# Preparing a mechanical oscillator in non-Gaussian quantum states

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We propose a protocol for coherently transferring non-Gaussian quantum states from optical field to a mechanical oscillator. The open quantum dynamics and continuous-measurement process, which can not be treated by the stochastic-master-equation formalism, are studied by a new path-integral-based approach. We obtain an elegant relation between the quantum state of the mechanical oscillator and that of the optical field, which is valid for general linear quantum dynamics. We demonstrate the experimental feasibility of such protocol by considering the cases of both large-scale gravitational-wave detectors and small-scale cavity-assisted optomechanical devices.

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*Introduction.*—It is becoming experimentally possible to prepare a macroscopic mechanical oscillator near its quantum ground state by either active feedback or passive cooling in optomechanical devices [1]. This activity has been motivated by (i) the necessity to increase the sensitivity of high-precision measurements with mechanical test bodies up to and beyond the *Standard Quantum Limit* (SQL) [2], and (ii) the test and interpretation of quantum theory, when macroscopic degrees of freedom are involved. However, for unequivocal evidences of quantum behavior, *merely* achieving quantum ground state, or preparing coherent/squeezed states, or overcoming the SQL is *insufficient*: In these situations, the oscillator initially occupies a Gaussian state and remains Gaussian, and therefore its Wigner function is positive and can always be interpreted in terms of a classical probability. *A true demonstration of the quantum behavior requires non-Gaussian quantum states or nonlinear measurements* [3, 4]. A natural approach is to create nonlinear coupling between a mechanical oscillator and external degrees of freedom, e.g., probing mechanical energy [5–8], coupling the oscillator to a qubit [9–11] or (low) cavity photon number [12–14]. For optomechanical devices, this generally requires zero-point uncertainty of the oscillator displacement $x_q$ to be comparable to the cavity linear dynamical range which is characterized by the optical wavelength $\lambda$ divided by the finesse $\mathcal{F}$, i.e.,

$$\lambda/(\mathcal{F}x_q) \lesssim 1. \tag{1}$$

Since $\lambda \sim 10^{-6}$m and $\mathcal{F} \lesssim 10^6$, we have $x_q \gtrsim 10^{-12}$ m, which is several orders of magnitude above the current technology ability.

In this article, we propose a protocol for preparations of non-Gaussian quantum states which *does not require* nonlinear optomechanical coupling. The idea is to inject a non-Gaussian optical state, e.g., a single-photon pulse created by cavity QED [15–17], into the optomechanical devices. Possible configurations are shown schematically

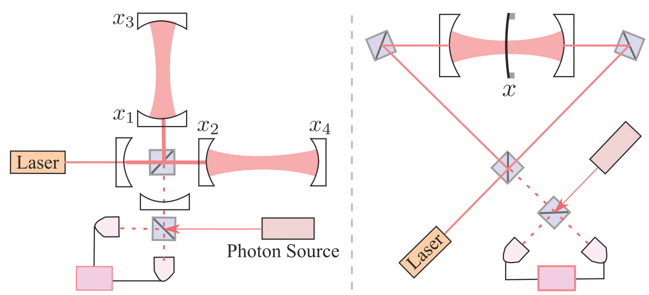


FIG. 1: (Color online) Possible schemes for preparing non-Gaussian quantum states of mechanical oscillators. The left is a Michelson interferometer, similar to an advanced gravitational-wave detector with kg-scale suspended test masses [18, 19]. The right panel shows a small coupled-cavity scheme with a ng-scale membrane inside a high-finesse cavity [7]. In both cases, a non-Gaussian optical state (a photon pulse) is injected into the dark port of the interferometer (local oscillator light for homodyne detection is not shown).

in Fig. 1. The radiation pressure induced by the photon pulse is coherently amplified by the classical pumping at the bright port, and the qualitative requirement for preparing a non-Gaussian state is

$$\lambda/(\mathcal{F}\,x_q) \lesssim \sqrt{N_\gamma}. \tag{2}$$

Here $N_\gamma = I_0\,\tau/(\hbar\omega_0)$ ($I_0$ the pumping laser power and $\omega_0$ the frequency) is the number of pumping photons within the duration $\tau$ of the single-photon pulse, and we gain a significant factor of $\sqrt{N_\gamma}$ compared with Eq. (1), which makes it experimentally achievable. This radiation-pressure-mediated optomechanical coupling is similar to what was considered in Refs. [2, 20–22]. However, there are significant differences: (i) This protocol includes both finite interaction time and photon shape, in which case neither the rotating-wave approximation [20] nor the three-mode approach [21] applies; (ii) To better model an actual experiment, we consider a continuous measurement process rather than a single measurement at some given instant as assumed in Ref. [22]. This takes into account all the information of the oscillator motion that is distributed in the output field, and thus allows us to prepare a nearly *pure* non-Gaussian quantum state of the oscillator; (iii) There are non-trivial quantum correlations at different times (non-Markovianity) due to the finite-duration photon pulse, which cannot be treated by the conventional *stochastic-master-equation* (SME) approach [23–27]. Here we develop a path-integral-based approach, and it applies to general linear quantum dynamics and continuous measurement process.

*A simple case.*—To illustrate the non-Gaussian state-preparation procedure, we first make an order-of-magnitude estimate of experimental requirements by considering a simple case where the cavity decay is much faster than all other time scales and the oscillator can be approximated as a free mass. The corresponding input-output relations, in the Heisenberg picture, simply read:

$$\dot{\hat{x}}(t) = \hat{p}(t)/m\,, \qquad \dot{\hat{p}}(t) = \alpha\,\hat{a}_1(t) + \hat{F}_{\rm th}(t)\,, \tag{3}$$

$$\hat{b}_1(t) = \hat{a}_1(t)\,, \qquad \hat{b}_2(t) = \hat{a}_2(t) + (\alpha/\hbar)\hat{x}(t)\,. \tag{4}$$

Here $\hat{x}$ and $\hat{p}$ are position and momentum; the coupling constant $\alpha \equiv 8\sqrt{2}(\mathcal{F}/\lambda)\sqrt{\hbar I_0/\omega_0}$; $\hat{a}_{1,2}$ and $\hat{b}_{1,2}$ are input and output optical amplitude and phase quadratures, with $[\hat{a}_1(t), \hat{a}_2(t')] = [\hat{b}_1(t), \hat{b}_2(t')] = i\,\delta(t-t')$; $\alpha\,\hat{a}_1$ is the back-action noise; $\hat{F}_{\rm th}$ is the force thermal noise.

Suppose at $t=-\tau$ the oscillator was prepared in some initial Gaussian state $|\psi_m\rangle = \int_{-\infty}^{\infty}\psi_m(x)|x\rangle\,dx$ (the procedure is detailed in Ref. [28]). Subsequently, a photon pulse is injected into the dark port of the interferometer and starts to interact with the oscillator. During this interaction, phase quadrature $\hat{b}_2(t)$ is continuously measured by a homodyne detection, until the photon pulse ends at $t=0$. If photon pulse (i.e., $\tau$) is short such that oscillator position almost does not change, we obtain:

$$\hat{X}(0) = \hat{X}(-\tau), \qquad \hat{P}(0) = \hat{P}(-\tau) + \kappa\,\hat{A}_1 + \hat{P}_{\rm th}\,, \tag{5}$$

$$\hat{B}_1 = \hat{A}_1, \qquad \hat{B}_2 = \hat{A}_2 + \kappa\,\hat{X}(0)\,. \tag{6}$$

We have normalized the oscillator position and momentum by their zero-point uncertainties: $\hat{X} \equiv \hat{x}/x_q$ [$x_q \equiv \sqrt{\hbar/(2m\omega_m)}$] and $\hat{P} \equiv \hat{p}/p_q$ [$p_q \equiv \sqrt{\hbar m\omega_m/2}$]; $\hat{A}_j = \sqrt{1/\tau}\int_{-\tau}^{0} dt\,\hat{a}_j(t)$ ($j=1,2$) which has an uncertainty of unity (i.e., $\Delta\hat{A}_j$=1); $\hat{B}_j = \sqrt{1/\tau}\int_{-\tau}^{0} dt\,\hat{b}_j(t)$; $\hat{P}_{\rm th} = \int_{-\tau}^{0} dt\,\hat{F}_{\rm th}(t)/p_q$; $\kappa \equiv \alpha\sqrt{\tau}/\hbar = 8\sqrt{2}\sqrt{N_\gamma}\mathcal{F}\,x_q/\lambda$.

Eqs. (5) and (6) describe the joint evolution of the oscillator, the optical field and heat bath in the Heisenberg picture (with $\hat{B}_j$ viewed as the evolved versions of $\hat{A}_j$). They transform back into an evolution operator of $\hat{U} = \exp[i(\kappa\hat{A}_1\hat{X} + \hat{P}_{\rm th}\hat{X})]$ in the Schrödinger picture. The corresponding density matrix of the system at $t=0$ is given by $\hat{\rho} = \hat{U}|\psi_o\rangle|\psi_m\rangle\hat{\rho}_{\rm th}\langle\psi_m|\langle\psi_o|\hat{U}^\dagger$, where $|\psi_o\rangle = \int_{-\infty}^{\infty}\psi_o(A_2)|A_2\rangle\,dA_2$ is the initial non-Gaussian optical state, and $\hat{\rho}_{\rm th}$ describes the heat bath associated with $\hat{F}_{\rm th}$. Given homodyne detection of $\hat{B}_2$ with a precise result $y$, the oscillator is projected into the following conditional state: $\hat{\rho}_m(y) = {\rm Tr}_{\rm th}\left[\langle y|\hat{U}|\psi_o\rangle|\psi_m\rangle\hat{\rho}_{\rm th}\langle\psi_m|\langle\psi_o|\hat{U}^\dagger|y\rangle\right]$. In the ideal case of negligible thermal noise, the conditional wave function $\psi_m^c(x)$ of the mechanical oscillator is simply

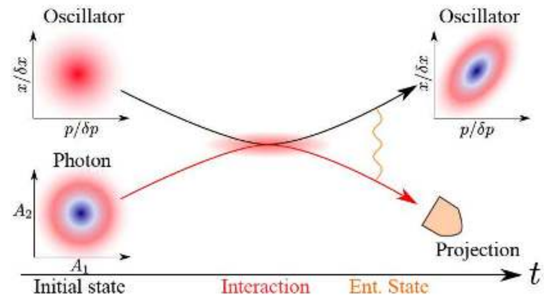


FIG. 2: (Color online) A schematic of the non-Gaussian state-preparation process. The interaction entangles the oscillator state and the optical state (depicted by their Wigner functions). Subsequent measurements of the optical fields disentangle the system and projects the oscillator into a non-Gaussian conditional quantum state.

$$\psi_m^c(x) = \psi_o(y-\kappa x)\psi_m(x) \tag{7}$$

—*the optical state is mapped onto the mechanical oscillator* as illustrated in Fig. 2. A complete mapping occurs when $\psi_m(x)\approx$ const, and this requires the momentum fluctuation due to optomechanical coupling be larger than the initial one, namely, $\kappa > 1$ or equivalently

$$\lambda/(\mathcal{F}\,x_q) < 8\sqrt{2}\sqrt{N_\gamma}, \tag{8}$$

which justifies Eq. (2).

When thermal noise is considered, non-Gaussianity can still remain, as long as thermal noise induces a smaller momentum fluctuation than the optomechanical interaction. This condition, in the high-temperature limit — $\langle\hat{F}_{\rm th}(t)\hat{F}_{\rm th}(t')\rangle = 4m\gamma_m k_B T\delta(t-t')$, reads

$$\lambda/(\mathcal{F}\,x_q)\sqrt{n_{\rm th}/Q_m}\sqrt{\omega_m\tau} < 8\sqrt{2}\sqrt{N_\gamma} \tag{9}$$

with $Q_m \equiv \omega_m/\gamma_m$ the mechanical quality factor and $n_{\rm th} \equiv k_B T/(\hbar\omega_m)$ the thermal occupation number. *These two conditions set the benchmarks for a successful non-Gaussian state-preparation experiment.* They can be satisfied with experimentally feasible specifications as shown in Table I, in which the first row is similar to the case of large-scale gravitational-wave detectors [19] and

TABLE I: Possible experimental specifications

| | $\lambda$ | $\mathcal{F}$ | $m$ | $\omega_m/2\pi$ | $Q_m$ | $T$ | $\tau$ |
|---|---|---|---|---|---|---|---|
| large scale | $1\mu$m | 6000 | 4kg | 1Hz | $10^8$ | 300K | 1ms |
| small scale | $1\mu$m | $10^4$ | 1ng | $10^5$Hz | $10^7$ | 4K | 0.01ms |

the second row is for small-scale optomechanical devices (e.g., the one in Ref. [7]). These qualitative results will be justified by a rigorous treatment below.

*General formalism.*—In general, the optomechanical interaction strength is finite and the oscillator has non-negligible displacement during the interaction, the cavity bandwidth can be comparable to the mechanical frequency, and thermal noises can be non-Markovian. All these factors obstruct finding a finite set of variables similar to $(\hat{X}, \hat{P}, \hat{A}_1, \hat{A}_2)$ that satisfy a closed set of equations [cf. Eqs. (5) and (6)]. It is therefore hard to determine, *a priori*, the finite number of observables that one has to measure to project the oscillator into a desired conditional state.

To address these issues, we adopt the Heisenberg picture starting from $t = -\infty$, and write down the initial density matrix as $\hat{\rho}_{in} = \hat{\rho}_m(-\infty) \otimes \hat{\rho}_o \otimes \hat{\rho}_{\rm th}$. Details of $\hat{\rho}_m(-\infty)$ for the oscillator and whether the initial state is truly a direct product, do not matter, because the system is stable, and the initial position and momentum will decay away after several mechanical relaxation times. For the optical state, we consider an arbitrary spatial mode given by $f(x/c)$, whose annihilation operator is

$$\hat{\Gamma} \equiv \textstyle\int_{-\infty}^0 dt f(t)[\hat{a}_1(t) + i\hat{a}_2(t)]/\sqrt{2}. \tag{10}$$

A general state of this mode can be written in the P-representation as $\hat{\rho}_o = \int d\boldsymbol{\zeta}\, P(\boldsymbol{\zeta})|\zeta\rangle\langle\zeta|$, where vector $\boldsymbol{\zeta} \equiv (\Re[\zeta], \Im[\zeta])$ and $|\zeta\rangle \equiv \exp[\zeta\,\hat{\Gamma}^\dagger - \zeta^*\hat{\Gamma}]|0\rangle$.

A continuous measurement of the output optical quadrature $\hat{y}(t) \equiv \cos\theta\,\hat{b}_1(t) + \sin\theta\,\hat{b}_2(t)$ for $t \in (-\infty, 0]$, *projects the entire system into a conditional state:*

$$\hat{\rho}_c[y(t)] = \hat{\mathcal{P}}_y\,\hat{\rho}_{in}\hat{\mathcal{P}}_y/\mathrm{Tr}[\hat{\mathcal{P}}_y\,\hat{\rho}_{in}\hat{\mathcal{P}}_y]. \tag{11}$$

The operator $\hat{\mathcal{P}}_y$ projects the output field into the subspace where $\hat{y}(t)$ agrees exactly with the measured results $y(t)$. To simplify output correlations at different times, we can *causally whiten* $\hat{y}(t)$ into $\hat{z}(t)$ such that $\langle\hat{z}(t)\hat{z}(t')\rangle = \delta(t-t')$, as detailed in Ref. [28]. Since the output quadratures at different times also commute, i.e., $[\hat{z}(t), \hat{z}(t')] = 0$, the projection $\hat{\mathcal{P}}_y$ can then be expressed as the product of Dirac-$\delta$ functions that project each $\hat{z}(t)$ into its measured value $z(t)$:

$$\begin{aligned}\hat{\mathcal{P}}_y = \hat{\mathcal{P}}_z &= \prod_{-\infty<t<0} \delta[\hat{z}(t) - z(t)] \\ &= \int \mathcal{D}[\xi] \exp\left\{i \textstyle\int_{-\infty}^0 dt\, \xi(t)[\hat{z}(t) - z(t)]\right\}. \end{aligned} \tag{12}$$

with $\int \mathcal{D}[\xi]$ denoting the path integral. This allows us to take the entire measurement history for $z$ (or equivalently $y$) and project into the corresponding subspace in a single step, instead of having to successively project output-field degrees of freedom continuously at each time step as in the case of SME approach, thereby allowing a non-Markvonian input field.

The generating function for the oscillator state is then

$$\mathcal{J}[\boldsymbol{\alpha}; z(t)] \equiv \mathrm{Tr}\left[e^{i\,\boldsymbol{\alpha}\,\hat{\boldsymbol{x}}_0'}\hat{\rho}_c[z(t)]\right], \tag{13}$$

where $\boldsymbol{\alpha} \equiv (\alpha_x, \alpha_p)$, $\hat{\boldsymbol{x}}_0 \equiv (\hat{x}(0), \hat{p}(0))$, and superscript $'$ denotes transpose. From Eqs. (11) and (12), we have

$$\mathcal{J} = \int d\boldsymbol{\zeta}\, P(\boldsymbol{\zeta}) \int \mathcal{D}[\xi]\, e^{i[\zeta^*\hat{\Gamma} - \zeta\,\hat{\Gamma}^\dagger,\, \hat{B}]}\langle 0|e^{i\hat{B}}|0\rangle \tag{14}$$

with $\hat{B} \equiv \boldsymbol{\alpha}\,\hat{\boldsymbol{x}}_0' + \int_{-\infty}^0 dt\,\xi(t)[\hat{z}(t) - z(t)]$. This can be evaluated by decomposing $\hat{\boldsymbol{x}}_0 \equiv \hat{\boldsymbol{R}} + \int_{-\infty}^0 dt\,\boldsymbol{K}(-t)\hat{z}(t)$ where $\boldsymbol{K} \equiv (K_x, K_p)$ are causal Wiener filters, $\boldsymbol{K}(-t) = \langle 0|\hat{z}(t)\hat{\boldsymbol{x}}_0|0\rangle$ and $\hat{\boldsymbol{R}} \equiv (\hat{R}_x, \hat{R}_p)$ are parts of displacement and momentum uncorrelated with the output: $\langle 0|\hat{R}_{x,p}\hat{z}|0\rangle = 0$. Completing path integral, we obtain

$$\mathcal{J} = \int d\boldsymbol{\zeta} e^{-[\boldsymbol{\alpha}\mathbb{V}_c\boldsymbol{\alpha}' + \|z - 2\,\boldsymbol{\zeta}\boldsymbol{L}'\|^2]/2 + i\,\boldsymbol{\alpha}\,(\zeta^*\gamma' + \zeta\gamma^\dagger + \boldsymbol{x}_c')} P(\boldsymbol{\zeta}). \tag{15}$$

Here $\|a\|^2 \equiv \int_{-\infty}^0 a(t)a^*(t)dt$ and we have defined vectors $\boldsymbol{\gamma} \equiv [\hat{\Gamma}, \hat{\boldsymbol{R}}]$ and $\boldsymbol{L} \equiv (\Re[L], \Im[L])$ with $L(t) \equiv [\hat{\Gamma}, \hat{z}(t)]$, which characterize the extent of photon mode influence on the fluctuations of $\hat{x}(0)$ and $\hat{p}(0)$, and output field $\hat{z}$; quantities $\mathbb{V}_c \equiv \langle 0|\hat{\boldsymbol{R}}^T\hat{\boldsymbol{R}}|0\rangle$ and $\boldsymbol{x}_c \equiv (x_c, p_c) = \int_{-\infty}^0 dt\boldsymbol{K}(-t)z(t)$ are the conditional covariance matrix and means of $\hat{x}(0)$ and $\hat{p}(0)$ when the optical state is vacuum. The resulting *conditional* Wigner function reads

$$W[\boldsymbol{x}; z(t)] = \int d\boldsymbol{\zeta} e^{-[\boldsymbol{\chi}\mathbb{V}_c^{-1}\boldsymbol{\chi}' + \|z - 2\,\boldsymbol{\zeta}\boldsymbol{L}'\|^2]/2} P(\boldsymbol{\zeta}) \tag{16}$$

with $\boldsymbol{\chi} \equiv \boldsymbol{x} - \boldsymbol{x}_c - \zeta^*\boldsymbol{\gamma} - \zeta\boldsymbol{\gamma}^*$. *This formula directly relates the injected optical state to the state of the mechanical oscillator.* In deriving it, we only use the linearity of quantum dynamics rather than specific equations of motion. For cavity-assisted optomechanical system, one can obtain $\boldsymbol{\gamma}$, $\mathbb{V}_c$, $\boldsymbol{K}$ and $L$ from input-output relations in Refs. [29–31] by using formalism developed in Ref. [28].

*Single-photon case.*—As an example, we consider the simplest case of a single-photon injection, with $\hat{\rho}_o = |1\rangle\langle 1|$ and $P(\boldsymbol{\zeta}) = e^{|\zeta|^2}\partial^2\delta^{(2)}(\zeta)/\partial\zeta\partial\zeta^*$. From Eqs. (16), it gives

$$\begin{aligned} W[\boldsymbol{x}; z(t)] &= \frac{1 - \boldsymbol{\gamma}\mathbb{V}_c^{-1}\boldsymbol{\gamma}^\dagger - \|L\|^2 + |\boldsymbol{\gamma}V_c^{-1}\delta\boldsymbol{x}' + Z|^2}{1 - \|L\|^2 + |Z|^2} \\ &\quad \frac{1}{2\pi\sqrt{\det\mathbb{V}_c}}\exp\left[-\frac{1}{2}\delta\boldsymbol{x}\mathbb{V}_c^{-1}\delta\boldsymbol{x}'\right] \end{aligned} \tag{17}$$

where $\delta\boldsymbol{x} \equiv \boldsymbol{x} - \boldsymbol{x}_c$ and $Z \equiv \int_{-\infty}^0 dt\, z(t)L(t)$. This Wigner function depends on the measurement result $z(t)$, $t \in (-\infty, 0]$ through four quantities, the two components of $\boldsymbol{x}_c$ (through $\delta\boldsymbol{x}$) and the real and imaginary parts of $Z$: $Z$ determines the shape of $W$, and $\boldsymbol{x}_c$ describes the translation of $W$. The random vector $\boldsymbol{Z} = (\Re[Z], \Im[Z])$ has a two-dimensional probability density of

$$w[\boldsymbol{Z}] = \frac{1 - \|L\|^2 + \boldsymbol{Z}\boldsymbol{Z}'}{2\pi\sqrt{\det\mathbb{V}_L}}\exp[-\boldsymbol{Z}\mathbb{V}_L^{-1}\boldsymbol{Z}'/2], \tag{18}$$

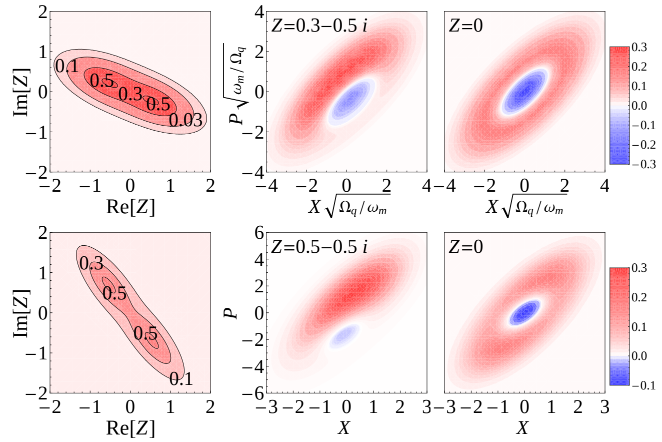

FIG. 3: (Color online) Distributions of measurement results (left panels) and the corresponding Wigner function of the oscillator given the most probable measurement results (middle panels) and less probable results but with a significant non-Gaussianity (right panels). The upper panels show the case of non-Gaussian state-preparation with future gravitational-wave detectors, and the lower panels for small-scale devices. We used normalized coordinates (with respect to $x_q$ and $p_q$) and introduced $\Omega_q \equiv \sqrt{\hbar m/\alpha^2}$.

where matrix $\mathbb{V}_L \equiv \int_{-\infty}^{0} dt \boldsymbol{L}'\boldsymbol{L}$.

The pre-factor in the Wigner function [cf. Eq. (17)] is a second-order polynomial in $\boldsymbol{x}$, which resembles that of a single-photon. For strong non-Gaussianity, significant $\boldsymbol{\gamma}$ and $\|L\|^2$ (making $\boldsymbol{\gamma}$ terms in the pre-factor to prevail) are essential — these physically correspond to requiring that the photon mode must influence the fluctuation of $\hat{x}$ and $\hat{p}$, as well as $\hat{z}$ strongly. It in turn requires the photon coherence time to be comparable to the *measurement time scale* characterized by $\boldsymbol{K}$. It is possible for small-scale optomechanical devices with high-frequency mechanical oscillators. The corresponding photon can be generated by a cavity QED scheme [15–17]. While for large-scale gravitational-wave detectors, the time scale is $\sim$ 10 ms and it is challenging to create photons with comparable coherent length. However, developments of low-frequency squeezing source [32] will eventually solve this issue.

With Eq. (17), we can justify the simple-case qualitative results. We use the same specifications listed in Table I. As an example, we assume a photon shape of $f(t) = \sqrt{2\gamma_f} e^{(\gamma_f + i\omega_f)t}$ and specify that $\omega_f/2\pi = \gamma_f/2\pi = 70$ Hz in the case of future gravitational-wave detectors, and $\omega_f/\omega_m = 0.1$, $\gamma_f/\omega_m = 0.3$ for small-scale experiments. The Wigner functions for some given measurement results are shown in Fig. 3. In both cases, there are negative regions in the Wigner function, which is a unique feature of the quantumness. The prepared non-Gaussian quantum state can be independently verified using the quantum tomography protocol developed in Ref. [33] that allows sub-Heisenberg accuracy of Wigner function reconstruction, which is crucial for revealing those negativity regions.

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