# PRESSURE LOSS AND LIQUID HOLDUP IN PACKED BED REACTOR WITH COCURRENT GAS－LIQUID DOWN FLOW＊ 

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#### Abstract

Paying attention to the discrepancy between available data，probably due to the uncertainty in void fraction，the entrance end effect and characteristics of the experi－ mental method，new experimental results were obtained for air－water flow in 65.8 and 122 mm I．D．columns packed with glass spheres of six different sizes between 2.59 and $24.3 \mathrm{~mm} \phi$ ．The two－phase pressure loss based on an energy（not momentum）balance was correlated in terms of Lockhart－Martinelli parameters as follows： $\phi_{l}=1.30+1.85 \chi^{-0.85}, 0.1<\chi<20$ The total liquid holdup $R_{l}$ was determined by weighing the excess weight of bed in operation over the dry column．The result was dependent on the specific surface area of the bed $a_{s}$ and correlated by $R_{l}=0.40 a_{s}{ }^{1 / 3} \chi^{0.22}\left(a_{s}\right.$ in mm ${ }^{-1}$ ） The present result on total holdup was higher than available results obtained by quick－ closing valve，which measured probably the operating holdup．Data of void fraction for spherical packings are appended and the differences in void fraction and its repro－ ducibility due to packing procedure are noted．


## 1．Introduction

Some gas－liquid reactions are processed in packed catalyst beds under cocurrent two－phase flow．For proper design and operation of such reactors，knowl－ edge of the fluid flow characteristics is of fundamental importance．Although experimental data have been collected on pressure loss and liquid holdup in cocur－ rent two－phase flow ${ }^{1,4,10,13,15,16)}$ ，the discrepancies between different investigations are not small．

The first probable reason for it is thought to be the uncertainty in the packing state．Void fraction should be treated as a statistical process because not only does it have local variation but also the overall value varies at every trial of dumping．In this work，prior to flow experiments the reproducibility in the overall viod fraction under several packing procedures was exam－ ined to find the method of packing with the most reproducible void fraction．

[^0]Secondly，the entrance end effect should be pointed out．The ratio of column length to column diameter in practice and in experiment is not so large，and dis－ regarding the entrance effect leads to the discrepancy between the separate investigations．In the present work，the inlet distributor was designed so that a short－ er entrance region was attained，and the pressure gradi－ ent was read in the region showing the linear decrease of pressure to the exclusion of the entrance region．

Measurement of liquid holdup may be carried out by various methods such as l）quick－closing valve ${ }^{10,13,15)}$ ， 2）tracer response，3）weighing the column，and so on． However，the possible difference in results due to dif－ ferent methods of measurement has not been paid attention to，which is a third possible reason for the discrepancies between investigations．The present work discusses this problem based on new experimental data measured by a weighing technique，which is ex－ pected to be the most accurate．

Taking the above into consideration，new data and correlations for pressure loss and liquid holdup are given in this work and are compared with existing correlations．

## 2．Packed Beds and Experimental Apparatus

Packed columns used were 1 m long transparent

Table 1 Packing spheres used

| Arithmetic mean <br> diameter $[\mathrm{mm}]$ <br> $d_{a} \pm \sigma$ | Volume-surface <br> mean diameter $[\mathrm{mm}]$ <br> $d_{v s}$ |
| :---: | :---: |
| $2.52 \pm 0.30$ | 2.59 |
| $5.55 \pm 0.41$ | 5.61 |
| $7.94 \pm 0.56$ | 8.01 |
| $12.17 \pm 0.33$ | 12.2 |
| $16.52 \pm 0.27$ | 16.5 |
| $24.27 \pm 0.26$ | 24.3 |



Fig. 1 Main section of experimental apparatus


Fig. 2 Detail of inlet distributor ( $D=65.8 \mathrm{~mm}$ )
acrylic tubings of 122 and 65.8 mm I.D. in which commercial glass spheres of six different sizes were packed separately. The arithmetic mean and the vol-ume-surface mean diameters were measured by a micrometer, as shown in Table 1. When the above two diameters were not the same for smaller particles, the volume-surface mean diameter was used for data processing because the specific surface area is more significant than the linear dimension. The void fraction of gently packed beds of spheres was calculated according to the proposed equation, Eq.(Al), in Appendix.

For pressure loss study, ten measuring points were positioned on the column as shown in Fig. 1, with longitudinal space intervals closer at the entrance region so as to observe the probable entrance end effects. At each measuring point, four pressure taps were mounted $90^{\circ}$ apart each on the circumference of the column wall and combined into a single pressure conveying tube to a manometer so that the circumferential variation of pressure should be averaged automatically and the manometer reading be more stabilized.

In two-phase operation, a gas-liquid separator was installed between the pressure-conveying tube and the manometer so that bubbles entering the tube accidentally were trapped in the upper chamber of the separator and the tube was filled with water during the experiment. Sometimes, especially in the case of twophase flow, the pressure gradient changed gradually in the entrance region ( $\sim 25 \mathrm{~cm}$ under experimental condition) and then reached a constant value, which was taken as the developed pressure gradient in the present study.

Liquid holdup was determined by a weighing method of which a schematic diagram is shown in Fig. 1. A dry packed column was suspended by piano wires and balanced by counterweights on saucers. When gas and liquid were introduced into the column through flexible P.V.C. tubes, the excess weight over the dry column was weighed by a spring balance ( 1 kg capacity) to within 1 gram and liquid holdup was determined after subtracting the weight of liquid contained in the liquid chamber. The effect on apparent weight of column due to the longitudinal momentum difference between inlet and outlet was checked to be negligibly small by both calculation and experiment.

The distributor head was designed as shown in Fig. 2 so that both gas and liquid were evenly distributed and the entrance developing region became shorter. Gas and liquid were introduced into separate chambers, from which they passed through bundles of 2 mm I.D. copper tubes and contacted in the layer of packed spheres.

All the measurements were carried out for air and water at room temperature.

Three distinct flow patterns, i.e. gas continuous flow, pulse flow and dispersed bubble flow, were observed as reported ${ }^{4)}$. However, there was no abrupt change
of pressure loss and liquid holdup for the transition from any one flow type to another. This suggests that correlation of the data should be possible, independent of the flow pattern.

## 3. Single Phase Pressure Loss

Since the wall effect was expected to be appreciable in the case of large ratio of particle to column diameter, the particle diameter divided by the wall correction factor $M^{12)}$

$$
\begin{equation*}
d^{\prime}=d / M \tag{1}
\end{equation*}
$$

in which

$$
\begin{equation*}
M=1+\frac{4 d}{6 D(1-\epsilon)} \tag{2}
\end{equation*}
$$

was used for the correlation of data in terms of the viscous friction factor $f_{v}$ and Reynolds number $R e$.

$$
\begin{gather*}
f_{v}=\left(\frac{\Delta P}{\Delta L}\right) g_{c} \frac{d^{\prime 2}}{\mu U} \frac{\epsilon^{3}}{(1-\epsilon)^{2}}  \tag{3}\\
R e=d^{\prime} U_{\rho} / \mu(1-\epsilon) \tag{4}
\end{gather*}
$$

Fig. 3 shows the present result together with some of the available correlations and the recalculated data of Wentz and Thodos ${ }^{17}$ ) for distended beds. Linear relationships of the type of $f_{v}=\alpha+\beta R e$, such as those of Ergun ${ }^{5)}$ and Handley and Heggs ${ }^{67}$, fail to correlate the data. Instead, the present data are most closely expressed by a nonlinear formula of Tallmadge ${ }^{14)}$

$$
\begin{equation*}
f_{v}=150+4.2 R e^{5 / 6} \tag{5}
\end{equation*}
$$

which was obtained by a simple interpolation between Kozeny equation ${ }^{8)}$ and Wentz-Thodos' results ${ }^{17}$ ) but has not been supported so far by many experimental results in the intermediate Reynolds number range of $500-5000$.

## 4. Two Phase Pressure Loss

When the kinetic energy difference may be zero for fully developed flow, the macroscopic equations of momentum and energy, respectively, become

$$
\begin{gather*}
\Delta P_{m}=-\Delta p+\bar{\rho}_{m}\left(g / g_{c}\right) \Delta L  \tag{6}\\
\Delta P_{e}=-\Delta p+\bar{\rho}_{f}\left(g / g_{c}\right) \Delta L \tag{7}
\end{gather*}
$$

in which $\bar{\rho}_{m}$ is the volume-average density of twophase mixture in the bed defined as

$$
\begin{equation*}
\bar{\rho}_{m}=\rho_{l} R_{l}+\rho_{g}\left(1-R_{g}\right) \tag{8}
\end{equation*}
$$

and $\bar{\rho}_{f}$ is the flow-average density of mixture defined as

$$
\begin{equation*}
\bar{\rho}_{f}=\frac{\rho_{l} U_{l}+\rho_{g} U_{g}}{U_{l}+U_{g}} \tag{9}
\end{equation*}
$$

i.e. the total mass supplied divided by the total volume supplied.

The term $\Delta P_{m}$ means the force of the fluid mixture on the solid per unit cross sectional area of the column while $\Delta P_{e}$ is the energy loss converted to heat irreversibly due to viscous dissipation per unit volume of fluid supplied to the bed. Both have the same value for single-phase flow but they are usually different in two-


Fig. 3 Correlation of single-phase pressure loss
phase down flow unless the slip velocity between phases is zero. Since pressure loss is an energy concept but not a momentum concept, Eq.(7) is used to express the pressure loss in the present work according to Hughmark \& Pressburg's suggestion ${ }^{\text { }}$, unlike most previous work, in which the momentum equation, Eq. (6) ${ }^{10,13,15)}$, or just the static pressure difference ${ }^{16)}$ was used. However, the difference due to the choice of Eq.(6) or Eq.(7) is not expected to be large because of smaller contribution of either potential head term relative to the static pressure term $\Delta p$ under reported experimental conditions.

In the final correlation, the pressure loss was expressed in terms of two-phase parameters, $\phi_{l}$ and $\chi$, which have been introduced for two-phase flow in open tubes by Lockhart and Martinelli ${ }^{11)}$ and then extended to packed beds by Larkins, White and Jeffrey ${ }^{10}$. These parameters are defined as

$$
\begin{align*}
\phi_{l} & =\sqrt{\Delta P_{l g} / \Delta P_{l}}  \tag{10}\\
\chi & =\sqrt{\Delta P_{l} / \Delta P_{g}} \tag{11}
\end{align*}
$$

in which $\Delta P_{l}$ and $\Delta P_{g}$, respectively, are the pressure loss that would exist if the liquid and the gas were assumed to flow alone separately in single-phase flow with the same flow rates as those in two-phase flow. They may be predicted by Eq.(5).

Using these variables the present results are shown in Fig. 4 and compared with various two-phase correlations for packed beds and open tubes. The present proposed correlation is given as

$$
\begin{equation*}
\phi_{l}=1.30+1.85 \chi^{-0.85} \text { for } 0.1<\chi<20 \tag{12}
\end{equation*}
$$

which is slightly higher than that of Larkins, White and Jeffrey ${ }^{10)}$ and slightly lower than that of Weekman and Myers ${ }^{16)}$. Eq.(12) is a purely empirical curve fitting in the specified range of $\chi$, and it should be noted that it cannot fulfill the following requirements at two extremes.

$$
\begin{align*}
& \phi_{l} \longrightarrow 1 \text { as } \chi \longrightarrow \infty \text { (liquid single phase) } \\
& \phi_{l} \longrightarrow \chi^{-1} \text { as } \chi \longrightarrow 0 \text { (gas single phase) } \tag{13}
\end{align*}
$$



Fig. 4 Correlation of two-phase pressure loss


Fig. 5 Alternate expression of two-phase pressure loss

Larkins, White and Jeffrey ${ }^{10}$ ) have proposed another form of correlation which satisfies the above conditions.

$$
\begin{equation*}
\log \frac{\Delta P_{l g}}{\Delta P_{l}+\Delta P_{g}}=\log \frac{\dot{\phi}_{l}^{2} \chi^{2}}{1+\chi^{2}}=\frac{0.416}{(\log \chi)^{2}+0.666} \tag{14}
\end{equation*}
$$

This equation is symmetrical about $\chi=1$ in the logarithmic scale, as shown in Fig. 5, and thus implies that the effect of liquid flow superimposed on gas singlephase flow and the effect of gas flow superimposed on liquid single-phase flow are the same with respect to two-phase pressure loss. However, a similar expression for the present data becomes

$$
\begin{equation*}
\log \frac{\Delta P_{l g}}{\Delta P_{l}+\Delta P_{g}^{-}}=\frac{0.70}{\{\log (\chi / 1.2)\}^{2}+1.00} \tag{15}
\end{equation*}
$$

which is symmetric about $\chi=1.2$ instead of $\chi=1$. Close inspection of data point of Larkins, White and Jeffrey ${ }^{10}$ ) shows that the symmetric axis shifts slightly to larger $\chi$ from unity and the results of Reiss ${ }^{13}$ show a similar trend in spite of large scatter of data points, both being consistent with the present result.

A comparison of the present correlation for packed beds with those for open tubes reveals a remarkable agreement with the viscous liquid-turbulent gas mode
of Lockhart-Martinelli correlation ${ }^{11)}$ in spite of complete difference in the flow geometry. Thus, it is probable that the mechanism of energy dissipation in twophase flow is very similar in both systems except that single-phase flow is controlled by different equations.

## 5. Liquid Holdup

Larkins, White and Jeffrey ${ }^{10}$ ) reported previously the holdup correlation in terms of a single parameter $\chi$ as follows

$$
\begin{equation*}
\log R_{l}=-0.774+0.525 \log \chi-0.109(\log \chi)^{2}, \tag{16}
\end{equation*}
$$

However, a similar plot of the present data resulted in a family of separate curves for different packing diameters, indicating the significant dependence of the specific surface area of beds, $a_{s}$. The smaller the packing, the greater the holdup at the same value of $\chi$. Taking this effect into account, the proposed equation becomes the following, as shown in Fig. 6.

$$
\begin{equation*}
R_{l}=0.40 a_{\varepsilon}^{1 / 3} \chi^{0.22} \text { for } 0.1<\chi<20 \tag{17}
\end{equation*}
$$

in which

$$
\begin{equation*}
a_{s}=6(1-\epsilon) / d^{\prime} \tag{18}
\end{equation*}
$$

Larkins' original data ${ }^{9}$ ) are apparently dependent on the specific surface area $a_{s}$ although he did not note this fact. The results of Reiss ${ }^{13)}$ and Bakos and Charpentier ${ }^{1 /}$ show the same trend. The correction of their data with $a_{s}^{1 / 3}$ gives better correlations with less scatter than the original ones. The broken and the chain line in Fig. 6, respectively, show thus corrected correlations of data of Larkins ${ }^{97}$ and Reiss ${ }^{13}$.
The present result, Eq.(17), is substantially higher than those of the other two investigators. This difference was brought about probably by the different methods of holdup measurement. Experiments of Larkins, White and Jeffrey ${ }^{10}$ ) and Reiss ${ }^{13)}$ were carried out by quick-closing of the column test section and


Fig. 6 Correlation of liquid holdup
successive draining of liquid holdup there. Thus obtained holdup is just the operating holdup which has drained from packings in a relatively short time, and does not include the static holdup which remains adhered on the packing surface or contained in the wedge of contacting points of packings for a long time after closing the column. On the other hand, the present method measures the total holdup, which is the total liquid in the packing under operating condition, i.e. the sum of the operating and the static holdup. The preliminary experiment showed that the static holdup amounted to as much as $10-15 \%$ even at several hours after stopping the running. This value of static holdup compares to the difference between the results of the present work and Larkins, White and Jeffrey ${ }^{10}$.

## Appendix: Void fraction of packed beds of sphere

The following three different methods of packing were employed for void measurement, since the packing method is reported to influence the void fraction significantly ${ }^{2)}$. They are
(a) gently dumped (without external impact).
(b) dumped with simultaneous vibration.
(c) one minute vibration after dumped.
in which the vibration was supplied by an unbalanced motor at a frequency of $57 \mathrm{sec}^{-1}$. The void fraction was measured for $6 \mathrm{com}-$ binations of three different glass spheres (2.59, 12.2 and $24.3 \mathrm{~mm} \phi$ ) and two different columns ( 65.8 and $122 \mathrm{~mm} \phi \times 1 \mathrm{~m}$ ). The result is given in Table 2, in which $\bar{\varepsilon}$ is the average of 60 repeated measurements and $\sigma$ is the standard deviation from it. It is seen that the use of vibration produces a more reproducible and denser packing state.

The void fraction in the present systems was found to be a linear function of the ratio of particle diameter to column diameter, $d / D$, for the same packing procedure. The resultant correlating equations are

$$
\begin{aligned}
& \epsilon=0.3517+0.4657(d / D) \text { for method (a). } d / D<0.4 \\
& \epsilon=0.3472+0.4417 \text { (A1) }(d / D) \text { for method (b). } d / D<0.4 \text { (A2) } \\
& \epsilon=0.3494+0.4381 \text { (d/D) for method (c). } d / D<0.4 \text { (A3) }
\end{aligned}
$$

## Nomenclature

| $a_{s}$ | $=$ specific surface area of bed | $[1 / \mathrm{mm}]$ |
| :--- | :--- | ---: |
| $D$ |  | $=$ column diameter |
| $d$ |  | $[\mathrm{~mm}]$ |
| $d^{\prime}$ |  | particle diameter |
| $f_{v}$ |  | modified diameter of particle (Eq.(1)) |
| $M$ |  | $[\mathrm{~cm}],[\mathrm{cm}],[\mathrm{mm}]$ |
|  | $=$ vall correction factor (Eq.(3)) | $[-]$ |
|  |  | $[-]$ |

Table 2 Void fraction of packed beds of spheres

| $D[\mathrm{~mm}]$ | $d[\mathrm{~mm}]$ | Method* | $\bar{\varepsilon} \pm \sigma$ |  |
| :---: | :---: | :---: | :---: | :---: |
| 65.8 | 2.59 | a | $0.3668 \pm 23 \times 10^{-4}$ |  |
|  |  | b | $\begin{aligned} & 0.3686 \pm 6 \\ & 0.3685 \pm 5 \end{aligned}$ |  |
|  |  | c |  |  |
|  | 12.2 | a | $0.4341 \pm 27$ | " |
|  |  | b | $0.4285 \pm 20$ | " |
|  |  | c | $0.4273 \pm 15$ | " |
|  | 24.3 | a | $0.5239 \pm 76$ | " |
|  |  | b | $0.5130 \pm 75$ | " |
|  |  | c | $0.5113 \pm 68$ | " |
| 122 | 2.59 | a | $0.3627 \pm 33$ | " |
|  |  | b | $0.3539 \pm 12$ | " |
|  |  | c | $0.3549 \pm 7$ | " |
|  | 12.2 | a | $0.4024 \pm 23$ | " |
|  |  | b | $0.3948 \pm 10$ | " |
|  |  | c | $0.3979 \pm 8$ | " |
|  | 24.3 | a | $0.4457 \pm 30$ | " |
|  |  | b | $0.4281 \pm 50$ | " |
|  |  | c | $0.4364 \pm 21$ | " |

* a: Gently dumped
b: Dumped with simultaneous vibration
c: One minute vibration after dumped

| $\Delta L$ | $=$ length of packed bed | cm] |
| :---: | :---: | :---: |
| $A^{2}$ | $=$ pressure loss | [G/ $\mathrm{cm}^{2}$ ] |
| $\Delta P_{g}, \Delta P_{l}$ | $=$ pressure loss which would appear if gas and liquid flowed separately | [G/cm ${ }^{2}$ ] |
| $\Delta P_{t g}$ | $=$ two-phase pressure loss . | [G/ $\mathrm{cm}^{2}$ ] |
| $R e$ | $=$ reynolds number (Eq.(4)) | [-] |
| $R_{i}$ | $=$ liquid holdup per unit volume of void | [-] |
| U | $=$ superficial velocity | [ $\mathrm{cm} / \mathrm{sec}]$ |
| $\epsilon$ | $=$ void fraction of packed bed | [-] |
| $\mu$ | $=$ viscosity of fluid | [g/cm $\cdot \mathrm{sec}$ ] |
| $\rho_{g}, \rho_{l}$ | $=$ density of gas and liquid | $\left[\mathrm{g} / \mathrm{cm}^{3}\right]$ |
| $\overline{\boldsymbol{\rho}}_{m}$ | $=$ volume-average density of two-phase mixtu (Eq. (8)) | $\left[\mathrm{g} / \mathrm{cm}^{3}\right]$ |
| $\bar{\rho}_{f}$ | $=$ flow-average density of two-phase mixture (Eq.(9)) | [g/cm ${ }^{3}$ ] |
| $\phi{ }_{l}$ | $=$ Lockhart-Martinelli parameter (Eq.(10)) | [-] |
| $\chi$ | $=$ Lockhart-Martinelli parameter (Eq.(11)) | [-] |

$=$ standard deviation around mean

## Litertaure Cited

1) Bakos, M. and J. C. Charpentier: Chem. Eng. Sci., 25, 1822 (1970)
2) Burke, S. P. and W. B. Plummer: Ind. Eng. Chem., 20, 1196
（1928）
3）Carman，P．C．：Trans．Instn．Chem．Engrs．（London），15，T150 （1937）
4）Charpentier，J．C．，C．Prost and P．L．Goff：Chem．Eng．Sci．，24， 1777 （1969）
5）Ergun，S．：Chem．Eng．Progr．，48，No．2， 89 （1952）
6）Handley，D．and P．J．Heggs：Trans．Instn．Chem．Engrs．（Lon－ don），46，T251（1968）
7）Hughmark，G．A．and B．S．Pressburg：A．I．Ch．E．Journal，7， 677 （1961）
8）Kozeny，J．：Sitz－Ber．Wienner Akad．，Abt．IIa，136， 271 （1927）
9）Larkins，R．P．：Ph．D．Thesis，Univ．Mich．，Ann Arbor（1959）
10）Larkins，R．P．，R．R．White and D．W．Jeffrey：A．I．Ch．E． Journal，2， 231 （1961）

11）Lockhart，R．W．and R．C．Martinelli：Chem．Eng．Progr．，45， No．1， 39 （1949）
12）Mehta，D．and M．C Hewley：Ind．Eng．Chem．，Process Design and Develop．，8， 280 （1969）
13）Reiss，L．P．：Ind．Eng．Chem．，Process Design and Develop．，6， 486 （1967）
14）Tallmadge，J．A．：A．I．Ch．E Journal，16， 1092 （1970）
15）Turpin，J．L ，and R．L．Huntington：A．I．Ch．E．Journal，13， 1196 （1967）
16）Weekman，V．W．，Jr．and J．E．Myers：A．I．Ch．E．Journal，6， 951 （1964）
17）Wentz，C．A．Jr．and G．Thodos：A．I．Ch．E．Journal，9， 81 （1963）

# DYNAMIC BEHAVIOUR OF TRANSFER COEFFICIENT IN PULSATING LAMINAR TUBE FLOW＊ 

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The frequency response of transfer coefficient to flow－rate change in laminar tube flow is studied experimentally and analytically．In the analysis，the linearized basic equation is solved both numerically and by using a similarity variable which is used in analysis of steady－state problem．In the experiment，the variations of local value and space－averaged value with respect to time are measured under two heating conditions， i．e．，constant wall heat－flux and constant wall temperature．

From these results it is concluded that the similarity relation is not preserved in unsteady state and that resonance occurs in variation of transfer coefficient．This reso－ nance point is characterized by a parameter $\operatorname{SnPrX} X^{2 / 3}$ which is derived from a similari－ ty relation．In the last part of this paper，the effect of pulsating flow on the time－aver－ aged transfer coefficient is discussed．

## Introduction

In unsteady flow，the heat or mass transfer rate at a wall is subjected to the change of velocity gradient at the wall．Accordingly，the character of response of transfer rate is often used in measuring the variation of velocity gradient．

There have been a number of experimental investi－ gations ${ }^{9,12)}$ concerning only the effect of pulsation in flow rate on the time－averaged rate of heat or mass transfer．However，these results are in conflict with each other．

[^1]An analytical approach to the problem was first made by Siegel and Perlmutter ${ }^{10}$ ．Using characteris－ tic method they obtained a solution for heat transfer of pulsating slug flow between parallel plates，and clari－ fied that the time－averaged heat transfer rate was not appreciably changed by the oscillations．Simultaneous－ ly，they pointed out that a node in wall temperature or heat flux variation might exist．A further elaborated numerical calculation was made by Mochizuki and Hatta ${ }^{7}$ by including the effect of the transverse veloc－ ity distribution．

Recently，from the viewpoint of dynamic re－ sponse，some analytical approaches to frequency re－ sponse of transfer coefficient to wall shear stress were made．Fagella－Alabastro and Hellums ${ }^{2)}$ made a calcu－ lation by straightforward numerical integration and， for the cases of very high and low frequency by a per－ turbation method similar to that used by Lighthill ${ }^{5 \text { ．}}$ ． Lebouché ${ }^{4}$ also used a perturbation method，but his results comprise a similarity relation which was used in analysis of the steady－state problem．Since his results


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