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PREPARATION OF MATHEMATICS TEACHERS: LESSONS FROM REVIEW OF LITERATURE ON TEACHERS' KNOWLEDGE, BELIEFS, AND TEACHER EDUCATION

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A B S T R A C T

Teachers' mathematics knowledge has been known to have a significant impact on instructional practices. This paper discusses research on teachers' mathematics knowledge. The paper has been summarised in five main areas: (a) the role of subject matter knowledge in teaching and learning, (b) teachers' beliefs about mathematics teaching and learning, (c) beliefs and beliefs-in-practice: inconsistencies, (d) teacher education and its impact on instructional practices, and (e) future research on teachers' mathematics knowledge. The review indicated that teachers are critical factors in the learning of mathematics and the extents of their content and pedagogical knowledge do determine students' achievement. Also, the paper acceded to the view that, a teacher's memories from the school years is a central influencing factor that affects its mathematics related beliefs, hence there is a need to enhance pre-service teachers' positive attitude towards mathematics during training. The paper suggested further areas of research should look at: different theoretically-and empirically distinction in content knowledge for teaching and investigate their relationship, separately and in combination, to student achievement; whether mathematics teachers' knowledge affects their lesson planning strategies and whether the provision of 'mathematical knowledge for teaching' by teacher training institutions improve pre-service teachers' beliefs about mathematics and mathematics teaching. The paper concluded that, mathematics teacher education programme should provide pre-service teachers with awareness of conception of mathematics which may influence their teaching.

Keywords: Mathematics teacher's knowledge, instructional practices, teacher's belief, teacher education.

INTRODUCTION

Mathematics subject knowledge especially at primary and secondary school levels has been recognized as an issue for some time within the mathematics community (Ball, 1990), by policymakers (Alexander, Rose, and Woodhead, 1992). A general concern raised in these studies as well as studies in other countries such as the USA, the UK, Botswana, Nigeria, and South Africa concerns the state of the teachers' mathematics knowledge, particularly in respect of the subject content knowledge (National Research Council [NRC], 2001; Ball, Lubienski, and Mewborn, 2001; Hodgen, 2003; Pendaeli, Ogunnyi and Mosothwane, 1993; Taylor and Vinjevold, 1999; Akinsola, 2013). A common factor identified in all these studies as necessary for effective education is the crucial role that the teacher plays in teaching and learning.

Research, however, suggests that pre-service elementary and secondary school teachers often lack a fundamental understanding of school mathematics (Cooney, Shealy, and Arvold, 1998; Ma, 1999; Simon, 1993; Akinsola, 2009). In particular, primary school teachers are not competent in mathematics, science, and pedagogy mainly due to having a very short pre-service teacher training period just after ten years of schooling (Mahmood, 2002; Akinsola and Ajiboye, 2009.). Teachers' inadequate subject matter, however, should not be surprising. These teachers themselves are the products of primary and secondary schools, in which research has shown that students rarely develop a deep

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understanding of the subject matter (Ball and McDiarmid, 1990; Boaler, 1998), and we seek to improve (Ball, Hill, and Bass, 2005).

Fennema and Franke (1992) contend that; "No one questions the idea that what a teacher knows is one of the most important influences on what is done in classrooms and, ultimately, on what students learn" (p. 147). Teachers' inadequate subject-matter issue has been more than one of simply ensuring teachers have "more" mathematics (Hodgen, 2003). An extensive research that focuses on teachers' mathematics knowledge has emerged over the last decades (Ball, 1988; Fennema and Franke, 1992; Ma, 1999; Shulman, 1987), in addition to the continuing public and policy concern about improving teachers' knowledge of mathematics as a means to improve instruction and maximize student learning (National Commission on Teaching and America's Future [NCTAF], 1996).

The American Council of Education [ACE] (1999) believes that; "A thorough grounding in college-level subject matter and professional competence in professional practice are necessary for good teaching…Students learn more mathematics when their teachers report having taken more mathematics" (p. 6). Weaver (1979), however, reports no demonstrated casual link between academic ability and teacher competency. Askew, Brown, Rhodes, Johnson, and William (1997) report that increased academic qualifications in mathematics have a slightly negative relation with mathematics teaching. Other studies, however, report of a demonstrated relationship between teachers' mathematical preparation and students' test performance (Chaney, 1995; Harbison and Hanushek, 1992; Rowan, Chiang and Miller, 1997; Mullens, Murname and Willet, 1996). Other studies also report a relation between teacher knowledge of the subject and instructional processes (Muijs and Reynolds, 2002; Ball, Lubienski and Mewborn, 2001). McDiarmid, Ball, and Anderson (1989) report that;

Research highlights the critical influence of teachers' subject matter understanding on their pedagogical orientations and dec isions…Teachers' capability to pose questions, select tasks, evaluate their pupils understanding, and make curricular choices all depend on how they themselves understand the subject matter. (p. 195-196).

Askew and his colleagues (1997) compared teachers with A-Level mathematics with those without A-Level mathematics. They found that the teaching of teachers who had studied mathematics A-Level was similar to the teaching of those who had not studied A-Level. Askew and his colleagues (1997) research into teachers of numeracy, also suggests that primary school teachers with a 'connectionist' approach to teaching were able to see connections in the mathematics they were teaching. 'Connectionist' teachers were considered to be more effective in teaching mathematics. Ma (1999) also compared Chinese elementary teachers who were mathematics specialist but had not studied mathematics beyond the age of 14, with USA generalist elementary teachers who studied mathematics at college level. She, however, found that the distinguishing feature of Chinese teachers was their attitude to and understanding of mathematics. Some of the Chinese teachers in her study appeared to have an extremely deep knowledge, something she defined as a 'Profound Understanding of Fundamental Mathematics (PUFM). Her study highlighted the possibility of primary teachers developing a deep understanding of mathematics.

Teacher in the Educational production Function Literature: The traditional education production function literature shows that researchers working in this tradition have typically measured teachers' knowledge using variables such as courses taken or degree attained. This is in contrast to another group of education researchers who have begun to conceptualise teachers' knowledge for teaching differently, arguing that teacher effects on student achievement are driven by teachers' ability to understand and use subject matter knowledge to carry out the task of teaching (Ball, 1990; Shulman, 1986; Wilson, Shulman, and Richert, 1987, Akinsola, 1999, Akinsola, 2013). In these researchers' views, mathematical knowledge for teaching goes beyond that captured in measures of mathematics courses taken or basic mathematical skills. Teachers of mathematics not only need to calculate correctly, but also know how to use diagrams to represent mathematics concepts and procedures to students, provide students with explanations for common rules and mathematical procedures, and analyze students' solutions and explanations (Hill, Rowan, and Ball, 2005).

By inadequately measuring teachers' knowledge, existing educational production function research could be limited in its conclusion, not only about the magnitude of the effects that teacher knowledge has on student learning, but also about the kind of teacher knowledge that produces effective student learning. Very few educational production function studies have, however, measured teachers' mathematical knowledge directly and used this as a predictor of student achievement (Mullens, Murnane and Willet, 1996; Rowan, Chiang, and Miller, 1997). Most other production function studies used tests of teacher verbal ability to predict students' achievement outcomes. Reviews of this work have disputed the extent to which variables such as teacher preparation and experience contribute to student achievement (Greenwald, Hedges, and Laine, 1996; Hanushek, 1996), with conflicting interpretations resting on the samples of the studies and methods used for conducting meta-analysis. Beyond these methodological issues, another potential reason for the uncertainties in these research findings might be that teacher preparation and job experience are poor proxies for the kinds of teacher knowledge and skills that matter most in helping students learn academic content.

In cognizant of this problem, a smaller number of production function studies (Harbison and Hanushek, 1992; Mullens et al., 1996) have sought to measure teachers' knowledge by looking at teachers' performance on certification examinations subject content competence. By using such measures, these studies have assumed a relationship between teacher content knowledge as measured by such assessment and the kind of teaching performance that produce improved student achievement. Studies using this approach are reported to have found positive effect of teacher knowledge, as measured by certification examinations of subject content competence on student achievement.

Although this is an important research finding, it cannot fully describe how teacher knowledge relates to student achievement. Measuring quality teachers through performance on tests of verbal or mathematics ability may overlook key elements in what produces quality teaching. Effectiveness in teaching does not simply depends on the knowledge a teacher holds, but how that knowledge is used in classrooms is very crucial. Teachers who are highly proficient in mathematics will only help other learn mathematics if they are able to use their knowledge to perform the tasks they must enact as teachers. For example, to listen to students, to select and make use of good assignments, to manage class discussions are some of the ingredients of effective teaching. Yet these additional content related abilities specific to teaching have not been included in the educational production function models of research.

Teachers' mathematics knowledge and its impact on instructional practices: Alongside production function research, an alternative research focuses directly on teacher knowledge and asks what teachers need to know about the subject-matter content in order to teach it to students (Hill, Rowan, and Ball, 2005). According to Hill and her colleagues the focus on subject-matter knowledge rose, at least in part, because of evidence suggesting that USA teachers lack essential knowledge for teaching mathematics (Ball, 1990; Ma, 1999) and because of evidence from the educational production function literature suggesting that teachers' intellectual resources significantly affect student learning. Despite this widespread interest of and concern what counts as "subject matter knowledge for teaching" and how it relates to student achievement remains inadequately specified in previous research (Hill et al, 2005).

Philosophical arguments as well as common sense, however, support the conviction that teachers' subject matter knowledge influences their efforts to help students learn subject matter. Seldon (2003) argues that; "One needs a solid understanding of the mathematics at, and beyond, the level at which students being observed are working" (p. 3). Post, Harel, Behr, and Lesh (1988) also argue that, "A firm grasp of the underlying concepts is an important and necessary framework for the elementary school teacher to possess…[when] teaching related concepts to children…[and] many teachers simply do not know enough mathematics" (pp. 210-213). Simmons (1993) also contends that, "In order to teach well the teacher needs to know about the subject matter in both width and depth to a degree unlikely to be found amongst those beginning a teacher training course" (p. 9).

When teachers' knowledge of the subject that they teach (subject content knowledge) is rich, integrated and accessible, they tend to teach the subject more dynamically by using more varied ways (pedagogical content knowledge) while encouraging and responding more fully to learners' questions and comments (Brophy, 1991). On the other hand, when teachers possess inaccurate information or conceive of knowledge in narrow ways, they may pass on these ideas to their students. They may fail to challenge students' misconceptions; they may use texts uncritically or may alter them inappropriately. Ball and her colleagues (2005), however, write that, "although many studies demonstrate that teachers' mathematical knowledge helps support increased student achievement, the actual nature and extent of that knowledge whether it is simply basic skills at the grades they teach, or complex and professionally specific mathematical knowledge is largely unknown"(p. 16).

The mathematical knowledge important for the work of teaching is a significant issue in mathematics education. The mathematical knowledge necessary to teach 'effectively' is recognized as a more complex issue than simply requiring as grasp of mathematics subject knowledge (Ball, 1990; Fennema and Franke, 1992). Teachers use mathematical knowledge not so much for the doing of mathematics, but rather for the teaching of mathematics. Hence a key aspect of primary school teachers' mathematics knowledge, even at early stage in their teaching career needs to be the teacherly transformation of the content knowledge into the knowledge sufficient for the teaching of mathematics (Hodgen, 2003).

Two decades ago, Shulman (1986) proposed three categories of teacher subject matter knowledge (SMK). His first category, *content knowledge or SMK*, was intended to denote "the amount and organization of knowledge…in the mind of teachers" (p. 9). According to Shulman, content knowledge includes both facts and concepts in the domain, but also why facts and concepts are true, and how knowledge is generated and structured in the discipline. The second category Shulman advanced was *pedagogical content knowledge (PCK)*. PCK consists of "the ways of representing the subject which makes comprehensible to others…[it] also includes an understanding of what makes the learning of specific topics easy or difficult…" (p. 9). PCK is essentially to conceptualize the hitherto missing link between knowing something for oneself and being able to enable others to know it. The last Shulman's category is the *curriculum knowledge*. This category involves awareness of how topics are arranged both within a school year and over time and ways of using curriculum resources, such as textbooks, to organize a program of study for students.

The relationship between SMK and PCK required for teaching is still not fully understood. For instance, in the USA Ball and her colleagues (2001) acknowledge that there is insufficient understanding of the mathematical knowledge it takes to teach well. In the UK, Aubrey (1997) argues for the central importance of disciplinary knowledge to good primary school teaching. Finally, Askew and his colleagues (1997) report that teachers whose students made the greatest gains in test scores were described as having knowledge and awareness of conceptual connections between the areas which they taught without necessarily having advanced mathematical qualifications.

There is a widespread consensus among those involved in mathematics education that teacher knowledge especially PCK is a determinant of mathematics instruction and student learning (Fennema and Franke, 1992). Mason and Spence (1999) reviewed a range of categorizations and focused particularly on teachers' knowledge as dynamic and evolving and of importance of knowing-to as it requires "relevant knowledge to come to the fore so it can be acted upon" (p. 139). It is at this point that knowledge of mathematics and instructional practice interact and knowledge can prove to be useful or otherwise. Brophy (1991) argues in relation to content knowledge that;

Where (teachers') knowledge is more explicit, better connected, and more integrated, they will tend to teach the subject more dynamically, represent it in more varied ways and encourage and respond fully to students' comments and questions. Where their knowledge is limited, they will tend to depend on the text foe content, deemphasize interactive discourse in favour of seatwork assignments, and in general, portray the subject as a collection of static, factual knowledge. (p. 352).

Carpenter, Fennema, and Franke (1996) describe teaching as, "a complex problem-solving activities that cannot be understood only by looking at the activities that teacher engage in as they teach" (p. 3). Carpenter and her colleagues investigated whether Cognitively Guided Instruction (CGI) that focuses on children's understanding of specific mathematical concepts can provide a basis for teachers to develop their pedagogical knowledge more broadly. Their study results show that when teachers consider how to integrate their emerging knowledge about children's thinking with their existing pedagogical knowledge, they question their pedagogical knowledge. In recognizing that students have knowledge worth listening to and building on, teachers evaluate their general philosophies about their role as the dispenser of knowledge. These teachers' analysis of their pedagogical knowledge, results in changes in their general pedagogical knowledge that goes beyond the teaching of mathematics (Fennema, and Franke, 1992). Carpenter, Fennema, and Franke's (1996) study also provides teachers a framework with which to construct a coherent, organized knowledge based that they can draw on to solve complex pedagogical problems they encounter in teaching primary school mathematics.

Carpenter, Fennema, Peterson, Chiang, and Loef (1989) also investigated teachers' use of knowledge from research on children's mathematical thinking and how their students' achievement is influenced. The results of the study show that experimental teachers taught problem solving significantly more and number facts less than did the control teachers. Experimental teachers also encouraged students to use a variety of problem solving strategies, and listened to processes their students used significantly than did the control teachers. Carpenter and his colleagues study also show that giving teachers access to research based knowledge about students' thinking and problem solving can affect teachers' beliefs about learning and instruction, their classroom practices, their knowledge about their students, and most important, their students' achievement and beliefs.

Teachers' beliefs and their impact on instructional practices: There is considerable research on the relationship between teacher beliefs and practices (Richardson, 2003; Zhihui, 1996). Teachers' classroom behaviour is influenced not only by the teachers' knowledge of the content to be taught, how students learn or the understanding of specific content and methods to teach specific content. The teachers' instructional practices are also influenced by the teachers' attitudes and beliefs about the subject and its teaching (Koehler and Grouws, 1992; Akinsola, 2009). Studies of teachers' beliefs in mathematics education have investigated their beliefs about the nature of mathematics (Ernest, 1989), as well as the general conceptions of mathematics teaching and learning (Cobb, Wood, and Yackel, 1990; Ball, 1990). Research has shown that teachers' beliefs about teaching and learning their subject area significantly influence their performance in the classroom and in their students' learning (Ball et al. 2001; Pajares, 1992; Thompson, 1984, 1992; Prawat, 1992). A teacher's memories from his school years seem to be a central influential factor on his mathematics related beliefs. A teacher who has experienced mathematics as awful during his school years may try to protect his students from mathematics (Gellert, 2000; Akinsola, 2008).

Furthermore, an extensive overview of empirical research has been carried out into teachers' beliefs and conceptions (Thompson, 1992; Middleton, 1995; Franke, 1990; Lindgren, 1995). In using Kogelman and Warren's (1978) framework of mathematics myth, Franke (1990) found that pre-service elementary school teachers share many of the mathematical beliefs held by math-anxious people. Lindgren (1995) also analyzed pre-service teachers' beliefs and conceptions about mathematics and its teaching using the three views on the nature of mathematics proposed by Ernest (1989). Thompson (1992, 1984) also studied the relationship between the conceptions and instructional practices of three junior high school teachers. She used case studies and a variety of techniques, including observations, audio-recorded lessons, and interviews.

Other empirical studies looked at beliefs in specific mathematical topics. Askew and his colleagues (1997) studied the link between (i) teachers' practices, beliefs, and knowledge, and (ii) pupils learning outcomes. The study explores the teachers' beliefs about what it means to be numerate, and how pupils become numerate. The group of participants were grouped as discovery, transmission, and connectionist. Pinto and Tall (1996) investigated teachers' conceptions of rational numbers and idiosyncratic beliefs about real numbers. The student teachers who participated in the study showed a diversity of imagery the students have about rational numbers. Brown's (2003) study also shows that student teachers consider mathematics as a subject filled with horror stories of their own schooling. Verschaffel (1996) also investigated pre-service elementary teachers' conceptions and beliefs about the role of real-world knowledge in arithmetical word problem solving. The results of his study indicate a strong positive correlation of the pupils' tendency to exclude real-word knowledge to analogous student-teachers exclusion as well as their exclusion of appreciation of the pupils' answers. Akinsola (2009),concluded that teachers' beliefs and expectations interact and influence mathematics teachers' planning, delivery of instruction, engagement of students and classroom management which may influence students' achievement.

Beliefs and beliefs-in-practice: inconsistencies: Pre-service teachers set out with well developed personal beliefs about learning and teaching (Joram and Gabriele, 1998). According to Daskalogianni and Simpson (2000), students also bring various forms of mathematical knowledge when moving from school to study mathematics degree as well as bring with them beliefs about the nature of the subject, which have been built up from their experience of school mathematics. The teachers' beliefs about mathematics are deeply rooted and peripheral changes such as curriculum or teaching materials cannot influence them (Furinghetti and Phkonen, 2002).

Although researchers generally report consistency between teacher beliefs and instructional practice (Thompson, 1992), some researchers, however, report inconsistencies between teachers' beliefs and classroom practices (Raymond, 1997; Jones, Henderson, and Cooney, 1986; Brown, 1985; Cooney, 1985; Shaw, 1989; Thompson, 1984). While most teachers' actions are based on implicit, tacit knowledge (Day, 1999), Argyris and Schon (1974) explain that integrating action with thought is a difficult task. In drawing a distinction between peoples' espoused theories and theories-in-use, Argyris and Schon write:

When someone is asked how he would behave under certain circumstances, the answer he usually gives is his espoused theory of action for the situation. This is the theory of action to which he gives allegiance, and which, upon request, he communicates to others. However the theory that actually governs his actions is his theory-in-use, which may or may not be compatible with his espoused theory; furthermore, the individual may or may not be aware of incompatibility of the two theories. (p. 6-7).

Shaw (1989) gives an example of a teacher who may believe that exploring mathematical situations is more important than rote practice. Yet often assign 50 exercises for students to work during class. These inconsistencies suggest that there is no simple cause-effect relationship between beliefs and practices and suggests that there exist other factors that influence both the professional practice and the institutional context (Cooney, 1985; Hoyles, 1992).

Recent studies, however, indicate that there may not be a direct link between teachers' beliefs and the instructional practices (Levitt, 2001; Wilcox-Herzog, 2002), and other studies supports the idea that there are sets of beliefs that are particular to specific disciplines (Cobb, 2002). Schoenfeld (2002) provides evidence supporting domain specificity using examples from elementary teachers' mathematics instruction. There is also some evidence that subject matter influences teaching practices in elementary school setting (Olafson and Schraw, 2006).

Teacher education and its impact on instructional practices: The complexity of learning to be a teacher is recognized in the research related to subject matter knowledge (Aubrey, 1997; Brown and McIntyre, 1993; Shulman, 1986). Cooney (1999) writes: "If one is trying to understand the status of what teachers know and to build a case for increasing teachers' knowledge of mathematics, this approach [i.e., examining what teachers know and believe] has merit" (p. 164). Along similar line of argument, Simon (1993) contends: "A research base with respect to prospective teachers' knowledge is essential if we are to develop instructional interventions that will help prospective teachers extend and modify their knowledge" (p. 233).

In view of these observations, Ball and her colleagues (2005) report that, "mathematical knowledge of many teachers is dismaying and thin" (p. 14). One of the many suggested solutions to this problem is to require teachers to study more mathematics, either by requiring an additional coursework, or even stipulating a subject matter major. The problem with this view is that it is not based on adequate research evidence. It has not been possible to link teachers' knowledge of the subjects to students' achievement. Ball and her colleagues further suggest other solutions to include more practice-oriented approach, preparing teachers in the more mathematics they will use on the job. This approach call for revamping mathematics methods coursework and professional development to focus more closely on the mathematics contained in classrooms, curriculum materials, and students' minds. Research and experience, however, consistently reveal that despite implementing the suggested approaches, students' achievement is still below accepted standards.

With the introduction of any new initiative into the mathematics classroom, there is often an assumption that it will produce measurable effects in teaching approaches and student progress. Some studies, however, suggest that asking teachers to move from one teaching approach to another cannot be regarded as a straight forward substitution because of various factors (Remillard and Bryans, 2004). The teachers' orientations and preferences are of significant influence (Brown, Askew, Millett, and Rhodes, 2003). Moving to a more learner-centred approach, however, places greater demands to teacher knowledge, as the lesson can take many possible directions, given the more responsive nature of teaching process, and students' strategies and reasoning could well challenge the teacher's mathematical "comfort zone" (Clarke, 2003).

Many countries, however, have acknowledged a shift in focus from transmission model of teaching to an emphasis on teaching for understanding (Fennema and Romberg, 1999). This shift is present in policy statements and curriculum documents (Government of Botswana, 1993, 1994; National Council of Teachers of Mathematics [NCTM], 1991; Australian Association of Mathematics Teachers [AAMT], 2000). In the UK, for example, the government specified for the first time a curriculum for Initial Teacher Training (ITT) setting out what was deemed to be the "knowledge and understanding of mathematics that teacher trainees need in order to underpin effective teaching of mathematics at primary level" (DfEE, 1998). Such a curriculum is founded on the belief that teachers' subject knowledge is an essential ingredient for successful teaching.

Since the work of Dan Lortie (Lortie, 1975) evidence has accumulated to indicate that teacher training is a minimal impact enterprise. Some of this evidence is reviewed in a meta-analysis of 40 previous studies (Kagan, 1992). Kagan considered the alteration in beliefs of pre-service teachers during their teacher education programs. She claims to have found a lack of change in the pre-existing beliefs, and that university courses are largely irrelevant in supplying adequate procedural knowledge for teaching. Furthermore, lack of impact of training is due to the weight of previous experiences of education that each student teacher brings with them to their course (Powell, 1992). This leads to a predisposition to teach in particular ways and entrenched beliefs about the nature of teaching and learning.

Teacher educators spend considerable time attempting to teach pre-service teachers how to teach so their students could learn. What the teacher educators often overlook is approaches to affect the pre-service teachers' beliefs and learning strategies. As Dembo (2001) points out, pre-service education should have two complementary goals. First, it should teach future teachers to become more effective learners themselves. Secondly, it should teach them to be more effective teachers.

Thompson (1992) writes: "The task of modifying long held, deeply rooted conceptions of mathematics and its teaching in the short period of a course in methods of teaching remains a major problem in mathematics teacher education" (p. 135). Studies by Pietila (2002) and Kaasala (2000), however, show that elementary student-teachers' self-confidence can be improved by providing a challenging and safe environment in the studies, by involving student-teachers to experience studies as beneficial, and by allowing them opportunities to elaborate on their negative experiences.

Future research on teachers' mathematics knowledge: Research on mathematics learning needs to be complemented by research on mathematics teaching (Ernest, 1989). The relationship between SMK and PCK required for teaching is still not fully understood (Goulding, 2003). In the USA for instance, Ball and her colleagues (2001) acknowledge that, we have insufficient understanding of the mathematical knowledge it takes to teach well. Promising insights, however, into the way in which a combination of SMK and PCK can inform teaching are emerging (Huckstep, Rowland, and Thwaites, 2002). Effective teaching requires knowing and understanding mathematics, students as learners, and pedagogical strategies (NCTM, 2000). For example, skilled science teachers are reported to "have special understandings and abilities that integrate their knowledge of science content, curriculum, learning, teaching, and students" (NRC, 1996, p. 62). Trainees who have several representations for mathematical ideas and whose knowledge is already richly linked will able to draw upon these both in planning and in spontaneous teaching interactions. In a study by Askew and his colleagues (1997), teachers whose pupils made greatest gains in test scores were described as having 'knowledge and awareness of conceptual connections between areas which they taught' without necessarily having advanced mathematical qualifications.

With such overwhelming evidence, it seems mathematics teacher education should shift the focus from 'give more content' to 'how the content could be taught effectively.' Future research on the impact of teachers' mathematics knowledge on instructional practices should follow three lines of inquiry. First, should different theoretically-and empirically distinction in content knowledge for teaching and investigate their relationship, separately and in combination, to student achievement. A second line of investigation could focus on investigating whether and how the instructional practices of mathematically knowledgeable and less knowledgeable teachers differ. For example, does teachers' mathematics knowledge affects instructional decision-making? Does teachers' mathematics knowledge affect their lesson planning strategies? Does teachers' mathematics knowledge influence the teachers' use of students' misconceptions? Finally, can the provision of 'mathematical knowledge for teaching' by teacher training institutions improve pre-service teachers' beliefs about mathematics and mathematics teaching? Can this approach by teacher training institutions lead to better instructional practice in schools?

CONCLUSION

Evidence suggests that few students develop conceptual understanding of mathematics (Howie, 2001; NRC, 1989), and many students are unable to use mathematics in situations outside the classroom context (Boaler, 1998). Two widely given explanations for why students do not learn mathematics are the inadequacy of their teachers' mathematics content knowledge and lack of rigorous certification requirements for teachers (Hare, 1999). Teacher training institutions have an active role to play in correcting the situation by considering and reviewing their current pre-service teacher training programs and also strengthening existing in-service programs to assist practising teachers in acquiring adequate content and pedagogical knowledge. Balancing SMK and PCK in mathematics teacher education is critical. Enhancing pre-service teachers' content and pedagogical knowledge in combination with collaboration and reflection can serve as a catalyst for improvement in instructional practice (Swafford, Jones, Thornton, Stump, and Miller, 1999).

There are, however, other influences on establishing teachers' professional development beyond teacher education programs. One of the central challenges of teacher education is to influence pre-service teachers' mathematical skills and beliefs, also their beliefs on themselves as learners of mathematics. Teachers tend to teach the way they were taught. Bramald, Hardman, and Leat, (1995), for example write that, "…student teachers have definite ideas about teaching and learning when they start out in their training, which have developed from their own educational experience and which shape their perceptions of teaching and developing practice." (p. 23). Teacher education alone, however, cannot change teachers' conceptions of mathematics and its teaching but can provide pre-service teachers with awareness of how their mathematics conceptions can influence their teaching.

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