# Present Value Formulation of Economic Lot-Size Model for Inventory System with Variable Deteriorating Rate of Items

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*The analysis of the deterministic economic order quantity problem for time dependent deterioration of units seeks to minimize the average cost of inventory systems. An alternative numerical analysis is described in this paper to examine the present value of the discounted costs over an infinite horizon. The deterioration rate is a continuous random variable and follows a two parameter weibull distribution. Also a sensitivity analysis is studied by some numerical illustrations. Comparison of two solutions is also discussed.*

***Keywords:*** *Time dependent deterioration rate, Discounted Cash Flow (DCF), EOQ.*

## 1 Introduction

Extensive work is done by researchers leading to possible expansions of the classical Economic Order Quantity (EOQ) model for various situations where its assumptions can be weakened. Trippi and Lewin (1974) gave the discounted cash-flows (DCF)/(Net Present Value (NPV)) approach for the analysis of the basic EOQ model. Kim, Philipratos and Chung (1986) extended above approach to various inventory systems. However, these two articles did not consider the realistic assumption that units in an inventory system are subject to deterioration. Chakrabarty et al (1998) derived model for deteriorating items with instantaneous supply, trended demand and shortages in inventory. They considered three parameters weibull distribution for deterioration of units. Chung and Lin (2000) derived mathematical model considering time value of money over a fixed planning horizon. They established convexity of total variable cost. Chung and Tsai (2001) also extended above concept for trended demand. Raafat (1991), Shah and Shah (2000), and Goyal and Giri (2001) gave up-to-date survey of literature for inventory models when units in inventory are subject to deterioration.

This paper extends the work of Trippi and Lewin as well the article of Kim et al by explicitly recognizing the effect of time dependent deterioration of units in an inventory system on the optimum order quantity using the DCF approach. The sensitivity analysis of interdependency of parameter is studied by numerical illustrations.

## 2. Assumptions and Notations

The mathematical model is developed under the following assumptions and notations.

1. The demand rate is R units per unit time.
2. The replenishment size (order quantity) is Q which is a decision variable.
3. The length of inventory cycle is T,(a decision variable)
4. C - denotes unit purchase cost of item.
5. A - denotes ordering cost per order.
6. Lead time is zero, replenishment is instantaneous and shortages are not allowed.
7. h – denotes inventory carrying cost per unit per time unit.
8. r – denotes discount rate (opportunity cost) per time unit.
9. The deterioration rate is given by the weibull distribution:

$$q(t) = a\, b\, t^{b-1} \qquad ,0 \le t \le T$$

where $\alpha$ - scale parameter, $0 \le \alpha < 1$; $b$ - shape parameter, $b \ge 1$; t - time to deterioration, t > 0

There is no repair or replacement of deteriorated inventory during a given cycle.

## 3. Mathematical Formulation:

Let Q(t) be the on-hand inventory at any instant of time $t (0 \le t \le T)$. Reviewing continuous system, it is assumed that depletion due to deterioration and due to demand will occur simultaneously. Then the differential equation governing the instantaneous state of Q (t) over the cycle time (0, T) is given by

$$\frac{dQ(t)}{dt} + q(t)Q(t) = -R,\ 0 \le t \le T \quad (1)$$

with initial conditions Q(0)=Q and Q(T)=0.

Then solution of (1) is

$$Q(t) = R\left[T - t + \frac{\alpha T}{\beta+1}\left(T^{\beta} - (1+\beta)t^{\beta}\right) + \frac{\alpha.\beta.t^{\beta+1}}{\beta+1}\right] \quad (2)$$

using $Q(0) = Q$, we get $Q = R\left[T + \frac{\alpha T^{\beta+1}}{\beta+1}\right]$ (3)

The total demand during one cycle is RT. Hence the number of units deteriorated during one cycle, D(T) is given by $D(T) = Q - RT = \frac{R\alpha T^{\beta+1}}{\beta+1}$ (4)

Under the DCF approach, at the beginning of each cycle there will be cash out flow of both the ordering cost $A$ and the purchase cost $CQ$.

Since the inventory carrying cost is proportional to the value of the inventory, the out flow inventory cost per unit time at any instant t is $hQ(t)$ and the present value of the out of pocket carrying cost is $h.Q(t).e^{-rt}$ (using Chung (1989)). Thus the present value of cash out flows PV (T) for the first cycle is

$$PV(T) = -\left(A + CQ + h\int_0^T Q(t)e^{-rt}dt\right)$$

$$= -A - CR\left[T + \frac{aT^{b+1}}{b+1}\right] - hR\left[\frac{T^2}{2} + \frac{abT^{b+2}}{(b+1)(b+2)} - \frac{rT^3}{6}\right]$$

(5).

The present value of all future cash – out – flows is given by $PV_{\infty}(T) = \sum_{n=0}^{\infty} PV(T).e^{-nrT}$

$$= \frac{PV(T)}{1-e^{-rT}} \quad \therefore PV_{\infty}(T) = \left(\frac{1}{rT} + \frac{1}{2} + \frac{rT}{4}\right)PV(T)$$

$$\therefore PV_{\infty}(T) = -\frac{A}{rT} - \frac{CR}{r}\left(1 + \frac{\alpha T^{\beta}}{\beta+1}\right) - \frac{hR}{r}\left(\frac{T}{2} + \frac{\alpha\beta T^{\beta+1}}{(\beta+1)(\beta+2)} - \frac{rT^2}{6}\right) - \frac{A}{2}$$
$$-\frac{CR}{2}\left(T + \frac{\alpha T^{\beta+1}}{\beta+1}\right) - \frac{hR}{2}\left(\frac{T^2}{2} - \frac{\alpha\beta T^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{rT^3}{6}\right) - \frac{ArT}{4} - \frac{CRrT^2}{4} - \frac{hRrT^3}{8} \quad (6)$$

The optimum value of T can be obtained from the solution of the following equation

$\frac{dPV_{\infty}(T)}{dT} = 0$ which gives

$$\frac{A}{rT^2} - \frac{CR\alpha\beta T^{\beta-1}}{r(\beta+1)} - \frac{hR}{r}\left(\frac{1}{2} + \frac{\alpha\beta T^{\beta}}{\beta+2} - \frac{rT}{3}\right) - \frac{CR}{2}\left(1 + \alpha T^{\beta}\right)$$
$$-\frac{hR}{2}\left(T + \frac{\alpha\beta T^{\beta+1}}{\beta+1} - \frac{rT^2}{2}\right) - \frac{Ar}{4} - \frac{CRrT}{2} - \frac{3hRrT^2}{8} = 0$$

(7)

Which is very difficult to solve for T and so we solve numerically as shown in the following tables.

Clearly the obtained value of T satisfies:

$$\frac{d^2PV_{\infty}(T)}{dT^2} = -\frac{2A}{rT^3} - \frac{CR\alpha\beta(\beta-1)T^{\beta-2}}{r(\beta+1)} - \frac{hR}{r}\left(\frac{\alpha\beta^2T^{\beta-1}}{\beta+2} - \frac{r}{3}\right) - \frac{CR\alpha\beta T^{\beta-1}}{2}$$
$$-\frac{hR}{2}\left(1 + \alpha\beta T^{\beta} - rT\right) - \frac{CRr}{2} - \frac{3}{4}hRrT \quad (8)$$

which is $< 0$ for all values of T.

Hence, relation (6) is minimum for the obtained value of T. Theoretically, it was difficult to find the optimum value of T because of non-linearity of relation (7). Hence, using series expansion of exponent and ignoring second and higher powers of both $\alpha T$ and $rT$, solution could be obtained.

Clearly, if discounted cash flow (DCF) approach is not applied, the underlined model will reduce to that of Covert and Philip (1973). Further, if $\alpha = 0$ and $\beta = 1$ i.e. there is no deterioration; the model reduces to classical EOQ model. (Naddor (1966)).

## 4. Numerical Analysis:

Consider an inventory system with following parametric values in appropriate units.

R = 2000 A = 200 C = 20 I = 0.15 h = C.I = 3

$\alpha$ = 0.02, 0.03, 0.04 $\beta$ = 1.5, 2.0, 2.5r = 0.03, 0.04, 0.05, 0.08, 0.10

In the table values T=Cycle time; Q=Procurement quantity and $P = PV_{\infty}(T)$ =Present value.

**Table 1.** Variations in $\alpha$ and $\beta$ for given value of r=0.03

| $\beta\rightarrow$ / $\alpha\downarrow$ | | 1.5 | 2.0 | 2.5 |
|---|---|---|---|---|
| 0.02 | T | 0.228 | 0.231 | 0.233 |
| | Q | 456.39 | 462.16 | 466.07 |
| | P | 1391258 | 1390541 | 1390254 |
| 0.03 | T | 0.224 | 0.229 | 0.232 |
| | Q | 448.57 | 458.24 | 464.10 |
| | P | 1391842 | 1390782 | 1390356 |
| 0.04 | T | 0.221 | 0.228 | 0.231 |
| | Q | 442.73 | 456.32 | 462.14 |
| | P | 1392414 | 1391018 | 1390456 |

**Table 2.** Variations in $\alpha$ and r for given value of $\beta$ =1.5

| $r\rightarrow$ / $\alpha\downarrow$ | | 0.03 | 0.04 | 0.05 | 0.08 | 0.10 |
|---|---|---|---|---|---|---|
| 0.02 | T | 0.228 | 0.222 | 0.217 | 0.210 | 0.204 |
| | Q | 456.39 | 444.37 | 434.35 | 420.32 | 408.30 |
| | P | 1391258 | 1044610 | 836600 | 524532 | 420469 |
| 0.03 | T | 0.225 | 0.219 | 0.214 | 0.205 | 0.201 |
| | Q | 450.58 | 458.54 | 428.51 | 410.46 | 402.43 |
| | P | 1391842 | 1045032 | 836927 | 524707 | 420605 |
| 0.04 | T | 0.221 | 0.216 | 0.211 | 0.200 | 0.197 |
| | Q | 442.73 | 432.69 | 422.65 | 400.57 | 394.55 |
| | P | 1392414 | 1045446 | 837247 | 524884 | 420734 |

**Table 3.** Variations in $\beta$ and r for given value of $\alpha$ =0.02

| $r\rightarrow$ / $\beta\downarrow$ | | 0.03 | 0.04 | 0.05 | 0.08 | 0.10 |
|---|---|---|---|---|---|---|
| 1.5 | T | 0.228 | 0.222 | 0.217 | 0.210 | 0.204 |
| | Q | 456.39 | 444.37 | 434.35 | 420.32 | 408.04 |
| | P | 1391258 | 1044610 | 836600 | 524532 | 420212 |
| 2.0 | T | 0.231 | 0.225 | 0.219 | 0.213 | 0.209 |
| | Q | 462.16 | 450.15 | 438.14 | 426.13 | 418.05 |
| | P | 1390541 | 1044088 | 836195 | 524300 | 420233 |
| 2.5 | T | 0.233 | 0.227 | 0.221 | 0.217 | 0.213 |
| | Q | 466.07 | 454.06 | 432.06 | 428.05 | 426.05 |
| | P | 1390254 | 1043882 | 836037 | 524210 | 420258 |

From the tables given above, it is observed that among scale parameter$(\alpha)$, shape parameter $(\beta)$ and discounting factor (r); discounting factor plays a very significant role in deciding T, Q and PV. From Table 1, it is clear that as $\beta$ increases for the given values of $\alpha$ and r, then T and Q increases but PV decreases. Similarly from Table 2, it can be observed that as $\alpha$ increases for the given values of r and $\beta$, then T and Q decreases but PV increases. Also it can be seen from Table 3, as r increases for the given value of $\alpha$ and $\beta$, then T and Q decreases and PV also decreases very rapidly i.e. discounting factor (r) plays more significant role in determination of PV than the scale parameter $(\alpha)$ and shape parameter $(\beta)$.

## Conclusion

The logical basis of considering inventory as an investment over infinite planning horizon seems to be real situation. Another natural aspect is that of opportunity cost of an item and time dependent deterioration of units in an inventory system. It appears that the present value cost function is less sensitive to order quantity to be procured and most sensitive to discounting factor. The present value model seems to be more realistic in present day market situations.

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