

1985  
NASA/ASEE SUMMER FACULTY RESEARCH FELLOWSHIP PROGRAM

MARSHALL SPACE FLIGHT CENTER  
THE UNIVERSITY OF ALABAMA

PRELIMINARY DESIGN METHODS FOR FIBER REINFORCED  
COMPOSITE STRUCTURES EMPLOYING A PERSONAL COMPUTER

Prepared by: Charles N. Eastlake, P.E.  
Academic Rank: Associate Professor  
University and Department: Embry-Riddle Aeronautical Univ.  
Aeronautical Engineering Dept.  
NASA/MSFC: Structures and Propulsion Lab  
Division: Structures Division  
Branch: Structural Development Branch  
EP-13  
MSFC Counterpart: Erich E. Engler  
Date: August 2, 1985  
Contract No.: NGT 01-008-021  
The University of Alabama in  
Huntsville

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BY

Charles N. Eastlake  
Associate Professor of Aeronautical Engineering  
Embry-Riddle Aeronautical University  
Daytona Beach, Florida

ABSTRACT

The objective of this project was to develop a user-friendly interactive computer program to be used as an analytical tool by structural designers. Its intent was to do preliminary, approximate stress analysis to help select or verify sizing choices for composite structural members. The approach to the project was to provide a subroutine which uses classical lamination theory to predict an effective elastic modulus for a laminate of arbitrary material and ply orientation. This effective elastic modulus can then be used in a family of other subroutines which employ the familiar basic structural analysis methods for isotropic materials. This method is simple and convenient to use but only approximate, as is appropriate for a preliminary design tool which will be subsequently verified by more sophisticated analysis.

Additional subroutines have been provided to calculate laminate coefficient of thermal expansion and to calculate ply-by-ply strains within a laminate.

## 1. INTRODUCTION

The NASA Marshall Space Flight Center has both research and hardware development within its mission. The responsibility for developing a variety of items of space flight hardware brings with it the necessity of doing structural design in-house. In the beginning stages of design concept definition it is helpful to have quick and convenient approximate stress analysis tools available to the designers so that a wide variety of structural configurations can be sketched out and checked for feasibility. At this stage of the design process the stress/strain analysis does not need to be particularly accurate because any configurations tentatively defined as feasible will later be analyzed in detail by stress analysis specialists. Besides, the sophisticated finite element methods such as NASTRAN which are industry analysis standards are simply too time-consuming and complex for practical use in preliminary design. The emphasis in preliminary design must be on fast, user-friendly methods so that rough but technically sound evaluation of a broad variety of conceptual designs can be accomplished. The development of a computer program to function as such a tool was the goal of this project.

## 2. OBJECTIVES: The broad concept of the computer program COMPSIZE

To maximize user convenience the desire of the sponsoring branch was from the very beginning to employ a personal computer located in the designers' offices. The preliminary designer must usually make frequent reference to drawings, manuals, handbooks, etc. as he progresses through his calculations. If the computer is located remote from his "home" office the author can attest from experience that the designer is almost certain to waste frustrating time going back after needed data which is not included in the bundle of information that he has carried to the computer area with him. To meet this goal, both a Digital Equipment Corp. DEC 350 and an IBM PC were possible candidate hardware. The IBM PC was chosen because a greater number of machines appear to be likely to be available in the user's facility for the near term future.

Several aspects of the practicality of the project were not certain at the outset and were to be clarified during the writing of this program.

1. Does a personal computer have adequate memory?
2. Is BASIC language adaptable enough and fast enough to handle problems of the type required?
3. Is it possible to build in sufficient user-friendliness to encourage widespread use of the program by the design personnel?

The final conclusions to these questions are presented in the Conclusions and Recommendations section. The rationale for the questions and some of the pertinent information are discussed here.

The IBM PC used for this project has dual 5.25" floppy disk drives employing double side, double density disks each having a storage capacity of 366 K bytes. The processor has a random access memory (RAM) of 512 K. This seemed likely to be adequate and turned out to be so. The disk storage space required to save the main program and six subprograms is approximately 40 K. Less than 1 K additional RAM is required to run the most memory-consuming portion of the program due to the fact that the number of variables used in the program is not particularly large. IBM Advanced BASIC requires 27 K, so the bare bones 64 K PC may not be adequate but 128 K versions should run the program without difficulty.

There was some speculation about the suitability of the BASIC language, a common discussion among the engineering community. BASIC was severely limited in capability in its early days, dating back to the mid-sixties. However, that

limitation does not appear to be a problem any longer. The IBM Advanced BASIC used in this project has 152 program commands, 15 built-in mathematical operations, user definable mathematical functions, color graphics, and sound production ability. It does not have for some reason the matrix manipulation functions frequently found in BASIC, which would have been useful for the composite laminate analysis portion of this program. At any rate the author did not feel any serious constraints due to the language employed.

User-friendliness goals are centered around the sad but true fact that many users will not read a user's manual until they get into some mess that they can't find their way out of. In addition, many users are unfamiliar with the operation of PC hardware. With that in mind, this program was written in such a way that if the user knows how to turn on the computer and initiate the program he should be able to solve his problem with no further guesswork even if he has not read the manual. The program uses a main menu format listing 15 major problem types which can be selected. Several of the major problem selections have sub-menus of up to 5 further specialized problems. In each problem subroutine all units are specified for both input and output. At the conclusion of each problem the user is asked whether he wants a printed copy of the results, then whether he wants another problem of the same type or to go back to the main menu. To speed operations, the user can answer "no" to any yes/no input requested by simply pressing the carriage return. The video game fascination is assuaged by a color graphics NASA logo during the initial sign-on. The author feels that the user-friendliness has worked out well and will be of value as well for classroom use of the program at his home institution. The programming of these prompts, messages, and fault-tolerant branches did, however, take possibly twice as much storage space and programming time as the solution of the equations.

### 3. A BRIEF REVIEW OF COMPOSITE ANALYSIS METHODOLOGY

Finite element methods (FEM) have become firmly established as the standard structural analysis method in the aerospace industry. The most widely known is NASTRAN, a contraction for NASA Structural Analysis. FEM methods when correctly applied are relatively reliable and accurate. However, the creation of the geometric model of the structure to be analyzed is very time consuming and something of an art for complex structural shapes. Automatic generation of FEM models directly from data bases assembled during the creation of drawings on Computer-aided Design (CAD) systems is close to operational availability and will broaden the use of FEM. But for now the tedium of model generation makes FEM analysis impractical for preliminary design. Even after automatic model generation is a reality the method will still require a significant amount of time expenditure in creation of a fairly detailed CAD drawing of the structure being studied. So the hand-drawn sketch method with quick-look calculations will probably endure forever in the early concept definition stages of structural preliminary design.

It is necessary even as part of FEM analysis to specify a layer-by-layer description of the stackup of plies in a laminated composite part. In its most accurate form this composite analysis technique calculates the mechanical strain separately in each ply of the composite part, which is typically .005 to .010 inches thick. The analysis is done by matrix algebra methods which the computer can do very quickly, but evaluation of the data can be a slow and painstaking process. The description of COMPSIZE main menu problem 2 in Section 4.2 gives a more detailed picture of the geometry and material description conventions used for that process.

A much faster though less accurate method widely used for preliminary design (Ref. 3,4,and 8) is to use the stackup of plies to calculate an effective stiffness of the laminated material and then analyze it using the much simpler equations which describe homogeneous and isotropic materials. This calculation of effective stiffness can be done using classical lamination theory which is employed in COMPSIZE problems 1, 2, and 3. Even this can be quickened considerably by solving for stiffness of a large variety of physical orientations of ply stackups and plotting the results on an easy-to-use figure known as a carpet plot. However, the designer must have the carpet plots on hand for the material combinations which he is considering using. A simple mathematical representation of a carpet plot (a single equation) developed by the author during this project is

presented in COMPSIZE problem 1 sub-menu 1. Another simple approximation in lieu of a carpet plot, from Reference 3, is presented in problem 1 sub-menu 3.

Estimation of coefficient of thermal expansion (CTE) of laminated composites is of great significance in space structures because of large size parts and large temperature differences between sunlight and shade on orbit. CTE can be estimated using an extension of stiffness estimation methods which is presented as problem 3.

#### 4. SPECIFIC DESCRIPTION OF THE "COMPSIZE" PROGRAM

It would be easiest to describe COMPSIZE by referring to a listing of the program. However, the listing would require about 30 pages, which is well beyond the acceptable limit for this report. Instead, replicas of the first several monitor screens will be presented to give the flavor of the program.

A detailed User's Manual was written as the culmination of the project. It includes a listing, description of the calculation procedures, and sample problem for each problem (subroutine). Readers seriously interested in using the program should contact the sponsoring branch listed on the title page to request copies of the program disk and manual.

Figure 1 shows the first screen to appear when the program is initiated. It displays the NASA logo and program name and asks if the user wants to see the program introduction. This introductory description, shown in Figure 2, is similar to the abstract of this report. The next screen, shown in Figure 3, lists the default values used in the program for safety factor. It also lists stiffness and allowable stress for a typical intermediate strength graphite/epoxy laminate in case the user does not have other data at hand and simply wants to input something to get ballpark numbers. Default material properties are only written into the actual calculation process for problem 1 sub-menu 1. Creation of files of default or selectable material properties would be a helpful next task in the continued development of this program.

Figure 4 shows the main menu, which appears next. Once a user is familiar with the program he can go directly to this point by entering RUN 298 instead of simply RUN. There are 15 problems listed, seven of which have sub-menus for further specialized problem selection. Each main menu item is described in detail in a separate section of the user's manual. Because this report is more descriptive in nature, Figure 5 shows the screen exactly as it would appear to a user solving problem 1 sub-menu 3. The choice is an approximate stiffness calculation using a set of equations from Reference 3. The exact layup of ply orientations is not required for this problem, only the percent of plies to be laid up at each angular orientation. As is common industry design practice 0,90, or +/-45 orientation is presumed. The stiffness properties of the individual layer material with all layers assumed to be the same are input and the output is the stiffness properties of the entire laminate.

The program asks if another problem of the same type is



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COMPSIZE  
7/19/85

COMPOSITE STRUCTURE PRELIMINARY SIZING

WANT AN INTRODUCTION? (Y/N) ? ■

FIGURE 1

## COMPSIZE

This program was written by Prof. Chuck Eastlake of Embry-Riddle Aeronautical University under the sponsorship of the 1985 NASA/ASEE Summer Faculty Fellowship Program. Work was performed at the NASA Marshall Space Flight Center, Structures and Propulsion Laboratory, Structural Development Branch, EP-13.

It is intended to be a user-friendly, interactive analytical tool for preliminary, approximate stress analysis by structural designers to select or verify sizing choices for composite structural members. Homogeneous materials can be analyzed as simplified cases. A flat laminate analysis routine is included for analysis of arbitrary layups. However, it should be sufficient for most users to presume a quasi-isotropic layup and use the familiar basic structural analysis methods for isotropic materials, after estimating an appropriate elastic modulus. All the equations used in this program assume elastic deformation. Intermediate strength graphite/epoxy is used as the default material selection. Example problems are included in the program manual.

PRESS ANY KEY TO CONTINUE ?

FIGURE 2

**DEFAULT VALUES USED IN THE PROGRAM:****Safety factor=2 for pressure vessels.****Safety factor=1.4 for all other problems.****Material properties for quasi-isotropic laminate of intermediate strength graphite/epoxy:****E=7.2E6 psi, G=2.8E6 psi, Poissons ratio=.32****ultimate tensile stress=54.6 ksi****ultimate compressive stress=52 ksi****ultimate in-plane shear stress=34 ksi****ultimate net tension in joint=27 ksi****ultimate bearing stress (e/D=2)=65 ksi**

**To speed up running of the program, you may press the RETURN/ENTER key to answer NO for any question requiring a (Y/N) answer.**

**PRESS ANY KEY TO CONTINUE ?**

FIGURE 3

MAIN MENU: PROBLEM TYPES

1. COMPOSITE MATERIAL PROPERTY ESTIMATION
2. LAMINATE ANALYSIS
3. LAMINATE COEFFICIENT OF THERMAL EXPANSION
4. AXIAL TENSION OR COMPRESSION (NO BUCKLING)
5. DIRECT SHEAR OR PUNCH-OUT
6. SYMMETRIC BENDING
7. COMBINED AXIAL AND BENDING
8. SHEAR INDUCED BY BENDING~
9. TORSIONAL SHEAR
10. COLUMN BUCKLING (EULER)
11. PRESSURE VESSELS
12. STRESS INTERACTIONS FOR COMBINED LOADS
13. FITTING STRESSES
14. SECTION AREA AND MOMENT OF INERTIA
15. SANDWICH PANEL ANALYSIS
20. QUIT

TYPE THE NUMBER YOU WANT ?

FIGURE 4

COMPOSITE MATERIAL PROPERTY ESTIMATION

MATERIAL PROPERTY ESTIMATION SUB-MENU:

1. CARPET PLOT READ-OFF
2. CALCULATED FROM A MATRIX
3. APPROXIMATE SOLUTION BASED ON LAYER ORIENTATION

ENTER NUMBER FOR THE PROCEDURE YOU WANT ? 3

ENTER PERCENT OF LAYERS TO BE SET AT 0 DEGREES ? 25

ENTER PERCENT OF LAYERS TO BE SET AT 90 DEGREES ? 25

ENTER PERCENT OF LAYERS TO BE SET AT +/-45 DEGREES ? 50

ENTER MAJOR ELASTIC MODULUS, E11 (MSI) ? 22

ENTER MINOR ELASTIC MODULUS, E22 (MSI) ? 1.5

ENTER SHEAR MODULUS, G12 (MSI) ? .52

ENTER MAJOR POISSONS RATIO, V12 ? .25

Ex (MSI)= 8.333333

Ey (MSI)= 8.333333

Gxy (MSI)

Vxy= .3197279

Vyx= .3197279

= 3.0825

ANOTHER CALCULATION OF THIS TYPE? Y/N ? N

MAIN MENU: PROBLEM TYPES

1. COMPOSITE MATERIAL PROPERTY ESTIMATION
2. LAMINATE ANALYSIS
3. LAMINATE COEFFICIENT OF THERMAL EXPANSION
4. AXIAL TENSION OR COMPRESSION (NO BUCKLING)
5. DIRECT SHEAR OR PUNCH-OUT
6. SYMMETRIC BENDING
7. COMBINED AXIAL AND BENDING
8. SHEAR INDUCED BY BENDING
9. TORSIONAL SHEAR
10. COLUMN BUCKLING (EULER)
11. PRESSURE VESSELS
12. STRESS INTERACTIONS FOR COMBINED LOADS
13. FITTING STRESSES
14. SECTION AREA AND MOMENT OF INERTIA
15. SANDWICH PANEL ANALYSIS
20. QUIT

TYPE THE NUMBER YOU WANT ? 20

THE END. HOPE YOU GOT GOOD NUMBERS!

Ok

FIGURE 5

desired. If the response is Y the program returns to the beginning of the same subroutine. If the response is anything else it returns to the main menu, from which the user may select another problem type or terminate the program.

Problems 1,2 and 3 are specifically devoted to design with composites. Problem 11 sub-menu 1 calculates the Tsai-Hill failure criterion which is specific to composites. Problem 15 is not necessarily a composite design tool, but sandwich structures are most frequently composite. Problem 15 is also the only subroutine which has an optimization process for minimum weight written in as a sub-menu choice. That is because this optimization is represented by a fairly simple equation having a closed form solution.

The remaining 10 problems are standard stress analysis methods for homogeneous and isotropic materials. They can be found in many basic texts on strength of materials like References 5 and 6. These should be familiar to almost any engineer or engineering student and thus do not warrant extensive explanation.

The first 3 subroutines are the heart of the composite application of this program and will be described in modest detail. Again, a more detailed explanation can be found in the User's Manual.

#### 4.1 SUBROUTINE 1 Composite Material Property Estimation

There are three methods known to the author for estimation of stiffness (elasticity) properties of laminated composites, represented by the three sub-menu options available in this subroutine. The calculated results of the three methods are compared in Table 1. The method which should be most accurate is that involving the calculation of the terms in the so-called A-matrix, so this is used as the standard of comparison. It is used in the subroutine named AMATRIX which is chained into the program by selection of problem 1 sub-menu 2.

The equations presented by Lubin (Ref. 3) are the simplest and are in reasonably good agreement except for the symmetric angle ply case. For that case, which unfortunately is typical of structures designed primarily for torsional loads, the agreement is quite bad. The difference is over 50 percent and is due to the fact that the equations inherently presume that the modulus of elasticity for a 45 degree angle ply laminate is identical to the transverse direction modulus of the ply material. For some materials this is a reasonable assumption but for many others it is not. This set of equations is available as problem 1 sub-menu 3.

The author felt it was appropriate because of this error to attempt to formulate another more accurate method which would still be simple enough to use on programmable calculators. The method which resulted is problem 1 sub-menu 1. This subroutine named CARPLOT solves a single equation, albeit a moderately long one, which represents a math model of a carpet plot. The equation, shown below, has a closed form solution. It requires no iterations or matrix algebra and could thus be solved on nearly any programmable hand calculator. Figure 6 and 7 show the accuracy achieved.

$$E_x = E_{11}P_0 + E_{22}(1-P_0) + \left( \frac{4QG_{12}}{Q+G_{12}} - E_{22} \right) P_{45} \\ + .06(1-P_0)E_{11} \sin \left[ \pi \left( \frac{P_{45}}{1.001 - P_0} \right) \right]$$

where  $Q = \frac{1}{4} \left[ \left( \frac{E_{11}}{E_{11} - \nu_{12}^2 E_{22}} \right) (E_{11} + E_{22} + 2\nu_{12}E_{22}) \right]$

$P_0$  = percent of 0 degree plies

$P_{45}$  = percent of +/-45 degree plies

$E_{11}, E_{22}, G_{12}, \nu_{12}$  = ply material properties

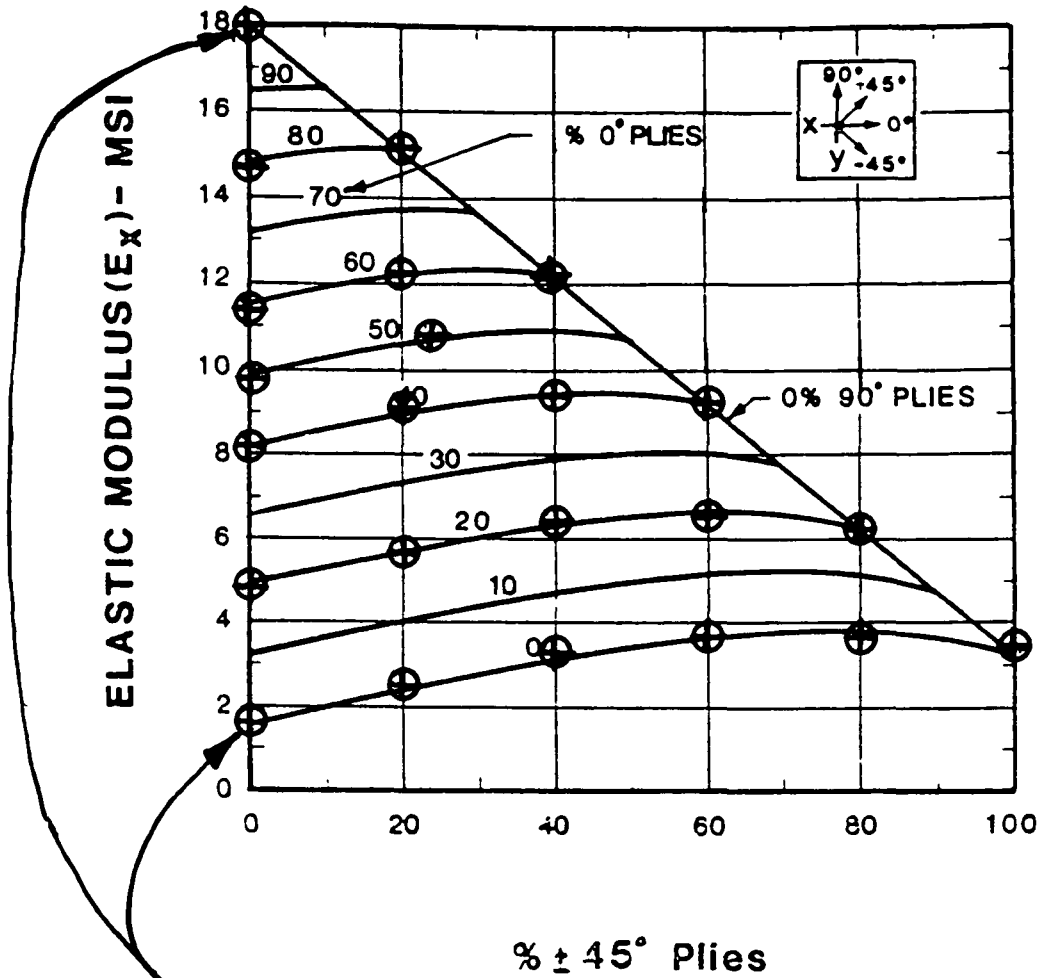
LAMINATE GEOMETRY	PERCENT 0/90/±45	"AMATRIX" E <sub>x</sub> (MSI)	"CARPLOT" E <sub>x</sub> /% VAR.	REF. 3 E <sub>x</sub> /% VAR.
QUASI-ISOTROPIC [0/45/45/90] <sub>s</sub>	25/25/50	7.362	7.284/-1.06%	7.067/-4.17%
SYM. ANGLE PLY [45/-45/45/-45] <sub>s</sub>	0/0/100	3.362	3.366/-0.12%	1.6/-52.4%
CROSS PLY [0/90/0/90] <sub>s</sub>	50/50/0	9.889	9.80/-0.90%	9.80/-0.90%
GENERAL [0 <sub>8</sub> /90/45/45] <sub>s</sub>	73/9/18	14.18	14.143/-0.26%	14.064/-0.82%

STIFFNESS ESTIMATES  
GRAPHITE/EPOXY

TABLE 1



# GRAPHITE/EPOXY

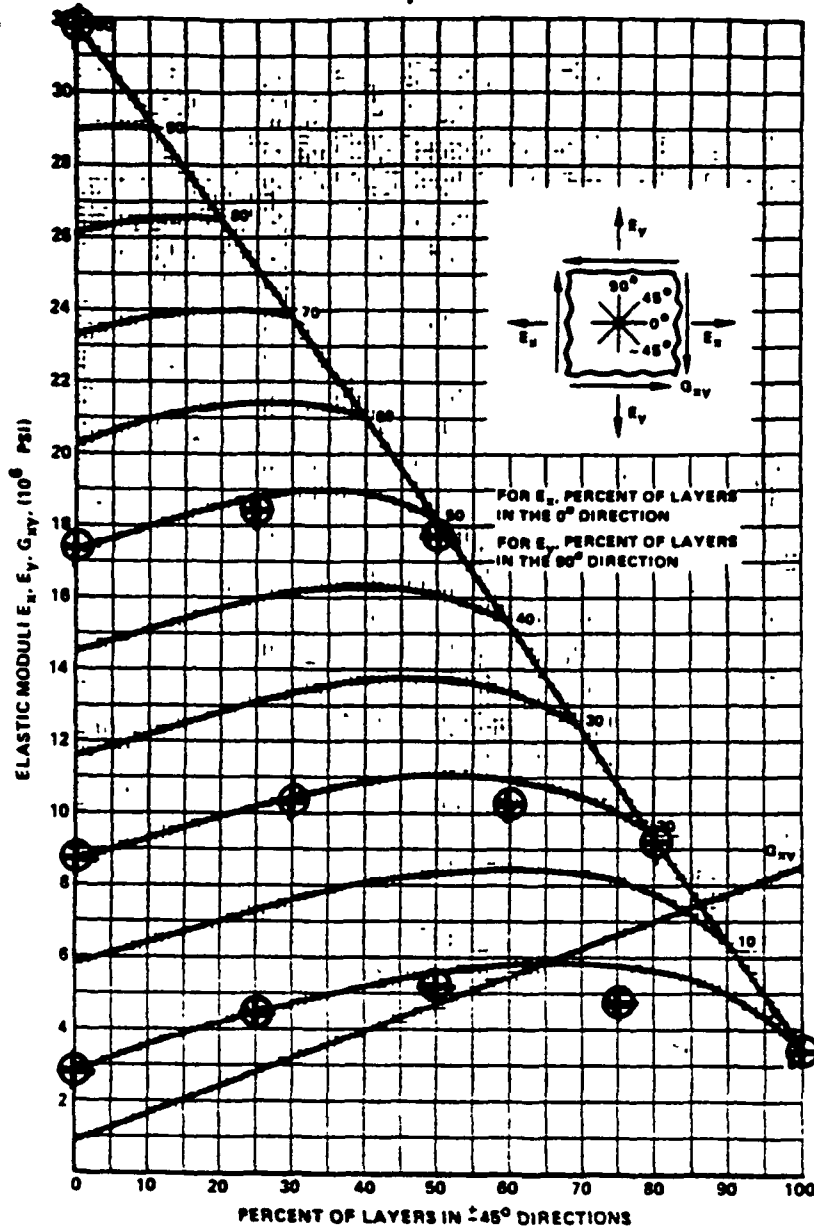


INPUT DATA POINTS

⊕ OUTPUT OF 'CARPLOT' SUBROUTINE

FIGURE 6 (FROM REF. 9)

# BORON/EPOXY



ELASTIC MODULI FOR  $[0_i/\pm 45_j/90_k]$  FAMILY. RT

⊕ OUTPUT OF "CARPLOT" SUBROUTINE

FIGURE 7 (FROM REF. 8)

#### 4.2 SUBROUTINE 2 Laminate Analysis

The solution of classical laminate theory as performed in this subroutine is described in detail in References 1-4. The notation of Ref. 1 is used in this description. It begins with the formulation of the 3-dimensional Hooke's Law in matrix notation:

$$(1) \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{Bmatrix}$$

The 6x6 matrix of  $C_{ij}$  terms is known as the stiffness matrix. When we limit our analysis to planar laminates it reduces to a 3x3 matrix because row and column 3, 4, and 5 become zero by definition. The equation is then written:

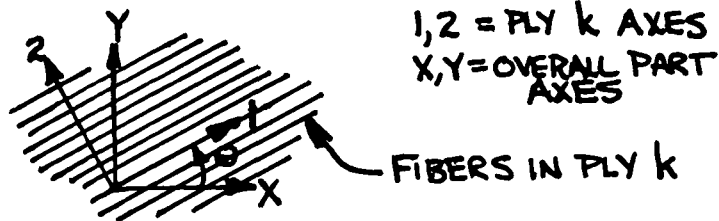
$$(2) \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

Stiffness terms are determinable from engineering constants which define the basic material properties of the plies:

$$(3) \quad Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}} \quad Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}} \quad Q_{66} = G_{12}$$

A further generalization of Hooke's Law is required, however, to allow for the possibility that the fibers in any individual layer, or ply, may not be aligned with the XY axes which are used to describe the overall size and shape of the finished composite part and which are also used to specify the applied loads.



The coordinate rotation is described by:

$$(4) \quad \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}$$

$$(5) \quad [T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

Figure 8 shows the stackup of a typical laminate to be analyzed and the axis and sign conventions used in the mathematical description of the laminate.

The concept of tensor strain, which is simply half of the actual strain, is introduced so that stress and strain can be transformed using the same matrix:

$$(6) \quad \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = [R] \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \frac{\gamma_{12}}{2} \end{Bmatrix} \quad \text{WHERE} \quad [R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Hooke's Law can now be written as

$$(7) \quad \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [T]^{-1} [Q_{ij}] [R] [T] [R]^{-1} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

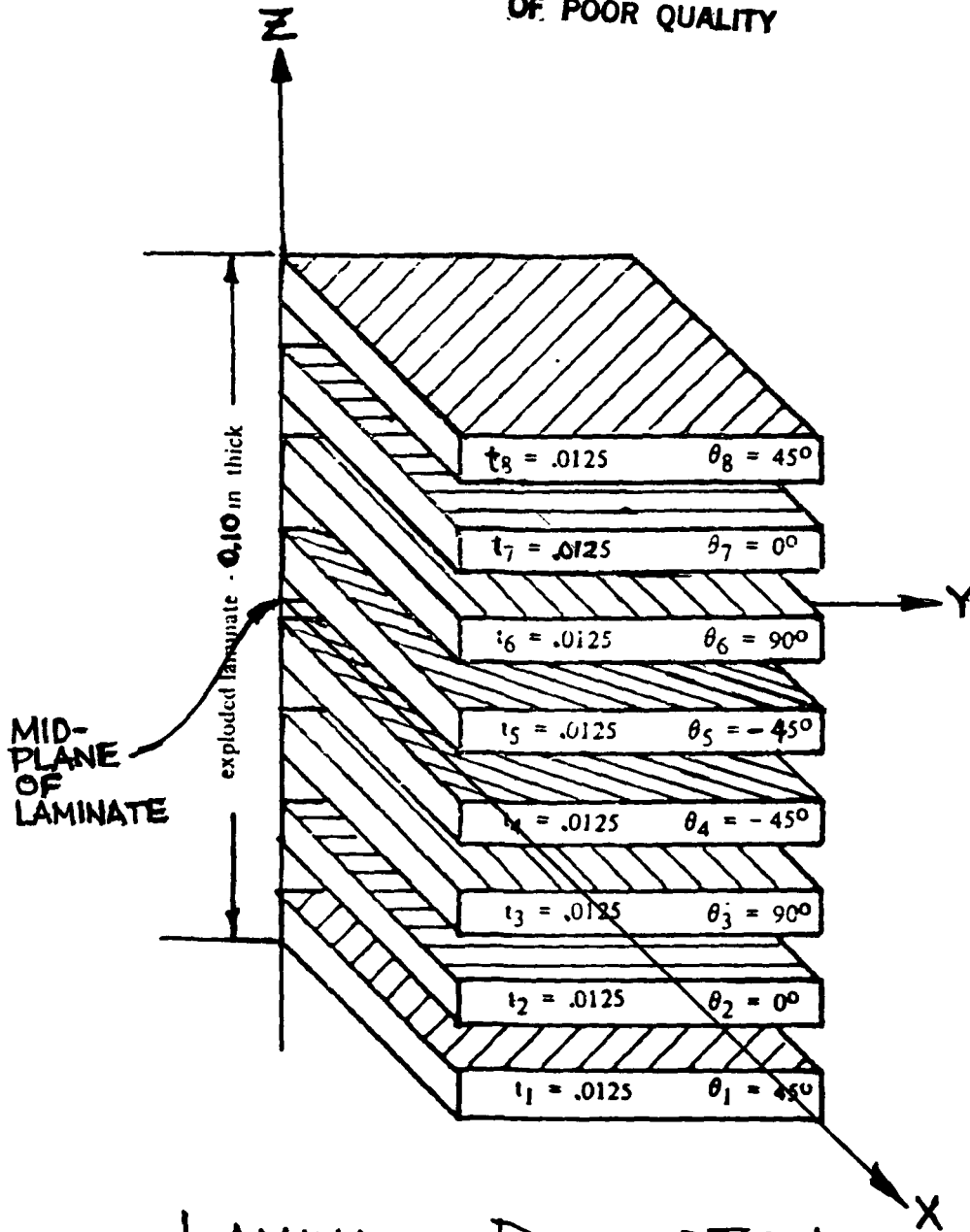
or, in shorthand notation

$$(8) \quad \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\bar{Q}_{ij}] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11} \cos^4\theta + 2(Q_{12} + 2Q_{66}) \sin^2\theta \cos^2\theta + Q_{22} \sin^4\theta \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66}) \sin^2\theta \cos^2\theta + Q_{12} (\sin^4\theta + \cos^4\theta) \\ \bar{Q}_{22} &= Q_{11} \sin^4\theta + 2(Q_{12} + 2Q_{66}) \sin^2\theta \cos^2\theta + Q_{22} \cos^4\theta \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin\theta \cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin^3\theta \cos\theta \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin^3\theta \cos\theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin\theta \cos^3\theta \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2\theta \cos^2\theta + Q_{66} (\sin^4\theta + \cos^4\theta) \end{aligned}$$

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### LAMINATE DESCRIPTION

$t, \theta; E_{11}, E_{22}, G_{12}, \nu_{12}$

FIGURE 8

Keep in mind that this relationship is only valid for a single layer. The terms in these equations must be recalculated for each ply.

The manner in which these relations are most frequently used employs the equations which result from integrating the stress distribution over the thickness of the built-up laminate. The equations thus derived relate inplane forces,  $N$ , and out of plane moments,  $M$ , to the extension strains,  $\epsilon$  and  $\gamma$ , and curvatures of the laminate center plane,  $K$ .

$$(9) \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

$$(10) \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} K_x \\ K_y \\ K_{xy} \end{Bmatrix}$$

$$(11) A_{ij} = \sum_{k=1}^N (\bar{Q}_{ij})_k t_k, \quad B_{ij} = \frac{1}{2} \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2), \quad D_{ij} = \frac{1}{3} \sum_{k=1}^N (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3)$$

The A, B and D-matrices contained in these equations are widely used descriptions of laminated composite behavior. In shorthand notation, the above equations are written as:

$$(12) \begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon \\ K \end{Bmatrix}$$

Since applied forces and moments are usually the known quantities, the above equation is frequently inverted to:

$$(13) \begin{Bmatrix} \epsilon \\ K \end{Bmatrix} = \begin{bmatrix} A' & B' \\ B' & D' \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix}$$

Using this equation, strains in each individual ply are calculated. A judgement of likelihood of failure is based upon comparison of strain in each ply to the allowable strain for the type of material used in that ply. If any ply fails, the part is considered to have failed.

A special note is necessary here. It is only true that  $A' = A^{-1}$  and  $D' = D^{-1}$  if  $B = B' = 0$ . The designer can cause the B-matrix to be identically zero by specifying a balanced and symmetric laminate. This means that any  $+\ominus$  layer orientation is balanced by a layer with  $-\ominus$  orientation. In other words, the number of  $+\ominus$  layers equals the number of  $-\ominus$

layers. Also, the distribution of layer orientations is symmetric above and below the center plane of the laminate. This does not occur automatically. It is up to the designer to be aware of this and to specify a proper layup. All aerospace companies known to the author instruct designers to use a balanced and symmetric layup except in very specialized designs.

The LAMINAT2 subroutine, which is problem 2 in the main menu, calculates the A, B, and D-matrix and inverts the A and D-matrix. It solves for the strains and curvatures presuming  $B=0$ , and prints a warning to that effect immediately after the B-matrix in the output. The designer can thus verify that he has input an appropriate layup.

#### 4.3 SUBROUTINE 3 Laminate Coefficient of Thermal Expansion

Thermal expansion is typically of far greater concern in the space part of aerospace design than it is in the aeronautical part. Space structures under development are quite large in some cases and temperature variations from sunlight to shade on orbit are typically 200 degrees Farenheit. Both of these factors can contribute to unacceptably large thermal deformations in spite of the fact that CTE's for commonly used graphite/epoxy are considerably smaller than CTE's for aluminum or steel. Proper latching of Space Shuttle payload bay doors and pointing accuracy and/or focus of the Space Telescope are examples of potential problems of this nature. For this reason a subroutine named CTE using the equations of Ref. 4 is included as problem 3. It calculates CTE in both longitudinal and transverse direction for a laminate of any arrangement of ply material and ply orientation. Stiffness properties and CTE's for individual plies must be known for input. This CTE calculation is probably inseparably coupled with the stiffness properties calculated using problem 1 or 2 in most design projects. Varying ply orientation in the stackup to minimize CTE must, of course, be done with care since it also changes the stiffness of the laminate. It may be impossible to optimize both CTE and stiffness for a particular structural part.



## 5. CONCLUSIONS AND RECOMMENDATIONS

The following conclusions and recommendations must be considered tentative in some cases because the COMPSIZE program is only becoming available to MSFC designers at the time of writing of this report. The author has demonstrated the program to most of the Structural Development Branch engineers for their initial impressions and suggestions but it has not yet been used for "production".

### Conclusions:

1. Based on the reactions of the branch design engineers and upon the author's experience as both designer and design teacher it seems highly likely that COMPSIZE will be a useful tool and will function as intended. Users adjust quickly to the program and appear satisfied with the type and quantity of answers they can get. The accuracy of calculated results in determining the practicality of preliminary design concepts can only be determined after it has been used for a period of time. It is currently in use on a Space Station project.

2. The employment of a personal computer for composite design seems to be a workable idea. The IBM PC on which the program was developed is located in a 4-man office, and its proximity noticeably improves the feelings of the branch personnel about its utility. The PC has more than adequate memory and its solution speed, as far as the user can tell, is instantaneous. BASIC language was adequate to produce a highly user-friendly program format and presented no difficulty in setting up solutions of the required equations.

### Recommendations:

Recommendations for further development of the program to increase its utility are as follows.

1. Add selectable files of material properties for composite and homogeneous materials typically used in space structures. This would significantly reduce the time required for data input, particularly for laminate analysis.

2. Develop methods for structural optimization, probably to minimize weight, maximize stiffness, and minimize CTE.

3. Add further problem types to the main menu. These might include:

- buckling of stiffened panels
- ring frames with inplane loads

thermal stresses  
expansion of composites due to moisture  
absorption  
unsymmetric bending  
normal loads on plates  
stress concentrations

4. Make COMPSIZE available to other users. The sponsoring branch listed on the title page is the appropriate contact.

## 6. REFERENCES

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