\phi\|^2\le\tau\|\mathcal{L}_1\|_F^2\bar{\rho}^2e^{\bar{\alpha}\tau}\int_{t-\tau}^{t}\left(\gamma_1^2\|\xi\|^2+\gamma_2^2\|\bar{e}_\omega\|^2\right)\mathrm{d}s.$$

This completes the proof. The proof of $\bar{\lambda}$ is similar to that of λ and hence omitted.

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