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AUTHOR	Heller, Joan I.; Reif, F.
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ABSTRACT

A theoretical model specifying the underlying knowledge and procedures whereby human subjects can generate effective initial descriptions of scientific problems was formulated. The model is prescriptive since it does not necessarily try to simulate the behavior of actual experts nor assume that their performance is optimal. The model, elaborated in the domain of mechanics, specifies explicit procedures for redescribing problems in terms of a relevant knowledge base. To test the model, carefully controlled experiments were devised where human subjects were induced to act in accordance with alternative models and where their resulting performance was observed in detail. Such experiments, carried out with undergraduate physics students, showed that the proposed model is sufficient to generate excellent problem descriptions, that these markedly improve subsequent problem solutions, and that most components of the model are indeed necessary for good performance. Detailed data analysis also showed how the model predictably prevents the occurence of many common errors. Such a validated model of effective problem description provides a useful basis for teaching students improved scientific problem-solving skills. (Author/JN)

University of California, Berkeley, California 94720
Physics Department and Group in Science and Mathematics Education

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PRESCRIBING EFFECTIVE HUMAN PROBLEM-SOLVING PROCESSES: PROBLEM DESCRIPTION IN PHYSICS

Joan I. Heller and F. Reif

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Abstract

We formulate a theoretical model specifying the underlying knowledge and procedures whereby human subjects can generate effective initial descriptions of scientific problems. This model is prescriptive since it does not necessarily try to simulate the behavior of actual experts nor assume that their performance is optimal. The model, elaborated in the domain of mechanics, specifies explicit procedures for redescribing problems in terms of a relevant knowledge base. To test this model we devised carefully controlled experiments where human subjects are induced to act in accordance with alternative models and where their resulting performance is observed in detail. Such experiments carried out with undergraduate physics students, show that the proposed model is sufficient to generate excellent problem descriptions, that these markedly improve subsequent problem solutions, and that most components of the model are indeed necessary for good performance. Detailed analysis of the data also shows how the model predictably prevents the occurrence of many common errors. Such a validated model of effective problem description provides a useful basis for teaching students improved scientific problem-solving skills.

Problem solving is a centrally important and intellectually demanding activity in any science. Hence it is a challenging task to design instructional methods for teaching students good scientific problem-solving skills. Effective instructional design, however, depends crucially on an adequate understanding of how good scientific problem-solving performance is achieved and of how novice students perform before instruction.

Recent years have witnessed considerable interest in analyzing cognitive processes and knowledge structures underlying problem-solving performance in several scientific domains, e.g., in geometry (Greeno, 1978), physics (Simon & Simon, 1978), or computer programming (Polson & Jeffries, 1981). In particular, problem solving in the domain of physics has been studied by an especially large variety of approaches. A number of investigations have examined the problem-solving performance of subjects of different levels of expertise (e.g., Simon & Simon, 1978; Larkin & Reif, 1979; Larkin, McDermott, Simon, & Simon, 1980a; Chi, Feltovich, & Glaser, 1981). Other studies have identified naive conceptions or misconceptions of physics students (e.g., Viennot, 1979, Champagne, Klopfer, & Anderson, 1980, McCloskey, Caramazza, & Green, 1980; Trowbridge & McDermott, 1980, 1981; Clement, 1982, diSessa, 1982). Efforts have also been made to develop process models of physics problem solving. These have included psychological models to describe and simulate the performance of human subjects of different levels of ability (e.g., Larkin, McDermott, Simon & Simon, 1980; Larkin, 1981), models recently summarized by Chi, Glaser, & Rees (1981). They have also included some artificial-intelligence models embodied in computer programs less concerned with simulating human performance (e.g., de Kleer, 1977, Novak 1977, Bundy, 1978; Bundy, Byrd, Luger, Mellish, & Palmer, 1979; Byrd & Borning, 1980; Luger, 1981).

Although studies of the thought processes of experts and novices have yielded valuable insights about effective problem solving, they have significant limitations. For example, it is unwise to assume that the performance of experts is necessarily optimal. Furthermore, educational efforts must do more than merely teach students to perform like experts. Instead it is often necessary to teach students to use explicit procedures to accomplish tasks which experts perform almost automatically because they recognize patterns familiar to them as a result of years of experience.

Accordingly, the work discussed in this paper has sought to study human problem solving from a more general point of view which transcends the investigation of naturally occurring intellectual functioning. In particular, our aim has been to specify cognitive processes and knowledge structures which lead to good human problem solving in a realistic scientific domain, <u>without</u> necessarily trying to simulate what actual experts do and <u>without</u> assuming that experts always perform optimally. Such a "prescriptive" approach is clearly more general than a descriptive one since it allows greater freedom for theoretical inventiveness and controlled experimental manipulations. For example, although a prescriptive theoretical model of good intellectual performance may be partly suggested by naturalistic observations of experts, it may also be proposed on the basis of purely theoretical task analyses. Correspondingly, the sole criterion of validity of such a prescriptive model is that it lead to predictably effective performance when implemented by a person, even if it does not mimic what actual experts do.

An analogy may help to clarify the distinction between a prescriptive point of view and a descriptive one. Imagine that a hypothetical cognitive scientist, working in the era of Julius Caesar, had been trying to formulate a theoretical model of good performance in arithmetical problem solving. If this model had proposed the use of the modern place-value representation of numbers, it would have led to very good arithmetical performance and would thus have been an excellent prescriptive model. However, it would have been an unsatisfactory descriptive model of the behavior of contemporary experts since they used Roman numerals.

A prescriptive approach is quite common in artificial intelligence (AI), but can also have considerable interest in work on human cognitive processes (Reif, 1979). In particular, by identifying essential knowledge required for good performance, it can help to make explicit expert knowledge which is often largely tacit. Furthermore, a prescriptive approach is of essential importance for any efforts aiming to improve human performance, to design effective instruction, or to exploit the potentialities of person-computer interaction.

The prescriptive approach adopted in our work on human problem solving has some similarities to work in AI. (a) As in AI, our main interest is in formulating models of effective functioning on complex intellectual tasks, without necessarily trying to simulate what human experts do. However, since our models are specifically designed to be implementable by human information processors rather than by computers, they presuppose explicitly that these information processors possess certain distinctly human limitations (e.g., limitations of short-term memory and processing speed) and human capabilities (e.g., abilities to understand natural language, to construct and interpret diagrams, etc.). A corresponding advantage is that one may then focus primary attention on centrally important problem-solving processes, <u>without</u> the need to deal explicitly with more basic prerequisite skills whose existence may be assumed without special analysis. (b) As in AI, the efficacy of a theoretical model of good performance is tested by expressing it in the form of an explicit "program" which can be implemented by an information processor. However, in our experiments (described more fully in later sections) the information processor is a human subject, rather than a computer, and the program expressing the model consists correspondingly of instructions designed to be reliably executable by a human subject. The advantage of this method (unlike some forms of computer simulation) is that a model of human cognitive functioning can be tested in a way which is clearly valid and relevant to human subjects. Furthermore, once such a model has been validated empirically, it is already in a form well adapted for human use. Hence it provides a ready basis for designing methods to teach problem-solving skills to human subjects.

In seeking to specify the knowledge and procedures leading to good human performance in a scientific domain, we have focused our attention on problem-solving in basic college-level physics, specifically in the field of mechanics. This scientific domain is realistically complex, often difficult for many students, and representative of other quantitative scientific or engineering fields. On the other hand, this domain is sufficiently simple and well-defined to be amenable to a detailed analysis of underlying cognitive processes. Furthermore, such an analysis can draw upon insights derived from previous observations of experts and novices in this domain.

PRESCRIPTIVE MODEL OF PROBLEM DESCRIPTION

We have presented elsewhere a prescriptive theoretical model of effective problem solving in physics (Reif & Heller, in press). This model specifies some general procedures to be used in conjunction with a knowledge base about a particular scientific domain. The general procedures subdivide the problem-solving process into three major stages: (a) the generation of an initial problem description and qualitative analysis designed to facilitate the subsequent construction of a problem solution; (b) the generation of the actual solution by methods facilitating judicious decisions needed for efficient search; and (c) the assessment and improvement of this solution. The domain-specific knowledge base has characteristics specifically designed to facilitate the implementation of these procedures. In particular, it contains declarative knowledge of concepts and principles useful in the particular scientific domain, is organized hierarchically and described at various levels of detail, and includes explicit guidelines facilitating appropriate application of the declarative knowledge.

The work described in the present paper has aimed to study specifically the generation of effective initial problem descriptions. Hence the study deals only with one aspect of a more encompassing model of effective problem solving. But this aspect is of crucial importance since the initial description of a problem often determines how easily the problem can subsequently be solved or whether it can be solved at all. Furthermore, observations of actual experts provide relatively little direct information about the description process since experts tend to describe problems

rapidly, and almost automatically, on the basis of large amounts of tacit knowledge.

According to our prescriptive model, the generation of an initial problem description can usefully be decomposed into two successive stages. The first of these uses general domain-independent knowledge to generate a "basic description" of a problem. This basic description merely aims to identify explicitly the information specified and wanted in the problem, to identify relevant processes and subprocesses in the problem situation, to introduce useful symbols, and to express the relevant information in various convenient symbolic representations (e.g., in pictorial as well as verbal forms).

The second stage of the description process aims to generate a "theoretical description" of the problem by deliberately redescribing the problem in terms of the special concepts provided by the knowledge base for the relevant domain. In particular, the generation of such a theoretical description involves identifying the special entities of interest in the problem, describing these entities in terms of the special concepts provided by the knowledge base, and exploiting the known properties of these concepts. Since all principles in the knowledge base are expressed in terms of these special concepts, the theoretical description of the problem makes all these principles readily accessible for the subsequent qualitative analysis and later solution of the problem. Hence the initial theoretical description of a problem greatly facilitates the search for its solution.

The basic description of a problem can usually be generated relatively easily by a procedure outlined in Reif and Heller (in press). The generation of the subsequent theoretical description of a problem is more complex and is the main topic discussed in the following pages. Thus we shall first outline a prescriptive model specifying the knowledge and procedures needed to generate good theoretical problem descriptions, with particular emphasis on problems in the prototype domain of mechanics. Then we shall delineate some special experimental methods useful for testing such a prescriptive model. After these general remarks, we shall describe in detail the implementation of these methods by specific experiments. Finally, we shall discuss the results of these experiments and some of their implications.

Theoretical Problem Description in Mechanics

As already mentioned, the theoretical description of a problem aims to redescribe the problem in terms of the concepts and information provided by the knowledge base for the relevant domain. An effective knowledge base about any particular domain should, therefore, have characteristics which facilitate the generation of a theoretical description of any problem encountered in this domain. Accordingly, any such knowledge base should specify the entities of particular interest in this domain, the special concepts most useful for describing these entities, and the important properties of these concepts. Furthermore, the knowledge base should include explicit guidelines specifying how the preceding declarative knowledge is to be used to describe any situation in this domain.

The preceding general comments can be elaborated into the following prescriptive model specifying the structure of the knowledge base and

associated procedures for generating a theoretical description of any problem in the domain of mechanics.

The knowledge base about this domain specifies that the entities of particular interest in this domain are particles (i.e., objects small or simple enough to be adequately described by single points) and more complex systems consisting of many such particles (e.g., strings, rigid bodies, etc.). As indicated in Figure 1, the knowledge base introduces two different kinds of special concepts to describe such particles, "individual descriptors" and "interaction descriptors". The individual descriptors describe particles by themselves, without regard to interactions between them. Some of these individual descriptors are merely "intrinsic descriptors" (such as "mass" or "electric charge") used to characterize any particle; the other individual descriptors are "motion descriptors" (such as "position", "velocity", "acceleration") specifically used to describe the motion of any particle. By contrast, the interaction descriptors do not describe individual particles, but the interaction between such particles. For example, the "force" exerted on a particle by some other particle is one such interaction descriptor, "potential energy" is another one.

Insert Figure 1 about here

The knowledge base for mechanics specifies important properties of the preceding descriptors. In particular, "interaction laws" specify how the interaction descriptors are related to the individual descriptors of the interacting particles (e.g., how the force on one particle by another is related to the intrinsic characteristics of the particles and to their

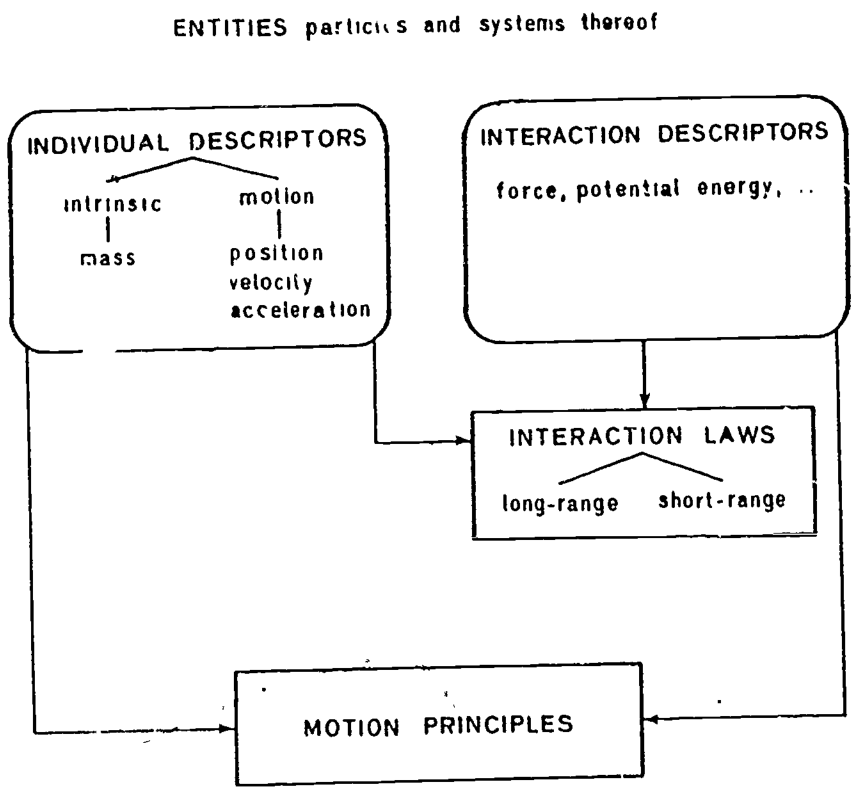

Figure 1. Overview of the knowledge base for mechanics

positions). Such interaction laws are specified for various kinds of interactions encountered in nature. These interactions can be classified into the following two types: Some of these interactions are "long-range" because they are appreciable even if the interacting particles are separated by an appreciable distance. (The prime example is the gravitational interaction of a particle with the earth.) The other interactions are "short-range" because they are only appreciable when the interacting particles are so close that they "touch" each other. (Examples are the interaction of a particle in contact with a string or with the surface of a solid object.)

Lastly, the knowledge base for mechanics specifies important "motion principles" which specify how the motion descriptors of particles change with time as a result of the interaction between particles (e.g., how the acceleration of a particle depends on the force on this particle by other particles). These motion principles provide the science of mechanics with its great predictive power.

The preceding factual knowledge in the knowledge base for mechanics is accompanied by explicit guidelines specifying how this knowledge is to be used for generating an explicit theoretical description of any problem in mechanics. According to our prescriptive model, these guidelines specify a description procedure consisting of the major steps summarized in Table 1.

Insert Table 1 about here

Note that the description procedure of Table 1 exploits extensively the specialized factual information in the knowledge base of mechanics

Table 1

Summary of Procedure for Generating a Theoretical Problem Description

- *Relevant times and systems*: At each relevant time (previously identified in the basic problem description) identify those systems (particles or other special systems of interest in mechanics) relevant in the problem because information about them is wanted, or because they interact with such systems directly or indirectly.
- *Description of relevant systems*: At each relevant time, describe each relevant system as follows, introducing convenient symbols and expressing simply related quantities in terms of the same symbol:
  - *Description of motion*: Draw a "motion diagram" indicating available information about the position, velocity, and acceleration of each particle.
  - *Description of forces*: Draw a "force diagram" indicating available information about all external forces on the system. Identify these forces as follows:
    - *Long-range forces*. Identify all other objects interacting with the given system by long-range interactions (ordinarily this is just the earth interacting by gravitational interaction). For each such interaction, indicate on the diagram the corresponding force and all available information about it.
    - *Short-range forces*: Identify every other object which touches the given system and thus interacts with it by short-range interaction. For each such interaction, indicate on the diagram the corresponding force and all available information about it.
- *Checks of description*: Check that the descriptions of motion and interaction are qualitatively consistent with known motion principles (e.g., that the acceleration of each particle has the same direction as the total force on it, as required by Newton's motion principle $m\underset{\sim}{a} = \underset{\sim}{F}$).

(see Figure 1). Thus it describes explicitly every system of interest in this domain in terms of the special motion descriptors (e.g., velocity and acceleration) and interaction descriptors (e.g., forces) specified by the knowledge base. It deliberately uses the classification of interactions into long-range and short-range types to specify explicit criteria for identifying all forces on a system. It also incorporates in the description known relations between all these descriptors. For instance, the knowledge base of Figure 1 is used to incorporate in the description known relations between individual descriptors (e.g., relations between motion descriptors such as acceleration and velocity), known relations between interaction descriptors (e.g., Newton's third law specifying that the mutual forces between interacting particles are of equal magnitude but opposite directions), and known relations specified by interaction laws (e.g., relations specifying how gravitational forces, or forces exerted by strings, are related to the properties of the interacting systems). Finally, the motion principles are used to check that the resulting theoretical description is internally consistent.

The description procedure of Table 1 will be elaborated more fully, and illustrated in specific cases, in connection with the experiments discussed later in this paper.

Our prescriptive theoretical model would predict that the implementation of the description procedure of Table 1 by a human subject should lead to the following important consequences:

(i) It should generate an explicit and detailed initial description of any mechanics problem in terms of the special concepts of this scientific domain. In particular, such a description should be appreciably more explicit than descriptions overtly generated by actual experts or than descriptions commonly presented in textbooks.

(2) The description procedure of Table 1 should help human subjects to avoid many of the errors commonly committed by students. For example, the explicitness of the procedure should help to avoid errors due to the omission of relevant forces or due to the enumeration of extraneous forces not produced by interaction with any discernible objects.

(3) The description procedure should sometimes lead to significantly easier reformulations of certain problems. (For example, a question asking "when a string becomes slack" would automatically be translated into a question asking "when the force by the string becomes zero". Similarly, a question asking "when a particle slides off a surface" would be translated into a question asking "when the force on the particle by the surface becomes zero". Such reformulated questions, involving the properties of familiar forces, are much more easily interpreted and answered).

(4) The explicit problem description generated by the procedure of Table 1 should appreciably facilitate the subsequent solution of a problem (since this description incorporates already much information needed for the subsequent generation of equations and since it helps to limit appreciably the range of plausible alternatives to be considered in the decisions needed to generate such equations). Indeed, the generation of the initial description of a problem may sometimes constitute the major difficulty of a problem and, once implemented, may make the subsequent solution of the problem fairly trivial.

Methods for Testing a Prescriptive Model

In the preceding sections we have outlined a prescriptive model whereby a human subject should reliably be able to generate useful initial problem descriptions. As mentioned previously, the ultimate criterion of validity of such model is not whether it simulates closely what actual experts do, but whether it leads to predictably good performance. Accordingly, the basic paradigm for testing the validity of such a prescriptive model is the following: Induce a human subject to act in accordance with the prescriptive model and observe whether the resulting performance is effective in the predicted ways.

The particular way in which we have sought to implement this general paradigm is in experiments in which a human subject is induced to act under "external control". To clarify this experimental method by an analogy, consider the familiar situation where a pilot lands his or her plane in bad weather while following directions from an air-traffic controller on the ground. Under these conditions, a human information processor (the pilot) makes extensive use of his or her sophisticated knowledge, but relegates higher-level control of this knowledge to external directions. This situation can be viewed as an experiment with the following interesting characteristics: (1) It allows a separation of high-level control knowledge from lower-level implementation knowledge. For example, if the plane were to crash, the information retrievable from the taped conversation between the pilot and ground control would allow one to distinguish whether the crash occurred as a result of appropriate control directions improperly executed by the pilot, or whether it occurred as a result of faulty control directions. By contrast, if a pilot crashed the plane while flying entirely under his or her own control, one could not distinguish whether the fault was in the pilot's higher-level control knowledge or lower-level implementation knowledge. (2) A set of control directions, specifying how to land a plane, can be viewed as a cognitive theory specifying how a human subject, with sophisticated human capabilities, can land a plane. In other words, such control directions would constitute a good theory of plane landing if, and only if, the correct execution of these directions leads to reliably effective landing of planes. (3) Such a validated theory of plane landing could ultimately be used as the basis of a theory of instruction for landing planes. In particular, such an instructional theory would need to teach human subjects to internalize, and carry out independently, the control directions which had previously been external.

Let us now turn from this analogy to external-control experiments designed to test other prescriptive models of human performance, e.g., models of effective problem description. To carry out such experiments, one needs first to design a "program" consisting of step-by-step directions, and associated knowledge, whereby a human subject can be guided to act in accordance with a specified model of performance. For example, such a program might guide a human subject explicitly to execute the description procedure outlined in Table 1. The program should be problem-independent, i.e., equally applicable to any problem in the specified domain. Furthermore, one must make sure that the directions in such a program are properly matched to the characteristics and pre-existing knowledge of the human subjects for whom the program is designed. In particular, the individual directions specified by the problem must be reliably interpretable and executable by the human subject. They must also be formulated at an appropriate level of detail, i.e., detailed enough to provide adequate guidance, but not so excessively detailed as to be burdensome or distracting to the subject.

In the actual experimental procedure an individual human subject is then asked to carry out specified tasks (e.g., the description and subsequent solution of various problems) by executing on-line successively stated directions according to a program specified by the model. While so doing, the subject is asked to talk out loud about his or her thought processes and the whole session is tape-recorded. Detailed data can thus be gathered about the subject's written output and verbalized thought processes while responding to the external control directions.

Such detailed observations allow one to obtain the following kinds of information to test the proposed model of performance:

(1) One can ascertain whether the proposed model of good performance is, in fact, sufficient to lead to good performance. This can be done by determining whether subjects, working under external control in accordance with the model, do indeed achieve good performance. (Note that such experiments do not imply that the proposed model is unique since other models might conceivably also lead to good, or even better, performance.)

(2) One can verify that the prerequisite basic knowledge, which the model presupposes of human subjects, is by itself not sufficient to produce good performance. This can be done by letting subjects, with such knowledge, work without external guidance of the model and observing that the resulting performance is poor.

(3) One can ascertain whether selected features of the proposed model are, in fact, necessary to achieve good performance. This can be done by comparative experiments where human subjects work under external

control of a modified model which lacks selected features of the proposed model of good performance. Predictable performance deficiencies should then occur.

(4) Finally, one can test whether the proposed model of good performance, when implemented, leads to specific predicted features in the resulting performance. For example, one can ascertain whether, and how, the occurrence of specific errors is prevented when human subjects act in accordance with the model.

It should be emphasized that the aim of such external-control experiments is to ascertain the merits of a proposed model of good performance, but not to teach. Subjects may, of course, learn incidentally while working under conditions of external control. However, such learning need not occur because external control directions may not become internalized. For example, a subject, performing very well while working under external control, might revert to poor performance when external control knowledge is subsequently removed.

EXPERIMENTS TESTING THE MODEL OF DESCRIPTION

We implemented the general approach, outlined in the preceding section, in specific experiments designed to test the proposed procedures for generating effective theoretical descriptions of problems in mechanics. In particular, these experiments were designed to answer the following questions: (a) Does the procedure lead to explicit and correct descriptions of the motion and interaction of systems in various problems? (b) Do the resulting theoretical descriptions indeed facilitate the subsequent generation of correct equations and hence of correct ultimate solutions of these problems?

In the following sections we describe the method, results, and implications of an experimental investigation to test these aspects of the prescriptive model.

Method

The experiments compared the problem-solving performance of individual subjects working under different external conditions. One group of subjects, the "model group" (M), worked on particular problems while guided by external control directions implementing the proposed model of good performance. (These control directions were read to them by the experimenter according to a written script.) Another group of subjects, the "modified-model group" (M^*), worked on these problems while guided by external control directions implementing a modified model different from the proposed model of good performance by the omission of certain selected features. (The performance of these subjects would be expected to deteriorate in predictable ways if these omitted features are necessary for good performance.) Finally a last group of subjects, the "comparison group" (C), worked on these same problems without any external guidance.

Subjects

The subjects in these experiments were 24 paid volunteers, all undergraduate students currently enrolled in the second course of an introductory physics sequence at the University of California in Berkeley. These students had previously studied, in the first course of this sequence, the relevant physics principles and types of problems used in our experimental study. Hence the subjects in this study could be assumed to have learned this relevant knowledge just a few months previously.

The subjects were selected randomly from those volunteers who had received a grade of B- or better in their last physics courses. These subjects were then randomly assigned to the three groups, eight in each group.

Procedure

A pretest, consisting of three mechanics problems, was first administered individually to each subject. Subjects were asked to talk aloud about what they were thinking while solving the problems, and their verbalized statements were recorded with their permission. During this and subsequent sessions, the subjects were provided with a printed summary of relevant mechanics principles to which they could refer at any time. Because our interest was not in the subjects' knowledge about algebra or trigonometry, apparent errors in the application of such knowledge were pointed out or corrected by the experimenter when they occurred.

Subjects in groups M and M* then received brief training to familiarize them with the directions they were subsequently going to be asked to follow. This training consisted of a single practice run through the major steps of the problem-solving procedure.

Each subject then returned for one or two subsequent sessions during which three problems, approximately equivalent to the pretest problems, were administered individually. Groups M and M* were guided through the solution of these problems, while Group C again worked without external

guidance. The subjects were again asked to talk out loud and were tape-recorded. The subjects' written work and verbalized comments comprised the data for this study.

Subjects working with external guidance were read the standard directions one step at a time. Each direction had to be implemented by the subject before the next one was read. If a step was not performed, the directions were repeated.

External Control Directions

Standard external control directions were developed for use with subjects in groups M and M*. These directions provide very specific guidance through problem solutions but are problem-independent--i.e., the same directions are applicable to any mechanics problem that can be solved by applications of Newton's motion principle (his "second law"). A summary and comparison of the kinds of knowledge included in the directions for groups M and M* is provided in Table 2.

Insert Table 2 about here

The external control directions specify procedures for accomplishing two major activities involved in problem solving: Constructing an initial theoretical problem description and subsequently synthesizing the problem solution by generating constraints in the form of equations or inequalities.

As indicated in Table 2, the modified M* version of the model consists essentially of a subset of the steps included in the full M version of the model of good performance. For example, the full model specifies explicit descriptions of both the motion and the interaction of systems; but the

Table 2

Major Components of External Control Directions for Model (M) and Modified Model (M*)

Components common to both models	Additional components in model M only.
Motion Description	
Explicit mention of motion when generating equations.	Direction to draw separate motion diagrams indicating position, velocity, and acceleration of each system. Special mention of knowledge about motion (e.g., knowledge about components of acceleration for circular motion).
Interaction Description	
Direction to draw separate force diagrams indicating all forces exerted on each system by all other systems.	Specific algorithm for enumerating forces. Special mention of knowledge about properties of interactions (e.g., explicit rules for determining directions of forces).
Checks on Descriptions	
Reminder to choose useful symbols.	Check for consistency of motion and interaction descriptions.
Check that all given information has been used.	Check that mutual forces are described correctly (equal in magnitude, opposite in direction).
Synthesis of Solution	
Assessment of current problem state.	
Explication of kinds of decisions to be made during application of motion principles (choice of principle, of system, of direction).	

modified model includes only a description of interaction. Furthermore, the full model specifies an explicit algorithm for enumerating all forces involved in the interaction; but the modified model specifies only that all forces acting on a system be indicated, without specifying in greater detail how these forces should be identified. Thus the modified model corresponds roughly to the kinds of suggestions provided by a typical physics textbook (i.e., to draw "free-body" force diagrams of systems, without providing explicit rules for identifying or describing relevant forces).

The full model also includes methods for checking that the motion and interaction of systems are correctly and conveniently described. One check involves the comparison of motion and interaction descriptions to ensure their consistency. (This is only possible in the full model where both motion and interaction have been explicitly described.) A second check involves examination of interaction descriptions to ensure that constraints implied by Newton's third law have been considered--namely, that mutual forces (i.e., "actions" and "reactions") are described as equal in magnitude and opposite in direction.

Directions for synthesizing solutions are essentially identical in both versions of the model. Of major interest here are the directions to choose explicitly a principle, a system, and a particular direction (or coordinate system) when applying Newton's motion principle to generate equations.

The way in which the differences between the full and modified models were actually implemented is exemplified in Table 3 which contains excerpts from the scripts used to direct subjects through the enumeration of forces acting on a chosen system.

Insert Table 3 about here

Problems for Assessing Performance

Three approximately matching pairs of mechanics problems (listed in the Appendix) were selected from commonly used introductory physics texts (French, 1971; Resnick & Halliday, 1977; Symon, 1971) and reworded slightly for increased clarity. The pairs of problems were split into two sets, A and B. Half of the subjects received one set as a pretest and the other set during treatment sessions; the other half of the subjects received these sets in opposite order.

All of the problems used in the study could be solved by application of one fundamental motion principle, Newton's second law ($m\underline{a} = \underline{F}$). Two of the three pairs of problems (1A, 1B; 3A, 3B) required non-trivial force descriptions because they involved several forces (both long-range and short-range). These problems were included to allow assessment of procedures for enumerating forces. The third pair of problems (2A and 2B) required non-trivial motion descriptions; they involved systems in circular motion, the analysis of which is frequently performed incorrectly by novices. These problems were included to allow assessment of procedures for describing motion.

Data Analysis

In order to assess the quality of students' problem-solving behavior, it was necessary to identify and define explicit performance measures. Table 4 summarizes the specific criteria used as measures of good performance--and the major classes of errors used to assess deficiencies of performance. Note that the first two measures in Table 4 were designed to reveal the adequacy of problem description, whether explicitly exhibited in diagrams (e.g., by subjects induced to act in accordance with the model) or only implicitly generated (e.g., by many subjects in the pretest or in the comparison group). Hence these measures assessed the completeness and correctness of the descriptive information incorporated in the subjects' equations, even if these subjects did not exhibit any explicit prior description.

Insert Table 4 about here

Results

The adequacy of every solution was assessed with respect to the performance measures listed in Table 4. The data in Table 5 and Figure 2 show the mean number of each student's solutions (on the three problems solved during pretest or treatment sessions) that were correct on each of these measures. The data are summarized for students in each of the three treatment groups M, M*, and C. The rightmost columns in Table 5 indicate which of the differences between these groups are statistically significant. Table 5 and Figure 2 also summarize the performance of all 24 students on the pretest. There were no significant differences between

Table 3

Excerpts of External Control Directions for Constructing Interaction Descriptions

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<u>Model M</u>:

E: Let's now draw diagrams describing the forces on each system of interest. Which system...do you wish to consider first/next?

S: (Names a system "X".)

E: First name each system that touches X, including those that exert applied forces. As you identify each system, indicate all external contact forces exerted on X by that system.

S: (Names systems and indicates contact forces.)

> **IF NAMED SYSTEM INTERACTS BY SURFACE CONTACT:
>
> E: Remember, the force exerted by a surface ordinarily, although not always, has two components, the normal force and friction. Check to be sure whether both components exist in this case.
>
> Also, remember that the normal force is perpendicular to, and directed away from, the surface exerting it. The friction force opposes the relative motion of the contact points; it opposes the motion of X relative to (interacting system).

E: Now name all external systems that directly interact with X without touching it or through any other physical contact. Then indicate the long-range forces exerted on X by each such system.

S: (Names systems and indicates long-range forces.)

E: Are there any other systems touching X?

S: (Indicates no others or names additional system(s) and indicates contact force(s).)

E: Are there any other systems directly interacting with X by long-range forces?

S: (Indicates no others or names additional system(s) and indicates long-range force(s).)

E: If not, you are finished describing all forces on X. DO NOT ADD ANY OTHERS.

Table 3 (cont'd)

<u>Modified Model M*</u>:

E: Let's now draw diagrams describing the forces on each system of interest. Which system...do you wish to consider first/next?

S: (Names a system "X".)

E: Draw a force diagram indicating <u>all</u> the forces exerted on X by all other systems.

S: (Draws a diagram.)

E: Are there any other forces exerted on X by any other systems?

S: (Indicates no others or draws additional forces.)

Note: E = experimenter, S = subject

Table 4

Performance Measures and Error Types

Performance Measure	Major Error Types
1. Adequacy of motion information: Was information about the magnitude and direction of each system's acceleration correctly included in the equations?	Wrong direction of acceleration. Wrong magnitude of acceleration.
2. Adequacy of interaction information: Were all required forces included in the equations? Were directions and magnitudes of those forces correctly indicated?	Missing forces(s) in equation. Wrong direction of a force. Wrong magnitude of a force.
3. Adequacy of constraint equations. Were the number and kinds of equations generated sufficient to determine a solution? Were all equations correctly instantiated?	Missing required equation. Incorrect information contained in equation. Meaningless equation (e.g., inconsistent choices of systems).
4. Correctness of final answer: Was correct answer obtained?	Incorrect (or no) final answer.

the various groups on this pretest, nor between these pretest results and the performance of the comparison group C in the treatment.

Insert Table 5 about here

Insert Figure 2 about here

Sufficiency of the Model

The purpose of this research was to evaluate selected aspects of the proposed model of good problem-solving performance in mechanics. The major question of interest is whether the kinds of procedures proposed by the model are sufficient for producing successful solutions. If the kinds of knowledge included in the model are sufficient, students working in accordance with the model would be expected to perform well.

The performance of subjects in group M, working under external control, indicates that the proposed procedures did indeed lead to very good performance. As is apparent from Table 5 and Figure 2, these students performed nearly perfectly: All of their solutions contained every required equation, and all their equations contained correct and complete information about motion and interaction. (The slightly lower incidence of correct final answers resulted from incorrect combination of equations on problem 2B; instead of performing a required vector addition, some students treated vectors like numbers.)

Table 5

Mean Number of Solutions with Correct Performance on Specified Measures

Performance measures	Pretest[a]	Treatment[b]			Statistical differences[c]		
		M	M*	C	M>M*	M*>C	M>C
Correct motion information	1.83	3.00	2.63	1.63			*
Correct force information	1.33	3.00	2.00	1.38	**		**
Sufficient and correct equations	0.83	2.88	1.63	0.75	**		**
Correct final answer	0.79	2.75	1.38	0.63	*		**

Note: Maximum score = 3.00.

[a] $n = 24$.

[b] $n = 8$ per group.

[c] Kruskal-Wallis Test results: $*p<0.01$; $**p<0.005$.

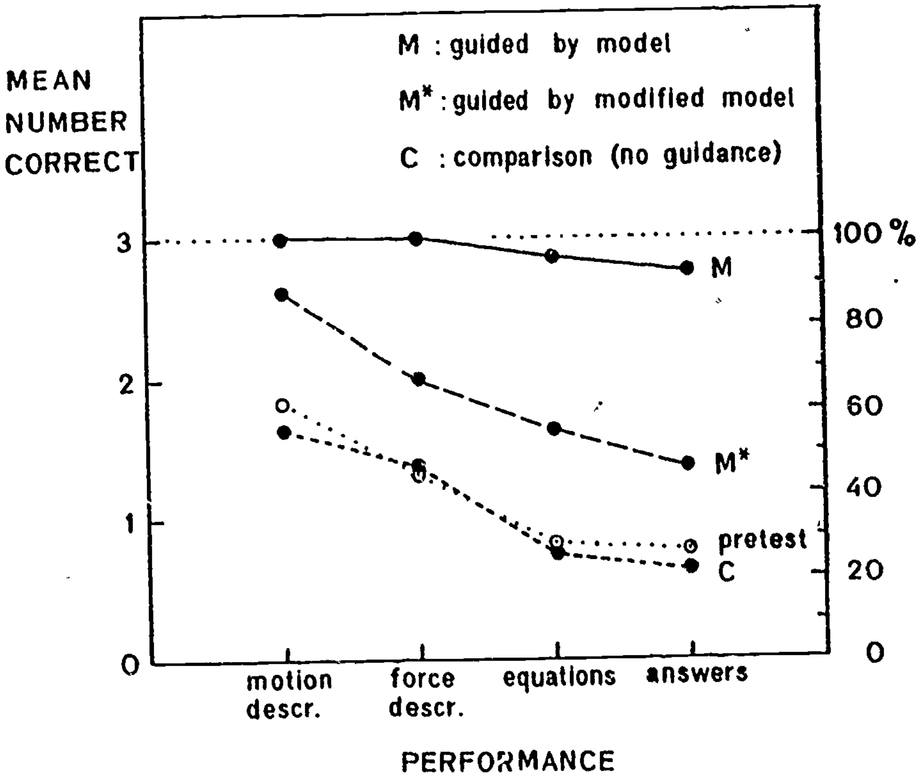

Figure 2. Graphs of mean number of solutions (out of three) with correct performance on specified measures.

Inadequacy of Performance Unguided by the Model

The previous results indicate that subjects, induced to act in accordance with the model, perform excellently. However, one may ask whether the subjects might not have performed equally well without guidance by the model, since they had studied mechanics in a previous course a few months before and might thus have the requisite knowledge to solve independently the fairly standard kinds of problems used in our study.

As indicated in Table 5 and Figure 2, the subjects' performance on the pretest, as well as the performance of the comparison group C, indicates that the subjects' prior knowledge was definitely *not* sufficient to solve these kinds of problems adequately. On the average, the subjects solved correctly only less than one third of the pretest problems. Furthermore, only less than one third of their solutions contained enough equations to achieve a solution, and less than one half of these solutions incorporated correct information about both the motion and interaction of the relevant systems. (The performance of the comparison group C in the treatment was virtually identical to the performance of all subjects on the pretest.)

The preceding results indicate that the kind of knowledge students acquire as a result of ordinary instruction in an introductory mechanics course is *not* sufficient to endow them with the ability to solve typical mechanics problems at the level of this course. However, it should be noted that the subjects in our study *did* have an adequate knowledge of basic physics concepts and principles, i.e., enough knowledge to interpret and implement the external control directions used in our experiments. However, the additional procedural and factual knowledge provided by these

directions was necessary to help the subjects achieve good problem-solving performance.

Necessity of Components of the Model

The results already discussed show that subjects, working in accordance with the proposed model, perform very well on problem-solving tasks. However, one may ask whether all components of this model are actually necessary. For example, it might be that some of the procedures and knowledge incorporated in the model are superfluous and that performance might be equally good (and perhaps more efficient) without these components.

This question can be answered by comparing the performance of group M, which worked in accordance with the proposed model, with the performance of group M* which worked in accordance with a modified model omitting certain components of the proposed model. (See Table 2.) If these knowledge components, contained in the full model but deleted from the model used to guide group M*, were in fact necessary for good performance, the observed performance of group M* should be less adequate than that of group M. In particular, since the major differences in the models, and in the experimental directions based on them, lay in the completeness and explicitness of the initial problem descriptions, the descriptions of motion and interaction by group M* would be expected to be inferior to those by group M. Correspondingly, the subsequent equations generated by subjects in group M*, and hence also the final problem answers obtained by them, should be less often correct than those generated by subjects in group M.

The data in Table 5 and Figure 2 reveal essentially this pattern of results. All results are statistically significant, except in the case of motion description, where the performance of Group M* was not significantly poorer than the perfect performance of Group M. It thus appears that the kinds of knowledge included in the model are both sufficient and necessary for achieving good problem solutions.

Detailed Analysis of Subjects' Performance

A closer examination of the subjects' performance provides insights into the ways in which the proposed model facilitates good performance. In this section we discuss some specific examples of typical difficulties subjects encounter during problem solving. For each such example, we indicate the particular components of the model that lead to good performance by reducing such difficulties and preventing common errors.

Omission of relevant forces. One of the most common errors committed by subjects, working unaided without external control, involved the omission of some relevant forces acting on a system. Indeed, about 75% of the subjects omitted some relevant forces in at least one of their pretest problem solutions.

These difficulties in problem description may be illustrated by problem 3B used in our study. This problem (illustrated in Figure 3 and described more fully in the Appendix) deals with two blocks A and B connected by a string passing over a fixed pulley. The block B may slide, with friction, relative to the horizontal floor beneath it and relative to the block A on top of it. It is desired to find the magnitude of the force F_0 needed to pull the block B to the left with constant velocity.

Insert Figure 3 about here

The description procedure, specified by the model summarized in Table 1, would describe the motion of block B by its velocity $\underset{\sim}{v}$ and acceleration $\underset{\sim}{a}$, as indicated in Figure 3. It would also describe the interaction of this block by using the procedure of Table 1 to identify all forces on this block. Thus it would first identify the long-range gravitational force $\underset{\sim}{F}_g$ on block B by the earth. Then it would identify all other objects which touch the block B and correspondingly identify all the short-range forces acting on B. As indicated in Figure 3, these are the force $\underset{\sim}{F}_0$ applied by the system pulling this block, the tension force $\underset{\sim}{T}$ by the string, the normal and friction forces ($\underset{\sim}{N}$ and $\underset{\sim}{f}$) by the floor, and the normal and friction force ($\underset{\sim}{N}'$ and $\underset{\sim}{f}'$) by block A.

Identification of all these forces presented particular difficulties for the subjects unguided by the model. For example, the friction force $\underset{\sim}{f}$ on B by block A was omitted in half of the pretest solutions of this problem. The normal force $\underset{\sim}{N}'$ on B by block A, and the tension force $\underset{\sim}{T}$ on B by the string, were omitted in 25% of the pretest solutions.

These errors were eliminated entirely in the solutions of subjects guided by the model. The reason is that the model (Table 1) contains an explicit algorithm which enumerates all short-range forces by identifying all objects which touch the system of interest--and the identification of objects touching a given system is trivial for human subjects. Furthermore, the algorithm includes an explicit reminder of factual knowledge in the knowledge base, i.e., that the force exerted by a surface consists ordinarily of two component forces (the normal and friction forces)

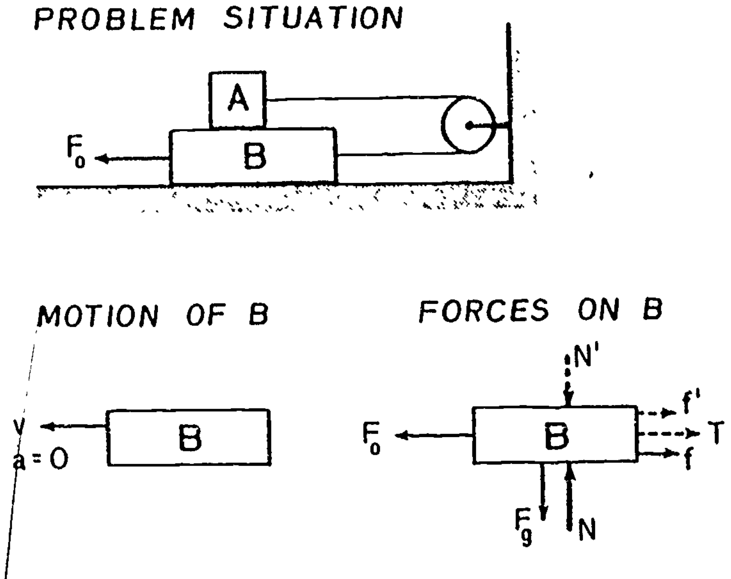

Figure 3. Problem 3A involving two blocks connected by a string, with motion and force descriptions of block B. (Forces frequently omitted by subjects are indicated by dashed arrows.)

perpendicular and parallel to the surface. Hence the explicit description eliminates automatically the common error of omitting some short-range forces acting on a system.

In this particular problem subjects in group M* also did not omit any forces on block B (although they were merely told to "indicate all the forces exerted on block B by all other systems" and then to check that they had identified all such forces). However, subjects in group M* did omit relevant forces in the solutions of the other problems (as indicated in Table 5 and Figure 2), while subjects in group M never omitted any relevant forces in any of the problems. Thus the detailed description procedure specified by the full model is far more reliable than the less explicit procedure provided to subjects in group M*. Nevertheless, the latter procedure still leads to better performance than that exhibited by subjects working without any external guidance.

Wrong directions of forces. A second very common error exhibited on the pretest was that of ascribing the wrong direction to a force. Half of the subjects made this error in at least one pretest solution. An example of this difficulty occurs in problem 1A illustrated in Figure 4 and described more fully in the Appendix. In this problem blocks A and B are connected by a thin light string which passes over a fixed pulley. It is specified that the block C, which lies on top of block A, remains at rest relative to A without sliding off.

Insert Figure 4 about here

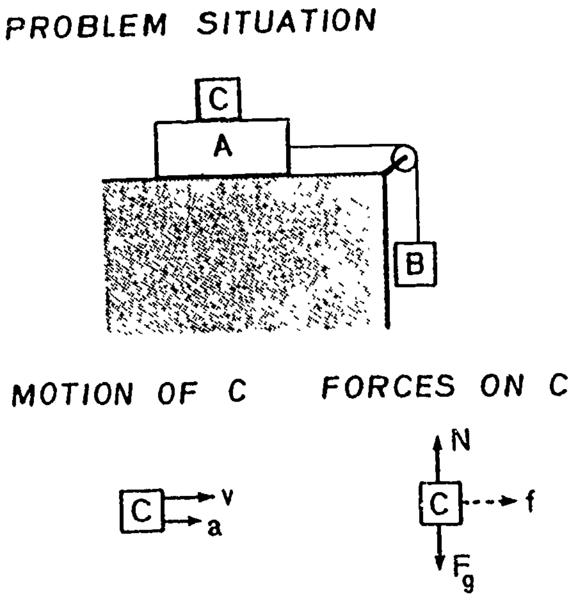

Figure 4. Problem 1A involving three blocks, with motion and force descriptions of block C. (The friction force $\underset{\sim}{f}$, indicated by a dashed arrow, is frequently ascribed the wrong direction, i.e., to the left.)

To solve this problem, all the forces on block C must be correctly identified and described. However, in 83% of the pretest solutions of this problem, subjects asserted that the friction force on C by A was directed to the <u>left</u>, when it is in fact directed to the right. (Indeed, if the friction force were directed to the left, block C would certainly slide off block A. It is only the friction force exerted on C by A which moves C to the right along with A.)

This error appears to be the result of the following inaccurate, and frequently verbalized, rule used by many subjects to determine the direction of the friction force: "Friction opposes the motion of C". This rule is too general and leads to correct force descriptions only under certain special conditions. The correct general rule is that "friction opposes the <u>relative</u> motion of the contact points"; i.e., in this case the friction force opposes the motion of C <u>relative</u> to A and must thus be directed to the <u>right</u>. Subjects in group M, who used this rule included in the model of description, never erred in ascribing the correct direction to the friction force. By contrast, subjects in groups M* and C made this error as frequently as the subjects on the pretest problems.

The model (Table 1) provides not only explicit rules for correctly describing forces, but includes also checks to ensure that forces have been described properly. One such check requires that the descriptions of the motion and interaction of each system by consistent. In order to perform this check, both the motion and interaction of each system must have been described explicitly, as required by the model. Such descriptions for selected systems have been indicated for the problems illustrated in Figures 3 and 4.

The check for consistency between motion and interaction is based on Newton's motion principle $m\underset{\sim}{a} = \underset{\sim}{F}$ which implies that the acceleration $\underset{\sim}{a}$ of any particle must have the same direction as the <u>total</u> force $\underset{\sim}{F}$ on it. Accordingly, the detailed program implementing the model for Group M contained the following direction for checking the consistency of motion and interaction descriptions: "In your diagrams, are the forces on the selected particle such that, with proper magnitudes, their vector sum can have the same direction as the particle's acceleration? If not, there is something wrong."

The power of the preceding checking procedure can be illustrated in the case of the problem of Figure 4. It is quite easy for subjects, describing the motion of block C, to determine that its acceleration is directed to the right. If such a subject, describing the interaction of block C, then claims that the friction force $\underset{\sim}{f}$ on this block is directed to the left, the checking procedure would immediately reveal that the direction of this force is inconsistent with that of the acceleration and must therefore be incorrect. Thus the check of consistency between motion and interaction provides a reliable means of detecting and correcting the common error of incorrectly ascribing the wrong direction to the friction force in this problem.

The explicit qualitative comparison of motion and force diagrams in our experiments also seemed to provide sutdents with a powerful graphic demonstration of the meaning of Newton's motion principle. Many of the students in group M spontaneously reacted to this comparison with comments indicating a new understanding of the implications of Newton's motion principle $m\underset{\sim}{a} = \underset{\sim}{F}$, e.g., with comments such as "Oh, that's neat! I hadn't thought about it that way before!". This kind of qualitative comparison procedure may thus have a substantial potential for enhancing students' understanding of physics principles, a potential which may be worthy of further investigation.

Another check on the initial theoretical description of a problem involves determining whether mutual forces between interacting particles (i.e., "actions" and "reactions") have been correctly described in a manner consistent with "Newton's third law". To be specific, subjects in group M were directed to do the following: "Check to make sure that all action-reaction pairs of forces are described as equal in magnitude and opposite in direction. For example, if systems A and B interact, the force on A by B in your force diagram of A should be opposite in direction, but should have the same magnitude, as the force on B by A in your diagram of B."

Most students, working independently <u>without</u> guidance in the problem of Figure 4, did correctly state that the friction force on block A by block C is directed to the left, even though many claimed incorrectly that the friction force on block C by block A is also directed to the left. Since the model incorporates an explicit check on the consistency of mutual forces, subjects working in accordance with this model detect the inconsistency of such force descriptions and make appropriate corrections.

DISCUSSION AND IMPLICATIONS

The work discussed in this paper has aimed to formulate and validate a prescriptive theoretical model specifying some of the knowledge and procedures leading to good human problem solving in a quantitative science such as physics. We have focused particular attention on the generation of effective initial problem descriptions which facilitate the subsequent solutions of such problems. Thus we sought to specify explicit procedures for generating a "theoretical problem description" which deliberately redescribes any situation in terms of the special concepts specified by the knowledge base for the relevant scientific domain. In the science of mechanics these procedures specify explicitly how to describe the motion of any system in terms of concepts such as velocity and acceleration, how to describe the interaction of any such system in terms of specified kinds of forces, how to exploit special knowledge about the properties of such forces, and how to check the resulting description by its consistency with known physics principles.

The special experiments, discussed in the preceding pages, show that human subjects, induced to follow such description procedures under carefully controlled conditions, do indeed reliably generate explicit and correct descriptions of the motion and interaction of systems in mechanics problems. Furthermore, these descriptions markedly facilitate the subsequent construction of correct problem solutions.

The generation of effective initial problem descriptions is far from trivial. Indeed, our experiments show that many students, after receiving good grades in a recent course where they received formal instruction in mechanics, nevertheless generate incomplete and/or incorrect descriptions of fairly routine problems--and thus fail to solve them properly.

As we have pointed out, these problem-solving deficiencies exist even if students have a good understanding of prerequisite physics concepts and principles. They still lack the more strategic kinds of knowledge specified in our prescriptive model, i.e., the meta-knowledge that it is

important to describe a problem with care before attempting to search for its solution, explicit knowledge about what types of information should be included in an effective description, and explicit systematic procedures specifying how to generate such a description. These kinds of knowledge are usually possessed by experts, although predominantly in tacit form, and are rarely taught explicitly in physics courses. The work discussed in the preceding pages shows that such knowledge can be made more explicit and that, if used by students, it can strikingly improve their problem-solving performance.

Our theoretical ideas about the generation of effective initial problem descriptions have been illustrated in the particular scientific domain of mechanics. However, they can readily be extended to other scientific domains (e.g., to electric circuits, or thermodynamics, or even to domains outside of physics) provided that they are used in conjunction with the particular knowledge base of the relevant scientific domain.

The generation of effective initial problem descriptions, discussed in the preceding pages, is very important to achieve effective problem solving, but is not sufficient. A complete prescriptive theoretical model of effective problem solving must, therefore, also deal with other central issues, e.g., with decision processes facilitating the efficient search for a solution, with useful forms of organization of the knowledge base, etc. We have outlined such a more encompassing problem-solving model elsewhere (Reif & Heller, in press) and hope to validate other aspects of this model by experimental methods similar to those used in our study of problem description.

The particular experimental methods, discussed in the preceding pages to study models of problem description, have involved the detailed observations of human subjects working under "external control" in accordance with prescriptive models of performance (either a proposed model of effective performance or alternative models). This method permits one to explore in detail the efficacy of any proposed model of human task performance and to manipulate experimentally various parameters of such a model. Accordingly, this method may be broadly useful to study theoretical models specifying cognitive processes and knowledge structures for achieving intellectual performance in a wide variety of domains.

The work discussed in this paper is highly relevant to the design of instruction for teaching students improved scientific problem-solving skills. Indeed, such instruction requires a well-validated prescriptive model specifying how good problem solving is to be achieved by students as a result of instruction. (As pointed out in the introductory paragraphs, such a model must do more than merely simulate the problem-solving behavior of actual experts.) Our model for generating effective problem descriptions, together with the experiments verifying its efficacy, is thus an essential prerequisite for teaching students important problem-description skills needed for good problem solving.

Such teaching efforts would require students to internalize, and learn to use habitually, control knowledge which was explicitly externalized in our experiments. In other words, instructional design must use insights about good performance and then deal explicitly with the processes whereby such performance can be learned. Indeed, our model of problem description has already been quite useful in some of our practical efforts to teach problem-solving skills to students in physics courses. We hope to go beyond such informal efforts to develop more explicit and systematic instructional methods based on our analysis of relevant cognitive processes.

APPENDIX

Problems Used in Experiments

The following are the three pairs of problems used in our experiments. Each problem was presented to a subject together with a tabular summary of the information specified in the statement of the problem.

Problem 1A

Figure 5 shows a cart A (of mass 2m) free to move without friction along a horizontal table. This cart is attached by a light string, which passes over a pulley of negligible mass and negligible friction, to a block B (of mass m_B) suspended from the other end of the string. A block C (of mass m) lies on top of cart A. The coefficient of static friction between A and C is μ. What is the maximum value of m_B for which block C will remain on the cart without sliding?

Insert Figure 5 about here.

Problem 1B

Figure 6 shows a cart A, of mass m_A, which moves with negligible friction along a horizontal floor when it is pushed to the right by an

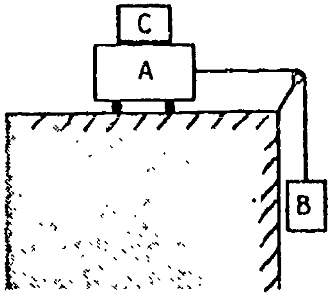

Figure 5. Diagram for problem 1A.

applied force of magnitude F_0. A small block B, of mass m_B, is in contact with the right vertical side of the cart. The coefficient of static friction between the block and the side of the cart has a value μ. How large must be the magnitude F_0 of the applied force so that the block remains at rest relative to the cart, without slipping down?

Insert Figure 6 about here.

Problem 2A

A pendulum bob, of mass m, swings in a vertical plane at the end of a string of negligible mass fastened to the ceiling. At the highest point of its swing, the pendulum is in the position shown in Figure 7, with the string at an angle θ from the vertical. What is the magnitude of the tension force exerted on the bob by the string at this instant?

Insert Figure 7 about here.

Problem 2B

An object of mass m slides along a circular track with negligible friction. When the object passes the point P in Figure 8, the magnitude of the force exerted on the object by the track is $3mg/\sqrt{2}$. What is the magnitude of the object's acceleration at that instant? (Use the values: $\sin 45^0 = \cos 45^0 = 1/\sqrt{2}$.)

Insert Figure 8 about here.

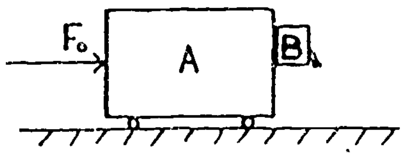

Figure 6. Diagram for problem 1B.

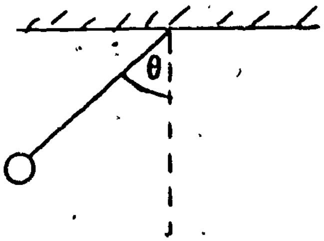

Figure 7. Diagram for problem 2A.

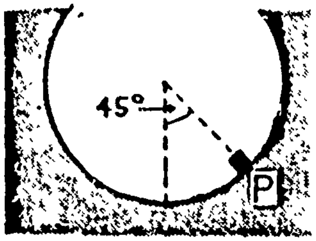

Figure 8. Diagram for problem 2B.

Problem 3A

A man, of mass m, stands on a board, of mass M, which he previously placed on a mud-covered hilly surface making an angle θ with the horizontal. The man holds on to a rope (of negligible mass and parallel to the surface of the hill) whose other end is fastened to a wall at the top of the hill. (See Figure 9.) The man finds, to his dismay, that the board beneath him starts sliding down the hill. The coefficient of sliding friction between the man's shoes and the board is μ_1, and the coefficient of sliding friction between the board and the surface of the hill is μ_2. What is the magnitude of the acceleration a_B with which the board beneath the man slides down the hill while the man, holding on to the rope, remains at rest relative to the ground?

Insert Figure 9 about here.

Problem 3B

Two blocks A and B are connected by a light flexible string passing around a frictionless pulley of negligible mass. (See Figure 10.) Block A has a mass m_A and block B has a mass m_B. The coefficient of sliding friction between the two blocks, and also between block B and the horizontal table below it, has a value μ. What is the magnitude F_0 of the force necessary to pull block B to the left at constant speed?

Insert Figure 10 about here.

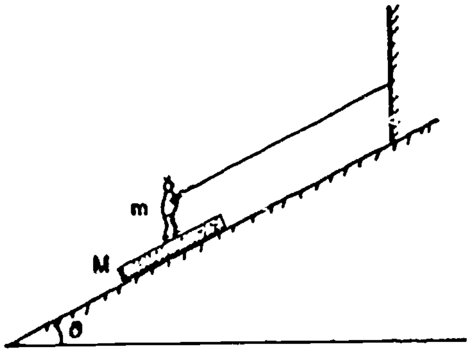

Figure 9. Diagram for problem 3A.

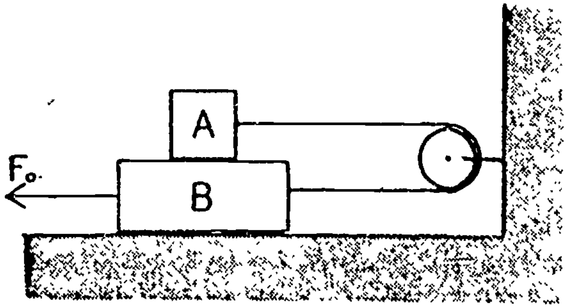

Figure 10. Diagram for problem 3B.

References

Bundy, A. Will it reach the top? Prediction in the mechanics world. Artificial Intelligence, 1978, 10, 129-146.

Bundy, A., Byrd, L., Luger, G., Mellish, C., & Palmer, M. Solving mechanics problems using meta-level inference. Proceedings of the 6th International Joint Conference on Artificial Intelligence, Tokyo, 1979.

Byrd, L., & Borning, A. Extending Mecho to solve statics problems (Tech. Rep. DAI 137). University of Edinburgh, Department of Artificial Intelligence, 1980.

Champagne, A.B., Klopfer, L.E., & Anderson, J.H. Factors influencing the learning of classical mechanics. American Journal of Physics, 1980, 8, 1074-1075.

Chi, M.T.H., Feltovich, P.J. & Glaser, R. Categorization and representation of physics problems by experts and novices. Cognitive Science, 1981, 5, 121-152.

Chi, M.T.H., Glaser, R., & Rees, E. Expertise in problem solving. In R. Sternberg (Ed.), Advances in the psychology of human intelligence (Vol. 1). Hillsdale, N.J.: Erlbaum, 1981.

Clement, J. Students' preconceptions in introductory mechanics. American Journal of Physics, 1982, 50, 66-71.

de Kleer, J. Multiple representations of knowledge in a mechanics problem solver. Proceedings of the 5th International Joint Conference on Artificial Intelligence. Cambridge, Mass.: MIT Press, 1977.

diSessa, A. Unlearning Aristotelian physics: A study of knowledge-based learning. Cognitive Science, 1982, 6, 37-75.

French, A.P. Newtonian mechanics. New York: W.W. Norton, 1971.

Greeno, J.G. Nature of problem solving abilities. In W.K. Estes (Ed.), Handbook of learning and cognitive processes (Vol. 5). Hillsdale, N.J.: Erlbaum, 1978.

Larkin, J.H. Enriching formal knowledge: A model for learning to solve textbook physics problems. In J. Anderson (Ed.), Cognitive skills and their acquisition. Hillsdale, N.J.: Erlbaum, 1981.

Larkin, J., McDermott, J., Simon, D.P., & Simon, H.A., Expert and novice performance in solving physics problems. Science, 1980, 208, 1335-1342. (a)

Larkin, J.H., McDermott, J., Simon, D.P., & Simon, H.A. Models of competence in solving physics problems. Cognitive Science, 1980, 4 317-345. (b)

Larkin, J.H. & Reif, F. Understanding and teaching problem solving in physics. European Journal of Science Education, 1979, 1, 191-203.

Luger, G.F. Mathematical model building in the solution of mechanics problems: Human protocols and the MECHO trace. Cognitive Science, 1981, 5, 55-77.

McCloskey, M., Caramazza, A., & Green, B. Curvilinear motion in the absence of external forces: Naive beliefs about the motion of objects. Science, 1980, 210, 1139-1141.

Novak, G.S., Jr. Representations of knowledge in a program for solving physics problems. Proceedings of the 5th International Joint Conference on Artificial Intelligence. Cambridge, Mass.: MIT Press, 1977.

Polson, P.G., & Jeffries, R. Problem solving as search and understanding. In R. Sternberg (Ed.), Advances in the psychology of intelligence (Vol. 1). Hillsdale, N.J.: Erlbaum, 1981.

Reif, F. Theoretical and educational concerns with problem solving: Bridging the gaps with human cognitive engineering. In D. Tuma and F. Reif (eds.), Problem solving and education: Issues in teaching and research. Hillsdale, N.J.: Lawrence Erlbaum Associates, 1979.

Reif, F. & Heller, J.I. Knowledge structure and problem solving in physics. Educational Psychologist, in press.

Resnick, R. & Halliday, D. Physics (3rd ed.). New York: Wiley, 1977.

Simon, D.P., & Simon, H.A. Individual differences in solving physics problems. In R.S. Siegler (Ed.), Children's thinking: What develops? Hillsdale, N.J.: Erlbaum, 1978.

Symon, K.R. Mechanics (3rd ed.). Reading, Mass.: Addison-Wesley, 1971.

Trowbridge, D.E. & McDermott, L.C. Investigation of student understanding of the concept of velocity in one dimension. American Journal of Physics, 1980, 48, 1020-1028.

Trowbridge, D.E. & McDermott, L.C. Investigation of student understanding of the concept of acceleration in one dimension. American Journal of Physics, 1981, 49, 242-253.

Viennot, L. Spontaneous reasoning in elementary dynamics. European Journal of Science Education, 1979, 1, 205-221.