Preliminary evaluation of a fiber-optic sensor for flow measurements in pulmonary ventilators

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Abstract — **A novel optical fiber air flow sensor was developed for monitoring flow rates supplied by infant ventilators. The device is based on a fiber optic sensing technique and overcomes some important shortcomings of biomedical applications, such as electromagnetic interference and possible electrical hazard. The sensing principle is based on measuring the displacement of an emitting optical fiber cantilever by means of a photodiode linear array: the detection of the illumination pattern makes the developed system less sensitive to light intensity source variations than intensity-based sensors. A preliminary evaluation of the relationship between displacement and flow rate is theoretically and experimentally conducted, as well as a measurement range up to 3.00•10^{-4} m^3/s (18.0 l/min) has been verified, in accordance to the flow range usual for tidal breathing of infants.**

Keywords: flow measurement; sensor; infant ventilation; optical instruments

I. INTRODUCTION

Over the years various technologies have been used to measure air flow in mechanically ventilated patients [1-2], such as systems based on differential pressure sensing (Fleisch, Lilly and Schaller pneumotachograph), drag force detection through strain gauge, ultrasonic flow meters, turbine flow meters, variable area flow meters and hot wire anemometers. The above mentioned systems are based on electrical or electromagnetic transducers and, therefore, represent a possible cause of risk for patient safety and are subject to electromagnetic interference, e.g. during a magnetic resonance imaging (MRI) examinations or electrosurgery operations. As the mentioned drawbacks (patient electrical safety, electromagnetic interference) affect all biomedical fields, several fiber optic sensing techniques have been proposed for biomedical application [3–5], because of their immunity to electromagnetic interference and electrical insulation, but also in dependence of their high sensitivity, large bandwidth, small size and multiplexing [6].

Despite several optical flow sensors have been proposed in the last few years [7–14], today it is difficult to find an exhaustive investigation on the design and testing of optical fiber air flow meters in mechanical ventilation applications. In this work, a novel optical system for low air flows measurements is proposed, e.g. air flows measured at the inlet port of an infant ventilator to support neonates in intensive care units.

The theory of operation of the proposed system is based on measuring transversal displacement of an emitting optical fiber cantilever due to fluid drag force. A similar approach was adopted for an intensity-based sensor design [15], where the tip of the emitting fiber is placed in front of a receiving fiber and the movement of the emitting fiber, due to flow, changes the amount of light energy coupled to the output fiber; as a consequence, the variation of light intensity is a function of the flow. However, intensity variations can be also due to aging of the light source, as well as variations in connector characteristics and offset of the reference position of the receiving/emitting fiber: for the above reasons the sensor signal could be confused with an unreal flow variation.

In order to develop a measurement system less sensitive to the above mentioned error sources, a new approach is proposed in this paper, where the fiber tip displacement is detected by a photodiode linear array that is placed in front of the entrance face of the emitting fiber, in order to detect its illumination pattern (Fig. 1a).

In particular, the intensity profile of the light emitted toward the photodiode array has a maximum [16] that shifts according to the change in flow rate (Fig. 1b): therefore, the position of the most illuminated photodiode of the array is related to flow rate. Thus, the sensed information is directly encoded into an intensity distribution profile, that does not depend neither on the total light level (as long as the detected intensity is over the noise level), nor on losses in fibers or couplers, nor on variation in light source intensity. Moreover, if intensity of the emitting fiber varies during a constant flow rate, e.g. due to light source fluctuations, and, consequently, the maximum intensity value changes, the most illuminated photodiode of the array remains practically unchanged, confirming the independence of the measured flow on light source intensity variations (Fig. 2).

II. SENSOR STRUCTURE AND MEASUREMENT PRINCIPLE

The sensor is based on an emitting optical fiber (transversally placed in a circular tube, in which the air flow is supplied by an infant ventilator), that can be modeled as a cantilever beam (Fig. 1): the emitting fiber is fixed to the tube wall and is bent by a fluid drag force, that displaces the fiber tip of a quantity measured by means of a photodiode linear array, whose axis is approximately aligned with the displacement direction (Fig. 1b): the flow rate is then evaluated from the position of the most illuminated photodiode on the array.

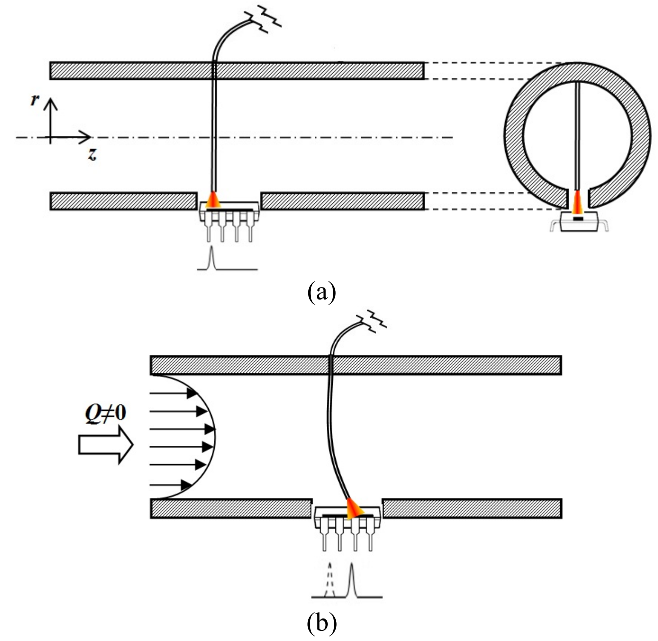

Figure 1. Optical fiber flow sensor scheme: (a) the light from the emitting fiber hits the photodiode array, built on an integrated circuit. At nul flow rate, the maximum ligh intensity emitted has a reference position corresponding to the maximum signal from the photodiodes; (b) as well as the flow rate Q changes, the emitting fiber bends and consequently the maximum moves from its reference position along the array direction.

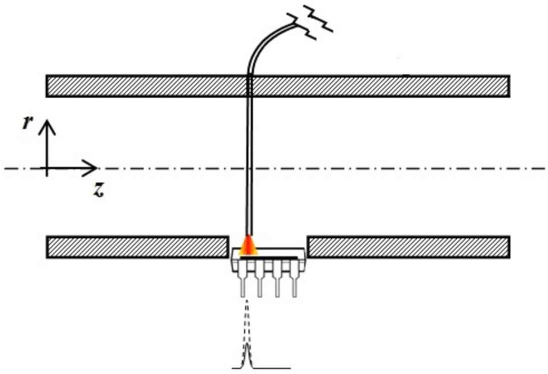

Figure 2. If the light intensity from the emitting fiber changes as a result of something unrelated to the flow rate variation, the position of the most illuminated photodiode of the array doesn't change significantly as well as the measured flow.

Numerical simulations and first experimental measurements were performed on an optical fiber arranged as in Fig. 1: preliminary results showed that the measurement system is not enough sensitive to detect small fiber tip displacements, due to resolution of photodiodes linear array (400 dots per inch) that allows a minimum detectable displacement equal to 63.5 µm.

In order to increase the device sensitivity, an alternative arrangement has been examined (Fig. 3), where the drag force acts on a target disk (about 10 mm in diameter) placed on the middle of the fiber (about 55 mm length) and perpendicular to the flow direction. In this arrangement, the measurement system is able to detect displacements up to about 7 mm for flow rates up to $3.00\cdot10^{-4}$ m^3/s (18.0 l/min), that are typical in infants with a maximum mass of 10 kg [17].

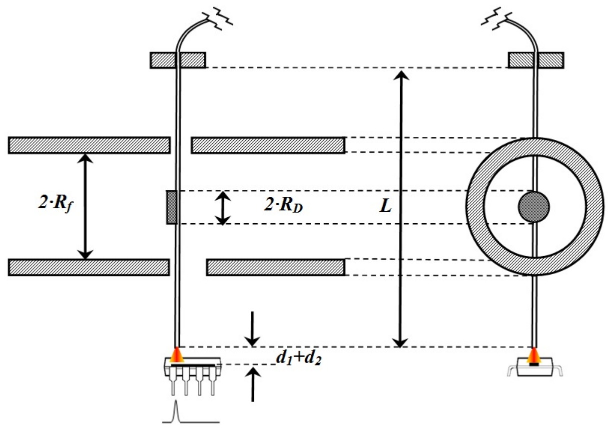

Figure 3. Alternative arrangement for the optical fiber flow sensor with main dimensions. R_f=10 mm; R_D=5mm; L=55 mm; d_1=0.5 mm; d_2=0.7 mm.

An approximated relationship between the fiber tip displacement δ and the air flow Q can be found from the equation for the wedge beam bending with a concentrated load F in the middle of the optical fiber cantilever [18]:

$$\delta = \frac{5}{48}\cdot\frac{F\cdot L^3}{E\cdot I} \tag{1}$$

where L is fiber length, E is the Young's modulus (E=71.8 GPa for an uncoated silica fiber and E=4.34 GPa for a fiber with polymer coating) [19], and I is the momentum of inertia of the circular fiber of outer diameter d_f [20], as in equation (2):

$$I = \frac{\pi\cdot d_f^2}{64} \tag{2}$$

As the maximum Reynolds number for the flow is approximately 1300, the relationship between the concentrated load F and the air flow Q can be found from velocity distribution v_z in the laminar flow:

$$v_z(r) = v_{z,\max} \cdot \left(1 - \left(\frac{r}{R}\right)^2\right) \quad (3)$$

with:

$$v_{z,\max} = 2 \cdot V = 2 \cdot \frac{Q}{S} = 2 \cdot \frac{Q}{\pi \cdot R^2} \quad (4)$$

where S is the cross-sectional area of the circular pipe, R is its radius, r is the radial distance from the center axis of the pipe in its cylindrical reference system and V is the mean flow velocity.

The laminar flow produces a drag force F, which can be evaluated by integration of the elementary drag force dF on each elementary annulus of area $2\pi \cdot r \cdot dr$ in the range $r \in [0, R_D]$:

$$F = \int_0^{R_D} C_D \cdot \left(\frac{1}{2}\rho \cdot [v_z(y)]^2\right) \cdot 2\pi \cdot r \cdot dr \quad (5)$$

where the quantity in round brackets is the dynamic pressure, R_D is the inner radius of the tube, ρ is the air density and C_D is the drag coefficient for a thin flat plate perpendicular to flow. There are many empirical formulas for C_D, which is dependent on Reynolds number and object shape, nevertheless for a thin plate perpendicular to flow, C_D value is about 2.0 for a wide range of Reynolds number [21].
Considering (3) and (4), the equation (5) can be written as:

$$F = \frac{2}{3} \cdot \frac{C_D \cdot \rho}{\pi \cdot R^8} \cdot R_D^2 \cdot (3 \cdot R^4 - 3 \cdot R^2 \cdot R_D^2 + R_D^4) \cdot Q^2 \quad (6)$$

Then, by substituting (6) into (1), the relationship between flow rate Q and fiber tip displacement δ is found:

$$Q = C \cdot \sqrt{\delta} \quad (7)$$

where C is a constant depending on the dimensional and mechanical characteristics of the optical fiber, as well as on the pipe and disk dimensions and on C_D and air density:

$$C = \left[\frac{9\pi^2}{40} \cdot \frac{E \cdot d_f^4 \cdot R^8}{C_D \cdot \rho \cdot L^3 \cdot R_D^2 \cdot (3 \cdot R^4 - 3 \cdot R^2 \cdot R_D^2 + R_D^4)}\right]^{1/2} \quad (8)$$

The above theoretical model doesn't take into account the gap between the tip of optical fiber and the sensitive surface of photodiode array, that is hit by the light from the fiber: the gap provides a displacement, measured by photodiode array, greater than the fiber tip has. The gap is made of two layer: the former is an air layer of thickness d_1, between the fiber tip and the external packaged surface of the photodiode array, and the latter is a sheet of nonconductive plastic with refractive index n=1.55, and thickness d_2=0.7 mm, which is placed onto the sensitive surface of photodiodes.

Taking into account the gap, the total displacement d measured by photodiode array is expressed in (9):

$$d = \delta + \delta_{gap} = \delta + \left(d_1 \cdot \tan\vartheta + \frac{d_2 \cdot \tan\vartheta}{n}\right) \quad (9)$$

where δ is the fiber tip displacement, the quantity in bracket δ_{gap} is the gap contribution to total displacement and θ is the fiber tip rotation, which in a first approximation can be described by (10):

$$\vartheta = \frac{F \cdot L^2}{8 \cdot E \cdot I} \quad (10)$$

Numerical simulations where d_1<1 mm, show that δ_{gap} is much less than δ, so the gap contribution to the measured displacement d can be neglected (e. g., for Q=18 l/min and d_1=0.500 mm, δ is about 7.5 mm and δ_{gap} is about 170 µm).

III. EXPERIMENTAL SET-UP

With reference to the scheme in Fig. 4, experimental tests were carried out in order to calibrate the device. The light emitted by an He-Ne laser L (632.8 nm, 5.0 mW) is collimated in a 50/125 µm multi-mode optical fiber OF with an aspheric collimator C; light intensity that propagates along the optical fiber, comes out from the tip of the fiber and is projected onto the photodiode array A (TSL1401R-LF, 400 dots per inch, 128 pixel), which is connect to its drive circuit D (Fig. 5). The intensity distribution profile of the light onto the photodiode array is converted into an electric signal and then acquired from the drive circuit (figure 5) by a data acquisition card that collects the array signal and sends it to a PC, where it is processed: the position of the most lighted photodiode in the array is detected and then related to the flow rate Q. The distance between the fiber tip and the array surface is adjusted by a travel stage which provides a resolution up to 1µm. Tests were performed for different air flow rates, set by adjusting the output pressure from a compressor so that the maximum Reynolds number is about 1300 and laminar flow occurs: in order to calibrate the proposed device, the input flow rate is measured by an air flow sensor Swema 3000 with an accuracy of 0.04 m/s in the range 0.10÷1.33 m/s and of 3% in the range 1.33÷30 m/s. Therefore the reference sensor (RS) allows to test the proposed device at a minimum air flow of 0.9 l/min.

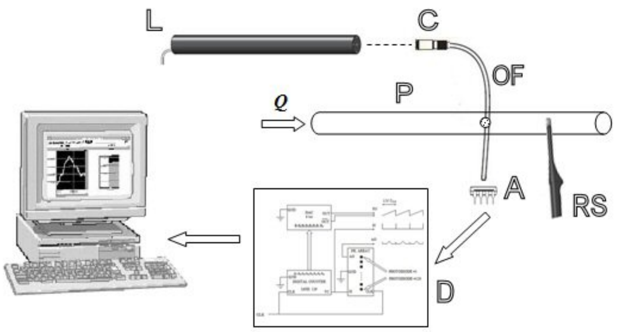

Figure 4. Experimental setup. L: laser; C: aspheric collimator; A: linear photodiodes array; D: drive circuit; RS: reference flow sensor; P: pipeline.

In particular, electric signals from the array are managed by the electronic drive circuit, which is able to generate a sawtooth waveform that is used to detect the position of the most lighted photodiode as a function of the input flow rate. In particular, a scanning session of the array begins by clocking in a high logic state on a Serial Input (SI); in the next rising edge of the clock, SI goes to a low state and an optical scanning of the 128 photodiodes starts (every photodiode is scanned in one clock cycle T_{CLK}, where T_{CLK} = 1μs); at the same time a voltage is generated on the Analog Output (AO), and it is proportional to the light intensity. After the 129th clock rising edge, the SI pulse comes back to a high level, AO assumes a high impedance state and a new scanning session begins.

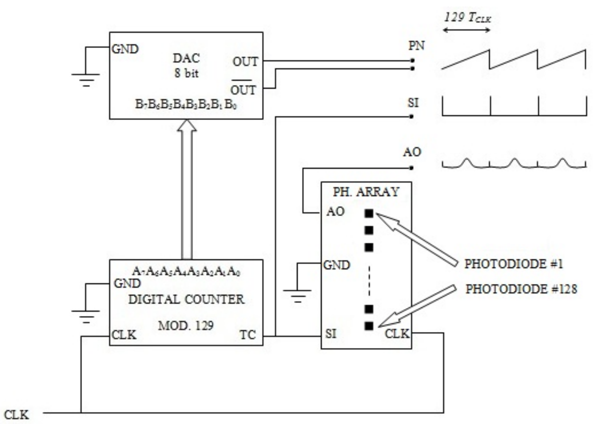

Figure 5. Drive circuit of the linear photodiode array.

The SI is generated by means of the terminal count signal (TC) of a modulo-129 digital counter (made up of a 8-bit loadable binary counter): the counter output is sent to a 8-bit Digital to Analog Converter (DAC) which provides an output voltage (PN). After a single scanning session (129 clock cycles), the AO signal is representative of the intensity profile projected on the photodiode array and the DAC output (Photodiode Number PN) is a sawtooth wave, whose amplitude is directly related to the position of the scanned photodiode at a well defined time. Finally, after a complete scanning session, the position of the most illuminated photodiode is determined measuring both the time instant when AO is maximum and the PN sawtooth amplitude at the same time. Then, the fiber tip displacement is measured as the difference (11) between the position δ_Q of the most lighted photodiode when flow rate is Q, and its reference position δ_{Q0} when the flow is minimum:

$$\delta = \delta_Q - \delta_{Q0} = (PN_Q - PN_{Q0}) \cdot \Delta \qquad (11)$$

where Δ is the distance between two adjacent photodetectors (63.5 μm), i.e. the minimum resolution of the array sensor, PN_Q is an integer indicating the position of a photodiode on the 128-elements linear array (PN_Q = 1, 2, ..., 128) and it can be derived from the PN signal. Therefore the flow Q is evaluated considering its relationship (7) with displacement δ.

IV. Results

In order to obtain the static calibration of the proposed air flow sensor, the set-up shown in Fig. 4 was used. Measurements were conducted up to $3.00 \cdot 10^{-4}$ m^3/s (18 .0 l/min), i.e. the maximum scale output determined by means of preliminary experimental tests that allowed to individuate the limit of the flow rate value when the last photodiode of the array (near to 128th photodiode) is the most illuminated one. This is the flow range normally encountered during tidal breathing of infants of mass lower than 10 kg [17].
Air flow rates are increased in step of $5.0 \cdot 10^{-5}$ m^3/s (3.0 l/min), by adjusting the working pressure of the compressor as long as the output of reference air flow sensor RS indicates the established input flow value.

In Fig. 6 some experimental data are shown: they show the fiber tip displacement δ, as the air flow Q increases, and the theoretical model (solid line), expressed by (7). There the coefficient of determination is r^2=0.997, so a good agreement between experimental data and the mathematical model can be deduced.

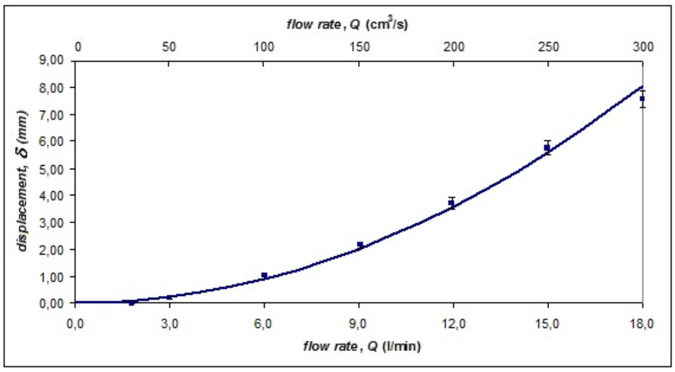

Figure 6. Measurements results: displacement δ as a function of flow rate Q.

Results show that the lower limit of the measuring interval is 2.0 l/min due to resolution of the photodiode array that allows a minimum detectable displacement equal to 63.5 µm and to relative standard measurement uncertainty of the air flow, which is, for air flow lower than 2.0 l/min, too high respect to usual characteristic.

In order to increase the sensitivity for low flow rates and extend the measurement range, another configuration is currently under development, where a single-mode optical fiber and a photodiode array with different resolution are used.

V. Evaluation of Uncertainty

An evaluation of measurement uncertainty in the calibration range has been performed according to [22].

In the equations (7) and (8), the theoretical model describes air flow rate Q, (not directly measured), as a function of different inputs :

$$Q = Q(E, \rho, d_f, R, C_D, L, R_D, \delta) \tag{12}$$

Taking into account that air density ρ can be expressed as a function of absolute temperature T and absolute pressure P, by the (13):

$$\rho = \frac{P}{R_S \cdot T} \tag{13}$$

where R_S =287.058 J/(kg·K) is the specific gas constant for dry air (the effects of relative humidity on ρ are neglected), the (10) becomes:

$$Q = Q(E, T, P, d_f, R, C_D, L, R_D, \delta) \tag{14}$$

The combined standard uncertainty δQ_c of the air flow rate Q, under the hypothesis of uncorrelated input quantities, is obtained from:

$$\delta Q_c = \left[\left(\frac{\partial Q}{\partial E}\cdot \delta E\right)^2 + \left(\frac{\partial Q}{\partial T}\cdot \delta T\right)^2 + \left(\frac{\partial Q}{\partial P}\cdot \delta P\right)^2 + \left(\frac{\partial Q}{\partial d_f}\cdot \delta d_f\right)^2 + \left(\frac{\partial Q}{\partial L}\cdot \delta L\right)^2 + \left(\frac{\partial Q}{\partial C_D}\cdot \delta C_D\right)^2 + \left(\frac{\partial Q}{\partial R_D}\cdot \delta R_D\right)^2 + \left(\frac{\partial Q}{\partial R}\cdot \delta R\right)^2 + \left(\frac{\partial Q}{\partial \delta}\cdot \delta\delta\right)^2\right]^{\frac{1}{2}} \tag{15}$$

where the partial derivatives are the sensitivity coefficients and the standard uncertainties, associated with the input directly measurable quantities, (confidence level 95%), have been found as follows:

- Young modulus E of the polymer coated optical fiber has been considered equal to 4.34 GPa with an uncertainty of 5% as in [19];
- air temperature T has been measured with a K type thermocouple IsoTech ITA11 (accuracy: 0.5%+2 K): $T \pm \delta T$=(295±2) K;
- for the pressure range normally encountered during pulmonary function tests in infants and young children [17], an absolute pressure of P=(101±7) kPa has been assigned;
- the outer diameter of optical fiber is $d_f \pm \delta d_f$=(245±7) µm (from the MM50 datasheet);
- the drag coefficient for a thin flat plate perpendicular to flow has been considered equal to $C_D \pm \delta C_D$=2.0±0.1, as indicated in [21];
- the length of the fiber L has been measured with a resolution of 1 mm; both the radius of the tube R and the radius of the circular disk target R_D have been derived from the measurement of the respective diameters by means of a decimal caliper;
- the displacement δ and its uncertainty $\delta\delta$, have been derived by a statistical analysis of the measurements, performed with the photodiode array, at different air flow rates Q.

Finally the measurement uncertainty δQ in the whole calibration range takes into account of the accuracy of the reference sensor.

In Table 1 the obtained uncertainty values and percentage errors $\varepsilon_{\%}$ are listed; the percentage error is obtained from (16):

$$\varepsilon_{\%} = \frac{|Q_{set} - Q|}{Q_{set}} \cdot 100\% \tag{16}$$

where Q_{set} is the air flow value measured by the reference sensor and Q is the flow evaluated by substituting the measured displacement δ in the theoretical model (7).

Results show that measurement uncertainty increases with Q and the percentage error varies from a minimum of about 1 percent at 15.0 l/min to a maximum of about 10 percent at 2 l/min.

TABLE I. MEASUREMENT UNCERTAINTY VALUES OF Q EVALUATED WITH THE LAW OF PROPAGATION OF UNCERTAINTY.

Q_{set} [l/min]	$\delta \pm \delta\delta$ [mm]	$Q \pm \delta Q$ [l/min]	$\varepsilon_{\%}$ (in percent)
2.0	0.06±0.03	2.2±0.4	10.0
3.0	0.19±0.03	2.8±0.5	6.7
6.0	1.02±0.03	6.4±0.6	6.7
9.0	2.15±0.13	9.3±0.9	3.3
12.0	3.71±0.22	12.2±1.1	1.7
15.0	5.75±0.25	15.2±1.3	1.3
18.0	7.54±0.30	17.4±1.5	3.3

Table 1 shows a constant displacement uncertainty for low flows (up to 6.0 l/min): from a series of independent observations, the most illuminated photodiode in the array is the same and so displacement uncertainty value is estimated by assuming a rectangular probability distribution (type B evaluation of standard uncertainty) with a width equal to photodiode longitudinal dimension (63.5 μm).

For air flows greater than 6.0 l/min, a series of independent observations of displacement provides a normal distribution: δ has been calculated by averaging a series of independent observations of displacements, and $\delta\delta$ has been calculated multiplying the standard deviation of the mean by the coverage factor associated to a level of confidence of about 95% (type A evaluation of standard uncertainty).

The measurement uncertainty δQ can be reduced improving the knowledge of the input quantities (e.g. performing a measurement of E, T and P with a lower uncertainty) and by means of a more accurate reference sensor at low flow rate.

VI. CONCLUSIONS

A novel optical fiber air flow sensor has been developed for measuring flow rates at the inlet port of a pulmonary ventilator for infants. The device is based on a fiber optic sensing technique, that overcomes drawbacks related to EM interference and patient electrical safety. The sensing principle is based on measuring the transversal displacement of an emitting optical fiber cantilever by means of a photodiode linear array. The measurement system is based on a measurement of the illumination pattern of an emitting fiber and is less sensitive to some influence quantities (e.g. intensity source variation) than intensity-based sensors because the flow rate is evaluated from the shift of the maximum of the intensity distribution profile of the light beam projected on the photodiode array, regardless of total light levels.

A static calibration of the sensor was performed and a measurement range up to $3.00 \cdot 10^{-4}$ m^3/s (18.0 l/min) was determined, in agreement to the flow range normally encountered during tidal breathing of infants of mass lower than 10 kg. Experimental data of static calibration are in accordance with the proposed theoretical model (the coefficient of determination r^2 is equal to 0.997).

Evaluation of measurement uncertainty shows that δQ increases with Q.

Other configurations are going to be developed to improve sensor performances and applications.

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