PRESSURE AND GAS FLOW GRADIENTS BEHIND THE PROJECTILE DURING THE INTERIOR BALLISTIC CYCLE<br>E.C. Hansen<br>Mechanical Engineering University of Florida Gaiesville, Fl 32611<br>O.K. Heiney<br>Rocketdyne Div.<br>Rockwell International Corp. 6633 Canoga Ave. Canoga Park, CA 91303<br>D/522-BA26

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A very important factor in determining the projectile acceleration in a lumped parameter ballistic model is the relationship between the computed parameter, the space mean pressure, and the parameter defining the projectile motion, the pressure acting on the base of the projectile. The gradients of pressure and density from the breech to projectile were postulated to depend on the term, projectile acceleration times axial position. The model compared favorably with experimental measurements of downbore to chamber pressure ratios.


## INTRODUCTION

The techniques for analytically describing the pressure gradient which exists from the static conditions in the breech to the turbulent area directly behind the projectile have been one of the central problems of interior ballistics from the inception of the science, to the present day. The closed form interior ballistics of the 1940's generally used a constant factor pressure decrement, based on the propellant charge to projectile mass ratio. These factors were derived from an integral approach and applied incrementally. Although they were nowhere correct on a stepwise basis, in the atgregate they were effective and suitable for the closed form calculations of the time.

In England the approach used by Corner [Ref. 1] and Hunt-Hinds [Ref. 2] was that of a shot base pressure ${ }^{P}$ to breech pressure ratio $P_{D}$ relationship of

$$
P_{s}=P_{D} /\left(1+\frac{1}{2} C / M\right)
$$

With a $C / M=.5$ this predicts

$$
P_{s}=.8 P_{b}
$$

In the United States, the Hirschfelder approach [Ref. 3] used an average pressure $P_{a v e}$ expression of the form
$P_{\text {ave }}=P_{3}\left(\frac{M+C / 3}{M}\right)$
which with a $C / M=0.5$ predicts $P_{s}=.87 \mathrm{P}_{\text {ave }}$ or in terms of $\mathrm{P}_{\mathrm{b}}$
$P_{s}=.76 P_{b}$
Both approaches recognized that these approximations became increasingly less accur te as the charge to mass ratio increased above 0.5 .

As computers became generally available in the early 1960's, other approaches and relationships were $p$ stulated for the lumped parameter interior ballistic formulations which were developed to modal the interior ballistic process while
avoiding solving coupled partial differential equations. An approach used for many years by one of the authors [Ref. 4] postulated that the pressure differential from the breech to the base of the projectile could be expressed in terms of the velocity of the projectile, by means of "quasi-isentropic" approaches, that is:

$$
\frac{P_{s}}{P_{b}}=\left[1+\left(\frac{\gamma-1}{2}\right)\left(\frac{U^{2}}{\gamma g R T}\right)\right]^{-\gamma / \gamma-1}
$$

or
$\frac{P_{s}}{P_{\text {ave }}}=\left[1+\left(\frac{\gamma-1}{2 \phi}\right)\left(\frac{U^{2}}{\gamma g R T}\right)\right]^{-\gamma / \gamma-1}$
with the value $U_{p}^{2} / Y g R T$ being the Mach number squared of the propellant combustion gases at the base of the projectile.

With a $C / M=.5$ this approach, when integrated, would give an overall effective $P_{s} / P_{b}$ ratio in the .75 to .8 range sifinilar to the old Corner and Hirschfelder approach. However, as $C / M$ values of very high performance gun systems started to approach and even exceed values of $C / M=1$ the velocity pressure postulate was found to theoretically under predict experimental gun firing experience.

More specifically, as in-bore velocity Doppler radar systems became available in the early 1970's and laser based visar in-bore velocity systems in the mid 1970 's, it was experimentally seen that substantially more $\Delta V$ was imparted to the projectile later in the ballistic cycle than predicted by the velocity based "quasiisentropic" postulate.

Clearly a more accurate postulate was required. From 1985 to 1987 the Air Force Armament

Laboratory sponsored and the Air Force Office of Scientific Research funded an effort at the University of Florida to study high velocity in-bore gas dynamics and generate a better postulate as to the mechanistic description of in-bore gas gradients.

The balance of this paper provides the results of the study and indicates that the in-bore pressure gradient is not driven by the velocity of the projectile but rather by the acceleration of the projectile.

## PHYSICAL MECHANISMS CAUSING PRESSURE AND DENSITY GRADIENTS

$\begin{array}{ccc}\text { The } & \text { physical } & \text { mechanism } \\ \text { postulated } & \text { to cause a pressure }\end{array}$ difference between the breech and projectile is the acceleration of the projectile. As the projectile accelerates, it produces a series of expansion waves behind it. The pressure drops from breech to projectile through these waves.

The ballistic cycle begins as combustion occurs within the chamber bounded by the breech, tube wall, and projectile and increases the pressure. Because the burning rate of the propellant increases with increased pressure, high pressures are quickly produced. These high pressures accelerate the projectile producing a series of exansion waves behind it. These wave accelerate the gases behind the projectile to the velocity of the projectile. They travel toward the breech, reflect, and return to the projectile distributing the velocities of the gases between the breech and the projectile.

As the projectile accelerates, the gases behind the projectile also accelerate. This acceleration produces a gradient of pressure, density, and temperature behind the projectile. Subsequently, the
pressure at the base of the projectile is lower than that at the breech.

The pressure drop is then governed by the acceleration of the piston. After the acceleration stops, the pressure drop from the stationary end of the cylinder to the moving piston goes to zero. In a typical ballistic system the acceleration begins low, it then increases rapidly to a peak and drops off with movement of the projectile. It would then be reasonable that the drop of pressure from breech to projectile would be dependent on the acceleration history of the recent past. The recent past would be defined as the time between formation of an expansion wave and when that wave has reflected and returned to the piston. Once the expansion wave has returned to the piston, its remaining effect is only to lower the pressure of the gas in the chamber.

## EQUATIONS OF MOTION FOR UNSTEADY FLOWS

The breech to projectile gradients of pressure, density and temperature are gorverned by the conservation equations of mass, momentum and energy. One dimensional forms of these equations were solved using simplifying assumptions to determine analytical relationships between the projectile acceleration and the gradients. The equations of motion for a mixture of gas and solids are the conservation of mass and the conservation of momentum.

$$
\begin{gathered}
\frac{\partial}{\partial t}\left(\rho_{s l}\left(1-v^{\prime}\right)+\rho v^{\prime}\right)+ \\
\frac{\partial}{\partial x}\left(\rho_{s l}\left(1-v^{\prime}\right) u_{s l}+\rho v^{\prime} u\right)=0 \\
\frac{\partial}{\partial t}\left(u_{s l} \rho_{s l}\left(1-v^{\prime}\right)+u \rho v^{\prime}\right)+
\end{gathered}
$$

$\frac{\partial}{\partial x}\left(u_{s l}^{2} \rho_{s l}\left(1-v^{\prime}\right)+u^{2} \rho v^{\prime}\right)=-\frac{\partial P}{\partial x}$
It was assumed that the velocity of the solids is related to the velocity of the gas by a constant ratio throughout the tube at any particular instant. No friction or heat transfer effects were included. The two equations were combined to give an equation which has a form similar to the unsteady Bernoulli equation.

$$
\begin{aligned}
& \frac{\rho_{m l}}{\rho_{m}} \int_{0}^{x} \frac{\partial u}{\partial t} d x+\left.\left(\frac{\rho_{m l} u}{\rho_{m}}\right)^{2} \frac{1}{2}\right|_{x}= \\
& -\int_{P_{b}}^{P} \frac{d P}{\rho_{m}} \\
& \rho_{m}=\rho_{s l}\left(1-v^{\prime}\right)+\rho v^{\prime} \\
& \rho_{m 1}=\rho_{s l}\left(1-v^{\prime}\right) u_{s l} / u+\rho v^{\prime}
\end{aligned}
$$

The terms on the left hand side of Eqn. 3 are acceleration terms. The integrated acceleration produces the pressure difference from breech to projectile. The right hand side represents the pressure gradient produced by the acceleration.

## Acceleration Term

The acceleration term was evaluated for solids and gases. $A$ linear profile of velocity with distance was assumed for both the velocities of gas and solids. The velocity of the solids was assumed to lag that of the gas by the ratio uslu. This approach was used bécause several different models including the XNOVA code [Ref. 5] indicated that the gas velocity profile was quite linear.

When the acceleration of the piston is zero, the linear velocity model seems to predict that the velocity past a point is always
decreasing. When $a_{p}>v^{2} / L$, the velocity past a point will increase with time. When $a_{p}<v^{2} / L$, the velocity past a point will decrease with time. This does not mean however, that the velocity of a particle is slowing down. As one follows a particle, it continues on at the same or greater velocity depending on the level of acceleration of the piston.

Assuming a linear velocity profile and integrating the acceleration side of equation 3 gave

$$
\begin{align*}
& {\left[\frac{\rho_{m l} L a_{p}}{\rho_{m}}+\frac{u_{p}}{2}\left(\left(\frac{\rho_{m l}}{\rho_{m}}\right)^{2}-\left(\frac{\rho_{m l}}{\rho_{m}}\right)\right)\right] \xi^{2}=} \\
& \quad-\int_{P_{b}}^{P_{m}} \frac{d P}{\rho_{m}}  \tag{4}\\
& \text { or } \frac{L a_{P}^{*}}{2} \xi^{2}=-\int_{P_{b}}^{P_{b}} \frac{d P}{\rho_{m}} \tag{5}
\end{align*}
$$

The most significant portion of the acceleration side is $L$ ap 2 . The factor $\rho_{m l} / \rho_{m}$ accounts for solids which are nôt accelerating as fast as the gas. The projectile velocity squared portion of the acceleration side will always have a minus sign in front of it because $\rho_{m l} / \rho_{m}$ is always less than 1. When there are no longer any solids, $\rho_{m l} / \rho_{m}$ goes to 1 and the entire acceleration term goes to $L a_{p} / 2$.

The ratio $\rho_{\mathrm{ml}} / \rho_{\mathrm{m}}$ is determined knowing the porosity, the gas density, and $u_{s l} / u$.
$u_{s 1} / u$ was estimated as a constant along the barrel at any instant in time. It was assumed to change with time. The ratio of average acceleration of the solids over the average acceleration of the gas was assumed to be equal to $u_{s l} / u$. The drag force on the cylindrical grain was assumed proportional to $\left(u-u_{s l}\right)^{2}$ and equal to the acceleration of the grain times its mass.
$u_{s l} / u$ was solved for as a function of a nondimensional acceleration number, Fa . Fa has been formulated to be a function of the easily determined parameters.

$$
\begin{gather*}
\mathrm{Fa}=\frac{2 C_{D} \rho U^{2}}{3 \pi r_{o} \rho_{s l}^{\prime} a_{p}} \\
\frac{u_{s l}}{u}=1+\frac{1}{2 F a}(1-\sqrt{4 \mathrm{Fa}+1}) \tag{6}
\end{gather*}
$$

The assumption of a linear velocity profile was tested using an inviscid computer model of gas flow in a tube between breech and projectile. The projectile was driven by estimates of accelerations from experimental shots 536 and 571 which will be discussed later. The ratio of the integrated particle acceleration (left hand side of equation 3) divided by $a_{p} L / 2$ was plotted against time in Figures 1 and 2. If the velocity profile was linear, the ratio would be 1 . In Figure 1 the ratio is 0.98 during the last 70\% of the time. This indicates that the integrated acceleration estimate of $a_{p} L / 2$ is 2\% high. Figure 2 is for a case with a $30 \%$ higher peak pressure. This higher pressure causes increased acceleration. This increased acceleration rate is the probable cause of the hump in Figure 2 that starts at 0.002 seconds. The peak of the hump, 1.07, indicates that the integrated acceleration estimate of $a_{p} L / 2$ is $7 \%$ low.

## Pressure Gradient Term

The pressure gradient term was evalated for the two cases of gas only and of gas and solids. The pressure was integrated by assuming an isentropic relationship between pressure and density.

The integration of the gas only
case stated in terms of breech temperature and density directly yielded equation 7a.
$\frac{\gamma}{\gamma-1}$
$\frac{P_{\xi}}{P_{b}}=\left(1-\frac{(\gamma-1)\left(1-p_{b} \eta\right) L a_{p} \xi^{2}}{2 \gamma R T_{b}}\right)$

When stated in terms of breech pressure and density it yielded

$$
\frac{P_{\xi}}{P_{b}}=\left(1-\frac{(\gamma-1) \rho_{b} L a_{p} \xi^{2}}{2 \gamma P_{b}}\right)^{\frac{\gamma}{\gamma-1}}
$$

(7b)
The gas and solids case was integrated using a linearized form of the term $\left(P / P_{b}\right)^{(1 / \gamma) \text {. The }}$ equation was stated in terms of breech temperature and density

$$
\begin{gather*}
\frac{P_{\xi}}{P_{b}}=1+\gamma\left(\frac{\rho_{s 1}\left(1 / v^{\prime}-1\right)}{\rho_{b}}+1\right)^{*} \\
{\left[\exp \left(\frac{-v^{\prime}\left(1-\rho_{b} n\right) L a_{p} \xi^{2}}{2 \text { RRT }_{b}}\right)-1\right](8 a)} \tag{8a}
\end{gather*}
$$

It was also stated in terms of breech pressure and density.

$$
\begin{gather*}
\frac{P_{\xi}}{P_{b}}=1+\gamma\left(\frac{\rho_{s l}\left(1 / v^{\prime}-1\right)}{\rho_{b}}+1\right) * \\
{\left[\exp \left(\frac{-v^{\prime} \rho_{b} L{ }_{p} \xi^{2}}{2 Y P_{b}}\right)-1\right] \quad \text { (8b) }} \tag{8b}
\end{gather*}
$$

The multiplier term
( $\left.\rho_{s t}\left(1 / v^{\prime}-1\right) / \rho_{b}+1\right)$ can greatiy inctease the pressure drop from breech to projectile over the gas only case. This is because the large forces are needed to accelerate the solids which have a
density much greater than the gases. The average pressure can be determined by integration of a series expansion of 8 b .

$$
\begin{align*}
\frac{P_{\text {ave }}}{P_{b}}=1 & +K_{1}\left(-\frac{K_{2}}{3}+\frac{K_{2}^{2}}{10}-\frac{K_{2}^{3}}{42}\right. \\
& \left.+\frac{K_{2}^{4}}{216}-\frac{K_{2}^{5}}{1320}\right)  \tag{9}\\
K_{1} & =\left(\frac{\rho_{s l}\left(1 / v^{\prime}-1\right)}{\rho_{b}}+1\right) \gamma \\
K_{2}= & \frac{v^{\prime}(1-\rho n) L^{*}}{2 \gamma r R T_{b}}
\end{align*}
$$

Kinetic Energy in the Gases
The ratio of kinetic energy in the gas solid mixture to the kinetic energy of the same mass moving at the projectile velocity is $1 / \delta . \delta$ was obtained using a linear velocity distribution and a parabolic density distribution obtained from the pressure distribution of Eqn. 8.

$$
\begin{equation*}
\delta=3 \frac{\frac{\rho_{s 1}\left(1-v^{\prime}\right)}{\rho_{b} \nu^{\prime}}+\left(1-\frac{\rho_{s}}{3 \rho_{b}}\right)}{\frac{\rho_{s 1}\left(1-v^{\prime}\right)}{\rho_{b} \nu^{\prime}}\left(\frac{u_{s}}{u}\right)+\left(1-\frac{3 \rho_{s}}{5 \rho_{b}}\right)} \tag{10}
\end{equation*}
$$

The case of gas only
$\delta=\frac{3\left(1-\frac{\rho_{s}}{3 \rho_{\mathrm{b}}}\right)}{\left(1-\frac{3}{5} \frac{\rho_{s}}{\rho_{\mathrm{b}}}\right)}$
COMPARISIONS OF ANALYTICAL MODEL WITH EXPERIMENTAL DATA

The equations just desoribed in were compared with experimental data
from Eglin AFB for firings with muzzle velocities of $1000 \mathrm{~m} / \mathrm{s}$. Comparisons were also made with computer modelled predictions obtained from Eglin AFB and the Ballistics Research Lab. The experimental data came from two series of shots of a 30 mm aircraft cannon. Each series consisted of four shots. These runs were evaluated by shifting the time scales to make the peak pressures coincide and then comparing the measured pressures in time. The two pressure time traces which best matched each other were then used as the standard data for each series.

Each set of data consisted of time and for pressure readings. The first reading was called the chamber pressure reading. It was not taken at the breech but just in front of the initial position of the rotating band. The three other readings were taken at three downbore locations.

The data in a nondimensional form were used for the comparison with analytical equations ( 7 and 8). The pressures at the three downbore locations were normalized by the chamber pressure reading. The resulting pressure ratio was less than 1 because the downstream pressures were always lower than the chamber pressure.

The analytical pressure ratio was determined using equation 8 to find the downoore pressure with respect to the breech pressure for both the chamber location and the downbore location. Values of the variables in equation 8 were obtained from an interior ballistics program of Heiney [Ref 4]. The variables needed were average porosity, breech density and pressure, projectile acceleration, and breech to projectile length. The analytical pressure ratio was determined as a function of time for each downbore pressure tap location beginning from the time that each
tap was passed by the projectile.
The results are quite encouraging. Comparisons with the 536 experimental data is shown in Figures 3-5. The experimental data is the jagged line. The other two lines are the results of calculations from equation 8 with the upper line being a gas only calculation and the lower line being the calculation when solids are included. At 1700 microseconds in Figure 3 one can see that the solids theory agrees quite well with the data.

At a later time of 2500 microseconds in Figure 4 the gas only theory seems to be a better estimate than that of the solids.
This is probably due to the fact that the solids do not follow directly behind the projectile but lag it. The $\xi$ squared term in equation 8 , causes the majority of the pressure drop to occur in the $30 \%$ of the length just behind the projectile. If the solids concentration is very low in this region, the pressure will drop less and will better follow the gas only theory.

The last figure in the 536 series is figure 5. It shows good agreement between theory and experiment for either gas only or gas and solids. The two theoretical lines are very close because at 3000 microseconds there are very few solids left.

Comparisons of 571 experimental data with theory are shown in Figures 6-8. This shot had a muzzle velocity 7\% higher than the previous 536 shot. This was produced by a thinner grain web thickness. Figure 6 shows the pressure ratio for downbore tap 2. Burning has occurred more quickly and few solids remain so that the gas and solids lines are very close. Predicted pressures are lower than those measured. One explanation may be lack of time for the velocity profile to become linear after the time of peak
pressure.
Figure 7 shows quite good agreement with the experimental data over the last half of the time of the run. Figure 8 shows a 'essure ratio higher than the data. This may te related to the hump in acceleration ratio shown in Figure 2

Two higher speed cases were compared with theory. Experimental data was not available for either of
these. However, computer predictions were available using two different codes which model the flow of both solid and gas as the solids burn.

The first case had a $1400 \mathrm{~m} / \mathrm{s}$ muzzle velocity. Combustion was complete and gas only theory was compared. Pressure at the projectile base over pressure at the breech was compared when the projectile was at the muzzle. The computer prediction of the pressure ratio was 0.707 and the pressure ratio of the theory of this work was 0.665

The second case was a large diameter round with a $1450 \mathrm{~m} / \mathrm{s}$ muzzle velocity. Combustion was complete and the pressure ratios was compared when the projectile was at the muzzle. The computer prediction of the pressure ratio was 0.731 and the pressure ratio of the theory of this work was 0.747.

In both of these cases the theory matched the computer predictions well. The Mach number of the gases behind the projectile in both of the last two cases is near 1.5 indicating that the flow was supersonic.

## SUMMARY

The gradient in pressure and density from breech to projectile was postulated to be caused by the acceleration of the gases behind the projectile. The major governing
parameter was the projectile acceleration times the axial position of the projectile.

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NOMENCLATURE

| $a_{p}$ | - | acceleration of projectile |
| :---: | :---: | :---: |
| $\mathrm{ap}^{*}$ | - | adjusted acceleration |
| $\mathrm{C}^{\text {p }}$ | - | change mass |
| Fa | - | acceleration number |
| L | - | distance $x$ between breech and projectile |
| M | - | projectile mass |
| P | - | pressure |
| $P_{\text {ave }}$ | - | average chamber pressure |
| R | - | gas constant |
| $t$ | - | time |
| T | - | temperature |
| u | - | velocity of gas |
| $u_{p}$ | - | velocity of projectile |
|  | - | kinetic energy of mass of gas at $u_{p}$ /kinetic energy of gas |
| $\gamma$ | - | specific heat ratio $C_{n} / C_{u}$ |
|  | - | covolume |
| $\nu^{\prime}$ | - | porosity |
| $\xi$ | - | x/L |
| $\rho_{m}$ | - | average density |
| $\rho_{m 1}$ | - | density times velocity |
|  |  | average |
| $p_{m 2}$ | - | density times velocity squared average |
|  | - | propellant grain |
| $p_{s 1}{ }^{\prime}$ | - | density of a grain |
|  |  | including gas within |


|  | perforation |
| :---: | :---: |
| ¢ | space mean factor |
| Subscripts |  |
| b - | breech |
| $s$ | projectile base, shot |
| 31 | solids |
| x | axial position beginning at breech |

## REFERENCES

1. Corner J., "Theory of the Interior Ballistics of Guns" John Wiley, 1950.
2. Hunt, F.R., and Hinds, G.H., "Internal Ballistics" HMSO London, 1951.
3. Hirschfelder, J.O., Et al, "Interior Ballistics" NDRC Report A-142, 1943.
4. Heiney, O.K., and West, R.J., "Interior Ballistics, Muzzle Flash and Gas Gradients of Aircraft Cannon", AFATL-TR-7634, 1976.
5. Gough, P.S., 'XNOVA -An Express Version of the NOVA Code", Contract Report N00174-82-M8048, Naval Ordinance Station, Indian Head, Maryland, November 1983.


Figure 1. Instantaneous Particle Acceleration Integrated from Breech to Projectile Over L ap/2 Calculated Using Inviscid Theory of a Compressible Gas Using Acceleretion of the Projectite from Shot 536.


Figure 2. Instantaneous Particle Acceleration Integrated from Breech to Projectile Over L ap/2 Calculated Using Inviscid Theory of a Compressible Gas Using Acceleration of the Projectile from Shot 571.


Figure 3. Pressure at Downbore Tap 2 Over Pressure al Chamber Tap 1 as a Function of Time


Figure 4. Pressure at Downbore Tap $\# 3$ Over Pressure al Chamber Tap "1 as a Function of Time


Figure 5. Pressure at Downbore Tap 4 Over Pressure at Chamber Tap 11 as a Function of Time.


Figure 6. Pressure at Downbore Tap 2 Over Pressure at Chamber Tap 1 as a Function of Time.


Figure 7. Pressure at Downbore Tap \#3 Over Fressure at Chamber Tap 1 as a Function of Time


Figure 8. Pressure at Downbore Tap 4 Over Piessure at Chamber Tap 1 as a Funclion of Time.

