Noname manuscript No.
(will be inserted by the editor)

Preference stability along time: The time cohesiveness measure

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Received: date / Accepted: date

Abstract This work introduces a non-traditional perspective about the problem of measuring the stability of agents' preferences. Specifically, the cohesiveness of preferences at different moments of time is explored under the assumption of considering dichotomous evaluations. The general concept of *time cohesiveness measure* is introduced as well as a particular formulation based on the consideration of any two successive moments of time, *the sequential time cohesiveness measure*. Moreover, some properties of the novel measure are also provided. Finally, and in order to emphasize the adaptability of our proposal to real situations, a factual case of study about Clinical Decision Making is presented. Concretely, the study of preference stability for life-sustaining treatments of patients with advanced cancer at end of life is analysed. The research considers patients who express their opinions on three life-sustaining treatments at four consecutive periods of time. The novel measure provides information of patients preference stability along time and considers the possibility of cancer metastases.

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Keywords Time cohesiveness measure · Dichotomous opinions · Preference stability · Patients' preferences

1 Introduction

Intertemporal decision making is an important scientific area and it has been obtaining attention from several research fields such as Economics, Health Economics, Social Choice, Psychology, Marketing, Decision Analysis, Neuroscience, and so on.

One of the main topics of this area is the study of preference stability that is often defined like the measurement of the choice consistency among options along time [8], [20], [28]. Traditionally, preferences have usually been considered permanent by theory [23], although there are also different studies to check if they are constant over time [4], [7], [11], [26]. Related to empirical literature on preference stability, most studies use small samples in short time periods and they are focused on a specific type of preferences, the *risk preferences* [27]. Recently, there has been an increment of works about time preference [12], [22], [24], while there are few contributions that study the stability of social preference [10].

From another point of view, a growing number of studies considers changes in preferences as a result of shocks such as illness, civil wars, natural disasters, etc. [9], [15], [21], [25].

The research to date has tend to explore preference stability by means of statistical approaches: from basic methods like descriptive analysis and multiple regression [22], [28] to more elaborate procedures like hierarchical generalized linear modelling [8] and others [29].

In order to enhance the preference stability topic, the aim of this contribution is to develop a new tool capable of measuring preference stability from a non-traditional perspective. For this purpose, the notion of preference stability is considered in the same vein that the notion of cohesiveness. This seems natural because the measurement of preference stability resembles the notion of measurement of cohesiveness over time, in the sense that the maximum value captures the notion of full stability, that is unanimity along time, while the minimum value captures the notion of total lack of stability, that is, total disagreement along time.

The cohesiveness or consensus measurement has been dealt in the Social Choice literature from Bosch's seminal work [6]. Subsequently, Alcalde-Unzu and Vorsatz [1], Alcantud et al. [2] and García-Lapresta and Pérez-Román [14] introduced several classes of consensus measures based on distances for ordinal information. Additionally, several studies related to consensus problem deal also with cardinal information like the approaches proposed by González-Arteaga et al. [16], González-Pachón and Romero [17], González-Pachón et al. [18], Herrera-Viedma et al. [19], and so on. From another point of view, Alcantud, de Andrés Calle and Cascón [3] introduced a cohesiveness measure when opinions are dichotomous.

Taking into account the previous contributions on preference stability and cohesiveness measure, this paper is focused on an inter-temporal decision making problem where a set of agents express their opinions on an alternative along different moments of time. To be precise, agents have to approve or disapprove the alternative under study at diverse point of time. Thus, the paper objective is to determine how much stability or cohesiveness agents' opinions conveys to the group on the alternative along time. In order to measure such stability, a new general approach is defined, the *time cohesiveness measure*. Following the Social Choice tradition, this measurement takes values in the unit interval considering value 1 full stability and value 0 total lack of stability. Moreover, an specific formulation of the time cohesiveness measure is introduced, the *sequential time cohesiveness measure* as well as a study of its analytic properties. Under this approach, the stability of preferences is understood like the probability that for a randomly chosen moment of time, two randomly chosen agents have the same opinion at such a time and its consecutive.

Furthermore, the measurement proposed is put in practice in a real case of study to emphasise its applicability. In particular, the stability of preferences for life-sustaining treatments in terminally cancer patients' last year of life is analysed.

The paper is structured as follows. It has been divided into three parts. The first part, Section 2, introduces our proposal to measure preference stability: the *time cohesiveness measure*. Moreover, an specific type of this measure, the *sequential time cohesiveness measure*, is presented as well as its properties. The second part, Section 3, includes an application of the novel approach to a real case of study. Finally, some concluding remarks are provided.

2 A new tool to measure preference stability: The time cohesiveness measure

This section is devoted to introduce some notation as well as our proposal of measurement of preference stability, namely, the *time cohesiveness measure*. Then, an specific formulation, *the sequential time cohesiveness measure*, is defined and its properties are examined.

2.1 Notation

Let $\mathbf{N} = \{1, 2, ..., N\}$ a set of agents or experts. Agents express their opinions on an alternative, x, at different time moments $\mathbf{T} = \{t_1, \dots, t_T\}$ by means of dichotomous opinions.

From now on, the notation used to formalize theses assessments is the following:

Definition 1 A *time preference profile* of a set of agents $\mathbf{N}$ on an alternative x at T different time moments is an $N \times T$ matrix

$$\mathbf{P} = \begin{pmatrix} P_{1t_1} & \dots & P_{1t_T} \\ \vdots & \ddots & \vdots \\ P_{Nt_1} & \dots & P_{Nt_T} \end{pmatrix}_{N\times T}$$

where P_{it_j} is the opinion of the agent i over alternative x at t_j moment, in the sense

$$P_{it_j} = \begin{cases} 1 \text{ if agent } i \text{ approves } x \text{ at the } t_j \text{ time,} \\ 0 \text{ otherwise.} \end{cases}$$

Let $\mathbb{P}_{N\times T}$ denote the set of all such $N \times T$ matrices. For simplicity of notation, $(1)_{N\times T}$ is the $N \times T$ matrix whose cells are universally equal to 1 and $(0)_{N\times T}$ is the $N \times T$ matrix whose cells are universally equal to 0.

A time preference profile $\mathbf{P}$ is *unanimous* if alternative x is approved (resp. disapproved) over $\mathbf{T}$ by all agents. In matrix terms, if the time preference profile $\mathbf{P} \in \mathbb{P}_{N\times T}$ is constant, $\mathbf{P} = (1)_{N\times T}$ (resp. $\mathbf{P} = (0)_{N\times T}$).

Any permutation σ of the agents $\{1, 2, ..., N\}$ determines a time preference profile $\mathbf{P}^\sigma$ by permutation of the rows of $\mathbf{P}$, that is, row i of the profile $\mathbf{P}^\sigma$ is row $\sigma(i)$ of the profile $\mathbf{P}$.

For each time preference profile $\mathbf{P}$, $\mathbf{P}_S$ is the restriction to a subset of agents, an *agent-subprofile* on the agents in $S \subseteq \mathbf{N}$, and it emerges from selecting the rows of $\mathbf{P}$ that are associated with the respective agents in S.

For each time preference profile $\mathbf{P}$, $\mathbf{P}^I$ is the restriction to a subset of consecutive moments of time, *time-subprofile* on the moments of time in $I \subseteq \mathbf{T}$, and it emerges from selecting consecutive columns of $\mathbf{P}$ that are associated with the respective moments of time in I. Any partition $\{I_1, \ldots, I_p\}$ of $\mathbf{P}$ generates a decomposition of $\mathbf{P}$ into time-subprofiles $\mathbf{P}^{I_1}, \ldots, \mathbf{P}^{I_p}$ where $\mathbf{P}^{I_1} \cup \ldots \cup \mathbf{P}^{I_p} = \mathbf{P}$.

An *extension* of a time preference profile $\mathbf{P}$ of a group of agents $\mathbf{N}$ at $\mathbf{T} = \{t_1, \ldots, t_T\}$ is a time preference profile $\overline{\mathbf{P}}$ at $\overline{\mathbf{T}} = \{t_1, \ldots, t_T, t_{T+1}, \ldots, t_{T+q}\}$ such that the restriction of $\overline{\mathbf{P}}$ to the first T moments of time of $\overline{\mathbf{T}}$ coincides with $\mathbf{P}$.

A *replication* of a time preference profile $\mathbf{P}$ of a group of agents $\mathbf{N}$ on alternative x is the time preference profile $\mathbf{P} \uplus \mathbf{P} \in \mathbb{P}_{2N\times T}$ obtained by duplicating each row of $\mathbf{P}$, in the sense that rows r and $N + r$ of $\mathbf{P} \uplus \mathbf{P}$ are row r of $\mathbf{P}$, for each $r = 1, ..., N$.

For each time preference profile $\mathbf{P}$ on alternative x, $n_0^{t_j}$ denotes the number of agents that disapprove x at the t_j moment of time, and $n_1^{t_j}$ denotes the number of agents that approve alternative x at the t_j moment of time. Therefore, $N = n_0^{t_j} + n_1^{t_j}$ for each $t_j \in \mathbf{T}$.

In addition, $n_{0,0}^{t_j,t_{j+1}}$ denotes the number of agents that disapprove alternative x at t_j and keep their opinion at the following point of time t_{j+1}. Similarly, $n_{1,1}^{t_j,t_{j+1}}$ denotes the number of agents that approve alternative x at t_j and keep their opinion at the following point of time t_{j+1}.

In this way, $n_{0,1}^{t_j,t_{j+1}}$ is the number of agents that disapprove alternative x at t_j but change their opinion at t_{j+1}, and $n_{1,0}^{t_j,t_{j+1}}$ is the number of agents that approve alternative k at t_j but change their opinion at t_{j+1}. For each $t_j \in \mathbf{T}$, $n_0^{t_j} = n_{0,0}^{t_j,t_{j+1}} + n_{0,1}^{t_j,t_{j+1}}$ and likewise $n_1^{t_j} = n_{1,1}^{t_j,t_{j+1}} + n_{1,0}^{t_j,t_{j+1}}$. See Table 1 for improving understanding.

For the purpose of clarifying the use of the previous notation, the following illustrative example is introduced.

t_j \ t_{j+1}	No	Yes	
No	$n_{0,0}^{t_j,t_{j+1}}$	$n_{0,1}^{t_j,t_{j+1}}$	$n_0^{t_j}$
Yes	$n_{1,0}^{t_j,t_{j+1}}$	$n_{1,1}^{t_j,t_{j+1}}$	$n_1^{t_j}$
	$n_0^{t_{j+1}}$	$n_1^{t_{j+1}}$	N

Table 1 Notation summary table

Example 1 Let $\mathbf{N} = \{1, 2, \ldots, 10\}$ be a set of ten agents that express their opinions on alternative x along four consecutive moments of time $\mathbf{T} = \{t_1, t_2, t_3, t_4\}$. Their time preference profile is:

$$\mathbf{P} = \begin{pmatrix} P_{1t_1} & \ldots & P_{1t_4} \\ \vdots & \ddots & \vdots \\ P_{10t_1} & \ldots & P_{10t_4} \end{pmatrix}_{10\times 4} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

This time preference profile can be summarized in a table containing the number of agents who approve or disapprove alternative x at each moment of time t_j as well as the number of agents that keep or change their opinion during consecutive time moments (see Table 2).

2.2 New approach to measure preference stability: Definition and properties

Influenced by Bosch's consensus approach [6], our proposal of cohesivenesses measure along time is introduced below.

Definition 2 A *time cohesiveness measure* for a group of agents $\mathbf{N} = \{1, ..., N\}$ on an alternative x is a mapping

$$\tau : \mathbb{P}_{N\times T} \to [0, 1]$$

that assigns a number $\tau(\mathbf{P}) \in [0, 1]$ to each time preference profile $\mathbf{P}$, with the properties:

i) $\tau(\mathbf{P}) = 1$ if and only if $\mathbf{P}$ is unanimous (full stability).
ii) $\tau(\mathbf{P}^\sigma) = \tau(\mathbf{P})$ for each permutation σ of the agents and $\mathbf{P} \in \mathbb{P}_{N\times T}$ (anonymity).

A time cohesiveness measure is a collection of time cohesiveness measures for each group of agents $\mathbf{N}$.

t_1 \ t_2	No	Yes	
No	$n_{0,0}^{t_1,t_2}=2$	$n_{0,1}^{t_1,t_2}=5$	$n_0^{t_1}=7$
Yes	$n_{1,0}^{t_1,t_2}=1$	$n_{1,1}^{t_1,t_2}=2$	$n_1^{t_1}=3$
	$n_0^{t_2}=3$	$n_1^{t_2}=7$	$N=10$

t_2 \ t_3	No	Yes	
No	$n_{0,0}^{t_2,t_3}=1$	$n_{0,1}^{t_2,t_3}=2$	$n_0^{t_2}=3$
Yes	$n_{1,0}^{t_2,t_3}=3$	$n_{1,1}^{t_2,t_3}=4$	$n_1^{t_2}=7$
	$n_0^{t_3}=4$	$n_1^{t_3}=6$	$N=10$

t_3 \ t_4	No	Yes	
No	$n_{0,0}^{t_3,t_4}=2$	$n_{0,1}^{t_3,t_4}=2$	$n_0^{t_3}=4$
Yes	$n_{1,0}^{t_3,t_4}=3$	$n_{1,1}^{t_3,t_4}=3$	$n_1^{t_3}=6$
	$n_0^{t_4}=5$	$n_1^{t_4}=5$	$N=10$

Table 2 Notation summary table for Example 1

Our proposal in contrast to Bosch's contribution does not require *neutrality* property, time moments can not be exchanged, due to the fact that time order is an essential aspect to measure the stability of preferences.

Now a particular time cohesiveness measure is introduced. Formally:

Definition 3 The *sequential time cohesiveness measure* for a group of agents $\mathbf{N}=\{1,...,N\}$ on an alternative x is the mapping $\tau_S:\mathbb{P}_{N\times T}\rightarrow[0,1]$ given by

$$\tau_S(\mathbf{P}) =$$

$$= \frac{1}{T-1}\cdot\frac{\sum_{j=1}^{j=T-1} n_{0,0}^{t_j,t_{j+1}}\cdot(n_{0,0}^{t_j,t_{j+1}}-1)}{N(N-1)}$$

$$+ \frac{1}{T-1}\cdot\frac{\sum_{j=1}^{j=T-1} n_{1,1}^{t_j,t_{j+1}}\cdot(n_{1,1}^{t_j,t_{j+1}}-1)}{N(N-1)}$$

Intuitively, it measures the probability that for a randomly chosen moment of time, two randomly chosen agents of a group have the same opinion upon an alternative at the moment of time selected and its consecutive.

It is easy to check that Definition 3 provides a time cohesiveness measure.

Hereunder, some desirable properties of the sequential cohesiveness measure are defined and proved.

Properties

Reversal invariance: This property shows that the main aspect of the time sequential cohesiveness measure is the stability of agents' opinions more than an specific value. If the 0's are changed for 1's and vice verse, then the sequential time cohesiveness measure reminds equal. Formally:

Let $\mathbf{P}^c$ be the complementary time preference profile of $\mathbf{P}$ defined by $\mathbf{P}^c=(1)_{N\times T}-\mathbf{P}$. If τ_S verifies reversal invariance then $\tau_S(\mathbf{P}^c)=\tau_S(\mathbf{P})$.

Proof Agents' opinions at $t_j,t_{j+1}\in\mathbf{T}$ do not change in $\mathbf{P}$ and $\mathbf{P}^c$, then τ_S does not change. That is, those agents whose opinions coincide at t_j and t_{j+1} in $\mathbf{P}$ have also coincident opinions at t_j and t_{j+1} in $\mathbf{P}^c$ although those opinions are different than in $\mathbf{P}$. Taking into account the Definition 3, τ_S does not change.

□

Time-reducibility: It means that the stability of a time preference profile is the average of the time cohesiveness measures of all its consecutive time-subprofiles of two consecutive moments of time. Formally:

Let $\mathbf{P}\in\mathbb{P}_{N\times T}$ be a time preference profile. We say that τ_S verifies time-reducibility if

$$\tau_S(\mathbf{P})=\frac{1}{T-1}\sum_{j=1}^{T-1}\tau_S(\mathbf{P}^{I_{j,j+1}})$$

where $\mathbf{P}^{I_{j,j+1}}\in\mathbb{P}_{N\times 2}$ is the time-subprofile of $\mathbf{P}$ containing the columns corresponding to times t_j and t_{j+1}.

Proof It is straightforward from the Definition 3 since

$$\tau_S(\mathbf{P}^{I_{j,j+1}}) =$$

$$= \frac{n_{0,0}^{t_j,t_{j+1}}(n_{0,0}^{t_j,t_{j+1}}-1)}{N(N-1)}$$

$$+ \frac{n_{1,1}^{t_j,t_{j+1}}(n_{1,1}^{t_j,t_{j+1}}-1)}{N(N-1)}$$

□

Replication monotonicity: When a non-unanimous time preference profile is replicated, its sequential time cohesiveness measure increases. Formally:

Let $\mathbf{P} \in \mathbb{P}_{N\times T}$ be a non unanimous time preference profile then

$$\tau_S(\mathbf{P} \uplus \mathbf{P}) > \tau_S(\mathbf{P})$$

Proof Using time-reducibility is enough to prove this property for only two moments of time. Consider $\mathbf{P}^{I_{j,j+1}} \in \mathbb{P}_{N\times 2}$ a time-subprofile for t_j and t_{j+1}.

$$\tau_S(\mathbf{P}^{I_{j,j+1}}) =$$

$$= \frac{n_{0,0}^{t_j,t_{j+1}}(n_{0,0}^{t_j,t_{j+1}}-1)}{N(N-1)} + \frac{n_{1,1}^{t_j,t_{j+1}}(n_{1,1}^{t_j,t_{j+1}}-1)}{N(N-1)}$$

$$\tau_S(\mathbf{P}^{I_{j,j+1}} \uplus \mathbf{P}^{I_{j,j+1}}) =$$

$$= \frac{2n_{0,0}^{t_j,t_{j+1}}(2n_{0,0}^{t_j,t_{j+1}}-1)}{2N(2N-1)} + \frac{2n_{1,1}^{t_j,t_{j+1}}(2n_{1,1}^{t_j,t_{j+1}}-1)}{2N(2N-1)}$$

It is enough that $\frac{2z-1}{2N-1} > \frac{z-1}{N-1}$ for each natural number $z \in \mathbb{N}$ with $z < N$. And this is easily checked. □

In addition, for an unanimous time preference profile $\mathbf{P} \in \mathbb{P}_{N\times T}$, by Definition 3, τ_S verifies

$$\tau_S(\mathbf{P} \uplus \mathbf{P}) = \tau_S(\mathbf{P}) = 1$$

Minimum time stability: If all agents express their opinions at a moment of time and change their opinions at the next moment of time, that is, all agents change their opinions along two successive moments of time, then the sequential time cohesiveness measure takes a zero value. It also happens when there are at most two agents that keep their opinion at two consecutive moments of time but their opinions do not coincide each other. Formally:

Let $\mathbf{P} \in \mathbb{P}_{N\times T}$ be a time preference profile such that there is at most one agent who has the same opinion at t_j and t_{j+1} for $j \in \{1, \dots T\}$, that is, $n_{0,0}^{t_j,t_{j+1}} \leq 1$ and $n_{1,1}^{t_j,t_{j+1}} \leq 1$ for all $j \in \mathbf{T}$. Then, $\tau_S(\mathbf{P}) = 0$.

Proof It is immediately from Definition 3. □

Leaving minimum time stability: In order to leave the minimum time stability it is needed that at least the opinions of two agents coincide at the same moment of time and the next one. Formally:

Let $\mathbf{P} \in \mathbb{P}_{N\times T}$ be a time preference profile such that there exists at least a k, $k \in \mathbf{T}$, such that $n_{0,0}^{t_k,t_{k+1}} > 1$ or $n_{1,1}^{t_k,t_{k+1}} > 1$, then $\tau_S(\mathbf{P}) > 0$.

Proof Using Definition 3 is straightforward. □

Time monotonicity: Consider two time preference profiles, $\mathbf{P}$ and $\mathbf{P'}$, that coincide in all their elements excepting the opinion of an agent $m \in \mathbf{N}$, at t_k and t_{k+1}. Concretely, this agent has different opinion at t_k and t_{k+1} in $\mathbf{P}$: $P_{mt_j} \neq P_{mt_{j+1}}$, and the agent's opinion is the same at t_k and t_{k+1} in $\mathbf{P'}$: $P'_{mt_j} = P'_{mt_{j+1}}$. In this case, the sequential time cohesiveness measure verifies $\tau_S(\mathbf{P'}) \geq \tau_S(\mathbf{P})$. Formally:

Let $\mathbf{P}, \mathbf{P'} \in \mathbb{P}_{N\times T}$ be time preference profiles such that:

a) $P_{it_j} = P'_{it_j}$, $i \in \{\mathbf{N} \setminus \{m\}\}$,
b) $P_{mt_k} \neq P_{mt_{k+1}}$, $m \in \mathbf{N}$, $t_k, t_{k+1} \in \mathbf{T}$,
c) $P'_{mt_k} = P'_{mt_{k+1}}$, $m \in \mathbf{N}$, $t_k, t_{k+1} \in \mathbf{T}$.

Then, $\tau_S(\mathbf{P'}) \geq \tau_S(\mathbf{P})$.

Proof It is enough to prove that $\tau_S(\mathbf{P'}) - \tau_S(\mathbf{P}) \geq 0$. Let $n_{1,1}^{t_k,t_{k+1}}$ and $n_{0,0}^{t_k,t_{k+1}}$ the number of agents that approve and disapprove alternative x at t_k and t_{k+1} from $\mathbf{P}$ and $(n_{1,1}^{t_k,t_{k+1}})'$ and $(n_{0,0}^{t_k,t_{k+1}})'$ the number of agents that approve and disapprove alternative x at t_k and t_{k+1} from $\mathbf{P'}$.

– If $P'_{mt_k} = P'_{mt_{k+1}} = 0$, then

$$(n_{0,0}^{t_k,t_{k+1}})' = n_{0,0}^{t_k,t_{k+1}} + 1$$

and

$$\tau_S(\mathbf{P'}) - \tau_S(\mathbf{P}) = = \frac{1}{T-1}\left(\frac{(n_{0,0}^{t_k,t_{k+1}}+1)((n_{0,0}^{t_k,t_{k+1}}+1)-1)}{N(N-1)}\right) - \frac{1}{T-1}\left(\frac{n_{0,0}^{t_k,t_{k+1}}(n_{0,0}^{t_k,t_{k+1}}-1)}{N(N-1)}\right) \geq 0$$

since for all $z \in \mathbb{N}$, $(z+1)z - z(z-1) \geq 0$.

– If $P'_{mt_k} = P'_{mt_{k+1}} = 1$, then

$$(n_{1,1}^{t_k,t_{k+1}})' = n_{1,1}^{t_k,t_{k+1}} + 1$$

and

$$\tau_S(\mathbf{P}') - \tau_S(\mathbf{P}) = = \tfrac{1}{T-1}\left(\tfrac{(n_{1,1}^{t_k,t_{k+1}}+1)((n_{1,1}^{t_k,t_{k+1}}+1)-1)}{N(N-1)}\right) - \tfrac{1}{T-1}\left(\tfrac{n_{1,1}^{t_k,t_{k+1}}(n_{1,1}^{t_k,t_{k+1}}-1)}{N(N-1)}\right) \geq 0$$

since for all $z \in \mathbb{N}$, $(z+1)z - z(z-1) \geq 0$.

□

Convergence to full stability: If new moments of times are repeatedly introduced into the problem and all agents have the same opinion at them, then the sequential time cohesiveness measure approaches 1. Formally:

Suppose that q moments of time $t_{T+1}, \ldots t_{T+q}$ are added to $\mathbf{T}$, and at these new moments of time the alternative x is unanimously approved (resp. unanimously disapproved) by all agents. If the introduction of new moments of time does not affect agents' opinions in past times, then the sequential time cohesiveness measure of the extended time preference profile $\overline{\mathbf{P}}^{(q)} \in \mathbb{P}_{N\times(T+q)}$ approaches 1 when q tends to infinity.

$$\lim_{q\to\infty} \tau_S(\overline{\mathbf{P}}^{(\mathbf{q})}) = 1$$

Proof Using time-reducibility,

$$\begin{aligned}
\tau_S(\overline{\mathbf{P}}^{(\mathbf{q})}) &= \\
&= \tfrac{1}{T+q-1} \sum_{j=1}^{T+q-1} \tau_S(\overline{\mathbf{P}}^{I_{j,j+1}}) = \\
&= \frac{1}{T+q-1}\sum_{j=1}^{T} \tau_S(\overline{\mathbf{P}}^{I_{j,j+1}}) \\
&+ \frac{1}{T+q-1}\sum_{j=T+1}^{T+q-1} \tau_S(\overline{\mathbf{P}}^{I_{j,j+1}}) = \\
&= \frac{1}{T+q-1}\sum_{j=1}^{T} \tau_S(\overline{\mathbf{P}}^{I_{j,j+1}}) \\
&+ \frac{1}{T+q-1}\sum_{j=T+1}^{T+q-1} 1 = \\
&= \frac{1}{T+q-1}\sum_{j=1}^{T} \tau_S(\overline{\mathbf{P}}^{I_{j,j+1}}) + \frac{q-2}{T+q-1}
\end{aligned}$$

Then when q tends to infinity the first term of $\tau_S(\overline{\mathbf{P}}^{(\mathbf{q})})$ tends to 0 and the second term tends to 1.

□

Convexity: It means the sequential time cohesiveness measure of a time preference profile is a weighted average of the measures of any decomposition of $\mathbf{P}$ into consecutive time-subprofiles. Formally:

For each time preference profile $\mathbf{P} \in \mathbb{P}_{N\times T}$, and each decomposition of $\mathbf{P}$ into two consecutive time-subprofiles, $\mathbf{P}^{I_1} \in \mathbb{P}_{N\times(k_1+1)}$ and $\mathbf{P}^{I_2} \in \mathbb{P}_{N\times(T-k_1)}$ with $I_1 = \{t_1, \ldots, t_{k_1+1}\}$ and $I_2 = \{t_{k_1+1}, \ldots, t_T\}$, and $(\mid I_1 \mid -1) + (\mid I_2 \mid -1) = T-1$

$$\tau_S(\mathbf{P}) = \frac{(\mid I_1 \mid -1)\cdot \tau_S(\mathbf{P}^{I_1}) + (\mid I_2 \mid -1)\cdot \tau_S(\mathbf{P}^{I_2})}{T-1}$$

Proof It is clear from time-reducibility taking into account the following

$$\tau_S(\mathbf{P}^{I_1}) = \frac{1}{\mid I_1 \mid -1}\sum_{j=1}^{k_1} \tau_S(\mathbf{P}^{I_{j,j+1}})$$

$$\tau_S(\mathbf{P}^{I_2}) = \frac{1}{\mid I_2 \mid -1}\sum_{j=k_1+1}^{T-1} \tau_S(\mathbf{P}^{I_{j,j+1}})$$

□

3 Comparative analysis of preference stability in Clinical Decision Making: The case of terminally cancer patients' last year of life

Since 1991, *Patient Self-Determination Acts* have become significant with specific regard to life support options [5]. In particular, patients can record their preferences about the type of care that they would like to receive or not in case of loss of decision-making capacity by means an official document called "living will".

In order to collect easily patients' preferences about life support choices, Beland and Froman [5] developed and validated an instrument capable of making patients easy to express their preferences about their options, the *Life Support Preferences Questionnaire* (LSPQ).

From the LSPQ beginnings to the present, several considerations have increased the significance of such tool. Among these, it can be highlighted the use of the questionnaire like a mechanism to educate patients and their families about the selection of life support choices [30]. In addition, the LSPQ can be used to make efficient and effective health care services at end of life [13] because population ageing are increasingly high health care costs. To tackle the aforementioned aims, it is necessary to achieve a detail study of patients' preferences and their preference stability along their illness.

In consequence, this contribution focuses on studying the stability of preferences for life-sustaining treatments of patients with advanced cancer. To do it, the sequential time cohesiveness measure is used taking into account three different treatments and the possibility of cancer metastases.

3.1 The setting of the study

So as to implement our proposal for measuring the stability of preferences along time of a group of agents, this contribution is inspired and motivated by the study of Tang et al. [29]. In this contribution, the authors examined the stability of life-sustaining treatment preferences at end of life of cancer patient's last year by means of an statistical approach. They explored longitudinal preference changes based on a sample of 257 patients recruited from March 2009 to December 2012 from the general medical inpatient units of a medical center in Northwest Taiwan and followed up until June, 2013.

Based on this study, a finite set of 257 patients $\mathbf{N} = \{1, 2, \ldots, N = 257\}$ is considered. Theses patients expressed their opinions on a finite set of 3 treatments for life-sustaining at end of life, $X = \{x_1, x_2, x_3\}$ being:

- $x_1 =$ cardiopulmonary resuscitation (CPR),
- $x_2 =$ dying in an intesive care unit (ICU),
- $x_3 =$ mechanical ventilation support (MSV).

For that purpose, patients' opinions were collected by means of an interview (an adapted LSPQ) where patients answer questions about their preferences of CPR, ICU and MSV treatment when life was in danger as Figure 1 shows.

Preferences for life-sustaining treatment questionnaire

Name ____________________________ Unit____________

I would now like to ask about your wishes in regard to some specific questions concern medical treatments:

1. If your heart were to stop beating and your life were in danger, your health-care professionals might provide CPR. CPR consists of electric shocks to the heart, pumping the chest to stimulate the heart, help with breathing, and heart medications given through the veins. If your life was in danger, would you want to receive CPR?
   (a) Yes, I want the treatment.
   (b) No, I do not want the treatment.
2. If you were dying and if you need intensive care, would you like to stay in an intensive care unit (ICU)? An ICU is an isolated care unit that heavily uses health technology to provide intensive care.
   (a) Yes, I want the treatment.
   (b) No, I do not want the treatment.
3. If you were dying and if you were unable to breathe on your own, would you want to be intubated with mechanical ventilation support (MVS)? In this situation, a tube would be placed through your mouth or nose into your lungs. This tube would be attached to a breathing machine. During that time, you would have to be continuously on the breathing machine and would be unable to talk and might be sedated.
   (a) Yes, I want the treatment.
   (b) No, I do not want the treatment.

Fig. 1 Adapted LSPQ

In the questionnaire patients expressed their preferences about approving o disapproving the aforementioned treatments at four different time moments along their illness, $\mathbf{T} = \{t_1, t_2, t_3, t_4\}$. To be precise, taking into account patients' time proximity to death:

- $t_1 = 181 - 365$ days,
- $t_2 = 91 - 180$ days,
- $t_3 = 31 - 90$ days,
- $t_4 = 1 - 30$ days.

Thus, patients' opinions can be formalized by means of a time preference profile for each treatment

$$\mathbf{P}^{CPR} = \begin{pmatrix} P^{CPR}_{1t_1} & \ldots & P^{CPR}_{1t_4} \\ \vdots & \ddots & \vdots \\ P^{CPR}_{257t_1} & \ldots & P^{CPR}_{257t_4} \end{pmatrix}_{257\times 4}$$

$$\mathbf{P}^{ICU} = \begin{pmatrix} P^{ICU}_{1t_1} & \ldots & P^{ICU}_{1t_4} \\ \vdots & \ddots & \vdots \\ P^{ICU}_{257t_1} & \ldots & P^{ICU}_{257t_4} \end{pmatrix}_{257\times 4}$$

$$\mathbf{P}^{MVS} = \begin{pmatrix} P^{MVS}_{1t_1} & \ldots & P^{MVS}_{1t_4} \\ \vdots & \ddots & \vdots \\ P^{MVS}_{257t_1} & \ldots & P^{MVS}_{257t_4} \end{pmatrix}_{257\times 4}$$

Suppose the information provided by the three previous time preference profiles can be group in Table 3.

Using Definition 3, the sequential time cohesiveness measure for each profile, that is, for each treatment can be computed. Table 4 shows such values including all moments of time and all patients and Figure 2 displays them.

As it can be seen in Table 4 and Figure 2, there is not much cohesiveness among patients about using life-sustaining treatments at end of live along their illness. The highest value is obtained for CPR treatment.

In order to explore in depth these results, the set of patients is partitioned, distinguish between patients with and without metastases. Table 5 shows the patients' opinions along time taking into account if they have metastases or not.

Taking into account data from Table 5, the values of the sequential time cohesiveness measure are computed and presented in Table 6.

Treatment	$n_{0,0}^{t_1,t_2}$	$n_{1,1}^{t_1,t_2}$
CPR	190	34
ICU	142	79
MSV	170	44

Treatment	$n_{0,0}^{t_2,t_3}$	$n_{1,1}^{t_2,t_3}$
CPR	210	24
ICU	156	63
MSV	187	38

Treatment	$n_{0,0}^{t_3,t_4}$	$n_{1,1}^{t_3,t_4}$
CPR	228	15
ICU	184	26
MSV	209	25

Table 3 Number of patients that approve and disapprove different treatments at different moments of time

Treatment	Profile	$\tau_S(\mathbf{P})$
CPR	$\mathbf{P}^{CPR}$	0.676
ICU	$\mathbf{P}^{ICU}$	0.449
MVS	$\mathbf{P}^{MVS}$	0.562

Table 4 Values of the sequential time cohesiveness measure for each treatment

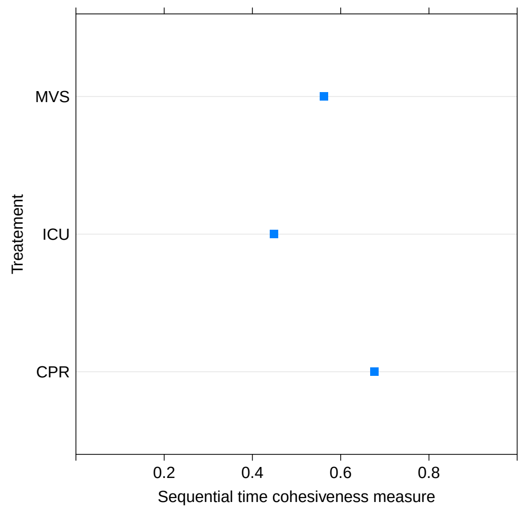

Fig. 2 Graphical display of sequential time cohesiveness measure for the three different treatment calculated from Table 3

Such as it can be observed in Table 6 and Figure 3, the sequential time cohesiveness measure for patients

	CPR		**ICU**		**MVS**	
Metastases	Yes	No	Yes	No	Yes	No
$n_{0,0}^{t_1,t_2}$	111	79	94	48	98	72
$n_{1,1}^{t_1,t_2}$	7	27	15	64	5	39
$n_{0,0}^{t_2,t_3}$	115	95	104	52	112	75
$n_{1,1}^{t_2,t_3}$	2	22	5	58	4	34
$n_{0,0}^{t_3,t_4}$	122	106	117	67	119	90
$n_{1,1}^{t_3,t_4}$	0	15	1	25	1	24

Table 5 Number of patients that approve and disapprove the three treatments at different moments of time distinguishing patients with and without metastases

Treatment		Agent-subprofile	$\tau_S(\mathbf{P})$
CPR	Metastases	$\mathbf{P}_M^{CPR}$	0.863
	No metastases	$\mathbf{P}_{NM}^{CPR}$	0.532
ICU	Metastases	$\mathbf{P}_M^{ICU}$	0.715
	No metastases	$\mathbf{P}_{NM}^{ICU}$	0.333
MVS	Metastases	$\mathbf{P}_M^{MVS}$	0.774
	No metastases	$\mathbf{P}_{NM}^{MVS}$	0.421

Table 6 Values of sequential time cohesiveness measure obtained for agent-subprofiles according to metastasis diagnoses

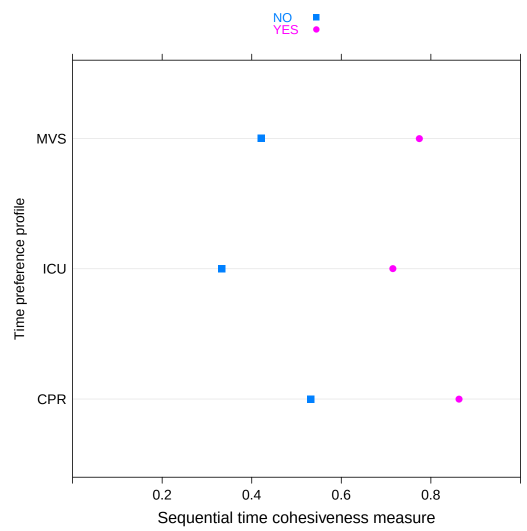

Fig. 3 Graphical display of sequential time cohesiveness measure for the three different treatment taking into account metastases diagnoses calculated from Table 5

suffering metastases is significant greater than the time cohesiveness measure for patients no suffering metastases. This is observed for the three treatments.

Now regarding the results obtained in Table 6 for CPR treatment, the study is focused on examining in detail the stability of patients' preferences at different time-subprofiles, $\mathbf{P}^{I_{t_1,t_2}}, \mathbf{P}^{I_{t_2,t_3}}, \mathbf{P}^{I_{t_3,t_4}}$, and considering two agent-subprofiles $\mathbf{P}_M$ and $\mathbf{P}_{NM}$.

Table 7 presents the number of agents that approve or disapprove CPR at different moments of time. From Table 7, the sequential cohesiveness measure is computed for the aforementioned time-subprofiles (see Table 8). Tables 9 and 10 show the values of the sequential time cohesiveness measure for the time-subprofiles distinguishing patients with and without metastases. Figure 4 represents values from Tables 8, 9 and 10.

t_1 \ t_2	No	Yes	
No	$n_{0,0}^{t_1,t_2} = 190$	$n_{0,1}^{t_1,t_2} = 8$	$n_0^{t_1} = 198$
Yes	$n_{1,0}^{t_1,t_2} = 25$	$n_{1,1}^{t_1,t_2} = 34$	$n_1^{t_1} = 59$
	$n_0^{t_2} = 215$	$n_1^{t_2} = 42$	$N = 257$

t_2 \ t_3	No	Yes	
No	$n_{0,0}^{t_2,t_3} = 210$	$n_{0,1}^{t_2,t_3} = 5$	$n_0^{t_2} = 215$
Yes	$n_{1,0}^{t_2,t_3} = 18$	$n_{1,1}^{t_2,t_3} = 24$	$n_1^{t_2} = 42$
	$n_0^{t_3} = 228$	$n_1^{t_3} = 29$	$N = 257$

t_3 \ t_4	No	Yes	
No	$n_{0,0}^{t_3,t_4} = 228$	$n_{0,1}^{t_3,t_4} = 0$	$n_0^{t_3} = 228$
Yes	$n_{1,0}^{t_3,t_4} = 14$	$n_{1,1}^{t_3,t_4} = 15$	$n_1^{t_3} = 29$
	$n_0^{t_4} = 242$	$n_1^{t_4} = 15$	$N = 257$

Table 7 Number of agents that approve or disapprove CPR at different moments of time

Treatment	Time-subprofile	$\tau_S(\mathbf{P})$
CPR	$\mathbf{P}^{I_{t_1,t_2}}{}^{CPR}$	0.563
	$\mathbf{P}^{I_{t_2,t_3}}{}^{CPR}$	0.675
	$\mathbf{P}^{I_{t_3,t_4}}{}^{CPR}$	0.790

Table 8 Values of sequential time cohesiveness measures for CPR according to different time-subprofiles

To conclude, it can be observed that preferences of patients with metastases are the most stable considering all moments of time and also for each time-subprofile.

Treatment	Time-subprofile	$\tau_S(\mathbf{P})$
CPR	$\mathbf{P}_M^{I_{t_1,t_2}}{}^{CPR}$	0.790
	$\mathbf{P}_M^{I_{t_2,t_3}}{}^{CPR}$	0.846
	$\mathbf{P}_M^{I_{t_3,t_4}}{}^{CPR}$	0.952

Table 9 Values of sequential time cohesiveness measures for CPR according to different time-subprofiles and for agent-subprofile of patients with metastases diagnoses

Treatment	Time-subprofile	$\tau_S(\mathbf{P})$
CPR	$\mathbf{P}_{NM}^{I_{t_1,t_2}}{}^{CPR}$	0.397
	$\mathbf{P}_{NM}^{I_{t_2,t_3}}{}^{CPR}$	0.543
	$\mathbf{P}_{NM}^{I_{t_3,t_4}}{}^{CPR}$	0.656

Table 10 Values of sequential time cohesiveness measures for CPR according to different time-subprofiles and for agent-subprofile of patients without metastases diagnoses

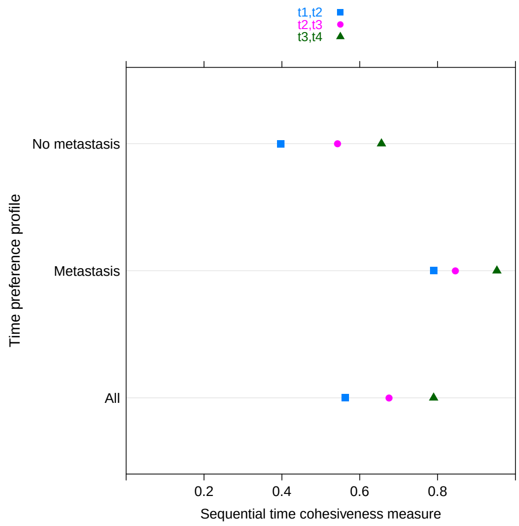

Fig. 4 Graphical display of sequential time cohesiveness measures for CPR, time-subprofiles and agent-subprofiles calculated from Tables 8, 9 and 10

4 Concluding remarks

Research on preference stability topic has advanced mainly in Economics. In this work, a non-traditional perspective is set out. The problem of measuring the degree of cohesiveness in a setting where agents express their opinions on an alternative at different times by means of an approval or disapproval evaluation is explored. A general concept of *time cohesiveness measure* is introduced and a particular formulation based on the consideration of any two successive times is proposed, namely *the sequential time cohesiveness measure*. Some properties which make our proposal appealing are also provided. Those properties are common in traditional consensus measures.

The applicability of our proposal to real situations is emphasized by means of adapting a factual problem in Clinical Decision Making. Concretely, the case of terminally cancer patients' last year of life is studied using the new sequential time cohesiveness measure.

Some straight lines of future research that could be addressed from the new approach are listed bellow:

- It could be interesting to analyse preference stability problem and its measure when the number of experts decreases along time because loss of experts to follow-up e.g., patients deaths before ended study.
- In some cases, experts could not be capable of expressing their opinion about an alternative, that is, they are undecided on it. Under this assumption it could be appealing to develop a specific time cohesiveness measure.
- Many problems from a diversity of fields could be tackled such as the consumers' preferences, Clinical Decision Making problems and so on.

Acknowledgements The authors thank the anonymous reviewers and S. García López and F. Herrera (Editors-in-Chief) for their valuable comments and recommendations. The authors acknowledge financial support by the Spanish Ministerio de Ciencia e Innovación under Project Project ECO2016-77900-P.

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