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PRELIMINARY DESIGN OPTIMIZATION FOR A SUPERSONIC TURBINE FOR ROCKET PROPULSION

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ABSTRACT

In this study, we present a method for optimizing, at the preliminary design level, a supersonic turbine for rocket propulsion system application. Single-, two- and three-stage turbines are considered with the number of design variables increasing from 6 to 11 then to 15, in accordance with the number of stages. Due to its global nature and flexibility in handling different types of information, the *response surface methodology* (RSM) is applied in the present study. A major goal of the present optimization effort is to balance the desire of maximizing aerodynamic performance and minimizing weight. To ascertain required predictive capability of the RSM, a two-level domain refinement approach has been adopted. The accuracy of the predicted optimal design points based on this strategy is shown to be satisfactory. Our investigation indicates that the efficiency rises quickly from single stage to 2 stages but that the increase is much less pronounced with 3 stages. A 1-stage turbine performs poorly under the engine balance boundary condition. A significant portion of fluid kinetic energy is lost at the turbine discharge of the 1-stage design due to high stage pressure ratio and high-energy content, mostly hydrogen, of the working fluid. Regarding the optimization technique, issues related to the *design of experiments* (DOE) has also been investigated. It is demonstrated that the criteria for selecting the data base exhibit significant impact on the efficiency and effectiveness of the construction of the response surface.

NOMENCLATURE

A_{ann}	: Exit Blade Annulus Area.
c	: Chord
D	: Diameter
h	: Blade Height
$\dot{m}$	: Mass flow
P	: Pressure
R^2	: Proportion of Variation
rms	: Root Mean Square
RPM	: Angular Speed
s^2	: Standard Deviation
se	: Standard Error
sr	: Stage Reaction
T_{in}	: Input Temperature
V_{pitch}	: Pitch Speed
W	: Weight
wf	: Work Fraction
Δpay	: Payload Increment
η	: Efficiency

1. INTRODUCTION

Supersonic turbine technologies are being actively investigated in the rocket propulsion community. Optimizing a multistage turbine is a labor-intensive task and it is desirable to develop efficient and effective techniques to undertake this task. In general, two types of optimization are needed, namely,

1. *Preliminary design*, in which simplified models employing loss correlations gleaned from experimental database and one-dimensional gasdynamic and thermodynamic considerations, and

2. *Detailed shape design of the turbine blades*, in which three dimensional computational fluid dynamics and detailed experimental information is employed.

In this study, we present an approach based on the response surface methodology[1] (RSM) for optimizing, at the preliminary design level, a supersonic turbine aimed for *Reusable Launching Vehicle* (RLV) propulsion system application. Single-, two- and three-stage turbines are considered with the number of design variables increasing in accordance with the number of stages. In the first step, the overall dimensions such as mean diameter and axial chords, the *RPM* and the number of stages are to be determined. In the second step, detail blade geometries will be optimized to achieve the best efficiency for a given overall gas path meridional geometry. This paper is limited to the preliminary optimization.

In the past such optimization tasks might take weeks to perform due to the large number of parameters involved. The systematic application of RSM computationally coupled with an appropriate turbine analysis code will in the future allow designers to cut this cycle time down to a few hours.

There are 2 types of design variables:

1. Geometric input needed to layout the turbine meridional geometry.
   - mean diameter
   - last rotor annulus area
   - blade height ratio between the 1st vane and the last rotor blade – linear distribution of blade heights is assumed between the 1st vane and the last rotor blade
   - vane and blade axial chords

2. Performance input used to calculate the turbine efficiency
   - *RPM*
   - number of stages
   - blade row reaction
   - work split (if more than 1 stage is investigated)

The above parameters are essentially sufficient to determine the geometry and the performance of a preliminary design. As indicated in Tables 1-3, for turbine with 1 stage, 2 stages or 3 stages the number of parameters are 6, 11, 15, respectively. Constraints are also part of the optimization process. There are 2 structural constraints, the blade centrifugal stress and the disk stress. The blade centrifugal stress was constrained by a limit placed on the lumped inertia measure (the product of the blade exit annulus area and the RPM^2). The disk stress was constrained by a limit placed on the pitchline velocity (the product of the *RPM* and the mean radius).

For rocket engine applications, maximizing the vehicle payload for a given turbine operating condition is the ultimate objective. Any gain in turbine efficiency will be reflected in a reduced propellant consumption, thus in an increase in payload. However, higher turbine performance usually entails multistage designs, which are heavier. A proprietary weight correlation is employed to estimate the impact of the turbine choices on the overall turbopump unit weight since a slowly rotating turbine is not only heavy by itself but also imposes a significant weight penalty on the pump side. An equation expressing the relationship between these opposing effects will be employed as a criterion to guide the optimization task. As will be presented in detail later, this composite objective function describes essentially the payload increment versus turbopump efficiency and weight. It is developed based on mission profile studies, engine balance perturbation and some detailed turbopump layout and stress information gained from other proprietary programs.

2. APPROACH

The overall approach to determine the optimum design is shown in Figure 1. The RSM is used to model the relationship between design variables and objective/constraint functions in function approximation stage of the overall approach.

The approach of RSM is to perform a series of experiments, based on numerical analyses, semi-empirical formulas, or experimental testing, for a prescribed set of design points, and to construct a global approximation of the measured quantity over the design space. In this effort, numerical analysis is based on the aerodynamic design software, *Meanline Flow Path Generator*, for rapid analyses of turbine flow fields. Using overall turbine and stage input, the *Meanline* code first generates a candidate turbine flow path and displays a plot of the elevation view. The code, then, runs a meanline analysis, calculating gas conditions, velocity triangles, and required number of airfoils, predicted efficiency and power output. A calculation to predict turbine weight is also included. The run time required for analysis of a three-stage turbine on the current version of the code is less than one second on a Pentium II PC. The sources of performance losses due to airfoil profile, secondary endwall, trailing edge blockage, trailing edge shock, leading edge shock and unshrouded blade tip leakage are also included.

Second-order polynomials are used for the response surface approximations for which extremal points are easily found by standard constrained optimization algorithms. The main advantages of RSM over other optimization tools such as gradient-based search algorithms are that it requires minimal interfacing with the analysis tools and avoids the need for expensive derivative calculations. To construct a second-order polynomial of N design variables, the number of coefficients to be fixed are $(N+1)(N+2)/2$.

As a first step of the overall approach of Figure 1, a generic design box obtained by coding all design variable to the range (-1,+1) is considered. Coding is based on the following formula:

$$x_i = \bar{x}_i \frac{(x_{max_i} - x_{min_i})}{2} + \frac{(x_{max_i} + x_{min_i})}{2} \quad (1)$$

where x_i: real value of the design variable
$\bar{x}_i$: normalized value of the design variable
x_{max}: maximum real value of the design variable
x_{min}: minimum real value of the design variable

The coding requires the information of the maximum and minimum values for each design variable that can be obtained from Tables 1-3.

The response surfaces of this study are generated by standard least-squares regression using JMP[2], a statistical analysis software having a variety of statistical analyses functions. The global fit and prediction accuracies of the response surfaces are assessed through statistical measures such as the t-statistic, or t-ratio, *rms-error*, variation[1,2,3]. The t-statistic is determined by

$$t = \frac{b_j}{se(b_j)} \quad (2)$$

where b_j: regression coefficient
$se(b_j)$: standard error of the regression coefficient and it is given by

$$se(b_j) = \hat{s}\sqrt{C_{jj}} \quad (3)$$

where C_{jj}: diagonal element of $(X'X)^{-1}$ corresponding to b_j (X is an nxp matrix of the levels of the independent variables where n is the number of observations and p is the number of terms in the model)
$\hat{s}$: unbiased estimator of the standard deviation of the observations and unbiased estimator of the *rms-error* in prediction based on the response surface and it is given by

$$\hat{s} = \sqrt{\frac{\sum e_i^2}{n-p}} \quad (4)$$

where e_i : difference between the observation, y_i, and the fitted value, $\hat{y}_i$.

The R^2 value is determined by

$$R^2 = \frac{SS_R}{SS_{yy}} = 1 - \frac{SS_E}{SS_{yy}} \quad (5)$$

where SS_E : sum of squares of the residuals or errors
SS_R: sum of squares due to regression
SS_{yy}: total sum of squares about the mean

$$SS_{yy} = SS_R + SS_E \quad (6)$$

R^2 measures the proportion of the variation in the response around the mean that can be attributed to terms in the model rather than to random error[3]. R_a^2 is an R^2 value adjusted to account for the degrees of freedom in the model and is given by

$$R_a^2 = 1 - \frac{SS_E/(n-p)}{SS_{yy}/(n-1)} = 1 - \left(\frac{n-1}{n-p}\right)(1-R^2) \quad (7)$$

Since R^2 will always increase as terms are added to the model, the overall assessment of the model may be better judged from R_a^2.

The polynomial-based RSM techniques are effective in representing the global characteristics of the design space. It can filter the noises associated with individual design data. On the other hand, depending on the order of polynomial employed and the shape of the actual response surface, the RSM can introduce a substantial error in certain region of the design space. An optimization scheme requiring large amounts of data and a large evaluation time to generate meaningful results is hardly useful.

The optimization technique follows qualitatively that adopted previously for optimizing fluid machinery such as diffuser, injector, and airfoil, as presented in Refs.[6-9]. The effect of numerical noise and the interaction between CFD models and RSM are addressed by Madsen et al[6]. In the paper by Tucker et al.[7], a first effort is made to apply RSM for injector optimization. Papila et al.[8] investigated the effect of data size and relative merits between RSM and neural networks in handling varying data characteristics. The neural network technique and the RSM are integrated to offer enhanced optimization capabilities by Shyy et al.[9]. A main focus in the present work is the interplay between the number of design variables and the predictive capability and input requirement of the RSM. To ascertain required predictive capability of the RSM, a two-level domain refinement approach has been adopted. As will be demonstrated in the following, the accuracy of the predicted optimal design points based on this strategy is satisfactory.

3. DESIGN OF EXPERIMENTS (DOE)

The response surface method is a collection of statistical and mathematical techniques useful for developing, improving, and optimizing processes and this provides an overall perspective of the system response within the design space[1]. The representation of the design space is, therefore, important. In order to help to minimize the effect of noise on the fitted polynomial, and to improve the representation of the design space, *design of experiments* (DOE) procedure can be used. There are a number of different DOE techniques reported in the literature[10-17]. For example, Unal et al[10] discussed the face centered composite designs and D-optimal designs for representation of the design space for wing-body configuration of a launch vehicle. They showed that D-optimal design provides an efficient approach for approximating model building and multidisciplinary optimization. Unal et al.[11] studied response surface model building using orthogonal arrays in computer experiments for reusable launch vehicle and illustrated that using this technique minimizes design, development, test and evaluation cost. Similar results were obtained using three level fractional factorial experimental design[12]. Unal and Dean[13] studied the robust design method based on the Taguchi method[14-15] to determine the optimum configuration of design parameters for performance, quality and cost. They demonstrated that using such a robust design method for selection of design points is a systematic and efficient approach for determining the optimum configuration.

The *face centered composite design* (FCCD) creates a design space composed of eight corners of the cube, four center of faces and the center of the cube. Figure 2 shows face centered composite design points for three design variables. The FCCD yields (2^N+2N+1) points, where N is the number of design variables. It is more effective when the number of design variables is modest, say, no larger than 5 or 6. The FCCD is widely used for fitting second-order response surface[1].

A *D-Optimal design* minimizes the generalized variance, [illegible] of the estimates which is equivalent to maximizing the determinant of the moment matrix, M[1].

$$|\mathbf{M}| = \frac{|\mathbf{X}'\mathbf{X}|}{N^p} \qquad (8)$$

The D-Optimal design approach requires the knowledge of the properties of polynomial model in selecting the design points.

An orthogonal array (OA) is a fractional factorial matrix that assures a balanced comparison of levels of any factor or interaction of factors. Because the points are not necessarily at vertices, the orthogonal array can be more robust than the face centered cubic design. Based on the design of experiments theory, OA can significantly reduces the number of experimental configurations.

In this study, although the majority of the work is based on the FCCD approach, alternative representations of the design space are performed by using D-Optimal design and OA design. We have considered 1-, 2- and 3-stage turbine. There are 6 design parameters for single- stage turbine case chosen as the mean diameter, *RPM*, blade annulus area, vane axial chord, blade axial chord, and stage reaction. For 2-stage turbine, mean diameter, *RPM*, exit blade annulus area, 1st blade height (% of exit blade), 1st vane axial chord, 1st blade axial chord, 2nd vane axial chord, 2nd blade axial chord, 1st stage reaction, 2nd stage reaction, and 1st stage work fraction, are chosen and there are, in total, 11 design parameters. There are 15 design parameters for 3-stage case determined as mean diameter, *RPM*, exit blade annulus area, 1st blade height (% of exit blade), 1st vane axial chord, 1st blade axial chord, 2nd vane axial chord, 2nd blade axial chord, 3rd vane axial chord, 3rd blade axial chord, 1st stage reaction, 2nd stage reaction, 3rd stage reaction, 1st stage work fraction, 2nd stage work fraction, and 3rd stage work fraction. Table 1-3 show the maximum and minimum values of these parameters as well as their baseline values.

With 6-input parameters of single-stage turbine, FCCD produces 77-data, but *Meanline* code produced results for 76-data excluding one unrealistic case. Therefore, 76-data is used to approximate the single-stage turbine characteristics. With 11-input parameters of 2-stage turbine, FCCD yields 2,071-data, but *Meanline* code produced results for 1990-data and this set is used to approximate the 2-stage turbine characteristics. For 3-stage having 15-input parameters, FCCD creates 32,799-data based on the formula of 2^N+2N+1 with $N=15$ demonstrating the *curse of dimensionality*. For such cases, a statistical method can be applied to reduce the number of data in an efficient way. In this effort, to reduce the data set of 3-stageturbine of 15-dimension, D-Optimal design is adopted to minimize the generalized variance of the estimates. With the D-optimal criterion, the number of data is reduced to 2500; the *Meanline* code worked for 2235 of them. In this paper, the orthogonal arrays are also applied for the 2-stage turbine case using a public domain software developed by Owen[18].

4. THE OPTIMIZATION PROBLEM

The equation describing the response as given by *JMP* is input to *Excel Solver*. *Solver* is an optimization toolbox included with Microsoft Excel which uses the *Generalized Reduced Gradient method*[5] to find the maximum or minimum of a function with given constraints.

The optimization problem at hand is a constrained optimization problem, which can be formulated as $min\{f(x)\}$ subject to $lb \le x \le ub$, where lb is the lower boundary vector and ub is the upper boundary vector of the design variables vector $\boldsymbol{x}$. Since the goal is to maximize objective function therefore $f(\boldsymbol{x})$ can be written as $-g(\boldsymbol{x})$, where $g(\boldsymbol{x})$ is the objective function. Minimizing of $f(x)$ gives the same solution as maximizing the objective function $g(x)$. Additional linear or nonlinear constraints can be incorporated if required.

In this study, the purpose is maximizing the turbine efficiency, η, and minimizing the overall weight, W, simultaneously. The response surface method can handle a multi-criteria optimization task in a straightforward manner by building a *composite response surface* from individual response surfaces. This composite response surface is referred to as the *desirability function*. The desirability function for η, i.e., d_1, can be defined which is to be maximized as

$$d_1 = \left(\frac{\eta - \eta_{min}}{\eta_{max} - \eta_{min}} \right)^s \qquad (9)$$

and the desirability function for W, i.e., d_2, can be defined which is to be minimized as

$$d_2 = \left(\frac{W - W_{max}}{W_{min} - W_{max}} \right)^t \qquad (10)$$

where powers s and t are weighting factors which are set according to the role of the response in composite desirability function, i.e., d, defined as follows:

$$d = \sqrt{d_1 \cdot d_2} \qquad (11)$$

Another way of finding optimum values of η and W simultaneously is to maximize payload increment, Δpay, which is a function of these two parameters in the following manner.

$$\Delta pay = c_1 \times 100 \times (\eta - \eta_b) - (W - W_b) \qquad (12)$$

where η_b is the baseline efficiency and W_b is the baseline weight. Δpay function represents the amount of increase in payload capacity. The results of both payload increment based and composite desirability function based optimization are illustrated for 1, 2, and 3-stage designs. For the composite desirability function based optimization, different combinations of the power of d_1 and d_2 are considered with different values of t and s.

The pitchline speed, V_{pitch}, and the lumped inertia measure, AN^2, are used as the design constraints when finding the optimum solutions.

$$V_{pitch} = D \times RPM \qquad (13)$$

$$AN^2 = Aann \times RPM^2 \qquad (14)$$

where D is meanline diameter.

5. RESULTS AND DISCUSSION

The properties of the response surfaces obtained for η, W, and Δpay are shown in Tables 4 and 5 for 1-,2-, and 3-stage turbine designs.

For the single-stage case, there are 28-unknown coefficients needed for constructing the 2nd-order response surface, 78 for the 2-stage and 136 for the 3-stage case. The quality of the fit can be evaluated by comparing the adjusted root mean square error *(rms-error)* shown in Table 5.

RSM-based approximations together with the *Excel Solver* to find the maximum or minimum of the objective function with a given constraints is used to find the optimum point are obtained for all cases. Different starting points are tried to avoid local maximum and the optimum values of η, W and Δpay with the corresponding design parameters are determined. Table 6 shows the optimum values of η, W and Δpay calculated for both Δpay (Eqn. 12) based optimization and composite desirability function based optimization (Eqn.11) for $(t=1, s=0)$, $(t=0, s=1)$, and $(t=1, s=1)$ cases for single-stage turbine. The case $(t=1, s=0)$ represents the optimization based on weight only, whereas $(t=0, s=1)$ represents the optimization based on efficiency only. The results shown in this table are comparable with the corresponding *Meanline* runs with the highest error of 5% for Δpay for single-stage turbine. Table 7 shows the optimum values of η, W and Δpay calculated for both Δpay based optimization and composite desirability function based optimization for $(t=1, s=0)$, $(t=0, s=1)$, and $(t=1, s=1)$ cases for the 2-stage turbine design.. When the results shown in this table are compared with the corresponding *Meanline* runs of the same design parameters, it is observed that the percentage error is increased up to 13.5% for Δpay. Table 8 shows the optimum values of η, W and Δpay calculated for the 3-stage turbine with the same approach. When the results shown in this table are compared with the corresponding *Meanline* runs, it is observed that the percentage error is increased up to 14.6 % for Δpay indicating that the accuracy of the response surfaces constructed for this case is poor.

Because the accuracy of the response surface is less than satisfactory for 2- and 3-stage cases, we have reduced the size of the parameter space with the intention of improving the fidelity of the response surface. The details of the 1/5 reduced design spaces are shown in Table 9 - 12. The new design space is based on the optimal values identified for η & W ($(t=1, s=0)$, $(t=0, s=1)$ & $(t=1, s=1)$) and Δpay based optimization cases. With these refined designed spaces, substantial improvement of the response surface fit accuracy is observed for 2-stage and also for 3-stage (Tables 13 and 14).

Based on the results obtained, the following observations can be made:

(i) To ascertain required predictive capability of the RSM, a two-level domain refinement strategy has been adopted. The accuracy of the predicted optimal design points based on this approach is shown to be satisfactory.

(ii) For Δpay-based optimization, the 2-stage turbine gives the best Δpay result. As the number of the stage increases, we see that efficiency improves while the weight increases also. According to the formula for Δpay, the improvement in efficiency can't compensate the penalty from higher weight. As shown in Figure 3, the mean diameter, speed, and the exit blade area exhibit distinct trends. Specifically, the diameter decreases, speed increases, and annulus area decreases with increasing number of stages. It is interesting to observe that none of these design parameters are toward the limiting values listed in Tables 1-3, indicating that the optimal designs result from compromises between competing parametric trends. For such cases, a formal optimizer such as the present response surface method is very useful.

(iii) For both weight only optimization *(t=1, s=0)* and efficiency only optimization *(t=0, s=1)*, as expected, the single-stage design gives the smallest weight. The efficiency also improves as the number of stages increases (see Tables 15 and 16, and Figure 4 for summaries). It is interesting to see that in both cases, the selections of the diameter, speed and annulus area are insensitive to the number of stages. For example, for weight only optimization *(t=1, s=0)*, *D* approaches minimum, and *RPM* approaches maximum, while the annulus area is governed by the design constraint between it and *RPM*.

(iv) On the other hand, for the efficiency only optimization *(t=0, s=1)*, for all three stage designs, the annulus area approaches maximum and *RPM* is governed by the design constraint. The diameter, *D*, on the other hand, takes middle values within the design range (Figure 5).

(v) Similar to the one based on *Δpay*, the case of *(t=1, s=1)* is also a compromise between weight and efficiency. However, because of the different mathematical formulas adopted, different design selections than the ones based on *Δpay* result. It seems clear that the precise definition of the optimal criterion substantially influence the selection of the optimal design.

In order to determine how the optimum solution for *Δpay* changes as the weighting constants, *t* and *s*, are changing, three designs of *(t=1, s=0)*, *(t=0, s=1)* and *(t=1, s=1)* denoted as Ω_1, Ω_2 and Ω_3, respectively, are selected as candidate vertex points to define a plane referred as α-Plane as follows.

$$\Omega = \alpha_1\Omega_1 + \alpha_2\Omega_2 + \alpha_3\Omega_3 \qquad (15)$$

where $\sum_i \alpha_i = 1$ and $0 \le \alpha_i \le 1$ for i=1, 2, 3

Equation 12 was used to obtain 66 design points distributed on the alpha plane for 2-stage turbine. Figure 6 illustrates the contour plots over the *α-Plane* for optimum *Δpay*. This figure shows that *Δpay* reaches its maximum value for a range of t, $0<t<1$ for $s=1$.

6. ORTHOGONAL ARRAYS FOR 2-STAGE TURBINE DESIGN

Although the majority of the present work is based on the FCCD approach, orthogonal arrays are constructed to investigate the efficiency of orthogonal array designs in representing the design space for 2-stage turbine. For this purpose, 249 design points are selected using OA designs. Table 17 shows the comparison of the statistics of the second-order response surfaces generated for η, W and *Δpay* by using 1990-data generated by face centered composite design and 249-data selected by OA method. This table illustrates that the fidelity of the response surface generated for design space of 249 data, based on orthogonal arrays, are comparable with that of 1990 data based on the face centered criterion. The response surface models are also assessed by using 78-test data to determine the predictive accuracy of these models. Table 18 presents that the testing *rms-errors* of response surfaces generated are 1.65% for η and 0.96% for W using 249-data, and 1.67% for η and 1.21% for W using 1990-data. The results of optimization based on Δpay and composite desirability function of η & W with 249-data selected by orthogonal arrays are shown in Table 19 for the original design space and in Table 20 for the refined design space. When these results are compared with the results of 1990-data presented in Tables 7 and 13, it is observed that the optimum η, W and *Δpay* are largely consistent. However, it is also observed from Figure 7 which shows the comparison of the design variables for optimization based on (*Δpay*), some of the design variables are different even though optimum η, W and *Δpay* are consistent. This shows that there are multiple points in the design space which yield comparable performance. Nevertheless, it remains true that the two-stage turbine is most suitable from a payload point of view.

7. CONCLUDING REMARKS

In summary, the result of the RSM computation indicates that indeed the efficiency rises quickly from 1 stage to 2 stages but the increase is much less pronounced with 3 stages. A 1-stage turbine performs poorly under the engine balance boundary condition. A significant portion of kinetic energy is lost at the turbine discharge of the 1-stage design due to high pressure ratio and high energy content, mostly hydrogen, of the working fluid. Adding a 2^{nd} stage recovers most of that wasted energy resulting in much better efficiency for a 2-stage turbine. An extra 3^{rd} stage only improves the efficiency slightly. Understandably, the turbopump weight also increases substantially from 1 to 3 stages even though the 3-stage turbine diameter is smaller. The smaller diameter is the direct outcome of higher *RPM* that is the result of lower exit annulus area satisfying the constraint defined for AN^2. The exit annulus area is smaller because of the lower pressure ratio per stage. However, the 3-stage turbine is much longer and requires a change in bearing configuration that adds significantly into the overall weight. The optimum 2-stage turbine resulting from the RSM optimization is consistent with a design produced by an experienced engineer; namely, that most of the work is done by the 1^{st} stage at very low reaction. By varying from 1- to 2-to 3-stages, we observe that while the size of the training data increase naturally with the number of design variables, the actual need is case dependent. Furthermore, it seems that the selection of the data distribution can be more critical than the data size. Present investigation has also demonstrated that the criteria for selecting the data base exhibit significant impact on the efficiency and effectiveness of the construction of the response surface.

8. ACKNOWLEDGEMENT

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Table 1. Design Space for Single-Stage Turbine (All geometric design variables are normalized by the baseline values)

Variable	Lower Limit	Upper Limit
Mean Diameter, D	0.50	1.50
Speed, *RPM*	0.70	1.30
Blade Annulus Area, A_{ann}	0.70	1.30
Vane Axial Chord, c_v	0.39	1.71
Blade Axial Chord, c_b	0.26	1.14
Stage Reaction, sr	0.0%	50%

Table 2. Design Space for 2-Stage Turbine (All geometric design variables are normalized by the baseline values)

Variable	Lower Limit	Upper Limit
Mean Diameter, D	0.50	1.50
Speed, *RPM*	0.70	1.30
Blade Annulus Area, A_{ann}	0.70	1.30
1st Blade Height (% of Exit Blade), h_1	0.90	1.50
1st Vane Axial Chord, c_{v1}	0.39	1.71
1st Blade Axial Chord, c_{b1}	0.26	1.14
2nd Vane Axial Chord, c_{v2}	0.21	1.41
2nd Blade Axial Chord, c_{b2}	0.17	1.13
1st Stage Reaction, sr_1	0.0%	50%
2nd Stage Reaction, sr_2	0.0%	50%
1st Work Fraction, wf_1	50%	85%

Table 3. Design Space for 3-Stage Turbine (All geometric design variables are normalized by the baseline values)

Variable	Lower Limit	Upper Limit
Mean Diameter, D	0.50	1.50
Speed, *RPM*	0.70	1.30
Blade Annulus Area, A_{ann}	0.70	1.30
1st Blade Height (% of Exit Blade) , h_1	0.90	1.50
1st Vane Axial Chord, c_{v1}	0.39	1.71
1st Blade Axial Chord, c_{b1}	0.26	1.14
2nd Vane Axial Chord, c_{v2}	0.21	1.41
2nd Blade Axial Chord, c_{b2}	0.17	1.13
3rd Vane Axial Chord, c_{v3}	0.21	1.41
3rd Blade Axial Chord, c_{b3}	0.17	1.13
1st Stage Reaction, sr_1	0.0%	50%
2nd Stage Reaction, sr_2	0.0%	50%
3rd Stage Reaction, sr_3	0.0%	50%
1st Work Fraction, wf_1	40%	80%
2nd Work Fraction, wf_2	30%	10%

Table 4. Response Surface Summary for 1, 2 and 3-Stage Turbine

	No. of Design Parameters	No. of Coefficients	No. of Design Points
1-Stage	6	28	76
2-Stage	11	78	1990
3-Stage	15	136	2235

Table 5. The quality of the Second-Order Response Surface obtained for η, W, and Δpay 1, 2 and 3-Stage Turbine (Mean values of η, W and Δpay are normalized by the baseline values)

		η	W	Δpay
1-Stage	R^2	0.998	0.999	0.998
	Ra^2	0.997	0.999	0.997
	rms- error	2.50%	0.82%	4.09%
	Mean	0.57	0.60	-0.43
2-Stage	R^2	0.995	0.996	0.995
	Ra^2	0.994	0.996	0.995
	rms- error	1.31%	2.56%	9.58%
	Mean	0.78	0.86	-0.24
3-Stage	R^2	0.989	0.989	0.994
	Ra^2	0.988	0.988	0.994
	rms- error	2.0%	2.9%	8.4%
	Mean	0.89	1.41	-0.26

Table 6. Optimization based on *Δpay* and composite desirability function of η and W for single-stage turbine (All geometric design variables and output parameters are normalized by the baseline values)

	η_{opt}	W_{opt}	Δpay_{opt}	D	*RPM*	A_{ann}	c_v	c_b	sr
RSM(*Δpay*)	0.766	0.731	-0.214	1.181	0.975	1.166	1.706	0.880	0
Meanline	0.797	0.733	-0.193						
Error % of mean	2.9	0.3	4.8						
RSM(*t=1, s=0*)	0.399	0.407	-0.611	0.502	1.284	0.699	0.394	0.264	0.5
Meanline	0.383	0.402	-0.623						
Error % of mean	1.8	1	2.7						
RSM(*t=0, s=1*)	0.781	0.762	-0.216	1.260	0.915	1.300	1.575	0.880	0
Meanline	0.797	0.762	-0.199						
Error % of mean	2.2	0.03	3.8						
RSM(*t=1, s=1*)	0.702	0.583	-0.261	0.895	1.284	0.699	1.706	0.264	0
Meanline	0.718	0.588	-0.239						
Error % of mean	3.1	0.8	5						

Table 7. Optimization based on *Δpay* and composite desirability function of η and W for 2-stage turbine for original design space (All geometric design variables and output parameters are normalized by the baseline values)

	η_{opt}	W_{opt}	Δpay_{opt}	D	*RPM*	A_{ann}	h_1	c_{v1}	c_{v2}	c_{b1}	c_{b2}	sr_1	sr_2	w_{f1}
RSM(*Δpay*)	1.10	1.05	0.11	1.16	0.99	1.14	1.50	1.57	0.97	0.71	0.68	0	0.50	0.85
Meanline	1.12.	1.05	0.14											
Error %of mean	2.50	0.00	10.60											
RSM(*t=1, s=0*)	0.65	0.66	-0.34	0.50	1.27	0.70	1.50	1.71	1.76	0.92	1.13	0.50	0.50	0.50
Meanline	0.65	0.65	-0.35											
Error %of mean	1.10	0.60	4.00											
RSM(*t=0, s=1*)	1.10	1.10	0.11	1.24	0.92	1.30	1.50	1.44	1.06	0.71	0.79	0	0	0.80
Meanline	1.13	1.10	0.14											
Error %of mean	3.30	0.41	13.50											
RSM(*t=1, s=1*)	0.99	0.85	0.03	0.91	1.27	0.70	1.50	1.71	1.76	0.85	1.13	0	0	0.50
Meanline	0.99	0.84	0.03											
Error %of mean	0.30	1.70	0.40											

Table 8. Optimization based on *Δpay* and composite desirability function of η and W for 3-stage turbine for original design space (All geometric design variables and output parameters are normalized by the baseline values)

	η_{opt}	W_{opt}	Δpay_{opt}
RSM (*Δpay*)	1.24	1.62	0.14
Meanline	1.21	1.57	0.10
Error %of mean	4.72	3.52	14.59
RSM (*t=1, s=0*)	0.85	1.13	-0.23
Meanline	0.83	1.13	-0.26
Error %of mean	2.13	0.34	14.39
RSM (*t=0, s=1*)	1.26	1.74	0.12
Meanline	1.23	1.69	0.09
Error %of mean	4.27	3.10	13.54
RSM (*t=1, s=1*)	1.08	1.33	0.05
Meanline	1.10	1.34	0.04
Error %of mean	2.16	0.59	4.11

	D	*RPM*	A_{ann}	h_1	c_{v1}	c_{v2}	c_{v3}	c_{b1}	c_{b2}	c_{b3}	sr_1	sr_2	sr_3	wf_1	wf_2
RSM (*Δpay*)	1.07	1.07	0.98	1.50	1.59	1.09	0.87	0.56	1.02	0.89	0	0.5	0.5	0.6	0.3
RSM (*t=1, s=0*)	0.50	1.28	0.70	0.90	1.71	1.76	1.41	0.73	1.41	0.17	0.5	0.5	0.5	0.4	0.3
RSM (*t=0, s=1*)	1.19	0.96	1.20	1.50	1.44	1.06	0.78	0.56	0.99	0.90	0	0.5	0.5	0.6	0.3
RSM (*t=1, s=1*)	0.91	1.29	0.70	1.50	1.71	0.62	0.78	0.17	0.21	1.13	0	0	0	0.6	0.1

Table 9. Upper and Lower Limits of the Design Parameters of the refined designed spaces for 2-stage turbine
(All geometric design variables are normalized by the baseline values)

		D	*RPM*	A_{ann}	h_1	c_{v1}	c_{v2}	c_{b1}	c_{b2}	sr_1	sr_2	w_{fl}
Original	Max	1.50	1.30	1.30	1.50	1.71	1.76	0.92	1.13	0.50	0.50	0.85
	Min	0.50	0.70	0.70	0.90	0.39	0.26	0.21	0.17	0.00	0.00	0.50
Refined	Max	1.26	1.05	1.20	1.50	1.71	1.11	0.79	0.79	0.05	0.50	0.85
(Δpay)	Min	1.06	0.93	1.08	1.44	1.44	0.81	0.64	0.60	0.00	0.45	0.82
Refined	Max	0.60	1.30	0.76	1.50	1.71	1.76	0.92	1.13	0.50	0.50	0.54
(*t=1, s=0*)	Min	0.50	1.21	0.70	1.44	1.57	1.61	0.85	1.03	0.45	0.45	0.50
Refined	Max	1.34	0.98	1.30	1.50	1.63	1.20	0.76	0.90	0.05	0.05	0.82
(*t=0, s=1*)	Min	1.14	0.86	1.24	1.44	1.36	0.91	0.62	0.70	0.00	0.00	0.75
Refined	Max	1.00	1.30	0.76	1.50	1.71	1.76	0.92	1.13	0.05	0.05	0.54
(*t=1, s=1*)	Min	0.80	1.21	0.70	1.44	1.57	1.61	0.80	1.03	0.00	0.00	0.50

Table 10. Center Coordinates of the refined designed spaces for 2-stage turbine
(All geometric design variables are normalized by the baseline values)

	D	*RPM*	A_{ann}	h_1	c_{v1}	c_{v2}	c_{b1}	c_{b2}	sr_1	sr_2	w_{fl}
Original	0	0	0	0	0	0	0	0	0	0	0
Refined (*Δpay*)	1.16	0.99	1.14	1.50	1.57	0.97	0.71	0.68	0	0.5	0.85
Refined (*t=1, s=0*)	0.50	1.27	0.70	1.50	1.71	1.76	0.92	1.13	0.5	0.5	0.5
Refined (*t=0, s=1*)	1.24	0.92	1.30	1.50	1.50	1.06	0.69	0.80	0.0	0.0	0.78
Refined (*t=1, s=1*)	0.91	1.27	0.70	1.50	1.71	1.76	0.85	1.13	0	0	0.50

Table 11. Upper and Lower Limits of the Design Parameters of the refined designed spaces for 3-stage turbine
(All geometric design variables are normalized by the baseline values)

		D	*RPM*	A_{ann}	h_1	c_{v1}	c_{v2}	c_{v3}	c_{b1}	c_{b2}	c_{b3}	sr_1	sr_2	sr_3	wf_1	wf_2
Original	Max	1.50	1.30	1.30	1.50	1.71	1.76	1.41	0.73	1.41	1.13	0.5	0.5	0.5	0.8	0.3
	Min	0.50	0.70	0.70	0.90	0.39	0.26	0.21	0.17	0.21	0.17	0.0	0.0	0.0	0.4	0.1
Refined	Max	1.17	1.13	1.05	1.50	1.71	1.24	0.99	0.62	1.14	0.99	0.05	0.50	0.50	0.62	0.30
(*Δpay*)	Min	0.97	1.01	0.92	1.44	1.46	0.94	0.75	0.51	0.90	0.79	0.00	0.45	0.45	0.56	0.26
Refined	Max	0.60	1.30	0.76	0.96	1.71	1.76	1.41	0.73	1.41	0.26	0.50	0.50	0.50	0.44	0.30
(*t=1, s=0*)	Min	0.50	1.22	0.70	0.90	1.57	1.61	1.29	0.68	1.29	0.17	0.45	0.45	0.45	0.40	0.28
Refined	Max	1.29	1.02	1.26	1.50	1.60	1.18	0.93	0.60	1.10	0.99	0.05	0.50	0.50	0.62	0.30
(*t=0, s=1*)	Min	1.09	0.90	1.14	1.44	1.34	0.88	0.69	0.49	0.86	0.80	0.00	0.45	0.45	0.56	0.26
Refined	Max	1.00	1.30	0.76	1.50	1.71	0.74	0.88	0.23	0.33	1.13	0.05	0.05	0.05	0.64	0.14
(*t=1, s=1*)	Min	0.80	1.23	0.70	1.44	1.57	0.44	0.64	0.17	0.21	1.03	0.00	0.00	0.00	0.58	0.10

Table 12. Center Coordinates of the refined designed spaces for 3-stage turbine (All geometric design variables are normalized by the baseline values)

	D	*RPM*	A_{ann}	h_1	c_{v1}	c_{v2}	c_{v3}	c_{b1}	c_{b2}	c_{b3}	sr_1	sr_2	sr_3	wf_1	wf_2
Original	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Refined (Δpay)	1.07	1.07	0.98	1.50	1.57	1.06	0.85	0.56	0.99	0.90	0.0	0.5	0.5	0.6	0.3
Refined ($t=1, s=0$)	0.50	1.28	0.70	0.90	1.71	1.76	1.41	0.73	1.41	0.17	0.5	0.5	0.5	0.4	0.3
Refined ($t=0, s=1$)	1.19	0.96	1.20	1.50	1.44	1.06	0.78	0.56	0.99	0.90	0.0	0.5	0.5	0.6	0.3
Refined($t=1, s=1$)	1.00	1.00	1.00	1.20	1.05	1.06	0.85	0.45	0.85	0.68	0.3	0.3	0.3	0.6	0.2

Table 13. Optimization based on Δpay and composite desirability function of η and W for 2-stage turbine for refined design space (All geometric design variables and output parameters are normalized by the baseline values)

	η_{opt}	W_{opt}	Δpay_{opt}	D	*RPM*	A_{ann}	h_1	c_{v1}	c_{v2}	c_{b1}	c_{b2}	sr_1	sr_2	w_{f1}
RSM (Δpay)	1.13	1.04	0.15	1.12	1.02	1.08	1.50	1.44	0.79	0.71	0.62	0.1	0.5	0.9
Meanline	1.13	1.04	0.15											
Error % of mean	0.03	0.02	0.16											
RSM($t=1, s=0$)	0.65	0.65	-0.35	0.50	1.27	0.70	1.50	1.71	1.76	0.92	1.13	0.5	0.5	0.5
Meanline	0.65	0.65	-0.35											
Error %of mean	0.00	0.00	0.01											
RSM($t=0, s=1$)	1.15	1.10	0.15	1.23	0.93	1.30	1.50	1.31	0.88	0.71	0.73	0.1	0	0.8
Meanline	1.15	1.10	0.15											
Error %of mean	0.02	0.01	0.09											
RSM($t=1, s=1$)	1.00	0.85	0.04	0.91	1.27	0.70	1.50	1.71	1.58	0.92	1.01	0	0	0.5
Meanline	1.00	0.85	0.04											
Error %of mean	0.08	0.04	0.30											

Table 14. Optimization based on Δpay and composite desirability function of η and W for 3-stage turbine for refined design space (All geometric design variables and output parameters are normalized by the baseline values)

	η_{opt}	W_{opt}	Δpay_{opt}
RSM (Δpay)	1.20	1.54	0.11
Meanline	1.21	1.54	0.11
Error %of mean	0.22	0.13	0.41
RSM ($t=1, s=0$)	0.82	1.13	-0.27
Meanline	0.82	1.13	-0.27
Error %of mean	0.04	0.02	0.31
RSM ($t=0, s=1$)	1.24	1.75	0.09
Meanline	1.23	1.72	0.09
Error %of mean	1.39	1.29	1.35

	D	*RPM*	A_{ann}	h_1	c_{v1}	c_{v2}	c_{v3}	c_{b1}	c_{b2}	c_{b3}	sr_1	sr_2	sr_3	wf_1	wf_2
RSM (Δpay)	1.03	1.11	0.92	1.50	1.46	0.94	0.75	0.51	0.90	0.79	0.04	0.5	0.5	0.62	0.28
RSM($t=1, s=0$)	0.50	1.27	0.70	0.96	1.71	1.76	1.41	0.73	1.41	0.17	0.5	0.5	0.5	0.4	0.3
RSM($t=0, s=1$)	1.20	0.94	1.26	1.47	1.35	0.88	0.69	0.54	0.86	0.80	0	0.45	0.45	0.6	0.3

Table 15. Optimization summary for 1, 2 and 3-stage turbine with response surface in original design space (All output parameters are normalized by the baseline values)

		η_{opt}	W_{opt}	Δpay_{opt}
	1-stage	0.77	0.73	-0.21
Δpay	2-stage	1.10	1.05	0.11
	3-stage	1.24	1.62	0.14
	1-stage	0.40	0.41	-0.61
(t=1, s=0)	2-stage	0.65	0.66	-0.34
	3-stage	0.85	1.13	-0.23
	1-stage	0.78	0.76	-0.22
(t=0, s=1)	2-stage	1.10	1.10	0.11
	3-stage	1.26	1.74	0.12
	1-stage	0.70	0.58	-0.26
(t=1, s=1)	2-stage	0.99	0.85	0.03
	3-stage	1.08	1.33	0.05

Table 16. Optimization summary for 1, 2 and 3-stage turbine with response surface in refined design space (All output parameters are normalized by the baseline values)

		η_{opt}	W_{opt}	Δpay_{opt}
	1-stage	0.77	0.73	-0.21
Δpay	2-stage	1.13	1.04	0.15
	3-stage	1.20	1.54	0.11
	1-stage	0.40	0.41	-0.61
(t=1, s=0)	2-stage	0.65	0.65	-0.35
	3-stage	0.82	1.13	-0.27
	1-stage	0.78	0.76	-0.22
(t=0, s=1)	2-stage	1.15	1.10	0.15
	3-stage	1.24	1.75	0.09
	1-stage	0.70	0.58	-0.26
(t=1, s=1)	2-stage	1.00	0.85	0.04

Table 17. The quality of the Second-Order Response Surface obtained for η, W and Δpay of 2-Stage Turbine for 1990-data (FCCD criterion) and 249-data (OA criterion)
(Mean values of η, W and Δpay are normalized by the baseline values)

		η	W	Δpay
	R^2	0.995	0.996	0.995
	Ra^2	0.994	0.996	0.995
1990-data	*rms- error*	1.31%	2.56%	9.58%
	Mean	0.78	0.86	-0.24
	R^2	0.995	0.998	0.994
	Ra^2	0.992	0.997	0.992
249-data	*rms- error*	2.128%	0.826%)	20.68%
	Mean	0.89	0.92	-0.11

Table 18. Testing of the Second-Order Response Surface obtained for η and W of 2-Stage Turbine for 1990-data (FCCD criterion) and 249-data (OA criterion) with 78- test data

# of design points	# of test data	*rms-error* for η (%)	*rms-error* for W(%)
249	78	1.65	0.96
1990	78	1.67	1.21

Table 19. Optimization based on Δpay and composite desirability function of η and W for 2-stage turbine for original design space with 249-data (OA criterion). (All geometric design variables and output parameters are normalized by the baseline values)

	η_{opt}	W_{opt}	Δpay_{opt}	D	*RPM*	A_{ann}	h_1	c_{v1}	c_{v2}	c_{b1}	c_{b2}	sr_1	sr_2	w_{f1}
RSM(*Δpay*)	1.13	1.04	0.15	1.13	1.02	1.10	1.50	1.71	0.44	0.92	0.62	0.0	0.4	0.9
Meanline	1.12	1.03	0.14											
Error %of mean	1.73	1.39	5.90											
RSM(*t=1, s=0*)	0.64	0.64	-0.36	0.50	1.27	0.70	0.90	0.39	0.26	0.92	1.13	0.5	0.5	0.5
Meanline	0.62	0.64	-0.38											
Error %of mean	1.85	0.62	7.15											
RSM(*t=0, s=1*)	1.13	1.09	0.14	1.21	0.95	1.27	1.50	1.71	0.53	0.85	0.68	0.0	0.3	0.9
Meanline	1.13	1.09	0.14											
Error %of mean	0.90	1.02	2.76											
RSM(*t=1, s=1*)	0.99	0.82	0.03	0.91	1.27	0.70	0.90	0.39	1.76	0.21	1.13	0.5	0.0	0.9
Meanline	0.96	0.83	0.00											
Error %of mean	2.67	0.49	11.88											

Table 20. Optimization based on Δpay and composite desirability function of η and W for 2-stage turbine for refined design space for 249-data (OA criterion). (All geometric design variables and output parameters are normalized by the baseline values)

	η_{opt}	W_{opt}	Δpay_{opt}	D	*RPM*	A_{ann}	h_1	c_{v1}	c_{v2}	c_{b1}	c_{b2}	sr_1	sr_2	w_{f1}
RSM (*Δpay*)	1.13	1.02	0.16	1.10	1.05	1.03	1.50	1.57	0.53	0.85	0.51	0.1	0.3	0.9
Meanline	1.12	1.02	0.15											
Error % of mean	0.04	0.01	0.37											
RSM(*t=1, s=0*)	0.62	0.64	-0.38	0.50	1.27	0.70	0.90	0.39	0.26	0.92	1.13	0.5	0.5	0.5
Meanline	0.63	0.64	-0.38											
Error %of mean	0.0031	0.0002	0.0002											
RSM(*t=0, s=1*)	1.13	1.10	0.15	1.23	0.93	1.30	1.50	1.57	0.70	0.78	0.62	0.1	0.2	0.9
Meanline	1.14	1.10	0.15											
Error %of mean	0.005	0.001	0.029											
RSM(*t=1, s=1*)	0.97	0.84	0.02	0.90	1.27	0.70	0.90	0.39	1.76	0.28	1.13	0.5	0.0	0.8
Meanline	0.98	0.84	0.02											
Error %of mean	0.08	0.05	6.08											

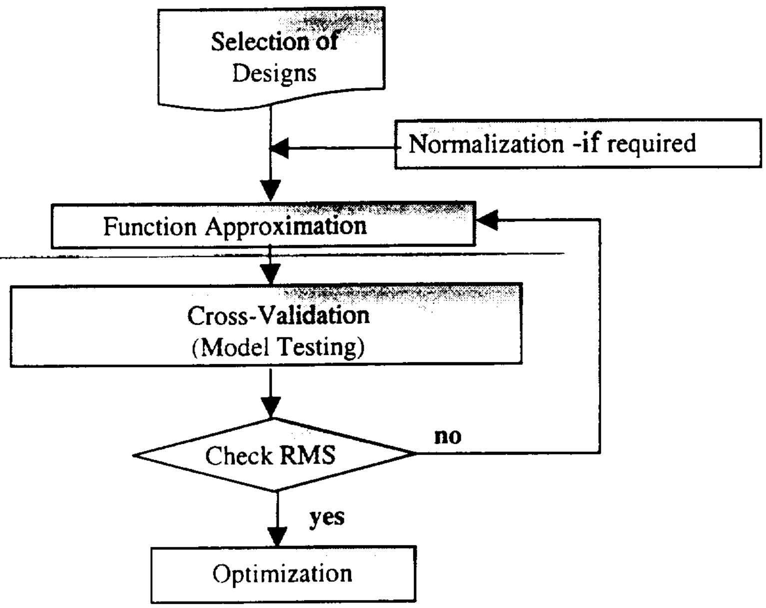


Figure 1. Function Approximation and Optimization Flow Chart

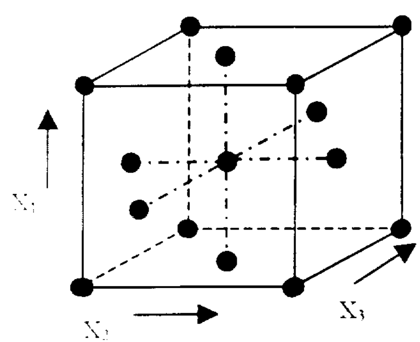


Figure 2. Face Centered Composite Designs (FCCD) for 3 Design Variables

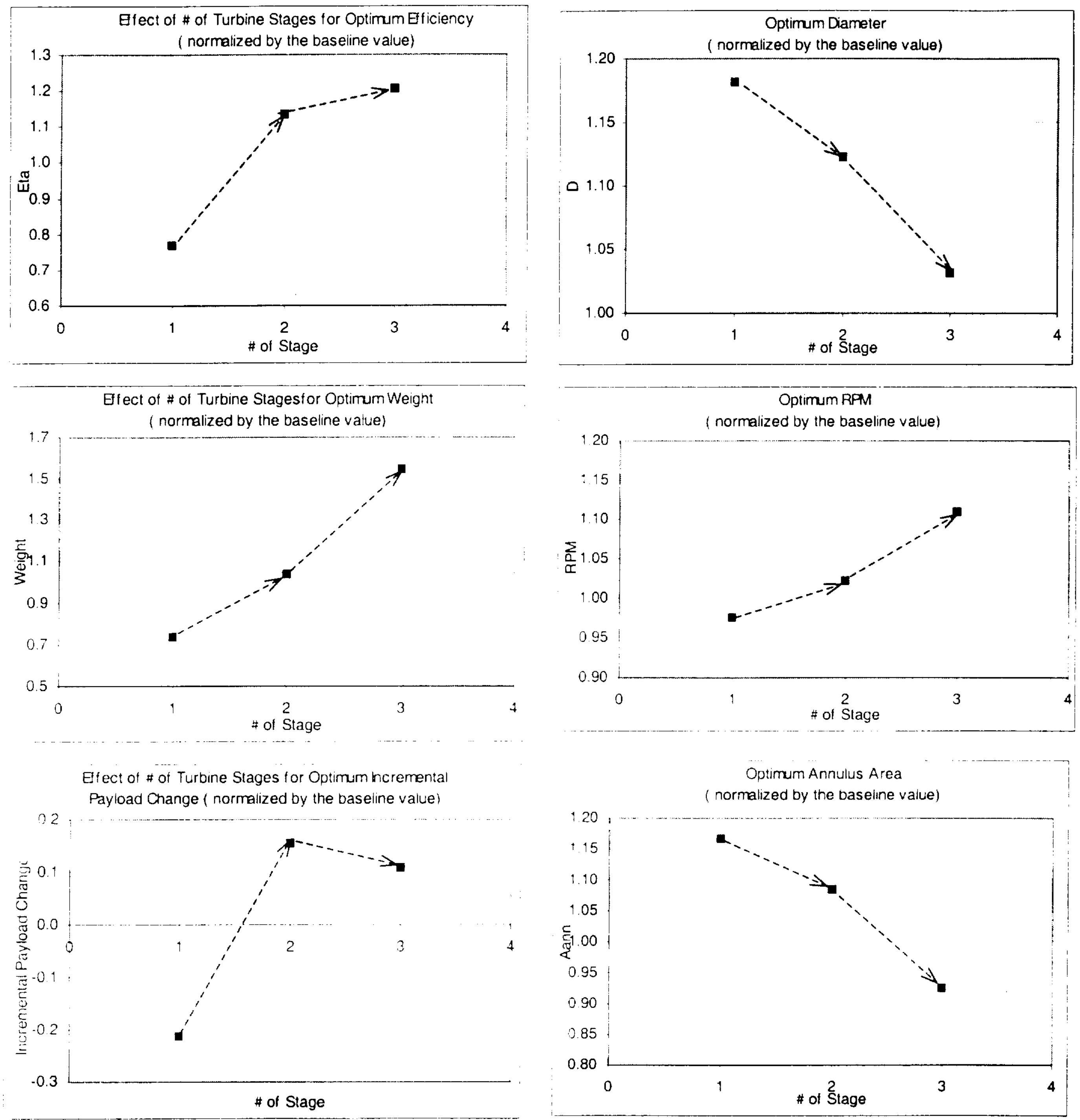


Figure 3. Effect of the number of turbine stage on optimum design parameters; D, RPM, and A_{ann} and optimum output parameters; η, W, and Δpay calculated for Δpay-based optimization
(All geometric design variables and output parameters are normalized by the baseline values)

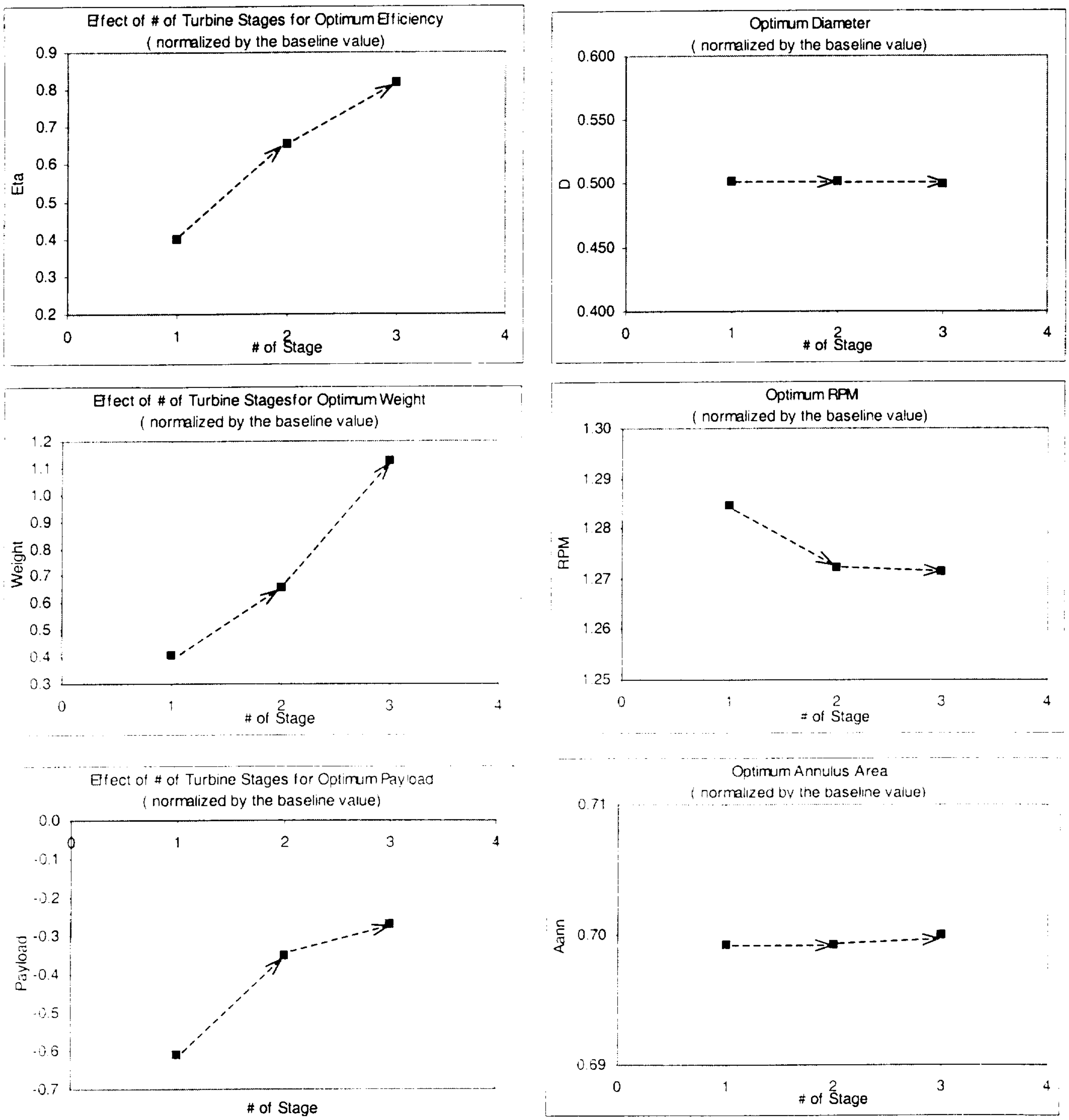


Figure 4. Effect of the number of turbine stage on optimum design parameters; *D*, *RPM*, and A_{ann} and optimum output parameters; η, *W*, and Δpay calculated for Weight-based optimization ($t=1$, $s=0$)

(All geometric design variables and output parameters are normalized by the baseline values)

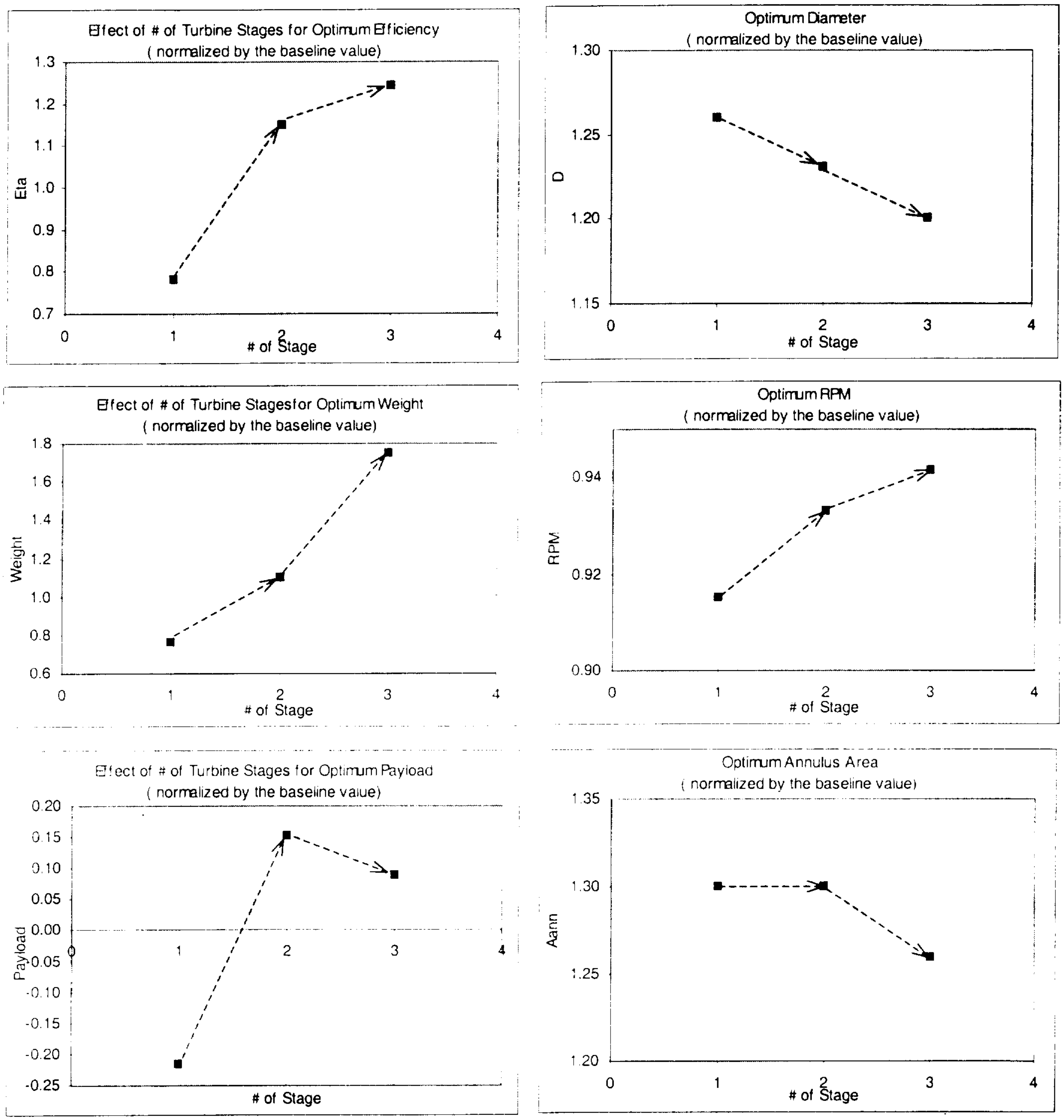


Figure 5. Effect of the number of turbine stage on optimum design parameters; *D*, *RPM*, and A_{ann} and optimum output parameters; η, *W*, and Δpay calculated for η-based optimization ($t=0$, $s=1$)
(All geometric design variables and output parameters are normalized by the baseline values)

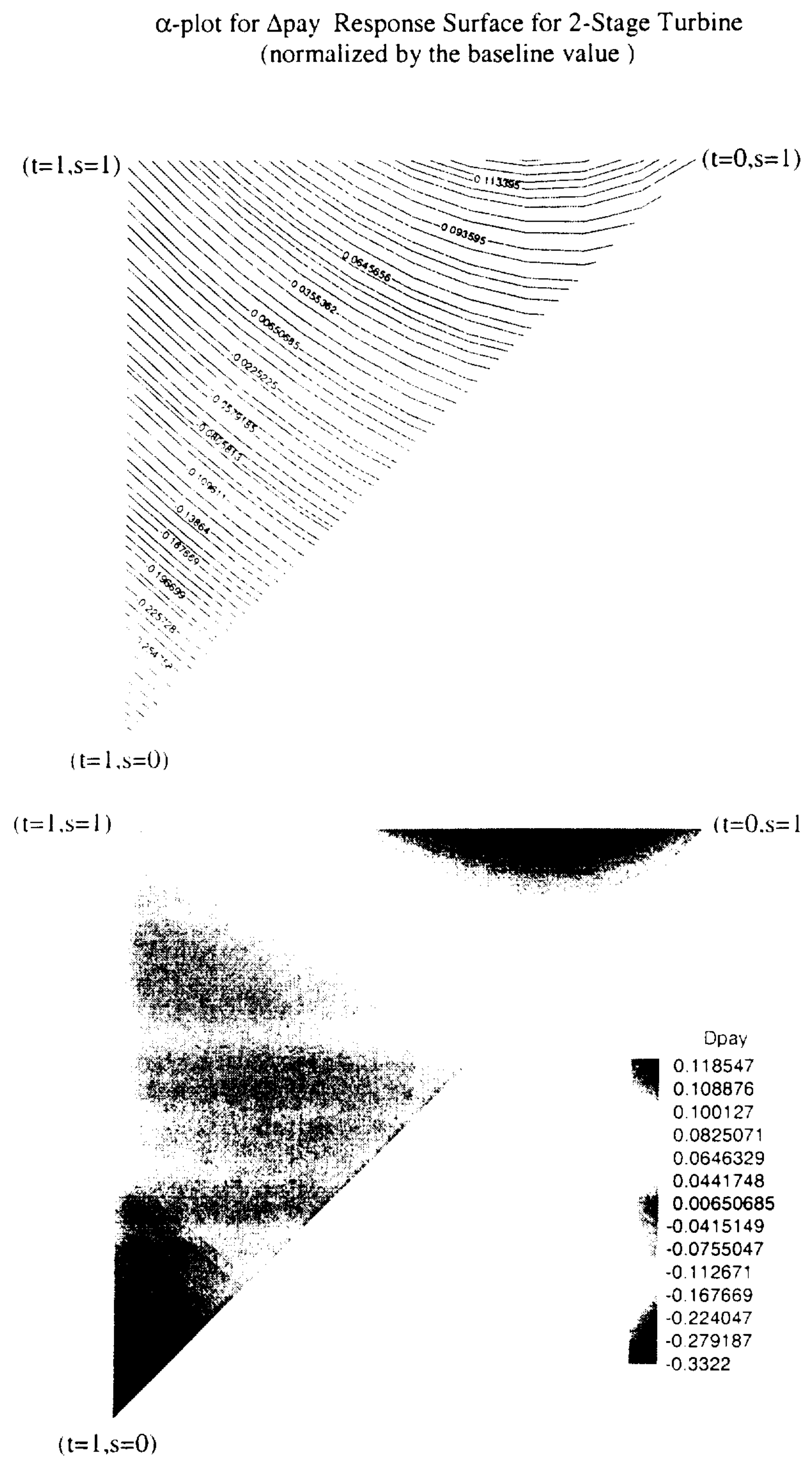
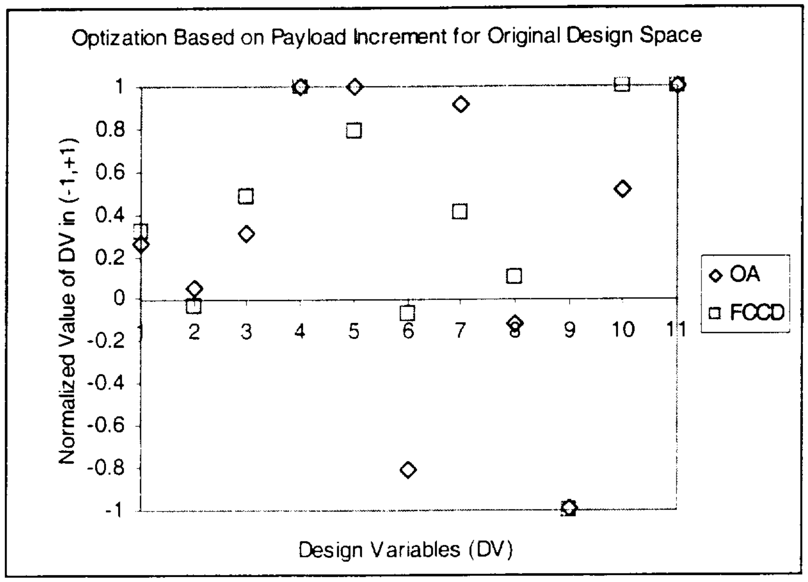
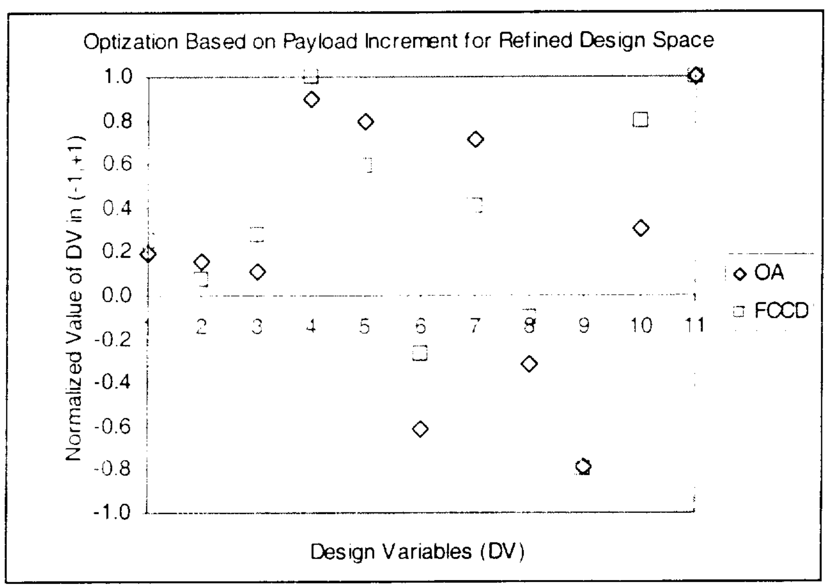


Figure 6. *α-Plot* for *Δpay* (normalized by the baseline values) Response Surface for 2-Stage Turbine based on composite desirability function optimization (Effect of values of *t* and *s* on optimum *Δpay*)



(a) Original Design Space



(b) Refined Design Space

Figure 7. Comparison of the Design Variables for Optimization based on Payload Increment (Δpay) using 1990-data (FCCD) and 249-data (OA) for both Original Design Space and Refined Design Space

(DV#1: D, DV#2: *RPM*, DV#3: A_{ann}, DV#4: h_1, DV#5: c_{v1}, DV#6: c_{v2}, DV#7: c_{b1}, DV#8: c_{b2}, DV#9: sr_1, DV#10: sr_2, and DV#11: w_{f1})